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Research Article

A New Nonlinear Controller Design for a TCP/AQM Network Based on Modified Active Disturbance Rejection Control

Anwer S. Aljuboury,[1,2] Subhi R. M. Zeebaree,[3] Firas Abedi,[4] Zahraa Sabah Hashim,[5] Rami Q. Malik,[6] Ibraheem Kasim Ibraheem,[7] and Ahmed Alkhayyat[8]

[1]*Continuing Education Center, Mustansiriyah University, Baghdad, Iraq*
[2]*Information Technology Unit, Hilla University College, Babylon, Iraq*
[3]*Energy Department, Technical College of Engineering, Duhok Polytechnic University, Duhok, Iraq*
[4]*Department of Mathematics, College of Education, Al-Zahraa University for Women, Karbala, Iraq*
[5]*Department of Electrical Engineering, College of Engineering, University of Baghdad, Baghdad, 10001, Iraq*
[6]*Department of Medical Instrumentation Techniques Engineering, Al-Mustaqbal University College, Hillah, 51001, Iraq*
[7]*Department of Computer Engineering Techniques, Al-Rasheed University College, Baghdad, 10001, Iraq*
[8]*College of Technical Engineering, The Islamic University, Najaf, Iraq*

Correspondence should be addressed to Ahmed Alkhayyat; ahmedalkhayyat85@iunajaf.edu.iq

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The main aim of this study was to address the problem of congestion in TCP nonlinear systems in the presence of mismatched exogenous disturbances. To achieve this problem, two methods are proposed: the first is active queuing management, based on two proposed controllers, an NLPID and STC-SM, while the second is the application of active queuing management-based anti-disturbance techniques such as active disturbance rejection control (ADRC) and the nonlinear disturbance observer (NLDO). The proposed ADRC consists of a new NLPID and a new super-twisting sliding mode controller (STC-SM), which functions as a novel NLSEF, and a proposed NLESO estimates the applied disturbance and cancels it in a responsive manner. A new tracking differentiator with a novel function is also used to generate a smooth and accurate reference signal and derivative. The NLDO is proposed to estimate the disturbance and combine this with the control signal of the designed nonlinear controller as a way to compensate for the disturbance. The simulation results for the proposed scheme (ADRC) as applied to a nonlinear model of the TCP network are thus found to provide smoother and more accurate tracking of the desired value, with high robustness against applied disturbance, as compared to the other schemes introduced in this study. The proposed scheme also shows a noticeable improvement in terms of the utilized performance indices and the OPI.

1. Introduction

The requirements for quick, high-speed, and reliable communication have become more intense with recent increases in the number of Internet users. To achieve the necessary reliable communication between the server and the client, TCP is thus widely used. TCP offers a connection-oriented packet switching method that provides a reliable, bidirectional connection between two endpoints; however, although TCP is more reliable than UDP, any significant increase in TCP flow may cause serious congestion in the router, which will reduce network communication quality. A network congestion control method must thus be utilized, and there are two main types of congestion control. The first is source-based TCP management, such as Sack, New Reno, and Vegas, while the second is router-based active queuing management (AQM). Issues with global synchronization caused by the first method mean, however, that it is the second method that has been most widely utilized, which has attracted the attention of most researchers [1].

Initially, AQM was proposed by [2] in a form known as random early detection; after that, Misra provided an analytical model for TCP/AQM in [3] using differential equations and fluid flow theory. AQM forms a method that actively drops the packet in the router buffer before it is full, ensuring that queuing length is always monitored: when congestion occurs, the queuing length becomes greater than the desired value, and the AQM uses this as an indication of congestion. At such times, the AQM provides effective, reliable, efficient, and fair communication between sender and receiver in the TCP network [1, 4].

AQM based on advanced control theory is now widely used, and it has thus attracted the attention of several researchers seeking to deal with the problem of congestion in TCP networks by achieving the desired trajectory of queuing length. The author in [1] presented the use of integral backstepping as an AQM for a multi-route TCP/AQM model as a way to reduce congestion, while in [5], the author proposed three controllers, H_∞, PSO-PID, and ACO-PID, as AQMs to reduce the effects of disturbance and uncertainty and to track the desired set point. In [6], the author introduced an AQM-based novel PD controller for both single and multiple bottleneck routers as a way to adjust the queuing length to the desired set point under small oscillations. The author in [7] designed a nonlinear disturbance observer with a backstepping controller to form a nonlinear TCP network system, while the author in [8] presented a backstepping controller that adopted a minimax approach to control congestion and avoid the influence of applied disturbance. In [9], the author introduced another controller, which utilized a combination of H_∞ theory and integral backstepping, to a nonlinear model of the TCP network system to control the congestion occurring in the network. The author in [10] further proposed the use of a PD controller as an AQM and linear disturbance observer (DOB), with a smith predictor (SP) and the linearized model of the TCP network used to avoid congestion in the TCP network and to eliminate the influence of or compensate for the time delay effect. A further finite time backstepping controller was proposed in [11], with the nonlinear model of the TCP network system encouraged to reach the desired value in a finite time; the author in [12] also proposed a self-tuning rate and queuing-based PI controller (SQR-PI) with a single/multiple bottleneck router model to control queuing length by estimating the rate of traffic and using this alongside a PI controller to map congestion levels and to dramatically reduce the probability of losing packets. A combination of finite time control, backstepping technique, prescribed performance, and fuzzy logic was then presented in [13] as a way to deal with applied disturbance and achieve queuing length in a finite time.

Although all the studies noted above provide robust controller techniques for a TCP network, several studies used the linearized model of the TCP network, while others considered the round trip time (RTT) as a constant. Further, no recent research has used the disturbance/uncertainty rejection technique, also known as active disturbance rejection control (ADRC). The ADRC is a powerful method, first proposed by [14], for dealing with the problem of exogenous disturbance, uncertainty, and unknown perturbations that may affect linear and nonlinear systems, whether SISO or MIMO. At present, while the ADRC is widely used in different fields, as introduced in [15–17], the effectiveness of the proposed methods as compared to the conventional one is unknown. Motivated by this survey, the researchers in this study used a modified version of the conventional ADRC technique as an AQM in this study.

The main aim of this study was to design an accurate control technique that can control congestion in the TCP nonlinear system and thus handle nonlinearity, disturbance, and uncertainty effects. A modified ADRC is thus proposed as an AQM in the time-delayed TCP network nonlinear model. The proposed method also contains two new controllers, NLPID and STC-SM, which are proposed as new NLSEFs, while a new fractional power nonlinear extended state observer is also proposed. Additionally, a new tracking differentiator is proposed using the sigmoid function, with three parts combined to form a modified ADRC that provides smooth, accurate, and excellent results. The parameters of the proposed controller, proposed NLESO, and the tracking differentiator were thus tuned using a genetic algorithm as an optimization technique [18], while a new multi-objective performance index was used in the minimization process. This includes the absolute of the control signals, the square of the control signals, the integral time absolute error, the integral time square error, and the mean square error.

The rest of this study is organized as follows: Section 2 presents the modelling of the TCP network, and then, the problem statement is illustrated in Section 3. Section 4 presents the design of the proposed ADRC, while Section 5 presents the design and convergence of NLDO, along with closed-loop stability analysis. Section 6 then illustrates the simulation results and offers a discussion of these simulations. Finally, Section 7 presents the conclusion of this study.

2. TCP/AQM Mathematical Models

Using fluid flow theory, the nonlinear model of a TCP network can be described using the following nonlinear differential equations with time-varying delays [19], assuming a single bottleneck router network topology as shown in Figure 1.

$$\begin{cases} \dot{W}(t) = \dfrac{1}{R(t)} - \dfrac{W(t)W(t-R(t))}{R(t)R(t-R(t))}\, p(t-R(t)), \\ \dot{q}(t) = \dfrac{N(t)}{R(t)} W(t) - C(t) + d(t), \end{cases} \tag{1}$$

$$\begin{cases} R(t) = \dfrac{q(t)}{C(t)} + \tau_p, \\ R(t-R(t)) = \dfrac{q((t-R(t)))}{C(t)} + \tau_p, \end{cases} \tag{2}$$

where $W(t)$ represents the average window size of the TCP network, $q(t)$ represents the average queuing length at the router, $R(t)$ is the round trip time, $C(t)$ is the link capacity, $N(t)$ is the number of TCP sessions, τ_p is the propagation

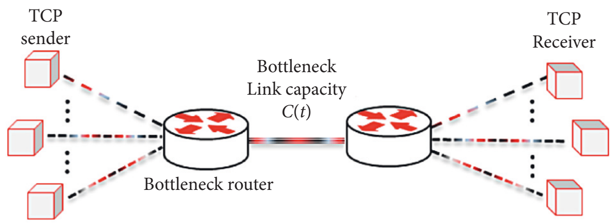

FIGURE 1: TCP network topology.

delay, and $p(t-R(t))$ is the probability of a packet marking that represents the AQM control strategy; additionally, $d(t)$ is the exogenous disturbance, denoted by the UDP unresponsive flows.

