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Research Article

Consensus of Time-Varying Interval Uncertain Multiagent Systems via Reduced-Order Neighborhood Interval Observer

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This work focuses on a multiagent system (MAS) with time-varying interval uncertainty in the system matrix, where multiple agents interact through an undirected topology graph and only the bounding matrices on the uncertainty in the system matrix are known. A reduced-order interval observer (IO), which is named the reduced-order neighborhood interval observer (NIO), is designed to estimate the relative state of each agent and those of its neighbors. It is shown that the reduced-order IO can guarantee the consensus of the uncertain multiagent system. Finally, simulation examples are proposed to verify the theoretical findings.

1. Introduction

With the discovery of the swarm intelligence and rapid development of the computer science [1–5], consensus of MASs has gained considerable attention [6–10], which means all the agents attain an agreement upon a common quantity of interest via distributed communications. Readers interested in consensus of MASs are referred to the great literature reviews on this topic, such as [11, 12].

Uncertainty exists widely in existing practical engineering, which can affect the stability of the control systems [13–15]. Most of the exisiting related works on MASs with uncertanities are carried out to eliminate the impact of uncertainties and achieve the consensus of the MASs [16, 17]. However, it is difficult or even impossible to get the specific information of the uncertainties. Thus, the acquirement of the bounding information on the uncertainties (BIU) is easier than that of the uncertainties. On the other hand, the states of the MASs cannot be achieved in some situations. Taking these two facts into consideration, IO is firstly proposed to single-agent systems [18, 19] to implement the state interval estimation and stabilization, where only the outputs and the BIU are related. An IO consists of two dynamical systems which are both in the form of Luenberger observer, where one is used to estimate the upper bound of the system state, while the other one aims at estimating the lower bound of the system state. Then, Wang et al. extended the IO to uncertain MASs and proposed some interesting results [20, 21]. According to the estimation objective, two kinds of IOs are defined for uncertain multiagent systems, the local IO [20–23] and the NIO [23, 24]. To be specific, the local IO is designed to do the estimation which relates only to the output information of the associated agent. Yet, the NIO is designed to estimate the relative states between agents and its neighbors, which relates to the sum of the relative outputs between each agent and its neighbors. In [20, 24], coordination control of MASs with uncertain disturbances is solved by introducing IO in MASs, including the local IO [20, 22] and the NIO [24]. In [21, 23], the IO-based consensus of MASs with time-varying interval uncertainties (TIUs) in the system dynamics is considered, by using only the outputs and the BIU, where the local full-order IO and neighborhood full-order IO are designed in [23], while local reduced-order IO is given in [21].

As stated above, the reduced-order NIO design problem of MASs with TIUs is not solved. This work pays attention to the reduced-order NIO design of MASs with TIUs and aims at estimating the sum of relative states between each agent and its neighbors and simultaneously achieving consensus. In this paper, the definition of reduced-order NIO is proposed in detail. It shows that the consensus is a by-part of the reduced-order NIO.

The rest of this paper is organized as follows: Problem formulation and some useful preliminaries used in this paper are introduced in Section 2. The main results are given in Section 3, and numerical simulations are presented in Section 4. Finally, conclusion is presented in Section 5.

2. Preliminaries and Problem Statement

2.1. Notation. $\mathbb{R}, \mathbb{N}, \mathbb{R}^{m\times n}$, and $\mathbb{R}_{+}^{m\times n}$ are denoted as the sets of real numbers, natural numbers, $m \times n$ real matrices, and $m \times n$ real matrices in which each element is a matrix with nonnegative entries, respectively. A square matrix is called to be Metzler when all its off-diagonal elements are nonnegative. All vector inequalities are understood element-wise, for instance, $v_a < (\le) v_b$ ($v_a = [\, v_{a1} \ \cdots \ v_{an} \,]$ and $v_b = [\, v_{b1} \ \cdots \ v_{bn} \,]$) if for all $i = 1, \ldots, n$, one has $v_{ai} < (\le) v_{bi}$. For any symmetric$A \in \mathbb{R}^{n\times n}$, $\lambda_i(A)$ denotes its eigenvalues, which is arranged as $\lambda_1(A) \le \lambda_2(A) \le \ldots \le \lambda_n(A)$. For any matrix $A = (a_{ij})$, $\|A\|$ denotes the 2-norm of A, $A^{+} = (a_{ij}^{+})$ (with $a_{ij}^{+} = \max\{a_{ij}, 0\}$), $A^{-} = A^{+} - A$ (similarly for vectors), and $|A| = (|a_{ij}|) = A^{+} + A^{-}$. A^T denotes its transpose matrix. I_N and 1_N denote an N-dimensional identity matrix and an $N \times 1$ vector with all the entries being 1, respectively. 0 denotes the number zero (or the zero matrix with compatible dimensions). $\text{diag}\{A1, \ldots, A_N\}$ denotes a block diagonal matrix, in which all the off-diagonal matrices are zeros and $A_i (i = 1, \ldots, N)$ is the i-th diagonal block.

2.2. Graph Theory. Let a triple $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$ be an undirected network, where $\mathcal{V} = \{v_1, \ldots, v_N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ are the node set and edge set, respectively, and $G = (g_{ij}) \in \mathbb{R}^{N\times N}$ is the adjacency matrix. If the information can be communicated between node v_i and node v_j, then $(v_i, v_j) \in \mathcal{E}$, that is, the edge (v_i, v_j) exists in $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$. The neighbor set of node i is $\mathcal{N}_i = \{v_j | (v_i, v_j) \in \mathcal{E}\}$. A path is a sequence of edges in $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$ of the form $(i_1, i_2) \ldots (i_2, i_3)$. If there exists a path from every node to every other node [9, 25, 26], it is said that $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$ is connected.

The adjacency matrix $G = (g_{ij}) \in \mathbb{R}^{N\times N}$ is defined as $g_{ij} = 1$ when $(v_i, v_j) \in \mathcal{E}$ and $g_{ij} = 0$ otherwise. The Laplacian matrix is defined as $L = (l_{ij}) \in \mathbb{R}^{N\times N}$, where $l_{ij} = -g_{ij}$ for $j \ne i$ and $l_{ij} = \sum_{j\ne i} g_{ij}$ for $j = i$. For this symmetric matrix L, in [9, 25, 26], it has exactly one zero eigenvalue with an associated eigenvector 1_N, and all the other ones are positive, if and only if $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$ is connected.

2.3. Problem Statement. Consider a continuous-time MAS with N agents and time-varying uncertainty in system matrix, where each agent moves in an n-dimensional Euclidean space and regulates itself based on the following dynamics:

$$\begin{cases} \dot{x}_i(t) = (A + \Delta A(t)) x_i(t) + B u_i(t), \\ y_i(t) = C x_i(t), i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n\times 1}, u_i(t) \in \mathbb{R}^{m\times 1}$, and $y_i(t) \in \mathbb{R}^{p\times 1}$ are the state, control input, and output of agent i, respectively. The matrices A, B, C are with compatible dimensions, while $\Delta A(t)$ (the uncertainty in system matrix) is a matrix-valued function of the time variable t. Moreover, $\Delta A(t)$ is time-varying interval uncertainty, which satisfies Assumption 1.

Assumption 1. There exist $\overline{\Delta A} \in \mathbb{R}^{n\times n}$ and $\underline{\Delta A} \in \mathbb{R}^{n\times n}$ with $\underline{\Delta A} \le \overline{\Delta A}$, such that $\Delta A(t) \in [\underline{\Delta A}, \overline{\Delta A}]$ for all $t \ge 0$.

Moreover, two technical assumptions are given.

Assumption 2. (A, B) is stabilizable.

Assumption 3. (C, A) is detectable.

Denote

$$w_i(t) = \sum_{j=1}^{N} l_{ij} x_j(t), \tag{2}$$

as the sum of relative state between the i-th agent and its neighbors. The main objective of this work is to realize the interval estimation on $w_i(t)$ on the basis of only the interval bound information of $\Delta A(t)$ given in Assumption 1, by using as few integrators as possible. Motivated by [27], a reduced-order NIO will be designed for system (1) to realize the interval estimation on $w_i(t)$. Again by [27], for C, there exists a matrix $D \in \mathbb{R}^{(n-p)\times n}$ to get a nonsingular matrix $P = [C/D] \in \mathbb{R}^{n\times n}$. Denote $P^{-1} = Q = [\, Q_1 \ \ Q_2 \,]$ with $Q_1 \in \mathbb{R}^{n\times p}$ and $Q_2 \in \mathbb{R}^{n\times (n-p)}$. With these matrices, one has $CQ_1 = I_p$ and $CQ_2 = 0$. For $i = 1, \ldots, N$, let $\widetilde{w}_i \triangleq P w_i = [\widetilde{w}_{iy} / \widetilde{w}_{iu}] = [\sum_{j=1}^{N} l_{ij} \widetilde{x}_{jy} / \sum_{j=1}^{N} l_{ij} \widetilde{x}_{ju}]$ with $\widetilde{w}_{iy} \in \mathbb{R}^{p}$ and $\widetilde{w}_{iu} \in \mathbb{R}^{n-p}$. Intuitively, $\widetilde{w}_{iy} = \sum_{j=1}^{N} l_{ij} \widetilde{x}_{jy} = \sum_{j=1}^{N} l_{ij} y_j$, so that there is no need to estimate $\widetilde{w}_{iy}$ but we have to estimate $\widetilde{w}_{iu}$. For simplicity, denote $PAP^{-1} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{bmatrix}$ and $PB = [\widetilde{B}_1 / \widetilde{B}_2]$ with $\widetilde{A}_{11} \in \mathbb{R}^{p\times p}, \widetilde{A}_{12} \in \mathbb{R}^{p\times (n-p)}, \widetilde{A}_{21} \in \mathbb{R}^{(n-p)\times p}, \widetilde{A}_{22} \in \mathbb{R}^{(n-p)\times (n-p)}, \widetilde{B}_1 \in \mathbb{R}^{p\times m}$, and $\widetilde{B}_2 \in \mathbb{R}^{(n-p)\times m}$. If ΔA in (1) is known, under Assumption 3, motivated by [27] and the full-order NIO constructed in [23], a neighborhood reduced-order observer can be designed as

