Hindawi
Complexity
Volume 2022, Article ID 4770439, 14 pages
https://doi.org/10.1155/2022/4770439

Research Article

Neural Network-Based Output Feedback Fault Tolerant Tracking Control for Nonlinear Systems with Unknown Control Directions

Kun Yan[1], **Chaobo Chen**[1], **Xiaofeng Xu**[2], **and Qingxian Wu**[2]

[1] *College of Electronic Information Engineering, Xi'an Technological University, Xi'an 710065, China*
[2] *College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*

Correspondence should be addressed to Kun Yan; yankun@xatu.edu.cn

Received 17 January 2022; Revised 17 March 2022; Accepted 29 April 2022; Published 29 May 2022

Academic Editor: Peter Giesl

In this study, an adaptive output feedback fault tolerant control (FTC) scheme is proposed for a class of multi-input and multioutput (MIMO) nonlinear systems with multiple constraints. The neural network (NN) is adopted to handle the unknown nonlinearity by means of its superior approximation capability. Based on it, the state observer is designed to estimate the unmeasured states, and the nonlinear disturbance observer is constructed to tackle the external disturbances. In addition, the Nussbaum function is utilized to cope with the actuator faults, which are coupled with the unknown control directions. Combining with the Lyapunov theory, a NN-based output feedback FTC law is developed for the MIMO nonlinear systems, and the boundedness of all closed-loop system error signals is proved. Simulation results on the unmanned helicopter are performed to demonstrate the effectiveness of the proposed controller.

1. Introduction

During these years, the control issue of multi-input and multioutput (MIMO) nonlinear systems has been drawing great attention due to their generalized and natural description for many practical applications, such as ocean vessel system, mobile robot system, and unmanned aerial vehicle system [1–4]. Since existing linear control methods do not match with the nonlinear systems, many effective nonlinear control approaches have been proposed and considerable results have been received. In [5–9] and the references therein, the adaptive backstepping control and sliding mode control schemes were, respectively, developed for a class of nonlinear systems to achieve acceptable control performance. Nevertheless, as the system structure becomes more and more complex, the unknown nonlinearity caused by nonlinear elements always appears and is worthy of further study.

Nowadays, in order to handle the unknown nonlinearity in the nonlinear systems, numerous control strategies have been presented, such as the neural network (NN) [10–13], extended state observer [14, 15], and fuzzy logic system [16–18]. In particular, the NNs are often combined with other control techniques to deal with the specific nonlinear systems due to their unique properties. In [10, 11], the adaptive backstepping-based neural control schemes were severally proposed to handle the unknown nonlinear functions for the helicopter and near space vehicle (NSV) systems. In [12], the NNs were combined with the model predictive control to overcome the so-called quadratic programming problem for a class of nonholonomic chained systems. In [13], a robust NN sliding mode control law was developed for a two-axis motion control system with unknown nonlinear terms and external disturbances. However, plenty of the above-mentioned literature are based on the available states, which are strict for some practical control systems. Actually, when the sensors malfunction, the signal usually cannot be accurately measured, and the state feedback approach is unreliable. Therefore, the high-quality controller needs to be further designed when the MIMO systems suffer from unmeasurable states.

The state observer is a common method to estimate the unmeasurable states in the academic community [19]. Meanwhile, fruitful and significant results can be found in the recent literature [20–26]. In [20], an adaptive NN output feedback optimized controller was designed for the nonlinear systems with unknown dead zones. In [21, 22], the output feedback control problem for uncertain stochastic nonlinear systems was investigated by employing the state observer approach. In [23], an observer-based finite time controller was developed for a class of second-order nonlinear homogenous systems. Considering the objective existence of performance constraint, the output feedback control schemes were separately presented for the single-input and single-output and MIMO nonlinear systems in [24, 25]. In [26], a stable fuzzy output feedback controller was constructed for a class of nonlinear systems by means of the small-AAIN approach. Nevertheless, it is worth noting that the control directions are usually unknown, and actuator faults occur frequently in many practical application [27, 28]. Therefore, for the purpose of maintaining admissible system performance, more attentions and deeper considerations should be paid to the issues of unknown control directions and actuator faults.

At present, Nussbaum gain theory, which was first proposed by Nussbaum in 1983 [29], has become one of the most valid methods to cope with the unknown control directions. Meanwhile, existing achievements show that the combination of Nussbaum gain control technology with nonlinear control method attains favorable control performance in practice [30–33]. In [30], a distributed consensus controller for uncertain nonlinear systems with unknown control directions was designed to complete control task. In [31], the adaptive protocol was presented for nonlinear systems when both control directions and parameters were all unknown. In [32], a predefined performance-based adaptive controller was proposed for MIMO nonlinear systems in presence of unknown control directions and unknown hysteresis nonlinearities. In [33], an adaptive fuzzy tracking controller was designed to assure the stability of the stochastic nonlinear systems. However, the actuator faults are neglected in the above literature, whose occurrence will worsen the system performance and even lead to instability. Over the years, a quantity of attentions have been focused on fault tolerant control (FTC), and some effective control strategies have been implemented to faulty system. In [34], a robust adaptive sliding mode FTC scheme was proposed for coaxial helicopter to deal with actuator faults. In [35], a fuzzy adaptive nonlinear FTC strategy was presented for hypersonic vehicles with actuator stuck and loss of effectiveness faults. In [36], an adaptive decentralized FTC algorithm was developed for NSV attitude dynamics with actuator faults and control surface damage. Nonetheless, besides the negative factors mentioned above, the time-varying disturbances derived from the outside world also should be further considered.

The problem of robust control has a long history, and a number of disturbance rejection methods have been developed in past decades [15, 19, 37–40]. Especially, the nonlinear disturbance observer (NDO) has been extensively employed in practice since it does not depend on complete information of the disturbance model. In [15], the NDO-based composite fuzzy control issue was investigated when the uncertain nonlinear systems suffered from unknown dead zone. In [19], a NDO-based output feedback control scheme was developed for uncertain nonlinear systems with unknown hysteresis and external disturbance. By combining the NDO and asymptotic tracking control techniques, a composed control approach was proposed for the spacecraft formation flying system under nonzero disturbances in [38]. However, when the unknown nonlinearity, unknown control directions, unmeasurable states, actuator faults, and external disturbances appear simultaneously, the control performance of the MIMO nonlinear system faces severe challenges, and it is of great significance to design high-quality control algorithms.

Motivated by above analysis, the radial basis function neural network (RBFNN), state observer, Nussbaum function, and NDO are combined with the backstepping technique to achieve satisfactory tracking control property. The main contributions can be summarized as follows:

(1) Different from the traditional state feedback control [7], an output feedback controller is designed to tackle the unmeasured states and unknown nonlinearity by means of the RBFNN and state observer;

(2) Compared with some direct adaptive fault estimation method using projection function [11], the presented Nussbaum-based FTC approach can overcome the singularity problem in a simpler way and reduce the complexity of controller design;

(3) The developed output feedback FTC scheme can guarantee satisfactory tracking performance for the MIMO nonlinear systems under multiple negative effects.

The rest of this paper is organized as follows. Problem formulation and preparation knowledge are presented in Section 2. Section 3 derives the main results. Simulation studies on unmanned helicopter are carried out in Section 4. Section 5 draws the conclusion.

2. Problem Formulation and Preparation

Consider a class of MIMO nonlinear systems with actuator faults and external disturbances as follows:

$$\begin{aligned} \dot{X}_i &= F_i(\overline{X}_i) + X_{i+1} + D_i(t), i = 1, 2, \ldots, n-1, \\ \dot{X}_n &= F_n(\overline{X}_n) + B\vartheta u + D_n(t), \\ y &= X_1, \end{aligned} \tag{1}$$

where $X_i \in \mathbf{R}^n$ and $\overline{X}_i = [X_1^T, X_2^T, \ldots, X_i^T]^T \in \mathbf{R}^{in}$ with $i = 1, 2, \ldots, n$ being the system state vectors. $u \in \mathbf{R}^n$ and $y \in \mathbf{R}^n$ denote the input vector and output vector, respectively. $F_i(\overline{X}_i) \in \mathbf{R}^n$ defines an unknown smooth nonlinear function. $B = \mathrm{diag}\{B_1, B_2, \ldots, B_n\}$ is the unknown nonzero constant control gain matrix. $\vartheta = \mathrm{diag}\{\vartheta_1, \vartheta_2, \ldots, \vartheta_n\}$, ϑ_i refers to the actuator fault factor, which describes the unknown remaining control efficiency of i^{th} actuator. $D_i(t)$ represents external time-varying disturbance. In this paper, it is assumed that only the system output y is available for measurement.

Remark 1. The considered MIMO nonlinear systems can be employed to describe many practical plants, such as ocean vessel system [2], unmanned helicopter system [4], and NSV system [11]. Moreover, it is worth mentioning that the control directions are unknown in many application requirements and usually coupled with actuator faults. Hence, for reflecting the system dynamics more practically, the unknown control directions and actuator faults are considered simultaneously in this work.

The control objective of this study is to design a robust adaptive NN output feedback FTC scheme, such that all error signals of the closed-loop system are convergent, and the output y can follow the desired trajectory y_d. To this end, the following definition, lemmas, and assumptions are introduced.

Definition 1 (see [33]). If a continuous function $H(\cdot)$ fulfills the performances as follows:

$$\begin{aligned} &\lim_{s\longrightarrow+\infty} \sup \frac{1}{s}\int_0^s H(\tau)\mathrm{d}\tau = +\infty, \\ &\lim_{s\longrightarrow-\infty} \inf \frac{1}{s}\int_0^s H(\tau)\mathrm{d}\tau = -\infty. \end{aligned} \tag{2}$$

Then $H(\cdot)$ is called a Nussbaum function. At present, the functions $\tau^2\cos(\tau)$, $e^{\tau^2}\cos(\tau)$, $\tau^2\sin(\tau)$, and $e^{\tau^2}\sin(\pi/2\tau)$ have been proven to be the Nussbaum-type functions in [41, 42]. In this paper, $H(\tau) = \tau^2\cos(\tau)$ is used.

Lemma 1 (see [33]). *Let $V(t)\geq 0$ and $\tau_i(t)$ be smooth functions defined on $[0,t_f)$, and let $H(\tau_i)$ be smooth Nussbaum-type function. If the following inequality holds:*

$$\dot{V}(t) \leq -b_0V(t) + b_1 + \sum_{i=1}^{n}\left[\psi_i H(\tau_i) + 1\right]\dot{\tau}_i, \tag{3}$$

where b_0, b_1 and ψ_i are suitable positive constants, then, $V(t)$, $\tau_i(t)$ and $\sum_{i=1}^{n}[\psi_i H(\tau_i)+1]\dot{\tau}_i$ must be bounded on $[0,t_f)$.

Lemma 2 (see [10]). *Owing to the powerful nonlinear approximation capability, RBFNN is frequently employed to approximate any unknown smooth nonlinear function $E(\gamma)$, which can be expressed as*

$$E(\gamma) = \widehat{\Phi}^T M(\gamma) + \sigma, \tag{4}$$

where $\gamma\in\mathbf{R}^n$ is the input vector, $\widehat{\Phi}\in\mathbf{R}^{j\times n}$ is the weight matrix, σ is the approximation error, and $M(\gamma) = [M(\gamma), M_2(\gamma), \ldots, M_j(\gamma)] \in \mathbf{R}^j$ is the basis function vector with $M_j(\gamma)$ being

$$M_j(\gamma) = \exp\left(\frac{-(\gamma-\nu_i)^T(\gamma-\nu_i)}{c_i^2}\right), \quad i = 1,2,\ldots,j, \tag{5}$$

where ν_i and c_i denote the center and width of the basis function, respectively.

