

Research Article

Metaverse, SED Model, and New Theory of Value

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The metaverse concept constructs a virtual world parallel to the real world. The social economic dynamics (SED) model establishes a systematic model for social economic dynamics simulation that integrates macroeconomy and microeconomy based on modeling mechanism of the new theory of value by analogy with Newtonian mechanics and the modeling approach of Agent-based computational economics. This article describes the SED model's modeling mechanisms, modeling rules, and behavior equations. At the same time, this article introduces the methods, testing standards, and some typical cases about using the SED model to generate the economic digital twin systems. By doing so, we hope to demonstrate that the method of computer simulation experiment based on the SED model is a scientific empirical method, which has more advantages than the existing empirical research methods in economics. The SED model, which can be fully used to form an economic engine and construct a virtual economic system by digital twin method, can be integrated with the extant physical engine in the metaverse concept to build a virtual world consisting of physics, economy, culture, and politics that is close to and coexists with reality.

1. Introduction

The term “metaverse” was first put forward by Neil Stephenson in his science fiction novel named “Snow Crash” in 1992. In this novel, the metaverse is known as a computer-generated universe parallel to the real world, where the users can enter as avatars by putting on goggles and earphones and connecting to the terminal [1]. On March 10th 2021, Roblox floated on the stock market, popularizing the concept of metaverse described as sustainable, shared 3D virtual spaces in the virtual universe in Roblox's prospectus. The Roblox platform has 8 critical features of the metaverse: identity, friends, immersive, anywhere, low friction, variety of content, economy, and safety. Since then, many tech giants have followed the example of Roblox to invest in the metaverse. For example, in April 2021, the chipmaker Nvidia released Omniverse, a virtual

working platform for the metaverse; in October 2021, the social network company Facebook announced that it had changed its company name to Meta, reflecting its growing ambitions towards the metaverse. While there is currently no unified definition of the word “metaverse,” academia and industry generally believe that the metaverse is a permanent, shared, open, and virtual space that blends with the real world. According to this general interpretation, the metaverse is not only a virtual world, a virtual space, a virtual reality, a digital and virtual economy, a game, a virtual theme park or Disneyland, a new app store, or a new user-generated content (UGC) platform [2]. The concept and vision of the metaverse will not only stimulate the advancement of related digital technologies but also accelerate their integration with the industries of military, real estate, manufacturing, art, education, etc. in addition to being used in social networking and entertainment [3].

According to Lee et al. [4], the development of the metaverse towards the fusion of virtual and reality needs to go through three progressive stages: digital twins, digital natives, and coexistence of physical-virtual reality. Digital twins in the first stage are large-scale and high-fidelity digital models capable of perceiving and mirroring the properties of physical entities or characteristics of human society. Based on digital twins, digital natives in the second stage concentrate on the native contents that are related to the real world or only exist in the virtual world. In the third stage of physical-virtual reality coexistence, the metaverse will become a self-sustaining and long-lasting virtual world that can interconnect multiple virtual worlds within it and blend with the real world. The concept of metaverse provides a promising vision of the development of a future virtual world. However, the current development of the metaverse is in a nascent stage, which requires constant progress in the underlying technology and improvement in the ecosystem. The pillars of the underlying technology consist of computer hardware technology (user interactivity, extended reality, robotics/IoT, edge/cloud computing, network, and hardware infrastructure) and software technology (computer vision, artificial intelligence, and blockchain), while the pillars of the ecosystem include avatars, virtual economy, content creation, social acceptability, security and privacy, trust, and responsibility.

Just like Karl Marx's famous assertion that "economic base determines the superstructure" [5], the economic system is also the cornerstone of the ecosystem in the metaverse. Like the real economic system, the metaverse economic system is also a complex economic system consisting of large-scale heterogeneous adaptive individual agents (consumers, producers, investors, etc.) whose interactions co-create the emergent aggregate behaviors [6, 7]. However, in the different development stages of the metaverse, its virtual economic system presents different characteristics. Unlike the digital twin economy in the first stage, a copy of the real economy, the digital native economy of the second state, and the economy of the third stage with the physical and virtual integration exhibit unique characteristics. Firstly, the economic agents do not merely refer to computer agents that obey the computerized rules of behaviors but to real human subjects who concurrently appear in the metaverse as avatars; secondly, the commodities in the metaverse include both valuable commodities that have their counterpart in the real economy and those commodities that are created in the virtual world, such as digital assets verified through non-fungible tokens (NFTs); thirdly, cryptocurrencies (note that cryptocurrencies are not legal tender, whose basic monetary functions, namely, value measure, means of payment, means of circulation, means of hoarding, and international currencies, may not necessarily be protected by laws of the country to which they belong, but even expressly prohibited or restricted by the laws) in the metaverse, such as Bitcoin and Ethereum, are not issued or supported by a central government or bank in the current world we reside, but technically backed by a peer-to-peer, decentralized blockchain supported by all users in the community [8]. In addition, unlike the money supply that can be freely controlled and adjusted by central banks and financial institutions, the cryptocurrency

supply is stable and slow-growing [4]. Thus, the metaverse economic system is closer to the liberal market economy system claimed by the Austrian School [9]. At present, "metaverse" has a physical engine that can virtualize machines, social networks, games, office environments, and various artificial intelligence products such as autopilot, smart medicine, smart transportation, and smart travel. However, "metaverse" still lacks a powerful economic engine and thus can neither mirror the real economic scenarios nor assist humans in scientific economic decisions. The social economic dynamics (SED) model can complement this function by generating a systematic and dynamic economic digital twin system close to various special economic scenarios to constitute the top layer of the future metaverse. Most importantly, the SED model is an economic engine with a deductive function established according to the principles of economics, which can generate an optimized economic digital twin system approaching the realistic global economic system with profit maximization as its guidance. The SED model can play a vital role in economic planning, forecasting, simulation deduction, virtual trial and error, intelligent decision support, storage optimization scheme, automatic upgrade of intelligent knowledge base, etc. Of course, as an economic engine run on a supercomputer, the SED model requires a significant number of objective calibrated parameters to generate the digital twin system approximating to reality. However, in the existing methods, these parameters obtained by statistical means suffer from various drawbacks such as time lags, information costs, information disconnections, and statistical errors. In the future, SED models can eliminate these drawbacks if they can complement the functional advantages of the current physical engines in the metaverse. At that time, the metaverse will have the function of endogenous intelligent optimization and gradually enter the highest realm of "the unity of heaven and man" in which the physical world and human intelligence are integrated (the relationship between the SED model and the metaverse is so complex that the in-depth and systematic analysis can only be established using the method of scientific theoretical paradigm, namely, mathematical economics; due to the limitations of the research focus and the length of the paper, the authors will not discuss it here for the time being; the authors are ready to delve into this important topic in a separate article).

Throughout history, countless economists have made ongoing explorations on the modeling of economic systems. Many mainstream macroeconomic theoretical models, such as the large-scale macroeconomic model, computable general equilibrium model (CGE model), and dynamic stochastic general equilibrium model (DSGE model), have achieved many theoretical and applied results. However, they have not yet met the requirements of modeling complex economic systems in the metaverse. These models constitute simultaneous equations based on Keynesian macroeconomic theory, neoclassical general equilibrium theory, and neoclassical/neo-Keynesian macroeconomic theory and can be applied to economic structure analysis, economic forecasting, and policy evaluation after model estimation and calibration [10]. However, the traditional Keynesian theory lacks microfoundations, and the neoclassical and neo-

Keynesian theories are usually characterized by homogenous agents. Consequently, macroeconomic, CGE, and DSGE models fail to depict heterogeneous microagents' behaviors directly so that their simulation results can not correspond to the microscenarios. In addition, the CGE and DSGE models usually assume that the microagents have perfect rationality or rational expectation, which does not reflect the real-world agents' bounded rationality or adaptive behaviors [11]. Although in recent years the DSGE model has been expanded to account for agents' heterogeneity and bounded rationality [12, 13], the degrees of heterogeneity and types of rationality of agents in these new DSGE models are mild [14], which in turn constrains its ability to reflect microscopic scenes. Finally, the CGE and DSGE models are equilibrium or dynamic equilibrium models, ignoring the dynamic adjustment processes of disequilibrium in the real world. Although the DSGE model features a dynamic adjustment towards the long-term equilibrium, this process is usually a result of exogenous disturbances, which ignores the inherent instability of the economic system caused by the non-linear interaction of economic agents [15]. Consequently, it lacks strong explanatory and predictive power about economic crashes or the bubbles [16, 17].

Agent-based computational economics (ACE) is the computational study of economies modeled as evolving systems of autonomous interacting agents from the bottom up [18]. The ACE approach can also model the metaverse economic systems characterized primarily by cryptocurrencies [19]. Three critical properties of ACE, namely, autonomous agents, social interaction, and macro-micro integration, are not only consistent with the connotations of complex economics pioneered by the Santa Fe Institute [20] but also pertinent to the ontological turn of economics advocated by many economic philosophers [21, 22]. The early representative ACE models were the ASPEN model developed by Sandia National Laboratory in the 1990s [23] and the EURACE model developed jointly by seven European universities in 2006. At present, the existing large-sized and medium-sized macro-ACE models can be divided into seven categories: (1) AGH model [24]; (2) CATS model (complex adaptive trivial systems model) [25]; (3) EUBI model (Eurace@Unibi model) derived from EURACE [26, 27]; (4) EUGE model, which is the frame model of EURACE [28]; (5) JAMEL model (Java agent-based macroeconomic laboratory model) [29]; (6) KS model ("Keynes Meeting Schumpeter" frame) [30]; and (7) LAGOM model [31]. These models introduce bounded rationality and endogenous market disequilibrium instead of perfect rationality and market equilibrium assumed in mainstream macroeconomics and establish a macro-micro integration model that is based on heterogeneous and autonomous agents. However, they often do not establish any new systematic and scientific economic theory, especially related to the theory of value, as the modeling basis, but instead adopt heuristics or rules of thumb based on experimental and empirical evidence [32]. In addition, owing to the lack of a suitable adjustment mechanism towards global equilibrium, these models suffer from inexplicable divergence problems [33], and the simulation results cannot

mirror the economic reality and generate digital twins system, but only reflect general economic law or trend (until now, apart from the SED model, only macroeconomic ABM models proposed by Polendna et al. [34] can compete with mainstream VAR and DSGE models in the forecasting accuracy of aggregate macroeconomic variables; however, the model is not comparable to the SED concerning size and complexity).

Nikola Tesla had argued, "Man, in the large, is a mass urged on by a force. Hence, the general laws governing movement in the realm of mechanics are applicable to humanity (excerpt from an article "A Machine to End War" by Nikola Tesla, published in Liberty magazine in February 1935)." This viewpoint can be traced back to the mechanistic materialist, such as Hobbes [35] and La Mettrie [36]. Although the forms and laws of the motion of matter they stressed are conducive for studying complex economic systems, we cannot simply equate man with machine or matter and ignore human subjective initiative or intelligence [37]. Since the mid-1990s, Chinese scholar Wu et al. have spent more than twenty years in establishing, developing, and maintaining a super-large, highly interacted agent-based social economic dynamics simulation model (SED), which can simulate the daily dynamics of more than 10 million agents and economic performances of hundreds of countries interacted through international trades and investments. The SED model is at the world's advanced level in terms of size, fidelity, speed, and operability. Compared with other macroeconomic ACE models, the SED model has the following characteristics and advantages. Firstly, the SED model is a model of economic mechanics using the innovative new theory of value as the modeling mechanism [38–40]. The new theory of value combines the labor theory and the utility theory of value and makes an analogy with Newtonian mechanics while fully considering the human subjective initiative or intelligence. It analyzes dynamic value calculation and pricing in market equilibrium and non-equilibrium under the invariable and variable labor productivity using the corresponding Euclidean or Riemann metric. Secondly, the SED model has a dynamic control mechanism backed up by the bottom-up free market and top-down government regulation to ensure global equilibrium and optimization. It can not only simulate the dynamics of various types of market economies but also ensure the stability of model operation. Thirdly, the SED model adopts the mechanism-based, rule-based, and data-based modeling approaches. Therefore, it allows for great flexibility in conducting a variety of simulation experiments, including simulation of completely rational agents, simulation of boundedly rational agents, and simulation of exogenous specific scenarios. Fourthly, the SED model with highly heterogeneous and interacted agents is run using high-performance parallel computing technology on the Tianhe-2 supercomputer located at the National Supercomputer Center in Guangzhou, China. The platform can be accessed through an independent website portal (<https://www.gzms.com/website/>) and has a user-friendly interface. These characteristics and advantages enable the SED model to simulate many stylized facts in different economies at the

macro, meso, and microlevels and also approximate the real economy, giving rise to the economic digital twin system and establishing the foundation for metaverse economic system. In recent years, the SED model has been applied to simulate the impacts of international events such as the Sino-US trade war on the global economy [33, 41], which catches the attention of international colleagues.

This paper will illustrate the modeling mechanism, modeling rules, and behavior equations of the SED model, as well as the methods, testing standards, and some typical cases of using the SED model to generate China's economic digital twin system. Other sections are organized as follows. In Section 2, we summarize the SED model's theoretical basis, modeling method, and control mechanism of global equilibrium. In Section 3, we describe in detail the modeling mechanisms of the SED model, such as the new theory of value and linear programming model of the commodity economy. Section 4 is devoted to a detailed description of the economic digital twin system based on the SED model. In Section 5, we present the methods, testing standards, and some typical cases of using the SED model to generate China's economic digital twin system. Section 6 concludes the paper.

2. Theoretical Basis, Modeling Method, and Control Mechanism of SED Model

The SED model is an economic simulation system covering international, national, and regional economies and can simulate economic performances of more than 200 countries and all regions inside each country. The SED national model consists of micro-subsystem, meso-subsystem, and macro-subsystem. The micro-subsystem has complete, systematic, and abundant heterogeneous near-reality economic agents, including thousands of enterprises, banks, shops, and securities that provide dozens of products or services with dozens of different grades, residents with different preferences, and the government with nearly ten different functional departments. The meso-subsystem comprises 42 different industries and 6 different commodities and financial markets, and commodities are exchanged between enterprises in different industries and between enterprises and residents through the corresponding market. Macro-subsystems influence meso-subsystems and micro-subsystems through macroeconomic policies and interact with every microeconomic agent. At present, the basic time step in the SED model is one day, and the model can run iteratively for more than 20 years. The SED model can perform a large number of low-cost, low-latency, and realistic simulations in tens of millions of segmented special economic scenarios.

