Hindawi
Complexity
Volume 2022, Article ID 6673618, 11 pages
https://doi.org/10.1155/2022/6673618

Research Article

Potential Effects of Delay on the Stability of a Class of Impulsive Neural Networks

Nan Zhan[1] and Ailong Wu[1,2]

[1]*College of Mathematics and Statistics, Hubei Normal University, Huangshi 435002, China*
[2]*Institute for Information and System Science, Xi'an Jiaotong University, Xi'an 710049, China*

Correspondence should be addressed to Ailong Wu; hbnuwu@yeah.net

Received 16 March 2022; Accepted 7 June 2022; Published 15 July 2022

Academic Editor: Miaomiao Wang

Aiming at the interference of the delay term in continuous dynamics to the impulsive systems, we study the potential effects of time delay on the stability of a class of impulsive neural networks (INNs) in this paper. Two cases of delay are considered. For the case of small delay, a sufficient condition for the stability of delayed INNs is obtained by virtue of the average impulsive interval (AII) method. The derived results illustrate that within limits, the convergence rate of the system becomes larger with the increase of time delay. For another case, a strict comparison principle is proposed to prove that the impulsive system still maintains the original stability for any large but bounded delay under certain conditions. In particular, as an extension, the stability of delayed INNs for hybrid impulses containing both stabilizing and destabilizing impulses is also discussed. Finally, three examples are simulated to demonstrate the validity of the theoretical results.

1. Introduction

As a mathematical model of information processing, neural network (NN) is one of the most active branches of computational intelligence and machine learning. There are many kinds of NNs, and as a special kind of NNs, impulsive neural networks (INNs) have unique research value. INN was first proposed by Alan Lloyd Hodgkin and Andrew Huxley in 1952. The simulation of its neurons is closer to reality because it characterizes the transient state mutation of neurons in neural networks at a certain moment. The impulsive system is a mixture of continuous dynamic system and discrete-time system, which is different from the pure continuous-time dynamic system and pure discrete-time system. It is suitable for studying a class of dynamic systems affected by sudden change or instantaneous disturbance [1]. Furthermore, impulsive phenomena exist in various fields such as secure communication, automatic control, and mechanical system. With the help of impulsive control, we can reduce a lot of application costs. So far, many interesting results have been obtained on INNs (see [2, 3] and their references).

Time delay is known to exist in many complex networks and control systems due to the influence of some practical situations. Over the past decades, time-delay systems have been vigorously studied because of their wide applications in NNs, sampling data control, biological modeling, and other fields. Meanwhile, various types of delays are discussed in NNs, such as distributed delay [4], time-varying delay [5], and state-dependent delay [6]. However, in previous studies, time delay is generally considered to be an important source of poor system performance and system instability. Few researchers have noticed that time delay may be beneficial to system stability. This is because our impression of time delay is so rigid that we ignore the stabilizing effects of time delay. Actually, we can also extract the stabilizing information of time delay through some analysis methods. For instance, in [7], the authors make a point that the increase of time delay has a dual effect on the stability of the system, that is, it may stabilize a previously unstable system or destabilize a previously stable system.

Combining the two points of time delay and impulsive effects, many scholars have done a lot of work on INNs with delay [8–12]. For example, in [8], Chen et al. utilized an auxiliary state variable to transform the impulsive delayed system into an equivalent augmented model. On the basis of this model, the stability criterion of the system was derived. In [9], Zhang et al. firstly designed an impulsive controller for the time-delay discrete system to guarantee that the system can achieve stability. In [11], Jiang et al. investigated the impacts of time delay in impulses on system stability through average impulsive delay and average impulsive interval (AII) methods. In [12], Li and Song focused on the stabilization of time-delay systems under impulsive control, and the results show that the delay term in impulses may be conducive to the stabilization of the system. Obviously, studying a system with both time delay and impulsive effects is challenging because we need to consider the interaction of the two on the system.

Furthermore, it can be observed that both references [11, 12] have investigated the impacts of the delay term in impulses on stability of the system, but few papers have studied the latent effects of the delay term in continuous dynamics on stability of the impulsive system. Knowing that an impulsive system is a combination of continuous and discrete subsystems, it is interesting to think about the overall effects of the delay term in the continuous subsystem and the impulsive effect in the discrete subsystem on the system stability. In addition, looking back at the fact that time delays may facilitate the stability of systems, a natural problem emerges: under what conditions does the delay term in continuous dynamics play a positive role in the stability of systems?

In view of the above discussion, this paper mainly studies the potential effects of delay term in continuous dynamics on the stability of a class of INNs. Compared with some existing results, this paper fully captures the information that time delay can enhance stability. With regard to small delay and large delay, the stability of INNs with delay is investigated by using AII condition, and the hidden role that delay plays in the stability of system is revealed. With regard to hybrid impulses, the AII condition is replaced by the dwell-time condition so as to deal with the impulsive parameters as a whole, and the stability criterion of INNs is also derived. On the whole, the main features of this paper can be generalized as follows:

(1) The time delay in two cases is considered. When the delay is small, we capture the stabilizing information of time delay by means of the impulsive delay inequality and then integrate it into the Lyapunov-based function. Finally, with the help of the AII condition, the stability criterion of a kind of general INNs is derived. The results show that in a certain range, the system converges more quickly when the delay value is larger.

(2) In order to handle the case of large delay, we adopt a strict comparison principle which is different from the comparison-like principle, and it is proved that these kinds of INNs are robust to any large but bounded delay.

(3) Considering the dual effects of impulses, we extend the ideas of the first two points to the hybrid INNs containing stabilizing and destabilizing impulses.

This paper is organized as follows. In Section 2, a kind of general INNs with delay is introduced, and some requisite definitions and assumptions are presented. In Section 3, the main theorem results of this paper are derived, which fully illustrate the latent effects of delay term in continuous dynamics on stability of a kind of INNs. In Section 4, three numerical examples are simulated to indicate the validity of the derived results. Finally, Sections 5 gives a brief conclusion and prospects of the feasibility of the future research.

2. Preliminaries

2.1. Notations. Let $\mathbb{R}$, $\mathbb{R}_+$, and $\mathbb{R}^n$ stand for the set of real numbers, the set of nonnegative real numbers, and the set of n–dimensional real-valued vectors, respectively. Denote $\mathbb{Z}_+$ and $\mathbb{Z}_+^0$ as the set of positive integer numbers and nonnegative integer numbers, respectively. For vector $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$, let $\|x\| = \sum_{i=1}^n |x_i|$. Denote $PC([-\tau, 0], \mathbb{R}^n)$ as the set of piecewise right continuous function ϕ: $[-\tau, 0] \longrightarrow \mathbb{R}^n$, where $\|\phi\|_\tau \triangleq \sup_{-\tau\le\theta\le 0}\|\phi(\theta)\|$. Denote the upper right-hand Dini derivative of function V as $D^+V(t) = \lim_{h\longrightarrow 0^+}\sup V(t+h) - V(t)/h$.

