

Research Article

Vertex-Edge-Degree-Based Topological Properties for Hex-Derived Networks

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A topological index can be focused on uprising of a chemical structure into a real number. The degree-based topological indices have an active place among all topological indices. These topological descriptors intentionally associate certain physicochemical assets of the corresponding chemical compounds. Graph theory plays a very useful role in such type of research directions. The hex-derived networks have vast applications in computer science, physical sciences, and medical science, and these networks are constructed by hexagonal mesh networks. In this paper, we determined the exact values of vertex-edge-degree-based topological descriptors for hex-derived networks HDN₁(*p*) and HDN₂(*p*), which are generated by the hexagonal network of dimension *p*.

1. Introduction

The applications of topological descriptors of chemical structure are nowadays a normal process in the education of structure-property relations, specifically in QSPR/QSAR studies. Topological indices play a dynamic part in the QSPR/QSAR study. They associate certain physicochemical assets of chemical compounds. Graph theory has provided pharmacists with an assortment of suitable apparatuses, such as topological indices. Chemicals and chemical compounds are frequently displayed by chemical graphs. A chemical graph is an illustration of the structural formula of a chemical compound in terms of graph theory, in which atoms are denoted with vertices and edges show the chemical bonding between them. Lately, a latest topic that has piqued the interest of researchers is cheminformatics, which is a composite of chemistry, information science, and mathematics, in which the QSAR/QSPR relationship, bioactivity, and classification of chemical compounds are investigated.

The topological descriptor is a real number associated with chemical compositions that maintains the correlation of chemical structures with a variety of physicochemical properties, chemical reactivity, or biological activity.

Topological indices are classified into three types: distance-based topological indices, degree-based topological indices, and counting-related topological indices. Numerous researchers have recently discovered topological indices for the study of fundamental properties of molecular graphs or networks. These networks have very appealing topological properties, which have been considered in various characteristics such as [1–8].

Chen et al. [9] explained the construction of hexagonal mesh networks that consist of triangles, as shown in Figure 1. Furthermore, we gather the *p*th hexagonal mesh by putting *p* triangles around the boundary of each hexagon. Imran et al. defined the new hex-derived networks, namely, first type HDN₁(*p*) (see Figure 2) and second type HDN₂(*p*) (see Figure 3); for detailed construction, see [10]. Simonraj and George [11] created the new network which is named as third type of hex-derived networks. Koam et al. [12] computed the vertex-edge-based topological indices of some hex-derived networks. There are some works related to hex-derived networks which can be seen in [13–15]. Related research papers that contain the theoretical as well as application aspects for new research directions can be found in [16–22].

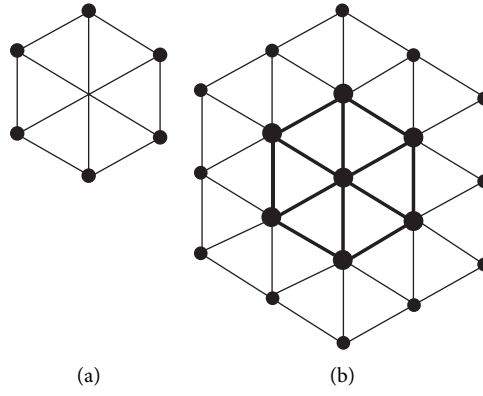


FIGURE 1: Hexagonal meshes: (a) $HX(2)$ and (b) $HX(3)$.

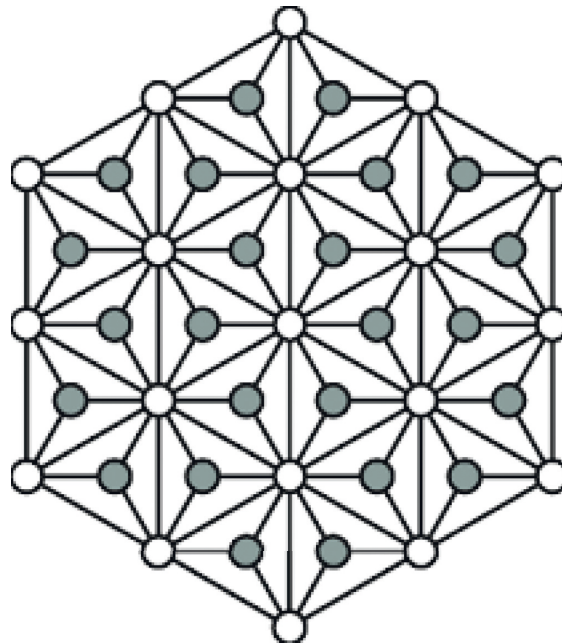


FIGURE 2: Hex-derived network $HDN_1(p)$ for $p = 3$.

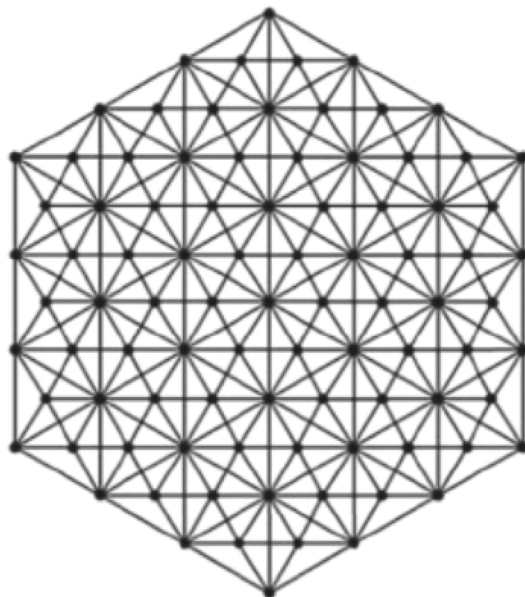


FIGURE 3: Hex-derived network $HDN_2(p)$ for $p = 4$.

2. Preliminaries

Let $G = (V, E)$ be a simple connected graph with E being the edge set and V being the vertex set. The symbol $\Psi(\theta)$ denoted the concept of degree of a vertex θ , and it is defined by the number of attached edges with θ . The symbol $N(\theta)$ denoted the number of all vertices adjacent to θ and is called as the open neighborhood of a vertex θ . On the contrary, the symbol $N[\theta]$ is the union of θ and $N(\theta)$ and called as the closed neighborhood of θ . The concept of ve -degree denoted by $\Psi_{ve}(\theta)$, and can be defined as follows: for any vertex θ

from the vertex set of a graph, is the number of different edges that are attached to any vertex from the closed neighborhood of θ . In this research work, we elaborated different ve -degree-associated topological descriptors. In [23], vertex/edge-degree-based topological indices are defined in which they computed the degree of an edge uv as $d_u + d_v - 2$. In this article, we consider ve -degree which is the degree of a vertex and is calculated by adding the degrees of its all-neighboring vertices.

$$\begin{aligned} \Gamma(G) &= \sum_{\theta\vartheta \in E(G)} \phi(\Psi_{ve}(\theta), \Psi_{ve}(\vartheta)), \\ \phi(\Psi_{ve}(\theta), \Psi_{ve}(\vartheta)) &= \sum_{\theta\vartheta \in E} (\Psi_{ve}(\theta) + \Psi_{ve}(\vartheta)), \text{ first } ve\text{-degree Zagreb } \beta\text{index } M_{\beta ve}^1, \\ \phi(\Psi_{ve}(\theta), \Psi_{ve}(\vartheta)) &= \sum_{\theta\vartheta \in E} (\Psi_{ve}(\theta) \times \Psi_{ve}(\vartheta)), \text{ second } ve\text{-degree Zagreb index } M_{\beta ve}^2, \\ \phi(\Psi_{ve}(\theta), \Psi_{ve}(\vartheta)) &= \sum_{\theta\vartheta \in E} \frac{2}{\Psi_{ve}(\theta) + \Psi_{ve}(\vartheta)}, \text{ } ve\text{-degree harmonic index } H_{ve}, \\ \phi(\Psi_{ve}(\theta), \Psi_{ve}(\vartheta)) &= \sum_{\theta\vartheta \in E} (\Psi_{ve}(\theta) \times \Psi_{ve}(\vartheta))^{-(1/2)}, \text{ } ve\text{-degree Randic index } R_{ve}, \\ \phi(\Psi_{ve}(\theta), \Psi_{ve}(\vartheta)) &= \sum_{\theta\vartheta \in E} (\Psi_{ve}(\theta) + \Psi_{ve}(\vartheta))^{-(1/2)}, \text{ } ve\text{-degree sum - connectivity index } \chi_{ve}, \\ \phi(\Psi_{ve}(\theta), \Psi_{ve}(\vartheta)) &= \sum_{\theta\vartheta \in E} \left(\frac{\Psi_{ve}(\theta) + \Psi_{ve}(\vartheta) - 2}{\Psi_{ve}(\theta) \times \Psi_{ve}(\vartheta)} \right)^{1/2}, \text{ } ve\text{-degree atom - bond connectivity index } ABC_{ve}, \\ \phi(\Psi_{ve}(\theta), \Psi_{ve}(\vartheta)) &= \sum_{\theta\vartheta \in E} \frac{2(\Psi_{ve}(\theta) \times \Psi_{ve}(\vartheta))^{1/2}}{\Psi_{ve}(\theta) + \Psi_{ve}(\vartheta)}, \text{ } ve\text{-degree geometric - arithmetic index } GA_{ve}. \end{aligned} \tag{1}$$

