Hindawi
Complexity
Volume 2022, Article ID 5136393, 10 pages
https://doi.org/10.1155/2022/5136393

Research Article

Robust H_∞ Control for Markovian-Jump-Parameters Takagi–Sugeno Fuzzy Systems

Wenzhao Qin[1], **Yukai Shen**[2], **and Lifang Wang**[1]

[1]*Department of Mathematics and Information Engineering, Dongchang College of Liaocheng University, Liaocheng 252000, China*
[2]*Mathematics Group, Liaocheng No. 1 High School of Shandong Province, Liaocheng 252000, China*

Correspondence should be addressed to Wenzhao Qin; qinwenzhao@lcudcc.edu.cn

Received 17 August 2022; Accepted 29 September 2022; Published 4 November 2022

Academic Editor: Atila Bueno

The H_∞ performance of a class of *T-S* fuzzy systems with Markovian-jump parameters is analyzed. A state feedback controller is designed for *T-S* fuzzy systems with Markovian-jump parameters. First, a new modal-dependent Lyapunov function composed of closed-loop functions is constructed, which can fully use system status information. Based on this function, the stability conditions with less conservatism are given by linear matrix inequalities (LMIS). At the same time, a design algorithm for a state feedback controller is proposed for the Markovian-jump-parameters *T-S* fuzzy systems, which ensures the system's stability under the condition of $H_\infty - \gamma$ performance. Simulation results demonstrate that the mentioned method is accurate and practical.

1. Introduction

Takagi–Sugeno (*T-S*) fuzzy systems can provide an effective description of complex nonlinear system problems [1]. The essence of *T-S* fuzzy systems is nonlinear systems, but each corresponding rule is linear, which is convenient for people to analyze the systems' stability and study the control problem. In recent decades, *T-S* fuzzy systems have become a research hotspot and an important method for nonlinear system control. The stability of a static output controller for a discrete-time *T-S* fuzzy system was studied by introducing relaxation variables and considering multiple Lyapunov matrices in [2]. The equivalence of the weight product index set for fuzzy rules was defined, and a new stability criterion with less conservatism was proposed to guarantee the system's stability using the equivalence relation in [3]. Two efficient design methods for fuzzy static output feedback controllers were proposed by combining Markovian Lyapunov functions and matrix inequality convexification techniques, and the robust stabilization of a nonlinear hyperbolic partial differential equation (PDE) system represented by a *T-S* fuzzy system is successfully solved in [4]. At the identical time, in order to make the system stable and ensure its good performance, many scholars have studied comprehensive control methods such as H_2 control and H_∞ control. In particular, H_∞ control can effectively suppress interference, so it is widely used in electric ground vehicle systems [5, 6] and electric train asynchronous motor systems [7]. A new augmented Lyapunov function was proposed to ensure the tracking performance of the *T-S* fuzzy networked control systems [8]. The nonlinear systems were transformed into continuous-time *T-S* fuzzy systems with uncertain parameters using the Wertinger integral inequality and the extended inverse convex matrix inequality, and the asymptotic stability condition with less conservativeness was proposed in [9]. A new bounded real lemma was constructed for a class of fuzzy time-delay systems, and a new Lyapunov–Krasovskii functional (LKF) was proposed in [10]. The robust output feedback control problem for a class of interval type 2 *T-S* fuzzy systems with multiple delays and disturbances was studied, and a delay-dependent dynamic output feedback controller was designed using the generalized redundancy method to make the closed-loop system asymptotically stable and obtain the performance in [11]. On the other side of research, the structure or parameters of increasingly complex automatic control systems inevitably jump randomly after a long period of operation, so stochastic hybrid systems with Markovian-jump parameters have received extensive attention. Markovian-jump systems are very effective in describing environmental changes, system failures, and repairs and therefore are widely used in circuit systems, aircraft systems, manufacturing systems [12–16], and networked control systems under cyber-attacks [17]. In recent years, many scholars are interested in the Markovian-jump fuzzy systems, and these studies have yielded a large number of meaningful results [18–20].

Moreover, as digital technology expands rapidly, research on sampled-data control has received high attention. The sampled-data control methods only need to use the instantaneous sampling information of the system, which greatly reduces the information transmission, so it can guarantee a better system performance and reduce the system operating costs. Sampled-data control methods include input delay method [21], discrete-time system method [22], and closed-loop function method [23]. It is worth mentioning that the looped function mentioned in the closed-loop function method is not necessarily a positive function that fully utilizes the information on the intervals $x(t_k)$ to $x(t)$. At the jumps, this Lyapunov function does not increase, and the whole sampling interval is sufficiently considered to yield less conservative conditions. These functionals are suitable for impulsive systems analysis as they allow one to express the discrete-time stability conditions in an affine manner, thus enabling the consideration of uncertain and time-varying systems. The research results of sampled-data control are numerous and significant [24–28]. However, the application of the looped function method to Markovian-jump *T-S* fuzzy systems with sampled data has been rarely investigated. In [29], a new Lyapunov function whose time scheme consists of an exponential closed-loop function was proposed, and linear interpolation and binning techniques were proposed to obtain criteria for mean-square exponential stability, but the conclusions were less conservative. A new input delay-dependent vector method proposed for fuzzy Markovian systems with sampled data was first mentioned in [30].

Based on the above discussion, our work is motivated by the issue of exploring the control of Markovian-jump *T-S* fuzzy systems based on sampled data. Some of the topics to be pondered are, for example, how to construct the Lyapunov functions to efficiently utilize the system information and how to modify the integral inequality to make the results less conservative. The nonfragile controllers [31–37], which are highly resistant to uncertainty in their parameters, have been widely used in many practical systems, but how we can apply them to Markovian-jump *T-S* fuzzy nonperiodic sampling control systems.

The contributions of this work consist of the following aspects: (1) A new mode-dependent Lyapunov function consisting of bilateral closed-loop functions is used in the *T-S* fuzzy systems with Markovian-jump parameters, this function fully covers the information on both intervals $[x(t_k), x(t)]$ and $[x(t), x(t_{k+1})]$, whereas the Lyapunov function constructed in the literature [38] only contained $\int_{t_k}^{t} x^T(t)\mathrm{d}s$ and does not contained $\int_{t}^{t_{k+1}} x^T(t)\mathrm{d}s$, however, our constructed function $V(t)$ includes both, thus the results derived in this study are less conservative. And we use a modified integral inequality with a free matrix to help deal with an integral part of the derivative of the closed-loop function, enabling the systems to be less conservative. (2) A state feedback controller design algorithm is proposed to ensure the system's stability under the H_∞ performance condition and a truck-trailer model is adopted to verify the effectiveness of the proposed method.

The rest of the article is organized as follows: some necessary lemmas and systems depiction are presented in Section 2; the stochastic stability condition for Markovian-jump *T-S* fuzzy nonperiodic sampling control systems is given under the $H_\infty - \gamma$ condition and a sampled-data controller with state feedback is designed in Section 3; and the effectiveness of our proposed method is verified with a truck-trailer model in Section 4; finally, we conclude this work with a summary in Section 5.

2. Preliminaries

2.1. Two Useful Lemmas

Lemma 1 (see [23]). *Let x be a differentiable function: $[\alpha, \beta] \longrightarrow \mathbb{R}^n$. If there is a vector $\eta \in \mathbb{R}^m$, matrix $U = U^T > 0$, $U \in \mathbb{R}^{n\times n}$, and M_1, $M_2 \in \mathbb{R}^{n\times n}$, then the following inequality holds:*

$$-\int_{\alpha}^{\beta} \dot{x}(s)^T U \dot{x}(s)\mathrm{d}s \leq \lambda\eta^T\left[M_1^T U^{-1} M_1 + \frac{\lambda^2}{3} M_2 U^{-1} M_1\right]\eta + 2\lambda\eta^T \lfloor M_2^T + 2\eta^T\left[M_1^T v - 2M_2^T \int_{\alpha}^{\beta} x(s)\mathrm{d}s\right], \quad (1)$$

where $\lambda = \beta - \alpha$, $v = x(\beta) - x(\alpha)$, and $\varrho = x(\beta) + x(\alpha)$.

Remark 1. This inequality is based on the free weight matrix inequality and is improved based on the inequality in [39]. The inequality contains $\int_{\alpha}^{\beta} x(s)\mathrm{d}s$ instead of $1/\beta - \alpha \int_{\alpha}^{\beta} x(s)\mathrm{d}s$, and this will facilitate the deduction of the main results and degrade the conservatism of the main results.