As this model incorporates time-varying delays, if $C(t)$ and $N(t)$ can be assumed to be constant (fixed) within a period of time, then $C(t)=C$ and $N(t)=N$ [19].

$$\begin{cases} p(t-R(t)) = u_0, \\ u = \aleph(u_0), \end{cases} \tag{3}$$

where $\aleph = \varepsilon + \varepsilon \tan h(u_0/\varepsilon)$ to ensure $u \in [0,1]$, and ε is a positive tuning parameter. Thus, equation (1) can be rewritten as follows:

$$\begin{cases} \dot{W}(t) = \dfrac{1}{R(t)} - \dfrac{W(t)W(t-R(t))}{R(t)R(t-R(t))}u, \\ \dot{q}(t) = \dfrac{N}{R(t)}W(t) - C + d(t). \end{cases} \tag{4}$$

The dynamic behavior of the window size in equation (1) is described by "addition increase multiplication decrease" [3]. The first term of $\dot{W}(t)$, which is $1/R(t)$, means that the window size increases by one for every round trip time $(R(t))$, while the second term, which is $W(t)W(t-R(t))/R(t)R(t-R(t))$, means that the window size is halved when congestion occurs, and the packet is lost. The first term of $\dot{q}(t)$, which is $N/R(t)W(t)$, thus refers to a newly arriving queuing packet. As the UDP shares the same link and channel with the TCP, the probability of losing a packet increases as the UDP continues sending packet even where congestion occurs: UDP unresponsive flow is thus considered to represent exogenous disturbance, $d(t)$.

3. Problem Statement

Let $x_1(t) = q(t)x_2(t) = N/R(t)W(t)$.

Remark 1. The model in equation (4) is different than the model introduced in [19], with the $\tan h(\cdot)$ function used as a limit function, with ε as a tuning parameter, rather than the $\mathrm{sat}(\cdot)$ function as a way to solve the problem of the sharp edge. In addition, the effect of both the disturbance and the time-varying delay is considered in this model, to approximately reflect the real behavior of the TCP network.

Based on the parameters from Remark 1, the equations for the TCP/AQM network can be represented as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t) + d(t), \\ \dot{x}_2(t) = f(\dot{x}_1, x_2, N, R(t), C, d(t)) + g_1(x)u, \\ y(t) = x(t), \end{cases} \tag{5}$$

where

$$f(x_1, x_2, N, R(t), C) = \frac{N}{R^2(t)} - \frac{x_2(t)\dot{x}_1(t)}{R(t)C},$$
$$g_1(x) = -\frac{x_2(t)+C}{2N}\left[(x_2(t) - R(t)) + C\right], \tag{6}$$

where $x = \{x_1, x_2\} \in \mathbb{R}^2$, representing the queuing length and the window size, respectively, $y(t) = x_1(t) \in \mathbb{R}$ is the measured output, and u is the control input, which is designed to stabilize and minimize the probability of packet loss to achieve the desired queuing length and reduce or avoid congestion when the exogenous disturbance $d(t)$ and parameter uncertainty are applied to the TCP/AQM network.

4. The Proposed ADRC Design

The ADRC is one of the most effective anti-disturbance methods, and it was first proposed by [14] in the late 1980s. The effectiveness of the ADRC is due to its ability to actively estimate disturbance, thus providing fast-tracking and accurate control. The design of the ADRC depends on its relative degree, and in this section, the design of the proposed ADRC, which consist of two options, a nonlinear controller and a tracking differentiator, and two schemes supporting the proposed nonlinear ESO, is introduced and examined.

4.1. The Proposed Tracking Differentiator. The tracking differentiator is that part of the ADRC used to generate the reference signal and the reference signal derivative, which must therefore offer a tuned and efficient response. The dynamic equation of the proposed tracking differentiator is thus given as

$$\begin{cases} \dot{r}_1(t) = r_2(t), \\ \dot{r}_2(t) = -a_1 R^2 \left(\dfrac{(r_1(t) - r(t)) + 2(r_1(t) - r(t))^3}{1 + \left| (r_1(t) - r(t)) + 2(r_1(t) - r(t))^3 \right|} \right) \\ -a_2 R r_2(t), \end{cases} \tag{7}$$

where $r_1(t)$ is the desired trajectory and $r_2(t)$ is its derivative and R, a_1, and a_2 are positive tuning parameters. It is worth noting that the function used in this equation (i.e., $((r_1(t)-r(t))+2(r_1(t)-r(t))^3/1+|(r_1(t)-r(t))+2(r_1(t)-r(t))^3|)$) was introduced in [20] in the form "*Rational functions and absolute value*" as a replacement for the function used in [14].

4.2. Proposed Nonlinear Controllers. In this subsection, the nonlinear controllers used in this study are introduced and defined:

(i) The first controller is the NLPID, which can be expressed as follows:

$$\begin{cases} u_1 = \dfrac{k_1}{1+\exp\left(e^2\right)}|e|^{\alpha_1}\operatorname{sign}(e), \\ u_2 = \dfrac{k_2}{1+\exp\left(\dot{e}^2\right)}|\dot{e}|^{\alpha_2}\operatorname{sign}(\dot{e}), \\ u_3 = \dfrac{k_3}{1+\exp\left(\int e^2 \mathrm{d}t\right)}\left|\int e\mathrm{d}t\right|^{\alpha_3}\operatorname{sign}\left(\int e\mathrm{d}t\right), \end{cases} \tag{8}$$

$$u_{0_{\mathrm{NLPID}}} = u_1 + u_2 + u_3. \tag{9}$$

(ii) The second controller is the proposed super-twisting sliding mode controller (STC-SM), expressed as

$$\begin{cases} \varsigma = \kappa e + \dot{e}, \\ u_{0_{\mathrm{STC-SM}}} = \kappa|\varsigma|^p \operatorname{sign}(\varsigma) + \xi \tan h\left(\dfrac{\varsigma}{\delta}\right), \end{cases} \tag{10}$$

where $(k_1, k_2, k_3, \alpha_1, \alpha_2, \alpha_3, \kappa, \xi, p, \delta)$ are the shared controller tuning parameters, ς is the sliding surface, and $e = r_1 - z_1$ and $\dot{e}$ are the reference error and its derivative.

4.3. The Proposed Nonlinear Extended State Observer. The nonlinear ESO is an improved nonlinear version of the linear ESO shown previously to effectively estimate disturbances in specific cases. The proposed NLESO is a modified version of the NLESO proposed by [21], and the mathematical representation of the first scheme of the modified NLESO is expressed as

$$\begin{cases} \dot{z}_1(t) = z_2(t) + \beta_1\widehat{e}_1(t), \\ \dot{z}_2(t) = z_3(t) + \beta_2\widehat{e}_2(t) + b_0 u(t), \\ \dot{z}_3(t) = \beta_3\widehat{e}_3(t). \end{cases} \tag{11}$$

For the nonlinear function,

$$\begin{cases} \widehat{e}_1(t) = \operatorname{sign}(e_1(t))|e_1(t)|^a + e_1(t), \\ \widehat{e}_2(t) = \operatorname{sign}(e_1(t))|e_1(t)|^{2a-1} + e_1(t), \\ \widehat{e}_3(t) = \operatorname{sign}(e_1(t))|e_1(t)|^{3a-2} + e_1(t), \end{cases} \tag{12}$$

where β_1, β_2, and β_3 are the observer gain, z_1 and z_2 are the estimated state, z_3 is the estimated total disturbance, and $0.67 < a < 1$ is a positive tuning parameter. Another scheme for NLESO is also proposed and used in this study. The mathematical representation of the second scheme for a modified NLESO can be expressed as follows:

$$\begin{cases} \dot{z}_1(t) = z_2(t) + \beta_1\widehat{e}_1(t), \\ \dot{z}_2(t) = z_3(t) + \beta_2\widehat{e}_2(t) + b_0 u(t), \\ \dot{z}_3(t) = \beta_3\widehat{e}_3(t), \end{cases} \tag{13}$$

where $\widehat{e}_i(t), i \in \{1,2,3\}$ is expressed as follows:

$$\begin{cases} \widehat{e}_1(t) = \operatorname{sign}(e_1(t))|e_1(t)|^a + Ae_1(t), \\ \widehat{e}_2(t) = \operatorname{sign}(e_1(t))|e_1(t)|^{a/2} + Ae_1(t), \\ \widehat{e}_3(t) = \operatorname{sign}(e_1(t))|e_1(t)|^{a/4} + Ae_1(t), \end{cases} \tag{14}$$

where $e_1(t) = q(t) - z_1(t)$, $e_1(t)$ is the estimation error, $z_1(t)$ is the estimation state of q, $\widehat{e}_1(t), \widehat{e}_2(t)$, and $\widehat{e}_3(t)$ are the nonlinear functions, a is a tuning parameter that should be less than 1, and $\mathscr{A}$ is another tuning parameter.

The complete diagrams of both the proposed controllers and the proposed ADRC with the TCP/AQM nonlinear model are shown in Figures 2 and 3. In this study, the tracking differentiator is used instead of the ordinary derivative as a way to access both the error and its derivative: thus, equations (7), (8), and (10) can be rewritten as follows.