2.2. Model. In this paper, based on relevant work in reference [13], we consider a class of INNs, the main form of which is as follows:

$$\left\{\begin{array}{c} \dot{x}_i(t) = -a_i x_i(t) + \sum_{j=1}^n b_{ij} f_j\big(x_j(t)\big) + \sum_{j=1}^n c_{ij} g_j\big(x_j(t-\tau)\big), t \ne t_k, t \ge t_0 \ge 0, \\ \Delta x_i(t_k) = U\big(k, x_i(t_k^-)\big), k \in \mathbb{Z}_+, \\ x_i(t_0+\theta) = \phi_i(\theta), \theta \in [-\tau, 0], \end{array}\right\} \tag{1}$$

where $a_i > 0, i = 1, 2, \ldots, n$ are constants, n is the number of neurons, $x_i(t)$ represents the state variable of the ith neuron at time t, $\dot{x}_i(t)$ represents the derivative of $x_i(t)$, τ is the transmission delay, $f_j(x_j(t))$ and $g_j(x_j(t-\tau))$ are the neuron activation functions at time t and $t-\tau$, respectively, b_{ij} and c_{ij} are real constants representing the connection weight, $\{t_k\}$ is the impulse sequence satisfying $0 \le t_0 < t_1 < \cdots < t_k < \cdots$ and $\lim_{k\longrightarrow+\infty} t_k = +\infty$, and

$\Delta x_i(t_k) \triangleq x_i(t_k^+) - x_i(t_k^-)$, where $x_i(t^+) = \lim_{t\longrightarrow t_k^+} x_i(t)$ and $x_i(t^-) = \lim_{t\longrightarrow t_k^-} x_i(t)$. Generally, we suppose that the solution of network (1) is right continuous, that is, $x_i(t_k^+) = x_i(t_k)$, and the sequence $\{t_k, U(k, x_i(t_k^-))\}$ is called the impulsive control rule. As a matter of convenience, we let $h_k(x_i(t_k^-)) \triangleq x_i(t_k^-) + U(k, x_i(t_k^-))$, which means $x_i(t_k^+) = h_k(x_i(t_k^-)), k \in \mathbb{Z}_+$. Define the solution of network (1) through (t_0, ϕ) as $x(t) = x(t, t_0, \phi)$, where $\phi \in PC([-\tau, 0], \mathbb{R}^n)$ represents the initial state.

For subsequent needs, we give some requisite definitions and assumptions as follows.

Definition 1 (see [14]). Suppose that there exist positive constants N_0 and T_a such that

$$N(t_2, t_1) \geq \frac{t_2 - t_1}{T_a} - N_0, \forall t_2 \geq t_1 \geq t_0, \tag{2}$$

where $N(t_2, t_1)$ represents the number of impulses in the interval $(t_1, t_2]$. Then, N_0 is called the elasticity number, and T_a denotes the AII constants.

Remark 1. The concept of AII is proposed to handle various types of impulses. In fact, AII condition (2) allows an upper bound, that is, $N(t_2, t_1) \leq t_2 - t_1/T_a + N_0$. Particularly, at least one impulse is required for each interval of length T_a in the case of $N_0 = 1$. For AII constant T_a, it can be observed that it contains more impulsive instant sequences when the elasticity number N_0 is larger.

Definition 2. For any given initial value $\phi \in PC([-\tau, 0], \mathbb{R}^n)$, if there exist positive numbers M and λ such that

$$\|x(t, t_0, \phi)\| \leq M\|\phi\|_\tau e^{-\lambda(t-t_0)}, \forall t \geq t_0, \tag{3}$$

holds for every sequence $\{t_k\} \in \mathcal{J}^*(T_a, N_0)$, then we can say that the network (1) is globally uniformly exponentially stable (GUES) over the class $\mathcal{J}^*(T_a, N_0)$.

Remark 2. $\mathcal{J}^*(T_a, N_0)$ mentioned in the above definition represents a collection of impulsive instant sequences $\{t_k\}$ that satisfy AII condition (2).

Assumption 1. The functions $f(\cdot), g(\cdot) \in \mathbb{R}^n$ satisfy $f(0) = 0$, $g(0) = 0$, and function $h_k: \mathbb{R} \longrightarrow \mathbb{R}$ satisfies $h_k(0) = 0$.

Remark 3. Clearly, Assumption 1 guarantees that $x = 0$ is an equilibrium point to network (1).

Assumption 2. The functions $f(\cdot), g(\cdot)$ are all Lipschitz continuous and meet

$$\begin{aligned} \left|f_j(\theta_1) - f_j(\theta_2)\right| &\leq \gamma_j\left|\theta_1 - \theta_2\right|, \\ \left|g_j(\theta_1) - g_j(\theta_2)\right| &\leq \overline{\gamma_j}\left|\theta_1 - \theta_2\right|, \end{aligned} \tag{4}$$

for all $\theta_1, \theta_2 \in \mathbb{R}, j = 1, 2, \ldots, n$, where $\gamma_j, \overline{\gamma}_j > 0$ are constants.

Assumption 3. The impulsive operator h_k meets

$$\left|h_k(\theta_1) - h_k(\theta_2)\right| \leq q\left|\theta_1 - \theta_2\right|, \tag{5}$$

for all $\theta_1, \theta_2 \in \mathbb{R}, k \in \mathbb{Z}_+$, where $q > 0$ is Lipschitz constant.

In order to facilitate the subsequent expression, we make

$$\begin{aligned} \alpha_1 &= \max_{1\leq i\leq n}\left(-a_i + \sum_{j=1}^n \left|b_{ji}\right|\gamma_i\right), \\ \alpha_2 &= \max_{1\leq i\leq n}\left(\sum_{j=1}^n \left|c_{ji}\right|\overline{\gamma}_i\right). \end{aligned} \tag{6}$$

3. Main Results

We will discuss the stability of INNs from the following three aspects in current section. Firstly, we consider the stability of a kind of INNs with small delay (the delay does not exceed any two consecutive impulsive time intervals, i.e., $\tau \leq t_k - t_{k-1}$). Besides, the latent effects of time delay are also explored. Secondly, the stability of INNs with arbitrarily finite delay is considered. Compared with small delay, we use large delay (which implies that the delay may be greater than a certain impulsive time interval, namely, $\tau \leq t_k - t_{k-1}$ may not be true) to represent relatively larger delay (which is collectively referred to as arbitrarily finite delay here). For arbitrarily finite delay, we analyze the robustness of the stability of INNs with delay and verify that the system can remain stable for any large but bounded delay under certain conditions. Finally, in view of the fact that the impulsive effects may promote or suppress system stability, we extend the ideas of the first two points to delayed INNs with hybrid impulses.

3.1. INNs with Small Delay. In what follows, we will discuss the case where the delay is small. We capture the stabilizing information of time delay with the help of the impulsive delay inequality and then integrate it into the Lyapunov-based function. Finally, through the AII method, we can derive the stability criterion of INNs.

Theorem 1. *If there exist constants $q \in (0, 1)$, $\eta^* = \max\{\alpha_1 + \alpha_2, \eta_0\}$ and the following conditions hold:*

$$\begin{aligned} &\alpha_1 + \alpha_2 > 0, \\ &\frac{\ln q}{T_a} + \eta^* < 0, \end{aligned} \tag{7}$$

where

$$\alpha_1 + \frac{\alpha_2}{q}e^{-\eta_0\tau} - \eta_0 = 0, \tag{8}$$

then under Assumptions 1–3, network (1) is GUES over the class $\mathcal{J}^*(T_a, N_0)$.