2.1. Theoretical Basis. The theoretical basis of the SED model is rooted in the new theory of value proposed by Wu [38–40]. This theory holds that commodity value is composed of labor value and utility value. Classical economists, represented by Smith, Ricardo, and Marx, believed that the value of commodities is determined by labor value, and on

this basis, they constructed the law of value, the law of supply and demand, the law of money circulation, the law of price, and the law of capital movement. However, the traditional theory of labor value does not thoroughly analyze the use value form of commodities. On the other hand, classical economists, represented by Jevons [42] and Walras [43], rejected the labor theory of value, put forward the cardinal utility theory of value, and argued that the value of commodities depends on the subjective utility evaluation of commodities, etc. However, the measurability of subjective utility has been questioned by many economists [44]. Although Hicks and Allen [45] later proposed the ordinal theory of utility to solve the problem of utility measurement encountered by classical cardinal theory of utility, the partial order relation of utility could not meet the research needs of economic theory in global optimization [38]. The new theory of value on which the SED model relies, harmonizing both the labor theory of value and the utility theory of value, puts forward a unified and dynamic measure of value and then constructs the commodity value with global optimization and total order relation structure. Further, it reveals the economic law of movement of commodities in the production, circulation, distribution, and consumption process in various socio-economic forms.

The mathematical model of the new theory of value is a fiber bundle of wealth value, consisting of 6 axiomatic assumptions, 339 definitions, and 253 theorems. The underlying economic concepts are time, quantity, and quality, and the derived dimensions of these concepts are velocity (efficiency), acceleration, the force of labor, the force of consumption, labor value, use value, exchange value, price, variable capital, constant capital, surplus value, profit, production capital, commercial capital, financial capital, national income, international trade, international finance, international capital, etc. Furthermore, through the relevant theorems that reflect the relationship among these concepts, the mathematical models of national economy integrating macroeconomy and microeconomy based on the base manifold of wealth value are constructed, including commodity production model, commodity exchange model, currency purchase model, commodity price/value model, production capital model, commercial capital model, financial capital model, government macroeconomic regulation model, etc. These models describe rational agents' behavior and economic laws in the general economic environment, which lay the theoretical foundation for the SED international, national, and regional models.

2.2. Modeling Method of SED. The SED model is an economic dynamic system model based on systematic simulation technology. The model relies on ABM, system dynamics, and discrete simulation technology and adopts an integrated modeling approach based on mechanisms, rules, and data, where economic mechanism-based modeling accounts for 70%, rule-based modeling accounts for 20%, and data-based modeling accounts for 10%.

Mechanism-based modeling is based on the deduction methodology, including axiom assumptions, theorem

reasonings, and case proof. The modeling mechanism of SED is economic mechanics based on the new theory of value. These mechanisms, in the form of corresponding mathematical formulas, influence the behaviors of thousands of heterogeneous agents in the model, who act without a unified plan but only rely on local economic information. Each agent pursues the maximization of its wealth value alone following the principle of profit maximization, and finally the entire market economy reaches the global equilibrium and optimization under free competition and spontaneous regulation. The mechanism-based modeling of the SED model solves the problem of computer simulation for the general law of economic systems.

Rule-based modeling is based on the regulations and statutes formulated and recognized by the public or those made by the representatives and obeyed by all members. Each independent agent in the model is bound by specific rules of conduct, including state institutions, industry regulations, and enterprise system norms, which are necessarily compatible with the above mechanisms based on the new theory of value. Under different rule constraints, each agent's behavior must follow special laws of motion and thus produce different economic results. Economic rules constrain the activities of rational and irrational agents and can help simulate various complex and special economic phenomena.

Data-based modeling is based on the functional relationship estimated using statistical data and econometric techniques. Although the agents' behaviors in the model are characterized by discreteness and complexity, their aggregate results are subject to the general statistical laws in economics, such as the law of large numbers and consumption functions derived from daily statistics. Data-based modeling can simulate some phenomena that take on a stable law or that cannot be explained by the economic theories, help validate the SED model's parameters, and construct data system of initial period for simulation model.

2.3. Global Equilibrium Control Mechanism of SED. The SED model is a dynamic economic simulation model with a large size and high coupling that unifies non-equilibrium and equilibrium. In the mechanism-based model, stability is a necessary condition to ensure that the system operates according to the mechanism under normal conditions. At the same time, it is also a necessary condition to verify that, under certain conditions, the system must show abnormal phenomena. Under the influences of the "invisible hand" of the market and the "visible hand" of the government, the model operation does not only have the characteristics of Lyapunov asymptotic stability [33, 46, 47] but also achieves the dynamic global optimization of macro and micro-economy. The details are as follows:

- (1) The output adjustment of enterprise under static conditions: under the condition that the external input of the model, especially the labor productivity, is constant and under the axiom that anyone's demand for a particular commodity is upper bounded (see Section 3.2), during the production process of

every commodity, the actual value of the commodity falls below a reasonable value when the supply of the commodity exceeds the rational demand of the market. At this time, any enterprise that produces this commodity will experience a decrease in revenue and will lose money if it continues the production. If the enterprise opts to downsize the production of the commodity so that the supply of the product is equal to its reasonable market demand, then the enterprise will stop losing. Otherwise, when the loss reaches a certain extent, the enterprise will go bankrupt, so that the quantity of social production is forced to restore generally in line with demand, and the commodity economy will asymptotically converge to a fixed point where the supply and demand of the market are rebalanced (a strict proof of dynamic output adjustment towards equilibrium is shown in Wu et al. [46]).

- (2) The enterprises' quality adjustment and entry or exit adjustment under dynamic conditions: under the condition that the external input of the model, especially the labor productivity, is increasing with time (see the axiomatic assumptions in Section 3.2), if enterprises in any industry continue to produce the products of the same quality (or grade) for too long, the overcapacity phenomenon will undoubtedly occur, i.e., the production capacity of enterprises for such products surpasses the market rational demand. If enterprises in this industry cannot improve product quality in time, it will lead to severe overcapacity. When most industries start experiencing severe overcapacity, it will give rise to large-scale losses and bankruptcies, a drop in workers' income or a spike in unemployment, and even a systemic recession or crisis. If enterprises in the industry can improve product quality in time, they can address excess capacity and drive economic recovery and prosperity, so that the industry reaches a new supply-demand balance at a higher product quality level. This economic development process does not only reflect the periodic movement of recession, crisis, recovery, and prosperity but also shows a continuously increasing wealth value in terms of utility properties.

Under the law of the average rate of profit among different industries, the cyclical fluctuations in the industry will be reduced. Cyclical fluctuations in different industries do not necessarily coincide, so there may be imbalances in the average rate of capital profit in different industries within the same economic system. For example, some industries are in recession or crisis, while others are in recovery and boom. The average rate of capital profit in the former industries is possibly lower than that of the latter. In this case, in order to chase higher profits, capital must flow from industries with a lower average rate of profit to industries with a higher average rate of profit. It will eventually restore the general

equilibrium of the economy by increasing the production capacity of industries in recovery and boom, while decreasing the production capacity of industries in recession and crisis.

- (3) The government's macroeconomic regulation: in addition to the spontaneous adjustment mechanism of the commodity economy system, government interventions are also a powerful control method to ensure the stability and optimization of the system. In the SED model, government's reasonable regulation towards the commodity economy system can avoid or reduce the economic instability and cyclical fluctuation caused by the market's spontaneous adjustment and then maximize national income and the value of social wealth. The government can implement monetary policies such as money supply and interest rate policies and fiscal policies such as tax, investment, and welfare policies. These policies can be contractionary or expansionary depending on the overcapacity situation of various industries in the model. When the entire society's production capacity outnumbers the bounded demand, the government adopts contractionary policies. Otherwise, the government adopts expansionary policies. Besides mitigating economic fluctuations, the government aims to promote long-run economic development by investing in science and technology and promoting new technologies and products.

3. Theoretical Foundation of the SED Model: The New Theory of Value

3.1. General Definition of the Value of a Commodity. Economics is the study of how societies use scarce resources to produce valuable commodities (goods and services) and distribute them among different individuals [48]. Thus, commodities are the vital research objects in economics. The core problem of studies concerning commodities is how their value is measured. The value of a commodity includes the exchange value determined by the amount of labor consumed during the production process of the commodities and the use value determined by the degree of satisfaction or the utility quantity obtained by the agents during the process of commodity consumption. Therefore, the key to measuring a commodity's value is to put forward a measure of the value of the amount of labor consumption and the utility quantity. However, neither the labor theory of value nor the utility theory of value has constructed a unified and dynamic measure of value [38].

The new theory of value provides a new definition of labor value and use value of commodities by analogy with Newtonian mechanics. Suppose that the labor consumed in the process of commodity production is equivalent to the "force" of labor, and that the utility obtained in the process of commodity consumption is equivalent to the "force" of consumption. Then, the labor value and use value are determined by the "force" of labor and the "force" of consumption, respectively. Suppose that before the advent of human beings, all natural matters are taking an uniform

linear motion and thus have no value. After the emergence of humankind, human beings began to engage in production activities to meet their own needs. The force of labor causes the displacement of particles in the commodity space to produce an acceleration, which determines the value of commodities. The force of labor is determined by three factors: product quantity (denoted by b), economic quality (or commodity grades and utility level, denoted by m), and time (denoted by t). By analogy with Newton's second law, the force of labor is $f = ma$ (the acceleration of labor production is $a = d^2b/dt^2$) and the work of labor is $w = fb$, which determines the value of commodities. Furthermore, humankind has the ability of improving the dexterity, i.e., "labor gravitational force," which leads to gravitational acceleration and generates surplus value in the production of commodities. The amount of surplus value or profit in the production of a commodity can be accurately measured by the Lagrangian $L = T - V$. On the other hand, the use value form of commodities has a dual relationship with some dimensions of labor value. For example, consumption volume is equal to the negative of product quantity; demand quantity is equal to the negative of labor productivity; utility quantity is equal to the inverse of negative labor productivity; utility level is equal to the negative of the economic quality; the force of consumption is equal to the negative of the force of labor; use value is equal to the negative of the labor value; and total market sales are equal to the total market purchasing power. Detailed definitions of labor values and use values in the new theory of values and their counterparts in Newtonian mechanics are listed in Table 1.

3.2. The Basic Axiom of the New Theory of Value. The new theory of value consists of six fundamental axiomatic assumptions. (1) Natural wealth has no value. In the motion of natural wealth without the participation of labor, there is no action of the force of labor on the displacement of a point during uniform linear motion in the quantity vector space formed by the quantity of natural wealth, i.e., $f = 0 \iff db/dt = 0$. (2) The acceleration in the production of wealth determines the force of labor. In the process of wealth production, a point in the quantitative vector space on which human beings use their labor and the means of production to work has a displacement, and the force of labor is equal to the product of economic quality and acceleration, i.e., $f = ma$. (3) The sum of the force of labor and the force of consumption is zero, by analogy with Newton's third law of motion, where the force of action (denoted by f) is equal to the force of reaction (denoted by $-f$). (4) Labor gravity must exist throughout the working process, but labor gravity is only a natural force instead of a force of labor. In the production process, humankind can learn by doing so that labor productivity will increase with time when producing the same product of any kind, resulting in acceleration of labor gravitational force generated in the production process, i.e., $g = \bar{m}(d^2\bar{b}/dt^2)$. (5) The force of labor determines the value of commodities. The commodity economic system is a social and economic system of wealth production through the division of labor and commodity exchange. In the commodity

TABLE 1: The definitions of labor value and use value in new theory of value and their counterpart in Newtonian mechanics.

Concepts in economics	Concepts in Newtonian mechanics	Mathematical notation	Dimension
<i>(A) Basics</i>			
Labor time	Time	t	(T)
Product quantity	Distance	b	(L)
Economic quality	Mass	m	(M)
Labor productivity	Velocity	$v_\alpha = db_\alpha/dt$	(LT ⁻¹)
Labor production acceleration	Acceleration	$a_\alpha = d^2b_\alpha/dt^2$	(LT ⁻²)
Force of labor	Force	$f_\alpha = m_\alpha a_\alpha$	(MLT ⁻²)
<i>(B) Labor value</i>			
Labor value of commodities	Work	$w = fb$	(L ² MT ⁻²)
Unit labor value	—	$\bar{w}_\alpha = w_\alpha/b_\alpha$	(L ² MT ⁻²)
Legal monetary aggregate	—	$M = \kappa(\sum_{\alpha=1}^n w_\alpha)$;	(kL ² MT ⁻²)
Commodity price	—	$p_\alpha = (\bar{w}_\alpha/w)M$	(kL ² MT ⁻²)
Kinetic energy of labor value	Kinetic energy	$E = \sum_{\alpha=1}^n (E_{\alpha 1} + E_{\alpha 2})$, Kinetic energy of labor value of means of production $E_{\alpha 1} = 1/2m_{\alpha 1}v_{\alpha 1}^2$, kinetic energy of labor value of the force of labor $E_{\alpha 2} = 1/2m_{\alpha 2}v_{\alpha 2}^2$,	(kL ² MT ⁻²)
Potential energy of labor gravitational force	Potential energy	$\varphi = \sum_{\alpha=1}^n m_\alpha g_\alpha b_\alpha$,	(L ² MT ⁻²)
Surplus labor value	Lagrangian	$\omega = \kappa(-E + \varphi)$	(kL ² MT ⁻²)
Constant capital (means of production)	Kinetic energy	$C = \sum_{\alpha=1}^n C_\alpha = \sum_{\alpha=1}^n \kappa E_{\alpha 1}$;	(kL ² MT ⁻²)
Variable capital (force of labor)	Kinetic energy	$V = \sum_{\alpha=1}^n V_\alpha = \sum_{\alpha=1}^n \kappa E_{\alpha 2}$;	(kL ² MT ⁻²)
Surplus value rate	—	$\tau = \omega/V$	(K ς)
Profit	—	$\Pi = \omega$	(kL ² MT ⁻²)
Profit rate	—	$\pi = \kappa[\omega/(C + V)]$	(K ς)
<i>(C) Use value</i>			
Consumption quantity	—	$-b$	(L)
Demand quantity	—	$\hat{v}_{\alpha\beta} = -db_{\alpha\beta}/dt$, where α represents the type of products and β represents the type of utility type with each type of products	(LT ⁻¹)
Utility quantity	—	$\hat{u}_{\alpha\beta} = -1/\hat{v}_{\alpha\beta}$	(LT ⁻¹)
Force of consumption	—	$\hat{f}_{\alpha\beta} = -f_{\alpha\beta}$	(MLT ⁻²)
Utility level	—	$\hat{m}_\alpha = m_\alpha = \sum_{\beta=1}^m m_{\alpha\beta}$	(M)
Use value	—	$\hat{w} = -w = -\hat{f}b$	(L ² MT ⁻²)
Total social market sales	—	$\hat{M} = -M = \sum_{\alpha=1}^n -M_\alpha = \sum_{\alpha=1}^n -\bar{w}_\alpha b_\alpha$, where M represents purchasing power of legal tender	(kL ² MT ⁻²)

Note. For the sake of simplicity, assume that the economic quality does not change over time; κ represents the conversion coefficient between the unit quantity of legal tender issued by a country under the non-gold standard and the total labor value of commodities, i.e., the price index.

economy system, when people transact commodities in the market, they evaluate the value of commodities exchanged according to the average necessary force of labor consumed in the production of commodities. The value of commodities is equal to the work done by the force of labor, that is, $w = fb$. (6) The quantity of demand for useful things is upper bounded. The rational demand for useful things is b^* , i.e., when $b_\alpha \leq b_\alpha^*$, $\hat{v}_{\alpha\beta} = -v_{\alpha\beta}$; when $b_\alpha > b_\alpha^*$, $\hat{v}_{\alpha\beta} = -v_{\alpha\beta}^*$, where α and β represent the type of product and the utility attribute of each product, respectively. Therefore, even in the case of excess supply, rational demand is a constant.