Then, the RBFNN (4) can approach any unknown smooth function $E(\gamma)$ in the form of

$$E(\gamma) = \Phi^{*T}M(\gamma) + \sigma^*, \tag{6}$$

where Φ^* is the optimal weight matrix, σ^* is the approximation error satisfying $\|\sigma^*\|\leq\overline{\sigma}$, and $\overline{\sigma}$ is a positive constant.

Assumption 1 (see [7]). For the bounded desired trajectory y_d and its derivatives $\dot{y}_d$ and $\ddot{y}_d$, there exists an unknown positive constant $\overline{l}_0$ making $\Pi_0\colon = \left\{(y_d,\dot{y}_d,\ddot{y}_d)\colon \|y_d\|^2 + \|\dot{y}_d\|^2 + \|\ddot{y}_d\|^2 \leq \overline{l}_0\right\}$ holds.

Assumption 2 (see [11]). The unknown external disturbance $D_i, i = 1,2\ldots,n$ is supposed to satisfy $\|D_i\|\leq\overline{D}_i$ and $\|\dot{D}_i\|\leq\zeta_i$, where $\overline{D}_i$ and ζ_i are unknown positive constants.

Assumption 3 (see [11, 33]). The control gain B_i is assumed to have the unknown but same sign with each other, and it satisfies $B_l\leq|B_i|\leq B_u$, where $B_l>0$ is the known constant. Moreover, the fault factor ϑ_i is assumed to be unknown constant and satisfies $0<\vartheta_m\leq\vartheta_i\leq 1$, where ϑ_m is the known lower bound.

Remark 2. For a practical system, the tracking mission should be realizable, and there should exist a feasible controller to achieve it, which means that assumption 1 is reasonable. In addition, assumption 2 is provided to illustrate that if the external disturbance is unbounded, it may result in that the system cannot provide enough energy to accomplish the specified object. Similarly, if the actuator loses too much effectiveness, the whole system may lose the FTC capacity. Therefore, it is reasonable for assumptions 1–3, and they have been extensively used in existing literature such as [4, 5, 7, 16–18, 40–42].

3. Control Design and Stability Analysis

In this section, a backstepping-based robust adaptive output feedback FTC control scheme will be developed for the MIMO nonlinear systems to deal with the unknown nonlinearity, unavailable states, unknown control directions, actuator faults, and external disturbances. The block diagram of the design thread is given in Figure 1.

3.1. Model Transformation and State Observer Design. Since the control gain B_i and fault factor ϑ_i of the i^{th} actuator are all unknown and coupled with the corresponding control input, it is difficult to design the controller directly. According to the properties of B_i and ϑ_i, it can be obtained that $0<B_l\vartheta_m\leq|B_i\vartheta_i|\leq B_u$. By defining $\hbar = B\vartheta$, it is known that $\hbar$ is an invertible matrix. In order to promote the control design, we introduce the new state variable $x_i = \hbar^{-1}X_i$, the new smooth function $f_i(\overline{x}_i) = \hbar^{-1}F_i(\overline{X}_i)$, and the new disturbance $d_i(t) = \hbar^{-1}D_i(t)$. Then, the MIMO nonlinear system (1) can be transformed into the following system:

$$\begin{aligned} \dot{x}_i &= f_i(\overline{x}_i) + x_{i+1} + d_i(t), i = 1,2,\ldots,n-1, \\ \dot{x}_n &= f_n(\overline{x}_n) + u + d_n(t), \\ y_1 &= x_1, \end{aligned} \tag{7}$$

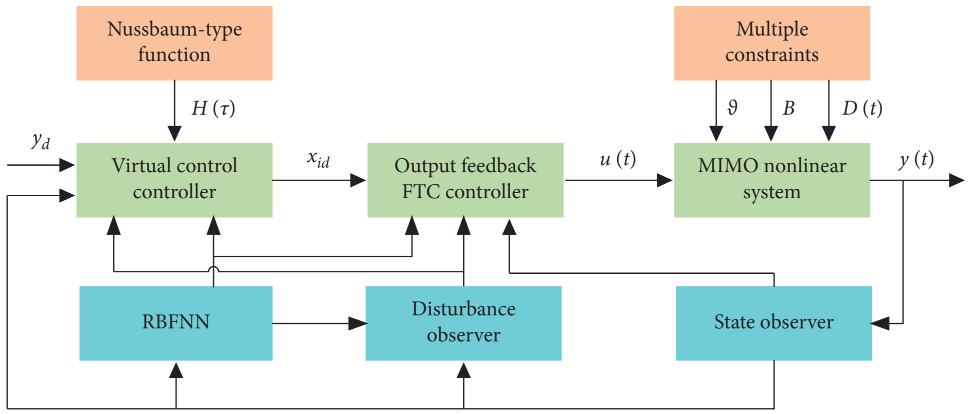

FIGURE 1: Block diagram of the proposed control scheme.

where $\bar{x}_i = [x_1^T, x_2^T, \ldots, x_i^T]^T \in \mathbf{R}^{in}$ and $y_1 \in \mathbf{R}^n$ are the new system state vector and output vector.

Remark 3. Considering assumption 2 and assumption 3, we obtain that the new disturbance $d_i(t)$ and its first derivative are also bounded. That is, there exist unknown positive constants $\bar{d}_i$ and ϱ_i such that $\|d_i\| \le \bar{d}_i$ and $\|\dot{d}_i\| \le \varrho_i$. Moreover, we should note that the state variable x_i and the output vector y_1 of the new MIMO nonlinear system (7) are all unavailable.

Here, the following RBFNN is adopted to approximate the unknown nonlinear term $L_i f_i(\bar{x}_i)$:

$$L_i f_i(\bar{x}_i) = \Phi_i^{*T} M_i(\bar{x}_i) + \sigma_i^*(\bar{x}_i), i = 1, 2, \ldots, n, \quad (8)$$

where $L_i = L_i^T > 0$ is the designed constant matrix, $M_i(\bar{x}_i)$ is the basis function, and Φ_i^* is the weight matrix.

Due to the unavailability of the system states x_i, the above function approximations (8) are invalid. Hence, we use the following approximation to design the state observer:

$$L_i \hat{f}_i(\hat{\bar{x}}_i) = \hat{\Phi}_i^T M_i(\hat{\bar{x}}_i), i = 1, 2, \ldots, n, \quad (9)$$

where $\hat{\bar{x}}_i = [\hat{x}_1^T, \hat{x}_2^T, \ldots, \hat{x}_i^T]^T$ is the estimation of $\bar{x}_i$, and $\hat{\Phi}_i$ is the estimation of Φ_i^*.

It is obvious that the approximation $L_i \hat{f}_i(\hat{\bar{x}}_i)$ depends on the available $\hat{\Phi}_i$ and $\hat{\bar{x}}_i$. In order to cope with the unknown system states in the MIMO nonlinear systems (7), the following state observer is designed [19]:

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{x}_{i+1} + \hat{f}_i(\hat{\bar{x}}_i) - k_i \hat{x}_1 + \chi_i y + \hat{d}_i, i = 1, \ldots, n-1, \\ \dot{\hat{x}}_n &= u + \hat{f}_n(\hat{\bar{x}}_n) - k_n \hat{x}_1 + \chi_n y + \hat{d}_n, \end{aligned} \quad (10)$$

where $k_i = \mathrm{diag}\{k_{i1}, k_{i2}, \ldots, k_{in}\}$ with $k_{ij} > 0$, $\chi_i = \mathrm{diag}\{\chi_{i1}, \chi_{i2}, \ldots, \chi_{in}\}$ with $\chi_{ij} > 0\,(j = 1, 2, \ldots n)$, $\hat{d}_i$ is the estimation of d_i. The corresponding updating laws with respect to $\hat{d}_i$ and $\hat{\Phi}_i$ will be given gradually along with the controller design process in the next subsection.

Define $z_i = x_i - \hat{x}_i$, $\lambda_i = L_i^{-1}\sigma_i^*(\bar{x}_i)$, and $\tilde{\Phi}_i = \Phi_i^* - \hat{\Phi}_i$. Considering (7–10) and differentiating z_i yield

$$\begin{aligned} \dot{z}_i &= x_{i+1} + L_i^{-1}\left(\Phi_i^{*T} M_i(\bar{x}_i) + \sigma_i^*(\bar{x}_i)\right) + d_i - \hat{x}_{i+1} \\ &\quad - L_i^{-1}\hat{\Phi}_i^T M_i(\hat{\bar{x}}_i) + k_i \hat{x}_1 - \chi_i y - \hat{d}_i, \\ &= -k_i z_1 + z_{i+1} + k_i x_1 + \lambda_i - \chi_i y + L_i^{-1}\tilde{\Phi}_i^T M_i(\hat{\bar{x}}_i) \\ &\quad + L_i^{-1}\Phi_i^{*T}\left(M_i(\bar{x}_i) - M_i(\hat{\bar{x}}_i)\right) - \varepsilon_i z_i + \tilde{\Delta}_i, \end{aligned} \quad (11)$$

$$\begin{aligned} \dot{z}_n &= -k_n z_1 + k_n x_1 + \lambda_n - \chi_n y + L_n^{-1}\tilde{\Phi}_n^T M_n(\hat{\bar{x}}_n) \\ &\quad + L_n^{-1}\Phi_n^{*T}\left(M_n(\bar{x}_n) - M_n(\hat{\bar{x}}_n)\right) - \varepsilon_n z_n + \tilde{\Delta}_n, \end{aligned} \quad (12)$$

where $\varepsilon_i = \varepsilon_i^T > 0$ is the designed diagonal matrix, and $\tilde{\Delta}_i$ is the estimation error of the auxiliary variable Δ_i, which will be given in the following.

Then, the observation error dynamics (11) and (12) can be rewritten as

$$\begin{aligned} \dot{Z} &= NZ + Kx_1 + L^{-1}\Phi^{*T}\Upsilon(\bar{x}, \hat{\bar{x}}) + \lambda \\ &\quad + \tilde{\Delta} + L^{-1}\tilde{\Phi}^T M(\hat{\bar{x}}) - hy. \end{aligned} \quad (13)$$

where $N = \begin{bmatrix} -k_1 - \varepsilon_1 & I_1 & \cdots & \cdots \\ -k_2 & -\varepsilon_2 & I_2 & \vdots \\ \vdots & \vdots & \ddots & I_{n-1} \\ -k_n & \cdots & \cdots & -\varepsilon_n \end{bmatrix}$, $Z = [z_1^T, z_2^T, \ldots, z_n^T]^T$, $\lambda = [\lambda_1^T, \lambda_2^T, \ldots, \lambda_n^T]^T$, $K = [k_1^T, k_2^T, \ldots, k_n^T]^T$, $L^{-1} = \mathrm{diag}\{L_1^{-1}, L_2^{-1}, \ldots, L_n^{-1}\}$, $\Phi^* = \mathrm{diag}\{\Phi_1^*, \Phi_2^*, \ldots, \Phi_n^*\}$, $\tilde{\Phi} = \mathrm{diag}\{\tilde{\Phi}_1, \tilde{\Phi}_2, \ldots, \tilde{\Phi}_n\}$, $\tilde{\Delta} = [\tilde{\Delta}_1^T, \tilde{\Delta}_2^T, \ldots, \tilde{\Delta}_n^T]^T$, $h = [\chi_1^T, \chi_2^T, \ldots, \chi_n^T]^T$, $M(\hat{\bar{x}}) = [M_1^T(\hat{\bar{x}}_1), M_2^T(\hat{\bar{x}}_2), \ldots, M_n^T(\hat{\bar{x}}_n)]^T$, $I_i \in \mathbf{R}^{n\times n}$ is the unit matrix, $\Upsilon(\bar{x}, \hat{\bar{x}}) = M(\bar{x}) - M(\hat{\bar{x}})$, $M(\bar{x}) = [M_1^T(\bar{x}_1), M_2^T(\bar{x}_2), \ldots, M_n^T(\bar{x}_n)]^T$.