(i) The tracking differentiator is as follows:

$$\left\{ \dot{\widetilde{e}}_1(t) = \widetilde{e}_2(t), \dot{\widetilde{e}}_2(t) = -a_1 R^2\left(\frac{(\widetilde{e}_1(t) - \widetilde{e}(t)) + 2(\widetilde{e}_1(t) - \widetilde{e}(t))^3}{1 + \left|(\widetilde{e}_1(t) - \widetilde{e}(t)) + 2(\widetilde{e}_1(t) - \widetilde{e}(t))^3\right|}\right) - a_2 R\widetilde{e}_2(t). \right. \tag{15}$$

(ii) The NLPID controller is as follows:

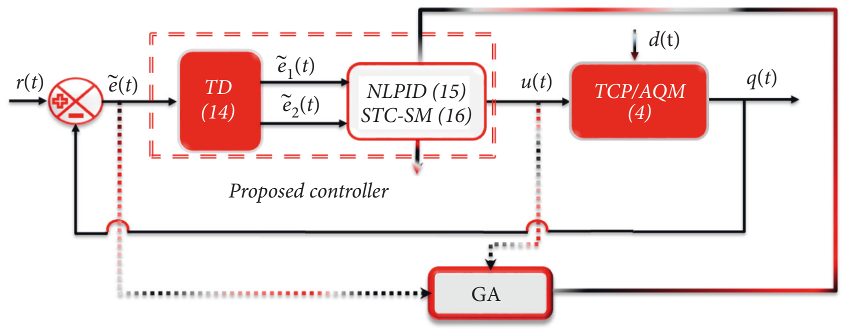

Figure 2: Completed diagram of the TCP/AQM based on the proposed controllers.

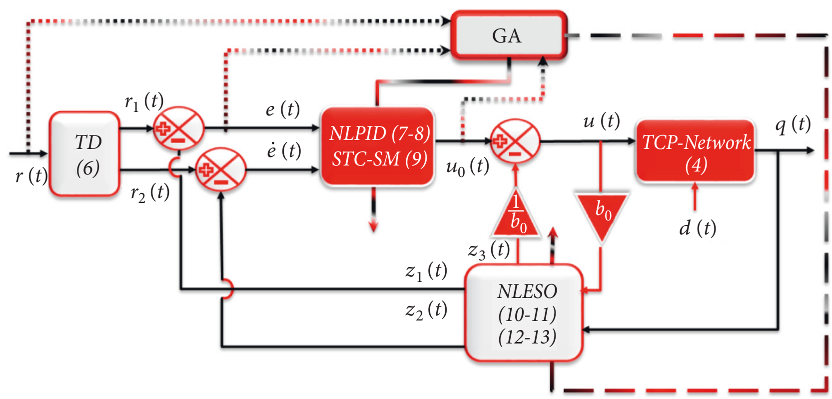

Figure 3: Completed diagram of the TCP/AQM based on the proposed ADRC.

$$\begin{cases} u_1 = \dfrac{k_1}{1+\exp\left(\tilde{e}_1^2\right)}\left|\tilde{e}_1\right|^{\alpha_1}\operatorname{sign}\left(\tilde{e}_1\right), \\ u_2 = \dfrac{k_2}{1+\exp\left(\tilde{e}_2^2\right)}\left|\tilde{e}_2\right|^{\alpha_2}\operatorname{sign}\left(\tilde{e}_2\right), \\ u_3 = \dfrac{k}{1+\exp\left(\int \tilde{e}_1^2\right)}\left|\tilde{e}_1\right|^{\alpha_3}\operatorname{sign}\left(\displaystyle\int \tilde{e}_1 \mathrm{d}t\right), \\ u_{0_{\text{NLPD}}} = u_1 + u_2 + u_3. \end{cases} \tag{16}$$

(iii) The STC-SM controller is as follows:

$$\begin{cases} \varsigma = \kappa\tilde{e}_1 + \tilde{e}_2, \\ u_{0_{\text{STC-SM}}} = \kappa|\varsigma|^a \operatorname{sign}(\varsigma) + \xi \tan h\left(\dfrac{\varsigma}{\delta}\right), \end{cases} \tag{17}$$

where $\tilde{e}_1$ and $\tilde{e}_2$ are the tracking error and its derivative and $\tilde{e}(t) = r(t) - q(t)$.

As mentioned previously, the design of the ADRC depends on the relative degree of the system, and as the TCP network is a SISO system, with a single input $u(t)$ and single output $q(t)$, the relative degree of the TCP/AQM network is thus $\rho = 2$.

5. The Proposed Nonlinear Disturbance Observer (NLDO) Design

The disturbance observer is one of the anti-disturbance techniques presented in [22], which can be linear or nonlinear; in this work, the nonlinear disturbance observer is thus presented, which is designed to estimate external disturbance, such as an unknown load, and the changes and UDP unresponsive flow are calculated so that the estimated value can be employed to compensate for the influence of the disturbance.

The NLDO can be expressed as follows [22]:

$$\begin{cases} \dot{\mathscr{Z}} = -l(x)\mathscr{G}_2(x)Z - l(x)\left[\mathscr{G}_2(x)P(x) + \mathscr{F}(x) + \mathscr{G}_1(x)u\right], \\ \hat{d} = P(x) + Z, \end{cases} \tag{18}$$

where $\hat{d}$ and $\mathscr{Z}$ are the estimated disturbance and the internal state of the nonlinear observer, respectively, and $P(x)$ is a nonlinear function to be designed, while $\ell(x)$ is the nonlinear observer gain where $x \in \{x_1, x_2\}$. To ensure that the NLDO is asymptotically stable, the nonlinear function $P(x)$ must be designed in such a way as to force the NLDO to be asymptotically stable.

5.1. Convergence of the Proposed NLDO

5.1. Convergence of the Proposed NLDO. To prove the effectiveness of the designed NLDO with the TCP network, an inclusive analysis was done using a Lyapunov stability approach [23].

As shown, equation (4) cannot fit the form of equation (18) due to the system in equation (4) having input in one channel and a disturbance in the other. To convert the system from a mismatched to a matched one, the following procedure must thus be applied:

Let $x_1(t) = q(t)x_2(t) = (N/R(t))W(t)$ so that

$$\begin{cases} \dot{x}_1(t) = x_2(t) - C + d(t), \\ \dot{x}_2(t) = \dfrac{N}{R^2(t)} - \dfrac{x_2(t)\dot{x}_1(t)}{R(t)C} \\ -\dfrac{x_2(t)+C}{2N}\left[(x_2(t)-R(t))+C\right]u(t), \\ y(t) = x_1(t). \end{cases} \tag{19}$$

For simplicity, $\phi(t-t_0) = \phi(t)$ is assumed; equation (19) can thus be rewritten as

$$\begin{cases} \dot{x}_1(t) = x_2(t) - C + d(t), \\ \dot{x}_2(t) = m_1 - m_2 x_2(t)u, \\ y(t) = x_1(t), \end{cases} \tag{20}$$

where $m_1 = (N/R^2(t)) - (x_2(t)\dot{x}_1(t)/R(t)C)$ and $m_2 = -((x_2(t)+C)^2/2N)$.

Equation (21) can then be transformed into the following form:

$$\begin{cases} \dot{x}_1(t) = \mathbb{F}_1(x_1,x_2) + bd(t), \\ \dot{x}_2(t) = \mathbb{F}_2(x_1,x_2) + b_2u(t), \\ y = x_1(t). \end{cases} \tag{21}$$

Differentiating the first equation of equation (21) yields

$$\ddot{x}_1 = \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_1}\dot{x}_1 + \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_2}\dot{x}_2 + b_1\dot{d}(t), \tag{22}$$

while substituting equations (21) into (22) produces

$$\begin{aligned} \ddot{x}_1 = {} & \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_1}\left[\mathbb{F}_1(x_1,x_2) + b_1d(t)\right] \\ & + \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_2}\left[\mathbb{F}_2(x_1,x_2) + b_2u(t)\right] + b_1\dot{d}(t), \end{aligned} \tag{23}$$

Rearranging equation (23) gives

$$\begin{aligned} \ddot{x}_1 &= \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_1}\mathbb{F}_1(x_1,x_2) + \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_2}\mathbb{F}_2(x_1,x_2) + \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_2}b_2u(t) + \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_1}b_1d(t) + b_1\dot{d}(t), \\ \ddot{x}_1 &= \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_1}\mathbb{F}_1(x_1,x_2) + \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_2}\mathbb{F}_2(x_1,x_2) + b_2\frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_2}\left[u(t) + \frac{\partial\mathbb{F}_1(x_1,x_2)/\partial x_1 b_1\mathrm{d}(t) + b_1\dot{\mathrm{d}}(t)}{\partial\mathbb{F}_1(x_1,x_2)/\partial x_2 b_2}\right]. \end{aligned} \tag{24}$$

Then,

$$\ddot{x}_1 = \widehat{\mathbb{F}}(x_1,x_2) + \widehat{b}(u+\mathbb{D}), \tag{25}$$

where

$$\begin{aligned} \widehat{\mathbb{F}}(x_1,x_2) &= \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial \mathrm{x}_1}\mathbb{F}_1(x_1,x_2) \\ &\quad + \frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_2}\mathbb{F}_2(x_1,x_2), \\ \widehat{b} &= b_2\frac{\partial\mathbb{F}_1(x_1,x_2)}{\partial x_2}, \\ d &= \frac{\partial\mathbb{F}_1(x_1,x_2)/\partial x_1 b_1\mathrm{d}(t) + b_1\dot{\mathrm{d}}(t)}{(\partial\mathbb{F}_1(x_1,x_2)/\partial x_2)b_2}. \end{aligned} \tag{26}$$

Let $x_1(t) = x_1(t)$ and $x_2(t) = \dot{x}_1(t)$; this allows equation (20) to be rewritten as

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = \widehat{\mathbb{F}}(x_1,x_2,x_2) + \widehat{b}(u+d), \\ y(t) = x_1(t). \end{cases} \tag{27}$$

where $x = \{x_1(t), x_2(t)\} \in \mathbb{R}^2$, $\widehat{\mathbb{F}}(x_1,x_2,x_2)$ is the matched nonlinear function, and $Y(t) \in \mathbb{R}$ is the output of the system.