Proof. Construct a function $V(t) \triangleq V(t, x(t)) = \|x(t)\| = \sum_{i=1}^n |x_i(t)|$ and make $\overline{V}(t_0) = \sup_{t_0-\tau\leq s\leq t_0} V(s)$. For any $\varepsilon > 0$, let $\eta = \eta^* + \varepsilon$, and design an auxiliary function

$$L(t) = \left\{ \begin{array}{c} V(t)e^{-\eta(t-t_k)}, t \in [t_k, t_{k+1}), \\ V(t), t_0 - \tau \le t \le t_0. \end{array} \right\} \quad (9)$$

To start with, let $\Omega_k = q^k \overline{V}(t_0)$, and then we will confirm that

$$V(t) \le \Omega_k e^{\eta(t-t_0)}, t \in [t_k, t_{k+1}], k \in \mathbb{Z}_+^0. \quad (10)$$

Together with (7) and (8),

$$L(t) \le \Omega_k e^{\eta(t_k - t_0)}, t \in [t_k, t_{k+1}]. \quad (11)$$

Firstly, we demonstrate that (9) is true for $k = 0$, namely, $L(t) \le \Omega_0 = \overline{V}(t_0), t \in [t_0, t_1]$. Note that $L(t_0) = V(t_0) \le \overline{V}(t_0) = \Omega_0$. If the above statement is incorrect, then there is an instant $t^* \in [t_0, t_1]$ such that

$$L(t^*) = \Omega_0, L(t) \le \Omega_0, t \in [t_0, t^*) \text{and} D^+ L(t^*) \ge 0. \quad (12)$$

When $s \in [t_0 - \tau, t_0]$, it is apparent that $L(s) = V(s) \le \overline{V}(t_0) = \Omega_0$, and in combination with (10), we derive $L(s) \le \Omega_0, t \in [t_0 - \tau, t^*]$. For $t \in [t_k, t_{k+1}], k \in \mathbb{Z}_+^0$, we can calculate that

$$\begin{aligned} D^+V(t) &= \sum_{i=1}^n \operatorname{sgn}(x_i(t)) \left[-a_i x_i(t) + \sum_{j=1}^n b_{ij} f_j(x_j(t)) + \sum_{j=1}^n c_{ij} g_j(x_j(t-\tau)) \right] \\ &\le \sum_{i=1}^n (-a_i |x_i(t)|) + \sum_{i=1}^n \sum_{j=1}^n |b_{ij}| \gamma_j |x_j(t)| + \sum_{i=1}^n \sum_{j=1}^n |c_{ij}| \overline{\gamma}_j |x_j(t-\tau)| \\ &\le \max_{1\le i\le n} \left(-a_i + \sum_{j=1}^n |b_{ji}| \gamma_i \right) \|x(t)\| + \max_{1\le i\le n} \left(\sum_{j=1}^n |c_{ji}| \overline{\gamma}_i \right) \|x(t-\tau)\| \\ &= \alpha_1 V(t) + \alpha_2 V(t-\tau). \end{aligned} \quad (13)$$

When $t^* - \tau \in [t_0 - \tau, t^*]$, combining the definition of $L(t)$ and conditions (6), (10), and (11), one has

$$\begin{aligned} D^+L(t)|_{t=t^*} &= e^{-\eta(t^*-t_0)} D^+V(t)|_{t=t^*} - \eta e^{-\eta(t^*-t_0)} V(t^*) \\ &\le e^{-\eta(t^*-t_0)} [\alpha_1 V(t^*) + \alpha_2 V(t^* - \tau)] \\ &\quad - \eta e^{-\eta(t^*-t_0)} V(t^*) \\ &= \alpha_1 L(t^*) + \alpha_2 L(t^* - \tau) e^{-\eta\tau} - \eta L(t^*) \\ &\le (\alpha_1 + \alpha_2 e^{-\eta\tau} - \eta) \Omega_0 < 0. \end{aligned} \quad (14)$$

Obviously, it could be observed that it contradicts $D^+L(t^*) \ge 0$, namely, (9) holds for $k = 0$.

Next, through mathematical induction method, we assume that (9) is true for $k \le p, p \in \mathbb{Z}_+^0$, i.e.,

$$L(t) \le \Omega_k e^{\eta(t_K - t_0)}, t \in [t_k, t_{k+1}), k \le p, \quad (15)$$

which means

$$L(t) \le \Omega_p e^{\eta(t_p - t_0)}, t \in [t_p, t_{p+1}). \quad (16)$$

Subsequently, we demonstrate that

$$L(t) \le \Omega_{p+1} e^{\eta(t_{p+1} - t_0)}, t \in [t_{p+1}, t_{p+2}). \quad (17)$$

Recall (7) and (14), and we obtain

$$\begin{aligned} L(t_{p+1}) &= V(t_{p+1}) = \|x(t_{p+1})\| \\ &= \|h_{p+1}(x(t_{p+1}^-)) - h_{p+1}(0)\| \\ &\le q L(t_{p+1}^-) e^{\eta(t_{p+1} - t_p)} \\ &= \Omega_{p+1} e^{\eta(t_{p+1} - t_0)}. \end{aligned} \quad (18)$$

Thus, (15) holds for $t = t_{p+1}$. On the contrary, it is assumed that there is an instant $t^* \in [t_{p+1}, t_{p+2}]$ which makes

$$\begin{aligned} L(t^*) = \Omega_{p+1} e^{\eta(t_{p+1} - t_0)}, L(t) \le \Omega_{p+1} e^{\eta(t_{p+1} - t_0)}, t \in \\ \cdot [t_{p+1}, t^*] \text{and} D^+L(t^*) \ge 0. \end{aligned} \quad (19)$$

If $t^* - \tau \ge t_{p+1}$, referring to (12), we derive that

$$\begin{aligned} D^+L(t)|_{t=t^*} \le \alpha_1 L(t^*) + \alpha_2 L(t^* - \tau) e^{-\eta\tau} - \eta L(t^*) \le \\ \cdot (\alpha_1 + \alpha_2 e^{-\eta\tau} - \eta) \Omega_{p+1} e^{\eta(t_{p+1} - t_0)} < 0. \end{aligned} \quad (20)$$

Similarly, what calls for special attention is that when $s \in [t_p, t_{p+1}]$, it follows from (14) that

$$\begin{aligned} L(s) \le \Omega_p e^{\eta(t_p - t_0)} &= \frac{\Omega_{p+1}}{q} e^{\eta(t_{p+1} - t_0)} e^{-\eta(t_{p+1} - t_p)} \\ &= \frac{L(t^*)}{q} e^{-\eta(t_{p+1} - t_p)} \le \frac{L(t^*)}{q} e^{-\eta\tau}. \end{aligned} \quad (21)$$

At this time, if $t^* - \tau < t_{p+1}$, on account of $t_p \le t_{p+1} - \tau \le t^* - \tau < t_{p+1}$, and together with (6), (16), and (17), we could compute that

$$\begin{aligned} D^+L(t)|_{t=t^*} &\le \alpha_1 L(t^*) + \alpha_2 L(t^* - \tau)e^{-\eta\tau} - \eta L(t^*) \\ &\le \alpha_1 L(t^*) + \alpha_2 \frac{L(t^*)}{q} e^{-\eta\tau} - \eta L(t^*) \\ &= \left(\alpha_1 + \frac{\alpha_2}{q} e^{-\eta\tau} - \eta\right)\Omega_{p+1} e^{\eta(t_{p+1} - t_0)} < 0, \end{aligned} \tag{22}$$

which contradicts $D^+L(t^*) \ge 0$, namely, (15) holds.