Lemma 2 ((Schur complement lemma) [40]). *If there are symmetric matrixes A, $_B$, and C, then the following three conditions are equivalent:*

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} < 0,$$
$$A < 0, B - C^T A^{-1} C < 0, \quad (2)$$
$$B < 0, A - CB^{-1} C^T < 0.$$

2.2. Depiction of T-S Fuzzy Systems with Markovian-Jump Parameters. Concerning the continuous-time *T-S* fuzzy systems with Markovian-jump parameters as follows, whose IF-THEN rules are described:

(1) Rule i: IF $\varphi_1(t)$ is ζ_{i1}, $\varphi_2(t)$ is ζ_{i2},. . ., $\varphi_p(t)$ is ζ_{ip}, THEN

$$\begin{cases} \dot{x}(t) = A_i(r(t))x(t) + B_{1,i}(r(t))u(t) + D_{1,i}(r(t))w(t), \\ z(t) = C_i(r(t))x(t) + B_{2,i}(r(t))u(t) + D_{2,i}(r(t))w(t), \end{cases} \quad (3)$$

where $i \in \mathscr{R} = (1, 2, \ldots, r)$, r is the number of IF-THEN rules, $\varphi_1(t)$, $\varphi_2(t)$, $\cdots$, $\varphi_p(t)$ are premise variables, $\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{ip}$ are fuzzy sets, $x(t)$ is the vector describing status of system, $u(t)$ is the vector indicates the control input, $w(t)$ is the perturbation input taking value in range $L_2[0, +\infty)$, $z(t)$ is the vector of control output, $r(t)$ is a Markovian process in continuous time taking value in a finite set $N = 1, 2, \ldots, s$, and $r(t)$ with switching transition probability matrix $\Pi = [\pi_{\iota\kappa}]_{s\times s}$ is described as follows:

$$P_r\{r(t+\Delta) = \kappa | r(t) = \iota\} = \begin{cases} \pi_{\iota\kappa}\Delta + o(\Delta), \iota \neq \kappa, \\ 1 + \pi_{\iota\kappa}\Delta + o(\Delta), \iota = \kappa, \end{cases} \quad (4)$$

where $\Delta > 0$, $\lim_{\Delta\longrightarrow 0}(o(\Delta)/\Delta) = 0$. When $\iota \neq \kappa$, $\pi_{\iota\kappa}(>0)$ indicate the rate of mode transition from mode ι at time t to mode κ at time $t + \Delta$, and $\pi_{\iota\iota} = -\sum_{\kappa=1,\kappa\neq\iota}^{s} \pi_{\iota\kappa}$.

According to fuzzy logic inference method, system (3) can be reconstructed as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\varphi(t))\left\{A_i(r(t))x(t) + B_{1,i}(r(t))u(t) + D_{1,i}(r(t))w(t)\right\}, \\ z(t) = \sum_{i=1}^{r} h_i(\varphi(t))\left\{C_i(r(t))x(t) + B_{2,i}(r(t))u(t) + D_{2,i}(r(t))w(t)\right\}, \end{cases} \quad (5)$$

where $\varphi(t) = [\varphi_1(t)\, \varphi_2(t) \ldots \varphi_p(t)]$, $h_i(\varphi(t)) = (\vartheta_i(\varphi(t))/\sum_{j=1}^{r} \vartheta_j(\varphi(t)))$, and $\vartheta_i(\varphi(t)) = \prod_{j=1}^{p} \zeta_{ij}(\varphi_j(t))$, $\zeta_{ij}(\varphi_j(t))$ denote the membership degree of $\varphi_j(t)$ in ζ_{ij} $(j = 1, 2, \ldots, p)$, we can easily find out that for all t, $\vartheta_i(\varphi(t)) \geq 0$, $\sum_{i=1}^{r} \vartheta_i(\varphi(t)) \geq 0$, $h_i(\varphi(t)) \geq 0$, and $\sum_{i=1}^{r} h_i(\varphi(t)) = 1$. For all the possible $r(t) = \iota$, we can simplify fuzzy system (5) as follows:

$$\begin{cases} \dot{x}(t) = A_\iota(t)x(t) + B_{1\iota}(t)u(t) + D_{1\iota}(t)w(t), \\ z(t) = C_\iota(t)x(t) + B_{2\iota}(t)u(t) + D_{2\iota}(t)w(t), \end{cases} \quad (6)$$

where

$$A_\iota(t) = \sum_{i=1}^{r} h_i(\varphi(t))A_{\iota i}, \; B_{1\iota}(t) = \sum_{i=1}^{r} h_i(\varphi(t))B_{1\iota i},$$
$$B_{2\iota}(t) = \sum_{i=1}^{r} h_i(\varphi(t))B_{2\iota i}, \; C_\iota(t) = \sum_{i=1}^{r} h_i(\varphi(t))C_{\iota i}, \quad (7)$$
$$D_{1\iota}(t) = \sum_{i=1}^{r} h_i(\varphi(t))D_{1\iota i}, \; D_{2\iota}(t) = \sum_{i=1}^{r} h_i(\varphi(t))D_{2\iota i}.$$

Consider the zero-order hold function, which gets a set of hold times $0 = t_0 < t_1 < \cdots < t_k < \cdots \lim_{k\longrightarrow\infty} t_k = +\infty$, and for any $k > 0$, $t_{k+1} - t_k = h_k \in [h_m, h_M]$, h_m indicate the lower boundary of the sampling periods, and h_M indicate the upper bound of the sampling periods, then, the state feedback controller for system (3) is designed as follows.

(2) Rule j : IF $\varphi_1(t)$ is ζ_{j1}, $\varphi_2(t)$ is ζ_{j2},. . ., $\varphi_p(t)$ is ζ_{jp}, THEN

$$u(t) = K_j x(t_k), \; i = 1, 2, \ldots, r. \quad (8)$$

Based on the fuzzy logic inference method, the state feedback controller $u(t)$ is reconstructed as follows:

$$u(t) = K(t)x(t_k), \quad (9)$$

where $K(t) = \sum_{j=1}^{r} h_j(\varphi(t))K_j$.

Substituting the $u(t)$ into system (6), the closed-loop system is as follows:

$$\begin{cases} \dot{x}(t) = A_\iota(t)x(t) + B_{1\iota}(t)K(t)x(t_k) + D_{1\iota}(t)w(t), \\ z(t) = C_\iota(t)x(t) + B_{2\iota}(t)K(t)x(t_k) + D_{2\iota}(t)w(t), \; t_k \leq t < t_{k+1}. \end{cases} \quad (10)$$

3. Main Results

We propose a sufficient condition to solve the control problem formulated in the previous section.

First, two definitions are necessary.

Definition 1 (see [41]). When $u(t) = 0$, the system (10) is stochastic stable, if the continuous function $\Phi(t) \in \mathbb{R}^n$ defined on the interval $[-\tau, 0]$, and the initial mode $r_0 \in N$, then the following inequality holds:

$$\lim_{t\longrightarrow\infty}\varepsilon\left\{\int_0^t x^T(s,\Phi,r_0)x(s,\Phi,r_0)ds\right\} < \infty, \tag{11}$$

where $\varepsilon\{\cdot\}$ denotes the mathematical expectation and $x(s,\Phi,r_0)$ indicates the solution of system (10) at the instant t under the initial condition $\Phi(t)$ and r_0.

Definition 2. For a control law that any $w(t) \neq 0$, $w(t) \in L_2[0,\infty)$, system (10) is stochastic stable under the $H_\infty - \gamma$ condition. That is to say, for some scalar $\gamma > 0$, system (10) is stochastic stable if the inequality

$$\varepsilon\left\{\int_0^\infty z^T(t)z(t)\mathrm{d}t\right\} < \gamma^2\int_0^\infty w^T(t)w(t)\mathrm{d}t, \tag{12}$$

holds under zero initial condition.

