Research Article

Stackelberg Game Perspective on Pricing Decision of a Dual-Channel Supply Chain with Live Broadcast Sales

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Focusing on the dual-channel supply chain with live broadcasts selling, this paper investigates the service overflow of live broadcasts with Stackelberg game perspective and the impact of retailers’ different market potentials on the pricing decisions of dual-channel members. Meanwhile, it also evaluates the pricing strategy of online retailers after introducing KOL (Key Opinion Leader) live broadcasts. The results show that when one of the dual-channel retailers adopts live broadcast sales, the live broadcast service overflow will have an adverse impact on it, but the degree of the impact depends on the market potential of supply chain members, and different power structures can be used to offset the adverse impact of live broadcast service overflow. Then, the live broadcast sales service overflow will have a certain beneficial impact on online retailers under certain circumstances. Furthermore, with the increase of live broadcast sales service overflow, the retail prices of both companies will decline, while the live broadcast sales service overflow is more beneficial for consumers.

1. Introduction

In recent years, with the widespread popularity of mobile networks and smartphones in China, live broadcasting has developed rapidly as a new mode of communication and gradually became a new way of people’s social life. According to the “44th Statistical Report on Internet Development in China” released by the China Internet Network Information Center (2019), as of June 2019, the number of users watching live webcasts in China has reached 433 million, more than half of the total number of Internet users. Affected by COVID-19 in early 2020, people’s offline life, such as consumption and education, has stagnated. The live broadcast once again uses its characteristics to penetrate people’s lives in an all-around way, such as e-commerce live broadcast, classroom live broadcast, live broadcast of epidemic prevention, and so on.

In 2016, e-commerce live broadcasts developed rapidly as a new sales model. E-commerce platforms gradually began to discover the profitability of live broadcasts and adopted a new model of “live broadcast + e-commerce,” which simply operated shopping pages and live broadcast pages to link each other. As a new business model, the live broadcast has changed the way of communication between merchants and consumers. For example, small businesses and individual sellers can publicize and display their products through live broadcasting. Live demonstrations of products provide richer information and a more interactive experience than the text descriptions and pictures on the web because live communication can not only show the appearance and functions of the product, but also demonstrate the use method and postuse effect of the product and even answer the questions of consumers during the live broadcast. These functions of live broadcasts greatly reduce consumers’ uncertainty about products because consumers can easily understand products (such as clothing) and infer whether goods meet their preferences. Therefore, the live broadcast has gradually developed into a new sales method, and many retailers have adopted live broadcast sales to display and introduce their products. Since the service effect of live broadcast sales is affected by many factors, such as consumers’ personal factors, the interaction with consumers
in the live broadcast room will affect consumers’ emotions, thereby affecting consumers’ information screening and purchasing decisions. At the same time, the anchor’s professionalism and the content of the live broadcast room affect the service effect of live broadcast sales. So, service overflow may occur, especially in dual-channel supply chains, when two channels provide different services in real life, one of the channels provides corresponding services in live broadcast sales, and services are provided between these two channels. The phenomenon of overflow has become more and more prominent, aggravating the conflict between the two channels and having a certain impact on the pricing of supply chain members and channel coordination issues.

This article uses mathematical modeling and numerical example simulation methods to establish a model to study a series of questions. When a retailer in a dual-channel supply chain uses live broadcast to sell and to discuss how the service overflow provided by live broadcast affects other members of the dual-channel supply chain and considers the two retailers’ different market potentials. At the same time, the impact of live broadcast service overflow on pricing under different market potentials of two retailers is studied, and the optimal decision-making under different power structures is compared, and the retailer’s profits are affected by live broadcast service overflow and market potential.

The literature involved in the research questions in the article is mainly divided into three categories: one is live sales. Reference [1] has integrated computer science, marketing, and other domain knowledge to study live broadcasts. They believe that social e-commerce is an Internet-based business application. Social interaction is a new social media that communicates with users to help consumers make decisions and obtain products and services on online channels. In order to study how live broadcast services affect user stickiness through user attachment, [2] developed a theoretical model based on social technology methods and attachment theory and found that both technical and social factors would increase user stickiness. Reference [3] thought that live shopping is a new social media model with high human-computer interaction (HCI). In the context of live sales in China, [4] studied the impact of celebrity product endorsement matching degrees on consumers’ purchasing attitudes and developed a comprehensive model of online celebrity endorsements. Based on the perspective of social presence, according to the characteristics of e-commerce live broadcasting, Zhou et al. [5] combined SOR (Stimulus-Organism-Response) theory and TAM (Technology Acceptance Model) to depict the consumers’ purchase intention in e-commerce live broadcasting and investigated the social presence factors affecting consumers’ purchase intention, which is of great significance to the development of e-commerce live broadcasting market. Regarding live broadcast sales, from the perspective of customer loyalty, Chen [6] found that e-commerce live broadcast sales should focus on traffic, scenes, and content, as well as cultivating consumer trust and loyalty.