$$\begin{aligned} \dot{z}_{iu} = {} & (\widetilde{A}_{22} - K\widetilde{A}_{12}) z_{iu} + \left[(\widetilde{A}_{22} - K\widetilde{A}_{12}) K + (\widetilde{A}_{21} - K\widetilde{A}_{11}) \right] \sum_{j=1}^{N} l_{ij} y_j \\ & + (\widetilde{B}_2 - K\widetilde{B}_1) \sum_{j=1}^{N} l_{ij} u_j + D\Delta A w_i - KC\Delta A w_i, \end{aligned} \tag{3}$$

where $K \in \mathbb{R}^{(n-p)\times p}$ is chosen to make $\widetilde{A}_{22} - K\widetilde{A}_{12}$ Hurwitz. On the other hand, for $\widetilde{w}_{iu}$ and $\widetilde{w}_{iy}$ defined above, by (1), one can get that

$$z_{iu} = \widetilde{w}_{iu} - K\widetilde{w}_{iy} = \widetilde{w}_{iu} - K\sum_{j=1}^{N} l_{ij}y_j. \quad (4)$$

However, in case that ΔA is unknown, the neighborhood reduced-order observer in (3) cannot be designed. Motivated by the local reduced-order IO given in [21], in this paper, we will solve the reduced-order NIO design for MASs steered by (1), where only the bounding information on ΔA is known. For better understanding, Definition 1 is given.

Definition 1. Consider two dynamical systems in the form of

$$\begin{aligned}
\dot{\overline{z}}_{iu} &= \overline{f}_1\left(A, B, C, \overline{\Delta A}, \underline{\Delta A}, \sum_{j=1}^{N} l_{ij}y_j, \sum_{j=1}^{N} l_{ij}u_j, \overline{z}_{iu}\right),\\
\dot{\underline{z}}_i &= \underline{f}_1\left(A, B, C, \overline{\Delta A}, \underline{\Delta A}, \sum_{j=1}^{N} l_{ij}y_j, \sum_{j=1}^{N} l_{ij}u_j, \underline{z}_{iu}\right), \quad (5)\\
\overline{z}_{iu}(0) &= \overline{f}_2\left(\overline{w}_i(0), \underline{w}_i(0)\right),\\
\underline{z}_{iu}(0) &= \underline{f}_2\left(\overline{w}_i(0), \underline{w}_i(0)\right),
\end{aligned}$$

with

$$u_i = f\left(\overline{z}_{iu}, \underline{z}_{iu}, y_j\right), j \in \mathcal{N}_i, \quad (6)$$

where $\overline{f}_1, \underline{f}_1, \overline{f}_2, \underline{f}_2$, and f are some differentiable continuous functions. Define

$$\begin{aligned}
\overline{w}_i &= \overline{f}_3\left(\overline{z}_{iu}, \underline{z}_{iu}, y_j\right),\\
\underline{w}_i &= \underline{f}_3\left(\overline{z}_{iu}, \underline{z}_{iu}, y_j\right),
\end{aligned} \quad (7)$$

with $j \in \mathcal{N}_i$; if $\underline{w}_i \le w_i \le \overline{w}_i$ holds for $t \ge 0$, it is said that $\overline{z}_{iu}$ and $\underline{z}_{iu}$ in (4) constitute a neighborhood reduced framer for (1). Beyond the holding $\underline{w}_i \le w_i \le \overline{w}_i$ for $t \ge 0$, we also have

$$\begin{aligned}
&\lim_{t\longrightarrow\infty}\left[\overline{w}_i - \sum_{j=1}^{N} l_{ij}x_j(t)\right] = 0,\\
&\lim_{t\longrightarrow\infty}\left[\underline{w}_i - \sum_{j=1}^{N} l_{ij}x_j(t)\right] = 0, i = 1, \ldots, N.
\end{aligned} \quad (8)$$

It is said that $\overline{z}_{iu}$ and $\underline{z}_{iu}$ in (4) constitute a reduced NIO for (1).

2.4. The Theory of Positive Systems. In order to realize the main objective of this paper, two lemmas about the positive systems theory are introduced.

Lemma 1 (see [28]). *Given a nonautonomous system described by $\dot{x} = Ax + B$, where A is a Metzler matrix and $B \ge 0$ for $t \ge 0$. Then, $x \ge 0$ for $t > 0$, provided that $x(0) \ge 0$.*

Lemma 2 (see [29]). *Let $x \in \mathbb{R}^{n\times 1}$ be a vector variable, $\underline{x} \le x \le \overline{x}$ for some $\overline{x}, \underline{x} \in \mathbb{R}^{n\times 1}$. Then,*

(1) For $A \in \mathbb{R}^{m\times n}$, $A^+\underline{x} - A^-\overline{x} \le Ax \le A^+\overline{x} - A^-\underline{x}$

(2) For $A \in \mathbb{R}^{m\times n}$ satisfying $\underline{A} \le A \le \overline{A}$ for some $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$ $\underline{A}^+\underline{x}^+ - \overline{A}^+\underline{x}^- - \underline{A}^-\overline{x}^+ + \overline{A}^-\overline{x}^- \le Ax \le \overline{A}^+\overline{x}^+ - \underline{A}^+\overline{x}^- - \overline{A}^-\underline{x}^+ + \underline{A}^-\underline{x}^-$

In the following, t will be omitted in all variables without confusion for notational simplicity. Similarly, we denote $x = \begin{bmatrix} x_1^T & \ldots & x_N^T \end{bmatrix}^T$ and give similar manners for y, u as well as other variables.

3. Main Results

Define

$$\begin{aligned}
\dot{\overline{z}}_{iu} =& \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\overline{z}_{iu} + \left[\left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)K + \left(\widetilde{A}_{21} - K\widetilde{A}_{11}\right)\right]\sum_{j=1}^{N} l_{ij}y_j\\
&+ \left(\widetilde{B}_2 - K\widetilde{B}_1\right)\sum_{j=1}^{N} l_{ij}u_j + D^+\overline{h}_i - D^-\underline{h}_i + (-KC)^+\overline{h}_i - (-KC)^-\underline{h}_i,\\
\dot{\underline{z}}_{iu} =& \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\underline{z}_{iu} + \left[\left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)K + \left(\widetilde{A}_{21} - K\widetilde{A}_{11}\right)\right]\sum_{j=1}^{N} l_{ij}y_j \quad (9)\\
&+ \left(\widetilde{B}_2 - K\widetilde{B}_1\right)\sum_{j=1}^{N} l_{ij}u_j + D^+\underline{h}_i - D^-\overline{h}_i + (-KC)^+\underline{h}_i - (-KC)^-\overline{h}_i,\\
\overline{z}_{iu}(0) =& D^+\overline{w}_i(0) - D^-\underline{w}_i(0) + (-KC)^+\overline{w}_i(0) - (-KC)^-\underline{w}_i(0),\\
\overline{z}_{iu}(0) =& D^+\underline{w}_i(0) - D^-\overline{w}_i(0) + (-KC)^+\underline{w}_i(0) - (-KC)^-\overline{w}_i(0),
\end{aligned}$$

where

$$\overline{h}_i \triangleq \overline{\Delta A}^{+}\overline{w}_i^{+} - \underline{\Delta A}^{+}\overline{w}_i^{-} - \overline{\Delta A}^{-}\underline{w}_i^{+} + \underline{\Delta A}^{-}\underline{w}_i^{-},$$
$$\underline{h}_i \triangleq \underline{\Delta A}^{+}\underline{w}_i^{+} - \overline{\Delta A}^{+}\underline{w}_i^{-} - \underline{\Delta A}^{-}\overline{w}_i^{+} + \overline{\Delta A}^{-}\overline{w}_i^{-}. \tag{10}$$

Then, one has Theorem 1.