Next, we focus on the following system:

$$\dot{x}(t) = A_\iota(t)x(t) + B_{1\iota}(t)K(t)x(t_k), \quad t_k \le t < t_{k+1}. \tag{13}$$

If the controller is a constant, we will give the stochastic stability conditions as follows:

Theorem 1. *Given a scalar $\rho > 0$, T-S fuzzy system (13) is stochastic stable if there are matrices $P_i > 0$, $U_1 > 0$, $U_2 > 0$, $H_1 = H_1^T$, H_2, $H_3 = H_3^T$, and H_4, and arbitrary matrices Y_1, Y_2, Y_3, $M_1(t,t_k)$, $M_2(t,t_k)$, $N_1(t,t_k)$, and $N_2(t,t_k)$ with appropriate dimensions such that the following inequation is true for each $i \in \mathbb{R}$, $\iota \in N$, $h_k \in \{h_m, h_M\}$:*

$$\Xi_q < 0, q = 1, 2, \tag{14}$$

where $\Xi_1 = \begin{bmatrix} \Delta_1(t,t_k) & \sqrt{h_k}M_1^T(t,t_k) & \sqrt{h_k}h_M M_2^T(t,t_k) \\ * & -U_2 & 0 \\ * & * & -3U_2 \end{bmatrix}$,
$\Xi_2 = \begin{bmatrix} \Delta_2(t,t_k) & \sqrt{h_k}N_1^T(t,t_k) & \sqrt{h_k}h_M N_2^T(t,t_k) \\ * & -U_1 & 0 \\ * & * & -3U_1 \end{bmatrix}$,
$\Delta_1(t,t_k) = \Theta_1(t,t_k) + \Theta_2(t,t_k) + h_k\Theta_3(t,t_k)$, $\Delta_2(t,t_k) = \Theta_1(t,t_k) + \Theta_2(t,t_k) + h_k\Theta_4(t,t_k)$, $\Theta_1(t,t_k) = Sym\{e_1^T P_i e_6 - \Pi_1^T H_2\Pi_2 + \Pi_4^T H_4\Pi_2 + M_1^T(t,t_k)\Pi_6 - 2M_2^T(t,t_k)e_4 - N_1^T(t,t_k)\Pi_7 - 2N_2^T(t,t_k)e_5\} + e_1^T\sum_{j=1}^s \pi_{ij}P_j e_1 - \Pi_1^T H_1\Pi_2 + \Pi_4^T H_3\Pi_4$, $\Theta_2(t,t_k) = Sym\{\Pi_{10}^T\Phi_1(t,t_k)\}$, $\Theta_3(t,t_k) = Sym\{\Pi_1^T H_1\Pi_3 + \Pi_3^T H_2\Pi_2 + N_2^T(t,t_k)\Pi_9\} + e_6^T U_1 e_6$, $\Theta_4(t,t_k) = Sym\{\Pi_4^T H_3\Pi_5 + \Pi_5^T H_2\Pi_2 + M_2^T(t,t_k)\Pi_8\} + e_6^T U_2 e_6$, $e_j = \left[0_{n\times(j-1)n}\ I_n\ 0_{n\times(6-j)n}\right]$, $j = 1,2,\ldots,6$, $\Pi_1 = [e_1^T - e_2^T\ e_4^T]^T$, $\Pi_2 = [e_2^T\ e_3^T]^T$, $\Pi_3 = [e_6^T\ e_1^T]^T$, $\Pi_4 = [e_1^T - e_3^T\ e_5^T]^T$, $\Pi_5 = [e_6^T - e_1^T]^T\Pi_6 = [e_1^T - e_2^T]^T$, $\Pi_7 = [e_1^T - e_3^T]^T$, $\Pi_8 = e_1 + e_2$, $\Pi_9 = e_1 + e_3$, $\Pi_{10} = [e_1^T Y_1 + e_2^T Y_2 + e_6^T Y_3]^T$, *and* $\Phi_1(t,t_k) = A_\iota(t)e_1 + B_{1\iota}(t)K(t_k)e_2 - e_6$.

Proof. Let us take the Lyapunov function for system (13) as follows:

$$\mathscr{W}(r_t,t) = V(r_t,t) + \mathscr{V}_0(t), \quad t \in [t_k,t_{k+1}), k \in N, \tag{15}$$

where $V(r_t, t)$ is a quadratic Lyapunov function defined as $V(r_t, t) = x^T(t)P(r(t))x(t)$, and $\mathscr{V}_0(t) = \sum_{p=1}^2 \mathscr{V}_p(t)$ with $\mathscr{V}_1(t) = (t_{k+1}-t)\eta_1^T[H_1\eta_1(t) + 2H_2\eta_3(t)] + (t-t_k)\eta_2^T[H_3\eta_2(t) + 2H_4\eta_3(t)]$, $\mathscr{V}_2(t) = (t_{k+1}-t)\int_{t_k}^t \dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s - (t-t_k)\int_t^{t_{k+1}}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s$, where $\zeta_1(t) = \int_{t_k}^t x(s)\mathrm{d}s$, $\zeta_2(t) = \int_t^{t_{k+1}} x(s)\mathrm{d}s$, $\zeta_3(t) = x(t) - x(t_k)$, $\zeta_4(t) = x(t) - x(t_{k+1})$, $\eta_1(t) = \mathrm{col}\{\zeta_3(t), \zeta_1(t)\}$, $\eta_2(t) = \mathrm{col}\{\zeta_4(t), \zeta_2(t)\}$, $\eta_3(t) = \mathrm{col}\{x_{t_k}, x_{t_{k+1}}\}$, and $\eta(t) = \mathrm{col}\{x(t), \eta_3(t), \zeta_1(t), \zeta_2(t), \dot{x}(t)\}$.

Then, for each $r(t) = \iota$, $\iota \in S$, we say that L is a weak infinitesimal generator of $\mathscr{W}(r_t,t)$, it follows that $LV_i(t) = 2\eta^T(t)e_1^T P_i e_6\eta(t) + \eta^T(t)e_1^T\sum_{j=1}^s \pi_{ij}P_j e_1\eta(t)$, $L\mathscr{V}_1(t) = \eta^T(t)\{-\Pi_1^T(H_1\Pi_2 + 2H_2\Pi_2) - 2(t_{k+1}-t)[\Pi_1^T H_1\Pi_3 + \Pi_3^T H_2\Pi_2] + \Pi_4^T(H_3\Pi_4 + 2H_4\Pi_2) + 2(t-t_k)[\Pi_4^T H_3\Pi_5 + \Pi_5^T H_4\Pi_2]\}\eta(t)$, $L\mathscr{V}_2(t) = (t_{k+1}-t)\eta(t)^T e_6^T U_1 e_6\eta(t) - \int_{t_k}^t \dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s + (t-t_k)\eta(t)^T e_6^T U_2 e_6\eta(t) - \int_t^{t_{k+1}}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s$,

By Lemma 1, the following two inequalities hold:

$$\begin{aligned} -\int_{t_k}^t \dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s \le \eta^T(t)\Big\{&(t-t_k)\left[M_1^T(t,t_k)U_1^{-1}M_1(t,t_k) + \frac{h^2}{3}M_2^T(t,t_k)U_1^{-1}M_2(t,t_k)\right] \\ &+ \mathrm{Sym}\left[M_1^T(t,t_k)\Pi_6 - 2M_2^T(t,t_k)e_3 + (t-t_k)M_2^T(t,t_k)\Pi_8\right]\Big\}\eta(t), \end{aligned} \tag{16}$$

$$\begin{aligned} -\int_t^{t_{k+1}} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s \le \eta^T(t)\Big\{&(t_{k+1}-t)\left[N_1^T(t,t_k)U_2^{-1}N_1(t,t_k) + \frac{h^2}{3}N_2^T(t,t_k)U_2^{-1}N_2(t,t_k)\right] \\ &+ \mathrm{Sym}\left[N_1^T(t,t_k)\Pi_7 - 2N_2^T(t,t_k)e_5 + (t_{k+1}-t)N_2^T(t,t_k)\Pi_9\right]\Big\}\eta(t), \end{aligned} \tag{17}$$

where $M_1(t,t_k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t_k))[M_{11,ij}, M_{12,ij},\ldots,M_{1k,ij}]$, $M_2(t,t_k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t_k))[M_{21,ij}, M_{22,ij},\ldots,M_{2k,ij}]$, $N_1(t,t_k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t_k))[N_{11,ij}, N_{12,ij},\ldots,N_{1k,ij}]$, and$N_2(t,t_k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t_k))[N_{21,ij}, N_{22,ij},\ldots,N_{2k,ij}]$.

Moreover, based on the system (13), for arbitrary matrices Y_1, Y_2, and Y_3 with appropriate dimensions, the following equation is true:

$$2\left[x(t)^T Y_1 + x(t_k)^T Y_2 + \dot{x}(t)^T Y_3\right]\left[-\dot{x}(t) + A_\iota(t)x(t) + B_{1\iota}(t)K(t)x(t_k)\right] = 0. \tag{18}$$

Observing the two inequalities (16) and (17), let's add $L\mathscr{W}_\iota(t)$ to the right side of the equation (18), thus, for any $t \in [t_k, t_{k+1})$, there is the following conclusion:

$$L\mathscr{W}_\iota(t) \le \eta(t)^T\left[\frac{t_{k+1}-t}{h_k}\Xi_1 + \frac{t-t_k}{h_k}\Xi_2\right]\eta(t). \tag{19}$$

where $\Xi_1 = \Delta_1(t,t_k) + h_k[N_1^T(t,t_k)U_2^{-1}N_1(t,t_k) + (h_M^2/3)N_2^T(t,t_k)U_2^{-1}N_2(t,t_k)]$ and $\Xi_2 = \Delta_2(t,t_k) + h_k[M_1^T(t,t_k)U_1^{-1}M_1(t,t_k) + (h_M^2/3)M_2^T(t,t_k)U_1^{-1}M_2(t,t_k)]$.