On dual-channel supply chain competition, [7], respectively, constructed the Stackelberg game model and Bertrand game model of the optimal price in the dual-channel supply chain to study the income distribution of members in the supply chain and proposed coordination strategies under different game models. Reference [8] studied the role of retail service levels in the dual-channel competitive market, and the results showed that improved retail services can effectively alleviate the competition and conflicts in the dual-channel. Reference [9] establishes the Stackelberg game and Bertrand game model to analyze the equilibrium strategy of manufacturers in the leading position in the dual-channel supply chain and further discussed the equilibrium prices and optimal returns of the supply chain participants. Reference [10] focused on the strategy of manufacturers to conduct dual-channel competition through direct sales channels and physical retail sales. They conducted a series of experiments to verify the model. The study found that the demand of the channel depends on the level of service provided by the channel, consumer product satisfaction, and shopping experience. Reference [11] investigated the competition of product types in a dual-channel supply chain composed of traditional retailers and online retailers. The results showed that online channels face more competition than traditional retailers when selling mainstream products in dual-channel competition. Reference [12] considered a dual-channel supply chain network composed of multiple competing manufacturers, multiple competing retailers, and multiple demand markets and established a dual-channel supply chain network equilibrium model to analyze the impact of three key factors on equilibrium and profits. Reference [13] studied the important role of channel coordination in the multichannel supply chain and provided manufacturers with a competitive advantage in opening online channels, deriving the best market strategies for multichannel manufacturers and retailers. Reference [14] established a model based on the theory of consumer utility to study dual channels, considering the ratio of free-riding consumers and the cost of consumer transfer, and compared the optimal pricing strategy and profit under decentralized decision-making. The study found out that the free-riding ratio and transfer cost have an important influence on the market game behavior of manufacturers and retailers. Chen et al. (2010) considered the service differences between channels and price factors, established a dual-channel competition model to evaluate the influence of channel service differences and the acceptance of network channels on dual channels, and compared the service levels and profits of different channels. The discovery of service competition made the dual-channel supply chain better than the single-channel one.

In the service spillover, [15] analyzed the optimal price of the entire dual-channel supply chain and the balanced competition among members of the dual-channel structure when there is free-riding behavior and the sharing of benefits. The results showed that revenue-sharing contracts could completely coordinate and disperse the entire dual-channel supply chain system. Revenue-sharing contracts and fixed price difference policies can coordinate and integrate dual-channel supply chains, while revenue-sharing contracts cannot fully coordinate and integrate the entire dual-
channel supply chain. Reference [16] investigated the degree of influence of the free-riding effect on the channel selection of manufacturers and analyzed the changes in sales, equilibrium pricing, and profits of manufacturers in different channels under the influence of the free-riding effect.

Through the literature review of live broadcast sales and dual-channel supply chain [17], it can be seen that the existing research on live broadcast sales is only from a partial perspective, and most of the existing literature only studies the phenomenon, flow, and development trend of live broadcast sales. From a theoretical point of view, the existing literature rarely uses quantitative models to analyze live broadcast sales. In order to better discover the impact of live broadcast service spillovers on dual channels, the model sets the cross-price elasticity in dual channels to 1, which is computationally more efficient, including convenience, and highlights the influence of the live broadcast service retention coefficient on dual channels; from a practical point of view, the model calculation results show that the service effect of the live broadcast room not only directly affects the live broadcast sales channel, but also affects the online sales channel. The model calculation results provide feasible suggestions on how retailers in the dual-channel supply chain can make full use of live broadcasting as a sales method.

The rest of this article is organized as follows. Section 2 is the research hypothesis and basic model, and Section 3 is the Stackelberg game and the Nash game research on the model. Then, Section 4 analyzes the comparison of different pricing decisions and returns. Section 5 introduces the KOL live broadcast sales model for extended analysis. Finally, we conclude the results and suggest topics for future research in Section 6.

2. The Basic Model

This article focuses on a dual-channel supply chain consisting of a live-streaming retailer and an online retailer. All retailers will sell products purchased from the same wholesaler to final consumers, and live-streaming retailer is committed to displaying products to consumers through live broadcasts. During live sales, the retailer provided demand enhancement services, including communication with customers, presale advice, product display, advertising, and promotion. Undoubtedly, this kind of service increases the potential market demand because consumers can learn about products in the living and decide whether to buy products from the same store or another online retailer. Therefore, the effort spent by the live broadcast retailer may increase the potential market demand for itself and another online retailer, but the cost of live service will be paid by the retailer that conducts live broadcast sales.

This article analyzes two scenarios, one is that the live-streaming retailer determines the price before the online retailer, that is, the live-streaming retailer as Stackelberg game leader (Scenarios LS), and the other is that the online retailer determines the price before the live broadcast retailer (Scenarios OS). Assuming that the manufacturer is non-strategic, the retailer purchases goods at the wholesale price determined by the market. Both retailers subsequently added their respective profits to the wholesale price to determine the selling price of the goods. Because retailers are strategic companies, the final retail prices they charge are not the same; \( p_l \) and \( p_e \) are the prices of the live-streaming retailer and the online retailer’s merchandise, respectively. The services provided by the live broadcast retailer during the live broadcast will help the live broadcast retailer and the online retailer to increase the demand. The needs of the live broadcast retailer are as follows:

\[
D_l(p_l, s) = a_l - p_l + p_e + y\sqrt{s},
\]

where \( a_l \) is the potential market demand of the live-streaming retailer, and the parameter \( s \) is the service provided by the live-streaming retailer to sell products to customers; \( s \) is a sales work carried out to show customers product features or answer customer questions related to the product, which may help increase sales. It is equivalent to the effort made by a live-streaming retailer, assuming that the increase in potential demand is diminishing returns to scale. The square root function used above is a concave function, which can depict the diminishing proportional returns. \( y\sqrt{s} \) is the part of the increase in demand for the service reservation of live broadcast retailer, which defines the reserved part of the sales activities of the live broadcast retailer; \( (1 - y)\sqrt{s} \) is the part of the live broadcast sales service overflowing to the demand growth of online retailer, and \( 0 \leq y \leq 1 \). Therefore, the demand for online retailers is as follows:

\[
D_e(p_e) = a_e - p_e + p_l + (1 - y)\sqrt{s},
\]

where parameter \( a_e \) is the potential market demand of online retailers, assuming that the cross-price elasticity of the two retailers is 1. The model assumes that the two retail channels facing consumers are symmetrical in all aspects, except that one is live sales, and the other is online sales. Therefore, it can be considered that consumers have a tendency to price products. Reference [18] applied similar demand function to investigate pricing decision of the "showrooming" in multichannel retail system.