The researchers in [24–32] detailed different ve -degree topological invariants. This research work contains the computational exact results of given above descriptors.

3. Hex-Derived Network $\text{HDN}_1(p)$

Let $\text{HDN}_1(p)$ be the notation for the hex-derived network of the first type, and it is shown in Figure 2. The original hex-derived network $\text{HDN}_1(p)$ contains $9p^2 - 15p + 7$ total number of vertices in which there are $6p^2 - 12p + 6$ vertices

of degree 3, 6 vertices of degree 5, $6p - 12$ vertices of degree 7, and $3p^2 - 9p + 7$ vertices of degree 12. There are $27p^2 - 51p + 24$ count of edges for the graph $\text{HDN}_1(p)$; all these edges are partitioned into eight subsets according to their degrees and corresponding ve -degrees of end vertices elaborated in equations (9)–(16).

Let $\Xi_{ve, \gamma, \delta}^{\alpha, \beta}$ be the edge partition of $\text{HDN}_1(p)$ according to its degrees (α, β) and ve -degrees (γ, δ) . It is defined as

$$\Xi_{ve, \gamma, \delta}^{\alpha, \beta} = \{\theta\vartheta \in E(\text{HDN}_1(p)) : d(\theta) = \alpha, d(\vartheta) = \beta, \Psi(\theta) = \gamma, \Psi(\vartheta) = \delta\}. \tag{2}$$

And $|\Xi_{ve, \gamma, \delta}^{\alpha, \beta}|$ represents the number of edges in $\Xi_{ve, \gamma, \delta}^{\alpha, \beta}$.

$$\left| \Xi_{ve,\gamma,\delta}^{3,5} \right| = 12, \quad \text{for } \gamma = 24, \delta = 32, \quad (3)$$

$$\left| \Xi_{ve,\gamma,\delta}^{3,7} \right| = \begin{cases} 12 & \text{if } \gamma = 24, \delta = 45, \\ 12 & \text{if } \gamma = 31, \delta = 45, \\ 12 & \text{if } \gamma = 26, \delta = 45, \\ 12p - 48 & \text{if } \gamma = 26, \delta = 47, \\ 6p - 24 & \text{if } \gamma = 31, \delta = 47, \end{cases} \quad (4)$$

$$\left| \Xi_{ve,\gamma,\delta}^{3,12} \right| = \begin{cases} 12 & \text{if } \gamma = 24, \delta = 73, \\ 12 & \text{if } \gamma = 31, \delta = 73, \\ 6p - 18 & \text{if } \gamma = 26, \delta = 80, \\ 12p - 36 & \text{if } \gamma = 31, \delta = 80, \\ 12 & \text{if } \gamma = 36, \delta = 73, \\ 18p - 54 & \text{if } \gamma = 36, \delta = 80, \\ 18p^2 - 90p + 114 & \text{if } \gamma = 36, \delta = 90, \end{cases} \quad (5)$$

$$\left| \Xi_{ve,\gamma,\delta}^{5,7} \right| = 12 \text{ if } \gamma = 32, \delta = 45, \quad (6)$$

$$\left| \Xi_{ve,\gamma,\delta}^{5,12} \right| = 6 \text{ if } \gamma = 32, \delta = 73, \quad (7)$$

$$\left| \Xi_{ve,\gamma,\delta}^{7,7} \right| = \begin{cases} 12 & \text{if } \gamma = 45, \delta = 47, \\ 6p - 30 & \text{if } \gamma = 47, \delta = 47, \end{cases} \quad (8)$$

$$\left| \Xi_{ve,\gamma,\delta}^{7,12} \right| = \begin{cases} 12 & \text{if } \gamma = 45, \delta = 73, \\ 12 & \text{if } \gamma = 45, \delta = 80, \\ 12p - 48 & \text{if } \gamma = 47, \delta = 80, \end{cases} \quad (9)$$

$$\left| \Xi_{ve,\gamma,\delta}^{4,18} \right| = \begin{cases} 12 & \text{if } \gamma = 73, \delta = 80, \\ 6 & \text{if } \gamma = 73, \delta = 90, \\ 6p - 24 & \text{if } \gamma = 80, \delta = 80, \\ 12p - 36 & \text{if } \gamma = 80, \delta = 90, \\ 9p^2 - 51p + 72 & \text{if } \gamma = 90, \delta = 90. \end{cases} \quad (10)$$

Given below are some ve -degree-based indices, for example, $M_{\beta ve}^1$ index, M_{ve}^2 index, R_{ve} index, ABC_{ve} index, GA_{ve} index, H_{ve} index, and χ_{ve} index for $HDN_1(p)$.

Theorem 1. Let $HDN_1(p)$ be the first type of hex-derived network; then,

$$(i) M_{\beta ve}^1(HDN_1(p)) = 3888p^2 - 10032p + 6276$$

$$(ii) M_{ve}^2(HDN_1(p)) = 131220p^2 - 404040p + 298614$$

$$(iii) H_{ve}(HDN_1(p)) = (27p^2/70) - (312850210871353p/920029146474984) + (1630411987161294780214346433/19563734063325532742904992750)$$

Proof. Let $HDN_1(p)$ be the notation for the hex-derived network of the first type, and it is shown in Figure 2. The original hex-derived network $HDN_1(p)$ contains $9p^2 - 15p + 7$ total number of vertices in which there are $6p^2 - 12p + 6$ vertices of degree 3, 6 vertices of degree 5, $6p - 12$ vertices of degree 7, and $3p^2 - 9p + 7$ vertices of degree 12. There are $27p^2 - 51p + 24$ count of edges for the graph $HDN_1(p)$; all these edges are partitioned into eight subsets according to their degrees and relative ve -degrees of both end vertices elaborated in equations (9)–(16). Evaluating equation (2), we can determine the first ve -degree Zagreb β index as

$$\begin{aligned} M_{\beta ve}^1(HDN_1(p)) &= \left| \Xi_{ve,\gamma,\delta}^{3,5} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{3,7} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{3,12} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{5,7} \right| (\gamma + \delta) \\ &\quad + \left| \Xi_{ve,\gamma,\delta}^{5,12} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{7,7} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{7,12} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{12,12} \right| (\gamma + \delta). \end{aligned} \quad (11)$$

$$M_{\beta ve}^1(HDN_1(p)) = 3888p^2 - 10032p + 6276. \quad (12)$$

Evaluating equations (9)–(16) and after simplifications, we will have the required results as