We can conclude from Lemma 2 and (14) that

$$\Xi_1 < -\frac{\rho I}{2}, \Xi_2 < -\frac{\rho I}{2}. \tag{20}$$

This means

$$LW_\iota(t) \le -\rho x(t_k)^2. \tag{21}$$

By Dynkin formula, for any $t \ge 0$, we get the following equation:

$$\varepsilon\{W(r_t,t)\} - \varepsilon\{W(r_0,t_0)\} \le -\rho\varepsilon\left\{\int_0^t x(s)^2 ds\right\}. \tag{22}$$

Hence,

$$\varepsilon\left\{\int_0^t x(s)^2 ds\right\} \le \rho^{-1}\varepsilon\{W(r_0,t_0)\}. \tag{23}$$

After the analysis above, we finally reach the conclusion.

$$\lim_{t\longrightarrow\infty}\varepsilon\left\{\int_0^t x(s)^2 ds\right\} \le d\sup_{-\infty_{<s\le 0}}\{|\Phi(s)|\}. \tag{24}$$

Then, we can conclude from Definition 1 that system (13) is stochastically stable. □

Remark 2. Inspired by literature [38], we propose a new Markovian modal-dependent Lyapunov function, this function fully utilizes the system sampled-date information from both intervals $[x(t_k), x(t)]$ and $[x(t), x(t_{k+1})]$, which satisfies the closed-loop function condition ($\mathscr{V}_0(t_k) = \mathscr{V}_0(t_{k+1}) = 0$). However, the Lyapunov function constructed in the literature [38] only contained $\int_{t_k}^{t} x^T(t)ds$ and does not contain $\int_t^{t_{k+1}} x^T(t)ds$ whereas our constructed function $V(t)$ includes both, thus the results derived in this study are less conservative.

Next, based on Theorem 1, we give the stochastic stability condition of system (10) under the $H_\infty - \gamma$ condition.

Theorem 2. *For a given scalar $\gamma > 0$, T-S fuzzy system (10) is stochastic stable under the $H_\infty - \gamma$ condition, if there are matrices $P_i > 0$, $U_1 > 0$, $U_2 > 0$, $H_1 = H_1^T$, H_2, $H_3 = H_3^T$, and H_4, and arbitrary matrices Y_1, Y_2, Y_3, $M_{1,ij}$, $M_{2,ij}$, $N_{1,ij}$, $N_{2,ij}$ such that inequation (25) is true for each $i \in \mathbb{R}$, $\iota \in N$, and $h_k \in \{h_m, h_M\}$:*

$$\Xi_{q,ij} < 0, q = 1,2, \tag{25}$$

where $$\Xi_{1,ij} = \begin{bmatrix} \Delta_{1,ij} & \sqrt{h_k}M_{1,ij}^T & \sqrt{h_k}h_M M_{2,ij}^T & \Phi_{2,j}^T \\ * & -U_2 & 0 & 0 \\ * & * & -3U_2 & 0 \\ * & * & * & -I \end{bmatrix},$$

$$\Xi_{2,ij} = \begin{bmatrix} \Delta_{2,ij} & \sqrt{h_k}N_{1,ij}^T & \sqrt{h_k}h_M N_{2,ij}^T & \Phi_{2,j}^T \\ * & -U_1 & 0 & 0 \\ * & * & -3U_1 & 0 \\ * & * & * & -I \end{bmatrix},$$

$\Delta_{1,ij} = \Theta_{1,ij} + \Theta_{2,ij} + h_k\Theta_{3,ij}$, $\Delta_{2,ij} = \Theta_{1,ij} + \Theta_{2,ij} + h_k\Theta_{4,ij}$, $\Theta_{1,ij} = Sym\{\hat{e}_1^T P_i\hat{e}_6 - \hat{\Pi}_1^T H_2\hat{\Pi}_2 + \hat{\Pi}_4^T H_4\hat{\Pi}_2 + M_{1,ij}^T\hat{\Pi}_6 - 2M_{2,ij}^T\hat{e}_4 - N_{1,ij}^T\hat{\Pi}_7 - 2N_{2,ij}^T\hat{e}_5\} + \hat{e}_1^T\sum_{j=1}^{s}\pi_{ij}P_j\hat{e}_1 - \hat{\Pi}_1^T H_1\hat{\Pi}_2 + \hat{\Pi}_4^T H_3 \quad \hat{\Pi}_4$, $\Theta_{2,ij} = Sym\left\{\hat{\Pi}_{10}^T\Phi_{1,ij}\right\} - \gamma^2\hat{e}_7^T\hat{e}_7$, $\Theta_{3,ij} = Sym\left\{\hat{\Pi}_1^T H_1\hat{\Pi}_3 + \hat{\Pi}_3^T H_2\hat{\Pi}_2 + N_{2,ij}^T\hat{\Pi}_9\right\} + \hat{e}_6^T U_1\hat{e}_6$, $\Theta_{4,ij} = Sym\left\{\hat{\Pi}_4^T H_3\hat{\Pi}_5 + \hat{\Pi}_5^T H_2\hat{\Pi}_2 + M_{2,ij}^T\hat{\Pi}_8\right\} + \hat{e}_6^T U_2\hat{e}_6$, $\hat{e}_j = \begin{bmatrix} 0_{n\times(j-1)n} & I_n & 0_{n\times(6-j)n} & 0_{n\times q}\end{bmatrix}$, $j = 1,2,\ldots,6$, $\hat{e}_7 = \begin{bmatrix} 0_{q\times 6n} & I_q\end{bmatrix}$, $\hat{\Pi}_1 = [\hat{e}_1^T - \hat{e}_2^T\ \hat{e}_4^T]^T$, $\hat{\Pi}_2 = [\hat{e}_2^T\ \hat{e}_3^T]^T$, $\hat{\Pi}_3 = [\hat{e}_6^T\ \hat{e}_1^T]^T$, $\hat{\Pi}_4 = [\hat{e}_1^T - \hat{e}_3^T\ \hat{e}_5^T]^T$, $\hat{\Pi}_5 = [\hat{e}_6^T - \hat{e}_1^T]^T$, $\hat{\Pi}_6 = [\hat{e}_1^T - \hat{e}_2^T]^T$, $\hat{\Pi}_7 = [\hat{e}_1^T - \hat{e}_3^T]^T$, $\hat{\Pi}_8 = \hat{e}_1 + \hat{e}$, $\hat{\Pi}_9 = \hat{e}_1 + \hat{e}_3$, $\hat{\Pi}_{10} = [\hat{e}_1^T Y_1 + \hat{e}_2^T Y_2 + \hat{e}_6^T Y_3]^T$, and$\Phi_{1,j} = A_\iota\hat{e}_1 + B_{1\iota}K_j\hat{e}_2 - \hat{e}_6 + D_{1\iota}\hat{e}_7$, $\Phi_{2,j} = C_\iota\hat{e}_1 + B_{2\iota}K_j\hat{e}_2 + D_{2\iota}\hat{e}_7$, and the definitions of $M_{1,ij}$, $M_{2,ij}$, $N_{1,ij}$, $N_{2,ij}$ are similar to that in Theorem 1.

Next, we give the proof of Theorem 2.

