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Research Article

Driven Force Induced Bifurcation Delay on the Chaotic Financial System

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To understand the variations in the financial characteristics, we examine the dynamical behaviors by considering the chaotic financial model with external force. First, the dynamical characteristics are analyzed by introducing the external driven force in the price index with commodity demand. We discover that the presence of an external force causes the alternate occurrence of oscillatory and steady states as a function of time. Interestingly, we find the existence of bifurcation delay (BD) during the transition from oscillatory (OS) to steady state (SS) or vice versa. Bifurcation delay is a phenomenon in which the bifurcation does not occur at the actual bifurcation point but rather at a later time, which is referred to as bifurcation delay. To confirm the delay in bifurcation, we estimate the actual bifurcation point and compare it to the observed bifurcation transition. Furthermore, to understand the variations in the bifurcation delay, we estimate the delay time between each consecutive cycle and find random fluctuations in the BD. Following that, the BD is virtualized via a transformed phase portrait. In addition, we show decreasing the value of average BD while increasing the frequency of external forcing. Second, the presence of BD is explored by incorporating external forces into the investment demand with unit investment cost. We discover the existence of a similar phenomenon with a constant bifurcation delay.

1. Introduction

Economic dynamics has found the interest of many researchers due to the rapid growth of the economy. Moreover, small changes in the environment can have a significant impact on micro and macroeconomics, resulting in irregular economic development [1–3]. For instance, unforeseeable global events such as wars, disasters, epidemics, and so on may provoke unanticipated changes in the investment environment or affect the fluctuations of economic development [4]. As [5] a result, modern economic research is increasingly interested in developing nonlinear financial models that incorporate a variety of relevant parameters such as interest rates, prices, saving amounts, and commodity demand, among others [6–8]. Typically, savings are determined by wealth and income. Investment and saving are separate decisions, and the relationship between these two is discussed in Ref. [9].

Besides, the dynamical behavior of the financial system has recently received special attention. For instance, the existence of periodic, quasiperiodic, strange nonchaotic, chaos behaviors and their transition route were described using a simple 3D financial system [10]. The construction of the 4D chaotic finance model and its significance were detailed [11]. The dynamics of the financial system have also been investigated by introducing investment incentives into

3D systems, resulting in a 4D financial system. The chaotic dynamics were observed using such a system with fractional order, which was confirmed using the $0-1$ test [12]. Further, the chaotic behavior was well studied in the fractional financial system with time-delay [13–16]. In addition, the control of the 4D hyperchaotic finance system was analyzed by adding an inverse optimal controller [17]. Thus, the dynamics of the financial system were thoroughly investigated by implementing a fractional order and introducing single and two delays into the financial systems [16].

On the other hand, many nonlinear systems can exhibit [18–23] slow-fast dynamics that can be modeled as slow-fast systems [18,20]. Such slow-fast systems exhibit many intriguing phenomena like bursting, mixed-mode oscillations, and bifurcation delay or slow passage effect, among others. Originally, the occurrence of bifurcation delay via Hopf bifurcation was identified by Baer et al. in a fast-slow system of FitzHugh–Nagumo (FHN) model [24]. Later, such occurrences were reported via various other bifurcation routes, including the pitchfork and saddle-node [25–27]. Further, the delayed bifurcation was reported in reaction-diffusion systems, as well as the bistable thermoacoustic system [28–30].

Furthermore, bifurcation delay is a fascinating phenomenon that can be found in a wide range of natural and engineering systems [31–34]. The delay in which the bifurcation occurs can be referred to as bifurcation delay. As a consequence, the existence of dynamic bifurcations as well as strange nonchaotic phenomena was delineated in [35] with single or two frequency driven nonlinear oscillators that is followed by the impact of propagation or processing delay on bifurcation delay reported in a network of slow-fast FHN oscillators [36]. It was identified that there is existence of various collective states including synchronization, chimera, and traveling wave when perturbing the frequency of a single node of the oscillator [37]. Furthermore, the effect of fractional order and noisy parameter on BD was also analyzed [38]. According to the above studies, the phenomenon of bifurcation delay has been identified in biological, physical, and chemical systems but has not yet been investigated in the financial system. With the above motivation, we investigated whether the chaotic financial model can exhibit the bifurcation delay phenomenon when introducing the external force into certain parameters. Since the price index with commodity demand and investment cost with unit investment cost is time-dependent, we analyzed the dynamical characteristics of the chaotic financial model by adding the external driving force with it.