Hence, we can get

$$L(t) \le \Omega_k e^{\eta(t_k - t_0)}, \forall t \in [t_k, t_{k+1}), k \in \mathbb{Z}_+^0, \tag{23}$$

which means (8) is true, namely, $V(t) \le \Omega_k e^{\eta(t-t_0)} = \Omega_k e^{(\eta^* + \varepsilon)(t-t_0)}, t \in [t_k, t_{k+1})$.

Let $\varepsilon \longrightarrow 0^+$, and then we obtain

$$V(t) \le \Omega_k e^{\eta^*(t-t_0)}, \forall t \in [t_k, t_{k+1}), k \in \mathbb{Z}_+^0. \tag{24}$$

Since $\{t_k\} \in \mathcal{J}^*(T_a, N_0)$, the AII method further yields that

$$\begin{aligned} V(t) &\le q^{N(t,t_0)} e^{\eta^*(t-t_0)} \overline{V}(t_0) \\ &\le q^{t-t_0/T_a - N_0} e^{\eta^*(t-t_0)} \overline{V}(t_0) \\ &\le q^{-N_0} e^{(\ln q/T_a + \eta^*)(t-t_0)} \overline{V}(t_0), \forall t \ge t_0, \end{aligned} \tag{25}$$

where $N(t, t_0)$ represents the number of impulses in the interval $(t_0, t]$.

According to (18) and the definition of $V(t)$, we have

$$\|x(t)\| \le M e^{-\lambda(t-t_0)} \|\phi\|_\tau, \forall t \ge t_0, \tag{26}$$

where $M = q^{-N_0}, \lambda = -(\ln q/T_a + \eta^*) > 0$. Until now, we have done the proof. □

Remark 4. From (6) and (8) in Theorem 1, we can notice the latent impacts of time delay τ on the decay rate of Lyapunov function $V(t)$. In particular, if $\eta_0 > \alpha_1 + \alpha_2$, then we have $\eta^* = \eta_0$. Meanwhile, the implicit function $\eta^*(\tau)$ is determined by $\alpha_1 + \alpha_2/qe^{-\eta^*\tau} - \eta^* = 0$, which decreases as τ increases. Obviously, the result derived from the above theorem shows that the convergence rate λ is related to parameter η^*, and in view of the relationship between η^* and delay τ, it can be concluded that the convergence rate of the system will become larger with the increase of delay, which means that we have captured the stabilizing effects of time delay. In addition, what needs special attention is that in the majority of the available literature about the stability of delayed INNs, we can see that the stability of the system tends to be destroyed as delay increases, but different results are obtained in this paper.

Remark 5. It should be noted that the conclusion of the relationship between time delay and system stability derived from Remark 4 is based on $\eta^* = \eta_0$, so the results may be conservative to some extent. Furthermore, the conclusion of Theorem 1 is a sufficient condition rather than a necessary condition; then, it is possible for the system to be stable when τ is small and does not meet the conditions of Theorem 1.

3.2. INNs with Arbitrarily Finite Delay.

For the case of arbitrarily finite delay, based on strict comparison principle and the concept of AII, a stability criterion of INNs is also derived.

Lemma 1 (see [13]). *Let $\alpha_1 \in \mathbb{R}, \alpha_2 \ge 0$ and $q > 0$. Suppose that $a(t), b(t) \in C([t_{k-1}, t_k], \mathbb{R}_+)$ meet*

$$\left\{\begin{array}{c} D^+a(t) \le \alpha_1 a(t) + \alpha_2 a(t-\tau), t \ge t_0, t \ne t_k, \\ a(t) \le q a(t^-), t = t_k, \end{array}\right\}, \tag{27}$$

$$\left\{\begin{array}{c} D^+b(t) = \alpha_1 b(t) + \alpha_2 b(t-\tau), t \ge t_0, t \ne t_k, \\ v(t) = q b(t^-), t = t_k, \end{array}\right\}, \tag{28}$$

for all $k \in \mathbb{Z}_+$. Then, $a(t) \le b(t), \forall t_0 - \tau \le t \le t_0$ implies that $a(t) \le b(t), \forall t \ge t_0$.

Theorem 2. *If there exists constant $q \in (0, 1)$ such that*

$$\alpha_1 + \frac{\alpha_2}{q^{N_0}} + \frac{\ln q}{T_a} < 0, \tag{29}$$

then under Assumptions 1–3, network (1) is GUES over the class $\mathcal{J}^*(T_a, N_0)$ for arbitrarily finite delay τ.

Proof. Construct a function $V(t) = \|x(t)\| = \sum_{i=1}^n |x_i(t)|$, and let $\overline{V}(t_0) = \sup_{t_0 - \tau \le s \le t_0} V(s)$.

Next, similar to (11), we have

$$D^+V(t) \le \alpha_1 V(t) + \alpha_2 V((t-\tau)), \forall t \in [t_{k-1}, t_k), k \in \mathbb{Z}_+. \tag{30}$$

When $t = t_k$, according to Assumptions 1–3, we could get

$$\begin{aligned} V(t_k) &= \|x(t_k)\| = \|h_k(x(t_k^-))\| = \|h_k(x(t_k^-)) - h_k(0)\| \\ &\le q\|x(t_k^-)\| = qV(t_k^-). \end{aligned} \tag{31}$$

Introduce an impulsive delayed system with $u(t)$ as its unique solution:

$$\left\{\begin{array}{c} D^+u(t) = \alpha_1 u(t) + \alpha_2 u(t-\tau), t \ge t_0, t \ne t_k, \\ u(t) = q u(t^-), t = t_k, \\ u(t) = \overline{V}(t_0), t \in [t_0 - \tau, t_0]. \end{array}\right\} \tag{32}$$

Apparently, $V(t) \le \overline{V}(t_0) = u(t)$ when $t \in [t_0 - \tau, t_0]$. In accordance with Lemma 1, we have

$$0 \le V(t) \le u(t), \forall t \ge t_0. \tag{33}$$

From the variable parameter formula, we obtain

$$u(t) = K(t, t_0) u(t_0) + \int_{t_0}^t K(t, s) \alpha_2 u(s - \tau) ds, t \ge t_0, \tag{34}$$

where $K(t,s)$ denotes the Cauchy matrix of the following system:

$$\begin{cases} D^{+}u(t) = \alpha_1 u(t), t \geq t_0, t \neq t_k, \\ u(t) = qu(t^{-}), t = t_k, k \in \mathbb{Z}_{+}. \end{cases} \tag{35}$$

According to the properties of the Cauchy matrix, combining $0 < q < 1$ and $\{t_k\} \in \mathcal{J}^{*}(T_a, N_0)$, we can obtain

$$\begin{aligned} K(t,s) &= e^{\alpha_1 (t-s)} \Pi_{s<t_k \leq t} q \leq e^{\alpha_1 (t-s)} q^{t-s/T_a - N_0} \\ &= q^{-N_0} e^{(\alpha_1 + \ln q/T_a)(t-s)} = q^{-N_0} e^{-c(t-s)}, \end{aligned} \tag{36}$$

where $c = -(\alpha_1 + \ln q/T_a)$, and it is evident that $c > 0$ by using condition (21).