Proof. Observe the statement in Theorem 2, assume that there is a scalar $\rho > 0$ satisfying

$$\overline{\Xi}_{q,ij} < 0, \quad q = 1,2, \tag{26}$$

where $$\overline{\Xi}_{1,ij} = \begin{bmatrix} \Delta_{1,ij} & \sqrt{h_k}M_{1,ij}^T & \sqrt{h_k}h_M M_{2,ij}^T \\ * & -U_2 & 0 \\ * & * & -3U_2 \end{bmatrix},$$

$$\overline{\Xi}_{2,ij} = \begin{bmatrix} \Delta_{2,ij} & \sqrt{h_k}N_{1,ij}^T & \sqrt{h_k}h_M N_{2,ij}^T \\ * & -U_1 & 0 \\ * & * & -3U_1 \end{bmatrix},$$

$\Delta_{1,ij} = \Theta_{1,ij} + \Theta_{2,ij} + h_k\Theta_{3,ij}$, $\Delta_{2,ij} = \Theta_{1,ij} + \Theta_{2,ij} + h_k\Theta_{4,ij}$, $\Theta_{1,ij} = \mathrm{Sym}\{e_1^T P_i e_6 - \Pi_1^T H_2\Pi_2 + \Pi_4^T H\Pi_2 + M_{1,ij}^T\Pi_6 - 2M_{2,ij}^T e_4 - N_{1,ij}^T\Pi_7 - 2N_{2,ij}^T e_5\} + e_1^T\sum_{j=1}^s \pi_{ij}P_j e_1 - \Pi_1^T H_1\Pi_2 + \Pi_4^T H_3\Pi_4$, $\Theta_{2,ij} = \mathrm{Sym}\{\Pi_{10}^T\Phi_{1,j}\}$, $\Theta_{3,ij} = \mathrm{Sym}\{\Pi_1^T H_1\Pi_3 + \Pi_3^T H_2\Pi_2 + N_{2,ij}^T\Pi_9\} + e_6^T U_1 e_6$, and $\Theta_{4,ij} = \mathrm{Sym}\{\Pi_4^T H_3\Pi_5 + \Pi_5^T H_2\Pi_2 + M_{2,ij}^T\Pi_8\} + e_6^T U_2 e_6$, $\Phi_{1,j} = A_\iota e_1 + B_{1\iota}K_j e_2 - e_6$.

Even more, the terms Ξ_1 and Ξ_2 in inequation (14) can be described as follows:

$$\Xi_q = \sum_{i=1}^{r} h_i(\varphi(t))\sum_{j=1}^{r} h_j(\varphi(t))\overline{\Xi}_{q,ij},\ q = 1,2. \tag{27}$$

Hence, we can conclude from inequation (26) that inequation (14) is true, that is to say, $\Xi_q < 0$, $(q = 1,2)$, then we know by Theorem 1 that system (10) is stochastically stable when $w(t) = 0$.

Next, we will prove that the system (10) is stochastic stable under the $H_\infty - \gamma$ condition.

Let

$$J_{zw} = L\mathscr{W}_\iota(t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t). \tag{28}$$

Using the same proof as in Theorem 1, for any $t \in [t_k, t_{k+1})$, the following inequation holds:

$$J_{zw} \le \begin{bmatrix}\eta(t)\\ w(t)\end{bmatrix}^T\left[\frac{t_{k+1}-t}{h_k}\Xi_1 + \frac{t-t_k}{h_k}\Xi_2\right]\begin{bmatrix}\eta(t)\\ w(t)\end{bmatrix}, \tag{29}$$

where

$$\Xi_1(t,t_k) = \begin{bmatrix}\Delta_1(t,t_k) & \sqrt{h_k}M_1^T(t,t_k) & \sqrt{h_k}h_M M_2^T(t,t_k) & \Phi_2^T(t,t_k)\\ * & -U_2 & 0 & 0\\ * & * & -3U_2 & 0\\ * & * & * & -I\end{bmatrix},$$

$$\Xi_2(t,t_k) = \begin{bmatrix}\Delta_2(t,t_k) & \sqrt{h_k}N_1^T(t,t_k) & \sqrt{h_k}h_M N_2^T(t,t_k) & \Phi_2^T(t,t_k)\\ * & -U_1 & 0 & 0\\ * & * & -3U_1 & 0\\ * & * & * & -I\end{bmatrix},$$

$\Delta_1(t,t_k) = \Theta_1(t,t_k) + \Theta_2(t,t_k) + h_k\Theta_3(t,t_k)$, $\Delta_2(t,t_k) = \Theta_1(t,t_k) + \Theta_2(t,t_k) + h_k\Theta_4(t,t_k)$, $\Theta_1(t,t_k) = \mathrm{Sym}\{\hat{e}_1^T P_i\hat{e}_6 - \hat{\Pi}_1^T H_2\hat{\Pi}_2 + \hat{\Pi}_4^T H_4\hat{\Pi}_2 + M_1^T(t,t_k)\hat{\Pi}_6 - 2M_2^T(t,t_k)\hat{e}_4 - N_1^T(t,t_k)\hat{\Pi}_7 - 2N_2^T(t,t_k)\hat{e}\}_5 + \hat{e}_1^T\sum_{j=1}^s \pi_{ij}P_j\hat{e}_1 - \hat{\Pi}_1^T H_1\hat{\Pi}_2 + \hat{\Pi}_4^T H_3\hat{\Pi}_4$,
$\Theta_2(t,t_k) = \mathrm{Sym}\{\hat{\Pi}_{10}^T\Phi_1(t,t_k)\} - \gamma^2\hat{e}_7^T\hat{e}_7$, $\Theta_3(t,t_k) = \mathrm{Sym}\{\hat{\Pi}_1^T H_1\hat{\Pi}_3 + \hat{\Pi}_3^T H_2\hat{\Pi}_2 + N_2^T(t,t_k)\hat{\Pi}_9\} + \hat{e}_6^T U_1\hat{e}_6$, $\Theta_4(t,t_k) = \mathrm{Sym}\{\hat{\Pi}_4^T H_3\hat{\Pi}_5 + \hat{\Pi}_5^T H_2\hat{\Pi}_2 + M_2^T(t,t_k)\hat{\Pi}_8\} + \hat{e}_6^T U_2\hat{e}_6$, $\Phi_1(t,t_k) = A_\iota(t)\hat{e}_1 + B_{1\iota}(t)K(t)\hat{e}_2 - \hat{e}_6 + D_{1\iota}(t)\hat{e}_7$, and $\Phi_2(t,t_k) = C_\iota(t)\hat{e}_1 + B_{1\iota}(t)K(t)\hat{e}_2 + D_{2\iota}(t)\hat{e}_7$.

We take

$$\overline{\Xi}_q = \sum_{i=1}^{R} h_i(\varphi(t))\sum_{j=1}^{R} h_j(\varphi(t))\Xi_{q,ij}, q = 1,2, \tag{30}$$

where $\Xi_{q,ij} < 0\,(q = 1,2)$; then, we know that

$$\overline{\Xi}_q < 0, q = 1,2. \tag{31}$$

Observing inequation (29), we conclude that

$$J_{zw} \le 0. \tag{32}$$

Therefore, for any $\tau \ge 0$, there must be a non-negative integer α such that $t_\alpha \le \tau < t_\alpha$; then let us observe formula (28) and inequation (32), and we obtain that

$$\int_0^\tau z^T(t)z(t)\mathrm{d}t - \gamma^2\int_0^\tau w^T(t)w(t)\mathrm{d}t \le -\mathscr{W}_\iota(\tau) \le 0. \tag{33}$$

By the Dynkin formula, for any $t \ge 0$, the following inequation holds:

$$\varepsilon\left\{\int_0^\infty z^T(t)z(t)\mathrm{d}t\right\} < -\gamma^2\int_0^\infty w^T(t)w(t)\mathrm{d}t. \tag{34}$$

As stated in Definition 2, we know that system (10) is stochastic stable under the $H_\infty - \gamma$ condition.

Next, let's give the sufficient condition for the existence of state feedback controllers for sampled-data follows according to Theorem 2. □

Theorem 3. *3.For a given scalar $\gamma > 0$, the T-S fuzzy system (10) is stochastic stable under the $H_\infty - \gamma$ condition, if there are matrices $\tilde{P}_i > 0$, $\tilde{U}_1 > 0$, $\tilde{U}_2 > 0$, $\tilde{H}_1 = \tilde{H}_1^T$, $\tilde{H}_2$, $\tilde{H}_3 = \tilde{H}_3^T$, $\tilde{H}_4$, $\tilde{Y}$, and L_ι, and arbitrary matrices $\tilde{M}_{1,ij}$, $\tilde{M}_{2,ij}$, $\tilde{N}_{1,ij}$, and $\tilde{N}_{2,ij}$ such that inequations (30) are true for each i, $j \in \mathbb{R}$, $\iota \in N$, $h_k \in \{h_m, h_M\}$, and*

$$\tilde{\Xi}_{q,ij} < 0, \quad q = 1,2, \tag{35}$$

where $$\tilde{\Xi}_{1,ij} = \begin{bmatrix}\tilde{\Delta}_{1,ij} & \sqrt{h_k}\tilde{M}_{1,ij}^T & \sqrt{h_k}h_M\tilde{M}_{2,ij}^T & \tilde{\Phi}_{2,j}^T\\ * & -\tilde{U}_2 & 0 & 0\\ * & * & -3\tilde{U}_2 & 0\\ * & * & * & -I\end{bmatrix},$$

$$\tilde{\Xi}_{2,ij} = \begin{bmatrix}\tilde{\Delta}_{2,ij} & \sqrt{h_k}\tilde{N}_{1,ij}^T & \sqrt{h_k}h_M\tilde{N}_{2,ij}^T & \tilde{\Phi}_{2,j}^T\\ * & -\tilde{U}_1 & 0 & 0\\ * & * & -3\tilde{U}_1 & 0\\ * & * & * & -I\end{bmatrix},$$