The live broadcast retailer will pay a certain cost to sell the products to consumers. This can be in the form of recruiting live broadcasters, who show consumers information about the product. This cost of work only occurred by the live-streaming retailer, and the cost is proportional to the amount of work invested. Therefore, the profit function of the live broadcast retailer is as follows:

\[
\pi_l(p_l, s) = D_l(p_l, s)(p_l - w) - s.
\]

Similarly, the profit function of an online retailer is

\[
\pi_e(p_e) = D_e(p_e)(p_e - w).
\]
To facilitate the distinction in the subsequent calculation and analysis process, the subscripts $l$ and $e$ are used to represent the decision-making models led by live-streaming retailers and online retailers, respectively, and the superscript "*" represents the optimal result.

3. Model Analysis

This article is based on the analysis of the different leadership positions of the two retailers in determining the prices. The framework of the model is the Stackelberg games and the Nash game. The Stackelberg leadership game is a strategic game in which the leader company acts first, and then the follower company makes the best response. The leader envisions the best response of the follower and determines the best action on this basis. References [19, 20] applied the Stackelberg model in a multichannel retail environment. In the model of this article, this leadership is the power to set prices first and get other retailers to respond.

3.1. Scenarios LS: Live-Streaming Retailers as Stackelberg Leader. First, we suppose that the live-streaming retailer will determine the price before the online retailer. At this time, the live-streaming retailer will play a leading role in the pricing role. Both retailers have determined the best response price based on given wholesale price $w$ and the retail prices of the other retailer. As this article assumes that both parties know all the information, in the game, the retailer will respond to the right price of the other retailer. The live broadcast retailer responds by setting the price and the service level of the live broadcast anchor, and the online retailer needs to set the price. To set the price, each retailer must maximize its own profits. The retail price charged by each retailer will be the sum of the retailer’s profit and the wholesale price. Therefore, the corresponding price will be set according to the profit and the wholesale price. Hereinafter, the corresponding price will be obtained by the above formula,

$$
\pi_l^* = \frac{8a_l + 4a_e}{8 - (1 + \gamma)^2},
$$

$$
\pi_e^* = \frac{(5 - \gamma^2)\alpha_l + (6 - \gamma^2)\alpha_e}{8 - (1 + \gamma)^2},
$$

$$
s_l^* = \frac{(1 + \gamma)(2\alpha_l + \alpha_e)}{8 - (1 + \gamma)^2}.
$$

Based on Lemma 1, we can get Proposition 1.

Proof: To see the appendix.

Proposition 1. Under the decision of the live broadcast retailer as the leader, $\partial\pi_l^*/\partial\gamma > 0$ and $\partial\pi_e^*/\partial\gamma > 0$. This means the higher the service level retention coefficient, the higher the retailer’s dual-channel sales price. When $a_l/a_e \geq \gamma^2 - 10\gamma - 3/6 - 2\gamma^2 + 20\gamma$, $\partial\pi_l^*/\partial\gamma \geq \partial\pi_e^*/\partial\gamma$, the level of live broadcast services has a greater impact on the prices of live sales channels than on online channels; otherwise, the opposite is true.

Proof: to see the appendix.

Proposition 1 shows that the improvement of live broadcast services level by live broadcast retailers will increase the cost of live broadcast channels. However, with live broadcast retailers as the leader, members of the supply chain make independent decisions. Under the situation of greater market potential for live-streaming retailers, they may raise prices on a certain basis to obtain higher profits after learning that online retailer may raise prices. Moreover, when live broadcast service overflow rarely occurs, it means that most of the consumers attracted by retailers to provide live broadcast services purchase goods through live broadcast sales channels. Therefore, with the increase in the reserved portion of live broadcast services, retailers increased live broadcast service costs that are more borne by consumers of live broadcast channel channels, and the price of live broadcast sales channels has increased even more. In the case of low service retention coefficient, when online retailer occupies a higher market potential, they will set higher prices and occupy a higher market demand share. In this case, online retailer already has a high market potential, and the overflow of live broadcast services has played a positive role in the sales of online retailer. Therefore, with higher market potential the online retailer will benefit more from the overflow of live broadcast services.

Proposition 2. Under the decision of the live broadcast retailer as the leader, the demand for online channels and live broadcast channels can be derived, and the partial derivation of the demand with respect to the service retention coefficient can be $\partial D_{l}^*/\partial\gamma > 0$ and $\partial D_{e}^*/\partial\gamma > 0$; as $\gamma$ increases, the demand for both channels will increase accordingly. When $a_l/a_e \geq \gamma^2 - 1 + 6\gamma/12\gamma - 2\gamma^2 - 2$, $\partial D_{l}^*/\partial\gamma > \partial D_{e}^*/\partial\gamma \geq 0$. When $a_l/a_e < \gamma^2 - 1 + 6\gamma/12\gamma - 2\gamma^2 - 2$, $\partial D_{l}^*/\partial\gamma > \partial D_{e}^*/\partial\gamma < 0$. And when $1 \geq \gamma \geq 3 - 2\sqrt{2}$, $a_l/a_e \geq 1$, the demand for
live channels is more affected by $\gamma$ than the online channel. Otherwise, the opposite is true.