The remaining sections of the article are as follows: In Sec. 2, we first present the dynamical model by introducing the driving force in the price index with commodity demand. We specifically discuss the existence of bifurcation delay and its characteristics. Followed by this, the occurrence of constant bifurcation delay is discussed in Sec. 3 when introducing the driving force into the investment demand with unit investment cost. Finally, in Section 4, the observed results are summarized.

2. Effect of Time-Varying Price Index with Commodity Demand

We consider a chaotic financial (CF) model as in Ref. [16] to exemplify the bifurcation delay in economic growth and its characteristics. Since the commodity demand and price indexes can vary depending on external factors, we modified the system to be a driven chaotic financial (DCF) model by including external forcing. The corresponding model equation is as follows:

$$\begin{aligned} \dot{x} &= z + (y - \alpha)x, \\ \dot{y} &= 1 - \beta y - x^2 - \beta x y, \\ \dot{z} &= -x - \gamma f(t) z, \end{aligned} \tag{1}$$

where x, y, and z are the system parameters that represent the interest rate, the investment demand, and price index, respectively. The constant parameters α, β, and γ denote the saving amount, the unit investment cost, and the elasticity of commodity demand, respectively. $f(t)$ is the external forcing, defined as $f(t) = (1 + f\sin(\omega t))$, where f is the amplitude of external force or the drive parameters and ω is the forcing frequency. The parameters values are fixed as $\alpha = 2.0$, $\beta = 0.1$, and $\gamma = 1.0$.

2.1. Bifurcation Delay (BD) and Its Transition. To demonstrate the occurrence of bifurcation delay (BD), we showed the time evolution of the x variable (represented by the red line) overlapped by $f(t) = 1 + f\sin(\omega t)$ (represented by the black line) in Figure 1(a). The time series signal clearly shows the continuous repetition of the oscillatory and steady state as a function of time. Further, to understand the bifurcation transition, the one-parameter bifurcation diagram (using XPPAUT Ref. [39]) is portrayed in Figure 1(b) as a function of f. The transition from an unstable steady (US) state to a stable steady (SS) state exists via subcritical Hopf bifurcation (HB). We also observed that unstable oscillations (OS) coexist with a stable steady state. From Figure 1(b), we obtained that the transition to steady state occurs at the Hopf bifurcation point $HB = -0.205$.

In addition to displaying the bifurcation delay clearly, we plotted a zoomed view of the time series signal with $f(t)$ as in Figure 1(c). The dashed line represents the Hopf bifurcation line, which is represented by using a point where HB occurs. Typically, the actual bifurcation occurs when the Hopf bifurcation point intersects the function $f(t)$, and then a steady state emerges. t_{HB_1} and t_{HB_2} are the time of first and second actual Hopf bifurcation which arise during the transition from OS to SS and SS to OS, respectively. But we observed that the transition to SS occurs at a time t_S and OS occurs at t_O. Therefore, the first and second bifurcation delay during OS state to SS state and SS state to OS state are obtained as $\tau_{b_1} = t_{HB_1} - t_S$ and $\tau_{b_2} = t_{HB_2} - t_O$, where t_S and t_O are the delay in bifurcation during the transition to steady state and oscillatory state, respectively. From the observation, it is clear that the existence of bifurcation delays is due to driving force. In the