$\tilde{\Delta}_{1,ij} = \tilde{\Theta}_{1,ij} + \tilde{\Theta}_{2,ij} + h_k\tilde{\Theta}_{3,ij}$, $\tilde{\Delta}_{2,ij} = \tilde{\Theta}_{1,ij} + \tilde{\Theta}_{2,ij} + h_k\tilde{\Theta}_{4,ij}$, $\tilde{\Theta}_{1,ij} = Sym\{\hat{e}_1^T\tilde{P}_i\hat{e}_6 - \hat{\Pi}_1^T\tilde{H}_2\hat{\Pi}_2 + \hat{\Pi}_4^T\tilde{H}_4\hat{\Pi}_2 + \hat{\Pi}_6^T\tilde{X}\hat{e}_6 - 2\tilde{M}_{2,ij}^T\hat{e}_4 - \tilde{N}_{1,ij}^T\hat{\Pi}_7 - 2\tilde{N}_{2,ij}^T\hat{e}_5\} + \hat{e}_1^T\sum_{j=1}^s \pi_{ij}\tilde{P}_j\hat{e}_1 - \hat{\Pi}_1^T\tilde{H}_1\hat{\Pi}_2 + \hat{\Pi}_4^T\tilde{H}_3\hat{\Pi}_4$, $\tilde{\Theta}_{2,ij} = Sym\{\hat{\Pi}_{10}^T\tilde{\Phi}_{1,j}\} - \gamma^2\hat{e}_7^T\hat{e}_7$,
$\tilde{\Theta}_{3,ij} = Sym\{\hat{\Pi}_1^T\tilde{H}_1\hat{\Pi}_3 + \hat{\Pi}_3^T\tilde{H}_2\hat{\Pi}_2 + \tilde{N}_{2,ij}^T\hat{\Pi}_9\} + \hat{e}_6^T\tilde{U}_1\hat{e}_6$,

$\widetilde{\Theta}_{4,ij} = Sym\left\{\widehat{\Pi}_4^T \widetilde{H}_3 \widehat{\Pi}_5 + \widehat{\Pi}_5^T \widetilde{H}_2 \widehat{\Pi}_2 + \widetilde{M}_{2,ij}^T \widehat{\Pi}_8\right\} + \widehat{e}_6^T \widetilde{U}_2 \widehat{e}_6$, $\widetilde{\Pi}_{10} = [\widehat{e}_1^T + \epsilon_1 \widehat{e}_2^T + \epsilon_2 \widehat{e}_6^T]^T$, $\widetilde{\Phi}_{1,j} = A_\iota \widetilde{Y}^T \widehat{e}_1 + B_{1\iota} L_j \widehat{e}_2 - \widetilde{Y}^T \widehat{e}_6 + D_{1\iota}\widehat{e}_7$, $\widetilde{\Phi}_{2,j} = C_\iota \widetilde{Y}^T \widehat{e}_1 + B_{1\iota} L_j \widehat{e}_2 + D_{2\iota}(t)\widehat{e}_7$, *and* $\widehat{\Pi}_\theta (\theta = 1, 2, \ldots, 9)$ *are the same definitions as in Theorem 2; in this case, we give the expected controller gains as follows:*

$$K_i = L_i Y^{-T}, \; i = 1, 2, \ldots, r. \tag{36}$$

Proof. .Let $\widetilde{Y} = Y_1^{-1}$, $L = K\widetilde{Y}^T$, $Y_2 = \epsilon_1 Y_1$, $Y_3 = \epsilon_2 Y_1$, $\widetilde{P}_i = \widetilde{Y} P_i \widetilde{Y}^T$, $\widetilde{U}_1 = \widetilde{Y} U_1 \widetilde{Y}^T$, $\widetilde{U}_2 = \widetilde{Y} U_2 \widetilde{Y}^T$, $\widetilde{H}_1 = \widetilde{Y} H_1 \widetilde{Y}^T$, $\widetilde{H}_2 = \widetilde{Y} H_2 \widetilde{Y}^T$, $\widetilde{H}_3 = \widetilde{Y} H_3 \widetilde{Y}^T$, $\widetilde{H}_4 = \widetilde{Y} H_4 \widetilde{Y}^T$, $\widetilde{M}_{1,ij} = \widetilde{Y} M_{1,ij} \widetilde{Y}^T$, $\widetilde{M}_{2,ij} = \widetilde{Y} M_{2,ij} \widetilde{Y}^T$, $\widetilde{N}_{1,ij} = \widetilde{Y} N_{1,ij} \widetilde{Y}^T$, $\widetilde{N}_{2,ij} = \widetilde{Y} N_{2,ij} \widetilde{Y}^T$, and

$$\Gamma = \mathrm{diag}\{\widetilde{Y}, \widetilde{Y}, \widetilde{Y}, \widetilde{Y}, \widetilde{Y}, \widetilde{Y}, I, \widetilde{Y}, \widetilde{Y}, I\}. \tag{37}$$

By the statements in Theorem 2, we know that $\Xi_{1,ij} < 0, \Xi_{2,ij} < 0$, let's pre-multiply $\Xi_{1,ij}$ by Γ, and post-multiply $\Xi_{2,ij}$ by Γ^T, we can conclude that inequations (35) are accurate. This concludes the proof process. □

4. Numerical Simulation Example

We provide an example in this work to verify that the proposed approach is effective.

Example: The Truck-Trailer model is mentioned in the literature [42, 43]. Here, we take it as an example to verify the results. The Truck-Trailer model is shown as follows:

$$\begin{cases} \dot{x}_1(t) = -\dfrac{\overline{w}_p \bar{t}}{L\bar{t}_0} x_1(t) + \dfrac{\overline{w}_p \bar{t}}{l_0 \bar{t}_0} u(t) + 0.1 w_1(t) + 0.1 w_2(t), \\ \dot{x}_2(t) = \dfrac{\overline{w}_p \bar{t}}{L\bar{t}_0} x_1(t) + 0.1 w_1(t) + 0.1 w_2(t), \\ \dot{x}_3(t) = \dfrac{\overline{w}_p \bar{t}}{\bar{t}_0} \sin\left(x_2(t) + \dfrac{\overline{w}_p \bar{t}}{2L} x_1(t)\right), \end{cases} \tag{38}$$

where $x_1(t)$ is the angle difference between the trailer and the truck, $x_3(t)$ is the vertical position of the rear of the trailer, $x_2(t)$ is the angle of the trailer, $w_1(t)$ and $w_2(t)$ are disturbance input, and the model parameters can be set as $\bar{t} = 2$, $\overline{w}_1 = -1$, $\overline{w}_2 = -1.05$, $\overline{w}_3 = -0.95$, $\bar{t}_0 = 0.5$, $l = 2.8$, $L = 5.5$. Assume that the switching process between the three modes follows a Markov process, and the transfer probability matrix is Π: $\Pi = \begin{bmatrix} -0.3 & 0.1 & 0.2 \\ 0.4 & -0.6 & 0.2 \\ 0.8 & 0.1 & -0.9 \end{bmatrix}$.

Let $\Theta(t) = x_2(t) + (\overline{w}\bar{t}/2L) x_1(t)$, the membership function is defined as follows:

$h_1(\Theta(t)) =$

$$\begin{cases} (\sin(\Theta(t)) - \overline{g}\Theta(t)/\Theta(t)(1-\overline{g})), & if\ \Theta(t) \neq 0, \\ 1, & if\ \Theta(t) = 0 \end{cases}$$ where

$\overline{g} = (10^{-2}/\pi)$, and let $h_2(\Theta(t)) = 1 - h_1(\Theta(t))$.