Theorem 1. *Under Assumptions 1 and 3, if $\overline{w}_i(0)$ and $\underline{w}_i(0)$ in (8) are chosen to satisfy $\underline{w}_i(0) \leq w_i(0) \leq \overline{w}_i(0)$ and there exists an observer gain $K \in \mathbb{R}^{(n-p)\times p}$ to make $\widetilde{A}_{22} - K\widetilde{A}_{12}$ Metzler, then $\overline{z}_{iu}$ and $\underline{z}_{iu}$ given in (8) with this K are considered a neighborhood reduced-order framer for the uncertain MAS described by (1), where $\underline{w}_i \leq w_i \leq \overline{w}_i$ holds for $i = 1, \ldots, N$ with*

$$\overline{w}_i \triangleq Q^{+}\begin{bmatrix} \sum_{j=1}^{N} l_{ij}y_j \\ \overline{z}_{iu} + K\sum_{j=1}^{N} l_{ij}y_j \end{bmatrix} - Q^{-}\begin{bmatrix} \sum_{j=1}^{N} l_{ij}y_j \\ \underline{z}_{iu} + K\sum_{j=1}^{N} l_{ij}y_j \end{bmatrix},$$
$$\underline{w}_i \triangleq Q^{+}\begin{bmatrix} \sum_{j=1}^{N} l_{ij}y_j \\ \underline{z}_{iu} + K\sum_{j=1}^{N} l_{ij}y_j \end{bmatrix} - Q^{-}\begin{bmatrix} \sum_{j=1}^{N} l_{ij}y_j \\ \overline{z}_{iu} + K\sum_{j=1}^{N} l_{ij}y_j \end{bmatrix}. \tag{11}$$

Proof. 1 By Lemma 2 (1), if the following holds:

$$\begin{bmatrix} \sum_{j=1}^{N} l_{ij}y_j \\ \underline{z}_{iu} + K\sum_{j=1}^{N} l_{ij}y_j \end{bmatrix} \leq \begin{bmatrix} \sum_{j=1}^{N} l_{ij}y_j \\ \widetilde{w}_{iu} \end{bmatrix} \leq \begin{bmatrix} \sum_{j=1}^{N} l_{ij}y_j \\ \overline{z}_{iu} + K\sum_{j=1}^{N} l_{ij}y_j \end{bmatrix}, \tag{12}$$

for $t \geq 0$, then one can get $\underline{w}_i \leq w_i \leq \overline{w}_i$ for $t \geq 0$. Therefore, the proof of Theorem 1 will be completed if the inequality in (11) holds for $t \geq 0$, that is to prove the establishment of.

$$\underline{z}_{iu} \leq \widetilde{w}_{iu} - K\sum_{j=1}^{N} l_{ij}y_j \leq \overline{z}_{iu}. \tag{13}$$

By (3), (12), and (13) is equivalent to,

$$\underline{z}_{iu} \leq z_{iu} \leq \overline{z}_{iu}, \tag{14}$$

where z_{iu} is given in (2).

In order to prove the relationship in (13), let $\overline{e}_{iu} = \overline{z}_{iu} - z_{iu}$ and $\underline{e}_{iu} = z_{iu} - \underline{z}_{iu}$. By (2) and (8), one has

$$\begin{aligned}
\dot{\overline{e}}_{iu} &= \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\overline{e}_{iu} + D^{+}\overline{h}_i - D^{-}\underline{h}_i + (-KC)^{+}\overline{h}_i - (-KC)^{-}\underline{h}_i - D\Delta A w_i + KC\Delta A w_i \\
&= \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\overline{e}_{iu} + D^{+}\left(\overline{h}_i - \Delta A w_i\right) + D^{-}\left(\Delta A w_i - \underline{h}_i\right) \\
&\quad + (-KC)^{+}\left(\overline{h}_i - \Delta A w_i\right) + (-KC)^{-}\left(\Delta A w_i - \underline{h}_i\right), \\
\dot{\underline{e}}_{iu} &= \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\underline{e}_{iu} + D\Delta A w_i - KC\Delta A w_i - D^{+}\underline{h}_i + D^{-}\overline{h}_i - (-KC)^{+}\underline{h}_i + (-KC)^{-}\overline{h}_i \\
&= \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\underline{e}_{iu} + D^{+}\left(\Delta A w_i - \underline{h}_i\right) + D^{-}\left(\overline{h}_i - \Delta A w_i\right) \\
&\quad + (-KC)^{+}\left(\Delta A w_i - \underline{h}_i\right) + (-KC)^{-}\left(\overline{h}_i - \Delta A w_i\right).
\end{aligned} \tag{15}$$

Under Assumption 1, similar to the proof of Lemma 5 in [21], there hold $\overline{h}_i \geq \Delta A^+ \overline{w}_i - \Delta A^- \underline{w}_i$ and $\underline{h}_i \leq \Delta A^+ \underline{w}_i - \Delta A^- \overline{w}_i$, so that

$$
\begin{aligned}
\dot{\overline{e}}_{iu} \geq{}& \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\overline{e}_{iu} + D^+\left[\Delta A^+\overline{w}_i - \Delta A^- \underline{w}_i - \Delta A w_i\right] \\
&+ D^- \left[\Delta A w_i - \left(\Delta A^+ \underline{w}_i - \Delta A^- \overline{w}_i\right)\right] \\
&+ (-KC)^+\left[\Delta A^+\overline{w}_i - \Delta A^- \underline{w}_i - \Delta A w_i\right] + (-KC)^- \left[\Delta A w_i - \left(\Delta A^+ \underline{w}_i - \Delta A^- \overline{w}_i\right)\right] \\
={}& \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\overline{e}_{iu} \\
&+ D^+\left[\Delta A^+\left(\overline{w}_i - w_i\right) + \Delta A^- \left(w_i - \underline{w}_i\right)\right] + D^- \left[\Delta A^+\left(w_i - \underline{w}_i\right) + \Delta A^- \left(\overline{w}_i - w_i\right)\right] \\
&+ (-KC)^+\left[\Delta A^+\left(\overline{w}_i - w_i\right) + \Delta A^- \left(w_i - \underline{w}_i\right)\right] \\
&+ (-KC)^- \left[\Delta A^+\left(w_i - \underline{w}_i\right) + \Delta A^- \left(\overline{w}_i - w_i\right)\right], \\
\dot{\underline{e}}_{iu} \geq{}& \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\underline{e}_{iu} + D^+\left[\Delta A w_i - \left(\Delta A^+ \underline{w}_i - \Delta A^- \overline{w}_i\right)\right] \\
&+ D^- \left(\Delta A^+\overline{w}_i - \Delta A^- \underline{w}_i - \Delta A w_i\right) \\
&+ (-KC)^+\left[\Delta A w_i - \left(\Delta A^+ \underline{w}_i - \Delta A^- \overline{w}_i\right)\right] + (-KC)^- \left(\Delta A^+\overline{w}_i - \Delta A^- \underline{w}_i - \Delta A w_i\right) \\
={}& \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\underline{e}_{iu} \\
&+ D^+\left[\Delta A^+\left(w_i - \underline{w}_i\right) + \Delta A^- \left(\overline{w}_i - w_i\right)\right] + D^- \left[\Delta A^+\left(\overline{w}_i - w_i\right) + \Delta A^- \left(w_i - \underline{w}_i\right)\right] \\
&+ (-KC)^+\left[\Delta A^+\left(w_i - \underline{w}_i\right) + \Delta A^+\left(\overline{w}_i - w_i\right)\right] \\
&+ (-KC)^- \left[\Delta A^+\left(\overline{w}_i - w_i\right) + \Delta A^- \left(w_i - \underline{w}_i\right)\right].
\end{aligned} \tag{16}
$$

That is,

$$
\begin{aligned}
\left[\dot{\overline{e}}_{iu}\ \dot{\underline{e}}_{iu}\right] ={}& \begin{bmatrix} \widetilde{A}_{22} - K\widetilde{A}_{12} & 0 \\ 0 & \widetilde{A}_{22} - K\widetilde{A}_{12} \end{bmatrix} \begin{bmatrix} \overline{e}_i \\ \underline{e}_i \end{bmatrix} \\
&+ \begin{bmatrix} D^+\Delta A^+ + (-KC)^+\Delta A^+ & D^+\Delta A^- + (-KC)^+\Delta A^- \\ D^-\Delta A^- + (-KC)^-\Delta A^- & D^-\Delta A^+ + (-KC)^-\Delta A^+ \end{bmatrix} \begin{bmatrix} \overline{w}_i - w_i \\ w_i - \underline{w}_i \end{bmatrix}.
\end{aligned} \tag{17}
$$

By (3) and (10), one has

$$
\begin{aligned}
\overline{w}_i - w_i &= Q_2^+\left(\overline{z}_{iu} - z_{iu}\right) + Q_2^-\left(z_{iu} - \underline{z}_{iu}\right), \\
w_i - \underline{w}_i &= Q_2^-\left(\overline{z}_{iu} - z_{iu}\right) + Q_2^+\left(z_{iu} - \underline{z}_{iu}\right),
\end{aligned} \tag{18}
$$

i.e.,

$$
\begin{bmatrix} \overline{w}_i - w_i \\ w_i - \underline{w}_i \end{bmatrix} = \begin{bmatrix} Q_2^+ & Q_2^- \\ Q_2^- & Q_2^+ \end{bmatrix} \begin{bmatrix} \overline{e}_{iu} \\ \underline{e}_{iu} \end{bmatrix}. \tag{19}
$$

Then, one has

$$
\begin{aligned}
\left[\dot{\overline{e}}_{iu}\ \dot{\underline{e}}_{iu}\right] \geq{}& \begin{bmatrix} \widetilde{A}_{22} - K\widetilde{A}_{12} & 0 \\ 0 & \widetilde{A}_{22} - K\widetilde{A}_{12} \end{bmatrix} \begin{bmatrix} \overline{e}_i \\ \underline{e}_i \end{bmatrix} \\
&+ \begin{bmatrix} D^+\Delta A^+ + (-KC)^+\Delta A^+ & D^+\Delta A^- + (-KC)^+\Delta A^- \\ D^-\Delta A^- + (-KC)^-\Delta A^- & D^-\Delta A^+ + (-KC)^-\Delta A^+ \end{bmatrix} \begin{bmatrix} Q_2^+ & Q_2^- \\ Q_2^- & Q_2^+ \end{bmatrix} \begin{bmatrix} \overline{e}_{iu} \\ \underline{e}_{iu} \end{bmatrix} \\
\triangleq{}& \begin{bmatrix} \widetilde{A}_{22} - K\widetilde{A}_{12} & 0 \\ 0 & \widetilde{A}_{22} - K\widetilde{A}_{12} \end{bmatrix} \begin{bmatrix} \overline{e}_i \\ \underline{e}_i \end{bmatrix} + \Pi \begin{bmatrix} \overline{e}_i \\ \underline{e}_i \end{bmatrix}.
\end{aligned} \tag{20}
$$

It is apparent that $\Pi \geq 0$.