Here, proper parameters k_i should be selected to ensure that N is Hurwitz. In other words, for a given matrix $P = P^T > 0$, there exists positive definite matrix $\Lambda = \Lambda^T > 0$ such that

$$N^T P + PN \le -\Lambda. \quad (14)$$

Due to the unknown disturbance d_i, the following auxiliary variable Δ_i is introduced:

$$\Delta_i = d_i + \varepsilon_i x_i, i = 1, 2, \ldots, n. \tag{15}$$

In light of (15), the NDOs are designed as

$$\begin{cases} \hat{d}_i = \hat{\Delta}_i - \varepsilon_i \hat{x}_i, i = 1, 2, \ldots, n-1, \\ \dot{\hat{\Delta}}_i = \varepsilon_i \left(\hat{f}_i (\hat{\overline{x}}_i) + \hat{x}_{i+1} - \varepsilon_i \hat{x}_i + \hat{\Delta}_i \right), \end{cases} \tag{16}$$

$$\begin{cases} \hat{d}_n = \hat{\Delta}_n - \varepsilon_n \hat{x}_n, \\ \dot{\hat{\Delta}}_n = \varepsilon_n \left(\hat{f}_n (\hat{\overline{x}}_n) + u - \varepsilon_n \hat{x}_n + \hat{\Delta}_n \right). \end{cases} \tag{17}$$

Define $\tilde{\Delta}_i = \Delta_i - \hat{\Delta}_i$ and $\tilde{d}_i = d_i - \hat{d}_i$. Then, we have

$$\tilde{\Delta}_i = d_i + \varepsilon_i x_i - \hat{d}_i - \varepsilon_i \hat{x}_i = \tilde{d}_i + \varepsilon_i z_i. \tag{18}$$

On the one hand, by invoking (15) and (16), we obtain

$$\begin{aligned} \dot{\tilde{\Delta}}_i &= \dot{d}_i + \varepsilon_i \dot{x}_i - \dot{\hat{\Delta}}_i \\ &= \dot{d}_i + \varepsilon_i \Big(L_i^{-1} \Phi_i^{*T} \Upsilon_i \left(\overline{x}_i, \hat{\overline{x}}_i \right) \\ &\quad + L_i^{-1} \tilde{\Phi}_i^T M_i \left(\hat{\overline{x}}_i \right) + \lambda_i + z_{i+1} - \tilde{\Delta}_i - \varepsilon_i z_i \Big), \\ &i = 1, 2, \ldots, n-1. \end{aligned} \tag{19}$$

On the other hand, by invoking (15) and (17), we obtain

$$\begin{aligned} \dot{\tilde{\Delta}}_n &= \dot{d}_n + \varepsilon_n \dot{x}_n - \dot{\hat{\Delta}}_n \\ &= \dot{d}_n + \varepsilon_n \Big(L_n^{-1} \Phi_n^{*T} \Upsilon_n \left(\overline{x}_n, \hat{\overline{x}}_n \right) \\ &\quad + L_n^{-1} \tilde{\Phi}_n^T M_n \left(\hat{\overline{x}}_n \right) + \lambda_n - \tilde{\Delta}_n - \varepsilon_n z_n \Big). \end{aligned} \tag{20}$$

Consider the following Lyapunov function candidate:

$$V_0 = Z^T P Z + \frac{1}{2} \sum_{i=1}^{n} \tilde{\Delta}_i^T \tilde{\Delta}_i. \tag{21}$$

Invoking (13), (14), (19) and (20), one obtains

$$\begin{aligned} \dot{V}_0 \leq & - Z^T \Lambda Z + 2 Z^T P K x_1 + 2 Z^T P \lambda + 2 Z^T P \tilde{\Delta} \\ & + 2 Z^T P L^{-1} \Phi^{*T} \Upsilon (\overline{x}, \hat{\overline{x}}) + 2 Z^T P L^{-1} \tilde{\Phi}^T M (\hat{\overline{x}}) \\ & - 2 Z^T P h y + \sum_{i=1}^{n} \tilde{\Delta}_i^T \dot{d}_i + \sum_{i=1}^{n} \tilde{\Delta}_i^T \varepsilon_i L_i^{-1} \Phi_i^{*T} \Upsilon_i \left(\overline{x}_i, \hat{\overline{x}}_i \right) \\ & + \sum_{i=1}^{n} \tilde{\Delta}_i^T \varepsilon_i L_i^{-1} \tilde{\Phi}_i^T M_i \left(\hat{\overline{x}}_i \right) + \sum_{i=1}^{n} \tilde{\Delta}_i^T \varepsilon_i \lambda_i \\ & - \sum_{i=1}^{n} \tilde{\Delta}_i^T \varepsilon_i \tilde{\Delta}_i - \sum_{i=1}^{n} \tilde{\Delta}_i^T \varepsilon_i^2 z_i + \sum_{i=1}^{n-1} \tilde{\Delta}_i^T \varepsilon_i z_{i+1}. \end{aligned} \tag{22}$$

Based on Young's inequality, the following inequalities can be obtained:

$$\begin{aligned} & 2 Z^T P K x_1 \leq \hbar_m Z^T P K K^T P^T Z + \| e_1 \|^2 + \varsigma_0^2, \\ & 2 Z^T P \lambda \leq Z^T P P^T Z + \sum_{i=1}^{n} \overline{\lambda}_i^2 \\ & 2 Z^T P L^{-1} \Phi^{*T} \Upsilon (\overline{x}, \hat{\overline{x}}) \\ & \leq Z^T P P^T Z + \sum_{i=1}^{n} \beta_i^2 \| L_i^{-1} \|^2 \| \Phi_i^* \|^2 \\ & 2 Z^T P L^{-1} \tilde{\Phi}^T M (\hat{\overline{x}}) \\ & \leq r_1 Z^T P P^T Z + \frac{1}{r_1} \sum_{i=1}^{n} \delta_i^2 \| L_i^{-1} \|^2 \| \tilde{\Phi}_i \|^2 \\ & 2 Z^T P \tilde{\Delta} \leq Z^T P P^T Z + \sum_{i=1}^{n} \tilde{\Delta}_i^T \tilde{\Delta}_i \\ & - 2 Z^T P h y \leq Z^T P h h^T P^T Z + \| e_1 \|^2 + \varsigma_0^2 \\ & \sum_{i=1}^{n} \tilde{\Delta}_i^T \dot{d}_i \leq \frac{1}{2} \sum_{i=1}^{n} \tilde{\Delta}_i^T \tilde{\Delta}_i + \frac{1}{2} \sum_{i=1}^{n} \rangle_i^2 \\ & \sum_{i=1}^{n} \tilde{\Delta}_i^T \varepsilon_i L_i^{-1} \Phi_i^{*T} \Upsilon_i \left(\overline{x}_i, \hat{\overline{x}}_i \right) \\ & \leq \frac{1}{2} \sum_{i=1}^{n} \tilde{\Delta}_i^T \tilde{\Delta}_i + \frac{1}{2} \sum_{i=1}^{n} \overline{\varepsilon}^2 \| L_i^{-1} \|^2 \beta_i^2 \| \Phi_i^* \|^2 \\ & \sum_{i=1}^{n} \tilde{\Delta}_i^T \varepsilon_i L_i^{-1} \tilde{\Phi}_i^T M_i \left(\hat{\overline{x}}_i \right) \\ & \leq \frac{r_2}{2} \sum_{i=1}^{n} \tilde{\Delta}_i^T \tilde{\Delta}_i + \frac{1}{2 r_2} \sum_{i=1}^{n} \overline{\varepsilon}^2 \| L_i^{-1} \|^2 \delta_i^2 \| \tilde{\Phi}_i \|^2, \\ & \sum_{i=1}^{n} \tilde{\Delta}_i^T \varepsilon_i \lambda_i \leq \frac{1}{2} \sum_{i=1}^{n} \tilde{\Delta}_i^T \tilde{\Delta}_i + \frac{1}{2} \sum_{i=1}^{n} \overline{\varepsilon}^2 \overline{\lambda}_i^2, \\ & - \sum_{i=1}^{n} \tilde{\Delta}_i^T \varepsilon_i^2 z_i \leq \frac{1}{2} \sum_{i=1}^{n} \tilde{\Delta}_i^T \tilde{\Delta}_i + \frac{\overline{\varepsilon}^4}{2} Z^T Z, \\ & \sum_{i=1}^{n-1} \tilde{\Delta}_i^T \varepsilon_i z_{i+1} \leq \frac{1}{2} \sum_{i=1}^{n} \tilde{\Delta}_i^T \tilde{\Delta}_i + \frac{\overline{\varepsilon}^2}{2} Z^T Z, \end{aligned} \tag{23}$$

where r_1 and r_2 are the designed positive constants, $\| y_d \| \leq \varsigma_0$, $\hbar_m = 1/B_l^2 \vartheta_m^2$, $\overline{\varepsilon} = \max(\| \varepsilon_i \|)$, $\| \lambda_i \| \leq \overline{\lambda}_i$, $\| \Upsilon_i (\overline{x}_i, \hat{\overline{x}}_i) \| \leq \beta_i$, $\| M_i (\hat{\overline{x}}_i) \| \leq \delta_i$, e_1 is the tracking error, which will be given subsequently.

Substituting the above inequalities into (22) produces

$$\dot{V}_0 \le -Z^T\left(\Lambda - \Lambda_1 - \frac{\bar{\varepsilon}^2 + \bar{\varepsilon}^4}{2}I_1\right)Z + 2\|e_1\|^2 - \sum_{i=1}^{n}\tilde{\Delta}_i^T\left(\varepsilon_i - \frac{7 + r_2}{2}I_1\right)\tilde{\Delta}_i + R_0 + \sum_{i=1}^{n}\left(\frac{1}{r_1} + \frac{\bar{\varepsilon}^2}{2r_2}\right)\|L_i^{-1}\|^2\delta_i^2\|\tilde{\Phi}_i\|^2, \tag{24}$$

where $\Lambda_1 = P(\hbar_m KK^T + hh^T + (r_1 + 3)I_1)P^T$, $R_0 = \sum_{i=1}^{n}(1 + 1/2\bar{\varepsilon}^2)\ \beta_i^2\|L_i^{-1}\|^2\|\Phi_i^*\|^2 + 2\varsigma_0^2 + \sum_{i=1}^{n}(1 + 1/2\bar{\varepsilon}^2)\ \lambda_i^2 + 1/2\sum_{i=1}^{n}\varrho_i^2$.

3.2. Robust Adaptive Neural Output Feedback Control Design

Step 1. Define the following tracking errors:

$$e_1 = y - y_d, \quad e_i = \hat{x}_i - x_{id}, i = 2, \ldots, n, \tag{25}$$

where x_{id} is the designed virtual control law.