Evaluating equation (3), we can compute the second ve -degree Zagreb index as

$$M_{ve}^2(\text{HDN}_1(p)) = \left| \Xi_{ve,\delta}^{3,5} \right| (\gamma \times \delta) + \left| \Xi_{ve,\delta}^{3,7} \right| (\gamma \times \delta) + \left| \Xi_{ve,\delta}^{3,12} \right| (\gamma \times \delta) + \left| \Xi_{ve,\delta}^{5,7} \right| (\gamma \times \delta) + \left| \Xi_{ve,\delta}^{5,12} \right| (\gamma \times \delta) + \left| \Xi_{ve,\delta}^{7,7} \right| (\gamma \times \delta) + \left| \Xi_{ve,\delta}^{7,12} \right| (\gamma \times \delta) + \left| \Xi_{ve,\delta}^{12,12} \right| (\gamma \times \delta). \tag{13}$$

Evaluating equations (9)–(16) and after simplifications, we will have the required result as

$$M_{ve}^2(\text{HDN}_1(p)) = 131220p^2 - 404040p + 298614. \tag{14}$$

Evaluating equation (4), we can determine the ve -degree harmonic index as

$$H_{ve}(\text{HDN}_1(p)) = \left| \Xi_{ve,\delta}^{3,5} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\delta}^{3,7} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\delta}^{3,12} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\delta}^{5,7} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\delta}^{5,12} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\delta}^{7,7} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\delta}^{7,12} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\delta}^{12,12} \right| \left(\frac{2}{\gamma + \delta} \right). \tag{15}$$

Evaluating equations (9)–(16) and after simplifications, we get

$$H_{ve}(\text{HDN}_1(p)) = \frac{27p^2}{70} - \frac{312850210871353p}{920029146474984} + \frac{1630411987161294780214346433}{19563734063325532742904992750}. \tag{16}$$

Theorem 2. Let $\text{HDN}_1(p)$ be the first type of hex-derived network; then,

- (i) $R_{ve}(\text{HDN}_1(p)) = ((\sqrt{10}/10) + (1/10))p^2 + (-2053/5640) + (6\sqrt{1222}/611) + (\sqrt{2}/10) + (6\sqrt{1457}/1457) + (3\sqrt{5}/20) + (3\sqrt{155}/155) + (3\sqrt{235}/235) - (\sqrt{10}/2) + (3\sqrt{130}/260)p + (12\sqrt{2263}/2263) - (24\sqrt{1222}/611) - (3\sqrt{2}/10) - (24\sqrt{1457}/1457) - (9\sqrt{5}/20) - (\sqrt{155}/31) - (8\sqrt{235}/235) + (11\sqrt{10}/15) - (\sqrt{130}/260) + (\sqrt{3}/4) + (\sqrt{438}/73) + (29/470) + (2\sqrt{73}/73) + (\sqrt{730}/365) + (3\sqrt{146}/292) + (7\sqrt{365}/365) + (\sqrt{30}/15)$
- (ii) $\chi_{ve}(\text{HDN}_1(p)) = ((3\sqrt{5}/10) + (3\sqrt{14}/7))p^2 + ((3\sqrt{10}/20) + (12\sqrt{73}/73) + (6\sqrt{170}/85) + (\sqrt{78}/13) -$

$$(17\sqrt{5}/10) + (9\sqrt{29}/29) + (3\sqrt{94}/47) + (4\sqrt{111}/37) + (12\sqrt{127}/127) - (15\sqrt{14}/7) + (3\sqrt{106}/53)p - (3\sqrt{10}/5) + (22\sqrt{14}/7) + (6\sqrt{19}/19) + (3\sqrt{26}/13) - (27\sqrt{29}/29) + (6\sqrt{23}/23) + (4\sqrt{17}/17) + (72\sqrt{5}/25) - (48\sqrt{127}/127) - (9\sqrt{106}/53) + (6\sqrt{118}/59) + (12\sqrt{77}/77) + (12\sqrt{71}/71) + (4\sqrt{69}/23) + (12\sqrt{97}/97) - (18\sqrt{170}/85) - (4\sqrt{78}/13) - (15\sqrt{94}/47) + (12\sqrt{109}/109) + (6\sqrt{163}/163) + (2\sqrt{105}/35) - (12\sqrt{111}/37) - (48\sqrt{73}/73)$$

Proof. The ve -degree Randic index can be determined by evaluating the edge partitions in equation (5):

$$R_{ve}(\text{HDN}_1(p)) = \left| \Xi_{ve,\delta}^{3,5} \right| (\gamma \times \delta)^{-\frac{1}{2}} + \left| \Xi_{ve,\delta}^{3,7} \right| (\gamma \times \delta)^{-(1/2)} + \left| \Xi_{ve,\delta}^{3,12} \right| (\gamma \times \delta)^{-(1/2)} + \left| \Xi_{ve,\delta}^{5,7} \right| (\gamma \times \delta)^{-(1/2)} + \left| \Xi_{ve,\delta}^{5,12} \right| (\gamma \times \delta)^{-(1/2)} + \left| \Xi_{ve,\delta}^{7,7} \right| (\gamma \times \delta)^{-(1/2)} + \left| \Xi_{ve,\delta}^{7,12} \right| (\gamma \times \delta)^{-(1/2)} + \left| \Xi_{ve,\delta}^{12,12} \right| (\gamma \times \delta)^{-(1/2)}. \tag{17}$$

The methodology of edge partitions can be determined from equations (9)–(16); after mathematical calculations, we get the following result:

$$\begin{aligned}
R_{ve}(\text{HDN}_1(p)) = & \left(\frac{\sqrt{10}}{10} + \frac{1}{10} \right) p^2 + \left(\frac{2053}{5640} + \frac{6\sqrt{1222}}{611} + \frac{\sqrt{2}}{10} + \frac{6\sqrt{1457}}{1457} + \frac{3\sqrt{5}}{20} + \frac{3\sqrt{155}}{155} \right. \\
& + \frac{3\sqrt{235}}{235} - \frac{\sqrt{10}}{2} + \frac{3\sqrt{130}}{260} \left. \right) p + \frac{12\sqrt{2263}}{2263} - \frac{24\sqrt{1222}}{611} - \frac{3\sqrt{2}}{10} - \frac{24\sqrt{1457}}{1457} - \frac{9\sqrt{5}}{20} - \frac{\sqrt{155}}{31} \\
& - \frac{8\sqrt{235}}{235} + \frac{11\sqrt{10}}{15} - \frac{\sqrt{130}}{260} + \frac{\sqrt{3}}{4} + \frac{\sqrt{438}}{73} + \frac{29}{470} + \frac{2\sqrt{73}}{73} + \frac{\sqrt{730}}{365} + \frac{3\sqrt{146}}{292} + \frac{7\sqrt{365}}{365} + \frac{\sqrt{30}}{15}.
\end{aligned} \tag{18}$$

The ve -degree sum-connectivity index can be determined by using the values from equations (9)–(16) in equation (6); we get the following:

$$\begin{aligned}
\chi_{ve}(\text{HDN}_1(p)) = & \left| \Xi_{ve,\gamma,\delta}^{3,5} \right| (\gamma + \delta)^{-(1/2)} + \left| \Xi_{ve,\gamma,\delta}^{3,7} \right| (\gamma + \delta)^{-(1/2)} + \left| \Xi_{ve,\gamma,\delta}^{3,12} \right| (\gamma + \delta)^{-(1/2)} + \left| \Xi_{ve,\gamma,\delta}^{5,7} \right| (\gamma + \delta)^{-(1/2)} \\
& + \left| \Xi_{ve,\gamma,\delta}^{5,12} \right| (\gamma + \delta)^{-(1/2)} + \left| \Xi_{ve,\gamma,\delta}^{7,7} \right| (\gamma + \delta)^{-(1/2)} + \left| \Xi_{ve,\gamma,\delta}^{7,12} \right| (\gamma + \delta)^{-(1/2)} + \left| \Xi_{ve,\gamma,\delta}^{12,12} \right| (\gamma + \delta)^{-(1/2)}.
\end{aligned} \tag{19}$$