On the other hand, by (3), one has $w_i = (D - KC)w_i$, so that there hold

$$\begin{aligned}
\overline{e}_{iu}(0) &= \overline{z}_{iu}(0) - z_{iu}(0)\\
&= D^{+}\overline{w}_i(0) - D^{-}\underline{w}_i(0) + (-KC)^{+}\overline{w}_i(0) - (-KC)^{-}\underline{w}_i(0) - (D-KC)w_i(0)\\
&= D^{+}\left(\overline{w}_i(0) - w_i(0)\right) + (-KC)^{+}\left(\overline{w}_i(0) - w_i(0)\right)\\
&\quad + D^{-}\left(w_i(0) - \underline{w}_i(0)\right) + (-KC)^{-}\left(w_i(0) - \underline{w}_i(0)\right)\\
&= \left[D^{+} + (-KC)^{+} \quad D^{-} + (-KC)^{-}\right]\begin{bmatrix}\overline{w}_i(0) - w_i(0)\\ w_i(0) - \underline{w}_i(0)\end{bmatrix},\\
\underline{e}_{iu}(0) &= z_{iu}(0) - \underline{z}_{iu}(0)\\
&= (D-KC)w_i(0) - D^{+}\underline{w}_i(0) + D^{-}\overline{w}_i(0) - (-KC)^{+}\underline{w}_i(0) + (-KC)^{-}\overline{w}_i(0)\\
&= D^{+}\left(w_i(0) - \underline{w}_i(0)\right) + D^{-}\left(\overline{w}_i(0) - w_i(0)\right)\\
&\quad + (-KC)^{+}\left(w_i(0) - \underline{w}_i(0)\right) + (-KC)^{-}\left(\overline{w}_i(0) - w_i(0)\right)\\
&= \left[D^{-} + (-KC)^{-} \quad D^{+} + (-KC)^{+}\right]\begin{bmatrix}\overline{w}_i(0) - w_i(0)\\ w_i(0) - \underline{w}_i(0)\end{bmatrix},
\end{aligned} \tag{21}$$

which further result in

$$\left[\overline{e}_{iu}(0)\underline{e}_{iu}(0)\right] = \begin{bmatrix} D^{+} + (-KC)^{+} & D^{-} + (-KC)^{-}\\ D^{-} + (-KC)^{-} & D^{+} + (-KC)^{+}\end{bmatrix}\begin{bmatrix}\overline{w}_i(0) - w_i(0)\\ w_i(0) - \underline{w}_i(0)\end{bmatrix}. \tag{22}$$

Since $\underline{w}_i(0) \leq w_i(0) \leq \overline{w}_i(0)$, by (18), one has $\left[\overline{e}_{iu}(0)\underline{e}_{iu}(0)\right] \geq 0$.

Therefore, by Lemma 1, if $\widetilde{A}_{22} - K\widetilde{A}_{12}$ is a Metzler matrix, then $\overline{z}_{iu}$ and $\underline{z}_{iu}$ given in (8) are considered a neighborhood reduced-order framer for the uncertain MAS described by (1).

This completes the proof.

For $\overline{w}_i$ and $\underline{w}_i$ in (10), construct

$$u_i = -B^T P_1\left(\overline{w}_i + \underline{w}_i\right), \tag{23}$$

where $P_1 > 0$ is the solution of the algebraic Riccati equation

$$A^T P_1 + P_1 A - \lambda_0 P_1 B B^T P_1 + \epsilon I = 0, \tag{24}$$

with $\lambda_0 \geq 2\lambda_2(L)$ and $\epsilon > 0$.

Define

$$\begin{aligned}
\overline{E}_i &= \overline{w}_i - w_i,\\
\underline{E}_i &= w_i - \underline{w}_i.
\end{aligned} \tag{25}$$

By (16), one has

$$\begin{bmatrix}\overline{E}_i\\ \underline{E}_i\end{bmatrix} = \begin{bmatrix} Q_2^{+} & Q_2^{-}\\ Q_2^{-} & Q_2^{+}\end{bmatrix}\begin{bmatrix}\overline{e}_{iu}\\ \underline{e}_{iu}\end{bmatrix}. \tag{26}$$

For u_i in (19), by (1), (21), and (22), one has

$$\begin{aligned}
\dot{x}_i &= (A + \Delta A)x_i - BB^T P_1\left(\overline{w}_i + \underline{w}_i\right)\\
&= (A + \Delta A)x_i - BB^T P_1\left(2w_i + \overline{E}_i - \underline{E}_i\right)\\
&= (A + \Delta A)x_i - 2BB^T P \sum_{j=1}^{N} l_{ij}x_j\\
&\quad - BB^T P_1 Q_2 \overline{e}_{iu} + BB^T P_1 Q_2 \underline{e}_{iu}.
\end{aligned} \tag{27}$$

On the other hand, by (14), there hold

$$\begin{aligned}
\dot{\overline{e}}_{iu} &= \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\overline{e}_{iu}\\
&\quad + \left[D^{+} + (-KC)^{+} \quad D^{-} + (-KC)^{-}\right]\begin{bmatrix}\overline{h}_i - \Delta A w_i\\ \Delta A w_i - \underline{w}_i\end{bmatrix},\\
\dot{\underline{e}}_{iu} &= \left(\widetilde{A}_{22} - K\widetilde{A}_{12}\right)\underline{e}_{iu}\\
&\quad + \left[D^{-} + (-KC)^{-} \quad D^{+} + (-KC)^{+}\right]\begin{bmatrix}\overline{h}_i - \Delta A w_i\\ \Delta A w_i - \underline{w}_i\end{bmatrix}.
\end{aligned} \tag{28}$$

Since

$$\begin{aligned}\overline{h}_i - \Delta A w_i &= \begin{bmatrix} \overline{\Delta A}^{+} & \overline{\Delta A}^{-} \end{bmatrix} \begin{bmatrix} \overline{E}_i \\ \underline{E}_i \end{bmatrix}^{+} - \begin{bmatrix} \underline{\Delta A}^{+} & \underline{\Delta A}^{-} \end{bmatrix} \begin{bmatrix} \overline{E}_i \\ \underline{E}_i \end{bmatrix}^{-} \\ &\quad + (\overline{\Delta A} - \Delta A) w_i^{+} + (\Delta A - \underline{\Delta A}) w_i^{-}, \\ \Delta A w_i - \underline{h}_i &= \begin{bmatrix} \underline{\Delta A}^{-} & \underline{\Delta A}^{+} \end{bmatrix} \begin{bmatrix} \overline{E}_i \\ \underline{E}_i \end{bmatrix}^{+} - \begin{bmatrix} \overline{\Delta A}^{-} & \overline{\Delta A}^{+} \end{bmatrix} \begin{bmatrix} \overline{E}_i \\ \underline{E}_i \end{bmatrix}^{-} \\ &\quad + (\Delta A - \underline{\Delta A}) w_i^{+} + (\overline{\Delta A} - \Delta A) w_i^{-},\end{aligned} \tag{29}$$

that is,

$$\begin{aligned}\begin{bmatrix} \overline{h}_i - \Delta A w_i \\ \Delta A w_i - \underline{w}_i \end{bmatrix} &= \begin{bmatrix} \overline{\Delta A}^{+} & \overline{\Delta A}^{-} \\ \underline{\Delta A}^{-} & \underline{\Delta A}^{+} \end{bmatrix} \begin{bmatrix} \overline{E}_i \\ \underline{E}_i \end{bmatrix}^{+} - \begin{bmatrix} \underline{\Delta A}^{+} & \underline{\Delta A}^{-} \\ \overline{\Delta A}^{-} & \overline{\Delta A}^{+} \end{bmatrix} \begin{bmatrix} \overline{E}_i \\ \underline{E}_i \end{bmatrix}^{-} \\ &\quad + \begin{bmatrix} \overline{\Delta A} - \Delta A \\ \Delta A - \underline{\Delta A} \end{bmatrix} w_i^{+} + \begin{bmatrix} \Delta A - \underline{\Delta A} \\ \overline{\Delta A} - \Delta A \end{bmatrix} w_i^{-},\end{aligned} \tag{30}$$

with (22) and (24), one has

$$\begin{aligned}\begin{bmatrix} \dot{\overline{e}}_{iu} \\ \dot{\underline{e}}_{iu} \end{bmatrix} &= \begin{bmatrix} \widetilde{A}_{22} - K\widetilde{A}_{12} & 0 \\ 0 & \widetilde{A}_{22} - K\widetilde{A}_{12} \end{bmatrix} \begin{bmatrix} \overline{e}_{iu} \\ \underline{e}_{iu} \end{bmatrix} \\ &\quad + \begin{bmatrix} D^{+} + (-KC)^{+} & D^{-} + (-KC)^{-} \\ D^{-} + (-KC)^{-} & D^{+} + (-KC)^{+} \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} \overline{\Delta A}^{+} & \overline{\Delta A}^{-} \\ \underline{\Delta A}^{-} & \underline{\Delta A}^{+} \end{bmatrix} \left(\begin{bmatrix} Q_2^{+} & Q_2^{-} \\ Q_2^{-} & Q_2^{+} \end{bmatrix} \begin{bmatrix} \overline{e}_{iu} \\ \underline{e}_{iu} \end{bmatrix} \right)^{+} \\ &\quad - \begin{bmatrix} \underline{\Delta A}^{+} & \underline{\Delta A}^{-} \\ \overline{\Delta A}^{-} & \overline{\Delta A}^{+} \end{bmatrix} \left(\begin{bmatrix} Q_2^{+} & Q_2^{-} \\ Q_2^{-} & Q_2^{+} \end{bmatrix} \begin{bmatrix} \overline{e}_{iu} \\ \underline{e}_{iu} \end{bmatrix} \right)^{-} \\ &\quad + \begin{bmatrix} \overline{\Delta A} - \Delta A \\ \Delta A - \underline{\Delta A} \end{bmatrix} w_i^{+} + \begin{bmatrix} \Delta A - \underline{\Delta A} \\ \overline{\Delta A} - \Delta A \end{bmatrix} w_i^{-}.\end{aligned} \tag{31}$$