Considering (7) and taking the time derivative of e_1 yield

$$\dot{e}_1 = \dot{X}_1 - \dot{y}_d = \hbar\dot{x}_1 - \dot{y}_d = F_1(y) + \hbar(x_{2d} + e_2 + z_2 + d_1) - \dot{y}_d. \tag{26}$$

Because of the unknown smooth function $F_1(y)$, the following RBFNN is used to approximate it:

$$F_1(y) = Q_F^{-1}\left(\Phi_F^{*T}M_F(y) + \sigma_F^*(y)\right), \tag{27}$$

where $Q_F = Q_F^T > 0$ is the designed matrix, $M_F(y)$ is Gaussian function, and Φ_F^* is the weight matrix.

Substituting (27) into (26) follows

$$\dot{e}_1 = Q_F^{-1}\Phi_F^{*T}M_F(y) + Q_F^{-1}\sigma_F^*(y) + \hbar x_{2d} + \hbar e_2 + \hbar z_2 + \hbar d_1 - \dot{y}_d \tag{28}$$

Consider the Lyapunov function candidate as

$$V_1 = V_0 + \frac{1}{2}e_1^Te_1 + \frac{1}{2}tr\left\{\tilde{\Phi}_1^T\Theta_1^{-1}\tilde{\Phi}_1\right\} + \frac{1}{2}tr\left\{\tilde{\Phi}_F^T\Omega_F^{-1}\tilde{\Phi}_F\right\}, \tag{29}$$

where $\Theta_1 = \Theta_1^T > 0$ and $\Omega_F = \Omega_F^T > 0$ are the designed matrixes, $\tilde{\Phi}_F = \Phi_F^* - \hat{\Phi}_F$.

Invoking (24) and (28), we obtain

$$\begin{aligned}
\dot{V}_1 \le & -Z^T\left(\Lambda - \Lambda_1 - \frac{\bar{\varepsilon}^2 + \bar{\varepsilon}^4}{2}I_1\right)Z + 2\|e_1\|^2 + R_0 \\
& + e_1^TQ_F^{-1}\Phi_F^{*T}M_F(y) - \sum_{i=1}^{n}\tilde{\Delta}_i^T\left(\varepsilon_i - \frac{7 + r_2}{2}I_1\right)\tilde{\Delta}_i \\
& + \sum_{i=1}^{n}\left(\frac{1}{r_1} + \frac{\bar{\varepsilon}^2}{2r_2}\right)\|L_i^{-1}\|^2\delta_i^2\|\tilde{\Phi}_i\|^2 + e_1^TQ_F^{-1}\sigma_F^*(y) \\
& + e_1^T\hbar x_{2d} + e_1^T\hbar e_2 + e_1^T\hbar z_2 + e_1^T\hbar d_1 - e_1^T\dot{y}_d \\
& - tr\left\{\tilde{\Phi}_1^T\Theta_1^{-1}\dot{\hat{\Phi}}_1\right\} - tr\left\{\tilde{\Phi}_F^T\Omega_F^{-1}\dot{\hat{\Phi}}_F\right\} \\
\le & -Z^T\left(\Lambda - \Lambda_1 - \frac{\bar{\varepsilon}^2 + \bar{\varepsilon}^4}{2}I_1 - C\right)Z + e_1^T\hbar x_{2d} \\
& + e_1^TQ_F^{-1}\Phi_F^{*T}M_F(y) - \sum_{i=1}^{n}\tilde{\Delta}_i^T\left(\varepsilon_i - \frac{7 + r_2}{2}I_1\right)\tilde{\Delta}_i \\
& + \sum_{i=1}^{n}\left(\frac{1}{r_1} + \frac{\bar{\varepsilon}^2}{2r_2}\right)\|L_i^{-1}\|^2\delta_i^2\|\tilde{\Phi}_i\|^2 + \left(\frac{5 + 3B_u^2}{2}\right)e_1^Te_1 \\
& + \frac{1}{2}\bar{\sigma}_F^2\|Q_F^{-1}\|^2 + \frac{1}{2}e_2^Te_2 + \frac{1}{2}\bar{d}_1^2 - tr\left\{\tilde{\Phi}_1^T\Theta_1^{-1}\dot{\hat{\Phi}}_1\right\} \\
& - tr\left\{\tilde{\Phi}_F^T\Omega_F^{-1}\dot{\hat{\Phi}}_F\right\} - e_1^T\dot{y}_d + R_0,
\end{aligned} \tag{30}$$

where $C = \mathrm{diag}\{0_{n\times n}, 1/2I_1, \ldots, 0_{n\times n}\}$, $\|\sigma_F^*\| \le \bar{\sigma}_F$.

Design the virtual control function x_{2d} and parameter adaptive laws as

$$x_{2d} = H(\tau)\mu, \tag{31}$$

$$\dot{\tau}_i = e_{1i}\mu_i, i = 1, 2, \ldots, n, \tag{32}$$

$$\mu = q_1e_1 - \dot{y}_d + Q_F^{-1}\hat{\Phi}_F^TM_F(y) - \hat{d}_1, \tag{33}$$

$$\dot{\hat{\Phi}}_1 = \Theta_1\left(M_1(\hat{\bar{x}}_1)e_1^TL_1^{-1} - \varpi_1\hat{\Phi}_1\right), \tag{34}$$

$$\dot{\hat{\Phi}}_F = \Omega_F\left(M_F(y)e_1^TQ_F^{-1} - \varpi_F\hat{\Phi}_F\right), \tag{35}$$

where $\tau = [\tau_1, \tau_2, \ldots, \tau_n]^T$, $\mu = [\mu_1, \mu_2, \ldots, \mu_n]^T$, e_{1i} is the i^{th} element of e_1, $H(\tau) = \mathrm{diag}\{H(\tau_1), H(\tau_2), \ldots, H(\tau_n)\}$,

$H(\tau_i)$ is Nussbaum-type function, which is chosen as $H(\tau_i) = \tau_i^2 \cos(\tau_i)$, $q_1 = \mathrm{diag}\{q_{11}, q_{12}, \ldots, q_{1n}\}$, q_{1i}, ϖ_1 and ϖ_F are designed positive constants.

Considering (31)–(35), the following facts can be obtained:

$$e_1^T \hbar x_{2d} = \sum_{i=1}^{n} \hbar_i H(\tau_i)\dot{\tau}_i = \sum_{i=1}^{n} [\hbar_i H(\tau_i) + 1]\dot{\tau}_i - e_1^T \mu, \quad (36)$$

$$\begin{aligned} e_1^T Q_F^{-1} \tilde{\Phi}_F^T M_F(y) - tr\left\{\tilde{\Phi}_F^T \Omega_F^{-1} \dot{\hat{\Phi}}_F\right\} &= tr\left\{\tilde{\Phi}_F^T \varpi_F \hat{\Phi}_F\right\} \\ &\leq -\frac{1}{2}\varpi_F \|\tilde{\Phi}_F\|^2 + \frac{1}{2}\varpi_F \|\Phi_F^*\|^2, \end{aligned} \quad (37)$$

$$\begin{aligned} -tr\left\{\tilde{\Phi}_1^T \Theta_1^{-1} \dot{\hat{\Phi}}_1\right\} &= -tr\left\{\tilde{\Phi}_1^T M_1(\hat{\bar{x}}_1) e_1^T L_1^{-1} - \tilde{\Phi}_1^T \varpi_1 \hat{\Phi}_1\right\} \\ &\leq e_1^T e_1 - \left(\frac{1}{2}\varpi_1 - \frac{\delta_1^2}{4}\|L_1^{-1}\|\right)\|\tilde{\Phi}_1\|^2 + \frac{1}{2}\varpi_1 \|\Phi_1^*\|^2, \end{aligned} \quad (38)$$

where $\hbar = \mathrm{diag}\{\hbar_1, \hbar_2, \ldots, \hbar_n\}$ with $\hbar_i$ being the diagonal element.

Substituting (36)–(38) into (30) gives

$$\begin{aligned} \dot{V}_1 \leq & -Z^T A Z - e_1^T \epsilon_1 e_1 - \left(\frac{1}{2}\varpi_1 - \frac{\delta_1^2}{4}\|L_1^{-1}\|^2\right)\|\tilde{\Phi}_1\|^2 \\ & -\frac{1}{2}\varpi_F \|\tilde{\Phi}_F\|^2 - \sum_{i=2}^{n} \tilde{\Delta}_i^T \left(\varepsilon_i - \frac{7 + r_2}{2} I_1\right)\tilde{\Delta}_i + \frac{1}{2} e_2^T e_2 \\ & - \tilde{\Delta}_1^T \left(\varepsilon_1 - \frac{8 + r_2}{2} I_1\right)\tilde{\Delta}_1 + \sum_{i=1}^{n} [\hbar_i H(\tau_i) + 1]\dot{\tau}_i \\ & + \sum_{i=1}^{n} \left(\frac{1}{r_1} + \frac{\bar{\varepsilon}^2}{2r_2}\right)\|L_i^{-1}\|^2 \delta_i^2 \|\tilde{\Phi}_i\|^2 + R_1, \end{aligned} \quad (39)$$

where $A = \Lambda - \Lambda_1 - \bar{\varepsilon}^2 + \bar{\varepsilon}^4/2I_1 - C - C_1$, $C_1 = \mathrm{diag}\{1/2I_1, 0_{n\times n}, \ldots, 0_{n\times n}\}$, $\epsilon_1 = q_1 - 9 + 3B_u^2 + \bar{\varepsilon}^2/2I_1$, $R_1 = R_0 + 1/2\varpi_F\|\Phi_F^*\|^2 + 1/2\bar{\sigma}_F^2\|Q_F^{-1}\|^2 + 1/2\varpi_1\|\Phi_1^*\|^2 + \bar{d}_1^2$.

Step 2. Considering (10) and differentiating e_2 give

$$\begin{aligned} \dot{e}_2 &= \hat{x}_3 + \hat{f}_2(\hat{\bar{x}}_2) - k_2\hat{x}_1 + \chi_2 y + \hat{d}_2 - \dot{x}_{2d} \\ &= x_{3d} + e_3 + L_2^{-1}\hat{\Phi}_2^T M_2(\hat{\bar{x}}_2) - k_2\hat{x}_1 + \chi_2 y \\ &\quad + \hat{d}_2 - \dot{x}_{2d}. \end{aligned} \quad (40)$$

In particular, the dynamic surface control technique is employed to avoid the repeated computation of $\dot{x}_{2d}$ and achieve its available derivative. Let x_{2d} pass the first-order filter η_2 [7].

$$h_2\dot{\eta}_2 + \eta_2 = x_{2d}, \eta_2(0) = x_{2d}(0), \quad (41)$$

where $h_2 = \mathrm{diag}\{h_{21}, h_{22}, \ldots, h_{2n}\} > 0$ is the time constant matrix of the filter.