Reviewing (26) and (28), we have

$$\begin{aligned} u(t) &\leq q^{-N_0} e^{-c(t-t_0)} u(t_0) \\ &\quad + \int_{t_0}^{t} q^{-N_0} e^{-c(t-s)} \alpha_2 u(s-\tau) \mathrm{d}s, \forall t \geq t_0. \end{aligned} \tag{37}$$

Since $c > 0, N_0 > 0$, and $0 < q < 1$, this implies

$$\begin{aligned} u(t) &= \overline{V}(t_0) = u(t_0) \leq u(t_0) e^{-c(t-t_0)} \\ &< q^{-N_0} u(t_0) e^{-\lambda (t-t_0)}, t \in [t_0 - \tau, t_0], \end{aligned} \tag{38}$$

where $\lambda = c - q^{-N_0} \alpha_2 e^{\lambda \tau}$. In fact, by using (21), we can obtain $\lambda > 0$. Then, we shall confirm that

$$u(t) < q^{-N_0} u(t_0) e^{-\lambda (t-t_0)}, t \geq t_0. \tag{39}$$

On the contrary, suppose (31) is untenable; then, there exists an instant $t^{*} > t_0$ which makes

$$u(t^{*}) \geq q^{-N_0} u(t_0) e^{-\lambda (t^{*}-t_0)}, \tag{40}$$

$$u(t) < q^{-N_0} u(t_0) e^{-\lambda (t-t_0)}, t < t^{*}. \tag{41}$$

Subsequently, from (29) and (33), one has

$$\begin{aligned} u(t^{*}) &\leq q^{-N_0} e^{-c(t^{*}-t_0)} u(t_0) + \int_{t_0}^{t^{*}} q^{-N_0} e^{-c(t^{*}-s)} \alpha_2 u(s-\tau) \mathrm{d}s \\ &< q^{-N_0} e^{-c(t^{*}-t_0)} u(t_0) + \int_{t_0}^{t^{*}} q^{-N_0} e^{-c(t^{*}-s)} \alpha_2 q^{-N_0} u(t_0) e^{-\lambda (s-\tau-t_0)} \mathrm{d}s \\ &= q^{-N_0} u(t_0) \left[e^{-c(t^{*}-t_0)} + q^{-N_0} \alpha_2 e^{\lambda \tau} e^{-ct^{*} + \lambda t_0} \int_{t_0}^{t^{*}} e^{(c-\lambda)s} \mathrm{d}s \right] \\ &= q^{-N_0} u(t_0) e^{-\lambda (t^{*}-t_0)}, \end{aligned} \tag{42}$$

which is in contradiction with (32). Thus, we can derive that (31) holds. Finally, combining (25), we have

$$V(t) \leq u(t) < q^{-N_0} u(t_0) e^{-\lambda (t-t_0)}, \forall t \geq t_0, \tag{43}$$

i.e.,

$$\|x(t)\| \leq q^{-N_0} e^{-\lambda (t-t_0)} \|\phi\|_{\tau}, \forall t \geq t_0. \tag{44}$$

So far, we have done the proof. □

Remark 6. It can be observed from Theorems 1 and 2 that the AII constant T_a should be small enough to meet conditions (5) and (21) in the case of $0 < q < 1$. The AII constant means the frequency of impulsive control. The smaller T_a is, the higher the impulsive frequency will be. In addition, it should be noted that the result derived from Theorem 2 involves elasticity number N_0, and equation (21) may not hold when N_0 is sufficiently large. However, for delay-free system (see [14]), the derived result does not involve N_0, so we cannot obtain such a conclusion. Moreover, we can obverse that the elasticity number N_0 is not involved in the result of Theorem 1, which is the difference between Theorems 1 and 2. Therefore, under the condition of AII, Theorem 2 further illustrates the internal relationship between large delay and system stability.

Remark 7. Although Theorems 1 and 2 are proposed for small and large delays, respectively, this does not mean that Theorem 2 and Theorem 1 are mutually exclusive. Actually, Theorem 2 is a supplement to Theorem 1 because the so-called large delay in Theorem 2 just means that $\tau \leq t_k - t_{k-1}$ may not be true, in which case it covers the case of small delay. Therefore, Theorem 2 is also applicable to the case of small delay.

3.3. Extension to INNs with Hybrid Impulses. In recent years, hybrid impulse as an important topic has attracted wide attention, and numerous meaningful results have emerged. Particularly, in [15], considering the influence of hybrid impulses on the synchronization process, the authors designed an effective hybrid impulsive controller so as to achieve the quasi-synchronization of NNs. In this case, a sufficient delay-dependent criterion for quasi-synchronization is obtained. Meanwhile, in [15, 16], the authors adopted AII and average impulsive gain methods to deal with the hybrid impulses. In this paper, an improved dwell-time condition is introduced to treat the hybrid impulses. In view of the above discussion, we extend the results of the first two theorems in this section.

First of all, in order to extend Theorem 1, we put forward Theorem 3 by referring to the processing procedure of the small delay case.

Theorem 3. *If Assumption 3 is replaced by the following condition:*

$$\left|h_k(\theta_1) - h_k(\theta_2)\right| \le e^{-\rho_k}\left|\theta_1 - \theta_2\right|, \tag{45}$$

where $\rho_k \in \mathbb{R}$, then under Assumptions 1 and 2 and (35), for given constants $\overline{\rho} > 0, \rho^* > 0, c > 0, \delta > 0$, the following conditions are satisfied:

$$\begin{aligned}
&(A_1)\alpha_1 + \alpha_2 > 0 \\
&(A_2)\sum_{k=1}^{N(t,t_0)} \rho_k > \overline{\rho}N(t,t_0) - \rho^* \\
&(A_3)\overline{\rho}N(t,t_0) \ge c(t-t_0) - \delta \\
&(A_4)\eta^* - c < 0,
\end{aligned} \tag{46}$$

where $\eta^* = \max\{\alpha_1 + \alpha_2, \eta_1\}$, and η_1 satisfies $\alpha_1 + \alpha_2 e^{\rho_{sup}} e^{-\eta_1 \tau} - \eta_1 = 0$ with $\rho_{sup} = \sup_{k \in \mathbb{Z}_+}\{\rho_k\} < \infty$; then, network (1) is GES.