3.3. *Linear Programming Model System of Commodity Economy.* This section describes the linear programming

model system that captures the key mechanism of the SED model. Several typical models detailed in Wu's monograph "On Wealth" are introduced [40], such as the models of production of means of subsistence and means of production (Chapter 12), the model of commodity exchange (Chapter 13), the model of commodity price (Chapter 14), the model of production and financial capital (Chapter 15), and the model of government's macroeconomic regulation (Chapter 16). The complete version of the linear programming model of SED analyzes the agents' behavior and market interaction in the environment where labor productivity is constant or time varying using the corresponding Euclidean or Riemann metrics as measure of value. Although the Riemann metric is more general than the Euclidean metric, the core mechanisms based on them are

almost the same. Admittedly, the agents in non-linear and dynamic SED model are more heterogeneous and interactive than those in the linear programming model system, the core behaviors of the agents in these two models are consistent, and the SED model asymptotically converges to the equilibrium and optimization state described by the linear theoretical model. Therefore, in illustrating the linear programming model, we retain the core behaviors of the agents in the commodity economy based on the value of Euclidean metric and abstract away the operation process of nonlinear, nonequilibrium, and dynamic evolution featured in the SED model.

3.3.1. Production of Enterprises' Means of Subsistence (or Consumer Goods). Here, the optimal production plan for means of subsistence is considered. Under the condition that labor productivity is constant, it is assumed that the following variables are known: (1) type and quantity of each factor of production, including force of labor, means of labor, and objects of labor; (2) labor productivity in the production process given those factors of production; and (3) amount and level of utility embedded in various means of subsistence produced by each person. Further, assume that everyone pursues the satisfaction of total utility for a specific usage before seeking a higher level of utility. The problem of optimal production planning is how to arrange the limited factors of production to produce the means of subsistence that meet the needs of all people and attain the highest level of utility. The value and quantity of commodities determined under the optimal production plan are the stable and equilibrium market value, product quantity, and rational demand of the whole commodity economy.

The linear production function of the α -th (n types in total) means of subsistence produced by the d -th individual (l individuals in total) using h factors of production is

$$b_{\alpha}^{(d)} = Fx^{(d)}, \quad (1)$$

where $F = (f_{\alpha\beta})_{n \times h}$, $f_{\alpha\beta}$ is the quantity of β -th (h types in total) factor of production required in the production of the α -th means of subsistence per unit quantity, $\alpha = 1 \dots n$, $\beta = 1 \dots h$. $b_{\alpha}^{(d)}$ is α -th (n types in total) means of subsistence produced by the d -th individual. $x^{(d)} = (x_1^{(d)}, \dots, x_h^{(d)})$ is the vector of h factors of production.

Let $i_k^{(d)}$ denote the number of utility attributes of all products for k -th usage owned by the d -th individual. The utility function is $A^{(d)} = (a_{\alpha k}^{(d)})_{n \times m}$, where $a_{\alpha k}^{(d)}$ represents the quantity required to attain per unit utility of the α -th means of subsistence for k -th usage; then,

$$i_k^{(d)} = \sum_{\alpha=1}^n A^{(d)}(Fx^{(d)}). \quad (2)$$

Let g_{α} be the utility level function; then, the utility level of α -th means of subsistence produced by the d -th individual is

$$j_{\alpha}^{(d)} = g_{\alpha}(Fx^{(d)}). \quad (3)$$

Under the production objectives and constraints above, it is required that the total utility of the products meets the needs of all people for all m usages, while the utility levels of the products are the highest. Then, the linear programming model for the production of means of subsistence is as follows.

The objective function is

$$\max j = \sum_{d\alpha} j_{\alpha}^{(d)} = \sum_{d\alpha} g_{\alpha}(Fx^{(d)}). \quad (4)$$

The constraints are

$$\begin{aligned} (a): i_k^{(d)} &= \sum_{\alpha=1}^n A^{(d)}(Fx^{(d)}) = 1, \quad k = 1 \dots m, d = 1 \dots l, \\ (b): 0 &< x_{\beta}^{(d)} < r^{(d)}, \\ (c): h &> m, \end{aligned} \quad (5)$$

where constraint (a) means that commodities for each class or each usage with different utility level must satisfy the demand of consumers for the bounded quantity of utility, constraint (b) means that the total amount of each factor of production is limited and fixed, and constraint (c) means that at least one commodity has a different utility level. The number of unknowns in these linear programming models is $h \times l$, and the number of independent equations is $h \times l - m \times l$. If the rank of the coefficient matrix is equal to the rank of the augmented matrix, the solutions to the linear equations exist but are not unique.

Thus, in any socio-economic form with limited amount of factors of production, if all members regard factors of production as a whole and take as a whole all their needs, i.e., the pursuit of both the amount and the level of utility, humankind can always find an optimal combination of the factors of production available, so that the produced means of subsistence can meet the needs of all people, and their utility level is the highest. In this case, the solutions to the model are not unique, but all production plans that lead to the optimal solutions are equivalent in terms of the total value of commodities.

3.3.2. Production of Means of Production (or Capital Goods) of the Enterprises. Here, the demand for means of production and the compensation of production materials required to produce the means of subsistence are considered. Given consumers' bounded demand and the input-output relationship between means of subsistence and means of production, the rational demand for means of production can be assumed to be known. At the same time, suppose that the costs of the means of production are known and that the reserve of means of production is sufficient. The problem of the optimal production plan of means of production is how to use the limited amount of factors of production to produce at the lowest cost as many products as rational demand for means of production.

Assume l persons own h means of production $X = (x_{\beta d})_{h \times l}$, where $x_{\beta d}$ denotes the β -th means of

production owned by the d -th person. $\bar{a} \in X$ represents a reasonable reserve of means of production. The production function of the means of production, i.e., the consumption matrix, $E = (e_{\alpha\beta})_{l \times h}$, $c \in C$, $c = (c_1, c_2, \dots, c_h)$, is the rational demand for means of production. Let $P = \sum_{d=1}^l P(x_{\beta d})$ be the production cost function of means of production, and the linear programming model for means of production is as follows.

The objective function is

$$\min P = P(x), \quad (6)$$

subject to the constraints that (a) $\sum_{d=1}^l P(x_{\beta d}) = c$ and (b) $\forall x_{\beta d} \in X, \bar{a} \leq x_{\beta d} \leq a$, where constraint (a) indicates that the produced means of production can satisfy the demand for the compensation for the means of production in the production process and constraint (b) means that the means of production used for production must be limited and sufficient for the production of means of production that meet rational demand.

The production of means of production can be combined with the production of means of subsistence to form a multi-objective linear programming model. This model can reasonably determine the reasonable allocation of the factors of production in the process of production of means of subsistence and means of production, so that means of subsistence produced can meet the needs of all people on total utility, attain the highest utility level, and consume the lowest cost of labor so that the production factors consumed in the production process are fully compensated, and the simple and repeated commodity production process is continued. In particular, if combined with Leontief's input-output theory, the model would have a unique optimal solution to the production plan of the entire society.

3.3.3. Exchange of Means of Subsistence by Households.

In any process of commodity exchange, individuals transact their commodities with others in pursuit of their private interests. Different individuals have distinct preferences for the same commodity, and commodities are redistributed among people by exchange so that everyone's demand for total utility is satisfied and the utility level of the final commodity portfolio reaches its maximum. The commodity exchange is carried out based on the principle of equivalence.

Assume l persons have the commodities with n usages and m types for each usage. Suppose that different individuals may obtain different levels of utility for the same category of commodities. Given the utility amount, utility level, and labor value of each commodity, the optimal commodity exchange plan is how to make the utility levels of the exchanged commodities for each person reach the maximum, given the constancy of the commodity quantity, the equivalence of labor value, and the satisfaction of everyone's needs.

The objective function is

$$\begin{aligned} \max j &= J(\bar{b}_{111}, \bar{b}_{112}, \dots, \bar{b}_{11l}, \bar{b}_{121}, \bar{b}_{122}, \dots, \bar{b}_{12l}, \dots, \bar{b}_{nml}) \\ &= f_j(\bar{B}^{(1)}, \bar{B}^{(2)}, \dots, \bar{B}^{(l)}) = \sum_{d=1}^l \text{tr} \bar{B}^{(d)} (J^{(d)})', \end{aligned} \quad (7)$$

subject to the constraints that

- (a) Equivalent exchange principle: $\text{tr} \bar{B}^{(d)} K^t = \text{tr} B^{(d)} K^t$.
- (b) The invariance principle of total quantity: $\sum_{d=1}^l \bar{B}^{(d)} = \sum_{d=1}^l B^{(d)}$.

- (c) The satisfaction principle of total utility for each

$$\text{usage: } \text{tr} \bar{B}^{(d)} I^t = \begin{bmatrix} 1 & \cdots & * \\ \vdots & \ddots & \vdots \\ * & \cdots & 1 \end{bmatrix}.$$

- (d) The quantity of commodities is non-negative: $B^{(d)}$ is non-negative, where $B^{(d)} = (b_{\alpha\beta d})_{n \times m \times l}$ is the d -th individual's commodity matrix before exchange, in which $b_{\alpha\beta d}$ represents the quantity of the β -th commodity with α -th usage owned by the d -th individual; $\bar{B}^{(d)} = (\bar{b}_{\alpha\beta d})_{n \times m \times l}$ is the d -th individual's commodity matrix after exchange, in which $\bar{b}_{\alpha\beta d}$ represents the quantity of the β -th commodity with α -th usage owned by the d -th individual; $K = (k_{\alpha\beta})_{n \times m}$ is the labor value matrix per unit of the commodity, in which $k_{\alpha\beta}$ represents the labor value of the β -th commodity with α -th usage; $I = (i_{\alpha\beta})_{n \times m}$ is the utility matrix of per unit of the commodity, in which $i_{\alpha\beta}$ represents the utility amount of β -th commodity with α -th usage; and $J^{(d)} = (j_{\alpha\beta d})_{n \times m \times l}$ is the utility level matrix of per unit of the commodity given to the individual, in which $j_{\alpha\beta d}$ denotes the utility level of the β -th commodity with α -th usage owned by the d -th individual.

3.3.4. *Exchange Value or Price of Commodities.* The exchange value of commodities, or the quantity of any other commodities for which it can exchange in the market, can be expressed as the proportion of the value of each commodity in unit quantity in the total value of all commodities in the entire society [39]. Under the non-gold standard system, the value of a commodity is expressed in money form. Money does not only perform as an information symbol of commodity value but can also have an impact on the production and exchange of commodities. If the unit value of commodities is constant and the distribution of money flows is reasonable, the price of each commodity is equal to the ratio of its value to the total value of the society multiplied by the total money flow:

$$p_\alpha = b_\alpha \frac{\bar{v}_\alpha / \sum_{\alpha=1}^n b_\alpha \bar{v}_\alpha}{b_\alpha} m, \quad (8)$$

where b_α is the quantity of the commodity, \bar{v}_α is the fixed value per unit of the commodity, and m is the total money flow. Here, it is shown that the prices of commodities are proportional to the currency supply.

According to the sixth axiomatic assumption, the commodities above the rational demand have no use value or utility and thus no labor value or exchange value. For a society with only one commodity, given the unit value and utility level and monetary aggregate remain unchanged, suppose the total commodity value is proportional to the total money flow in the society, namely, $m/b_\alpha \bar{v}_\alpha = k$ is a constant; then, the relationship between the price and quantity of the commodity is

$$p_\alpha = p_\alpha(b_\alpha) = \begin{cases} k\bar{v}_\alpha, & b_\alpha \leq b_\alpha^*, \\ \frac{kb_\alpha^* \bar{v}_\alpha}{b_\alpha}, & b_\alpha > b_\alpha^*, \end{cases} \quad (9)$$

where b_α^* is the rational demand or satiety point of consumption. When b_α does not exceed b_α^* , the price of the commodity p_α remains constant. When b_α exceeds b_α^* , the price of the commodity is inversely proportional to the quantity of the commodity.

For a society with multiple commodities, the relationship between the price and the quantity of the commodity is

$$p_\alpha = p_\alpha(b_\alpha) = \begin{cases} \frac{k\bar{v}_\alpha}{s_\alpha + b_\alpha \bar{v}_\alpha}, & b_\alpha \leq b_\alpha^*, \\ \frac{kb_\alpha^* \bar{v}_\alpha}{b_\alpha (s_\alpha + b_\alpha^* \bar{v}_\alpha)}, & b_\alpha > b_\alpha^*, \end{cases} \quad (10)$$

where $s_\alpha = \sum_{\beta \neq \alpha} b_\beta \bar{v}_\beta$ is the total value of the commodities other than the α -th commodity. Obviously, when considering multiple commodities, the unit price of a particular commodity decreases monotonically as the quantity increases and falls faster after the quantity surpasses the critical point b_α^* .

3.3.5. Production Capital Investment. The model of capital investment allocation reflects the best allocation of resources among different industries. Capital is the general representation of the wealth production process in the commodity economic system, and its movement is a repeated

process of commodity production, exchange, and monetary medium. Various forms of capital include production capital, commercial capital, and financial capital, whose goal is to chase profits.

Under the condition that labor productivity is constant, given the production function, sales revenue function, and cost function of various commodities, further assume capital flows freely between industries, and the problem of optimal allocation of production capital is how to allocate capital in the production of different commodities to maximize profits.