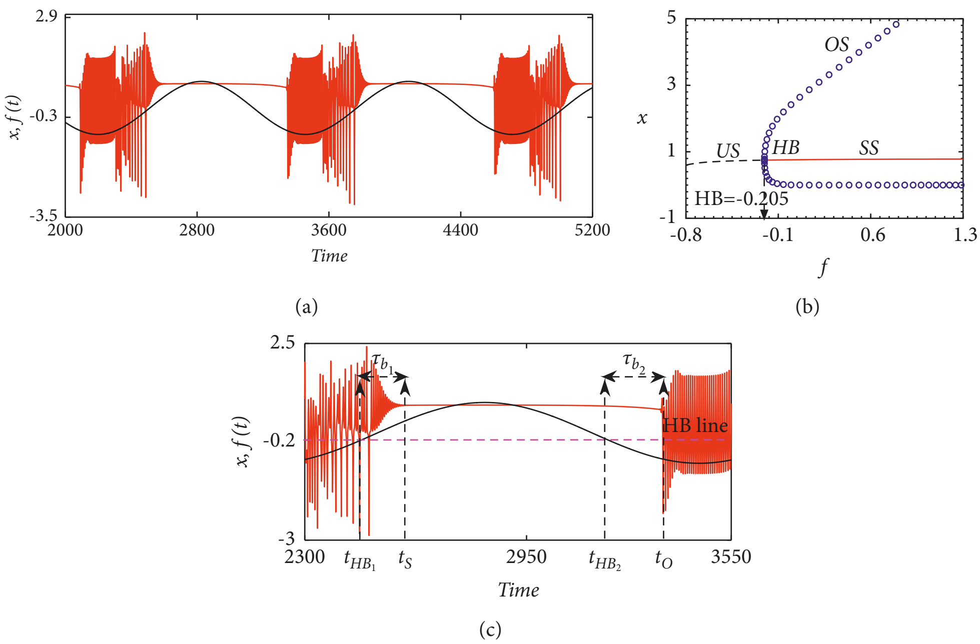

Figure 1: (a) Time series signal of a driven chaotic financial system overlapped with external force $f(t)$. (b) One-parameter bifurcation diagram (using XPPAUT) as a function of forcing amplitude. US and SS are unstable and stable steady state represented by dashed (black) and solid (red) lines, respectively. OS is the oscillatory state (denoted blue open circles) and HB is the Hopf bifurcation point. (c) Zoomed view of time series x and external force $f(t)$, where $\tau_{b_1} = t_{HB_1 - t_S}$ and $\tau_{b_2} = t_{HB_2 - t_O}$. Other system parameters are $\alpha = 2.0$, $\beta = 0.1$, $f = 0.85$, $\omega = 0.02$, and $\gamma = 2.5$.

following, we analyze whether the bifurcation delays τ_{b_1} and τ_{b_2} manifest fluctuations in long time series signals and their probability density functions.

2.2. Variation of BD and Its Characteristics. To determine the fluctuations in the bifurcation delay, we computed the bifurcation delay for each subsequent periodic cycle (n) in the long time series signals in Figure 2. In particular, Figures 2(a) and 2(c) are plotted for variation of first and second bifurcation delays τ_{b_1} and τ_{b_2}. We can see the random fluctuations (irregular motion) in both bifurcation delays τ_{b_1} as well as τ_{b_2}. However, it is also clear that the average mean value of the bifurcation delay is distributed around $(\tau_{b_1} = 134)$ and $(\tau_{b_2} = 149)$ for first and second BD, respectively. Furthermore, we estimate the probability distribution function (PDF) for the signal corresponding to Figures 2(a) and 2(c) in 2(b) and 2(d). The probability distribution function is estimated by finding the number of events (each point in the signal can be considered as an event) in the signal lying between a specific magnitude of bifurcation delay in the entire cycles in time series signal. It is observed that both the BDs follow the Gaussian distribution in the probability distribution function.

For a more clear understanding of the delay in bifurcation, we plotted transformed phase portrait in $(f(t), x)$ space as in Figure 3. Furthermore, to detect the bifurcation point, the bifurcation diagram (Figure 1(b)) is superimposed on the transformed phase portrait. The bifurcation point HB is where the actual bifurcation transition takes place. From Figure 3, it is evident that there is a delay in bifurcation, which means that the bifurcation OS-SS transition does not occur at the actual bifurcation point but rather after some time. Thus, it clearly depicts the occurrence of bifurcation delay.

In addition, the average bifurcation delay is estimated in Figure 4 by varying the forcing frequency. We can observe that the magnitude of bifurcation delays τ_{b_1} is reduced when increasing the frequency ω, as seen in Figure 4(a). We can note the second BD τ_{b_2} also manifests similar dynamical behaviors as shown in Figure 4(b).Futhermore, we also look the emergence of bifurcation delay when applying external forcing as time-varying investment demand with unit investment cost in the following section.