Let $\eta_i = \begin{bmatrix} x_i^T & \overline{e}_{iu}^T & \underline{e}_{iu}^T \end{bmatrix}^T$. It follows from (15) and (23) that

$$\begin{aligned}\dot{\eta}_i &= \begin{bmatrix} A & -BB^T P_1 Q_2 & BB^T P_1 Q_2 \\ 0 & \widetilde{A}_{22} - K\widetilde{A}_{12} & 0 \\ 0 & 0 & \widetilde{A}_{22} - K\widetilde{A}_{12} \end{bmatrix} \eta_i \\ &\quad - 2 \begin{bmatrix} BB^T P_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sum_{j=1}^{N} l_{ij} \eta_j \\ &\quad + \Gamma_2 \mathscr{M}_1 (\Gamma_1 \eta_i)^{+} - \Gamma_2 \mathscr{M}_2 (\Gamma_1 \eta_i)^{-} \\ &\quad + \Gamma_2 \mathscr{M}_3 \left(\sum_{j=1}^{N} l_{ij} \eta_j \right)^{+} + \Gamma_2 \Gamma_0 \mathscr{M}_3 \left(\sum_{j=1}^{N} l_{ij} \eta_j \right)^{-},\end{aligned} \tag{32}$$

where

$$\begin{aligned}\Gamma_0 &= \begin{bmatrix} I_n & 0 & 0 \\ 0 & 0 & I_n \\ 0 & I_n & 0 \end{bmatrix}, \\ \Gamma_1 &= \begin{bmatrix} I_n & 0 & 0 \\ 0 & Q_2^{+} & Q_2^{-} \\ 0 & Q_2^{-} & Q_2^{+} \end{bmatrix}, \\ \Gamma_2 &= \begin{bmatrix} I_n & 0 & 0 \\ 0 & D^{+} + (-KC)^{+} & D^{-} + (-KC)^{-} \\ 0 & D^{-} + (-KC)^{-} & D^{+} + (-KC)^{+} \end{bmatrix}, \\ \mathscr{M}_1 &= \begin{bmatrix} \Delta A & 0 & 0 \\ 0 & \overline{\Delta A}^{+} & \overline{\Delta A}^{-} \\ 0 & \underline{\Delta A}^{-} & \underline{\Delta A}^{+} \end{bmatrix}, \\ \mathscr{M}_2 &= \begin{bmatrix} \Delta A & 0 & 0 \\ 0 & \underline{\Delta A}^{+} & \underline{\Delta A}^{-} \\ 0 & \overline{\Delta A}^{-} & \overline{\Delta A}^{+} \end{bmatrix}, \\ \mathscr{M}_3 &= \begin{bmatrix} 0 & 0 & 0 \\ \overline{\Delta A} - \Delta A & 0 & 0 \\ \Delta A - \underline{\Delta A} & 0 & 0 \end{bmatrix},\end{aligned} \tag{33}$$

which induces

$$\begin{aligned}\dot{\eta} &= \left(I_N \otimes \mathscr{A} - L \otimes \mathscr{B} \right) \eta \\ &\quad + \left(I_N \otimes \Gamma_2 \mathscr{M}_1 \right) \left((I_N \otimes \Gamma_1) \eta \right)^{+} \\ &\quad - \left(I_N \otimes \Gamma_2 \mathscr{M}_2 \right) \left((I_N \otimes \Gamma_1) \eta \right)^{-} \\ &\quad + \left(I_N \otimes \Gamma_2 \mathscr{M}_3 \right) \left(\left(L \otimes I_{(3n-2p)} \right) \eta \right)^{+} \\ &\quad + \left(I_N \otimes \Gamma_2 \Gamma_0 \mathscr{M}_3 \right) \left(\left(L \otimes I_{(3n-2p)} \right) \eta \right)^{-},\end{aligned} \tag{34}$$

where

$$\begin{aligned}\mathscr{A} &= \begin{bmatrix} A & -BB^T P_1 Q_2 & BB^T P_1 Q_2 \\ 0 & \widetilde{A}_{22} - K\widetilde{A}_{12} & 0 \\ 0 & 0 & \widetilde{A}_{22} - K\widetilde{A}_{12} \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 2BB^T P_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.\end{aligned} \tag{35}$$

Since L is with an undirected graph, there is an orthogonal matrix $\mathscr{U} = \begin{bmatrix} 1/\sqrt{N}\, 1_N & \mathscr{U}_2 \end{bmatrix}$ with $\mathscr{U}_2 \in \mathbb{R}^{N \times (N-1)}$ such that $\mathscr{U}^T L \mathscr{U} = \operatorname{diag}\{\lambda_1(L), \ldots, \lambda_N(L)\}$, where $\lambda_i(L) > 0$ for $i = 2, \ldots, N$, if $\mathscr{G} = (\mathscr{V}, \mathscr{E}, W)$ is connected. For better understanding, denote

$$\Lambda = \operatorname{diag}\{\lambda_2(L), \ldots, \lambda_N(L)\} = \mathscr{U}_2^T L \mathscr{U}_2. \tag{36}$$

Let $\eta_u = (\mathscr{U}^T \otimes I_{(3n-2p)})\eta$ and $\eta_{uu} = (\mathscr{U}_2^T \otimes I_{(3n-2p)})\eta = \begin{bmatrix} \eta_{u2}^T & \cdots & \eta_{uN}^T \end{bmatrix}^T$, and then one has

$$\begin{aligned}\dot{\eta}_{uu} =& \left(\mathscr{U}_2^T \otimes \mathscr{A} - \mathscr{U}_2^T L \otimes \mathscr{B}\right)\eta \\ &+ \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_1\right)\left((I_N \otimes \Gamma_1)\eta\right)^+ - \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_2\right)\left((I_N \otimes \Gamma_1)\eta\right)^- \\ &+ \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_3\right)\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^+ + \left(\mathscr{U}_2^T \otimes \Gamma_2 \Gamma_0 \mathscr{M}_3\right)\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^- .\end{aligned} \tag{37}$$

Since $\mathscr{U}_2^T \mathscr{U}_2 = I_{N-1}$, $\mathscr{U}_2 \mathscr{U}_2^T = I_N - 1/N 1_N 1_N^T$ and $L 1_N = 0$, one has

$$\begin{aligned}\dot{\eta}_{uu} =& \left(I_{N-1} \otimes \mathscr{A}\right)\left(\mathscr{U}_2^T \otimes I_{(3n-2p)}\right)\eta - \left(\mathscr{U}_2^T L\left(\mathscr{U}_2 \mathscr{U}_2^T + \frac{1}{N} 1_N 1_N^T\right) \otimes \mathscr{B}\right)\eta \\ &+ \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_1\right)\left((I_N \otimes \Gamma_1)\eta\right)^+ - \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_2\right)\left((I_N \otimes \Gamma_1)\eta\right)^- \\ &+ \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_3\right)\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^+ + \left(\mathscr{U}_2^T \otimes \Gamma_2 \Gamma_0 \mathscr{M}_3\right)\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^- \\ =& \left(I_{N-1} \otimes \mathscr{A} - \Lambda \otimes \mathscr{B}\right)\eta_{uu} \\ &+ \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_1\right)\left((I_N \otimes \Gamma_1)\eta\right)^+ - \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_2\right)\left((I_N \otimes \Gamma_1)\eta\right)^- \\ &+ \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_3\right)\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^+ + \left(\mathscr{U}_2^T \otimes \Gamma_2 \Gamma_0 \mathscr{M}_3\right)\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^- .\end{aligned} \tag{38}$$

Construct a Lyapunov function:

$$V = \eta_{uu}^T \left(I_{N-1} \otimes \mathscr{P}\right)\eta_{uu}, \tag{39}$$

where $\mathscr{P} = \text{diag}\{P_1, P_2, P_2\}$ with P_1 in (20), $P_2 \succ 0$ being the solution of the Lyapunov equation

$$\begin{aligned}&\left(\tilde{A}_{22} - K\tilde{A}_{12}\right)^T P_2 + P_2\left(\tilde{A}_{22} - K\tilde{A}_{12}\right) \\ &\quad + \epsilon I_{n-p} + Q_2^T P_1 B B^T P_1 Q_2 = 0.\end{aligned} \tag{40}$$