Remark 2. As seen from equation (27), $\widehat{\mathbb{F}}$ depends on x_2; thus, to find an expression for x_2, the first equation of (19) can be used, and the x_2 thus found substituted into equation (27) to transform the system in the term $(x_1(t), x_2(t))$. $x_2(t) = \dot{x}_1(t) + C - d(t) \longrightarrow x_2(t) = x_2(t) + C - d(t)$. This means that equation (27) can be rewritten as

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = \widehat{\mathbb{F}}(x_1,x_2,C,d(t)) + \widehat{b}(u+D), \\ y(t) = x_1(t). \end{cases} \tag{28}$$

Adding $\pm b_0u$ to the second equation of (28) yields

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = \mathsf{F} + b_0u, \\ y(t) = x_1(t), \end{cases} \tag{29}$$

where $\mathsf{F} = \widehat{\mathbb{F}}(x_1,x_2,C,d(t)) + \widehat{b}D + (\widehat{b}-b_0)u$.

Theorem 1. *Assuming a TCP network as given in equation (28), the proposed NLDO will be asymptotically stable if* $P(\mathbb{X})$

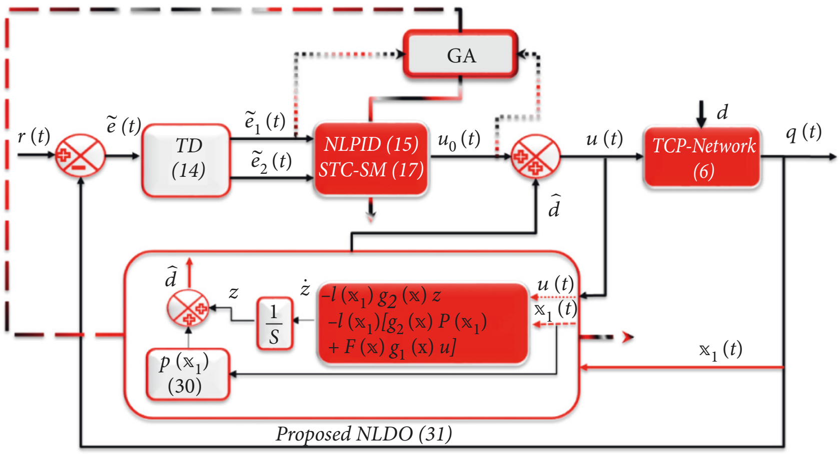

FIGURE 4: Completed diagram of TCP/AQM based on the proposed NLDO.

is designed appropriately. An appropriate nonlinear function $P(\mathbb{X})$ is proposed as

$$\begin{cases} P(x) = k + kx, \\ l(x) = \dfrac{\partial P(x)}{\partial x}, \end{cases} \tag{30}$$

where $x = x_1 \in \mathbb{R}h$ is a positive tuning parameter.

Proof. Assuming a system as given in equation (28), the proposed NLDO can be expressed as follows:

$$\begin{cases} \dot{\mathscr{Z}} = -l(x_1)g_2(x)Z - l(x_1)\left[g_2(x)P(x_1) + \widehat{\mathbb{F}}(x) + g_1(x)u\right], \\ \hat{d} = P(x_1) + Z. \end{cases} \tag{31}$$

Differentiating $\hat{d}$ gives

$$\dot{\hat{d}} = \dot{P}(x_1) + \dot{\mathscr{Z}}. \tag{32}$$

Differentiating the first equation of (30) yields

$$\dot{P}(x_1) = \frac{\mathrm{d}P}{\mathrm{d}x_1} = hx_1. \tag{33}$$

Substituting equation (33) into the first equation of (31) creates

$$\begin{aligned} \dot{\hat{d}} = {} & h\dot{x}_1 - \ell(x)g_2(x)\mathscr{Z} - g_2(x)P(x)\ell(x) \\ & - \ell(x)\widehat{\mathbb{F}}(x)\ell(x) - \ell(x)g_1(x)u. \end{aligned} \tag{34}$$

Simplifying equation (34) yields

$$\dot{\hat{d}} = hg_2(x)d(t) - hg_2(x)\mathscr{Z} - hg_2(x)P(x). \tag{35}$$

Substituting $\hat{d} = P(x) + \mathscr{Z}$ into equation (35) gives

$$\dot{\hat{d}} = hg_2(x)[d(t) - \hat{d}(t)], \tag{36}$$

where $d(t) - \hat{d}(t) = e_d$ represents the disturbance observer error, $d(t) = D$ is the applied exogenous disturbance, and $\hat{d}(t)$ is the estimated disturbance.

$$e_d = d(t) - \hat{d}(t). \tag{37}$$

Differentiating equation (37) allows the error dynamics to be expressed as

$$\dot{e}_d = \dot{d}(t) - \dot{\hat{d}}(t). \tag{38}$$

Assuming a constant disturbance, $\dot{d}(t) = 0$, the dynamic of the disturbance observer error can thus be given as

$$\dot{e}_d = -hg_2(x)e_d. \tag{39}$$

□

Remark 3. The NLDO is asymptotically stable if the estimated error converges to zero as $t \longrightarrow \infty$. To achieve this, a Lyapunov stability approach can be utilized [23].

Taking the Lyapunov function $V_{\mathrm{NLDO}} = (1/2)e_d^T e_d$,

$$\begin{aligned} \dot{V}_{\mathrm{NLDO}} &= e_d^T \dot{e}_d, \\ \dot{V}_{\mathrm{NLDO}} &= -e_d^T[hg_2(x)e_d], \\ \dot{V}_{\mathrm{NLDO}} &< -hg_2(x)e_d^2. \end{aligned} \tag{40}$$

The system in equation (30) is thus asymptotically stable when the following conditions are satisfied:

(i) V_{NLDO} is positive definite, $V_{\mathrm{NLDO}}(e_d) > 0$ for $e_d \neq 0$

(ii) $\dot{V}_{\mathrm{NLDO}}(e_d) < 0$ for $e_i \neq 0$

Thus, the NLDO is asymptotically stable if $hg_2(\mathbb{X}) > 0$, $h > 0$.

The complete diagram of the proposed NLDO with TCP network nonlinear model is shown in Figure 4.

5.2. Closed-Loop Stability. The overall stability analysis of the proposed ADRC with a TCP network nonlinear model is presented in this subsection. The ESO in the ADRC converts the system into a chain of integrators; however, the TCP network given in equation (7) cannot be converted into a chain of integrators due to the mismatch in the disturbance. To redress this, the TCP network given in equation (7) must be transformed from a mismatched system into matched one, as noted previously in equation (30).

Assuming that $\mathbb{L} = \mathsf{F} = x_3(t)$, equation (30) can be rewritten as

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = x_3(t) + b_0 u, \\ \dot{x}_3(t) = \dot{\mathbb{L}}, \\ y(t) = \dot{x}_1(t), \end{cases} \tag{41}$$

where $x_3(t)$ and $\dot{\mathbb{L}}$ represent the total disturbance and its derivative, respectively, and b_0 is a rough approximation of $\hat{b}$ within the range of ±50% [14].

Assumption 1 (see [15]). The total disturbance $\mathbb{L}$ should satisfy the following conditions:

(i) $\mathbb{L}$ and $\dot{\mathbb{L}}$ are bounded as
$\sup_{0\leq t\leq\infty}\mathbb{L} \leq c_1$ and $\sup_{0\leq t\leq\infty}\dot{\mathbb{L}} \leq c_2$

(ii) $\mathbb{L}$ and $\dot{\mathbb{L}}$ are constant at the steady state such that
$\lim_{t\longrightarrow\infty}\mathbb{L} = c_3$ and $\lim_{t\longrightarrow\infty}\dot{\mathbb{L}} \leq 0$
where $c_1 c_2$ and c_3 are positive constants.

Theorem 2. *Suppose an n order system with relative degree $\rho(\rho\leq n)$ is given as*

$$\begin{cases} \dot{x}_1(t) = f_1(x), \\ \vdots, \\ \dot{x}_\rho(t) = f_\rho(x) + b(u+d), \\ y = x. \end{cases} \tag{42}$$

where $x \in \{x_1, x_1, \ldots x_{\rho-1}\}$, and $f_1, \ldots, f_\rho$ are the system nonlinear functions.