**Proof:** to see the appendix.

Proposition 2 shows that when the live broadcast service retention coefficient is large enough, the higher the live broadcast service level is, the greater the demand for live sales channels will be affected by the service retention coefficient, and the market potential of live broadcast sales is also greater than that of online channels. Then the demand for online channels will be less affected. If the initial market potential of the live-streaming retailer is higher than that of the online retailer, then even if the live-streaming sales effect is poor, the online retailer will not be able to obtain market share from the live-streaming retailer. In this case, the live broadcast retailer can overcome the higher live broadcast service overflow, charge higher prices, and generate higher demand. This can be seen in real life. Large-scale live broadcast room sales with a loyal customer base and strong market share are rarely affected by service overflow, even if the live broadcast sales price is higher than other online retail stores. When the live service retention coefficient is less than $3 - \sqrt{2}$, the service overflow part will increase; at this time, the market potential of live broadcast sales is smaller than that of online channels. Because online channels have greater market potential than live broadcast sales channels, with greater level of impact on online channels, the final impact level is also related to the retention coefficient of live broadcast services. In real life, small live broadcast rooms will also cause service spillovers, causing consumers to shop at large online retailers.

3.2. Scenarios OS: Live-Streaming Retailers as Stackelberg Leader. Consider that the online retailer determines the price before the live broadcast retailer, so it is in a leading position in determining the price. Online retailer will pre-conceive the response of the live broadcast retailer and set their own response. The online retailer is the leader of Stackelberg. For any given wholesale price $w$ and online per retail profit $m_o$, the live broadcast retailer responds by setting $m_l$ and $s$. The live broadcast retailer calculates the optimal profit and then sets the price as the sum of the wholesale price and the profit. By maximizing its profit function, the best response of the live broadcast retailer can be obtained. Considering the best response of the live broadcast retailer to its profit function, the online retailer maximizes its profit by setting its profit. Therefore, for a given $w$, the best responses $m_l, m_o$, and $s$ are obtained. The following lemma summarizes the best pricing results for live-streaming retailers and online retailer and the best live-streaming service level for live-streaming retailers.

**Lemma 2.** Under Scenarios OS, the optimal $p_l, p_o, s$ can be obtained by the above formula.

\[
\begin{align*}
p_{lE}^* &= \frac{(3 + \gamma)a_l + (2 + \gamma)a_e}{4 - \gamma^2} \\
p_{oE}^* &= \frac{(1 + \gamma)a_l + (2 + \gamma)a_e}{2} \\
s_{E}^* &= \left(\frac{(3 + \gamma)a_l + (2 + \gamma)a_e}{2(4 - \gamma^2)}\right)^2.
\end{align*}
\]

**Proof:** to see the appendix.

**Proposition 3.** The partial derivative of the retailer’s price with respect to the service retention coefficient and the price is affected by the service retention coefficient; the higher the service level, the higher the retailer’s dual-channel sales price; \(\partial p_{lE}^*/\partial \gamma > 0\) and \(\partial p_{oE}^*/\partial \gamma > 0\). When $a_l/a_e \geq \sqrt{8 + 10/\gamma - 8\gamma}$, the partial derivative of the retailer’s price with respect to the service retention coefficient can be obtained: \(\partial p_{lE}^*/\partial \gamma > 0\). When $a_l/a_e < \sqrt{8 + 10/\gamma - 8\gamma}$, the partial derivative of the retailer’s price with respect to the service retention coefficient can be obtained: \(\partial p_{oE}^*/\partial \gamma > 0\). When $a_l/a_e = \sqrt{8 + 10/\gamma - 8\gamma}$, the partial derivative of the retailer’s price with respect to the service retention coefficient can be obtained: \(\partial p_{lE}^*/\partial \gamma > 0\). When $a_l/a_e > \sqrt{8 + 10/\gamma - 8\gamma}$, the partial derivative of the retailer’s price with respect to the service retention coefficient can be obtained: \(\partial p_{oE}^*/\partial \gamma > 0\).

**Proof:** to see the appendix.

Proposition 3 shows that, under the model of online retailers as leaders, retailers will increase the sales prices of the two channels to make profits. When online retailer has greater market potential, they may increase prices on a certain basis after they understand that live broadcast retailer may raise prices. At the same time, the overflow of live broadcast services will have a greater impact on the pricing of live broadcast sales. Furthermore, when the live broadcast sales effect is poor and the service retention coefficient is low, when the online retailer occupies a higher market potential, it will set high prices and occupy a higher market demand share. Under this circumstance, online retailer already has a high market potential, and the overflow of live broadcast services has played a positive role in the sales of online retailer. Therefore, online retailer with higher market potential will benefit more from the overflow of live broadcast services.

**Proposition 4.** With the decision of live sales as the leader, the demand for online channels and live broadcast channels can be derived, and the partial derivation of the demand with respect to the service retention coefficient can be obtained: \(\partial D_{DE}^*/\partial \gamma > 0\) and \(\partial D_{EE}^*/\partial \gamma > 0\). It is concluded that, under the pricing decision dominated by online retailers, with the increase in the retention coefficient of live broadcast services, the demand for both channels will increase accordingly; otherwise, the opposite is true.

**Proof:** to see the appendix.

Proposition 4 shows that the demand for both channels will increase with the increase of the live broadcast service.
retention coefficient, and the impact on the live broadcast channel has always been greater than that on the online channel.