After simplification, we obtain

$$\begin{aligned}
\chi_{ve}(\text{HDN}_1(p)) = & \left(\frac{3\sqrt{5}}{10} + \frac{3\sqrt{14}}{7} \right) p^2 + \left(\frac{3\sqrt{10}}{20} + \frac{12\sqrt{73}}{73} + \frac{6\sqrt{170}}{85} + \frac{\sqrt{78}}{13} - \frac{17\sqrt{5}}{10} + \frac{9\sqrt{29}}{29} \right. \\
& + \frac{3\sqrt{94}}{47} + \frac{4\sqrt{111}}{37} + \frac{12\sqrt{127}}{127} - \frac{15\sqrt{14}}{7} + \frac{3\sqrt{106}}{53} \left. \right) p - \frac{3\sqrt{10}}{5} + \frac{22\sqrt{14}}{7} + \frac{6\sqrt{19}}{19} + \frac{3\sqrt{26}}{13} \\
& - \frac{27\sqrt{29}}{29} + \frac{6\sqrt{23}}{23} + \frac{4\sqrt{17}}{17} + \frac{72\sqrt{5}}{25} - \frac{48\sqrt{127}}{127} - \frac{9\sqrt{106}}{53} + \frac{6\sqrt{118}}{59} + \frac{12\sqrt{77}}{77} + \frac{12\sqrt{71}}{71} \\
& + \frac{4\sqrt{69}}{23} + \frac{12\sqrt{97}}{97} - \frac{18\sqrt{170}}{85} - \frac{4\sqrt{78}}{13} - \frac{15\sqrt{94}}{47} + \frac{12\sqrt{109}}{109} + \frac{6\sqrt{163}}{163} + \frac{2\sqrt{105}}{35} - \frac{12\sqrt{111}}{37} - \frac{48\sqrt{73}}{73}.
\end{aligned} \tag{20}$$

Theorem 3. Let $\text{HDN}_1(p)$ be the third type of hex-derived network; then,

$$\begin{aligned}
(i) \quad ABC_{ve}(\text{HDN}_1(p)) = & ((\sqrt{178}/10) + (\sqrt{310}/5)) p^2 + \\
& (-17\sqrt{178}/30) + (3\sqrt{570}/20) + (12\sqrt{23}/47) + (3\sqrt{16895}/155) + (15\sqrt{47}/47) - \sqrt{310} + (3\sqrt{5}/5) + \\
& (3\sqrt{158}/40) + (6\sqrt{86762}/611) + (2\sqrt{21}/5) + \\
& (12\sqrt{27683}/1457) p - (24\sqrt{86762}/611) + \\
& (2\sqrt{8970}/65) + (12\sqrt{230826}/2263) - (9\sqrt{5}/5) + \\
& (9\sqrt{2}/4) + (12\sqrt{94}/47) + (\sqrt{41610}/73) + \\
& (\sqrt{30}/2) - (60\sqrt{23}/47) + (19\sqrt{310}/15) +
\end{aligned}$$

$$\begin{aligned}
& (8\sqrt{10585}/365) + (4\sqrt{11470}/155) + (\sqrt{2010}/15) - \\
& (3\sqrt{158}/10) - (6\sqrt{21}/5) - (48\sqrt{27683}/1457) + \\
& (4\sqrt{178}/5) + (\sqrt{123}/5) - (9\sqrt{570}/20) - \\
& (60\sqrt{47}/47) + (2\sqrt{7811}/73) + (\sqrt{117530}/365) + \\
& (3\sqrt{15038}/292) + (3\sqrt{55115}/365) - (9\sqrt{16895}/155)
\end{aligned}$$

Proof. The numerical descriptor of the ve -degree atom-bond connectivity index can be calculated by evaluating the values of edge partitions in equation (7):

$$\begin{aligned}
ABC_{ve}(\text{HDN}_1(p)) = & \left| \Xi_{ve,\gamma,\delta}^{3,5} \right| \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} + \left| \Xi_{ve,\gamma,\delta}^{3,7} \right| \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} + \left| \Xi_{ve,\gamma,\delta}^{3,12} \right| \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} + \left| \Xi_{ve,\gamma,\delta}^{5,7} \right| \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \\
& + \left| \Xi_{ve,\gamma,\delta}^{5,12} \right| \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} + \left| \Xi_{ve,\gamma,\delta}^{7,7} \right| \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} + \left| \Xi_{ve,\gamma,\delta}^{7,12} \right| \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} + \left| \Xi_{ve,\gamma,\delta}^{12,12} \right| \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2}.
\end{aligned} \tag{21}$$

Evaluating equations (9)–(16) and after mathematical calculations, we will have

$$\begin{aligned}
ABC_{ve}(\text{HDN}_1(p)) = & \left(\frac{\sqrt{178}}{10} + \frac{\sqrt{310}}{5} \right) p^2 + \left(-\frac{17\sqrt{178}}{30} + \frac{3\sqrt{570}}{20} + \frac{12\sqrt{23}}{47} + \frac{3\sqrt{16895}}{155} \right. \\
& + \frac{15\sqrt{47}}{47} - \sqrt{310} + \frac{3\sqrt{5}}{5} + \frac{3\sqrt{158}}{40} + \frac{6\sqrt{86762}}{611} + \frac{2\sqrt{21}}{5} + \frac{12\sqrt{27683}}{1457} \left. \right) p - \frac{24\sqrt{86762}}{611} + \frac{2\sqrt{8970}}{65} \\
& + \frac{12\sqrt{230826}}{2263} - \frac{9\sqrt{5}}{5} + \frac{9\sqrt{2}}{4} + \frac{12\sqrt{94}}{47} + \frac{\sqrt{41610}}{73} + \frac{\sqrt{30}}{2} - \frac{60\sqrt{23}}{47} + \frac{19\sqrt{310}}{15} + \frac{8\sqrt{10585}}{365} \\
& + \frac{4\sqrt{11470}}{155} + \frac{\sqrt{2010}}{15} - \frac{3\sqrt{158}}{10} - \frac{6\sqrt{21}}{5} - \frac{48\sqrt{27683}}{1457} + \frac{4\sqrt{178}}{5} + \frac{\sqrt{123}}{5} - \frac{9\sqrt{570}}{20} - \frac{60\sqrt{47}}{47} \\
& + \frac{2\sqrt{7811}}{73} + \frac{\sqrt{117530}}{365} + \frac{3\sqrt{15038}}{292} + \frac{3\sqrt{55115}}{365} - \frac{9\sqrt{16895}}{155}.
\end{aligned} \tag{22}$$

The ve -degree geometric-arithmetic index can be calculated by evaluating the methodology of edge partitions in (1):

$$\begin{aligned}
GA_{ve}(\text{HDN}_1(p))1 = & \left| \Xi_{ve_{\gamma,\delta}}^{3,5} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) + \left| \Xi_{ve_{\gamma,\delta}}^{3,7} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) + \left| \Xi_{ve_{\gamma,\delta}}^{3,12} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \\
& + \left| \Xi_{ve_{\gamma,\delta}}^{5,7} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) + \left| \Xi_{ve_{\gamma,\delta}}^{5,12} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) + \left| \Xi_{ve_{\gamma,\delta}}^{7,7} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) + \left| \Xi_{ve_{\gamma,\delta}}^{3,12} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \\
& + \left| \Xi_{ve_{\gamma,\delta}}^{12,12} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) + \left| \Xi_{ve_{\gamma,\delta}}^{5,12} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) + \left| \Xi_{ve_{\gamma,\delta}}^{7,7} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) + \left| \Xi_{ve_{\gamma,\delta}}^{7,12} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \\
& + \left| \Xi_{ve_{\gamma,\delta}}^{12,12} \right| \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right).
\end{aligned} \tag{23}$$

Evaluating equations (9)–(16) and after mathematical calculations, we will have

$$\begin{aligned}
GA_{ve}(\text{HDN}_1(p)) = & \left(\frac{36\sqrt{10}}{7} + 9 \right) p^2 + \left(-39 + \frac{24\sqrt{1222}}{73} + \frac{144\sqrt{2}}{17} + \frac{2\sqrt{1457}}{13} + \frac{216\sqrt{5}}{29} \right. \\
& + \frac{32\sqrt{155}}{37} + \frac{96\sqrt{235}}{127} - \frac{180\sqrt{10}}{7} + \frac{24\sqrt{130}}{53} \left. \right) p + \frac{3\sqrt{2263}}{13} - \frac{96\sqrt{1222}}{73} - \frac{432\sqrt{2}}{17} - \frac{8\sqrt{1457}}{13} \\
& - \frac{648\sqrt{5}}{29} - \frac{1158\sqrt{155}}{703} - \frac{6546\sqrt{235}}{2921} + \frac{2796\sqrt{10}}{77} - \frac{1296\sqrt{130}}{3763} + \frac{48\sqrt{3}}{7} + \frac{48\sqrt{438}}{97} + \frac{144\sqrt{73}}{109} \\
& + \frac{36\sqrt{730}}{163} + \frac{16\sqrt{146}}{35} + \frac{3724\sqrt{365}}{3009} + \frac{48\sqrt{30}}{23} + \frac{738}{25}.
\end{aligned} \tag{24}$$

□

4. Hex-Derived Network $HDN_2(p)$

Let $HDN_2(p)$ be the notation of the hex-derived network of the second type, and it is shown in Figure 3. This hex-derived network $HDN_2(p)$ contained $9p^2 - 15p + 7$ total number of vertices in which there are $6p$ vertices of degree 5, $6p^2 - 18p + 12$ vertices of degree 6, $6p - 12$ vertices of degree 7, and $3p^2 - 9p + 7$ vertices of degree 12.