Suppose a production system that produces n products, $\alpha = 1, 2, \dots, n$. Let C be the total amount of capital and b_α^0 and b_α^* be the rigid demand and rational demand for the α -th product. Assume that the variable cost of the α -th product is proportional to the quantity with the proportional coefficient λ_α and c is the fixed cost, and the relationship between the cost of producing α -th product and the output is

$$\hat{p}_\alpha = \hat{p}_\alpha(b_\alpha) = \lambda_\alpha b_\alpha + c. \quad (11)$$

Assume that the sales revenue of the α -th product is proportional to the quantity that does not exceed the rational demand b_α^* with the proportional coefficient $\bar{\lambda}_\alpha$ for rigid demand and $\bar{\lambda}_\alpha$ for normal demand. Suppose that the unit price of each commodity within the rigid demand range is higher than that of any product within the normal demand range, i.e., $\bar{\lambda}_\alpha > \max\{\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_\alpha\}$. The total value and sale revenues of commodities will cease to increase if there is excess production over the rational demand b_α^* . The relationship between the sales revenue and quantity of the α -th product is

$$\check{p}_\alpha = \check{p}_\alpha(b_\alpha) = \begin{cases} \bar{\lambda}_\alpha b_\alpha, & 0 \leq b_\alpha \leq b_\alpha^0, \\ \bar{\lambda}_\alpha b_\alpha + (\bar{\bar{\lambda}}_\alpha - \bar{\lambda}_\alpha) b_\alpha^0, & b_\alpha^0 \leq b_\alpha \leq b_\alpha^*, \\ \bar{\lambda}_\alpha b_\alpha^* + (\bar{\bar{\lambda}}_\alpha - \bar{\lambda}_\alpha) b_\alpha^0, & b_\alpha > b_\alpha^*. \end{cases} \quad (12)$$

Then, the relationship between profit and quantity can be expressed as

$$\pi_\alpha = \pi_\alpha(b_\alpha) = \check{p}_\alpha(b_\alpha) - \hat{p}_\alpha(b_\alpha) = \begin{cases} \bar{\lambda}_\alpha b_\alpha - \lambda_\alpha b_\alpha - c, & 0 \leq b_\alpha \leq b_\alpha^0, \\ \bar{\lambda}_\alpha b_\alpha + (\bar{\bar{\lambda}}_\alpha - \bar{\lambda}_\alpha) b_\alpha^0 - \lambda_\alpha b_\alpha - c, & b_\alpha^0 \leq b_\alpha \leq b_\alpha^*, \\ \bar{\lambda}_\alpha b_\alpha^* + (\bar{\bar{\lambda}}_\alpha - \bar{\lambda}_\alpha) b_\alpha^0 - \lambda_\alpha b_\alpha - c, & b_\alpha > b_\alpha^*. \end{cases} \quad (13)$$

The relationship between profit rate and quantity is

$$\kappa_\alpha = \kappa_\alpha(b_\alpha) = \frac{\pi_\alpha(b_\alpha)}{\bar{P}_\alpha(b_\alpha)} = \begin{cases} \frac{\bar{\lambda}_\alpha b_\alpha - \lambda_\alpha b_\alpha - c}{\lambda_\alpha b_\alpha + c}, & 0 \leq b_\alpha \leq b_\alpha^0, \\ \frac{\bar{\lambda}_\alpha b_\alpha + (\bar{\lambda}_\alpha - \bar{\lambda}_\alpha) b_\alpha^0 - \lambda_\alpha b_\alpha - c}{\lambda_\alpha b_\alpha + c}, & b_\alpha^0 \leq b_\alpha \leq b_\alpha^*, \\ \frac{\bar{\lambda}_\alpha b_\alpha^* + (\bar{\lambda}_\alpha - \bar{\lambda}_\alpha) b_\alpha^0 - \lambda_\alpha b_\alpha - c}{\lambda_\alpha b_\alpha + c}, & b_\alpha > b_\alpha^*. \end{cases} \quad (14)$$

As can be seen from equations (13) and (14), for the capital of a single industry, both profit and profit rates are increasing until rational demand is reached and will decline as product quantity outnumbers rational demand. Furthermore, set the total capital profit function as

$$\pi = \sum_{\alpha=1}^n \pi_\alpha(b_\alpha). \quad (15)$$

It can be demonstrated that if the marginal returns of capital are equal across industries and the average profit rate is the same, the profits of the entire society must be maximized at the satiety point of demand b_α^* . Since industrial capital is a function of product quantity, given the optimal product quantity of each industry, the optimal allocation of capital investment in each industry is determined.

3.3.6. Financial Capital Investments. Let m be the total amount of money used by the entire society for direct investment, C be the value of capital, κ be the value-added rate or average profit rate of capital investment, σ be the risk factor of the stock market, γ be the amount of money flowing to the deposit market, and γ be the deposit interest rate; then, the monetary capital value of the existing stock market is C/m . The expected unit value of monetary capital is

$$\frac{C(1 + \kappa) - C\sigma(1 + \kappa)}{m - \gamma}. \quad (16)$$

When the interest rate on bank deposits is known, the equilibrium equation of the yield of capital investment is

$$\frac{C(1 + \kappa) - C\sigma(1 + \kappa)/m - \gamma - C/m}{C(1 + \kappa) - C\sigma(1 + \kappa)/m - \gamma} = \gamma, \quad (17)$$

which means that the yield of money capital is equalized between the stock market and the deposit market. Further, it can be inferred that

$$y^* = m[1 - (1 - \gamma)(1 + \kappa)(1 - \sigma)]. \quad (18)$$

When the interest rate on deposits or risk factor is 100%, all of monetary capital will flow into the bank. When real rate of return on capital, i.e., the return on capital net of risk

factors, is greater than the deposit rate, more monetary capital will flow to the stock market. Moreover, all of the monetary capital will flow to the stock market if and only if $1 - (1 - \gamma)(1 + \kappa)(1 - \sigma) = 0$.

3.3.7. Macroeconomic Regulation by the Government. In the case of supply-demand imbalance, especially overcapacity, the government's macroeconomic regulation helps the market quickly restore the balance of supply and demand so that national income and the total value of wealth reach the maxima. Standard macroeconomic regulation policies are monetary and fiscal policy instruments. Monetary policy instruments include interest rate and money supply policies, while fiscal policy instruments include government purchase, government investment, and tax rate policies.

Suppose that the total income of the d -th economic agent is composed of wage income and disposable profit, i.e., $n^{(d)} = U^{(d)} + \pi^{(d)}$, where $U^{(d)}$ and $\pi^{(d)}$ are the wage income and disposable profit of the d -th person, respectively.

Suppose that the factor of production by d -th economic agent in the δ -th firm is $(x_1^{(\delta d)}, x_2^{(\delta d)}, \dots, x_h^{(\delta d)}) = (u_i^{(\delta d)}, q_o^{(\delta d)}, z_\rho^{(\delta d)})$, where $u_i^{(\delta d)}$, $q_o^{(\delta d)}$, and $z_\rho^{(\delta d)}$ are the vectors of force of labor, labor materials, and labor objectives, respectively. Accordingly, the output of the first α -th commodity produced is $b_\alpha^{(\delta d)} = f_{l_\alpha}(u_i^{(\delta d)}, q_o^{(\delta d)}, z_\rho^{(\delta d)})$, where $\bar{P}_{ui}^{(\delta)}$, $\bar{P}_{qo}^{(\delta)}$, and $\bar{P}_{z\rho}^{(\delta)}$ represent the cost of force of labor, labor materials, and labor objectives, respectively. The cost of the factors of production used by d -th economic agent in the δ -th firm is $D^{(\delta d)} = \sum_i \bar{P}_{ui}^{(\delta)} c_i^{(\delta d)} + \sum_i \bar{P}_{qo}^{(\delta)} q_o^{(\delta d)} + \sum_i \bar{P}_{z\rho}^{(\delta)} z_\rho^{(\delta d)}$, where the total wage income of the d -th person is $U^{(d)} = \sum_i \bar{P}_{ui}^{(\delta)} u_i^{(\delta d)}$. The dividends that the d -th person receives are calculated below.

The unit value of the α -th commodity is determined by the piecewise function of commodity value, i.e.,

$$\bar{v}_\alpha = f_{\bar{v}}(b_\alpha, D_\alpha) = \begin{cases} \bar{D}_\alpha, & b_\alpha \leq b_\alpha^*, \\ \frac{\bar{D}_\alpha b_\alpha^*}{b_\alpha}, & b_\alpha > b_\alpha^*, \end{cases} \quad (19)$$

where $b_\alpha = \sum_{\delta d} b_\alpha^{(\delta d)}$ is the quantity of the α -th commodity; $\bar{D}_\alpha = \sum_{\delta d} D_\alpha^{(\delta d)} / \sum_{\delta d} b_\alpha^{(\delta d)}$ is the unit cost of the α -th commodity; and b_α^* is the rational demand of the α -th commodity by all the people in the society. Given the money flow m , the price of the α -th commodity is calculated as

$$p_\alpha = \frac{\bar{v}_\alpha}{\sum_\alpha \bar{v}_\alpha b_\alpha} m = \begin{cases} \frac{\bar{D}_\alpha}{\sum_\alpha \bar{D}_\alpha b_\alpha} m, & b_\alpha \leq b_\alpha^*, \\ \frac{\bar{D}_\alpha b_\alpha^* / b_\alpha}{\sum_\alpha \bar{D}_\alpha b_\alpha^*} m, & b_\alpha > b_\alpha^*. \end{cases} \quad (20)$$

Let $b_\alpha = \min(b_\alpha, b_\alpha^*)$ denote the actual demand for the α -th commodity, and then the sales rate for the α -th commodity is $\bar{\lambda}_\alpha = a_\alpha / b_\alpha$. The total sales of the δ -th firm are $S^{(\delta)} = \sum_\alpha \bar{\lambda}_\alpha b_\alpha^{(\delta d)} p_\alpha$, and then the profit from the sales is

$$\bar{n} = \sum_d n^{(d)} = \sum_{\delta d \alpha} \frac{\bar{\lambda}_\alpha b_\alpha^{(\delta d)} \bar{v}_\alpha}{\sum_\alpha \bar{v}_\alpha b_\alpha} m \varepsilon^{(\delta d)} - \sum_{\delta d \alpha} \left(\sum_i \bar{P}_{qo}^{(\delta)} q_o^{(\delta d)} + \sum_i \bar{P}_{z\rho}^{(\delta)} z_\rho^{(\delta d)} \right) \varepsilon^{(\delta d)}. \quad (23)$$

When $\partial \bar{n} / \partial b_\alpha^{(\delta d)} \partial b_\alpha^{(\delta d)} / \partial x_\alpha^{(\delta d)} = 0$, national income is maximized, but $\sum_{\delta d} b_\alpha^{(\delta d)}$ is not necessarily equal to b_α^* . It means that when national income reaches the maximum, the value of social wealth is not necessarily the highest unless $\sum_{\delta d} b_\alpha^{(\delta d)} = b_\alpha^*$.

The government revenues and expenditures affect national income. The government incomes include sales tax, income tax, and other taxes. Sales tax T_s is the product of sales revenue and sales tax rate, i.e., $T_s = t_1 S$. Income tax T_n is the sum of profits and wages multiplied by the income tax rate, i.e. $T_n = t_2 (\pi + U)$. Other taxes are calculated as $T_\pi = t_3 S$. Government revenues are calculated as $F_I = T_s + T_n + T_\pi$. The government expenditures are a function of total government revenue. The government

$$\pi^{(\delta)} = S^{(\delta)} - \sum_{\delta d} D_\alpha^{(\delta d)}. \quad (21)$$

The proportion of shares held by the d -th person in the δ -th firm is the share of the cost of his factors of production in the total cost of the enterprise, i.e.,

$$\varepsilon^{(\delta d)} = \frac{D_\alpha^{(\delta d)}}{\sum_{\delta d} D_\alpha^{(\delta d)}}. \quad (22)$$

The profit that the d -th person in the δ -th firm gets is $\pi^{(\delta d)} = \varepsilon^{(\delta d)} \pi^{(\delta)}$. The total profit earned by the d -th person is $\pi^{(d)} = \sum_\delta \pi^{(\delta d)}$. The income of all economic agents in the entire society, i.e., national income, is

expenditures can be divided into the government infrastructure investments $G_I = f_1(F_I)$ and the government science and technology investment $G_S = f_2(F_I)$. The expansionary fiscal policy function is $G_\pi = G_I + G_S$. Let $G_K = f_3(F_I)$ represent the government welfare expenditure.

Taking into account government taxes, fiscal expenditures, monetary policies, resident reinvestments, and savings, the national income is equal to the current product quantity multiplied by the sales rate and multiplied by the price and currency issuance growth rate, plus the government's expansionary fiscal policy expenditures and protective fiscal policy expenditures, plus the monetary savings, minus the government revenues and the costs of production, i.e.,

$$\pi = \sum_{d\alpha} b_\alpha (\bar{\pi}, i, \bar{t}) \bar{\lambda}_\alpha p_\alpha \varepsilon^{(d)} \kappa_M + \underline{g}(G_\pi) + \underline{f}(G_K) + M(\bar{\pi}, i, \bar{t}) - F_I - \sum_{\delta d \alpha} \left(\sum_i \bar{P}_{qo}^{(\delta)} q_o^{(\delta d)} + \sum_i \bar{P}_{z\rho}^{(\delta)} z_\rho^{(\delta d)} \right) \varepsilon^{(\delta d)}, \quad (24)$$

where $\bar{\pi}$, i , and \bar{t} denote the rate of profit, the interest rate, and the composite tax rate, respectively, which jointly determine the output b_α . κ_M is the growth rate of currency issuance, $\underline{g}(G_\pi)$ is the revenue function of the government's expansionary fiscal policies, $\underline{f}(G_K)$ is the revenue function of the government's protective fiscal policies, and $M(\bar{\pi}, i, \bar{t})$ represents the households' savings.

Under different supply-demand imbalance, the government can help restore supply-demand balance and reach the maximum of the national income and wealth value by implementing the macroeconomic policies of interest rates, tax rates, welfare, and investment expenditures.

For interest rate policies, (1) when $b > b^*$ and $\partial \pi / \partial b > 0$, the economy is in a state of false prosperity, which implies

that $\partial \pi / \partial i < 0$. In order to reduce false profits, interest rates should be increased to reduce the production; (2) when $b > b^*$ and $\partial \pi / \partial b < 0$, which implied $\partial \pi / \partial i > 0$, interest rates should be raised to reduce production and increase profits; (3) when $b < b^*$ and $\partial \pi / \partial b > 0$, interest rates should be reduced to increase production and decrease profits; (4) when $b < b^*$ and $\partial \pi / \partial b < 0$, interest rates should be reduced to increase production and decrease profits.