According to equation (42), equation (30) can thus be represented as a chain of integrators in the form as follows:

$$\begin{cases} \dot{x}_1(t) = \dot{x}_2(t), \\ \vdots \\ \dot{x}_\rho(t) = x_{\rho+1}(t) + b_0 u, \\ \dot{x}_{\rho+1}(t) = \dot{\mathbb{L}}, \\ y(t) = x_1(t), \end{cases} \tag{43}$$

where $x_{\rho+1}(t)$ is the generalized disturbance.

If Assumption 1 is satisfied, then the system described by equation (43) is asymptotically stable when the estimated error of the proposed NLESO, as seen in equations (13) and (14) and expressed in the form $e_i = x_i - z_i i \in \{1, 2, \ldots, \rho+1\}$, approaches zero as $t \longrightarrow \infty$.

Proof. Let the estimated error e_i be

$$e_i = x_i - z_i, \tag{44}$$

where $i \in \{1, 2, \ldots, \rho+1\}$, ρ is the relative degree of the system, e_i is the estimated error, and z_i is the estimated state of x_i.

$$\begin{cases} e_1 = x_1 - z_1, \\ e_2 = x_2 - z_2, \\ e_\rho = x_\rho - z_\rho, \\ e_{\rho+1} = x_{\rho+1} - z_{\rho+1}. \end{cases} \tag{45}$$

Differentiating equation (45) produces

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{z}_1, \\ \dot{e}_2 = \dot{x}_2 - \dot{z}_2, \\ \vdots \\ \dot{e}_\rho = \dot{x}_\rho - \dot{z}_\rho, \\ \dot{e}_{\rho+1} = \dot{x}_{\rho+1} - \dot{z}_{\rho+1}. \end{cases} \tag{46}$$

Substituting (43) into (46) gives

$$\begin{cases} \dot{e}_1 = x_2 - z_2 - \beta_1\hat{e}_1, \\ \dot{e}_2 = \dot{x}_3 - z_3 - \beta_2\hat{e}_2, \\ \vdots \\ \dot{e}_\rho = x_{\rho+1} + b_0 u - z_{\rho+1} - b_0 u - \beta_\rho\hat{e}_\rho, \\ \dot{e}_{\rho+1} = \dot{\mathbb{L}} - \beta_{\rho+1}\hat{e}_{\rho+1}. \end{cases} \tag{47}$$

Simplifying equation (47) yields

$$\begin{cases} \dot{e}_1 = e_2 - \beta_1\hat{e}_1, \\ \dot{e}_2 = e_3 - \beta_2\hat{e}_2, \\ \dot{e}_\rho = e_{\rho+1} - \beta_\rho\hat{e}_\rho, \\ \dot{e}_{\rho+1} = \dot{\mathbb{L}} - \beta_{\rho+1}\hat{e}_{\rho+1}. \end{cases} \tag{48}$$

Expressing equation (48) in matrix form gives

$$\dot{e} = A_0 e + A_d\dot{\mathbb{L}}. \tag{49}$$

Assume $\hat{e}_1 = K_1(e_1), \ldots, \hat{e}_\rho = K_\rho(e_1)$ and $\hat{e}_{\rho+1} = K_{\rho+1}(e_1)$.

Then,

$$A_0 = \begin{bmatrix} -\beta_1 K_1(e_1) & 1 & \cdots & 0 & 0 \\ -\beta_2 K_2(e_1) & 0 & \ddots & 0 & 0 \\ -\beta_\rho K_\rho(e_1) & \vdots & \cdots & \vdots & \vdots \\ \vdots & 0 & \cdots & 0 & 1 \\ -\beta_\rho K_{\rho+1}(e_1) & 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \tag{50}$$

$$\dot{e} = \left[\dot{e}_1 \ \cdots \ \dot{e}_{\rho+1}\right],$$

$$e = \left[\dot{e}_1 \ \cdots \ \dot{e}_{\rho+1}\right].$$

The NLESO is asymptotically stable if the estimated error converges to zero as $t \longrightarrow \infty$. To check this, the Lyapunov stability can be used [23]. Taking the Lyapunov function $V_{\text{NLESO}} = 1/2e^T e$ gives $\dot{V}_{\text{NLESO}} = e^T \dot{e}$.

For the TCP network, the relative degree $\rho = 2$; hence,

$$\dot{V}_{\text{NLESO}} = \left[e_1 \ \ e_2 \ \ e_3\right] \begin{bmatrix} -\beta_1 K_1(e_1) & 1 & 0 \\ -\beta_2 K_2(e_1) & 0 & 1 \\ -\beta_3 K_3(e_1) & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \dot{\mathbb{L}}. \tag{51}$$

According to Assumption 1, $\dot{\mathbb{L}}$ converges to zero as $t \longrightarrow \infty$, so that the quadric form $\dot{V}_{\text{NLESO}} = e^T Q \dot{e}$ is asymptotically stable if Q is a negative definite matrix, and thus, the system as a whole is asymptotically stable. To check whether the matrix Q_i is negative definite or not, the Routh stability criteria can be utilized. This first requires computing the characteristic equation for matrix Q:

$$|\lambda I - Q| = 0,$$

$$\begin{vmatrix} \lambda + \beta_1 K_1(e_1) & -1 & 0 \\ \beta_2 K_2(e_1) & \lambda & -1 \\ \beta_3 K_3(e_1) & 0 & \lambda \end{vmatrix} = 0. \tag{52}$$

$\lambda^3 + \beta_1 K_1(e_1)\lambda^2 + \beta_2 K_2(e_1)\lambda + \beta_3 K_3(e_1) = 0$; using the Routh stability criteria thus yields

$$\begin{array}{lll} \lambda^3 & 1 & \beta_2 K_2(e_1) \\ \lambda^2 & \beta_1 K_1(e_1) & \beta_3 K_3(e_1) \\ \lambda^1 & \dfrac{\beta_1 K_1(e_1)\beta_2 K_2(e_1) - \beta_3 K_3(e_1)}{\beta_1 K_1(e_1)} = c_1 & 0 \\ \lambda^0 & \dfrac{\beta_3 K_3(e_1) c_1}{c_1} = \beta_3 K_3(e_1) & 0 \end{array}, \tag{53}$$

$$\beta_1 K_1(e_1)\beta_2 K_2(e_1) - \beta_3 K_3(e_1) > 0, \beta_1 K_1(e_1) < \frac{\beta_2 K_2(e_1)}{\beta_3 K_3(e_1)}, \beta_3 K_3(e_1) > 0, \beta_3 > 0.$$

Q is thus negative definite if the observer gain $\beta_1, \beta_2, \beta_3 > 0$, which also leads to the NLESO being asymptotically stable. Generally, the error dynamics of the closed-loop system can be written as

$$\begin{cases} \tilde{e}_1 = r - z_1, \\ \tilde{e}_2 = \dot{r} - z_2, \\ \vdots \\ \tilde{e}_\rho = r^{\rho-1} - z_\rho. \end{cases} \tag{54}$$

Differentiating equation (54) gives

$$\left\{ \dot{\tilde{e}}_1 = \dot{r} - \dot{z}_1, \dot{\tilde{e}}_2 = \ddot{r} - \dot{z}_2, \ \vdots \ , \dot{\tilde{e}}_\rho = r^\rho - \dot{z}_\rho. \right. \tag{55}$$

Simplifying equation (55) yields

$$\left\{ \dot{\tilde{e}}_1 = \tilde{e}_2, \dot{\tilde{e}}_2 = \tilde{e}_3, \ \vdots \ , \dot{\tilde{e}}_\rho = -z_{\rho+1} - b_0 u. \right. \tag{56}$$

□

Assumption 2. The tracking differentiator in equation (7) tracks the reference signal with only a very small error, which thus approaches zero with $r^\rho = 0$.

$$\lim_{t\longrightarrow\infty}\left|r_{1,\ldots,\rho}-r^{\rho-1}\right|=0. \tag{57}$$

Assumption 3. The NLESO in equations (12) to (15) estimates the states of the nonlinear system completely.

$$\lim_{t\longrightarrow\infty}e_{1,2,\ldots,\rho+1}=0. \tag{58}$$

Theorem 3. *Given the nonlinear system in (43) and the tracking differentiator given in (7) in conjunction with the NLPID given in (8) and (9) and the NLESO presented in (12) to (15), based on Assumptions 2 and 3, the closed-loop system is stable if* $\{\mathscr{K}_1(\tilde{e}_1)\tilde{e}_1, \mathscr{K}_2(\tilde{e}_2)\tilde{e}_2\}$ *is chosen in such a way that the Q matrix is negative definite and satisfies the characteristic equation* $\lambda^2+\mathscr{K}_2'\lambda+\mathscr{K}_1=0$*, which is Hurwitz.*