Proof. The same analysis method as Theorem 1 is used here, except that the impulsive parameters are changed, so we shall take advantage of (A_1) and (A_4) to acquire the following statement:

$$V(t) \le \Pi_{k=1}^{N(t,t_0)}(e^{-\rho_k})e^{\eta^*(t-t_0)}\overline{V}(t_0), \forall t \ge t_0. \tag{47}$$

Then, by using (A_2) and (A_3), one has

$$\begin{aligned}
V(t) &\le e^{\left(-\Sigma_{k=1}^{N(t,t_0)} e^{-\rho_k}\right)} e^{\eta^*(t-t_0)}\overline{V}(t_0) \\
&\le e^{-\overline{\rho}N(t,t_0)+\rho^*} e^{\eta^*(t-t_0)}\overline{V}(t_0) \\
&\le e^{-c(t-t_0)+\delta+\rho^*} e^{\eta^*(t-t_0)}\overline{V}(t_0) \\
&\le e^{\delta+\rho^*} e^{-(c-\eta^*)(t-t_0)}\overline{V}(t_0), \forall t \ge t_0.
\end{aligned} \tag{48}$$

That is,

$$\|x(t)\| \le e^{\delta+\rho^*} e^{-(c-\eta^*)(t-t_0)}\|\phi\|_\tau, \forall t \ge t_0. \tag{49}$$

The proof is completed.

Next, referring to the analysis method of arbitrarily finite delay case, we obtain Theorem 4 as an extension of Theorem 2. □

Theorem 4. *Suppose that the parameter q in Assumption 3 is replaced by $e^{-\rho_k}$, then under Assumptions 1 and 2 and modified Assumption 3, for given constants $\overline{\rho} > 0, \rho^* > 0, c > 0, \delta > 0$, the following conditions are fulfilled:*

$$\begin{aligned}
&(\widehat{A}_1)\sum_{k=1}^{N(t,t_0)} \rho_k > \overline{\rho}N(t,t_0) - \rho^* \\
&(\widehat{A}_2)\overline{\rho}N(t,t_0) \ge (\alpha_1 + c)(t-t_0) - \delta \\
&(\widehat{A}_3)\alpha_2 e^{\delta+\rho^*} - c < 0.
\end{aligned} \tag{50}$$

Then, network (1) is GES.

Proof. It is easy to prove this theorem by combining the analytical methods of Theorems 2 and 3, so we leave out it for brevity. □

Remark 8. The parameter $e^{-\rho_k}$ in Theorems 3 and 4 is used to describe the variable of hybrid impulses in impulsive control systems. As you can see, q in Theorems 1 and 2 is required to be $0 < q < 1$, but $e^{-\rho_k}$ in Theorem 3 and 4 satisfies $\rho_k \in \mathbb{R}$, that is, $e^{-\rho_k} < 1$ if $\rho_k > 0$ and $e^{-\rho_k} > 1$ if $\rho_k < 0$. It implies that stabilizing impulses and destabilizing impulses may exist at the same time. Furthermore, in order to handle these parameters overall, we propose conditions (A_2) and $(\widehat{A}_1)$, and in a sense, the parameter $\overline{\rho}$ may be approximately regarded as the "average value" of ρ_k. In fact, condition (A_2) combined with (A_3) or $(\widehat{A}_1)$ combined with $(\widehat{A}_2)$ can be considered as an improvement of dwell-time condition.

Remark 9. Compared with reference [11], this paper studies the effects of time delay in continuous dynamics on system stability. Bear in mind that the relationship between time delay in continuous dynamics and the stability of impulsive systems is not easy to find, and the derived results based on AII method contain both time delay and the AII constant T_a, which is not obtained in previous results. Furthermore, we extend the results to systems with hybrid impulses. In addition, when discussing the delay effects in reference [12], the authors limit the time delay to be less than any two consecutive impulsive time intervals. However, we loosen the condition of time delay in this paper, that is, the time delay can be smaller than any two consecutive impulsive intervals, or it can be greater than any two consecutive impulsive time intervals.

Remark 10. On the basis of this paper, we could also investigate the case of neural network models with time-varying delays. Actually, the results of this paper are still valid after the constant delay is replaced by time-varying delay in the model. We will continue to explore this question in depth in future studies.

4. Illustrative Examples

Finally, for the purpose of verifying the above achievements, we put forward the following three examples in current section.

Example 1. Consider a 2-dimensional INN with small delay:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = -\begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0.2 & 0.4 \\ 0.03 & 0.01 \end{bmatrix}\begin{bmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{bmatrix} + \begin{bmatrix} 0.25 & 0.25 \\ 0.05 & 0.02 \end{bmatrix}\begin{bmatrix} \tanh(x_1(t-\tau)) \\ \tanh(x_2(t-\tau)) \end{bmatrix}, \quad (51)$$

under impulsive control

$$\begin{bmatrix} x_1(t_k) \\ x_2(t_k) \end{bmatrix} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}\begin{bmatrix} x_1(t_k^-) \\ x_2(t_k^-) \end{bmatrix}, \quad (52)$$

where $t \geq 0, k \in \mathbb{Z}_+, t_k - t_{k-1} \geq \tau > 0$.

Obviously, $\gamma_1 = \gamma_2 = 1, \overline{\gamma}_1 = \overline{\gamma}_2 = 1, q = 0.01$. By calculating,

$$\alpha_1 = \max_{1\leq i\leq n}\left(-a_i + \sum_{j=1}^{n}\left|b_{ji}\right|\gamma_i\right) = 0.11,$$
$$\alpha_2 = \max_{1\leq i\leq n}\left(\sum_{j=1}^{n}\left|c_{ji}\right|\overline{\gamma}_i\right) = 0.3. \quad (53)$$

Here, we set the initial value $\phi = [3, -0.4]^T$ and choose impulsive instants $t_k = 5k, k \in \mathbb{Z}_+$, which means $T_a = 5$. When $\tau \in [4, 5]$, we can figure out $\eta^* \leq 0.91$ and $\ln q/T_a + \eta^* < 0$, and it can be tested that all conditions in Theorem 1 hold. Therefore, we can derive that systems (39) and (40) are GUES when $\tau \in [4, 5]$. In addition, according to Remark 4, it can be seen that the system may converge faster with the increase of delay, which corresponds to the simulation results in Figures 1–3. Moreover, we calculate its corresponding parameter η^* and estimate its convergence rate for different time delays $\tau = 4, 4.5, 5$, which are shown in Table 1. More importantly, it also reveals the potential stabilizing effect of time delay.

Example 2. Consider another 2-dimensional INN with large delay:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = -\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0.08 & 0.5 \\ 0.2 & 0.35 \end{bmatrix}\begin{bmatrix} \tanh\left(\frac{x_1(t)}{2}\right) \\ \tanh\left(\frac{x_2(t)}{2}\right) \end{bmatrix} + \begin{bmatrix} 0.03 & 0.1 \\ 0.1 & 0.07 \end{bmatrix}\begin{bmatrix} \tanh\left(\frac{x_1(t-\tau)}{8}\right) \\ \tanh\left(\frac{x_2(t-\tau)}{2}\right) \end{bmatrix}, \quad (54)$$

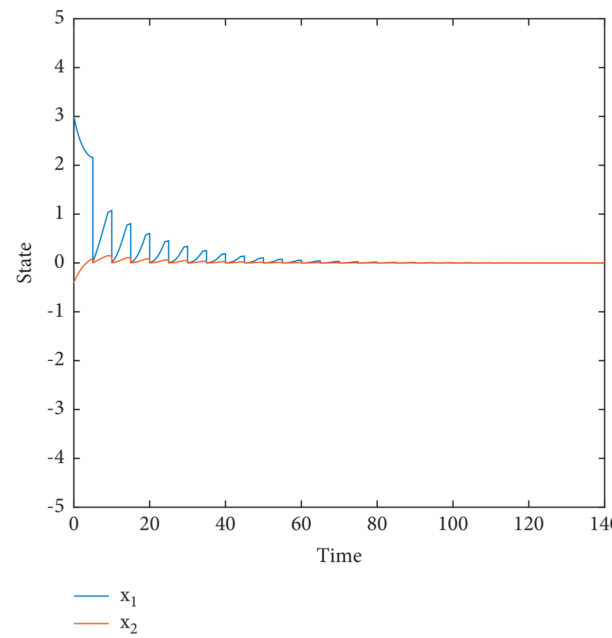

Figure 1: The state of (39) and (40) with $\tau = 4$.