For tax policies, (1) if $b > b^*$ and $\partial \pi / \partial b > 0$, the economy is in a state of false prosperity. As $\partial f(E) / \partial F_I < 0$ and $\partial F_I / \partial b > 0$, then $\partial \pi / \partial F_I < 0$. Therefore, the government should raise the tax rates and reduce profits and production. (2) Suppose $b > b^*$ and $\partial \pi / \partial b < 0$. As $\partial f(E) / \partial F_I < 0$ and $\partial F_I / \partial b > 0$, then $\partial \pi / \partial F_I < 0$. Therefore, the government

should stabilize the tax rate and increase investment in science and technology because lowering tax rates will increase production, which will lead to a decrease in profits, causing the enterprises to decrease production spontaneously. On the other hand, increasing tax rate will lead to bankruptcy of the enterprises. (3) If $b < b^*$, $\partial\pi/\partial b > 0$, $\partial g(E)/\partial F_I - \partial f(E)/\partial F_I < 0$, then $\partial\pi/\partial F_I > 0$, and the government should raise taxes to enlarge the size of production and increase revenues. (4) If $b < b^*$, $\partial\pi/\partial b > 0$, and $\partial g(E)/\partial F_I - \partial f(E)/\partial F_I > 0$, then $\partial\pi/\partial F_I < 0$, and the government should reduce the taxes in order to expand the size of production and increase revenues.

For money supply policies, (1) if $b > b^*$ and $\partial\pi/\partial b > 0$, the economy is in a state of false prosperity; then, $\partial\pi/\partial\kappa_M > 0$, and the central bank should reduce currency issuance, which will shrink the scale of overproduction. (2) If $b > b^*$ and $\partial\pi/\partial b < 0$, then $\partial\pi/\partial\kappa_M > 0$, and the central bank should stabilize currency issuance. (3) If $b < b^*$ and $\partial\pi/\partial b > 0$, then $\partial\pi/\partial\kappa_M > 0$, and the central bank should stabilize currency issuance so that the size of production is enlarged, profits and national income are increased, and the risk of inflation is excluded. (4) If $b < b^*$ and $\partial\pi/\partial b < 0$, then $\partial\pi/\partial\kappa_M < 0$, and the central bank should reduce currency issuance.

4. China's Economic Digital Twin System Based on the ABM-SED Model: The Relationship between the Virtual Network World and the Real Society

4.1. National Economic Simulation. A state or country is the most basic economic simulation unit, where the central government manages 8 major industries and 42 types of products. The daily flows of commodities, labor, and capital for every industry, enterprise, and resident inside the state constitute the digital twin system close to the real economic and social system (see Figure 1).

- (1) National model framework: the boxes in the flow-chart in Figure 1 above represent the economic objects described by the simulation model. These objects include the main economic entities, government entities, and various forms of wealth, which can be further divided into enterprises, residents, markets, banks, securities, currencies, commodities, prices, government departments, external environment, etc.

The arrows between the boxes represent the behaviors and directions of the economic objects, that is, the processes and flow rate of the simulation system. Specifically, they are the flows of commodities, labors, and capital in the real economic and social system. The boxes having outward arrows without inwards represent the start point of exogenous variables. The boxes having inward arrows without outwards represent the endpoint of these variables. The remaining variables with double arrows represent endogenous variables. All endpoint

variables and endogenous variables are output variables of the system.

The arrows and the boxes along which the arrows circulate constitute an internal loop of the economy. In the process of economic system simulation, each internal loop is at least one time step apart. Multiple internal loops can form a subsystem, and all subsystems form the time series relationship between every economic object in the whole economic simulation system.

- (2) Main blocks: according to the classification of the main economic objects, the SED model can be divided into six blocks: enterprises, residents, markets, banks, securities, and government. These blocks have some subblocks. For example, there are hundreds of subblocks in the resident block, hundreds of subblocks in the enterprise block to produce 42 products, and 6 markets (raw materials, equipment, living materials, banking, securities, and labor) in the market block.

4.1.1. Enterprise Block. In the enterprise block of the SED model, there are marketing department, planning department, production department, quality inspection department, procurement department, warehousing department, sales department, capital department, finance department, personnel department, and new product development department. Each department is also well equipped with full functionalities (see Figure 2).

Each enterprise can produce multiple products, each of which has 4 grades at any period. The product can upgrade along with the enterprise's production capacity and market demands, accompanied by the elimination of the products of older grades. The production plan of an enterprise is arranged according to market demands and profits. The enterprise would increase the production of highly profitable products, maintain or reduce the production of slightly profitable products, and turn to produce new high-grade products instead of those with continuous losses. Each type of equipment is specific and helps produce the products of various varieties, grades, and quantities by a fixed production function with other inputs after a specific time. Enterprises in different industries produce different types of products. There are 33 types of products in total. 7 of them are means of subsistence: basic food, basic daily necessities, basic durables, general consumables, general durables, development consumables, and development durables. 26 of them are means of production: the production equipment of means of subsistence and raw materials, specific equipment for banks, securities, government, and equipment for production equipment.

As far as the employment and wages of the enterprise's labor force are concerned, the personnel department shall employ high-skilled, medium-skilled, and low-skilled labor force and temporary low-skilled labor force in proportion to the equipment of different grades. The workers will work

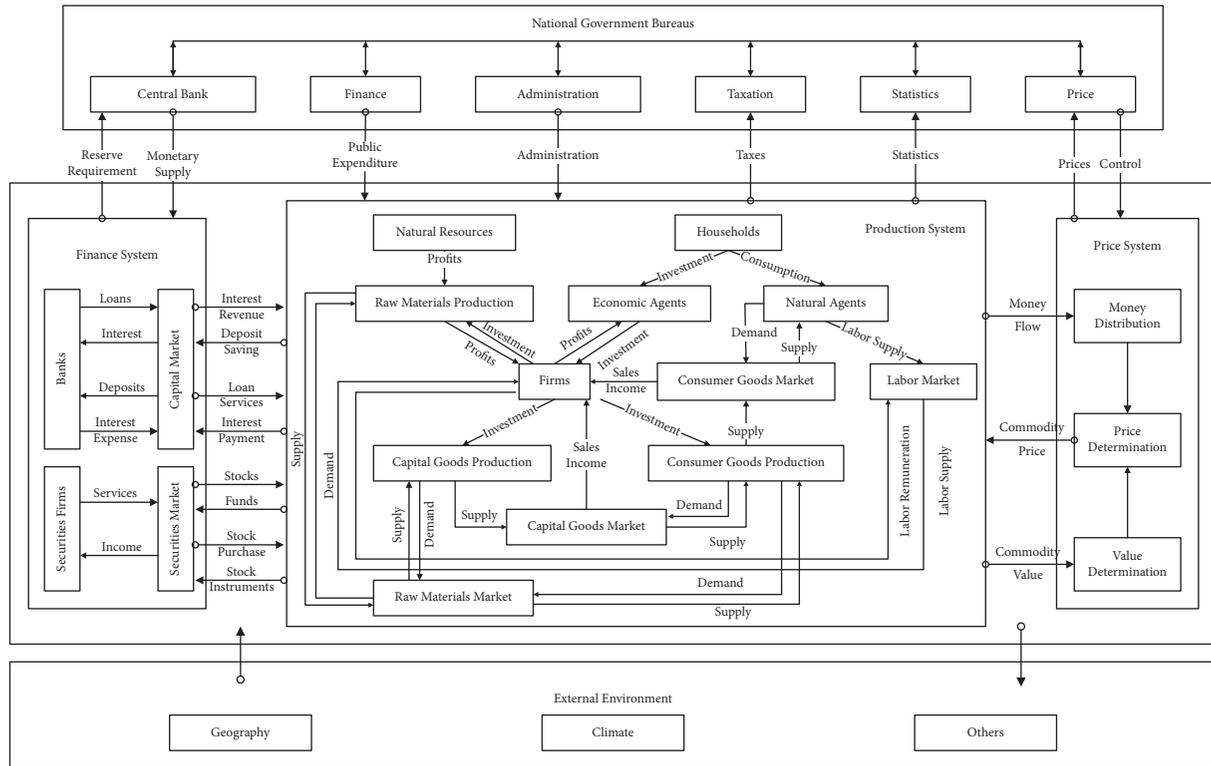


FIGURE 1: National economic simulation system (source: On Wealth I [40]).

overtime when the workload is heavy. Thus, the employment and wages of workers are changeable according to the business conditions of the enterprise.

Regarding the enterprise funds, the capital department allocates funds to the procurement department and personnel department according to the production tasks. If funds are insufficient, enterprises can apply for loans from the banks. The enterprise's funds come from shareholders' capital, sales revenue, and bank loans, which can be used to purchase equipment and raw materials, hire labor, distributed as profits, and pay taxes. Enterprises with sufficient funds can expand production, while those with insufficient funds will reduce or stop production. If many enterprises lose money for a long time leading to the fact that they will not run appropriately, the socio-economic system will not function as usual if this situation does not improve within a period of time.

The finance department is responsible for managing funds and preparing financial statements. The accounts of the financial statements in the SED model correspond to the actual financial statements used in real life.

The rest of the departments of the enterprise block in the SED model include marketing, planning, quality inspection, procurement, warehouse, sales, and new product development departments, whose functions are close to reality. The business properties of each department consist of factors such as population, amount of money, prices, number of equipment, and products' quantities, which frequently change at different times of a period.

4.1.2. Resident Block. The resident block of the SED model includes subblocks of population, employment, income, funds allocation, procurement, consumption, and warehouse. Each subblock is fully functional (see Figure 3).

Each resident block has initialized variables from outside, such as population size, population growth rate, education level, professional and technical level, capital reserve, and so on. At the same time, there are variables from other blocks, such as employment, wage income, market merchandise sales, prices, bank balances, bank interests, stock prices, stock price-earnings ratio, and so on. Each resident block in the SED model simulates a specific resident as its object. The resident object chooses between equity investments, bank savings, and consumption subject to the initial value of his property plus subsequent income. The purpose of residents' use of funds is to obtain more capital income in the later period of time in order to meet their demand for the utility of means of subsistence and means of subsistence with higher grades. Note that the enterprises can produce various products, each of which has 4 grades at any time. The product will upgrade with respect to the enterprise's production capacity and market demand changes, accompanied by the elimination of products of older grades. Therefore, residents' consumption and market purchase plans should be arranged according to the production plans of the enterprise. The residents die with age, and after the residents die, their property will be inherited by the successors.

Regarding the employment, the residents of the SED model are equal to the workforce. Each resident block

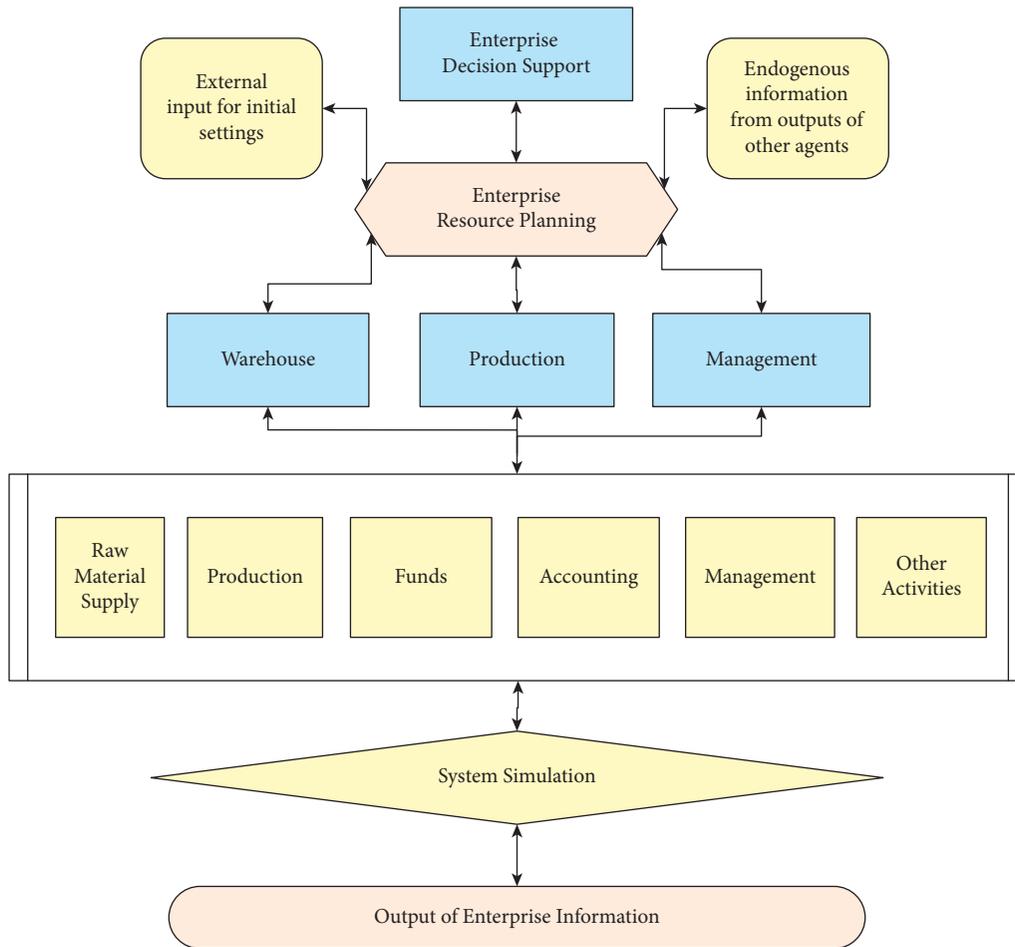


FIGURE 2: Overview of the process and structure of enterprise system (source: Guangzhou Milestone Software Co., Ltd., China).

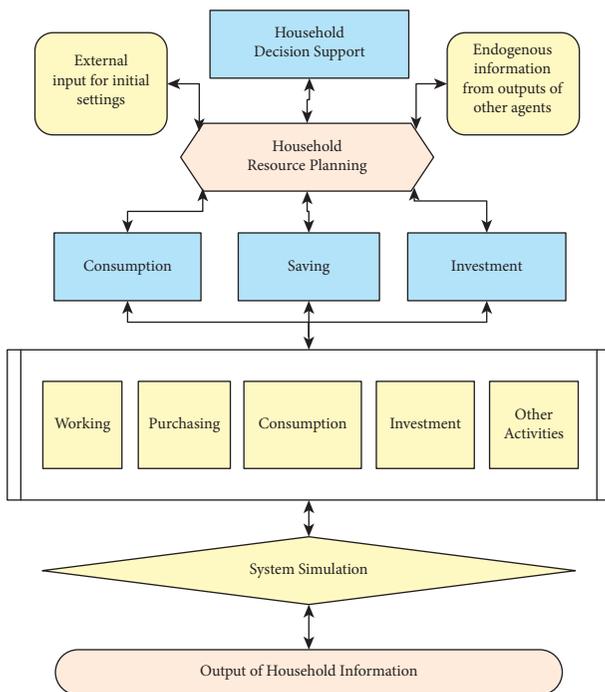


FIGURE 3: Overview of the process and structure of resident system (source: Guangzhou Milestone Software Co., Ltd., China).

simulates a labor object. A resident’s choice to work depends on whether his wages can meet the basic living needs or not. He will opt to work if the wages are sufficient to satisfy the basic living needs; otherwise, he will stay unemployed. In the case of multiple jobs, the resident chooses the most rewarding job. At the same time, the labor force itself is able to learn labor skills. Therefore, as long as each labor force can continue to work, his labor ability level will increase, so will his wages. If residents are unemployed for a long time, the government will pay out unemployment benefits.