Proof. Taking $u=u_0-(z_{\rho+1}/b_0)$, (56) can be rewritten as

$$\begin{cases}\dot{\tilde{e}}_1=\tilde{e}_2,\\ \dot{\tilde{e}}_2=\tilde{e}_3,\\ \vdots\\ \dot{\tilde{e}}_\rho=-z_{\rho+1}-b_0\left[u_0-\dfrac{z_{\rho+1}}{b_0}\right],\end{cases} \tag{59}$$

$$\begin{cases}\dot{\tilde{e}}_1=\tilde{e}_2,\\ \dot{\tilde{e}}_2=\tilde{e}_3,\\ \vdots\\ \dot{\tilde{e}}_\rho=-z_{\rho+1}-b_0u_0+z_{\rho+1}.\end{cases} \tag{60}$$

Simplifying (60) gives

$$\begin{cases}\dot{\tilde{e}}_1=\tilde{e}_2,\\ \dot{\tilde{e}}_2=\tilde{e}_3,\\ \vdots\\ \dot{\tilde{e}}_\rho=-b_0u_0,\end{cases} \tag{61}$$

$$\begin{cases}\dot{\tilde{e}}_1=\tilde{e}_2,\\ \dot{\tilde{e}}_2=\tilde{e}_3,\\ \vdots\\ \dot{\tilde{e}}_\rho=-b_0\left[u_1(\tilde{e}_1)+\cdots\cdots+u_\rho(\tilde{e}_\rho)\right].\end{cases} \tag{62}$$

□

Assumption 4. Assume $\mathscr{K}_i(\tilde{e}_i)=(k_i/1+\exp(\tilde{e}_i^2))i\in\{1,2\ldots\ldots,\rho+1\}$ and that $\alpha_1,\alpha_2,\ldots,\alpha_\rho$ approaches unity: based on these assumptions, the term $|\mathbb{S}|\text{sign}(\mathbb{S})$ in equation (8) is approximately equal to $\mathbb{S}$.

Based on Assumption 4, equation (62) can thus be rewritten as

$$\begin{cases}\dot{\tilde{e}}_1=\tilde{e}_2,\\ \dot{\tilde{e}}_2=\tilde{e}_3,\\ \vdots\\ \dot{\tilde{e}}_\rho=-b_0\left[\mathscr{K}_1(\tilde{e}_1)\tilde{e}_1+\cdots+\mathscr{K}_\rho(\tilde{e}_\rho)\tilde{e}_\rho\right].\end{cases} \tag{63}$$

Expressing equation (63) in matrix form gives

$$\dot{\tilde{e}}=A_{C_{\text{NLPID}}}\tilde{e}, \tag{64}$$

where

$$A_{C_{\text{NLPID}}}=\begin{bmatrix}0&1&\cdots&0&0\\0&0&\cdots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&0&1\\-\mathscr{K}_1'&-\mathscr{K}_2'&\cdots&-\mathscr{K}_{\rho-1}'&-\mathscr{K}_\rho'\end{bmatrix},\ \tilde{e}=\begin{bmatrix}\tilde{e}_1\\\tilde{e}_2\\\vdots\\\tilde{e}_{\rho-1}\\\tilde{e}_\rho\end{bmatrix},$$

$\mathscr{K}_i'=b_0\mathscr{K}_i(\tilde{e}_i)$, $i\in\{1,2,\ldots,\rho+1\}$.

A Lyapunov function can be used to check the stability of the closed-loop system: $V_{cl}=1/2\tilde{e}^T\tilde{e}$. Then, $\dot{V}_{cl}=\tilde{e}^T\dot{\tilde{e}}$.

$$\dot{V}_{cl}=\left[\tilde{e}_1,\tilde{e}_2,\ldots,\tilde{e}_\rho\right]\begin{bmatrix}0&1&\cdots&0&0\\0&0&\cdots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&0&1\\-\mathscr{K}_1'&-\mathscr{K}_2'&\cdots&-\mathscr{K}_{\rho-1}'&-\mathscr{K}_\rho'\end{bmatrix}\begin{bmatrix}\dot{\tilde{e}}_1\\\dot{\tilde{e}}_2\\\vdots\\\dot{\tilde{e}}_\rho\end{bmatrix}. \tag{65}$$

The quadric form $\dot{V}_{cl}=\tilde{e}^TQ\dot{\tilde{e}}$ is stable if Q is a negative semi-definite matrix, at which point the system is stable.

Finding the characteristic equation for matrix Q using the Routh stability criteria allows a check on the negative definiteness of matrix Q,

$$|\lambda I-Q|=0-\begin{vmatrix}\lambda&\cdots&0\\0&\cdots&0\\\vdots&\ddots&\vdots\\0&\cdots&\lambda\end{vmatrix}\begin{bmatrix}0&1&\cdots&0&0\\0&0&\cdots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&0&1\\-\mathscr{K}_1'&-\mathscr{K}_2'&\cdots&-\mathscr{K}_{\rho-1}'&-\mathscr{K}_\rho'\end{bmatrix},$$

$$\lambda^\rho+\mathscr{K}_\rho'\lambda^{\rho-1}+\cdots+\mathscr{K}_2'\lambda+\mathscr{K}_1'=0. \tag{66}$$

Table 1: TCP network model parameters.

Parameter	Description	Value	Unit
$q(t)_{\mathrm{des}}$	The desire queuing length	200	packets
$N(t)$	Load factor	100	unitless
$C(t)$	Link capacity	3750	packets/sec
τ_p	The propagation delay	0.195	Sec

Table 2: Descriptions and mathematical representations of the performance indices.

Performance index (PI)	Description	Mathematical representation
ITAE	Integral time absolute error	$\int_0^{tf} t\lvert e(t)\rvert dt$
IAU	Integral absolute of the control signal	$\int_0^{f} \lvert u(t)\rvert dt$
ISU	Integral square of the control signal	$\int_0^{f} u(t)^2 dt$
IAE	Integral square error	$\int_0^{tf} e(t)^2 dt$
ISE	Integral absolute error	$\int_0^{tf} \lvert e(t)\rvert dt$
MSE	Mean square error	$(1/T)\int_0^{tf} e(t)^2 dt$

Table 3: LPID parameters.

Controller	Parameter	Value	Parameter	Value
LPID	k_p	0.025000	k_i	0.015000
	k_d	0.010000	δ	3.039957

For $\rho = 2$,

$$
\begin{gathered}
\lambda^2 + \mathscr{K}_2'\lambda + \acute{\mathscr{K}}_1 = 0,\\
\begin{array}{lll}
\lambda^2 & 1 & \acute{\mathscr{K}}_1 \\
\lambda^1 & \mathscr{K}_2' & 0 \\
\lambda^0 & \dfrac{\mathscr{K}_1'\mathscr{K}_2'}{\mathscr{K}_2'} & \lambda^2
\end{array},\\
\acute{\mathscr{K}}_2 > 0, \frac{\mathscr{K}_1'\mathscr{K}_2' - 0}{\mathscr{K}_2'} > 0 \Rightarrow \mathscr{K}_1'\mathscr{K}_2' > 0 \Rightarrow \mathscr{K}_1' > 0.
\end{gathered}
\tag{67}
$$

The system is thus stable if the nonlinear function gains $\mathscr{K}_1'$ and $\mathscr{K}_2'$ satisfy the conditions mentioned above.

6. Simulation Results

The TCP network nonlinear model and the proposed controllers, the modified ADRC, and the proposed NLDO were designed and simulated using a MATLAB/Simulink environment. In addition, the parameters of all schemes mentioned previously and those proposed were tuned using a genetic algorithm (GA) [18]. Finally, the multi-objective performance index was utilized to investigate the performance and accuracy of the designed and proposed schemes. The TCP network model parameters are listed in Table 1, while the multi-objective performance index (OPI) is given as follows:

Table 4: NLPID parameters.

Controller	Parameter	Value	Parameter	Value
NLPID	k_1	0.562500	k_2	3.147000
	α_1	0.895600	α_2	0.779600
	k_3	7.193000	α_3	0.548450
TD	R	24.680000	a_2	7.842000
	a_1	2.912000	δ	10.326000

Table 5: STC-SM parameters.

Controller	Parameter	Value	Parameter	Value
STC-SM	κ	0.146000	ξ	0.000200
	p	0.954500	δ	0.426500
TD	R	6.190000	a_2	9.703000
	a_1	2.501000	—	—

$$
\begin{aligned}
\mathrm{OPI} = {} & w_1 * \frac{\mathrm{ITAE}}{\mathscr{N}_1} + w_2 * \frac{\mathrm{IAU}}{\mathscr{N}_2} + w_3 * \frac{\mathrm{ISU}}{\mathscr{N}_3} + w_4 * \frac{\mathrm{ISE}}{\mathscr{N}_4} \\
& + w_5 * \frac{\mathrm{IAE}}{\mathscr{N}_5} + w_6 * \frac{\mathrm{MSE}}{\mathscr{N}_6},
\end{aligned}
\tag{68}
$$

where $w_1, w_2, \ldots, w_6$ are the weighting factors that satisfy $w_1 + w_2 + \cdots + w_6 = 1$. These are thus set to $w_1 = 0.3$, $w_2 = 0.2$, $w_3 = 0.1$, $w_4 = 0.2$, $w_5 = 0.1$, and $w_6 = 0.1$, with $\mathscr{N}_1, \mathscr{N}_2, \ldots, \mathscr{N}_6$ as the nominal values of the individual objective functions, with values set to $\mathscr{N}_1 = 772$, $\mathscr{N}_2 = 1000$, $\mathscr{N}_3 = 20$, $\mathscr{N}_4 = 86014.936857$, $\mathscr{N}_5 = 260$, and $\mathscr{N}_6 = 860$.