By defining $\omega_2 = \eta_2 - x_{2d}$, we have

$$\begin{aligned} \dot{\omega}_2 &= \dot{\eta}_2 - \dot{x}_{2d} \\ &= -h_2^{-1}\omega_2 + \left(-\frac{\partial x_{2d}}{\partial \tau}\dot{\tau} - \frac{\partial x_{2d}}{\partial e_1}\dot{e}_1 - \frac{\partial x_{2d}}{\partial y_d}\dot{y}_d \right. \\ &\quad \left. - \frac{\partial x_{2d}}{\partial \hat{\Phi}_F}\dot{\hat{\Phi}}_F - \frac{\partial x_{2d}}{\partial y}\dot{y} - \frac{\partial x_{2d}}{\partial \hat{d}_1}\dot{\hat{d}}_1\right), \\ &= -h_2^{-1}\omega_2 + l_2(e_1, e_2, z_2, y, \dot{y}_d, \ddot{y}_d, \hat{\Phi}_F, \hat{d}_1), \end{aligned} \quad (42)$$

where $l_2(\bullet)$ is smooth function vector in regard to $\Pi_2(\bullet)$. Since the set $\Pi_2(\bullet)$ is compact, the smooth function $l_2(\bullet)$ has a maximum $\bar{l}_2$ on set $\Pi_2(\bullet)$ for the given initial conditions.

Considering (41), the virtual control law x_{3d} is designed as

$$\begin{aligned} x_{3d} = & -q_2 e_2 + \dot{\eta}_2 + k_2\hat{x}_1 - \chi_2 y \\ & - \hat{d}_2 - L_2^{-1}\hat{\Phi}_2^T M_2(\hat{\bar{x}}_2), \end{aligned} \quad (43)$$

where $q_2 = \mathrm{diag}\{q_{21}, q_{22}, \ldots, q_{2n}\}$, q_{2i} is the designed positive constant.

The parameter adaptive law is proposed as

$$\dot{\hat{\Phi}}_2 = \Theta_2\left(M_2(\hat{\bar{x}}_2)e_2^T L_2^{-1} - \varpi_2\hat{\Phi}_2\right), \quad (44)$$

where $\Theta_2 = \Theta_2^T > 0$ is the designed matrix, and $\varpi_2 > 0$ is the designed constant.

Invoking (43), we obtain

$$\dot{e}_2 = -q_2 e_2 + e_3 - h_2^{-1}\omega_2 + l_2(\bullet). \quad (45)$$

Choose the following Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2}e_2^T e_2 + \frac{1}{2}tr\left\{\tilde{\Phi}_2^T \Theta_2^{-1}\tilde{\Phi}_2\right\} + \frac{1}{2}\omega_2^T \omega_2. \quad (46)$$

The time derivative of V_2 is

$$\dot{V}_2 = \dot{V}_1 + e_2^T \dot{e}_2 - tr\left\{\tilde{\Phi}_2^T \Theta_2^{-1}\dot{\hat{\Phi}}_2\right\} + \omega_2^T \dot{\omega}_2. \quad (47)$$

Considering (44), we obtain

$$-tr\left\{\widetilde{\Phi}_2^T\Theta_2^{-1}\dot{\widehat{\Phi}}_2\right\} = -tr\left\{\widetilde{\Phi}_2^T M_2\left(\widehat{\overline{x}}_2\right)e_2^T L_2^{-1} - \widetilde{\Phi}_2^T \varpi_2\widehat{\Phi}_2\right\}$$
$$\leq e_2^T e_2 - \left(\frac{1}{2}\varpi_2 - \frac{\delta_2^2}{4}\|L_2^{-1}\|^2\right)\|\widetilde{\Phi}_2\|^2 + \frac{1}{2}\varpi_2\|\Phi_2^*\|^2. \tag{48}$$

Substituting (39), (45) and (48) into (47), we have

$$\begin{aligned}\dot{V}_2 \leq & -Z^T AZ - \sum_{i=1}^{2} e_i^T \epsilon_i e_i - \sum_{i=1}^{2}\left(\frac{1}{2}\varpi_i - \frac{\delta_i^2}{4}\|L_i^{-1}\|^2\right)\|\widetilde{\Phi}_i\|^2 \\ & - \frac{1}{2}\varpi_F\|\widetilde{\Phi}_F\|^2 - \sum_{i=2}^{n}\widetilde{\Delta}_i^T\left(\varepsilon_i - \frac{7+r_2}{2}I_1\right)\widetilde{\Delta}_i + e_2^T e_3 \\ & - \widetilde{\Delta}_1^T\left(\varepsilon_1 - \frac{8+r_2}{2}I_1\right)\widetilde{\Delta}_1 + \sum_{i=1}^{n}[\hbar_i H(\tau_i)+1]\dot{\tau}_i + R_2 \\ & + \sum_{i=1}^{n}\left(\frac{1}{r_1} + \frac{\overline{\varepsilon}^2}{2r_2}\right)\|L_i^{-1}\|^2\delta_i^2\|\widetilde{\Phi}_i\|^2 - \omega_2^T\left(h_2^{-1} - I\right)\omega_2,\end{aligned} \tag{49}$$

where $R_2 = R_1 + 1/2\varpi_2\|\Phi_2^*\|^2 + \overline{l}_2^2$, $\epsilon_2 = q_2 - 2I - 1/2\|h_2^{-1}\|^2 I$.

Step 3. ($3 \leq i \leq n-1$): Considering (10) and differentiating e_i give

$$\begin{aligned}\dot{e}_i = & x_{(i+1)d} + e_{i+1} + L_i^{-1}\widehat{\Phi}_i^T M_i\left(\widehat{\overline{x}}_i\right) - k_i\widehat{x}_1 \\ & + \chi_i y + \widehat{d}_i - \dot{x}_{id}.\end{aligned} \tag{50}$$

Similar to Step 2, let x_{id} pass the following first-order filter η_i [7]:

$$h_i\dot{\eta}_i + \eta_i = x_{id}, \eta_i(0) = x_{id}(0), \tag{51}$$

where $h_i = \text{diag}\{h_{i1}, h_{i2}, \ldots, h_{in}\} > 0$ is the time constant of the filter.

Define $\omega_i = \eta_i - x_{id}$. Differentiating ω_i yields

$$\begin{aligned}\dot{\omega}_i &= -h_i^{-1}\omega_i + \left(-\frac{\partial x_{id}}{\partial e_{i-1}}\dot{e}_{i-1} - \frac{\partial x_{id}}{\partial e_{i-2}}\dot{e}_{i-2} - \frac{\partial x_{id}}{\partial \widehat{x}_{i-1}}\dot{\widehat{x}}_{i-1} - \frac{\partial x_{id}}{\partial \widehat{\Phi}_{i-1}}\dot{\widehat{\Phi}}_{i-1} - \frac{\partial x_{id}}{\partial y}\dot{y} - \frac{\partial x_{id}}{\partial \widehat{d}_{i-1}}\dot{\widehat{d}}_{i-1}\right) \\ &= -h_i^{-1}\omega_i + l_i\left(e_i, e_{i-1}, y, \widehat{\overline{x}}_{i-1}, \widehat{\Phi}_{i-1}, \widehat{d}_{i-1}\right),\end{aligned} \tag{52}$$

where $l_i(\bullet)$ is a smooth function vector in regard to $\Pi_i(\bullet)$. Since the set $\Pi_i(\bullet)$ is compact, the smooth function $l_i(\bullet)$ has a maximum $\overline{l}_i$ on set $\Pi_i(\bullet)$ for the given initial conditions.

Considering (51), the virtual control law is developed as

$$\begin{aligned}x_{(i+1)d} = & -q_i e_i - e_{i-1} + \dot{\eta}_i + k_i\widehat{x}_1 - \chi_i y \\ & - \widehat{d}_i - L_i^{-1}\widehat{\Phi}_i^T M_i\left(\widehat{\overline{x}}_i\right),\end{aligned} \tag{53}$$

where $q_i = \text{diag}\{q_{i1}, q_{i2}, \ldots, q_{in}\}$, $q_{ij}, j = 1, 2, \ldots, n$ is the designed positive constant.

The parameter adaptive law is designed as

$$\dot{\widehat{\Phi}}_i = \Theta_i\left(M_i\left(\widehat{\overline{x}}_i\right)e_i^T L_i^{-1} - \varpi_i\widehat{\Phi}_i\right), \tag{54}$$

where $\Theta_i = \Theta_i^T > 0$ is the designed matrix, and $\varpi_i > 0$ is the designed constant.

Invoking (53), we obtain

$$\dot{e}_i = -q_i e_i - e_{i-1} + e_{i+1} - h_i^{-1}\omega_i + l_i(\bullet). \tag{55}$$

Select the Lyapunov function candidate as

$$V_i = V_{i-1} + \frac{1}{2}e_i^T e_i + \frac{1}{2}tr\left\{\widetilde{\Phi}_i^T\Theta_i^{-1}\widetilde{\Phi}_i\right\} + \frac{1}{2}\omega_i^T\omega_i, \tag{56}$$

By invoking (54), it outputs

$$-tr\left\{\widetilde{\Phi}_i^T\Theta_i^{-1}\dot{\widehat{\Phi}}_i\right\} = -tr\left\{\widetilde{\Phi}_i^T M_i\left(\widehat{\overline{x}}_i\right)e_i^T L_i^{-1} - \widetilde{\Phi}_i^T\varpi_i\widehat{\Phi}_i\right\}$$
$$\leq e_i^T e_i - \left(\frac{1}{2}\varpi_i - \frac{\delta_i^2}{4}\|L_i^{-1}\|^2\right)\|\widetilde{\Phi}_i\|^2 + \frac{1}{2}\varpi_i\|\Phi_i^*\|^2. \tag{57}$$

Differentiating (56), we obtain

$$\begin{aligned}\dot{V}_i \leq & -Z^T AZ - \sum_{i=1}^{n-1} e_i^T \epsilon_i e_i - \sum_{i=1}^{n-1}\left(\frac{1}{2}\varpi_i - \frac{\delta_i^2}{4}\|L_i^{-1}\|^2\right)\|\widetilde{\Phi}_i\|^2 \\ & - \frac{1}{2}\varpi_F\|\widetilde{\Phi}_F\|^2 - \sum_{i=2}^{n}\widetilde{\Delta}_i^T\left(\varepsilon_i - \frac{7+r_2}{2}I_1\right)\widetilde{\Delta}_i + e_i^T e_{i+1} \\ & - \widetilde{\Delta}_1^T\left(\varepsilon_1 - \frac{8+r_2}{2}I_1\right)\widetilde{\Delta}_1 + \sum_{i=1}^{n}[\hbar_i H(\tau_i)+1]\dot{\tau}_i \\ & + \sum_{i=1}^{n}\left(\frac{1}{r_1} + \frac{\overline{\varepsilon}^2}{2r_2}\right)\|L_i^{-1}\|^2\delta_i^2\|\widetilde{\Phi}_i\|^2 + R_i \\ & - \sum_{i=2}^{n-1}\omega_i^T\left(h_i^{-1} - I_1\right)\omega_i,\end{aligned} \tag{58}$$

where $R_i = R_{i-1} + 1/2\varpi_i\|\Phi_i^*\|^2 + \bar{l}_i^2$, $\epsilon_i = q_i - 3/2I_1 - 1/2\|h_i^{-1}\|^2 I_1$.

Step 4. Taking the derivative of e_n yields

$$\dot{e}_n = u + L_n^{-1}\hat{\Phi}_n^T M_n(\hat{\bar{x}}_n) - k_n\hat{x}_1 + \chi_n y + \hat{d}_n - \dot{x}_{nd}. \quad (59)$$

Similarly, let x_{nd} pass the following filter η_n [7]:

$$h_n\dot{\eta}_n + \eta_n = x_{nd}, \eta_n(0) = x_{nd}(0). \quad (60)$$

where $h_n = \text{diag}\{h_{n1}, h_{n2}, \ldots, h_{nn}\} > 0$ is the time constant of the filter.