□

Theorem 2. *Consider an uncertain MAS (1) communicating through an undirected network $\mathscr{G} = (\mathscr{V}, \mathscr{E}, G)$ and suppose that Assumptions 1, 2, and 3 hold. If $K \in \mathbb{R}^{(n-p)\times p}$ is given to make $\tilde{A}_{22} - K\tilde{A}_{12}$ Hurwitz and Metzler, and*

$$\theta_1 \theta_2 < \frac{\epsilon^2}{16}, \tag{41}$$

where

$$\begin{aligned}\theta_1 &= \left\|\Gamma_1\right\|^2 + \lambda_N(L), \\ \theta_2 &= \left(\gamma^2 + 2\left\|\overline{\Delta A} - \underline{\Delta A}\right\|^2\right)\left\|\Gamma_2\right\|^2 \max\left\{\lambda_n(P_1), \lambda_{3n-2p}(P_2)\right\},\end{aligned} \tag{42}$$

with $\gamma = \max\left\{\|\overline{\Delta A}\|, \|\underline{\Delta A}\|, \left\|\begin{bmatrix} \overline{\Delta A}^+ & \overline{\Delta A}^- \\ \underline{\Delta A}^- & \underline{\Delta A}^+ \end{bmatrix}\right\|\right\}$, *then $\overline{z}_{iu}$ and $\underline{z}_{iu}$ given in (8) with the control algorithm (19) constitute a reduced-order NIO for (1), provided that $\underline{w}_i(0) \le w_i(0) \le \overline{w}_i(0)$.*

Proof. 2 For the Lyapunov function given in (29), its derivative according to (28) yields

$$\begin{aligned}\dot{V} =& \sum_{i=2}^{N} \eta_{uu,i}^T \Phi_i \eta_{uu,i} \\ &+ \left[\left((I_N \otimes \Gamma_1)\eta\right)^+\right]^T \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_1\right)^T \left(I_{N-1} \otimes \mathscr{P}\right)\eta_{uu} \\ &+ \eta_{uu}^T \left(I_{N-1} \otimes \mathscr{P}\right)\left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_1\right)\left((I_N \otimes \Gamma_1)\eta\right)^+ \\ &- \left[\left((I_{N-1} \otimes \Gamma_1)\eta\right)^-\right]^T \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_2\right)^T \left(I_{N-1} \otimes \mathscr{P}\right)\eta_{uu} \\ &- \eta_{uu}^T \left(I_{N-1} \otimes \mathscr{P}\right)\left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_2\right)\left((I_N \otimes \Gamma_1)\eta\right)^- \\ &+ \left[\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^+\right]^T \left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_3\right)^T \left(I_{N-1} \otimes \mathscr{P}\right)\eta_{uu} \\ &+ \eta_{uu}^T \left(I_{N-1} \otimes \mathscr{P}\right)\left(\mathscr{U}_2^T \otimes \Gamma_2 \mathscr{M}_3\right)\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^+ \\ &+ \left[\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^-\right]^T \left(\mathscr{U}_2^T \otimes \Gamma_2 \Gamma_0 \mathscr{M}_3\right)^T \left(I_{N-1} \otimes \mathscr{P}\right)\eta_{uu} \\ &+ \eta_{uu}^T \left(I_{N-1} \otimes \mathscr{P}\right)\left(\mathscr{U}_2^T \otimes \Gamma_2 \Gamma_0 \mathscr{M}_3\right)\left(\left(L \otimes I_{(3n-2p)}\right)\eta\right)^- \\ \le& \sum_{i=2}^{N} \eta_{uu,i}^T \Phi_i \eta_{uu,i} \\ &+ c\left[\left((I_N \otimes \Gamma_1)\eta\right)^+\right]^T \left(\mathscr{U}_2 \mathscr{U}_2^T \otimes I_{(3n-2p)}\right)\left((I_N \otimes \Gamma_1)\eta\right)^+ \\ &+ \frac{\lambda_{3n-2p}(\mathscr{P})}{c}\left\|\Gamma_2\right\|^2 \left\|\mathscr{M}_1\right\|^2 \eta_{uu}^T \eta_{uu} \\ &+ c\left[\left((I_N \otimes \Gamma_1)\eta\right)^-\right]^T \left(\mathscr{U}_2 \mathscr{U}_2^T \otimes I_{(3n-2p)}\right)\left((I_N \otimes \Gamma_1)\eta\right)^- \\ &+ \frac{\lambda_{3n-2p}(\mathscr{P})}{c}\left\|\Gamma_2\right\|^2 \left\|\mathscr{M}_2\right\|^2 \eta_{uu}^T \eta_{uu}\end{aligned}$$

$$\begin{aligned}
&+ c\left[\left(\left(L\otimes I_{(3n-2p)}\right)\eta\right)^{+}\right]^{T}\left(\mathscr{U}_2\mathscr{U}_2^T\otimes I_{(3n-2p)}\right)\\
&\cdot\left(\left(L\otimes I_{(3n-2p)}\right)\eta\right)^{+}\\
&+\frac{\lambda_{3n-2p}\mathscr{P}}{c}\left\|\Gamma_2\right\|^2\left\|\mathscr{M}_3\right\|^2\eta_{uu}^T\eta_{uu}\\
&+c\left[\left(\left(L\otimes I_{(3n-2p)}\right)\eta\right)^{-}\right]^{T}\\
&\cdot\left(\mathscr{U}_2\mathscr{U}_2^T\otimes I_{(3n-2p)}\right)\left(\left(L\otimes I_{(3n-2p)}\right)\eta\right)^{-}\\
&+\frac{\lambda_{3n-2p}\mathscr{P}}{c}\left\|\Gamma_2\right\|^2\left\|\Gamma_0\right\|^2\left\|\mathscr{M}_3\right\|^2\eta_{uu}^T\eta_{uu},
\end{aligned}\tag{43}$$

where $c>0$ is constant to be determined and

$$\begin{aligned}
\Phi_i &= \left(\mathscr{A}-\lambda_i(L)\mathscr{B}\right)^T\mathscr{P}+\mathscr{P}\left(\mathscr{A}-\lambda_i(L)\mathscr{B}\right)\\
&=\begin{bmatrix}
A^TP_1+P_1A-4\lambda_i(L)P_1^TBB^TP_1 & -P_1^TBB^TP_1Q_2 & P_1^TBB^TP_1Q_2\\
-Q_2^TP_1^TBB^TP_1 & \begin{matrix}\left(\widetilde{A}_{22}-K\widetilde{A}_{12}\right)^TQ\\+Q\left(\widetilde{A}_{22}-K\widetilde{A}_{12}\right)\end{matrix} & 0\\
Q_2P_1^TBB^TP_1 & 0 & \begin{matrix}\left(\widetilde{A}_{22}-K\widetilde{A}_{12}\right)^TQ\\+Q\left(\widetilde{A}_{22}-K\widetilde{A}_{12}\right)\end{matrix}
\end{bmatrix},
\end{aligned}\tag{44}$$

with $i=2,3,\ldots,N$.

Since the eigenvalues of $\mathscr{U}_2\mathscr{U}_2^T$ are no less than 0, one has

$$\begin{aligned}
\dot{V} &\le \sum_{i=2}^{N}\eta_{uu,i}^T\Phi_i\eta_{uu,i}\\
&\quad+c\left(\left(I_N\otimes\Gamma_1\right)\eta\right)^T\left(\mathscr{U}_2\mathscr{U}_2^T\otimes I_{(3n-2p)}\right)\left(I_N\otimes\Gamma_1\right)\eta\\
&\quad+c\left(\left(I_N\otimes\Gamma_1\right)\eta\right)^T\left(\mathscr{U}_2\mathscr{U}_2^T\otimes I_{(3n-2p)}\right)\left(I_N\otimes\Gamma_1\right)\eta\\
&\quad+c\left(\left(L\otimes I_{(3n-2p)}\right)\eta\right)^T\left(\mathscr{U}_2\mathscr{U}_2^T\otimes I_{(3n-2p)}\right)\left(L\otimes I_{(3n-2p)}\right)\eta\\
&\quad+c\left(\left(L\otimes I_{(3n-2p)}\right)\eta\right)^T\left(\mathscr{U}_2\mathscr{U}_2^T\otimes I_{(3n-2p)}\right)\left(L\otimes I_{(3n-2p)}\right)\eta\\
&\quad+\frac{\lambda_{3n-2p}(\mathscr{P})}{c}\left\|\Gamma_2\right\|^2\left(\left\|\mathscr{M}_1\right\|^2+\left\|\mathscr{M}_2\right\|^2+\left\|\mathscr{M}_3\right\|^2+\left\|\Gamma_0\right\|^2\left\|\mathscr{M}_3\right\|^2\right)\eta_{uu}^T\eta_{uu}\\
&=\sum_{i=2}^{N}\eta_{uu,i}^T\Phi_i\eta_{uu,i}\\
&\quad+2c\eta_{uu}^T\left(I_{N-1}\otimes\Gamma_1^T\Gamma_1\right)\eta_{uu}+2c\eta^T\left(L\mathscr{U}_2\mathscr{U}_2^TL\otimes I_{(3n-2p)}\right)\eta\\
&\quad+\frac{\lambda_{3n-2p}(\mathscr{P})}{c}\left\|\Gamma_2\right\|^2\left(\left\|\mathscr{M}_1\right\|^2+\left\|\mathscr{M}_2\right\|^2+\left\|\mathscr{M}_3\right\|^2+\left\|\Gamma_0\right\|^2\left\|\mathscr{M}_3\right\|^2\right)\eta_{uu}^T\eta_{uu}.
\end{aligned}\tag{45}$$