The total number of edges of this network is $36p^2 - 72p + 36$, and they can be partitioned into ten

different subsets according to their degrees and associated with ve -degrees of both end vertices, which are shown in equations (17)–(26). Given below are some ve -degree-based indices, for example, $M_{\beta ve}^1$ index, M_{ve}^2 index, R_{ve} index, ABC_{ve} index, GA_{ve} index, H_{ve} index, and χ_{ve} index for the $HDN_2(p)$ network. Let $\Xi_{ve,\gamma,\delta}^{\alpha,\beta}$ be the edge partition of $HDN_2(p)$ according to its degrees (α, β) and ve -degrees (γ, δ) . It is defined as

$$\Xi_{ve,\gamma,\delta}^{\alpha,\beta} = \{\theta\vartheta \in \mathbb{E}(HDN_2(p)) : d(\theta) = \alpha, d(\vartheta) = \beta, \Psi(\theta) = \gamma, \Psi(\vartheta) = \delta\}. \quad (25)$$

And $|\Xi_{ve,\gamma,\delta}^{\alpha,\beta}|$ represents the number of edges in $\Xi_{ve,\gamma,\delta}^{\alpha,\beta}$.

$$|\Xi_{ve,\gamma,\delta}^{12,12}| = \begin{cases} 12 & \text{if } \gamma = 89, \delta = 97, \\ 6 & \text{if } \gamma = 89, \delta = 108, \\ 6p - 34 & \text{if } \gamma = 97, \delta = 97, \\ 12p - 36 & \text{if } \gamma = 97, \delta = 108, \\ 9p^2 - 51p + 72 & \text{if } \gamma = 108, \delta = 108, \end{cases} \quad (26)$$

$$|\Xi_{ve,\gamma,\delta}^{7,12}| = \begin{cases} 12 & \text{if } \gamma = 52, \delta = 89, \\ 12 & \text{if } \gamma = 52, \delta = 97, \\ 12p - 48 & \text{if } \gamma = 54, \delta = 97, \end{cases} \quad (27)$$

$$|\Xi_{ve,\gamma,\delta}^{7,7}| = \begin{cases} 12 & \text{if } \gamma = 52, \delta = 54, \\ 12p - 30 & \text{if } \gamma = 54, \delta = 54, \end{cases} \quad (28)$$

$$|\Xi_{ve,\gamma,\delta}^{6,12}| = \begin{cases} 12 & \text{if } \gamma = 47, \delta = 89, \\ 12p - 36 & \text{if } \gamma = 47, \delta = 97, \\ 12 & \text{if } \gamma = 54, \delta = 89, \\ 18p - 54 & \text{if } \gamma = 54, \delta = 97, \\ 18p^2 - 90p + 114 & \text{if } \gamma = 54, \delta = 108, \end{cases} \quad (29)$$

$$|\Xi_{ve,\gamma,\delta}^{6,7}| = \begin{cases} 12 & \text{if } \gamma = 47, \delta = 52, \\ 6p - 24 & \text{if } \gamma = 47, \delta = 54, \end{cases} \quad (30)$$

$$|\Xi_{ve,\gamma,\delta}^{6,6}| = \begin{cases} 6p - 12 & \text{if } \gamma = 47, \delta = 54, \\ 9p^2 - 39p + 42 & \text{if } \gamma = 54, \delta = 54, \end{cases} \quad (31)$$

$$|\Xi_{ve,\gamma,\delta}^{5,12}| = \begin{cases} 6 & \text{if } \gamma = 36, \delta = 89, \\ 12 & \text{if } \gamma = 35, \delta = 89, \\ 6p - 18 & \text{if } \gamma = 38, \delta = 97, \end{cases} \quad (32)$$

$$|\Xi_{ve,\gamma,\delta}^{5,7}| = \begin{cases} 12 & \text{if } \gamma = 36, \delta = 52, \\ 12 & \text{if } \gamma = 35, \delta = 52, \\ 12p - 48 & \text{if } \gamma = 38, \delta = 54, \\ 12 & \text{if } \gamma = 38, \delta = 52, \end{cases} \quad (33)$$

$$|\Xi_{ve,\gamma,\delta}^{5,6}| = \begin{cases} 12 & \text{if } \gamma = 35, \delta = 47, \\ 12p - 36 & \text{if } \gamma = 38, \delta = 47, \end{cases} \quad (34)$$

$$|\Xi_{ve,\gamma,\delta}^{5,5}| = \begin{cases} 6 & \text{if } \gamma = 35, \delta = 35, \\ 12 & \text{if } \gamma = 35, \delta = 36. \end{cases} \quad (35)$$

Theorem 4. Let $HDN_2(p)$ be the second type of the hex-derived network; then,

- (i) $M_{\beta ve}^1(HDN_2(p)) = 5832p^2 - 15132p + 9528$
- (ii) $M_{ve}^2(HDN_2(p)) = 236196p^2 - 723330p + 532842$
- (iii) $H_{ve}(HDN_2(p)) = (17p^2/36) - (2483413922281p/4268774116260) + (162959160788410914818964603487/747847045658086922387267678250)$

Proof. Let $HDN_2(p)$ be the second type of the hex-derived network which is shown in Figure 3. The hex-derived network $HDN_2(p)$ has $9p^2 - 15p + 7$ vertices. There are $36p^2 - 72p + 36$ number of edges of $HDN_2(p)$ which are partitioned into ten partitions based on their degrees and corresponding ve -degrees of end vertices given in equations (17)–(26).

The first ve -degree Zagreb β index can be calculated as

$$M_{\beta ve}^1(HDN_2(p)) = \left| \Xi_{ve,\gamma,\delta}^{12,12} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{7,12} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{7,7} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{6,12} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{6,7} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{6,6} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{5,12} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{5,7} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{5,6} \right| (\gamma + \delta) + \left| \Xi_{ve,\gamma,\delta}^{5,5} \right| (\gamma + \delta). \tag{36}$$

Evaluating equations (17)–(22) and after simplifications, we will have the final result as follows:

$$M_{\beta ve}^1(HDN_2(p)) = 5832p^2 - 15132p + 9528. \tag{37}$$

The second ve -degree Zagreb index can be calculated as

$$M_{ve}^2(HDN_2(p)) = \left| \Xi_{ve,\gamma,\delta}^{12,12} \right| (\gamma \times \delta) + \left| \Xi_{ve,\gamma,\delta}^{7,12} \right| (\gamma \times \delta) + \left| \Xi_{ve,\gamma,\delta}^{7,7} \right| (\gamma \times \delta) + \left| \Xi_{ve,\gamma,\delta}^{6,12} \right| (\gamma \times \delta) + \left| \Xi_{ve,\gamma,\delta}^{6,7} \right| (\gamma \times \delta) + \left| \Xi_{ve,\gamma,\delta}^{6,6} \right| (\gamma \times \delta) + \left| \Xi_{ve,\gamma,\delta}^{5,12} \right| (\gamma \times \delta) + \left| \Xi_{ve,\gamma,\delta}^{5,7} \right| (\gamma \times \delta) + \left| \Xi_{ve,\gamma,\delta}^{5,6} \right| (\gamma \times \delta) + \left| \Xi_{ve,\gamma,\delta}^{5,5} \right| (\gamma \times \delta). \tag{38}$$