3. Effect of Time-Varying Investment Demand with Unit Investment Cost

In addition to the preceding analysis, in realistic situations, the investment demand with unit investment cost can fluctuate over time. As a result, we include the additional external force $f(t)$ in the β variable, and the dynamical model could be written as

$$\begin{aligned} \dot{x} &= z + (y - \alpha)x, \\ \dot{y} &= 1 - \beta f(t) y - x^2 - \beta x y, \\ \dot{z} &= -x - \gamma z. \end{aligned} \tag{2}$$

To show the dynamical transition, we plotted the one-parameter bifurcation diagram in Figure 5(a) by varying the forcing amplitude f. The bifurcation transition illustrates that the transition from stable periodic oscillation to the

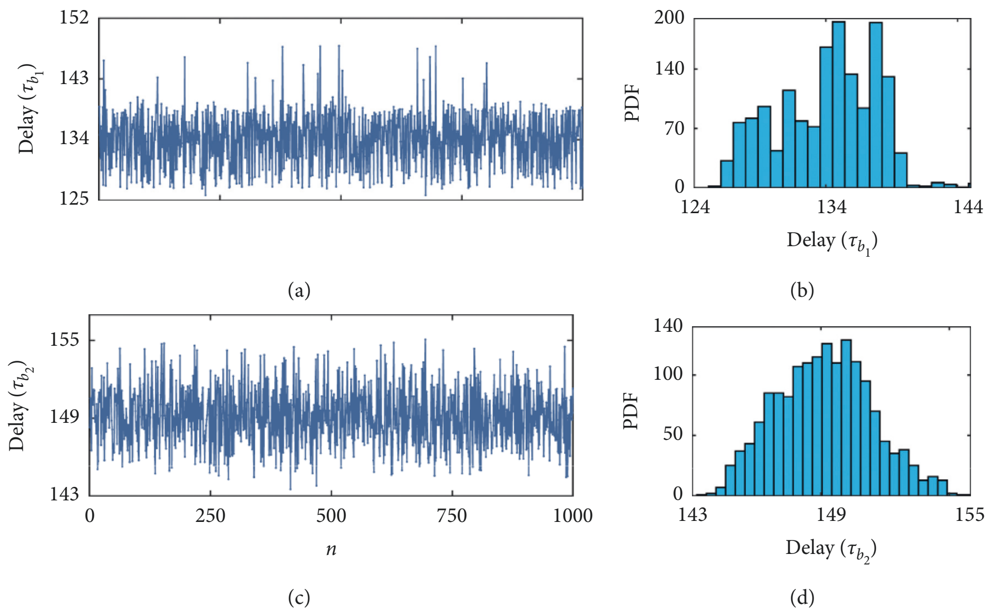

Figure 2: Bifurcation delay of consecutive periodic cycles for (a) first bifurcation delay τ_{b_1} and (c) second bifurcation delay τ_{b_2}. (b) and (d) are the corresponding probability distribution of both the bifurcation delays. Here, the average mean bifurcation delay is (a) $\tau_{b_1} = 134$ and (b)$\tau_{b_2} = 149$. Other parameter values are fixed same as in Figure 1.

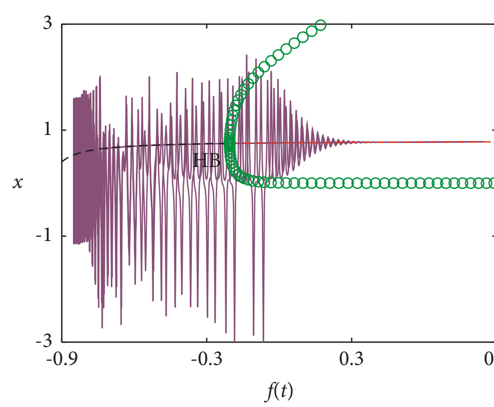

Figure 3: Transformed phase portrait on the plane of $(f(t) - x)$. In the figure, the HB is the Hopf bifurcation point. The red and black dashed lines correspond to stable and unstable steady state, respectively. The green unfilled points represent the amplitude of the limit cycle oscillation. Other parameter values are fixed same as in Figure 1.