Define $\omega_n = \eta_n - x_{nd}$. Taking the derivative of ω_n yields

$$\begin{aligned}\dot{\omega}_n &= -h_n^{-1}\omega_n - \frac{\partial x_{nd}}{\partial e_{n-1}}\dot{e}_{n-1} - \frac{\partial x_{nd}}{\partial e_{n-2}}\dot{e}_{n-2} - \frac{\partial x_{nd}}{\partial \hat{\bar{x}}_{n-1}}\dot{\hat{\bar{x}}}_{n-1} \\ &\quad - \frac{\partial x_{nd}}{\partial \hat{\Phi}_{n-1}}\dot{\hat{\Phi}}_{n-1} - \frac{\partial x_{nd}}{\partial y}\dot{y} - \frac{\partial x_{nd}}{\partial \hat{d}_{n-1}}\dot{\hat{d}}_{n-1} \\ &= -h_n^{-1}\omega_n + l_n\left(e_n, e_{n-1}, y, \hat{\bar{x}}_{n-1}, \hat{\Phi}_{n-1}, \hat{d}_{n-1}\right),\end{aligned} \quad (61)$$

where $l_n(\bullet)$ is smooth function vector in regard to $\Pi_n(\bullet)$. Since the set $\Pi_n(\bullet)$ is compact, the smooth function $l_n(\bullet)$ has a maximum $\bar{l}_n$ on set $\Pi_n(\bullet)$ for the given initial conditions.

Design the actual control input and parameter adaptive law as

$$\begin{aligned}u &= -q_n e_n - e_{n-1} + \dot{\eta}_n + k_n\hat{x}_1 - \chi_n y \\ &\quad - \hat{d}_n - L_n^{-1}\hat{\Phi}_n^T M_n(\hat{\bar{x}}_n),\end{aligned} \quad (62)$$

$$\dot{\hat{\Phi}}_n = \Theta_n\left(M_n(\hat{\bar{x}}_n)e_n^T L_n^{-1} - \varpi_n\hat{\Phi}_n\right), \quad (63)$$

where $q_n = \text{diag}\{q_{n1}, q_{n2}, \ldots, q_{nn}\}$ with $q_{nj}(j = 1, 2, \ldots, n)$ being positive constant, $\Theta_n = \Theta_n^T > 0$ is the designed matrix, and $\varpi_n > 0$ is the designed constant.

Define the Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{2}e_n^T e_n + \frac{1}{2}tr\left\{\tilde{\Phi}_n^T\Theta_n^{-1}\tilde{\Phi}_n\right\} + \frac{1}{2}\omega_n^T\omega_n. \quad (64)$$

Differentiating V_n outputs

$$\dot{V}_n = \dot{V}_{n-1} + e_n^T\dot{e}_n - tr\left\{\tilde{\Phi}_n^T\Theta_n^{-1}\dot{\hat{\Phi}}_n\right\} + \omega_n^T\dot{\omega}_n. \quad (65)$$

Considering (60)–(63), we obtain

$$\dot{e}_n = -q_n e_n - e_{n-1} - h_n^{-1}\omega_n + l_n(\bullet), \quad (66)$$

$$\begin{aligned}-tr\left\{\tilde{\Phi}_n^T\Theta_n^{-1}\dot{\hat{\Phi}}_n\right\} &\le e_n^T e_n + \frac{1}{2}\varpi_n\|\Phi_n^*\|^2 \\ &\quad - \left(\frac{1}{2}\varpi_n - \frac{\delta_n^2}{4}\|L_n^{-1}\|^2\right)\|\tilde{\Phi}_n\|^2,\end{aligned} \quad (67)$$

where ρ_n is the designed constant.

Substituting (66) and (67) into (65), we have

$$\begin{aligned}\dot{V}_n &\le -Z^T AZ - \sum_{i=1}^n e_i^T\epsilon_i e_i - \sum_{i=1}^n\left(\frac{1}{2}\varpi_i - \frac{\delta_i^2}{4}\left(\|L_i^{-1}\|^2\right)\right)\|\tilde{\Phi}_i\|^2 \\ &\quad - \frac{1}{2}\varpi_F\|\tilde{\Phi}_F\|^2 - \sum_{i=2}^n\tilde{\Delta}_i^T\left(\varepsilon_i - \frac{7+r_2}{2}I_1\right)\tilde{\Delta}_i + R_n \\ &\quad - \tilde{\Delta}_1^T\left(\varepsilon_1 - \frac{8+r_2}{2}I_1\right)\tilde{\Delta}_1 + \sum_{i=1}^n\left[\hbar_i H(\tau_i) + 1\right]\dot{\tau}_i \\ &\quad + \sum_{i=1}^n\left(\frac{1}{r_1} + \frac{\bar{\varepsilon}^2}{2r_2}\right)\|L_i^{-1}\|^2\delta_i^2\|\tilde{\Phi}_i\|^2 - \sum_{i=2}^n\omega_i^T\left(h_i^{-1} - I_1\right)\omega_i \\ &\le -Z^T AZ - \sum_{i=1}^n\epsilon_i e_i^T e_i - \tilde{\Delta}_1^T\left(\varepsilon_1 - \frac{8+r_2}{2}I_1\right)\tilde{\Delta}_1 \\ &\quad - \frac{1}{2}\varpi_F\|\tilde{\Phi}_F\|^2 - \sum_{i=2}^n\tilde{\Delta}_i^T\left(\varepsilon_i - \frac{7+r_2}{2}I_1\right)\tilde{\Delta}_i + R_n \\ &\quad - \sum_{i=1}^n\left(\frac{1}{2}\varpi_i - \left(\frac{1}{4} + \frac{1}{r_1} + \frac{\bar{\varepsilon}^2}{2r_2}\right)\delta_i^2\|L_i^{-1}\|^2\right)\|\tilde{\Phi}_i\|^2 \\ &\quad - \sum_{i=2}^n\omega_i^T\left(h_i^{-1} - I_1\right)\omega_i + \sum_{i=1}^n\left[\hbar_i H(\tau_i) + 1\right]\dot{\tau}_i,\end{aligned} \quad (68)$$

where $R_n = R_{n-1} + 1/2\varpi_n\|\Phi_n^*\|^2 + \bar{l}_n^2$, $\epsilon_n = q_n - 3/2I_1 - 1/2\|h_n^{-1}\|^2 I_1$.

3.3. Stability Analysis. Hereto, the following theorem is proposed to reveal the main results of this work.

Theorem 1. *Consider a class of MIMO nonlinear systems (1) satisfying assumption 1–3. The state observer is designed as (10), and the NDOs are developed as (16-17), and the virtual control laws are proposed as (31), (43), and (53). Under the presented output feedback FTC scheme (62), all tracking error signals of the closed-loop system are uniformly ultimately bounded.*

Proof. From (68), it is observed that

$$\dot{V}_n \le -sV_n + \Gamma + \sum_{i=1}^n\left[\hbar_i H(\tau_i) + 1\right]\dot{\tau}_i, \quad (69)$$

where s and Γ are given by

$$s=\min\left\{\begin{array}{c}\frac{\lambda_{\min}(A)}{\lambda_{\max}(P)},2\lambda_{\min}(\epsilon_i),\lambda_{\min}(2\varepsilon_1-(8+r_2)I_1),\frac{\varpi_F}{\lambda_{\max}\left(\Omega_F^{-1}\right)},\\ \lambda_{\min}(2\varepsilon_1-(7+r_2)I_1),2\lambda_{\min}\left(h_i^{-1}-I_1\right),\\ \frac{\varpi_i-\left(1/2+2/r_1+\bar{\varepsilon}^2/r_2\right)\delta_i^2\left\|L_i^{-1}\right\|^2}{\lambda_{\max}\left(\Theta_i^{-1}\right)}\end{array}\right\},$$

$$\begin{aligned}\Gamma={}&\sum_{i=1}^{n}\left(1+\frac{1}{2}\bar{\varepsilon}^2\right)\beta_i^2\left\|L_i^{-1}\right\|^2\left\|\Phi_i^*\right\|^2+\sum_{i=1}^{n}\left(1+\frac{1}{2}\bar{\varepsilon}^2\right)\bar{\lambda}_i^2\\&+\frac{1}{2}\varpi_F\left\|\Phi_F^*\right\|^2+\frac{1}{2}\bar{\sigma}_F^2\left\|Q_F^{-1}\right\|^2+\bar{d}_1^2+2\varsigma_0^2\\&+\sum_{i=1}^{n}\frac{1}{2}\varpi_i\left\|\Phi_i^*\right\|^2+\sum_{i=2}^{n}\bar{l}_i^2+\frac{1}{2}\sum_{i=1}^{n}\varrho_i^2.\end{aligned} \tag{70}$$

According to Lemma 1, it can be obtained that V_n and $\sum_{i=1}^{n}[\hbar_i H(\tau_i)+1]\dot{\tau}_i$ are bounded over $[0,t_f]$. By defining $\Gamma_1=\max_{t\in[0,t_f]}\sum_{i=1}^{n}[\hbar_i H(\tau_i)+1]\dot{\tau}_i$, we obtain

$$\dot{V}_n\leq -sV_n+\bar{\Gamma}, \tag{71}$$

where $\bar{\Gamma}=\Gamma+\Gamma_1$.

Integrating (71) over $[0,t]$ yields

$$V_n\leq\frac{\bar{\Gamma}}{s}+\left(V_n(0)-\frac{\bar{\Gamma}}{s}\right)e^{-st}. \tag{72}$$

From (72) and the definition of V_n, we obtain that V_n is exponentially convergent, i.e., $\lim_{t\longrightarrow\infty}V_n=\bar{\Gamma}/s$. Hence, all the error signals of the closed-loop remain bounded. This concludes the proof. □

Remark 4. In this paper, the actuator faults, which are coupled with the unknown control gains, are considered, and the Nussbaum function is utilized to eliminate their adverse effect. Compared with some direct adaptive method [11], the developed control algorithm can effectively avoid the problem of singularity and simplify the proof process. Compared with some existing observer-based output feedback control methods [18, 20, 43], the proposed robust FTC scheme can guarantee the closed-loop stability for the MIMO nonlinear system with multiple negative effects simultaneously, including unmeasured states, unknown nonlinearity, unknown control directions, actuator faults, and external disturbances.

Remark 5. In the control design, different design parameters will result in different system performance. Choosing larger ϵ_i will increase the control gains and is beneficial for tracking results within bounds. However, excessive control gains require more control energy and may cause input saturation. Consequently, in order to achieve applicable control performance, the selection of corresponding design parameters should satisfy the following conditions and reach a compromise:

$$\begin{aligned}&A>0,\epsilon_i>0,2\varepsilon_1-(8+r_2)I_1>0,h_i^{-1}-I_1>0,\\&\varpi_F>0,\varpi_i-\left(\frac{1}{2}+\frac{2}{r_1}+\frac{\bar{\varepsilon}^2}{r_2}\right)\delta_i^2\left\|L_i^{-1}\right\|^2>0.\end{aligned} \tag{73}$$

4. Simulation Examples

In this section, simulations studies on the unmanned helicopter position movement are given to illustrate the feasibility of the obtained results. The position dynamic model of unmanned helicopter with actuator faults and external disturbances can be described as [44]

$$\begin{aligned}\dot{P}&=V,\\ m\dot{V}&=B\vartheta F_R+mg\xi+D(t),\\ y&=P,\end{aligned} \tag{74}$$

where g is the gravitational acceleration, $\xi=[0,0,1]^T$, and m defines the gross mass of the unmanned helicopter. $P=[X,Y,Z]^T$ and $V=[u,v,w]^T$ denote the position and velocity vectors, respectively. $B=\mathrm{diag}\{B_1,B_2,B_3\}$ is the unknown constant control gain matrix, $\vartheta=\mathrm{diag}\{\vartheta_1,\vartheta_2,\vartheta_3\}$, ϑ_i is the remaining control efficiency, $D(t)$ is external time-varying unknown disturbance, $F_R=FR$, $F=[F_x,F_y,F_z]^T$ is the control input, and R refers to the rotation matrix whose definition can be found in [44].