Evaluating equations (17)–(22) and after simplifications, we will have the final result as follows:

$$M_{ve}^2(HDN_2(p)) = 236196p^2 - 723330p + 532842. \tag{39}$$

The ve -degree harmonic index can be determined as

$$H_{ve}(HDN_2(p)) = \left| \Xi_{ve,\gamma,\delta}^{12,12} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\gamma,\delta}^{7,12} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\gamma,\delta}^{7,7} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\gamma,\delta}^{6,12} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\gamma,\delta}^{6,7} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\gamma,\delta}^{6,6} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\gamma,\delta}^{5,12} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\gamma,\delta}^{5,7} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\gamma,\delta}^{5,6} \right| \left(\frac{2}{\gamma + \delta} \right) + \left| \Xi_{ve,\gamma,\delta}^{5,5} \right| \left(\frac{2}{\gamma + \delta} \right). \tag{40}$$

Evaluating equations (17)–(22) and after some calculations, we have the final result as follows:

$$H_{ve}(HDN_2(p)) = \frac{17p^2}{36} - \frac{2483413922281p}{4268774116260} + \frac{162959160788410914818964603487}{747847045658086922387267678250}. \tag{41}$$

Theorem 5. Let $HDN_2(p)$ be the second type of the hex-derived network; then,

$$(i) R_{ve}(HDN_2(p)) = ((1/4) + (\sqrt{2}/6))p^2 + ((12\sqrt{4559}/4559) + (2\sqrt{282}/141) - (5\sqrt{2}/6) + (3\sqrt{3686}/1843) + (2\sqrt{291}/291) + (5\sqrt{582}/291) + (2\sqrt{57}/57) + (6\sqrt{1786}/893) - (1189/1164))p + (2\sqrt{534}/267) - (2\sqrt{282}/47) + (12\sqrt{4183}/4183) + (12\sqrt{8633}/8633) + (12\sqrt{3115}/3115) - (8\sqrt{57}/57) + (3\sqrt{494}/247) - (9\sqrt{3686}/1843) - (36\sqrt{4559}/4559) + (24838/30555) + (6\sqrt{611}/611) + (\sqrt{267}$$

$$/267) + (6\sqrt{1157}/1157) + (6\sqrt{455}/455) + (\sqrt{13}/13) + (12\sqrt{1645}/1645) - (18\sqrt{1786}/893) + (2\sqrt{35}/35) + (19\sqrt{2}/18) + (\sqrt{89}/89) - (2\sqrt{291}/97) + (6\sqrt{1261}/1261) + (\sqrt{78}/39) - (17\sqrt{582}/291)$$

$$(ii) \chi_{ve}(HDN_2(p)) = ((\sqrt{3}/2) + \sqrt{2} + (\sqrt{6}/4))p^2 + (1 + (30\sqrt{151}/151) - (11\sqrt{3}/6) - 5\sqrt{2} + (12\sqrt{101}/101) - (17\sqrt{6}/12) + (6\sqrt{23}/23) + (12\sqrt{85}/85) + (2\sqrt{15}/15) + (12\sqrt{205}/205) + (3\sqrt{194}/97))p + (4\sqrt{11}/11) + (3\sqrt{34}/17) + 2\sqrt{6} - (24\sqrt{23}/23) - 3 + (6\sqrt{31}/31) + (3/11\sqrt{22}) + (2\sqrt{10}/5) - (2\sqrt{15}/5) - (36\sqrt{205}/205) + (12\sqrt{149}/149) + (6\sqrt{106}/53) + (12\sqrt{71}/71) - (12\sqrt{194}/97) - (102\sqrt{151}/151) + (12\sqrt{143}/143) - (36\sqrt{101}/101) + (6\sqrt{197}/197) + (4\sqrt{141}/47) + (2\sqrt{186}/31) + (4\sqrt{87}/29) + (6\sqrt{82}/41) - (36\sqrt{85}/85) + (3\sqrt{70}/35) + (2\sqrt{3}/3) + (6\sqrt{5}/25) + (19\sqrt{2}/3)$$

Proof. The ve -degree Randic index is measured by evaluating equations (17)–(26) in the following:

$$R_{ve}(HDN_2(p)) = \left| \Xi_{ve,\gamma,\delta}^{12,12} \right| (\gamma \times \delta)^{-1/2} + \left| \Xi_{ve,\gamma,\delta}^{7,12} \right| (\gamma \times \delta)^{-1/2} + \left| \Xi_{ve,\gamma,\delta}^{7,7} \right| (\gamma \times \delta)^{-1/2} + \left| \Xi_{ve,\gamma,\delta}^{6,12} \right| (\gamma \times \delta)^{-1/2} + \left| \Xi_{ve,\gamma,\delta}^{6,7} \right| (\gamma \times \delta)^{-1/2} + \left| \Xi_{ve,\gamma,\delta}^{6,6} \right| (\gamma \times \delta)^{-1/2} + \left| \Xi_{ve,\gamma,\delta}^{5,12} \right| (\gamma \times \delta)^{-1/2} + \left| \Xi_{ve,\gamma,\delta}^{5,7} \right| (\gamma \times \delta)^{-1/2} + \left| \Xi_{ve,\gamma,\delta}^{5,6} \right| (\gamma \times \delta)^{-1/2} + \left| \Xi_{ve,\gamma,\delta}^{5,5} \right| (\gamma \times \delta)^{-1/2}. \tag{42}$$

After simplification, we obtain

$$\begin{aligned}
 R_{ve}(\text{HDN}_2(p)) = & \left(\frac{1}{4} + \frac{\sqrt{2}}{6}\right)p^2 + \left(\frac{12\sqrt{4559}}{4559} + \frac{2\sqrt{282}}{141} - \frac{5\sqrt{2}}{6} + \frac{3\sqrt{3686}}{1843} + \frac{2\sqrt{291}}{291} + \frac{5\sqrt{582}}{291}\right. \\
 & + \left.\frac{2\sqrt{57}}{57} + \frac{6\sqrt{1786}}{893} - \frac{1189}{1164}\right)p + \frac{2\sqrt{534}}{267} - \frac{2\sqrt{282}}{47} + \frac{12\sqrt{4183}}{4183} + \frac{12\sqrt{8633}}{8633} + \frac{12\sqrt{3115}}{3115} \\
 & - \frac{8\sqrt{57}}{57} + \frac{3\sqrt{494}}{247} - \frac{9\sqrt{3686}}{1843} - \frac{36\sqrt{4559}}{4559} + \frac{24838}{30555} + \frac{6\sqrt{611}}{611} + \frac{\sqrt{267}}{267} + \frac{6\sqrt{1157}}{1157} + \frac{6\sqrt{455}}{455} \\
 & + \frac{\sqrt{13}}{13} + \frac{12\sqrt{1645}}{1645} - \frac{18\sqrt{1786}}{893} + \frac{2\sqrt{35}}{35} + \frac{19\sqrt{2}}{18} + \frac{\sqrt{89}}{89} - \frac{2\sqrt{291}}{97} + \frac{6\sqrt{1261}}{1261} + \frac{\sqrt{78}}{39} - \frac{17\sqrt{582}}{291}.
 \end{aligned} \tag{43}$$