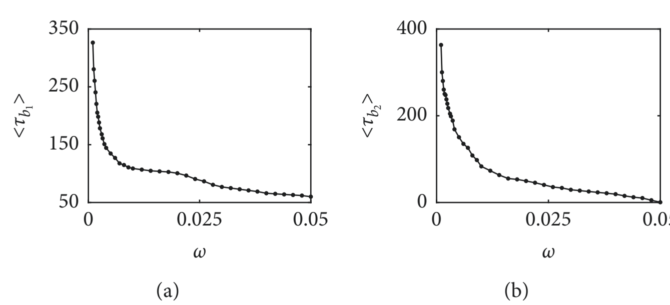

Figure 4: Average bifurcation delay as a function of forcing frequency (ω) for (a) first Hopf bifurcation $\langle\tau_{b_1}\rangle$ and (b) second Hopf bifurcation $\langle\tau_{b_2}\rangle$. Other parameter values are fixed same as in Figure 1.

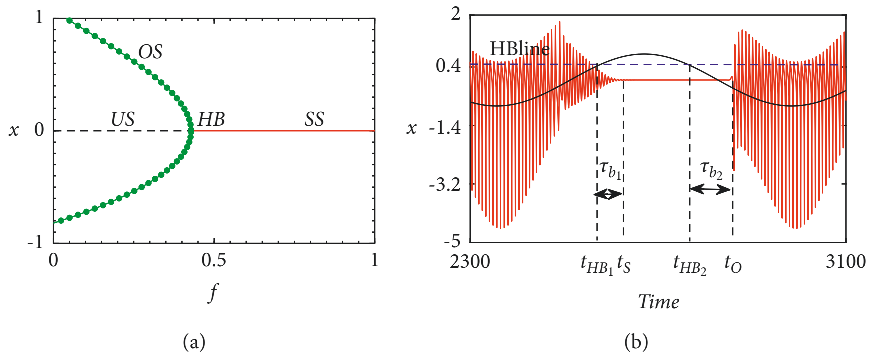

Figure 5: (a) One-parameter bifurcation diagram as a function of amplitude of external forcing f. Green filled circles represent the stable oscillations. The dashed (black) and solid (red) lines denote the unstable and stable steady state, respectively. (b) Time series signal overlapped with the function $f(t)$. Other parameter values are fixed same as in Figure 1.

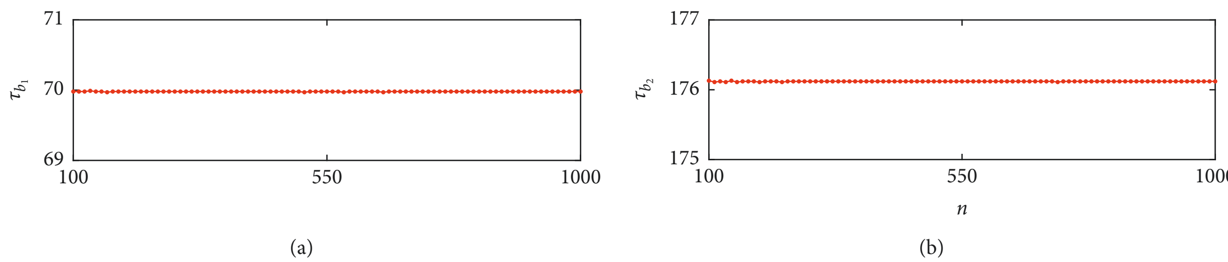

Figure 6: Variation in bifurcation delay (a)τ_{b_1} and (b)τ_{b_2} for each consecutive periodic cycles n.

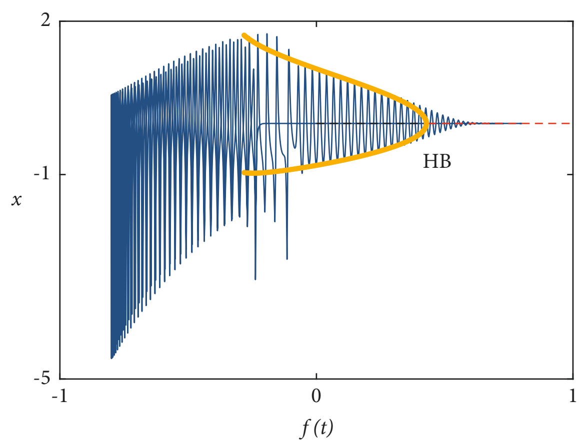

Figure 7: Transformed phase portrait on the plane of $(f(t) - x)$. In the figure, the HB is the supercritical Hopf bifurcation point, where the black line corresponds to stable trivial steady state and the green line represents the amplitude of the limit cycle oscillation. Other parameter values are fixed same as in Figure 1.