In the simulation, we suppose that the unmanned helicopter is operated in constant attitude angles. Furthermore, the initial positions are assumed as $[X_0,Y_0,Z_0]^T=[1,1,0]^T$ m. Here, the unmanned helicopter is required to track the following desired trajectories

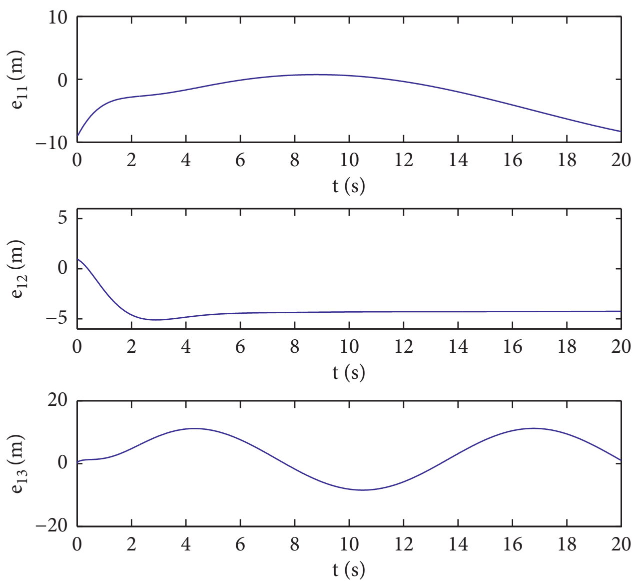

Figure 2: Tracking errors e_{1i} without robust FTC.

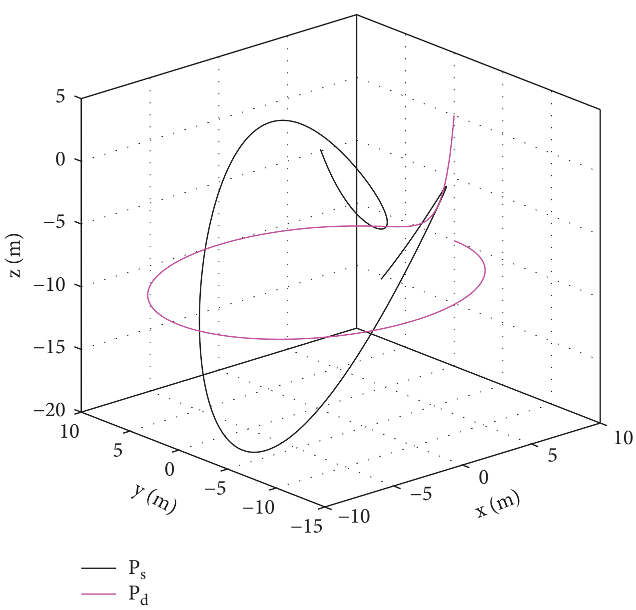

Figure 3: Tracking trajectories y and y_d without robust FTC.

$$\begin{cases} X_d = 10\cos\left(\dfrac{\pi}{10}t\right)m, \\ Y_d = 10\sin\left(\dfrac{\pi}{10}t\right)m, \\ Z_d = -10\left(1 - e^{-3t}\right)m, \end{cases} \tag{75}$$

The external time-varying disturbances are given by

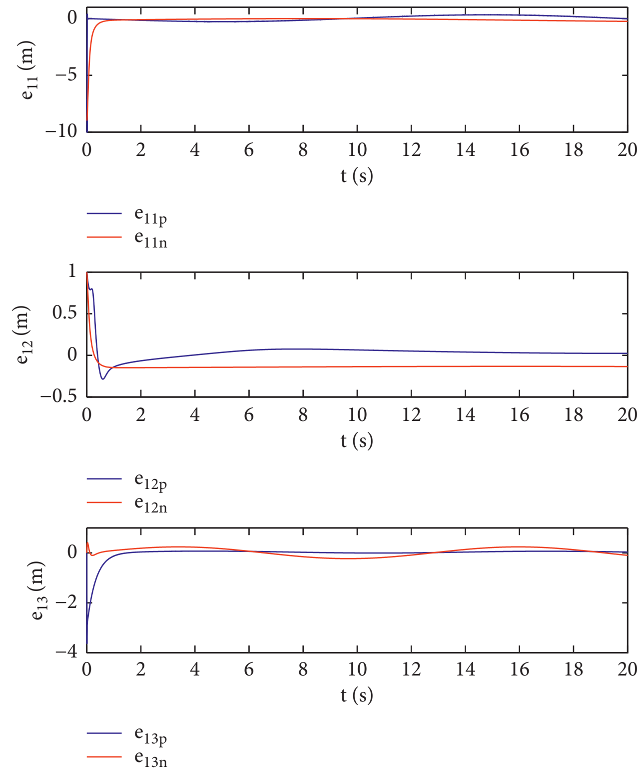

Figure 4: Contrastive results of tracking errors under proposed controller and adaptive FTC method [11].

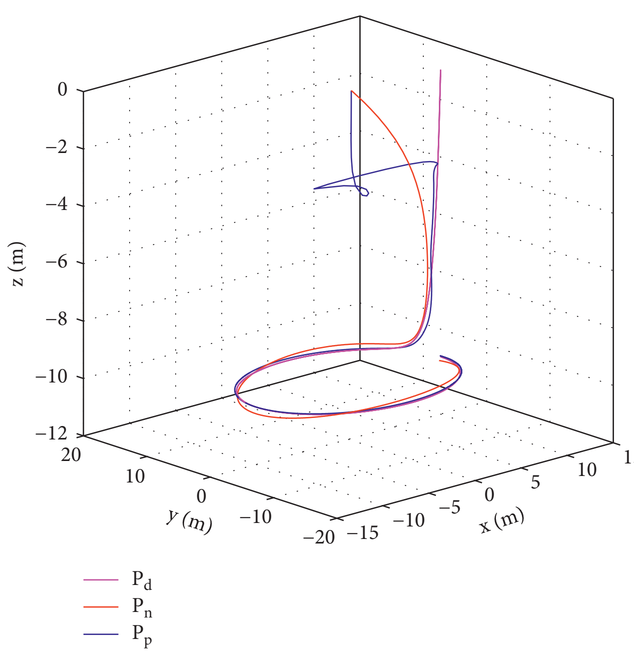

Figure 5: Contrastive results of output y under the proposed controller and the adaptive FTC method [11].

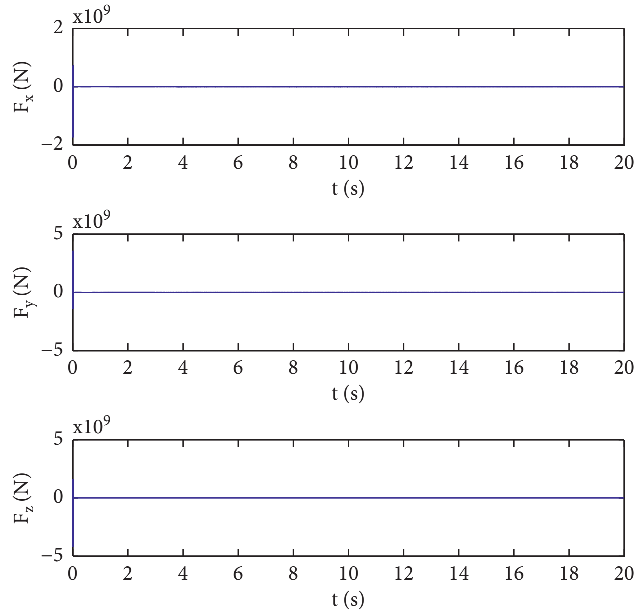

Figure 6: Input signals of the proposed controller.

$$D = \begin{bmatrix} 10\sin(0.2t) \\ 1.2\sin(0.1t) \\ 15\sin(0.5t) \end{bmatrix}. \tag{76}$$

In view of the analysis process in Section 3, the control object is to design control input vector F to make the output y track the desired trajectory y_d as closely as possible. The basic parameters of the unmanned helicopter are selected as $m = 800$ kg, $g = 9.8$, $\phi = 10^\circ$, $\theta = 15^\circ$ and $\psi = 20^\circ$. The control gains and actuator faults are assumed as $B = \text{diag}\{1.1, 1.2, 1.3\}$ and $\vartheta = \text{diag}\{0.8, 0.7, 0.6\}$, respectively. The relevant designed parameters are selected as $q_1 = \text{diag}\{5, 5, 5\}$, $q_2 = \text{diag}\{1000, 200, 500\}$, $k_1 = \text{diag}\{2, 2, 2\}$, $k_2 = \text{diag}\{1, 1, 1\}$, $\chi_1 = \text{diag}\{2, 2, 2\}$, $\chi_2 = \text{diag}\{1, 1, 1\}$, $\varpi_1 = 0.001$, $L_1 = \text{diag}\{20, 20, 20\}$.

First, the position tracking error $e_1 = y - y_d$ without robust FTC is shown in Figure 2. It can be noted that all tracking errors deviate from the origin severely. To reflect the movement of the helicopter more visually, Figure 3 is presented, where the magenta lines P d represent the desired trajectories, and the black lines P s are the actual outputs. From Figure 3, it can be seen that if there is no effective robust FTC scheme, satisfactory trajectory tracking performance cannot be obtained. Simulation results of Figures 2 and 3 reveal that the actuator faults, which are coupled with the unknown control directions and external disturbances, have a great negative influence on the control performance of the helicopter system. If these negative effects can not be tackled timely, they will lead to degradation of system performance and even catastrophic consequences.

In order to ensure the safe flight for the unmanned helicopter with unavailable states, unknown control directions, actuator faults, and external disturbances, a robust adaptive NN output feedback FTC scheme is developed, and the corresponding simulations are given by Figures 4–7.

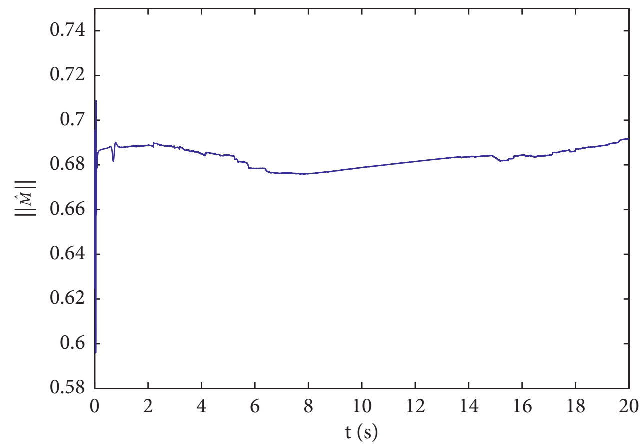

Figure 7: The norm of the RBFNN weight matrix estimation $\widehat{M}$.