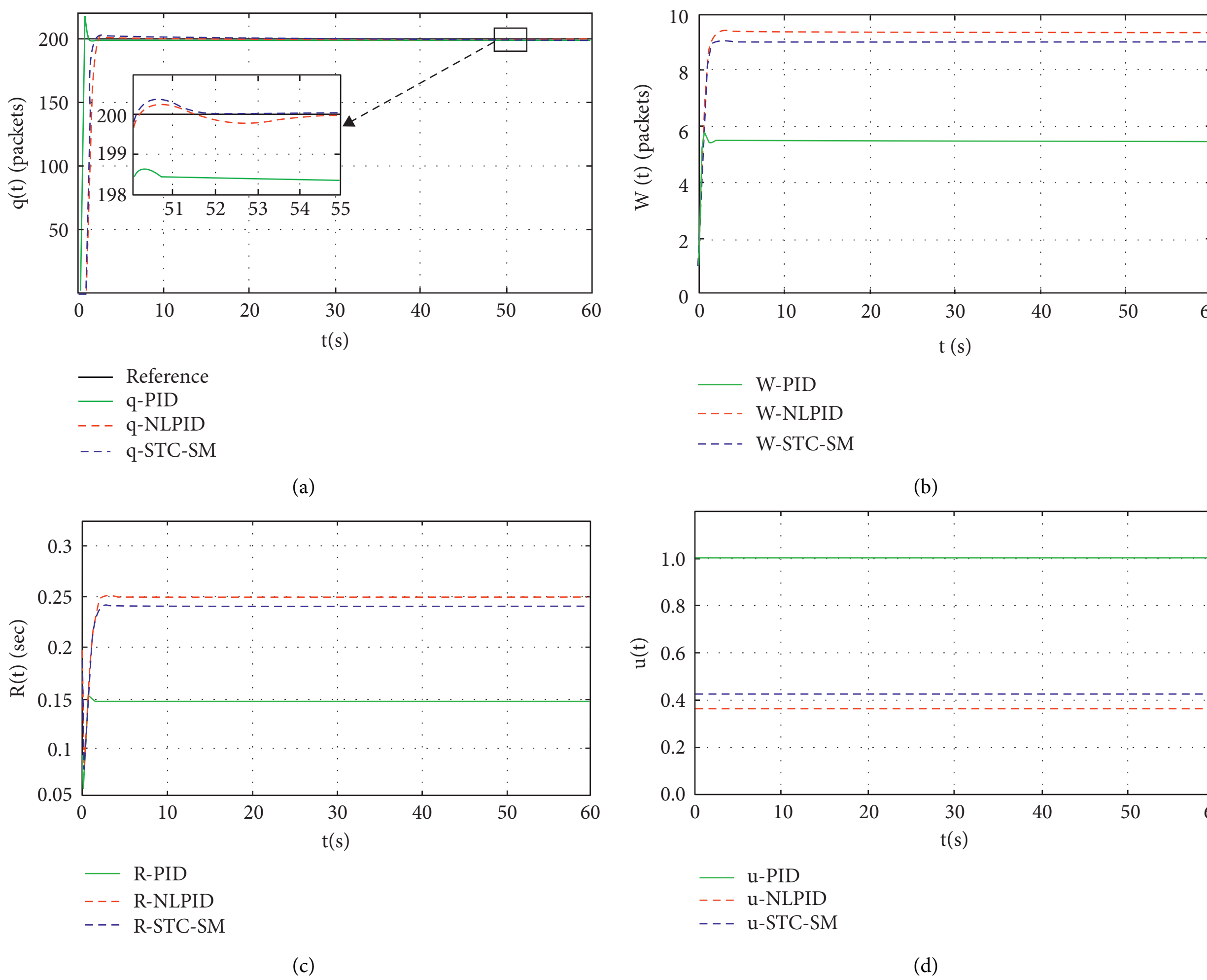

Figure 5: The output response when the disturbance is applied at $t = 50$ s. (a) The average queuing length $q(t)$. (b) The window size $W(t)$. (c) The round trip time $R(t)$ (d). The probability of losing packets $u(t)$. The performance index values are shown in Table 6. As demonstrated, the proposed controllers (NLPID and STC-SM) show noticeable improvements in OPI of 63.7363% and 72.52524%, respectively. The proposed controllers thus demonstrate effectiveness based on smooth response and minimized OPI.

Table 6: Performance indices.

PI	LPID	NLPID	STC-SM
ITAE	609.745670	544.413440	424.866163
IAU	5233.023924	544.539821	112.530207
ISU	132.195167	2.694587	0.184691
IAE	323.637549	301.475409	273.064131
ISE	53979.519643	48988.720573	44771.045158
MSE	4065.828712	1632.413215	1491.870882
OPI	2.078752	0.753616	0.571132

The mathematical representations of the performance indices used are presented more clearly in Table 2.

6.1. Simulation Using the Proposed Controllers. In this subsection, the simulation results from using the proposed controllers only are introduced. The obtained results are then compared with the LPID, which can be expressed as follows:

Table 7: LADRC parameters.

ADRC parts	Parameter	Value	Parameter	Value
LPID	k_p	10.237000	k_i	1.015000
	k_d	10.237000	δ	10.326000
TD	R	100	—	—
LESO	ω_0	10.326000	b_0	−69505.943979

$$u_{0_{PID}} = k_p e + k_i \int_0^T e dt + k_d \frac{de}{dt}, \quad (69)$$

where $k_p, k_i,$ and k_d are the proportional, integral, and derivative gains, respectively. The parameters of the LPID, NLPID, and STC-SM controllers are listed in full in Tables 3–5.

The simulation results using only controllers within the TCP network nonlinear model under the presence of an exogenous disturbance (UDP flow) are shown in Figure 5. Figure 5(a) shows the output response for queuing length: here, the queuing length reaches the desired or steady-state

Table 8: NLPID-ADRC parameters.

ADRC parts	Parameter	Value	Parameter	Value
NLPID	k_1	3.842000	k_2	3.386000
	α_1	0.998900	α_2	0.905400
	k_3	0.339800	α_3	0.043300
TD	R	11.150000	a_2	9.656000
	a_1	0.391000	δ	0.365300
NLESO	ω_0	50.335000	b_0	−4283.212200
	a_1	0.269550	$\mathscr{A}$	7.043000

Table 9: STC-ADRC parameters.

ADRC parts	Parameter	Value	Parameter	Value
STC-SM	κ	0.000200	ξ	0.000100
	p	0.300500	δ	0.586600
TD	R	49.340000	a_2	1.950000
	a_1	5.210000	—	—
NLESO	ω_0	56.334000	b_0	−1061.793374
	a_1	0.997100	—	—

Table 10: NLDO parameters.

	Parameter	Value	Parameter	Value
NLPID	k_1	0.059000	k_2	0.013400
	α_1	0.071600	α_2	0.441500
	k_3	0.417750	α_3	0.784600
TD	R	95.030000	a_2	9.573000
	a_1	6.030000	δ	0.383700
NLDO	h	4.049500	—	—

value at about 3.61 s under both the NLPID and the STC-SM, while in the LPID, the queuing length did not reach the desired value despite coming close to it. In addition, when applying a disturbance of 5 at a time 50 s after starting the simulation, the proposed controllers (NLPID and STC-SM) show greater robustness against the resulting disturbance, with STC-SM showing an overshoot of 0.2% of the steady-state value for about 1 s before moving back to the steady-state value and NLPID showing an overshoot and undershoot of 0.15% and 0.1%, respectively, of the steady state for about 4 s before returning to the steady-state value. The LPID is clearly more significantly affected by the applied disturbance; however, Figures 5(b) and 5(c) show the output response of the window size and the round trip time, while Figure 5(d) shows the control signal. As these indicate, comparing the performance of the LPID, NLPID, and the STC-SM shows that the NLPID and STC-SM give better responses with minimum packet loss, while the LPID packet loss rate is at maximum, exceeding the predefined limits of packet loss: LPID is thus clearly weakest at handling both complexity and delay in TCP networks.

6.2. Simulation with Anti-Disturbance Methods. In this subsection, the simulation results of the anti-disturbance methods (modified ADRC and NLDO) are introduced. The results of the proposed methods are also compared with linear ADRC (LADRC) that utilizes a conventional TD as proposed by [14], the linear ESO (LESO), and the LPID. The dynamics of the conventional TD and LESO are thus given as follows:

(i) For conventional TD [14],

$$\begin{cases} \dot{r}_1(t) = r_2(t), \\ \dot{r}_2(t) = -R\operatorname{sign}\left(r_1(t) - r(t) + \dfrac{r_2(t)\left|r_2(t)\right|}{2R}\right), \end{cases} \tag{70}$$

where $r_1(t)$ is the desired trajectory and $r_2(t)$ is its derivative. R is an application that depends on the other parameters [14].