3.3. Nash Equilibrium Analysis. Given the optimal response function of both parties, prove the existence and uniqueness of the Nash equilibrium solution and then further elaborate the decision-making behavior of both parties in the equilibrium state. Given the optimal response function of both parties,

\[ m_c = \frac{\alpha_e + m_l + (1 - \gamma)\sqrt{s}}{2} \]
\[ m_l = \frac{2(a_l + m_c)}{4 - \gamma^2} \]
\[ s = \left(\frac{y(a_l + m_c)}{4 - \gamma^2}\right)^2. \]

We can solve the optimal prices of the two retailers under Nash game.

**Proposition 5.** Under Nash game,  
\[ p_{lE}^* = 2a_l + \alpha_e + (1 - \gamma)\sqrt{s}/3 - \gamma^2, \]
\[ p_{E}^* = 4a_l + (4 - \gamma^2)\alpha_e + (1 - \gamma)(4 - \gamma^2)\sqrt{s}/6 - 2\gamma^2. \]

Calculating the partial derivatives of \( s \) for the prices of two retailers, respectively, we can get
\[ \frac{\partial p_{lE}^*}{\partial s} = (1 - \gamma)(4 - \gamma^2)/4(3 - \gamma^2)\sqrt{s}, \]
\[ \frac{\partial p_{E}^*}{\partial s} = 1 - \gamma/4(3 - \gamma^2)\sqrt{s}, \]

and then \( \frac{\partial p_{E}^*}{\partial \gamma} > 0, \)
\[ \frac{\partial p_{lE}^*}{\partial \gamma} > 0 \]

Taking the retailer’s price into the profit function for derivation, we can get \( \frac{\partial \pi_{lE}^*}{\partial \gamma} > 0 \) and \( \frac{\partial \pi_{E}^*}{\partial \gamma} > 0. \) Currently, the game between the two parties constitutes a supermodel game, and the Nash equilibrium solution exists and is unique.

Proposition 5 proves the existence and uniqueness of Nash equilibrium. Meanwhile, from the economic meaning of the supermodel game, we know that the decisions of online retailers are positively correlated with those of live broadcast retailers. Although the live broadcast service efforts of live broadcast retailers will increase the demand of online retailers, at this time online retailers will choose to follow the live broadcast retailers to increase prices and pursue an increase in marginal profits.

4. Comparative Analysis of Different Power Structures

By comparing the profits of retailers in the dual-channel supply chain affected by live broadcast service overflow and market potential, it can be clearly illustrated by drawing a picture.

Figure 1 shows the impact of market potential ratio \( \Omega = \frac{a_e}{a_l} \) and service retention coefficient on the profits of live broadcast retailer and online retailer under the situation of live broadcast retailer as the leader and the determines the best areas for the profits of the two retailers. At the top of the curve, the profit of live broadcast retailer is greater than that of online live retailers, and the bottom of the curve is that the profits of online live retailers are greater than the profits of live broadcast retailer. For the part with a low live broadcast service retention coefficient (higher service spillover effect) and the market potential of online retailer being less than that of live broadcast retailer, their profits will be greater than the profits of live broadcast sales. With the increase of the live broadcast service retention coefficient and the decrease of \( \Omega \), the profits of live broadcast sales begin to exceed the profits of online retailer.

Figure 2 shows that the profits of live broadcast retailer and online retailer under the leadership of online retailer are affected by market potential and service retention coefficient. At the top of the curve the profit of online retailer is greater than that of live retailers, and at the bottom of the curve the profit of live retailers is greater than that of online live retailers. It can be found from the figure that even for a low live broadcast service retention coefficient when the market potential of an online retailer is far greater than that of a retailer, its profit will be greater than that of live broadcast sales.

From Figures 1 and 2, we can draw the following conclusions.

**Conclusion 1.** With a higher relative market potential and a lower level of live broadcast sales service spillover, live broadcast retailer gets higher profits than online retailer. If the market potential of live-streaming retailers is higher than that of online retailer, with a lower level of live-streaming service overflow, live-streaming retailers will get higher profits than online retailers by higher prices and higher profits.

**Conclusion 2.** With a high relative market potential and a high level of live broadcast sales service overflow, online retailer gets more profits than live broadcast retailer. In the case of a higher level of live broadcast sales service overflow, online retailer will benefit from
the live broadcast services provided by live broadcast sales retailers.

When the market potential of online retailer is low, the advantage of a higher level of the overflow of sales services can be offset by the disadvantage of online retailer’ low market potential. When online retailers have higher market potential and a higher level of live broadcast service overflow, they will get higher profits than live broadcast retailer.

Therefore, when the live broadcast retailer has a high degree of its live broadcast service, it will be more beneficial to the live broadcast retailer. When it has higher market potential, it will increase this benefit. On the other hand, when the level of live broadcast sales service overflow is high, online retailer also begins to obtain more revenue, which is supplemented by its higher market potential. Therefore, the interaction between the market potential of live broadcast sellers and online retailer determines which retailer gets higher profits. When live broadcast sales have high market potential, the overflow of live broadcast sales services will have less impact on retailers that conduct live broadcast sales and can give full play to its certain advantages. At this time, the live broadcast retailer can set a higher profit margin, obtain higher market demand, and obtain higher profits.

Figure 3 shows that the profits of live broadcast retailer under different power structures are affected by the market potential and service retention coefficient. At the top of the curve, the profits of online retailer when pricing first are greater than those of live retailers when pricing first, and the bottom of the curve is the opposite. Figure 4 shows the impact of online retailer’ profits under different power structures.

From Figures 3 and 4, we can draw the following conclusions.

Conclusion 3. When the relative market potential of live broadcast retailer is high, it is more beneficial for online retailer to price live sales first. At the same time, live broadcast retailer has high relative market potential and can charge higher prices. This also leads online retailer to respond at higher prices and obtain higher profits. However, when the relative market potential of live retailers is low, it will not be able to charge a higher price by setting the price first. At this time, online retailer has obtained higher profits, because online retailer has higher market potential.