Once residents have decided on the allocation of funds over the consumption, they begin to spend them in the market procurement. Given the commodities’ varieties, quantities, grades, and prices, residents will buy the commodities with the highest grade on the premise that they use up the procurement funds and their demand for utility quantities is satisfied. Each resident’s purchasing behavior allows for personal preference. A portion of purchased commodities is stored in the warehouse in case of short supply of them in the market sometime in the future.

In the SED model, residents’ statistics constitute the financial statements, whose accounts align with those in reality. The business properties of each resident block in the SED model are composed of population size, money amount, prices, equipment’ quantities, and products’

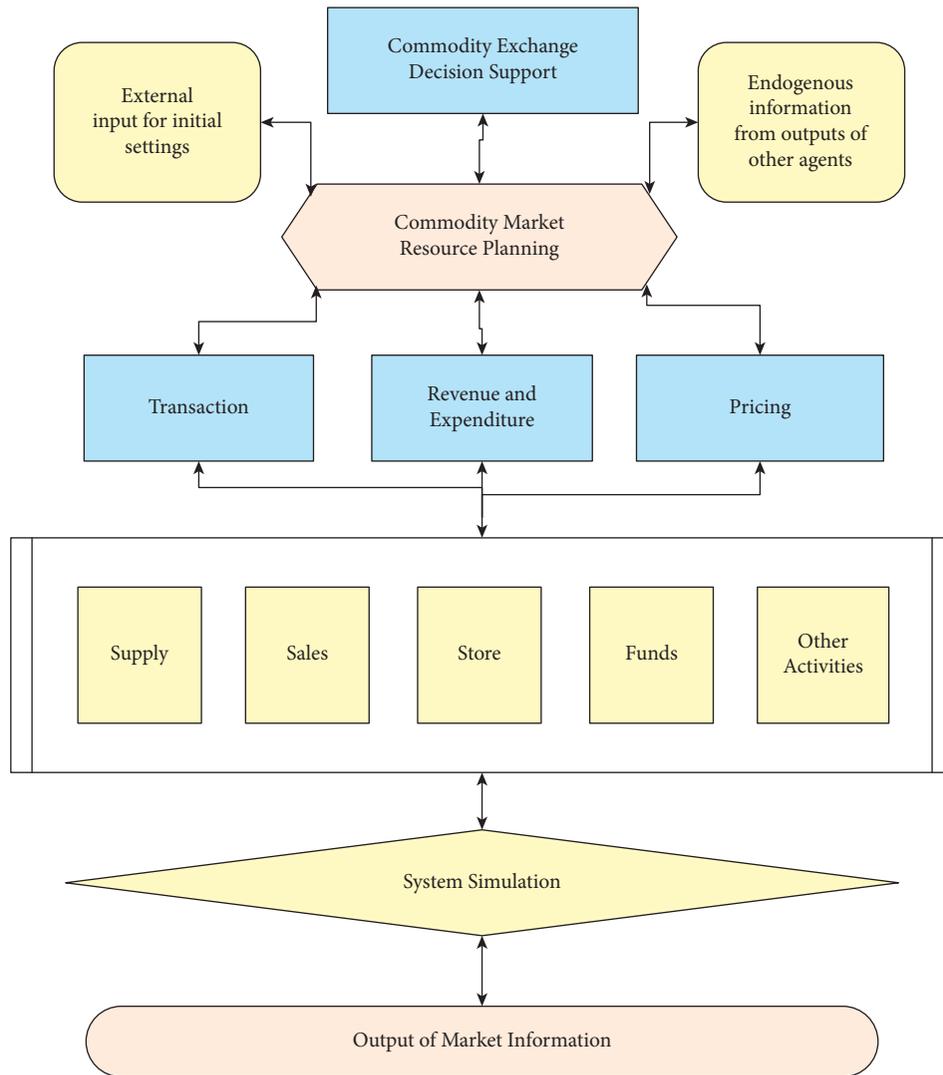


FIGURE 4: Overview of the process and structure of market system (source: Guangzhou Milestone Software Co., Ltd., China).

quantities, which frequently change at different times of a period. These residents' statistics will be outputted regularly from the SED model.

4.1.3. Market Block. The market block of the SED model includes subblocks of buyer, seller, commodity circulation, fund flow, commodity prices, and commodity storage. Each subblock is fully functional (see Figure 4).

The market block of the SED model has 6 subblocks: means of subsistence, means of production, raw materials, labor force, securities, and banks' deposit and loan markets. Each market subblock has external inputs such as commodity sales capacity, commodity reserve capacity, initial liquidity, inventories' varieties, quantities and costs, and so on. At the same time, there are variables inputted from other blocks such as commodities' varieties and quantities, ex-factory prices, liquidity, and market demands. Each market block of the SED model simulates a specific commodity object traded. During the trading process of that commodity, the price of each commodity is

determined by the commodity price model according to consumer demands, supply conditions of commodity markets, production costs, and procurement funds. Then, each commodity trader exchanges commodities under the principle of equivalent exchange so that the consumers' demands are satisfied and the commodities' grades are optimized. Obviously, there may be excess commodities in the commodity trading processes in this case. Therefore, not all traders can benefit from the commodity trades. Some commodities' trading can generate profits, while others do not.

There are multiple commodity market subblocks, which simulate the commodity trading process by the rules designated above. By simulating each transaction, the variety, quantity, grade, and price of commodities traded in each market at any moment can be known, and then the operation and profitability of each commodity trader can be learned. At the same time, the model can also calculate the variables' information such as inflation rate, supply, and demand. All this information becomes the outputs of various commodity market blocks of the SED model.

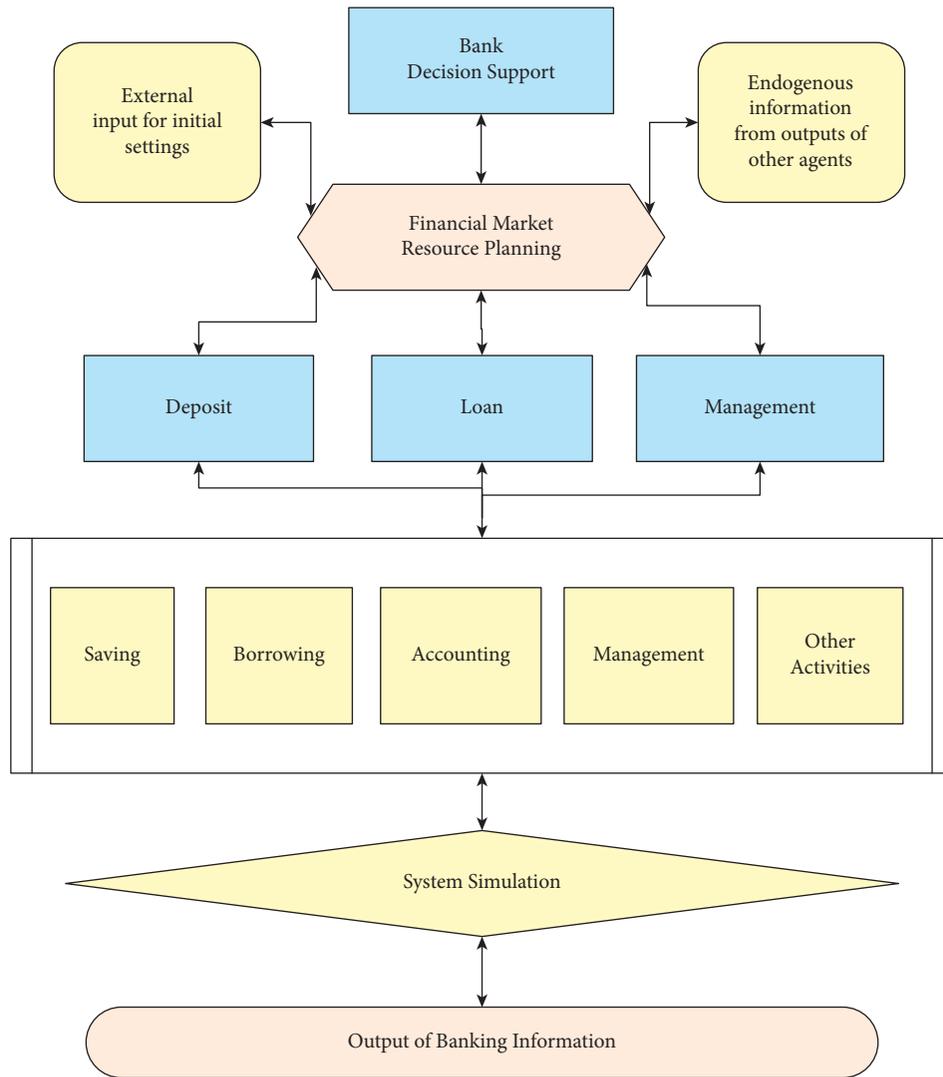


FIGURE 5: Overview of the process and structure of banking system (source: Guangzhou Milestone Software Co., Ltd., China).

4.1.4. Bank Block. The bank block of the SED model is a computer simulation block to simulate various financial business behaviors of banks and clients in the liquid market. The block has savings, credits, funds, finance, and business decision-making departments. The functionality of each department's simulation block is complete (see Figure 5).

Each bank block has the initialized variables from outside the model, such as banking capacity, own funds, technical level, deposit, and loan balances. At the same time, there are variables from other blocks, such as varieties and quantities of savings and borrowings in demand, deposits, loans, and reserve requirements. Each bank block in the SED model simulates a specific object of commercial banks. Each bank operates its loan business in the financial market based on the bank's own initial funds plus subsequent income, as well as deposits. The direct purpose of banks using funds is to obtain interest income under the premise of ensuring the safety of funds. However, the deposit and loan business of banks can improve the utilization rate of funds by making full use of the idle funds of deposit clients, transferring them

to borrowing clients who need capital to expand their business, and can indirectly create the value of social wealth by balancing the monetary value of different commodities in production and circulation markets.

As for the savings of the financial market, clients of the bank block are enterprises, residents, securities, government, and inter-banks. Once a bank has obtained deposit funds, it can transfer them to another bank or enterprise (the SED model does not currently have a resident loan subblock). Banks evaluate the creditworthiness of each enterprise and make loans after the credit has reached a certain threshold. The interest rates on deposits and loans of banks are subject to the provisions of the central bank. Banks are also required to pay reserves.

In the SED model, there are financial statements based on statistics of bank businesses, whose accounts correspond to those in reality. The business properties of each bank block in the SED model are composed of population size, money amount, commissions, interest rates, equipment quantities, deposit, and loan balances, which are changeable

frequently at different times of a period. These bank statistics of the SED model are outputted from the model regularly.

4.1.5. Securities Block. The securities block of the SED model simulates the business process of issuance of shares on the primary market and stock trading on the secondary market by securities companies in the capital market. The securities block has subblocks of stock issuance, stock trading, client account management, fund allocation, finance, and management. Each subblock is fully functional (see Figure 6).

Each securities block has the initialized variables from outside the model, such as the securities business capability, own funds, technical level, and market turnover ratio. There are also relevant variables from other blocks, such as the varieties and quantities of the public offerings and purchased stocks, as well as the central bank's policy provisions related to securities business. Each securities block in the model simulates a specific object of a securities company. Each securities company carries out securities business on the securities stock market by disposing of those funds from the company's initial own funds plus subsequent income, as well as borrowings. The direct purpose of using funds by securities companies is to obtain service charges under the premise of ensuring the stability of the stock market. At the same time, the securities business of securities companies can improve the utilization rate of funds by making full use of the idle funds of clients involved in stock trading so that funds can flow freely in different capital markets and can directly create the value of social wealth by balancing the monetary value of different commodities in production and circulation markets.

In the primary securities market, the clients of the securities blocks are all enterprises, that is, all enterprises simulated by the SED model are listed companies. In the actual simulation process, it is assumed that each enterprise can be listed through a securities company if it is continuously profitable and meets the listing conditions. After the enterprise applies for listing, the securities company will evaluate its listing qualification and arrange for listing only if it meets the criteria of listing. Once the enterprise has obtained the listing funds, it can use them to expand the business. After a company goes public, its stock circulates in the secondary stock market. As a result, residents can set up stock trading accounts in securities companies and buy and sell stocks at their own discretion. The stock price is determined by factors including stock price-earnings ratio and the investment demand of the residents. The SED model has a stock buying and selling submodel, where the rules of buying and selling are basically in line with the status quo of the real stock market. At the same time, each listed enterprise can issue new shares to expand the scale of production.

The service fee standards of securities companies shall be subject to the provisions of the central bank. The securities business statistics of the SED model constitute financial statements, whose accounts correspond to those in reality.

In the SED model, business properties of each securities block consist of population size, money amount, commissions, equipment quantities, the balance of stock buying and

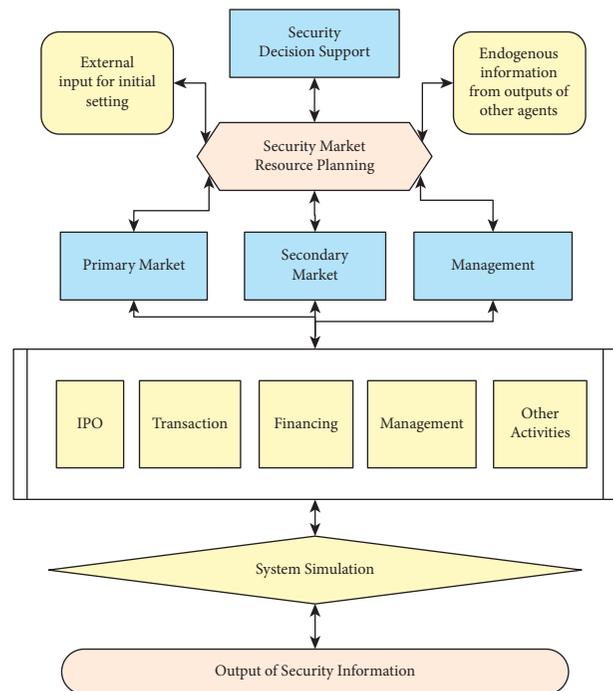


FIGURE 6: Overview of the process and structure of securities system (source: Guangzhou Milestone Software Co., Ltd., China).

selling, and so on, which are changeable frequently at different times of a period. These securities statistics of the SED model are outputted from the model regularly.