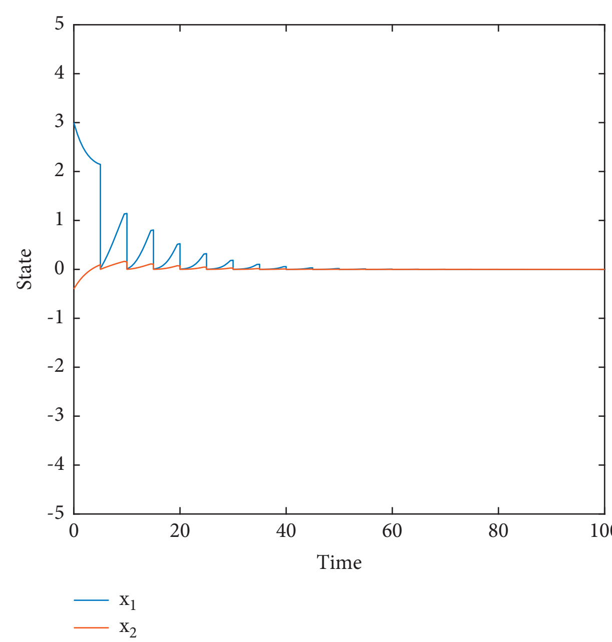

Figure 2: The state of (39) and (40) with $\tau = 4.5$.

under impulsive control

$$\begin{bmatrix} x_1(t_k) \\ x_2(t_k) \end{bmatrix} = \begin{bmatrix} \sin\left(\frac{x_1(t_k^-)}{5}\right) \\ \sin\left(\frac{x_2(t_k^-)}{5}\right) \end{bmatrix}, \quad (55)$$

where $t \geq 0, k \in \mathbb{Z}_+$ and $\tau = 25, t_k = 2k$. Here, we set the initial value $\phi = [0.4, 3]^T$. As shown in Figure 4, the impulse-free system is unstable.

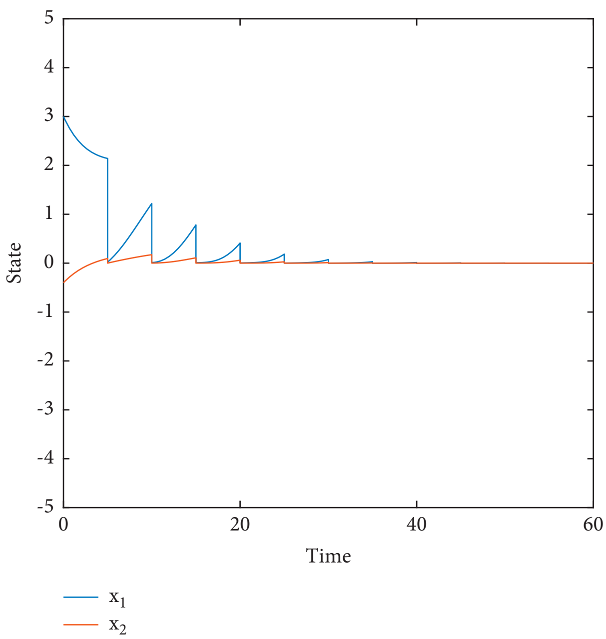

FIGURE 3: The state of (39) and (40) with $\tau = 5$.

TABLE 1: The convergence rate λ for different time delays.

Time delay τ	Parameter η^*	Convergence rate λ
4	0.9070	0.0140
4.5	0.8291	0.0919
5	0.7649	0.1561

In addition, it is apparent that $\gamma_1 = \gamma_2 = 1/2, \overline{\gamma}_1 = 1/8, \overline{\gamma}_2 = 1/2, q = 1/5, T_a = 2, N_0 = 1$. By calculating,

$$\begin{aligned} \alpha_1 &= \max_{1\le i\le n}\left(-a_i + \sum_{j=1}^{n} \left|b_{ji}\right|\gamma_i\right) = 0.325, \\ \alpha_2 &= \max_{1\le i\le n}\left(\sum_{j=1}^{n} \left|c_{ji}\right|\overline{\gamma}_i\right) = 0.085. \end{aligned} \tag{56}$$

Therefore, we can conclude that $\alpha_1 + \alpha_2/q^{N_0} + \ln q/T_a \approx -0.05 < 0$. That is to say, the conditions in Theorem 2 are fulfilled, and it is deduced that systems (43) and (44) are GUES, which is well reflected in Figure 5. By the way, the time delay could be much larger and the system would still be stable. In what follows, we calculate the corresponding convergence rate for different time delays $\tau = 25, 60, 100$, which are shown in Table 2. Actually, the delay is not limited to 100, and it can even be greater than 100. As long as the delay is bounded under certain conditions, the initial stability of impulsive system can be guaranteed. Here we only calculate the convergence rate of the system when the delay increases to 100.

Example 3. Consider a 3-dimensional INN:

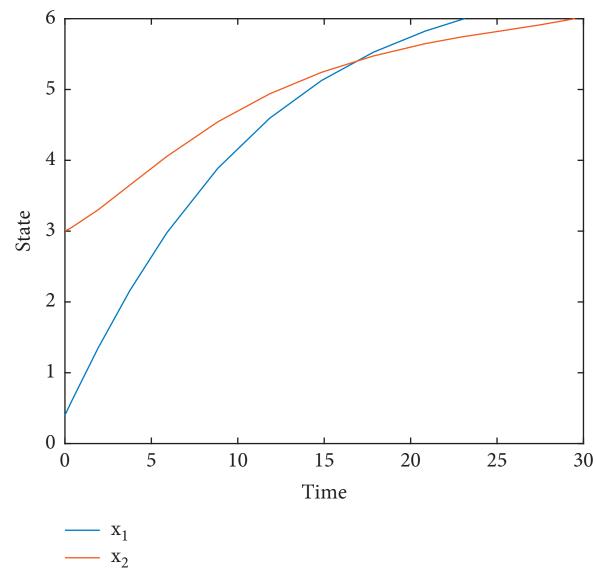

FIGURE 4: The state of (41) without impulse.