Figure 5 shows that the profits of live broadcast retailers and online retailers under the Nash equilibrium are affected by market potential and service retention coefficient. The upper part of the curve shows that the profits of live broadcast retailers are greater than the profits of online live broadcast retailers, and the bottom of the curve shows that the profits of online broadcast retailers are greater than the profits of live broadcast.

5. Extended Models: KOL Introduction

The combination of “Internet celebrity economy” and live broadcast forms a new live broadcast sales model. This kind of live broadcast form adds celebrities or Internet celebrities to sell goods on the basis of traditional live
broadcast sales, so it often causes consumers to consume impulsively. At the same time, due to the greater influence of Internet celebrities, KOL (Key Opinion Leader) is introduced, and this behavior can be called e-commerce Internet celebrity live broadcast. Therefore, introducing the variable of KOL online celebrity live broadcast may have an impact on the total profit of the supply chain. This article studies the impact of the introduction of Internet celebrity live broadcast by online retailer on their profits.

Assume that online retailer channels introduce KOL live sales and need to hire celebrities or Internet celebrities with a certain fan base for live sales. Assuming that the fixed hire fee of the hired celebrity Internet celebrity is $a$, the hire fee is a concave function of the number of fans. After the introduction of webcast sales, the new demand will increase by $\rho$ times the original market size, which is a concave function of $N$. The sales price of the products in the webcast room is $p_k$, which is given exogenously. At this time, the prices of other channels are $p_l, p_c$ determined in the previous section, which have been given by known parameters and satisfy $p_k < p_l$. Due to the existence of KOL live-streaming offering better prices at this time, some consumers who originally chose the live-streaming sales channel will turn to the KOL live-streaming channel, assuming that the information is asymmetrical, assuming that $\theta$ percentage of consumers will actually change the purchase channel. On the basis of the demand function, this article considers that, in the process of cooperating with celebrity Internet celebrity anchors, merchants will pay a commission according to a certain percentage of sales. During the sales process, whenever an Internet celebrity appeals, so the influence of KOL on the channel will be smaller, and the lower price. The price of the ordinary Internet celebrity live broadcast room is lower due to the lower fan cohesion and appeal, so the influence of KOL on the channel will be smaller, resulting in no price concessions in the KOL live broadcast room. Merchants are also more willing to provide more favorable prices to the more prestigious KOL.

**Proposition 6.** Without considering the optimal restriction of $\theta$, the sales price of the KOL live sales channel decreases with the increase of the influence coefficient of the KOL channel.

**Proof:** to see the appendix.

Proposition 6 shows that, under the model of joining the KOL live sales channel, the greater the influence of the KOL on the channel is, the lower the price may be. To gain profits, manufacturers can consider asking better KOL to conduct live broadcasts for them. At the same time, they will also reduce the prices of KOL during live broadcasts. In reality, the more famous and the more appealing the KOL live broadcast room is, the lower the price may be. Because of its strong appeal and fan cohesion, KOL has a greater impact on the channel and lower price. The price of the ordinary Internet celebrity live broadcast room is lower due to the lower fan cohesion and appeal, so the influence of KOL on the channel will be smaller, resulting in no price concessions in the KOL live broadcast room. Merchants are also more willing to provide more favorable prices to the more prestigious KOL.

**Proposition 7.** Under the decision of introducing online celebrity live sales, the demand for the online celebrity live sales channel can be obtained. The partial derivation of the demand on the retention coefficient of the online celebrity service can be obtained as $\partial D_{K}^{*}/\partial \theta = (1 - r)(m_l + D_l) - m_c/2(1 - r)$, and the size $\partial D_{K}^{*}/\partial \theta$ is related to $(1 - r)(m_l + D_l) - m_c$. When $(1 - r)(m_l + D_l) - m_c > 0$, the demand for online celebrity live sales will increase with the influence of the influence of the Internet celebrity channel; otherwise, the opposite is true.

**Proof:** to see the appendix.

**Lemma 4.** We can calculate $p_k, \theta$ which is the best response to online celebrity live sales:

$$p_k = \begin{cases} \frac{b - \sqrt{b^2 - 4c}}{2}, & \text{if } b > \sqrt{b^2 - 4c} \\ \frac{b + \sqrt{b^2 - 4c}}{2}, & \text{if } b < \sqrt{b^2 - 4c} \end{cases},$$

$$\theta^* = \frac{\rho}{\sqrt{b^2 - 4c}}.$$

(12)
Proposition 8. When \( b^2 - 4c < 0 \), the profit of the online celebrity live broadcast room will decrease as \( \theta \) increases; when \( b^2 - 4c > 0 \), the positive or negative of the partial derivative of profit to \( \theta \) is determined by the exogenously given \( p^*_b \); when \( p^*_b \in (b - \sqrt{b^2 - 4c}/2, b + \sqrt{b^2 - 4c}/2) \), the profit of the retailer increases with the increase of \( \theta \), and vice versa.

Proof: to see the appendix.

The proposition shows that when \( b^2 - 4c > 0 \), and only when \( p^*_b \in (b - \sqrt{b^2 - 4c}/2, b + \sqrt{b^2 - 4c}/2) \), the retailer’s profit will increase with the increase of \( \theta \). When the KOL pricing is not within this range, the retailer’s profit will decrease with the increase of \( \theta \).