Since

$$
\begin{aligned}
&2c\eta^{T}\left(L\mathscr{U}_{2}\mathscr{U}_{2}^{T}L\otimes I_{(3n-2p)}\right)\eta \\
&\quad = 2c\eta^{T}\left(\mathscr{U}_{2}\mathscr{U}_{2}^{T}+\frac{1}{N}1_{N}1_{N}^{T}\right)\left(L\mathscr{U}_{2}\mathscr{U}_{2}^{T}L\otimes I_{(3n-2p)}\right)\left(\mathscr{U}_{2}\mathscr{U}_{2}^{T}+\frac{1}{N}1_{N}1_{N}^{T}\right)\eta \\
&\quad = 2c\eta_{uu}^{T}\Lambda^{2}\eta_{uu} \\
&\quad \dot{V}\leq\sum_{i=2}^{N}\eta_{uu,i}^{T}\Phi_{i}\eta_{uu,i} \\
&\qquad +\left[2c\left(\left\|\Gamma_{1}\right\|^{2}+\lambda_{N}^{2}(L)\right)+\frac{\lambda_{3n-2p}(\mathscr{P})}{c}\left\|\Gamma_{2}\right\|^{2}\left(\left\|\mathscr{M}_{1}\right\|^{2}+\left\|\mathscr{M}_{2}\right\|^{2}+\left\|\mathscr{M}_{3}\right\|^{2}+\left\|\Gamma_{0}\right\|^{2}\left\|\mathscr{M}_{3}\right\|^{2}\right)\right]\eta_{uu}^{T}\eta_{uu}.
\end{aligned}
\tag{46}
$$

there holds

$$
\Phi_{i}\leq\begin{bmatrix}
A^{T}P_{1}+P_{1}A-2\lambda_{i}(L)P_{1}^{T}BB^{T}P_{1} & 0 & 0 \\
-0 & \left(\widetilde{A}_{22}-K\widetilde{A}_{12}\right)^{T}Q+Q\left(\widetilde{A}_{22}-K\widetilde{A}_{12}\right)+\frac{1}{\lambda_{i}(L)}Q_{2}^{T}P_{1}BB^{T}P_{1}Q_{2} & 0 \\
0 & 0 & \left(\widetilde{A}_{22}-K\widetilde{A}_{12}\right)^{T}Q+Q\left(\widetilde{A}_{22}-K\widetilde{A}_{12}\right)+\frac{1}{\lambda_{i}(L)}Q_{2}^{T}P_{1}BB^{T}P_{1}Q_{2}
\end{bmatrix},
\tag{47}
$$

with $i=2,3,\ldots,N$.

Therefore, one can get that

$$
\begin{aligned}
\dot{V}\leq&-\left\{\epsilon-\left[2c\left(\left\|\Gamma_{1}\right\|^{2}+\lambda_{N}^{2}(L)\right)\right]\right\}\eta_{uu}^{T}\eta_{uu} \\
&-\left\{\left[\frac{\lambda_{3n-2p}(\mathscr{P})}{c}\left\|\Gamma_{2}\right\|^{2}\left(\left\|\mathscr{M}_{1}\right\|^{2}+\left\|\mathscr{M}_{2}\right\|^{2}+\left\|\mathscr{M}_{3}\right\|^{2}+\left\|\Gamma_{0}\right\|^{2}\left\|\mathscr{M}_{3}\right\|^{2}\right)\right]\right\}\eta_{uu}^{T}\eta_{uu}.
\end{aligned}
\tag{48}
$$

Since $\Gamma_0^{-1} = \Gamma_0$ and $\|\Gamma_0\| = 1, \mathscr{M}_2 = \Gamma_0 \mathscr{M}_1 \Gamma_0$ and

$$\left\|\mathscr{M}_1\right\| = \max\left\{ \|\overline{\Delta A}\|, \|\underline{\Delta A}\|, \left\|\begin{bmatrix} \overline{\Delta A}^{+} & \overline{\Delta A}^{-} \\ \overset{-}{\underline{\Delta A}} & \overset{+}{\underline{\Delta A}} \end{bmatrix}\right\| \right\} = \gamma, \quad (49)$$

$\|\mathscr{M}_3\|^2 \leq 2\|\overline{\Delta A} - \underline{\Delta A}\,\|^2$, so that

$$\dot{V} \leq -\left[\epsilon - 2\left(\theta_1 c + \frac{\theta_2}{c}\right)\right]\eta_{uu}^T \eta_{uu}. \quad (50)$$

Choose $c = \sqrt{\theta_2/\theta_1}$, and under (31), one has $\epsilon - 2(\theta_1 c + \theta_2/c) > 0$, which induces that $\dot{V} \leq 0$, where the equality sign holds if and only if $\eta_{uu} = 0$. Thus, with u_i in (19), uncertain MAS (1) can achieve consensus. □

Remark 1. By Theorem 2, one has $\lim_{t \longrightarrow \infty} w_i^{+} = 0$ and $\lim_{t \longrightarrow \infty} w_i^{-} = 0$ for $i = 1, \ldots, N$. Thus, by (25), as $t \longrightarrow \infty$, one can get that

$$\begin{bmatrix} \dot{\overline{e}}_{iu} & \dot{\underline{e}}_{iu} \end{bmatrix} = \mathsf{F}_1 \begin{bmatrix} \overline{e}_{iu} \\ \underline{e}_{iu} \end{bmatrix} + \mathsf{F}_2 \left\| \begin{bmatrix} Q_2^{+} & Q_2^{-} \\ Q_2^{-} & Q_2^{+} \end{bmatrix} \begin{bmatrix} \overline{e}_{iu} \\ \underline{e}_{iu} \end{bmatrix} \right\|, \quad (51)$$

where

$$\mathsf{F}_1 = \begin{bmatrix} \widetilde{A}_{22} - K\widetilde{A}_{12} & 0 \\ 0 & \widetilde{A}_{22} - K\widetilde{A}_{12} \end{bmatrix} + \frac{1}{2}\begin{bmatrix} D^{+} + (-KC)^{+} & D^{-} + (-KC)^{-} \\ D^{-} + (-KC)^{-} & D^{+} + (-KC)^{+} \end{bmatrix}\left(\begin{bmatrix} \overline{\Delta A}^{+} & \overline{\Delta A}^{-} \\ \overset{-}{\underline{\Delta A}} & \overset{+}{\underline{\Delta A}} \end{bmatrix} + \begin{bmatrix} \overset{+}{\underline{\Delta A}} & \overset{-}{\underline{\Delta A}} \\ \overline{\Delta A}^{-} & \overline{\Delta A}^{+} \end{bmatrix}\right)\begin{bmatrix} Q_2^{+} & Q_2^{-} \\ Q_2^{-} & Q_2^{+} \end{bmatrix},$$

$$\mathsf{F}_2 = \frac{1}{2}\begin{bmatrix} D^{+} + (-KC)^{+} & D^{-} + (-KC)^{-} \\ D^{-} + (-KC)^{-} & D^{+} + (-KC)^{+} \end{bmatrix}\left(\begin{bmatrix} \overline{\Delta A}^{+} & \overline{\Delta A}^{-} \\ \overset{-}{\underline{\Delta A}} & \overset{+}{\underline{\Delta A}} \end{bmatrix} - \begin{bmatrix} \overset{+}{\underline{\Delta A}} & \overset{-}{\underline{\Delta A}} \\ \overline{\Delta A}^{-} & \overline{\Delta A}^{+} \end{bmatrix}\right). \quad (52)$$

Consequently, by Lyapunov stability theory, $[\overline{e}_{iu}/\underline{e}_{iu}]$ may approach 0 as t goes to ∞. This result will be established, if F_1, F_2, and $[Q_2^{+}/Q_2^{-}Q_2^{-}/Q_2^{+}]$ in (33) meet some conditions. In this case, the interval on which the sum of the relative information of each agent is located can be estimated by $\overline{w}_i$ and $\underline{w}_i$ in (10).

Remark 2. The main results are provided under the premise that $\widetilde{A}_{22} - K\widetilde{A}_{12}$ is Hurwitz and Metzler. If there exists a K to make $\widetilde{A}_{22} - K\widetilde{A}_{12}$ Hurwitz and Metzler, it can be acquired according to Lemma 4 in [21]. However, if such K does not exist, the time-invariant transformation and time-varying transformation in [20, 21], respectively, can be introduced to carry out the problem of reduced-order NIO design.