(ii) For LESO,

$$\begin{cases} \dot{z}_1(t) = z_2(t) + \beta_1(e_1), \\ \dot{z}_2(t) = z_3(t) + \beta_2(e_1) + b_0 u(t), \\ \dot{z}_3(t) = \beta_3(e_1). \end{cases} \tag{71}$$

The parameters of the proposed methods and the LADRC are listed in Tables 7–10.

A step function of $5u(t-50)$ was applied to the system as an exogenous disturbance. The simulation results when applying a UDP data flow as an exogenous disturbance to the

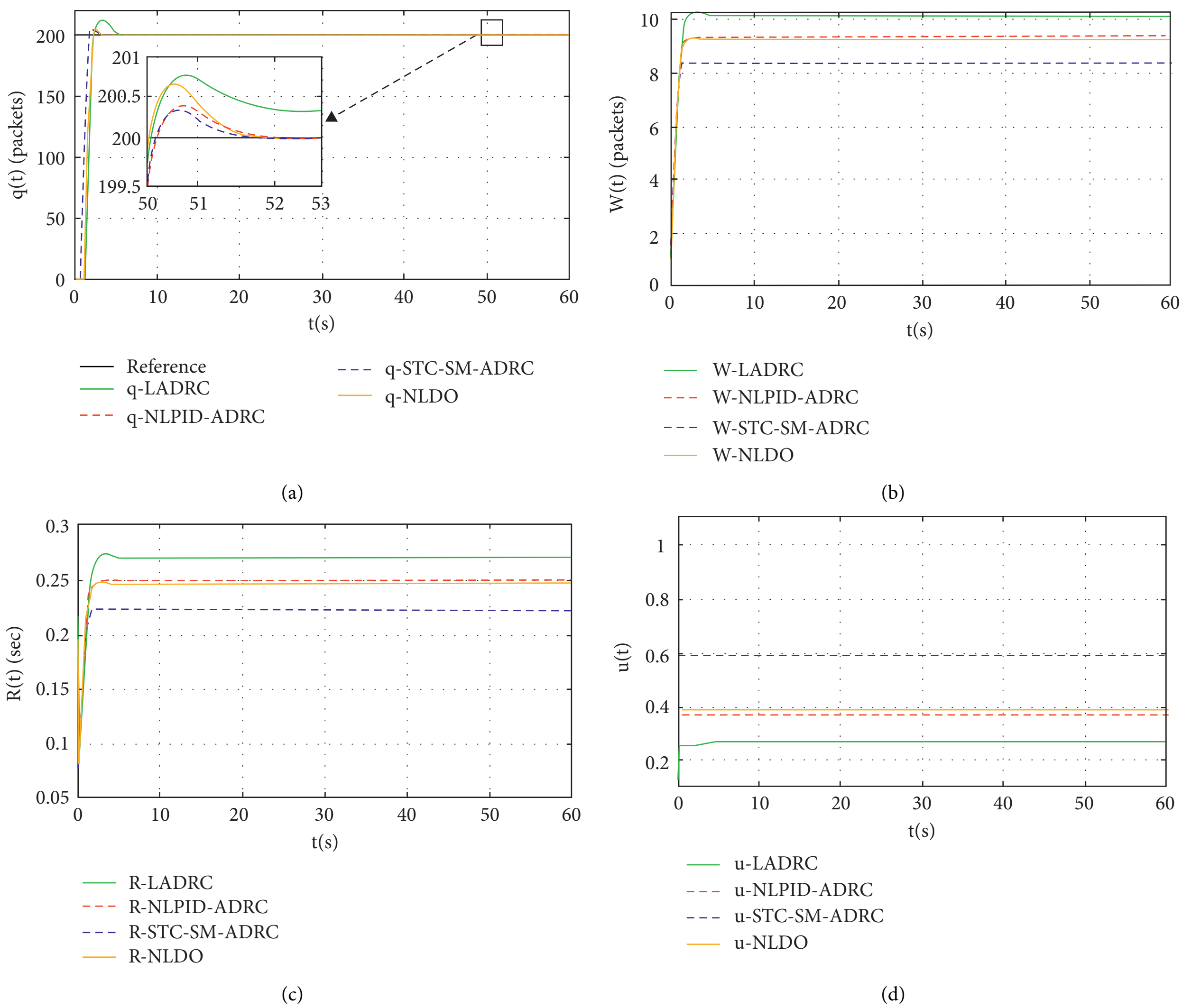

FIGURE 6: The output response when the disturbance is applied at $t = 50$ s. (a) The average queuing length $q(t)$. (b) The window size $W(t)$. (c) The round trip time $R(t)$ (d). The probability of losing packets $u(t)$.

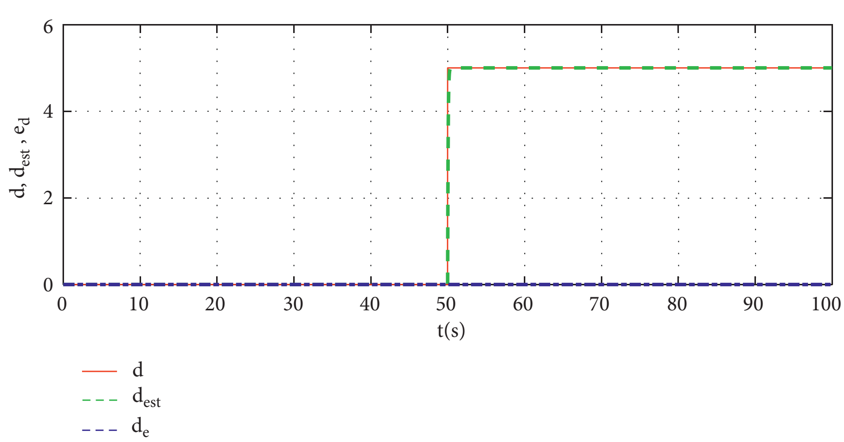

FIGURE 7: The applied disturbance $d(t)$, the estimated disturbance $\widehat{d}(t)$, and the disturbance error e_d.

Table 11: Performance indices.

PI	LADRC	NLDO	NLPID-ADRC	STC-SM-ADRC
ITAE	492.215585	275.662813	269.542633	209.239930
IAU	754.090790	71.626049	1.560232	0.315629
ISU	1.897476	0.019749	0.011761	0.000003
IAE	354.656276	278.636823	285.147109	221.044196
ISE	58172.631469	47530.946990	48837.954709	37063.347796
MSE	1938.441568	1583.836954	1627.389361	1235.033249
OPI	0.848649	0.523400	0.5176	0.396178

TCP network nonlinear model with the anti-disturbance methods (ADRC and NLDO) are shown in Figures 6 and 7. As shown in Figures 6(a)–6(c), the proposed methods (NLPID-ADRC and STC-ADRC) show excellent performance in terms of tracking the desired value and attenuating the disturbance as compared with the LADRC and NLDO options. Moreover, both NLPID-ADRC and STC-SM-ADRC reach a steady state in less than 2 s that lasts for about 1.5 s and 1.9 s, respectively, with movement back to the steady-state value after overshoots of 0.15% and 0.2% of the steady-state value. The NLDO lasted about 2.5 s before returning to the steady-state value after an overshoot of 0.35%. Finally, the LADRC is shown to be the weakest method in terms of disturbance attenuation as compared to the other methods. Figure 6(d) shows the control signal, which makes it clear that nearly all the presented methods show only small packet loss. Figure 7 shows the applied disturbance, the estimated disturbance, and the disturbance observer error, demonstrating that the proposed NLDO can perfectly estimate the applied disturbance.

The performance index values are shown in Table 11. The proposed controllers NLPID and STC-SM show noticeable improvement in OPI of 39.00894% and 53.30579%, respectively, in addition to improvements across all performance indices. Based on this, the proposed methods (NLPID-ADRC and STC-SM-ADRC) are thus shown to have improved accuracy and effectiveness based on smooth response and minimized OPI.

7. Conclusion

In this study, the time-delayed TCP network nonlinear model was utilized to design an accurate AQM using the ADRC approach to deal with the congestion problem and mismatched disturbances by stabilizing the nonlinear system. Two methods to achieve the main aim of this work were thus proposed in this study. A new NLPID and a new STC-SM were proposed to control and reduce congestion in the TCP network and stabilize the nonlinear system, and then, an NLDO was proposed to handle the problem of mismatched exogenous disturbance. Finally, a modified ADRC, consisting of the new NLPID and the new STC-SM as controller and tracking differentiator, respectively, was proposed to control congestion in the TCP network and to stabilize the nonlinear system as well as eliminating and rejecting the disturbance applied to the nonlinear system. The simulation results support the effectiveness of the modified ADRC in terms of congestion reduction and disturbance rejection. The modified ADRC was shown to provide a better performance, with smooth responses and minimum OPI, as compared to all the other methods introduced in this study. Moreover, the closed-loop stability and the convergence of NLESO were confirmed. Further studies related to this work could include using another optimization technique to tune the parameters of the modified ADRC.

Data Availability

All data are included within the manuscript.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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