4.1.6. Government Block. The government block of the SED model simulates the process of social and economic movement managed by the state of an independent economy through various macroeconomic policies and administrative means. The government block has subblocks of public finance, taxation, central bank, planning, price management, infrastructure investment, investment in science and technology, and social welfare. Each subblock is fully functional (see Figure 7).

The government block only has one simulation object. In this block, each department subblock has initialized variables from the outside of the model such as population, production capacity, natural resources, capital funds, and so on. In the meantime, there are variables from other blocks, such as employment, wage income, merchandise sales in markets, prices, bank balances of the deposits, interest rates of banks, stock prices, stock price-earnings ratios, and so on. The government chooses among investments, reserves, and consumptions based on the initial national wealth plus subsequent national income by administering these monetary funds, production capacity, and natural resources. The purpose of government management is to obtain more wealth value in the later period of time, that is, not only to meet the residents' demands for utility quantity but also to encourage their pursuit of higher utility grades. Therefore, in the SED model, according to the new theory of value, the government is not in the mere pursuit of GDP growth or improvement of other several economic indicators such as

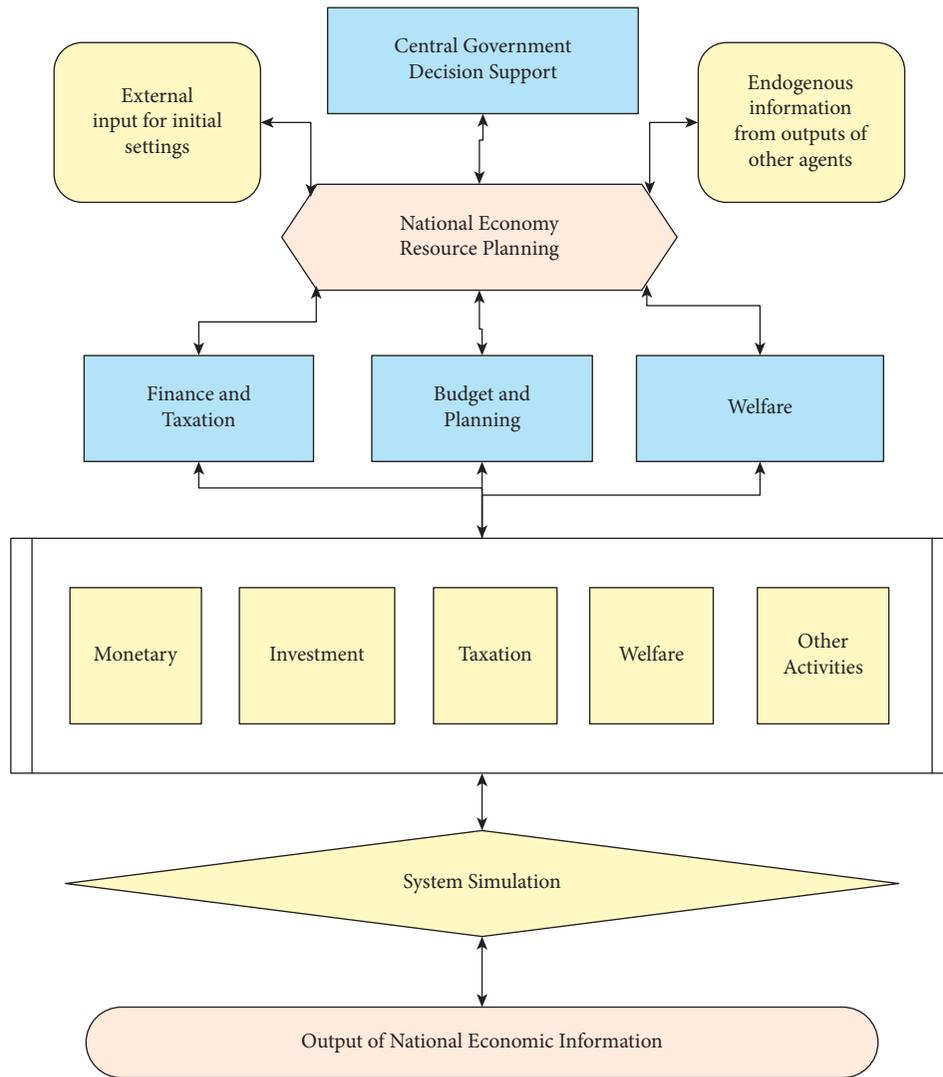


FIGURE 7: Overview of the process and structure of government module (source: Guangzhou Milestone Software Co., Ltd., China).

employment rate but in the pursuit of an improvement in the total value of social wealth.

The government manages socio-economic activities, mainly through macroeconomic policies and corresponding administrative means. According to the new theory of value, there are two main types of government macroeconomic policies: contractionary policy and expansionary policy. The SED model establishes an automatic control model for the government’s macroeconomic management, whose mechanisms have been detailed in Section 3. The automatic control model adopts contractionary and expansionary policies to different extents according to the overcapacity situation of various industries in the model operation. When the society has an excess production capacity, the government adopts contractionary macroregulation policies. When the society’s production capacity is insufficient, the government adopts expansionary macroregulation policies. The government macroregulation policies aim at balancing the supply and demand for production capacity in all sectors as much as possible. In order to fully simulate the real socio-

economic movement process, two different control methods are designed in the simulation operation of the SED model, either by entering macromanagement policy measures from the outside of the model or by using the automatic control model. Note that in the course of specific socio-economic simulations, under Adam Smith’s well-known assumption of “improvement of the dexterity of the workman,” it is always assumed that labor productivity is a strictly monotonically increasing function of time in the social production process of every product. Therefore, if the products of various industries cannot be upgraded, then the phenomenon of excess production of such products will emerge after a period of time. Therefore, the government’s macroregulation policies should also consider the factors driving sustainable economic development, encouraging timely investments in science and technology and promoting new technologies and products.

In addition, the government has a vital function in socio-economic management, which is to guarantee the right of life of every citizen. The SED model has a dedicated welfare

block that sends relief to a resident from the government's social welfare fund following the relevant provisions when the resident's income falls below the poverty line set by the government. Of course, the government allocates funds to the social welfare fund appropriately in each period. Other basic government subblocks include public finance, taxation, central bank, planning, price management, and infrastructure investment, all of which operate realistically. Here, it will not be explained in detail.

In the SED model, there are financial statements based on government statistics, whose accounts correspond to those in reality. The business properties of the government block include population size, money amount, prices, equipment quantities, number of services, and so on, which are changeable frequently at different times of a period. These government statistics of the SED model are outputted from the model regularly.

4.2. Regional Economic Simulation. Based on the national economic simulation, regional economic simulation is a simulator of the national economic system consisting of the local government system and the local economic system under the unified management of the central government (see Figure 8). There are subordinate relationships between local economic regions at different levels and parallel relationships for regions at the same level, i.e., relatively independent but economically connected (see Figure 9).

- (1) Regional model framework.
- (2) Main blocks.

The primary block function of the regional economy is similar to that of the national economic system (see Figure 10), except that local governments do not have the function block of the central bank.

Under the unified macroregulation policies of the central government, the local governments manage according to local conditions. There is a correlation between regional economy and other regional economies and the national economy. Also, there are more specific problems, i.e., (1) the static optimal planning of resource allocation within the regional economic system; (2) the dynamic optimal planning of resource allocation within the regional economic system; (3) the interaction between the developments of various industries and the optimal allocation of resources among industries within the regional economy; (4) the optimal development of the regional economy under the interaction between regional economic development, and the economic developments of other regions as well as the country; (5) the evaluation of the effect of local government macroregulation policies on the internal economic development of the region; (6) the evaluation of the effect of local government macroregulation policies on the economic development of other regions and the country; (7) the evaluation of the effect of the central government's overall macroregulation policies on regional economic development; and (8) the unified planning of regional and national economic developments.

4.3. International Economic Simulation. Based on the national economic simulation, international economic simulation is a parallel simulator of national economic systems linking all independent countries into one whole through international trade, international capital, and international financial markets, while at the same time each country is an independent economy.

- (1) International model framework: the SED international model constructs a global economic system, which can simulate economic performances of multiple countries in the world, economic dynamics of international trade, and international finance between different countries (see Figure 11).

The basic configuration of the international model is the approach of two-country and three-party, that is, to realize the simulation of the global economic system composed of two specific countries and the third party, which is the rest of countries except these two countries.

On this basis, the international model supports expanding the number of regions up to 200 economies. The model can simulate the global economic system accompanied by the economic simulation of up to 199 independent countries, together with all countries except those 199 countries as the third party in the global economic system.

- (2) Main blocks: the core feature of the SED international model is the international economic management block, mainly including international economic system, international trade management, foreign exchange management, and international investment management. It uses modern simulation technology to simulate the performance of the real global economy, by constructing international trade, international capital, and international financial markets to connect independent countries as a whole, while at the same time each country is an independent economy (see Figure 12).

4.3.1. The International Economic System. By using specific parameters and inputting characteristic data, the SED international model can simulate the actual economic situation of different countries, as well as their historical process, current economic conditions, and future trends in the global economy in the form of independent countries or economic unions (for details of the economic operation within an independent country, refer to Section 4.1).

4.3.2. International Trade Management. International trade management is divided into two functional modules: (1) tariff management—the management of import and export tariffs on different commodities from different countries, mainly the setting of tariff rates; (2) import and export commodity management—customs management and restrictions of the types and quantities of imported and exported commodities, mainly the total annual quota and allocation plan.

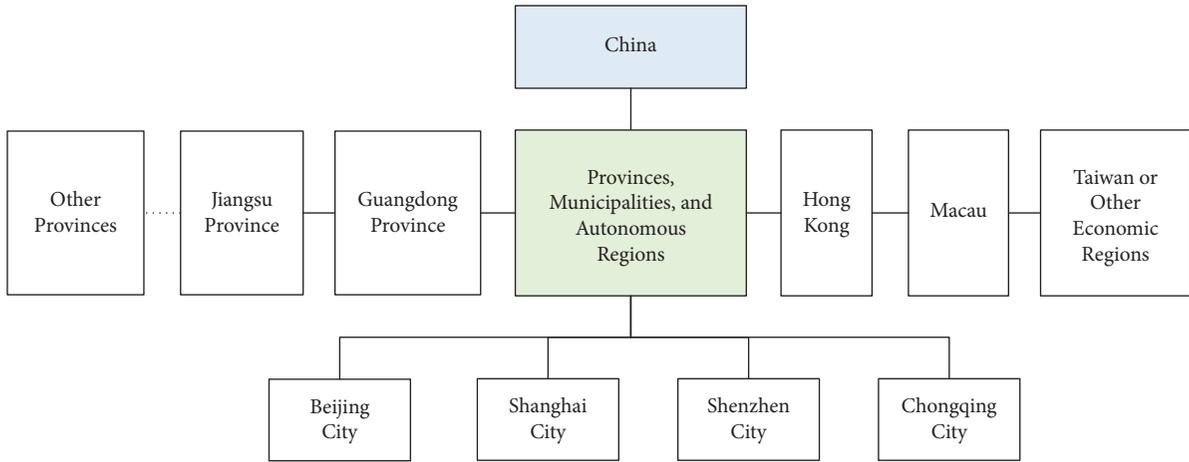


FIGURE 8: Regional economy simulation system (source: Guangzhou Milestone Software Co., Ltd., China).

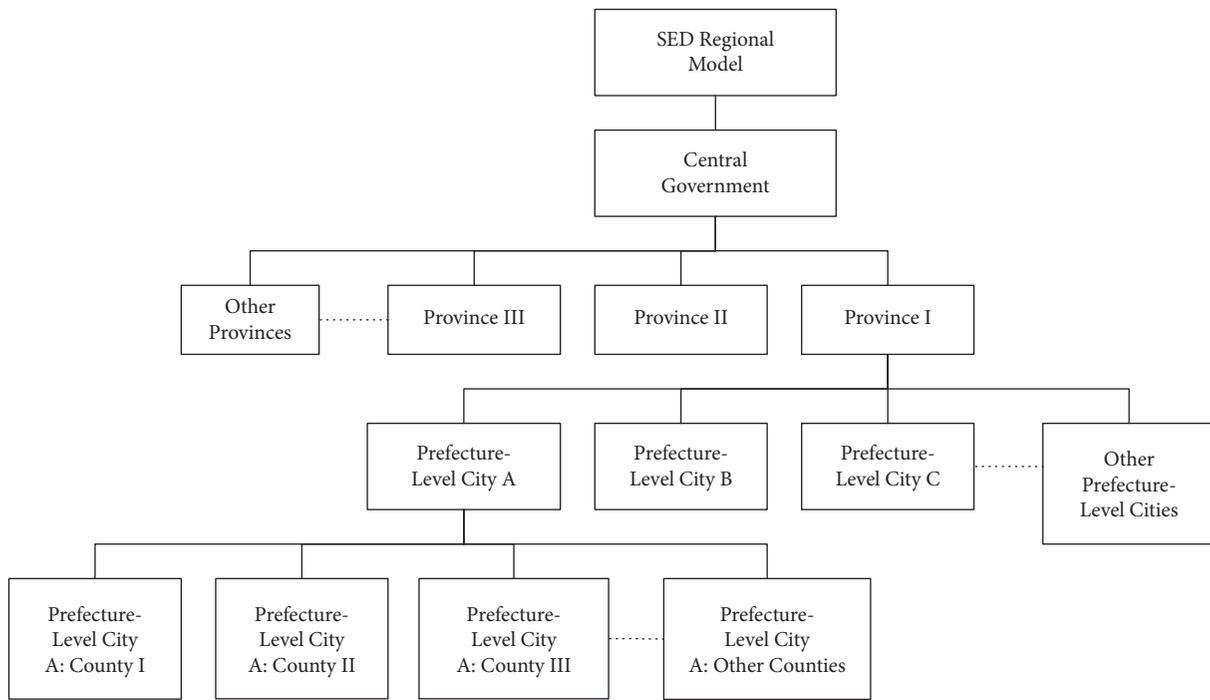


FIGURE 9: Multi-level structure of administrative regions (source: Guangzhou Milestone Software Co., Ltd., China).

4.3.3. *Foreign Exchange Management.* Foreign exchange management simulates the foreign exchange controls and regulations from relevant departments of each country, including foreign exchange income and expenditure, transactions, debits and credits, transfer payments, and international settlements, as well as controls exchange rates and foreign exchange markets, which include the setting of foreign exchange reserves, foreign exchange rates, etc.