The *T-S* fuzzy systems of the model (38) can be described as follows:

$$\dot{x}(t) = \sum_{i=1}^{3} h_\iota(x_1(t)) \left[A_{i,\iota} x(t) + B_{1i,\iota} u(t) + D_{1i,\iota} w(t)\right]. \tag{39}$$

The parameters of which we refer to the literature [42, 43] are defined as follows: $A_{1,\iota} = \begin{bmatrix} -(\overline{w}_p \bar{t}/L\bar{t}_0) & 0 & 0 \\ (\overline{w}_p \bar{t}/L\bar{t}_0) & 0 & 0 \\ (\overline{w}_p^2 \bar{t}^2/2L\bar{t}_0) & (\overline{w}_p \bar{t}/\bar{t}_0) & 0 \end{bmatrix}$, $A_{2,\iota} = \begin{bmatrix} -(\overline{w}_p \bar{t}/L\bar{t}_0) & 0 & 0 \\ (\overline{w}_p \bar{t}/L\bar{t}_0) & 0 & 0 \\ (\overline{g w}_p^2 \bar{t}^2/2L\bar{t}_0) & (\overline{g w}_p \bar{t}/\bar{t}_0) & 0 \end{bmatrix}$,

$B_{11,\iota} = B_{12,\iota} = \begin{bmatrix} (\overline{w}_p \bar{t}/l\bar{t}_0) \\ 0 \\ 0 \end{bmatrix}$, $D_{11,\iota} = D_{12,\iota} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \\ 0 & 0 \end{bmatrix}$.

Then, the output vector $z(t)$ is reconstructed as follows:

$$z(t) = \sum_{i=1}^{3} h_\iota(x_1(t)) \left[C_{i,\iota} x(t) + B_{2i,\iota} u(t) + D_{2i,\iota} w(t)\right], \tag{40}$$

where $C_{1,\iota} = C_{2,\iota} = \begin{bmatrix} 0.1 & 0.2 & 0.2 \\ 0.3 & 0.2 & 0.1 \end{bmatrix}$, $B_{21,\iota} = B_{22,\iota} = \begin{bmatrix} 1.2 \\ 1.1 \end{bmatrix}$, and $D_{21,\iota} = D_{22,\iota} = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.2 \end{bmatrix}$.

First of all, the uncertain sampling period is considered. Based on Theorem 3, let $\varepsilon_1 = \varepsilon_2 = 0.5$, and $h_m = 0$, $\gamma_{\min}$ with different largest sampling periods h_M are given in Table 1. It i's easy to find that a larger h_M corresponds to a larger $\gamma_{\min}$.

Next, the situation of a specific sampling period is analyzed. In other words, $h_m = h_M = h$, the H_∞ performances $\gamma_{\min}$ with different h are given in Table 2. It is easy to find that a larger h corresponds to a larger $\gamma_{\min}$.

From the two tables above, we can see that when the sampling period increases, the corresponding performance $\gamma_{\min}$ also increases. This phenomenon inspires us to improve the H_∞ performance of the system by increasing the sampling frequency.

In addition, we select sampling period $h_m = 0.1$, $h_M = 0.3$ to verify Theorem 3, and we can obtain the following controller: $K_1 = [\,12.1392 \;\; -22.3714 \;\; 4.0967\,]$ and $K_2 = [\,11.7478 \;\; -20.2820 \;\; 4.1563\,]$.

Based on the above controller, the initial value of the system is selected as $x_0 = [\,-0.5\pi \;\; -0.75\pi \;\; -4\,]$. Based on the nonperiodic sampled-data controller obtained from Theorem 3, the state response curve and Markovian-jump signals of the closed-loop system in Figure 1 show that the system state response is approaching zero. Figure 2 shows the sampled-data control input and the system aperiodic sampling intervals, as we see that the synchronization error of the Truck-Trailer model is approaching zero. Furthermore, each stem represents the sampling time t_k, and the value of each stem indicates the range of the sampling interval h_k. We can thus conclude that the closed-loop system is stable under the action of our proposed nonperiodic sampled-data controller, which shows the effectiveness of our proposed method.

Table 1: Different performance $\gamma_{\min}$ for different h_M, where $h_m = 0$.

h_M	0.05	0.15	0.25	0.35
$\gamma_{\min}$	1.7317	1.7678	1.8251	1.9298

Table 2: Different performance $\gamma_{\min}$ for different h, where $h_m = h_M = h$.

h	0.10	0.15	0.20	0.25
$\gamma_{\min}$	1.7427	1.7578	1.7760	1.7986

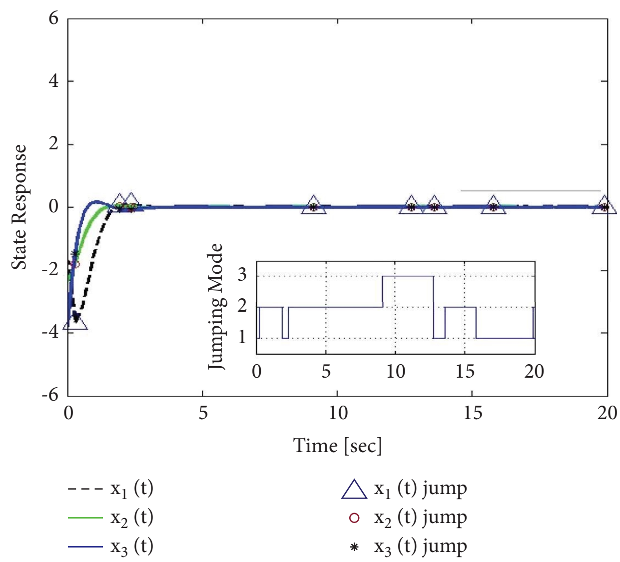

Figure 1: System state response and Markovian-jump signals in example.

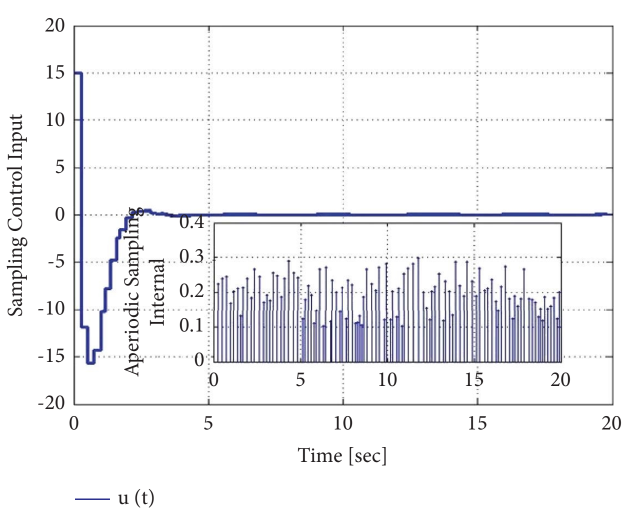

Figure 2: Sampled-data control input and aperiodic sampling intervals in example.

5. Conclusion

In this paper, a design algorithm of the state feedback controller is proposed, which ensures the stability of the *T-S* fuzzy systems with Markovian-jump parameters under H_∞ performance condition, and the stability condition with less conservative is given in linear matrix inequality form. At the same time, a Truck-Trailer model is used to simulate the *T-S* fuzzy systems with Markovian-jump parameters, and the simulation results also prove that our presented method is real and effective.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this article.

Acknowledgments

This work was supported by the Special Project in Higher Education Research of Shandong Higher Education Association (SDGJ212027), the Industry-School Cooperative Education Program of the Ministry of Education of China (202102030024), and the Planning Project for Five-Year Plan of Shandong Educational Sciences (2021ZC063).

References

[1] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.

[2] M. Chadli and T. M. Guerra, "LMI solution for robust static output feedback control of discrete Takagi-Sugeno fuzzy models," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 6, pp. 1160–1165, 2012.

[3] J. Dong, G. H. Yang, and H. Zhang, "Stability analysis of T-S fuzzy control systems by using set theory," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 4, pp. 827–841, 2015.

[4] J. Qiu, S. Ding, H. Gao, and S. Yin, "Fuzzy-model-based reliable static output feedback H_∞ control of nonlinear hyperbolic PDE systems," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 2, pp. 388–400, 2014.

[5] H. Zhang, G. Zhang, and J. Wang, "H_∞ observer design for LPV systems with uncertain measurements on scheduling variables: application to an electric ground vehicle," *IEEE*, vol. 21, no. 33, pp. 1659–1670, 2016.

[6] H. Zhang and J. Wang, "Active steering actuator fault detection for an automatically-steered electric ground vehicle," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 5, pp. 3685–3702, 2017.

[7] G. Rigatos, P. Siano, P. Wira, and F. Profumo, "Nonlinear H-infinity feedback control for asynchronous motors of

electric trains," *Intelligent Industrial Systems*, vol. 1, no. 2, pp. 85–98, 2015.

[8] H. Xiao, Y. He, M. Wu, S. Xiao, and J. She, "New results on H_∞ tracking control based on the T–S fuzzy model for sampled-data networked control system," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 6, pp. 2439–2447, 2015.