The ve -degree sum-connectivity index is measured by evaluating equations (17)–(26) in the given following formula:

$$\begin{aligned}
 \chi_{ve}(\text{HDN}_2(p)) = & \left|\Xi_{ve,\gamma,\delta}^{12,12}\right|(\gamma + \delta)^{-(1/2)} + \left|\Xi_{ve,\gamma,\delta}^{7,12}\right|(\gamma + \delta)^{-(1/2)} + \left|\Xi_{ve,\gamma,\delta}^{7,7}\right|(\gamma + \delta)^{-(1/2)} + \left|\Xi_{ve,\gamma,\delta}^{6,12}\right|(\gamma + \delta)^{-(1/2)} \\
 & + \left|\Xi_{ve,\gamma,\delta}^{6,7}\right|(\gamma + \delta)^{-(1/2)} + \left|\Xi_{ve,\gamma,\delta}^{6,6}\right|(\gamma + \delta)^{-(1/2)} + \left|\Xi_{ve,\gamma,\delta}^{5,12}\right|(\gamma + \delta)^{-(1/2)} + \left|\Xi_{ve,\gamma,\delta}^{5,7}\right|(\gamma + \delta)^{-(1/2)} + \left|\Xi_{ve,\gamma,\delta}^{5,6}\right|(\gamma + \delta)^{-(1/2)} \\
 & + \left|\Xi_{ve,\gamma,\delta}^{5,5}\right|(\gamma + \delta)^{-\frac{1}{2}}.
 \end{aligned} \tag{44}$$

After simplification, we obtain

$$\begin{aligned}
 \chi_{ve}(\text{HDN}_2(p)) = & \left(\frac{\sqrt{3}}{2} + \sqrt{2} + \frac{\sqrt{6}}{4}\right)p^2 + \left(1 + \frac{30\sqrt{151}}{151} - \frac{11\sqrt{3}}{6} - 5\sqrt{2} + \frac{12\sqrt{101}}{101} - \frac{17\sqrt{6}}{12}\right. \\
 & + \left.\frac{6\sqrt{23}}{23} + \frac{12\sqrt{85}}{85} + \frac{2\sqrt{15}}{15} + \frac{12\sqrt{205}}{205} + \frac{3\sqrt{194}}{97}\right)p + \frac{4\sqrt{11}}{11} + \frac{3\sqrt{34}}{17} + 2\sqrt{6} - \frac{24\sqrt{23}}{23} - 3 \\
 & + \frac{6\sqrt{31}}{31} + \frac{3}{11}\sqrt{22} + \frac{2\sqrt{10}}{5} - \frac{2\sqrt{15}}{5} - \frac{36\sqrt{205}}{205} + \frac{12\sqrt{149}}{149} + \frac{6\sqrt{106}}{53} + \frac{12\sqrt{71}}{71} - \frac{12\sqrt{194}}{97} \\
 & - \frac{102\sqrt{151}}{151} + \frac{12\sqrt{143}}{143} - \frac{36\sqrt{101}}{101} + \frac{6\sqrt{197}}{197} + \frac{4\sqrt{141}}{47} + \frac{2\sqrt{186}}{31} + \frac{4\sqrt{87}}{29} + \frac{6\sqrt{82}}{41} - \frac{36\sqrt{85}}{85} \\
 & + \frac{3\sqrt{70}}{35} + \frac{2\sqrt{3}}{3} + \frac{6\sqrt{5}}{25} + \frac{19\sqrt{2}}{3}.
 \end{aligned} \tag{45}$$

Theorem 6. Let $\text{HDN}_2(p)$ be the second type of the hex-derived network; then,

(i) $ABC_{ve}(\text{HDN}_2(p)) = ((\sqrt{214}/12) + (4\sqrt{5}/3) + (\sqrt{106}/6))p^2 + ((2\sqrt{3102}/47) - (17\sqrt{214}/36) - (11\sqrt{106}/18) + (2\sqrt{570}/19) - (20\sqrt{5}/3) + (6\sqrt{148238}/893) + (3\sqrt{1358}/97) + (12\sqrt{647378}/4559) + (2\sqrt{59073}/291) + (48\sqrt{3}/97) + (5\sqrt{86718}/291))p - (6\sqrt{3102}/47) + (12\sqrt{560522}/4183) - (8\sqrt{570}/19) + (12\sqrt{380030}/3115) + (12\sqrt{2717}/247) - (18$

□

$\sqrt{148238}/893) - (9\sqrt{1358}/97) - (36\sqrt{647378}/4559) + (12\sqrt{17}/35) + (2\sqrt{106}/9) - (188\sqrt{3}/291) + (76\sqrt{5}/9) - (2\sqrt{59073}/97) + (42\sqrt{3783}/1261) + (2\sqrt{2415}/35) - (17\sqrt{86718}/291) + (\sqrt{1118}/13) + (\sqrt{10947}/89) + (6\sqrt{160823}/1157) + (24\sqrt{397118}/8633) + (6\sqrt{1547}/91) + (48\sqrt{329}/329) + (2\sqrt{8366}/89) + (6\sqrt{59267}/611) + (2\sqrt{214}/3) + (\sqrt{5785}/89)$

(ii) $GA_{ve}(\text{HDN}_2(p)) = (18 + 12\sqrt{2})p^2 + (-78 + (72\sqrt{282}/101) - 60\sqrt{2} + (4\sqrt{3686}/45) + (\sqrt{4559}/6) +$

$$\begin{aligned} & (144\sqrt{291}/205) + (180\sqrt{582}/151) + (36\sqrt{57}/23) + \\ & (24\sqrt{1786}/85)p + 76\sqrt{2} + (72\sqrt{89}/125) - \\ & (432\sqrt{291}/205) + (48\sqrt{1261}/149) + (72\sqrt{78}/53) - \\ & (612\sqrt{582}/151) + 66 + (72\sqrt{534}/143) - \\ & (216\sqrt{282}/101) + (3\sqrt{4183}/17) + (4\sqrt{8633}/31) + \\ & (6\sqrt{3115}/31) - (144\sqrt{57}/23) + (8\sqrt{494}/15) - \\ & (4\sqrt{3686}/15) - (\sqrt{4559}/2) + (16\sqrt{611}/33) + \end{aligned}$$

$$\begin{aligned} & (72\sqrt{267}/197) + (16\sqrt{1157}/47) + (16\sqrt{455}/29) + \\ & (36\sqrt{13}/11) + (12\sqrt{1645}/41) - (72\sqrt{1786}/85) + \\ & (144\sqrt{35}/71) \end{aligned}$$

Proof. The ve -degree atom-bond connectivity index is measured as

$$\begin{aligned} ABC_{ve}(\text{HDN}_2(p)) = & \left| \Xi_{ve,\gamma,\delta}^{12,12} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right| + \left| \Xi_{ve,\gamma,\delta}^{7,12} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right| + \left| \Xi_{ve,\gamma,\delta}^{7,7} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right| \\ & + \left| \Xi_{ve,\gamma,\delta}^{6,12} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right| + \left| \Xi_{ve,\gamma,\delta}^{6,7} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right| + \left| \Xi_{ve,\gamma,\delta}^{6,6} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right| + \left| \Xi_{ve,\gamma,\delta}^{5,12} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right| \\ & + \left| \Xi_{ve,\gamma,\delta}^{5,7} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right| + \left| \Xi_{ve,\gamma,\delta}^{5,6} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right| + \left| \Xi_{ve,\gamma,\delta}^{5,5} \left(\frac{\gamma + \delta - 2}{\gamma \times \delta} \right)^{1/2} \right|. \end{aligned} \quad (46)$$

By putting the values from equations (17)–(26) and after simplification, we obtain

$$\begin{aligned} ABC_{ve}(\text{HDN}_2(p)) = & \left(\frac{\sqrt{214}}{12} + \frac{4\sqrt{5}}{3} + \frac{\sqrt{106}}{6} \right) p^2 + \left(\frac{2\sqrt{3102}}{47} - \frac{17\sqrt{214}}{36} - \frac{11\sqrt{106}}{18} \right. \\ & + \frac{2\sqrt{570}}{19} - \frac{20\sqrt{5}}{3} + \frac{6\sqrt{148238}}{893} + \frac{3\sqrt{1358}}{97} + \frac{12\sqrt{647378}}{4559} + \frac{2\sqrt{59073}}{291} + \frac{48\sqrt{3}}{97} + \frac{5\sqrt{86718}}{291} \left. \right) \\ & p - \frac{6\sqrt{3102}}{47} + \frac{12\sqrt{560522}}{4183} - \frac{8\sqrt{570}}{19} + \frac{12\sqrt{380030}}{3115} + \frac{12\sqrt{2717}}{247} - \frac{18\sqrt{148238}}{893} - \frac{9\sqrt{1358}}{97} \\ & - \frac{36\sqrt{647378}}{4559} + \frac{12\sqrt{17}}{35} + \frac{2\sqrt{106}}{9} - \frac{188\sqrt{3}}{291} + \frac{76\sqrt{5}}{9} - \frac{2\sqrt{59073}}{97} + \frac{42\sqrt{3783}}{1261} + \frac{2\sqrt{2415}}{35} \\ & - \frac{17\sqrt{86718}}{291} + \frac{\sqrt{1118}}{13} + \frac{\sqrt{10947}}{89} + \frac{6\sqrt{160823}}{1157} + \frac{24\sqrt{397118}}{8633} + \frac{6\sqrt{1547}}{91} + \frac{48\sqrt{329}}{329} \\ & + \frac{2\sqrt{8366}}{89} + \frac{6\sqrt{59267}}{611} + \frac{2\sqrt{214}}{3} + \frac{\sqrt{5785}}{89}. \end{aligned} \quad (47)$$