4. Numerical Simulation

Some numerical simulations are proposed to verify the theoretical results in this section. Similar to [21], the system matrices are given as

$$A = \begin{bmatrix} -1 & 4 & 0 \\ 0 & 3 & -1 \\ 0 & 1 & -2 \end{bmatrix},$$
$$B = \begin{bmatrix} -5 \\ 1 \\ -1 \end{bmatrix}, \quad (53)$$
$$C = \begin{bmatrix} -1 & -2 & 1 \end{bmatrix}.$$

Obviously, (C, A) is observable, and (A, B) is stabilizable. The time-varying uncertainty is

$$\Delta A(t) = 10^{-2}\begin{bmatrix} \frac{\sin(t)}{10} & -\cos^2(t) & \frac{1}{11} \\ 0 & \frac{\sin(t/3)}{5} & 0 \\ \frac{1}{20} & 0 & \frac{\cos^2(2t)}{20} \end{bmatrix}, \quad (54)$$

which meets Assumption 1 and

$$\overline{\Delta A} = 10^{-2}\begin{bmatrix} \frac{1}{10} & 0 & \frac{1}{11} \\ 0 & \frac{1}{5} & 0 \\ \frac{1}{20} & 0 & \frac{1}{20} \end{bmatrix},$$
$$\underline{\Delta A} = 10^{-2}\begin{bmatrix} -\frac{1}{10} & -1 & 0 \\ 0 & -\frac{1}{5} & 0 \\ \frac{1}{20} & 0 & 0 \end{bmatrix}. \quad (55)$$

For $P = \begin{bmatrix} -1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, one has $\widetilde{A}_{21} = [0/0]$, $\widetilde{A}_{22} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$, $\widetilde{A}_{11} = [\,-1\,]$, and $\widetilde{A}_{12} = [\,-11 \quad 1\,]$. Here, we

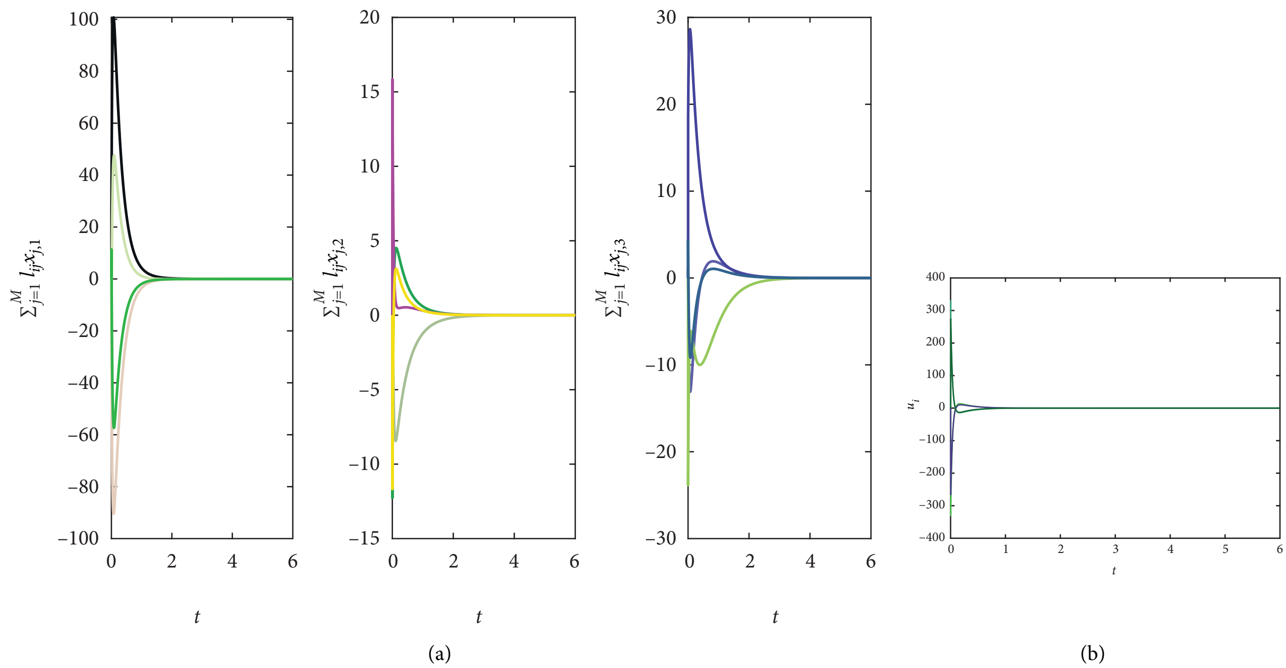


Figure 1: Reduced-order NIO-based consensus. (a) Convergence of $\sum_{i=10}^{N} l_{ij}x_j^t$. (b) Convergence of u_i.

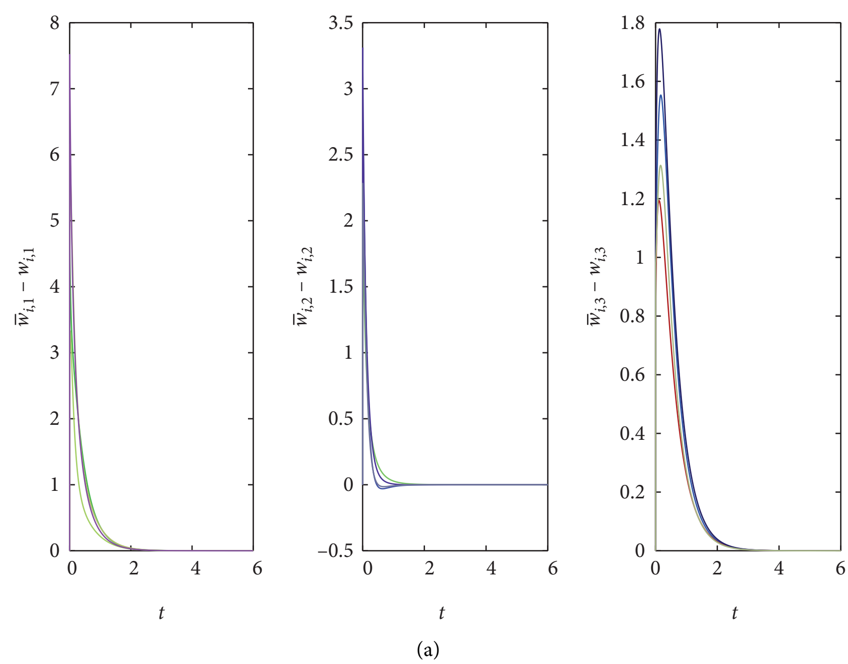
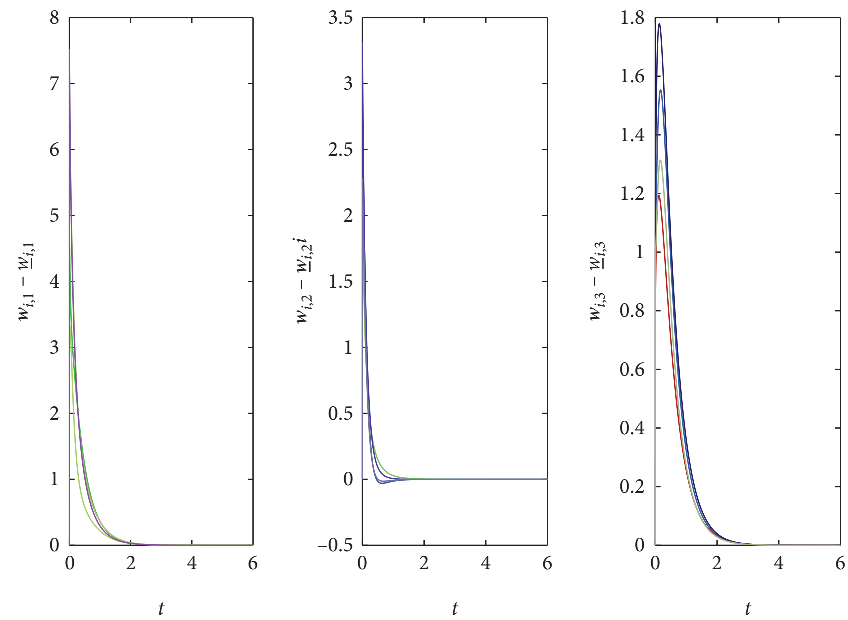


Figure 2: Continued.



(b)

FIGURE 2: Reduced-order NIO-based consensus. (a) Trajectory of $\overline{w}_i - w_i$. (b) Trajectory of $\overline{w}_j - w_j$.

can choose $K = [-1/1/4]$, so that $\widetilde{A}_{22} - K\widetilde{A}_{12}$ is Hurwitz and Metzler. A multiagent system consists of $N = 4$ agents, which communicates through a connected graph with Laplacian matrix as follows:

$$L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}. \tag{56}$$

The initial state of system is chosen randomly from $[-2020] \times [-2020] \times [-2020], \overline{w}_i(0) = w_i(0) + \alpha$, and $\underline{w}_i(0) = w_i(0) - \alpha$, where α is a vector with each element choosing from [01]. Thus, the relationship $\underline{w}_i(0) \leq w_i(0) \leq \overline{w}_i(0)$ holds.

For multiagent system with above details, choose $\epsilon = 9$; then, (31) is satisfied. That is, the premises in Theorem 2 are satisfied.

Figure 1 is given to verify Theorem 2. As shown in Figure 1(a), $\sum_{j=1}^{N} l_{ij}x_j$ converges to 0 for $i = 1, \ldots, N$, and simultaneously, Figure 1(b) shows that the control input u_i can also converge to 0 for $i = 1, \ldots, N$. Both figures imply the consensus. Figures 2(a) and 2(b) display the trajectories of $\overline{w}_i - w_i$ and $w_i - \underline{w}_i$, respectively. As shown in these two figures, $\overline{w}_i - w_i$ and $w_i - \underline{w}_i$ are guaranteed to be nonnegative, provided that $\underline{w}_i(0) \leq w_i(0) \leq \overline{w}_i(0)$ holds. Therefore, Theorem 1 is established. Further, as shown in Figure 2, $\overline{w}_i - w_i$ and $w_i - \underline{w}_i$ approach 0 as time goes to ∞. That is, the interval on which the sum of the relative information of each agent associated with the uncertain multiagent system in this example is located can be estimated by the reduced-order NIO given in this paper. Consequently, Remark 1 holds.

5. Conclusions

In this paper, the reduced-order NIO is designed for MASs with TIUs in system matrix to implement the interval estimation, by using only the outputs and the bounding information of the uncertain system matrix. Consensus of this kind of uncertain multiagent systems can be achieved as a by-part of the reduced-order NIO design. This work is an important complement to the IO design for MASs with TIUs.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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