6. Conclusion

With the popularity of online retail and e-commerce platforms, today’s consumers can choose between multiple purchase channels. The dual-channel supply chain model that combines online sales channels and traditional retail channels has demonstrated its advantages. However, with the development of technology, the live broadcast sales model began to occupy a large market. Live broadcast sales have gradually become an important means. Consumers can learn about products through the live broadcast room, but they may choose not to buy goods in the live broadcast room. Retailers have created market demand through live broadcast services, but the demand is met through other channels. In the continuously developing live broadcasting market, this paper determines the best pricing decisions of live broadcasting retailers and online retailers and the live broadcast service level of live broadcast retailer according to the factors such as pricing order, market potential, and live broadcast sales service overflow. Through the analysis and research, three main conclusions are drawn. Firstly, the research shows that live broadcast sales retailers are adversely affected by live broadcast service spillover, but the extent of this impact depends on their market potential. The live broadcast sales retailers with good market potential are relatively less affected by the live broadcast service overflow. Similarly, online retailer with higher market potential will be able to obtain more benefits of live broadcast service overflow than online retailer with lower market potential. Second, contrary to the popular view that live-streaming sales will only affect retailers that conduct live-streaming sales, the research results show that online retailer will be negatively affected by live-streaming sales service overflow. This paper infers that live-streaming sales have an impact on both retailers. The results show that, with the increase of live broadcast sales service spillover, the prices of both retailers will decline. Third, different pricing sequences also have a certain impact on retailers’ pricing and profits. When live broadcast retailer has higher market potential, it is more beneficial to set prices first. Different pricing orders can also be used to offset the adverse effects of live broadcast service overflow. From the perspective of consumers, sales services overflow is beneficial because it can reduce overall retail prices. When online retailer introduces KOL live broadcast sales, their profits are affected not only by KOL live broadcasts, but also by the previous pricing. This paper just establishes a general model to analyze multichannel retail under the influence of live broadcast. Customers are nonstrategic, and the model can be extended to places where customers can formulate strategies according to the showroom strength of the price charged by retailers, and there is still more room for expansion.

Appendix

Proof of Lemma 1

The authors know that the traditional retailer’s demand function is \( D_l(p_t, s) = a_t - p_t + p_e + \gamma \sqrt{s} \), and the profit function of the live broadcast retailer is

\[
\pi_l(p_t, s) = (a_t - p_t + p_e + \gamma \sqrt{s}) (p_t - w) - s,
\]

\[
\pi_l(m_t, s) = a_m m_t - (w + m_t) m_t + s(w + m_e) m_t + s m_t \gamma \sqrt{s} - s.
\]

Similarly, the demand function of an online retailer is \( D_e(p_e) = a_e - p_e + p_t + (1 - \gamma) \sqrt{s} \). The profit function of an online retailer is

\[
\pi_e(p_e) = (a_e - p_e + p_t + (1 - \gamma) \sqrt{s}) (p_e - w),
\]

\[
\pi_e(m_e) = a_e m_e - (w + m_e) m_e + (w + m_t) m_t + m_e (1 - \gamma) \sqrt{s}.
\]

In this setting, the profit function of the online retailer will be solved to find the best response function. Subsequently, the live broadcast retailer found its best response function. The best profit for an online retailer is

\[
\text{Max } \pi_l(m_t, s) = a_t m_t - m_t^2 + m_t m_e + m_e (1 - \gamma) \sqrt{s}.
\]

The first-order condition is \( \frac{\partial \pi_l}{\partial m_t} = a_t - 2 m_t + m_e + (1 - \gamma) \sqrt{s} \), and the second derivative is \( \frac{\partial^2 \pi_l}{\partial m_t^2} = -2 < 0 \). Therefore, the profit function of an online retailer is a concave function of \( m_t \), so the existence of \( m_t \) maximizes the profit. Setting the first-order condition equal to 0, the authors get \( m_t = a_t + m_e + (1 - \gamma) \sqrt{s}/2 \). Based on the above response function, the authors can find the best response of the live sales retailer,

\[
\text{Max } \pi_l(m_t, s) = a_t m_t - m_t^2 + m_t \left[ a_t + m_t + \frac{(1 - \gamma) \sqrt{s}}{2} \right] + m_t \gamma \sqrt{s} - s.
\]

Due to \( \frac{\partial \pi_l}{\partial m_t} = 2 a_t - 4 m_t + a_e + 2 m_t + (1 + \gamma) \gamma \sqrt{s} \), \( \frac{\partial \pi_l}{\partial s} = 1/2 \left( (1 + \gamma) m_t / 2 \sqrt{s} - 2 \right) \), the second-order condition is \( \frac{\partial^2 \pi_l}{\partial m_t^2} = -2 < 0 \), \( \frac{\partial^2 \pi_l}{\partial m_t \partial s} = -(1 + \gamma) m_t / 8 s^{3/4} < 0 \), and \( \frac{\partial^2 \pi_l}{\partial m_t \partial s} = (1 + \gamma) / 4 \sqrt{s} \). The authors can find that the Hessians of the second-order condition is

\[
H = \left[ \begin{array}{cc}
\frac{\partial^2 \pi_l}{\partial m_t^2} & \frac{\partial^2 \pi_l}{\partial m_t \partial s} \\
\frac{\partial^2 \pi_l}{\partial m_t \partial s} & \frac{\partial^2 \pi_l}{\partial s^2}
\end{array} \right] = \frac{1}{4} \left[ \begin{array}{cc}
-2 & -(1 + \gamma) / 4 \sqrt{s} \\
-(1 + \gamma) / 4 \sqrt{s} & -1 / 8 s^{3/4}
\end{array} \right] < 0.
\]

Therefore, the profit function of the live broadcast retailer is the concave function of \( m_t \) and \( s \), and there is a unique optimal \( m_t^* \) and \( s^* \) to maximize it. Setting the first-order condition of the retailer’s profit function to 0, the authors get two sets of equations: \( m_t = 2 a_t + a_e + \ldots \)
(1 + γ)/√2 and m_t(1 + γ) = 4√γ. By solving these two sets of equations and substituting them, the optimal value is obtained.