stable steady state occurs via Hopf bifurcation. Since the transition from stable limit cycle oscillation to the stable steady state, the associated bifurcation point is supercritical Hopf bifurcation, and the Hopf bifurcation point is identified as $HB = 0.4286$. Using the observed HB point, the HB line (denoted by dashed line) is plotted in Figure 5(b). Comparing the time series signal with actual bifurcation point (intersection of $f(t)$ with t_{HB_1} or t_{HB_2}), we observed that there is the delay in bifurcation $\tau_{b_1} = t_{HB_1} - t_S$ and $\tau_{b_2} = t_{HB_2} - t_O$ during the OS state to SS state or OS state to SS state, respectively.

Further, it is also inspected whether the bifurcation delay can have any fluctuations in the successive cycle in the time series signal. Therefore, the bifurcation delays τ_{b_1} and τ_{b_2} of each consecutive cycles are portrayed in Figures 6(a) and 6(b). Due to periodic repetition of oscillation, we observed constant bifurcation delay during OS-$SS(\tau_{b_1})$ and SS-$OS(\tau_{b_2})$ transitions.

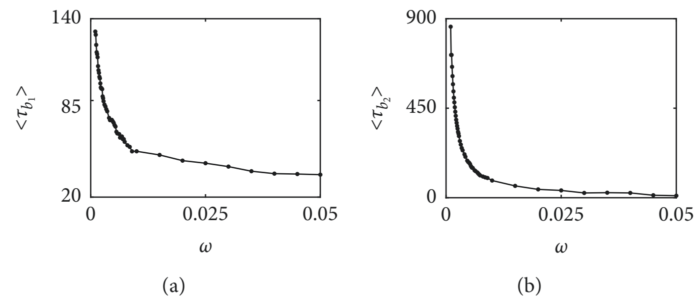

FIGURE 8: Average bifurcation delay as a function of forcing frequency (ω) for (a) first Hopf bifurcation $\langle \tau_{b1} \rangle$ and (b) second Hopf bifurcation $\langle \tau_{b2} \rangle$. Other parameter values are fixed same as in Figure 1.

Following that, the transformed phase portrait is illustrated in $(f(t), x)$ space by overlapping bifurcation plot (shown in Figure 5(a)) to show the delay in bifurcation during the transition from the oscillatory to the steady state. Figure 7 depicts the transition to SS occurring after the actual (HB) bifurcation. Thus, it is clear that the occurrence of bifurcation delay while the transition from oscillatory to steady state.As shown in Figure 4, the average bifurcation delays $\langle \tau_{b1} \rangle$ and $\langle \tau_{b2} \rangle$ decrease as forcing frequency increases, as shown in Figures 8(a) and 8(b).

4. Conclusion

In this study, we have investigated the influence of a time-varying parameter in a chaotic financial model. First, we explored the dynamical behavior of the price index with commodity demand by applying external forcing. Surprisingly, we discovered that the external force had a bifurcation delay in the system. When the bifurcation transition to the steady state or oscillatory state occurs, some delay time can be found as bifurcation delay. We discovered that the price index with commodity demand can result in the BD, which shows random fluctuations in each successive cycle. The relevant probability distribution function (PDF) was also estimated, and we discovered that it follows the Gaussian distribution function. The existence of BD was also investigated using the transformed phase portrait, which clearly shows that the transition from OS to SS occurs after some time when compared to the actual bifurcation point. In addition, we carried out a similar analysis by incorporating external forces in investment demand with unit investment cost. In the time evolution of the signal, we detected a continuous bifurcation delay between each successive cycle. Using the transformed phase portrait, the BD was further validated. Finally, it was discovered that similar to the prior case, raising the frequency range reduces the range of bifurcation delay. Thus, based on the observations, one may infer that an external effect on a certain parameter may produce a bifurcation delay in a chaotic financial system. Our research will offer insight into the bifurcation transition in financial systems.

Data Availability

Data generated during the current study will be made available at reasonable request.

Conflicts of Interest

The authors declare that there are no conflicts of interest in publishing this paper.

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