The ve -degree geometric-arithmetic index is measured as

$$\begin{aligned} GA_{ve}(\text{HDN}_2(p)) = & \left| \Xi_{ve,\gamma,\delta}^{12,12} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right| + \left| \Xi_{ve,\gamma,\delta}^{7,12} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right| + \left| \Xi_{ve,\gamma,\delta}^{7,7} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right| + \left| \Xi_{ve,\gamma,\delta}^{6,12} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right| \\ & + \left| \Xi_{ve,\gamma,\delta}^{6,7} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right| + \left| \Xi_{ve,\gamma,\delta}^{6,6} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right| + \left| \Xi_{ve,\gamma,\delta}^{5,12} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right| + \left| \Xi_{ve,\gamma,\delta}^{5,7} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right| \\ & + \left| \Xi_{ve,\gamma,\delta}^{5,6} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right| + \left| \Xi_{ve,\gamma,\delta}^{5,5} \left(\frac{2(\gamma \times \delta)^{1/2}}{\gamma + \delta} \right) \right|. \end{aligned} \quad (48)$$

By putting the values from equations (17)–(22) and after simplification, we obtain

TABLE 1: Numerical comparison of $M_{\beta ve}^1$, M_{ve}^2 , H_{ve} , R_{ve} , χ_{ve} , ABC_{ve} , and GA_{ve} for the first type of hex-derived network $HDN_1(p)$.

| p | $M_{\beta ve}^1$ | M_{ve}^2 | H_{ve} | R_{ve} | χ_{ve} | ABC_{ve} | GA_{ve} |
|-----|------------------|------------|----------|----------|-------------|------------|-----------|
| 5 | 53316 | 1558914 | 8.0260 | 8.5241 | 41.752 | 87.893 | 418.85 |
| 6 | 86052 | 2598294 | 11.929 | 12.703 | 63.506 | 134.04 | 649.96 |
| 7 | 126564 | 3900114 | 16.603 | 17.715 | 89.809 | 189.87 | 931.59 |
| 8 | 174852 | 5464374 | 22.049 | 23.559 | 120.67 | 255.44 | 1263.8 |
| 9 | 230916 | 7291074 | 28.266 | 30.233 | 156.08 | 330.69 | 1646.4 |
| 10 | 294756 | 9380214 | 35.254 | 37.743 | 196.02 | 415.68 | 2079.7 |
| 11 | 366372 | 11731794 | 43.014 | 46.083 | 240.52 | 510.37 | 2563.2 |
| 12 | 445764 | 14345814 | 51.546 | 55.259 | 289.56 | 614.76 | 3097.5 |
| 13 | 532932 | 17222274 | 60.848 | 65.263 | 343.16 | 728.87 | 3682.5 |
| 14 | 627876 | 20361174 | 70.923 | 76.103 | 401.30 | 852.69 | 4317.7 |

TABLE 2: Numerical comparison of $M_{\beta ve}^1$, M_{ve}^2 , H_{ve} , R_{ve} , χ_{ve} , ABC_{ve} , and GA_{ve} for the second type of hex-derived network $HDN_2(p)$.

| p | $M_{\beta ve}^1$ | M_{ve}^2 | H_{ve} | R_{ve} | χ_{ve} | ABC_{ve} | GA_{ve} |
|-----|------------------|------------|----------|----------|-------------|------------|-----------|
| 5 | 79668 | 2821092 | 9.1146 | 9.3555 | 50.648 | 103.29 | 559.92 |
| 6 | 128688 | 4695918 | 13.727 | 14.093 | 77.722 | 158.57 | 874.85 |
| 7 | 189372 | 7043136 | 19.284 | 19.803 | 110.59 | 225.70 | 1259.7 |
| 8 | 261720 | 9862746 | 25.786 | 26.484 | 149.24 | 304.64 | 1714.8 |
| 9 | 345732 | 13154748 | 33.232 | 34.134 | 193.67 | 395.46 | 2239.3 |
| 10 | 441408 | 16919142 | 41.622 | 42.760 | 243.88 | 498.07 | 2834.1 |
| 11 | 548748 | 21155928 | 50.957 | 52.356 | 299.89 | 612.51 | 3498.8 |
| 12 | 667752 | 25865106 | 61.237 | 62.921 | 361.68 | 738.81 | 4233.2 |
| 13 | 798420 | 31046676 | 72.461 | 74.458 | 429.25 | 876.91 | 5037.7 |
| 14 | 940752 | 36700638 | 84.629 | 86.968 | 502.60 | 1027.0 | 5912.3 |

$$\begin{aligned}
GA_{ve}(HDN_2(p)) = & (18 + 12\sqrt{2})p^2 + \left(-78 + \frac{72\sqrt{282}}{101} - 60\sqrt{2} + \frac{4\sqrt{3686}}{45} + \frac{\sqrt{4559}}{6} \right. \\
& + \frac{144\sqrt{291}}{205} + \frac{180\sqrt{582}}{151} + \frac{36\sqrt{57}}{23} + \frac{24\sqrt{1786}}{85} \left. \right) p + 76\sqrt{2} + \frac{72\sqrt{89}}{125} - \frac{432\sqrt{291}}{205} + \frac{48\sqrt{1261}}{149} \\
& + \frac{72\sqrt{78}}{53} - \frac{612\sqrt{582}}{151} + 66 + \frac{72\sqrt{534}}{143} - \frac{216\sqrt{282}}{101} + \frac{3\sqrt{4183}}{17} + \frac{4\sqrt{8633}}{31} + \frac{6\sqrt{3115}}{31} \\
& - \frac{144\sqrt{57}}{23} + \frac{8\sqrt{494}}{15} - \frac{4\sqrt{3686}}{15} - \frac{\sqrt{4559}}{2} + \frac{16\sqrt{611}}{33} + \frac{72\sqrt{267}}{197} + \frac{16\sqrt{1157}}{47} + \frac{16\sqrt{455}}{29} \\
& + \frac{36\sqrt{13}}{11} + \frac{12\sqrt{1645}}{41} - \frac{72\sqrt{1786}}{85} + \frac{144\sqrt{35}}{71}.
\end{aligned} \tag{49}$$

5. Conclusion

In this paper, certain ve -degree-based topological indices, namely, first ve -degree Zagreb $\beta(M_{\beta ve}^1)$ index, second ve -degree Zagreb (M_{ve}^2) index, ve -degree harmonic (H_{ve}) index, ve -degree Randic (R_{ve}) index, ve -degree sum-connectivity (χ_{ve}) index, ve -degree atom-bond connectivity (ABC_{ve}) index, and ve -degree geometric-arithmetic (GA_{ve}) index, for the first type of hex-derived network $HDN_1(p)$ and second type of hex-derived network $HDN_2(p)$ are studied. We calculated exact formulas of the aforementioned ve -degree-based topological indices for these derived networks. These results provide a basis to comprehend the deep network topology of these vital networks. The numerical comparison of the first type of hex-derived network

$HDN_1(p)$ and second type of hex-derived network $HDN_2(p)$ is shown in Tables 1 and 2. □

Data Availability

No data were associated with this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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