Proof: of Proposition 1
\[
\frac{\partial p_{el}^*}{\partial y} = (16y + 16a_1 + (8γ + 8)a_s/ (7 - 2y - γ^2))^2 > 0, \\
\frac{\partial p_{el}^*}{\partial y} = (2y - γ + 10a_1 + \left(γ^2 - 2y + 5a_s/ (7 - 2y - γ^2)^2 > 0, \\
\frac{\partial p_{el}^*}{\partial y} = (20y + 6 - 2γ^2)a_1 + (10y + 3 - 2γ^2)a_s/ (7 - 2y - γ^2)^2 > 0.
\]
Therefore, when \(a_1/a_s \geq γ^2 + 10 - 3/6 \geq 20γ \), \(\frac{\partial p_{el}^*}{\partial y} \geq \frac{\partial p_{el}^*}{\partial y}; \) otherwise, it is true.

Proof: of Proposition 2
Due to \( D_{IL}^* = 4a_1 + 2a_s/7 - 2y - γ^2 \) and \( D_{cl}^* = (5 - γ^2)a_1 + (6 - γ - γ^2)a_s/7 - 2y - γ^2 \), the authors can get the Hessian (determinant) of the second-order condition as
\[
H = \begin{bmatrix}
\frac{\partial^2 \pi_l}{\partial m_l^2} & \frac{\partial^2 \pi_l}{\partial m_l \partial s} \\
\frac{\partial^2 \pi_l}{\partial m_l \partial s} & \frac{\partial^2 \pi_l}{\partial s^2}
\end{bmatrix} = \begin{bmatrix}
-2 & \frac{γ}{2\sqrt{γ}} \\
\frac{γ}{2\sqrt{γ}} & \frac{m_tγ}{4\sqrt{γ}}
\end{bmatrix} < 0. \quad (A.5)
\]

Therefore, the profit function of the live broadcast retailer is the concave function of \( m_t \) and \( s \). There is a unique optimal \( m_t^* \) and \( s^* \) to maximize it. Setting the first-order condition of the profit function of the live broadcast retailer to 0, the authors get two sets of equations, \( 2m_l = a_1 + m_s + m_tγ/\sqrt{γ} \) and \( m_t = 2\sqrt{γ} \). Solving them at the same time, the authors get \( m_t = 2(\alpha_1 + m_s)/4 - y^2 \) and \( s = (γ(\alpha_1 + m_s)/4 - y^2)^2 \).

Now, the authors find the best response of the online retailer based on the above response function. The first-order condition is \( \frac{\partial \pi_l}{\partial m_s} = \alpha_1 - 2m_s + (2 + γ(1 - γ)) (\alpha_1 + 2m_s)/4 - y^2 = 0 \). The authors get the optimal solution.

Proof: of Proposition 3
Due to \( \frac{\partial p_{el}^*}{\partial y} = (4 + 6y + γ^2)a_1 + (4 + 4y + γ^2)a_s/ (4 - y^2)^2 > 0 \), \( \frac{\partial p_{el}^*}{\partial y} = a_1 + a_2/2 > 0 \), the authors can get
\[
\frac{\partial p_{el}^*}{\partial y} = \frac{(12γ + 10y^2 - γ^4 - 8)\alpha_1 + (8γ + 10y^2 - γ^2)\alpha_s}{2(4 - γ^2)^2} \quad (A.6)
\]

Therefore, when \( a_1/a_s \geq γ^4 + 8 \), \( \frac{\partial p_{el}^*}{\partial y} \geq \frac{\partial p_{el}^*}{\partial y}; \) otherwise, it is true.

Proof: of Proposition 4
Due to \( D_{IE}^* = (3 + γ)a_1 + (2 + γ)a_s/4 - y^2 \) and \( D_{cl}^* = (2 + γ - γ^2)a_1 + (4 - γ^2)a_s/2(4 - γ^2) \), the authors can get
\[
\frac{\partial D_{IE}^*}{\partial y} = (4 + 6y + γ^2) a_1 + (γ^2 + 4y^4 + 4a_s/ (4 - γ^2)^2 > 0, \\
\frac{\partial D_{IE}^*}{\partial y} = (4 - 4γ^2)a_1/2(4 - γ^2)^2 > 0, \\
\frac{\partial D_{IE}^*}{\partial y} = (4 + 16γ + γ^2) a_1 + (8 + 8y + 2γ^2)a_s/2(4 - γ^2)^2. \]
Therefore, when \( 0 \leq γ \leq 1, \frac{\partial D_{IE}^*}{\partial y} > \frac{\partial D_{cl}^*}{\partial y} \cdot \frac{\partial D_{IE}^*}{\partial y} \). The authors can get the optimal result by the first-order necessary condition easily.

Proof: of Proposition 7
Due to \( D_k = \rho + \theta(\rho + \theta(\rho - p_k)) + \theta((1 - r)(p_k - w)(p + \theta(\rho - p_k + D_l)) - a) \), the authors can get
\[
\frac{\partial \pi_k}{\partial \theta} = -p_k - w)θ + r(1 - r)θ(1 - r)(p_k - p_k + D_l).
\]
The authors can get the optimal result by the first-order necessary condition easily.

Data Availability
All data supporting the findings of this study are available within the article.

Conflicts of Interest
All authors declare no possible conflicts of interest.

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