4.3.4. *International Investment Management.* International investment management refers to the international economic activities in which international enterprises invest their capital abroad to obtain certain economic benefits. The model includes the management of foreign

investment in the country and the management of the overseas direct investment.

5. The Methods, Testing Standards, and Typical Cases of Using SED Model to Generate China’s Economic Digital Twin System

The steps of using the SED model to produce China’s economic digital twin system for studying real-world problems are as follows. Firstly, the model initialization and parameterization are based on the official data, professional data, and empirical data (official data refer to the international, industrial, and regional data officially released by international economic organizations, institutions, or

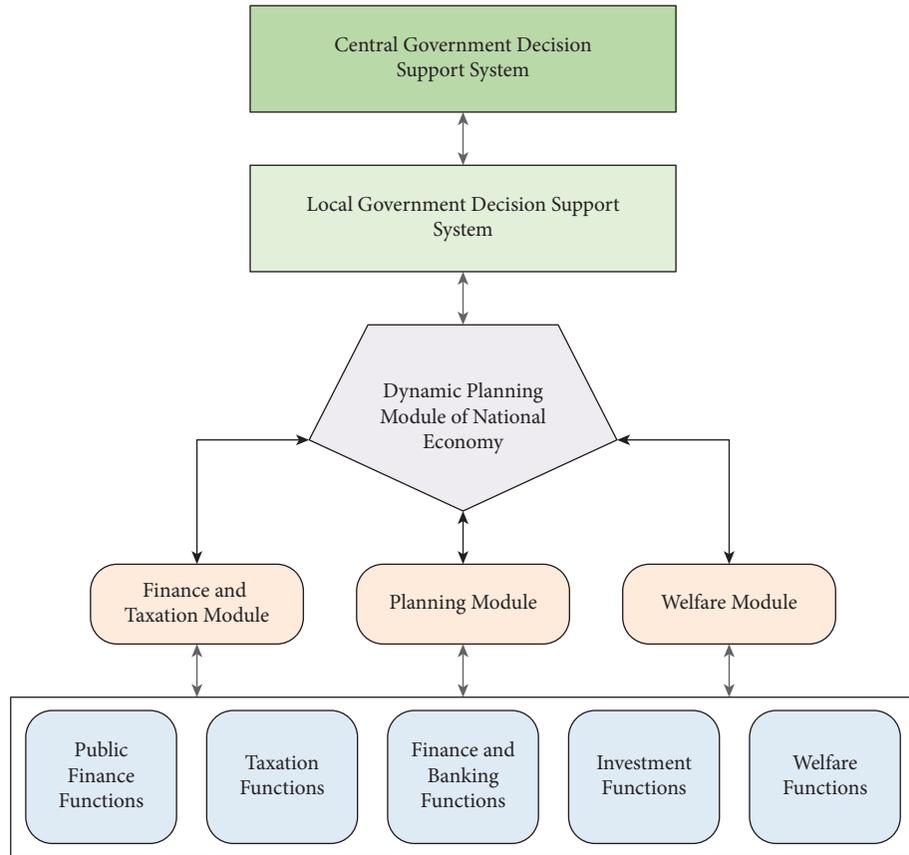


FIGURE 10: Structure of government decision support system (source: Guangzhou Milestone Software Co., Ltd., China).

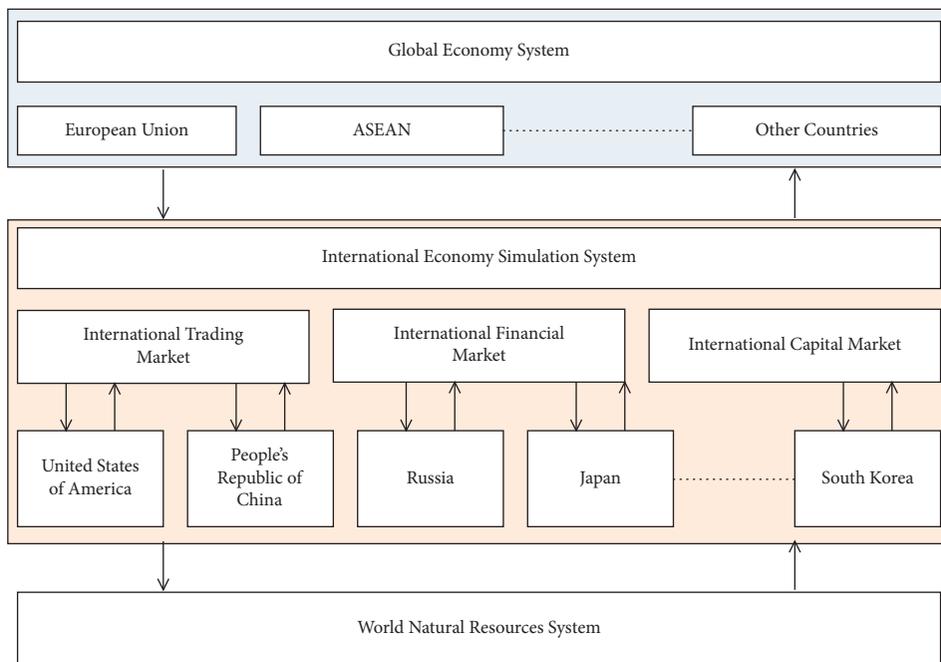


FIGURE 11: International economy simulation system (source: Guangzhou Milestone Software Co., Ltd., China).

departments of various countries; professional data refer to data in the report data of industry associations, industry research institutes, industry experts, statistics institutions,

and colleges and universities recognized by the society; empirical data refer to data reference suggestions that are obtained by authoritative experts in professional fields in

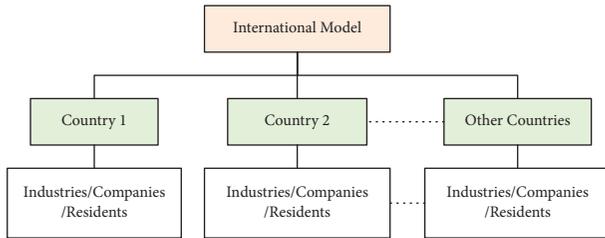


FIGURE 12: Framework of international model (source: Guangzhou Milestone Software Co., Ltd., China).

various countries through a comprehensive analysis of the two types of data as mentioned above in accordance with the law of economy and society) that are authoritative, stable, and public and on the principle of data disaggregation, normalization, and value consistency (data disaggregation principle means that if the simulated objects, such as the enterprises' fixed assets and liquidity, have no corresponding microdata but meso or macrodata, the meso or macrodata can be disaggregated by the proportions (statistical law) among enterprises; normalization is the process of converting some of the dimensional data to dimensionless data, which is convenient for data processing and the acceleration of model convergence; the principle of value consistency means that product quantity or price is calculated by the formula of "price \times quantity = total value" given product price or quantity under the condition that the total value of the product remains unchanged). Secondly, the model itself is authenticated, calibrated, and verified to meet the standards and requirements for simulation, i.e., (1) the simulation accuracy of crucial economic indicators, such as GDP, unemployment rate, and inflation rate, reaches more than 90% in three consecutive years; (2) macro and microresults of simulation conform to the empirical law of economics, such as Phillips curve, Lorenz curve, and Engel curve; (3) the simulation results are in line with the model mechanism; and (4) the simulation results pass the balance sheet test and monetary base test (balance sheet test means that in each simulation step of the entire model, assets and debts for each microagent are kept balanced, i.e., the sum of assets and debts is equal to zero; monetary base test refers to the test where in each simulation step of the entire model, the sum of money owned by each microagent is equal to the total money stock issued by the central bank). Finally, simulation operations, decision deductions, and result analysis are carried out following the mechanism and rules. At present, the SED model has been successfully applied in the simulation analysis of the international economy, macroeconomy, industrial economy, and regional economy, which is sufficient to verify the scientific nature of the modeling mechanism of the SED model and the feasibility of the SED model in simulation analysis.

For international economic simulation, the SED model has been used in a collaborative study based on the SED model to simulate the Sino-US trade war, by simulating the impact of import tariffs imposed by China and the United States on their GDP, respectively, which was carried out by the Central Institute of Economics and Mathematics of the Russian

Academy of Sciences and Wu [33, 41]. On the basis that the deviation of the simulated macroeconomic variables such as GDP, unemployment rate, and CPI from the known statistics did not exceed 3%, the scenarios of raising import tariffs by 5, 10, and 15 percentage points, respectively, between China, Russia, Europe, and the United States were calculated. The results showed that the trade war would lead to a decline in GDP in China, Russia, Europe, the United States, and the rest of the world, but the lowest decline in China's GDP.

For China's macroeconomic simulation, Wu et al. [47] used the SED model to carry out the research project, namely, "Analysis of the Effect of Four-Trillion Fiscal Investment in the Context of Global Financial Crisis 2008 and the Application of SED Model (see details about macro, industrial, and regional economic simulation in the policy reports on the following website: <https://www.gzmss.com/website/>)." By keeping the discrepancy between the simulated GDP and the real statistical data lower than 5% for 3 consecutive years, 2007–2009, and conforming simulation results to primary economic laws (e.g., Phillips curve and Engel's law), the following scenarios were simulated: (1) the effect of fiscal stimulus policies under the 2008 financial crisis, (2) the macroeconomic effects of the financial crisis itself, and (3) the difference of the macroeconomic effects between the fiscal stimulus policies implemented and the composite fiscal policies in the globally automatic module of the SED model. The simulation results showed that (1) after 2008 financial crisis, the adoption of expansionary fiscal policies would inevitably lead to the acceleration of GDP growth and inflation; (2) financial crisis and the subsequent macroregulation policies have not only slowed down China's economic growth rate but also increased the inflation rate and Gini coefficient, compared with the scenario without the 2008 financial crisis; and (3) a simple loose fiscal policy cannot promote the long-run development of the national economy, compared with the composite fiscal policies that can generally balance the revenues and expenditures.

For China's industrial economic simulation, Wu et al. carried out a research project, namely, "Using SED Model to Analyze the Impact of Strategic Emerging Industries on China's GDP." By keeping the deviation of the GDP simulated from the actual statistical data lower than 5% for 3 consecutive years, 2010–2012, the following scenarios were simulated: (1) under the current tendency whether the development of emerging industries, their added-value proportion to GDP, in 2014–2015, and (2) different impacts of different government policies on strategic emerging industries. The simulation results were as follows: (1) the development of emerging industries, their added-value proportion to GDP, would meet the planned target in 2014–2015 would be 6.7% and 7.2%, respectively, failing to reach the planned target of 8% by 2015; (2) appropriate government investments in science and technology and price subsidies would have a significantly positive impact on both GDP and strategic emerging industries.

For the regional economic simulation, Wu et al. carried out the research project called "Applying Innovative Technology of SED Model to Establish a Long-Term

Mechanism of Macroeconomic Decision Support System in Guangdong Province.” Ensuring that the discrepancy between Guangdong’s primary economic indicators such as GDP simulated and the statistics of the National Bureau of Statistics was under 5% for 3 consecutive years, 2010–2012, and the simulation results of SED were in line with Phillips law and Engel’s law, an overall economic simulation and forecast of Guangdong Province in 2013 and 2014 was conducted. The results showed that the model made an accurate prediction of the main economic indicators of Guangdong, especially its GDP, whose prediction error was controlled within 2%. Moreover, the global optimization module of the SED model showed that the overall economy of Guangdong province still has room for continuous improvement. Cooperating with experts and scholars from Guangdong Key Laboratory of High Performance Computing, Wu et al. are continuing to study the application of high-performance computing (especially parallel computing) in regional economic simulation.

6. Conclusion and Prospect

The method of computer simulation experiment based on the SED model is a scientific empirical method, which has the advantages of low cost, low delay, high precision, fine granularity, strong systematic property, and strong objectivity compared with the mainstream empirical methods. The SED model, which can be fully used to form an economic engine and construct a virtual economic system by digital twin method, can be integrated with the extant physical engine in the metaverse concept to build a virtual world consisting of physics, economy, culture, and politics that is close to and coexists with reality. With the development of the metaverse, the SED model has been constantly improving in terms of the theoretical foundation of modeling and computer simulation technology. In the new theory of value, although Wu [38, 40] proposed the Riemann metric space of n -types of commodity values with $2n$ -dimension saddle Riemann manifold as the base manifold, as well as the corresponding measure of value in the Riemann metric space, so that economics can measure commodity value by using time-varying measure of value under the initial state of the differential equation of each commodity value, they did not give a dynamic measure of value under the more general continuous conditions for the field of complex numbers. The paper by Chen and Wang [49] on the convergence of high-dimension Kähler–Ricci flow solved the problem of convergent differential equations based on the metric tensor, which varies with the time parameter in the field of complex numbers. These latest research results lay the foundation for the new theory of value to finally solve the problem of perfect and unified measure of value under variable labor productivity proposed by David Ricardo [50]. In computer simulation technology, the SED model’s functions can be upgraded in the following dimensions: (1) the automatic calibration of model parameters; (2) the automatic generation system of initial data; (3) the parallel control system of model simulation; (4) the establishment and upgrades of case knowledge base for models; and (5) the

economic decision support system based on AlphaGo artificial intelligence technology.

The SED model can provide a simulation platform for research on China’s and global economic development. Note that in 2021, the National Natural Science Foundation of China announced a major special research project, namely, “The Basic Theories and Empirical Evidences on the Law of China’s Economic Development,” which is to supplement the research on this subject since the founding of the People’s Republic of China in 1949, especially since the reform and opening-up in 1978. Ten topics were released in the first phase of the project: (1) the theory of China’s economic growth and structural transformation; (2) the theory of macroregulations with Chinese characteristics; (3) China’s financial system’s reformation and financial security; (4) population and China’s economic development; (5) rural reformation, the theory of poverty reduction and rural revitalization; (6) regional policies and China’s economic development; (7) the coordinated development of resources and environment under the target of “carbon peak and carbon neutral”; (9) international trades, international investments, and China’s economic development; and (10) modeling and simulation of China’s economic system. The SED model can deduce, simulate by trials and errors, and predict the complex and realistic economic system in the digital virtual world, to reveal the interaction between different economic agents and its effects, to explain the phenomenon of China’s economic development and its evolutionary characteristics, to quantitatively analyze the influencing mechanism, paths, and duration of economic policies, as well as the impact of economic policies on social welfare, and to provide methodologies and toolboxes for intelligent analysis and evaluation for the formulation and implementation of future economic policies. The SED model can provide an accurate, efficient, and operable intelligent simulation platform for the research on the first nine topics mentioned above, relying on the modules of governmental macroregulations, finance, population, rural areas, industries, technologies, environment, international trades, and investments.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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