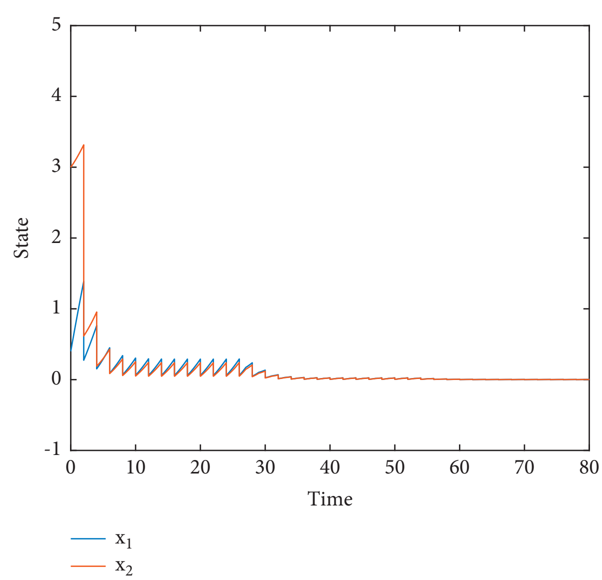

FIGURE 5: The state of (41) with impulsive effects (42).

TABLE 2: Stability of systems with different time delays.

Time delay τ	Stable or not	Convergence rate λ
25	✓	0.0045
60	✓	0.0019
100	✓	0.0012

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Cg(x(t-\tau)), \tag{57}$$

under hybrid impulsive control

$$x(t_k) = e^{-\rho_k} x(t_k^-), t_k = 8k, k \in \mathbb{Z}_+, \tag{58}$$

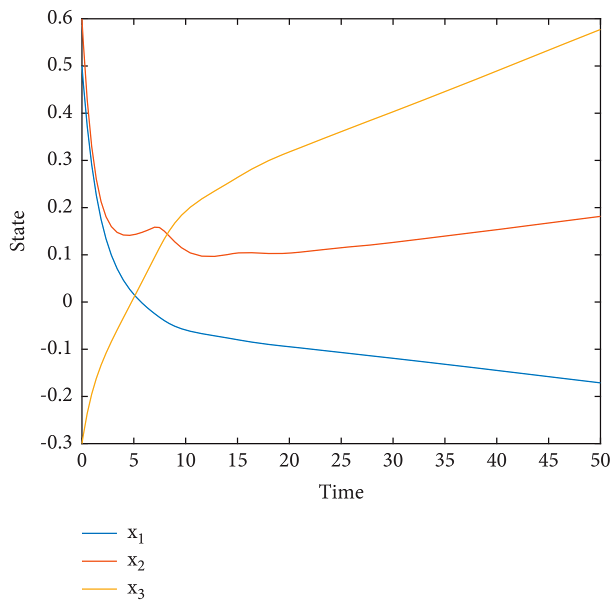

Figure 6: The state of (43) without impulse.

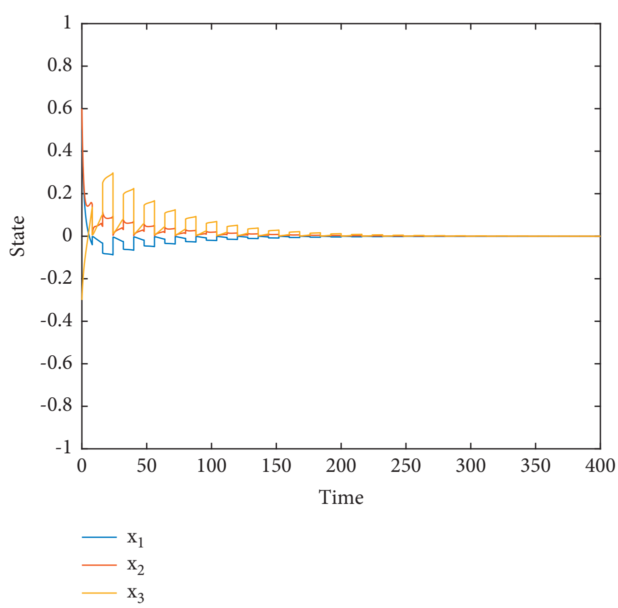

Figure 7: The state of (43) under hybrid impulsive control (44).

where

$$A = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}, B = \begin{bmatrix} 0.24 & -0.3 & -0.2 \\ -0.2 & -0.24 & 0.3 \\ -0.4 & 0.7 & 0.9 \end{bmatrix}, C = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, \tag{59}$$

$$\rho_k = \left\{ \begin{matrix} 3.1, k = 2l - 1 \\ -0.9, k = 2l \end{matrix} \right\}, l \in \mathbb{Z}_+. \tag{60}$$

Suppose that $f(x) = (f_1(x_1), f_2(x_2), f_3(x_3))^T$, $g(x) = (g_1(x_1), g_2(x_2), g_3(x_3))^T$ and $f_i(x_i) = \tanh(x_i(t)/2)$, $g_i(x_i) = \tanh(x_i(t))$, $i = 1, 2, 3$. We give the initial value $\phi = [0.5, 0.6, -0.3]^T$ and take time delay $\tau = 7$. As shown in Figure 6, the impulse-free system is unstable.

Additionally, it is evident that $\gamma_1 = \gamma_2 = \gamma_3 = 1/2, \overline{\gamma}_1 = \overline{\gamma}_2 = \overline{\gamma_3} = 1, \rho_{\text{sup}} = 3.1, \overline{\rho} = 1.1 > 0$. By calculating,

$$\begin{aligned} \alpha_1 &= \max_{1 \le i \le n} \left(-a_i + \sum_{j=1}^{n} \left| b_{ji} \right| \gamma_i \right) = 0.1, \\ \alpha_2 &= \max_{1 \le i \le n} \left(\sum_{j=1}^{n} \left| c_{ji} \right| \overline{\gamma}_i \right) = 0.2. \end{aligned} \tag{61}$$

Furthermore, we can figure out $\eta_1 \approx 0.3899$ by Matlab. That is, $\eta^* = \max\{\alpha_1 + \alpha_2, \eta_1\} \approx 0.39$ and $c = 0.4$ in conditions $(A_1) - (A_4)$ of Theorem 3 are fulfilled. At the same time, a series of conditions in Theorem 3 are completely true. Therefore, systems (43) and (44) are GES, which is well illustrated in Figure 7. From the simulation results, the impulsive effects indeed have both stabilizing and destabilizing effects.

5. Conclusion

In this paper, we have discussed the stability of a kind of INNs with delay. Particularly, the internal relation between time delay and system stability has been revealed. Firstly, we have investigated the case where the delay is small. By constructing Lyapunov function, combining the impulsive delay inequality and AII condition, we have obtained a sufficient condition to assure the exponential stability of INNs. The results have shown that within limits, the system converges more quickly with the increase of time delay. Secondly, we have explored the case where the delay is arbitrarily large but bounded and derived a Lyapunov-based stability criterion by virtue of the strict comparison principle. Finally, as an extension, we have considered the case where INN is a system with hybrid impulses. In future studies, we may discuss the delay effects of a kind of INN with state-dependent delay.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study was supported by the Natural Science Foundation of China under grant no. 61976084, the Natural Science Foundation of Hubei Province of China under grant no. 2021CFA080, and the Young Top-Notch Talent Cultivation Program of Hubei Province of China.

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