Figure 4 provides the contrastive results of tracking errors, where the blue lines e_{1ip} denote the tracking errors under the proposed controller, and the red lines e_{1in} denote these under adaptive FTC strategy proposed in [11]. According to Figure 4, it can be observed that both control schemes can guarantee that the tracking errors converge to a small boundary. However, compared with the adaptive FTC strategy proposed in [11], the developed control algorithm has faster convergence rate and smaller tracking error. Meanwhile, the three-dimensional graph of the output trajectory y is provided in Figure 5, where the blue lines P p denote the system output under the proposed controller, and the red lines P n refer to these under adaptive FTC strategy proposed in [11]. From Figure 5, we can see that the presented control scheme can make the system output reach the desired trajectory faster, which reflects the same conclusion as Figure 4. The curve of designed fault tolerant controller is provided in Figure 6. Figure 6 manifests that the control command signals are convergent and can adjust dynamically to restrain the adverse impacts. Furthermore, the RBFNN weight matrix changes in reasonable boundary as time goes on, which is displayed in Figure 7.

5. Conclusion

In this paper, a NN-based adaptive output feedback FTC scheme has been proposed for uncertain MIMO nonlinear systems with unmeasured states, unknown control directions, actuator faults, and external disturbances. The RBFNN has been utilized to approximate the unknown nonlinear function, and the NDO has been constructed to estimate the external disturbances. Especially, the unknown actuator faults and control directions have been handled by the constructed Nussbaum function. The stability of the closed-loop system has been proved, and the performance of the presented controller has been confirmed through simulation results on unmanned helicopter position movement. In the future, the novel robust FTC strategy for the variable gain nonlinear systems with random faults and disturbances is worthy of further exploration, which are the other common difficulties in practice.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 62103315, the Industrial Project from Science and Technology Department of Shaanxi Province under Grant 2022GY-243, and the Special Project from Education Department of Shaanxi Province under Grant 21JK0686.

References

[1] M. Chen, P. Shi, and C.-C. Lim, "Robust constrained control for MIMO nonlinear systems based on disturbance observer," *IEEE Transactions on Automatic Control*, vol. 60, no. 12, pp. 3281–3286, 2015.

[2] B. Xiao, X. Yang, and X. Huo, "A novel disturbance estimation scheme for formation control of ocean surface vessels," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 6, pp. 4994–5003, 2017.

[3] Z. Li, J. Deng, R. Lu, Y. Xu, J. Bai, and C.-Y. Su, "Trajectory-tracking control of mobile robot systems incorporating neural-dynamic optimized model predictive approach," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 6, pp. 740–749, 2016.

[4] H. Yang, B. Jiang, H. Yang, and H. H. T. Liu, "Synchronization of multiple 3-DOF helicopters under actuator faults and saturations with prescribed performance," *ISA Transactions*, vol. 75, pp. 118–126, 2018.

[5] X.-J. Li and G.-H. Yang, "Adaptive decentralized control for a class of interconnected nonlinear systems via backstepping approach and graph theory," *Automatica*, vol. 76, pp. 87–95, 2017.

[6] X. Cao, P. Shi, Z. Li, and M. Liu, "Neural-network-based adaptive backstepping control with application to spacecraft attitude regulation," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 9, pp. 4303–4313, 2018.

[7] K. Yan, M. Chen, Q. Wu, and R. Zhu, "Robust adaptive compensation control for unmanned autonomous helicopter with input saturation and actuator faults," *Chinese Journal of Aeronautics*, vol. 32, no. 10, pp. 2299–2310, 2019.

[8] S. Tong, L. Zhang, and Y. Li, "Observed-based adaptive fuzzy decentralized tracking control for switched uncertain nonlinear large-scale systems with dead zones," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 1, pp. 37–47, 2016.

[9] X.-G. Yan, S. K. Spurgeon, and C. Edwards, "Memoryless static output feedback sliding mode control for nonlinear systems with delayed disturbances," *IEEE Transactions on Automatic Control*, vol. 59, no. 7, pp. 1906–1912, 2014.

[10] K. Yan, M. Chen, Q. Wu, Y. Wang, and R. Zhu, "Prescribed performance fault tolerant control for uncertain nonlinear systems with input saturation," *International Journal of Systems Science*, vol. 51, no. 2, pp. 258–274, 2020.

[11] B. Jiang, D. Xu, P. Shi, and C. C. Lim, "Adaptive neural observer-based backstepping fault tolerant control for near space vehicle under control effector damage," *IET Control Theory & Applications*, vol. 8, no. 9, pp. 658–666, 2014.

[12] Z. Li, H. Xiao, C. Yang, and Y. Zhao, "Model predictive control of nonholonomic chained systems using general projection neural networks optimization," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 45, no. 10, pp. 1313–1321, 2015.

[13] C.-F. Hsu and C.-W. Chang, "Intelligent dynamic sliding-mode neural control using recurrent perturbation fuzzy neural networks," *Neurocomputing*, vol. 173, pp. 734–743, 2016.

[14] M. Ran, Q. Wang, and C. Dong, "Active disturbance rejection control for uncertain nonaffine-in-control nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5830–5836, 2017.

[15] M. Ran, Q. Wang, and C. Dong, "Stabilization of a class of nonlinear systems with actuator saturation via active disturbance rejection control," *Automatica*, vol. 63, pp. 302–310, 2016.

[16] Y.-J. Liu and S. Tong, "Adaptive fuzzy control for a class of unknown nonlinear dynamical systems," *Fuzzy Sets and Systems*, vol. 263, pp. 49–70, 2015.

[17] B. Xu, F. Sun, Y. Pan, and B. Chen, "Disturbance observer based composite learning fuzzy control of nonlinear systems with unknown dead zone," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 1854–1862, 2017.

[18] S. Tong, B. Huo, and Y. Li, "Observer-based adaptive decentralized fuzzy fault-tolerant control of nonlinear large-scale systems with actuator failures," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 1, pp. 1–15, 2014.

[19] M. Chen and S. S. Ge, "Adaptive neural output feedback control of uncertain nonlinear systems with unknown hysteresis using disturbance observer," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 12, pp. 7706–7716, 2015.

[20] Y. Li, Y. Liu, and S. Tong, "Observer-based neuro-adaptive optimized control of strict-feedback nonlinear systems with state constraints," *IEEE Transactions on Neural Networks and Learning Systems*, pp. 1–15, 2021.

[21] H. Wang, P. X. Liu, and P. Shi, "Observer-based fuzzy adaptive output-feedback control of stochastic nonlinear multiple time-delay systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2568–2578, 2017.

[22] S. Tong, T. Wang, Y. Li, and H. Zhang, "Adaptive neural network output feedback control for stochastic nonlinear systems with unknown dead-zone and unmodeled dynamics," *IEEE Transactions on Cybernetics*, vol. 44, no. 6, pp. 910–921, 2014.

[23] H. Du, Y. He, and Y. Cheng, "Finite-time synchronization of a class of second-order nonlinear multi-agent systems using output feedback control," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 61, no. 6, pp. 1778–1788, 2014.

[24] Y. Li, S. Tong, L. Liu, and G. Feng, "Adaptive output-feedback control design with prescribed performance for switched nonlinear systems," *Automatica*, vol. 80, pp. 225–231, 2017.

[25] Y. Li, S. Tong, and T. Li, "Hybrid fuzzy adaptive output feedback control design for uncertain MIMO nonlinear systems with time-varying delays and input saturation," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 4, pp. 841–853, 2016.

[26] S. Shao-Cheng Tong, X. Xiang-Lei He, and H. Hua-Guang Zhang, "A combined backstepping and small-gain approach

to robust adaptive fuzzy output feedback control," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 5, pp. 1059–1069, 2009.

[27] J. Huang, Y. Song, W. Wang, C. Wen, and G. Li, "Fully distributed adaptive consensus control of a class of high-order nonlinear systems with a directed topology and unknown control directions," *IEEE Transactions on Cybernetics*, vol. 48, no. 8, pp. 2349–2356, 2018.

[28] B. Huo, Y. Xia, L. Yin, and M. Fu, "Fuzzy adaptive fault-tolerant output feedback attitude-tracking control of rigid spacecraft," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 1898–1908, 2017.

[29] R. D. Nussbaum, "Some remarks on a conjecture in parameter adaptive control," *Systems & Control Letters*, vol. 3, no. 5, pp. 243–246, 1983.

[30] W. Wang, D. Wang, Z. Peng, and T. Li, "Prescribed performance consensus of uncertain nonlinear strict-feedback systems with unknown control directions," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 9, pp. 1279–1286, 2016.

[31] H. Rezaee and F. Abdollahi, "Adaptive consensus control of nonlinear multiagent systems with unknown control directions under stochastic topologies," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 8, pp. 3538–3547, 2018.

[32] L. Liu, Z. Wang, Z. Huang, and H. Zhang, "Adaptive predefined performance control for MIMO systems with unknown direction via generalized fuzzy hyperbolic model," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 3, pp. 527–542, 2017.

[33] Y. Li, S. Tong, and T. Li, "Observer-based adaptive fuzzy tracking control of MIMO stochastic nonlinear systems with unknown control directions and unknown dead zones," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 4, pp. 1228–1241, 2015.

[34] S. Zeghlache, H. Mekki, A. Bouguerra, and A. Djerioui, "Actuator fault tolerant control using adaptive RBFNN fuzzy sliding mode controller for coaxial octorotor UAV," *ISA Transactions*, vol. 80, pp. 267–278, 2018.

[35] C. Hu, X. Zhou, B. Sun, W. Liu, and Q. Zong, "Nussbaum-based fuzzy adaptive nonlinear fault-tolerant control for hypersonic vehicles with diverse actuator faults," *Aerospace Science and Technology*, vol. 71, pp. 432–440, 2017.

[36] D. Xu, B. Jiang, and P. Shi, "Robust NSV fault-tolerant control system design against actuator faults and control surface damage under actuator dynamics," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 9, pp. 5919–5928, 2015.

[37] B.-Z. Guo, Z.-H. Wu, and H.-C. Zhou, "Active disturbance rejection control approach to output-feedback stabilization of a class of uncertain nonlinear systems subject to stochastic disturbance," *IEEE Transactions on Automatic Control*, vol. 61, no. 6, pp. 1613–1618, 2016.

[38] D. Lee, "Nonlinear disturbance observer-based robust control for spacecraft formation flying," *Aerospace Science and Technology*, vol. 76, pp. 82–90, 2018.

[39] Y. Zou, "Adaptive trajectory tracking control approach for a model-scaled helicopter," *Nonlinear Dynamics*, vol. 83, no. 4, pp. 2171–2181, 2016.

[40] M. Chen, S. S. Ge, and B. Ren, "Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints," *Automatica*, vol. 47, no. 3, pp. 452–465, 2011.

[41] Q. Hu, X. Shao, and L. Guo, "Adaptive fault-tolerant attitude tracking control of spacecraft with prescribed performance," *IEEE*, vol. 23, no. 1, pp. 331–341, 2018.

[42] S. S. Ge, F. Hong, and T. H. Lee, "Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients," *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics)*, vol. 34, no. 1, pp. 499–516, 2004.

[43] S. Tong, S. Sui, and Y. Li, "Fuzzy adaptive output feedback control of MIMO nonlinear systems with partial tracking errors constrained," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 4, pp. 729–742, 2015.

[44] K. Yan, M. Chen, B. Jiang, and Q. Wu, "Robust adaptive active fault-tolerant control of UAH with unknown disturbances and actuator faults," *International Journal of Adaptive Control and Signal Processing*, vol. 33, no. 4, pp. 684–711, 2019.