[9] Z. Lian, Y. He, C. K. Zhang, P. Shi, and M. Wu, "Robust H_∞ control for T–S fuzzy systems with state and input time-varying delays via the delay-product-type functional method," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 10, pp. 1917–1930, 2019.

[10] X. Wang, S. Ding, X. Zhang, and X. Fan, "Further studies on robust H_∞ control for a class of Takagi–Sugeno fuzzy time-delay systems with application to continuously stirred tank reactor problems," *Proceedings of the Institution of Mechanical Engineers - Part I: Journal of Systems & Control Engineering*, vol. 233, no. 2, pp. 103–117, 2019.

[11] W. Zheng, Z. Zhang, F. Sun, S. Wen, X. Li, and H. Wang, "Robust H_∞ output feedback control for type-2 Takagi-Sugeno fuzzy systems with multiple time-delays and disturbances: a descriptor redundancy approach," *International Journal of Robust and Nonlinear Control*, vol. 31, no. 13, pp. 6095–6122, 2021.

[12] J. Leng, H. Zhang, D. Yan, Q. Liu, X. Chen, and D. Zhang, "Digital twin-driven manufacturing cyber-physical system for parallel controlling of smart workshop," *Journal of Ambient Intelligence and Humanized Computing*, vol. 10, no. 3, pp. 1155–1166, 2019.

[13] L. Li, V. A. Ugrinovskii, and R. Orsi, "Decentralized robust control of uncertain Markov jump parameter systems via output feedback," *Automatica*, vol. 43, no. 11, pp. 1932–1944, 2007.

[14] F. Abdollahi and K. Khorasani, "A decentralized markovian jump ${\cal H}_{\infty}$ control routing strategy for mobile multi-agent networked systems," *IEEE Transactions on Control Systems Technology*, vol. 19, no. 2, pp. 269–283, 2011.

[15] G. Chen, J. Xia, and G. Zhuang, "Delay-dependent stability and dissipativity analysis of generalized neural networks with Markovian jump parameters and two delay components," *Journal of the Franklin Institute*, vol. 353, no. 9, pp. 2137–2158, 2016.

[16] O. L. V. Costa, M. D. Fragoso, and M. G. Todorov, *Continuous-Time Markov Jump Linear Systems*, Springer, Berlin, Germany, 2013.

[17] X. Li, G. Wei, D. Ding, and S. Liu, "Recursive filtering for time-varying discrete sequential systems subject to deception attacks: weighted try-once-discard protocol," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 6, pp. 3704–3713, 2022.

[18] H. Shen, Z. Wang, X. Huang, and J. Wang, "Fuzzy dissipative control for nonlinear Markovian jump systems via retarded feedback," *Journal of the Franklin Institute*, vol. 351, no. 7, pp. 3797–3817, 2014.

[19] H. Shen, Y. Z. Men, Z. G. Wu, and J. H. Park, "Nonfragile H∞ Control for Fuzzy 466 Markovian Jump Systems Under Fast Sampling Singular Perturbation," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 12, pp. 2058–2069, 2018.

[20] F. Li, P. Shi, C. C. Lim, and L. Wu, "Fault detection filtering for nonhomogeneous markovian jump systems via a fuzzy approach," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 1, pp. 131–141, 2018.

[21] E. Fridman, "A refined input delay approach to sampled-data control," *Automatica*, vol. 46, no. 2, pp. 421–427, 2010.

[22] H. Shen, J. H. Park, L. Zhang, and Z. G. Wu, "Robust extended dissipative control for sampled-data Markov jump systems," *International Journal of Control*, vol. 87, no. 8, pp. 1549–1564, 2014.

[23] H. B. Zeng, K. L. Teo, Y. He, H. Xu, and W. Wang, "Sampled-data synchronization control for chaotic neural networks subject to actuator saturation," *Neurocomputing*, vol. 260, pp. 25–31, 2017.

[24] Y. K. Shen and J. W. Xia, "Aperiodic-sampled-data-based $\mathbf{H}_\infty$ control of T-S fuzzy systems," *Journal of Liaocheng university*, vol. 2022, no. 4, pp. 1–8, Article ID 6401231, 2022.

[25] R. Saravanakumar, R. Datta, and Y. Cao, "New insights on fuzzy sampled-data stabilization of delayed nonlinear systems," *Chaos, Solitons & Fractals*, vol. 154, Article ID 111654, 2022.

[26] Y. Yang and M. Zheng, "Sampled-data control for a class of singular takagi-sugeno fuzzy systems with application in truck-trailer system," *Symmetry*, vol. 14, no. 9, p. 1762, 2022.

[27] P. Nirvin, F. A. Rihan, R. Rakkiyappan, and C. Pradeep, "Impulsive sampled-data controller design for synchronization of delayed T–S fuzzy Hindmarsh–Rose neuron model," *Mathematics and Computers in Simulation*, vol. 201, pp. 588–602, 2022.

[28] R. Surendar, M. Muthtamilselvan, and R. Rakkiyappan, "LMI based sampled-data controller for synchronization on the time-delay Darcy-Brinkman model," *Journal of the Franklin Institute*, vol. 359, no. 12, pp. 5865–5890, 2022.

[29] G. Chen, J. Sun, and J. Chen, "Mean square exponential stabilization of sampled-data Markovian jump systems," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 18, pp. 5876–5894, 2018.

[30] R. Zhang, X. Liu, D. Zeng, S. Zhong, and K. Shi, "A novel approach to stability and stabilization of fuzzy sampled-data Markovian chaotic systems," *Fuzzy Sets and Systems*, vol. 344, pp. 108–128, 2018.

[31] L. Yan, J. Shun, and P. Feng, "Non-fragile robust H_∞ control for networked control system with packet dropouts," *Information and Control*, vol. 45, no. 3, pp. 320–327, 2016.

[32] J. J. Ren, X. Z. Liu, H. Zhu, and S. Zhong, "Passivity-based non-fragile control for Markovian jump delayed systems via stochastic sampling," *International Journal of Control*, vol. 92, no. 4, pp. 755–777, 2019.

[33] H. Yu and F. Hao, "Design of event conditions in event-triggered control systems: a non-fragile control system approach," *IET Control Theory & Applications*, vol. 10, no. 9, pp. 1069–1077, 2016.

[34] S. Fu, J. Qiu, and W. Ji, "Non-fragile control of fuzzy affine dynamic systems via piecewise Lyapunov functions," *Frontiers of Computer Science*, vol. 11, no. 6, pp. 937–947, 2017.

[35] J. Xia, H. Gao, M. Liu, G. Zhuang, and B. Zhang, "Non-fragile finite-time extended dissipative control for a class of uncertain discrete time switched linear systems," *Journal of the Franklin Institute*, vol. 355, no. 6, pp. 3031–3049, 2018.

[36] D. Yue and J. Lam, "Non-fragile guaranteed cost control for uncertain descriptor systems with time-varying state and input delays," *Optimal Control Applications and Methods*, vol. 26, no. 2, pp. 85–105, 2005.

[37] Y. Q. Wu, H. Su, R. Lu, Z. G. Wu, and Z. Shu, "Passivity-based non-fragile control for Markovian jump systems with aperiodic sampling," *Systems & Control Letters*, vol. 84, pp. 35–43, 2015.

[38] H. B. Zeng, K. Teo, and Y. He, "A new looped-functional for stability analysis of sampled-data systems," *Automatica*, vol. 82, pp. 328–331, 2017.

[39] H. B. Zeng, Y. He, M. Wu, and J. She, "Free-matrix-based integral inequality for stability analysis of systems with time-varying delay," *IEEE Transactions on Automatic Control*, vol. 60, no. 10, pp. 2768–2772, 2015.

[40] S. Boyd, L. Ghaoui, E. Feron, and V. Balakrishnan, "Linear matrix inequalities in systems and control theory," *SIAM Studies in Applied Mathematics*, vol. 15, pp. 7–9, 1994.

[41] Y. Zhao and H. Gao, "Fuzzy-model-based control of an overhead crane with input delay and actuator saturation," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 1, pp. 181–186, 2012.

[42] H. B. Zeng, K. L. Teo, Y. He, and W. Wang, "Sampled-data-based dissipative control of T-S fuzzy systems," *Applied Mathematical Modelling*, vol. 65, pp. 415–427, 2019.

[43] Z. G. Wu, P. Shi, H. Su, and R. Lu, "Dissipativity-based sampled-data fuzzy control design and its application to truck-trailer system," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 5, pp. 1669–1679, 2015.