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Research Article

Distributed Continuous-Time Containment Control of Heterogeneous Multiagent Systems with Nonconvex Control Input Constraints

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This paper focuses on studying containment control problem with switching communication graphs of continuous-time heterogeneous multiagent systems where the control inputs are constrained in a nonconvex set. A nonlinear projection algorithm is proposed to address the problem. We discuss the stability and containment control of the system with switching topologies and nonconvex control input constraints under three different conditions. It is shown that all agents converge to the convex hull of the given leaders ultimately while staying in the nonconvex set under the premise that at least one directed path from leaders to the agents exists in each bounded time interval. Finally, the validity of the results obtained in this paper is verified by simulation.

1. Introduction

With the in-depth research of autonomous control of multiagent systems in recent years, many cooperative control problems of multiagent systems have become the focus of many fields, such as control theory, biology, robotic systems, and spacecraft systems [1–5]. The study and exploration of multiagent systems provided a unified framework and theoretical basis for various practical problems such as unmanned aerial vehicles, formation aircrafts, multiple robots, and other practical applications. A Lyapunov-based approach was proposed to address the consensus problem in [6]. Lin et al. [7] emphasized on the consensus control problem in consideration of nonconvex control input constraints. The consensus control problem of multiagent systems with time delays was studied in consideration of external interference in [8]. And the work [9] was centered on the containment control problem. Lin et al. [10] studied distributed optimization problems for continuous-time and discrete-time multiagent systems with different constraints.

As an important branch of control theory, there were many articles focusing on the study of containment control [11–20]. Ji et al. and Li et al. [11, 13] solved the containment control problem with fixed topology. The output formation-containment problem of heterogeneous system was investigated in [14, 15]. For double-integrator multiagent systems, containment control problem with fixed communication topology and position measurements was addressed in [16, 17]. Two distributed algorithms were proposed for containment control in the case of only using absolute position measurements and relative position measurements, respectively. However, the ‘sign’ function was employed in [17], which may lead to the chattering phenomenon. Cheng et al. [18] avoided employing the ‘sign’ function, and both disturbance and measurement noise were also taken into consideration. Notarstefano et al. and Zhang et al. [19, 20] investigated containment control with switching topologies. Cao et al. [12] studied the solution of containment control in consideration of switching topologies and fixed simultaneously.

In many practical systems, the control inputs were generally constrained in a convex or a nonconvex hull, while most of existing works studied the multiagent problem without considering the constraint of control inputs. The authors of [21–24] studied the consensus problem with position constraints on the basis of the property of stochastic matrices. Nevertheless, these approaches could not be applied to the realization of containment control with the nonconvex control inputs' constraints. The containment control problem with input saturation of the second-order agent system was studied in [25, 26]. However, control input of each agent was supposed to be in a hypercube. In reality, on account of the physical limitation, the control inputs were often constrained in a convex or a nonconvex region. For instance, the maximum driving forces of quadrotors, which formed a nonconvex region, were in the direction of the diagonal axis. A multiagent system model and a projection consensus algorithm were introduced in [23], and it placed emphasis on the effects of control input constraints. The algorithm was executed locally by each agent and its relationship with the alternate projection method was discussed in [23]. Yang et al. and Lin et al. [27, 28] took nonconvex velocity and control inputs' constraints into consideration, but what they emphasized on was the consensus control problem which meant all the followers had to reach a consensus.

In [29], all agents were assumed to be in the form of the second-order dynamics. In practical applications, the multiagent systems might contain different kinds of agents. Our focus of this paper is to study the containment control for heterogeneous multiagent systems with nonconvex input constraints. Since the agents in this paper have different dynamics, the analysis for the case with all identical agents in [29] cannot be applied to this paper directly. In this study, we expand the results of [29] and mainly focus on the containment control problem of continuous-time heterogeneous multiagent systems, given the nonconvex control input constraints and switching communication graphs. Li et al. [30] studied the containment control of heterogeneous multiagent systems. However, they did not consider the constraints of control inputs and heterogeneous multiagent systems at the same time. To analyze the stability and convergence of the system, a nonlinear projection algorithm is proposed to address the problem. Then, a model transformation is introduced and estimates the distance from the followers to the nonconvex hull by using the Lyapunov function we construct. We prove that the distance decreases and ultimately all followers converge to the nonconvex hull.

1.1. Notations. Assume that $\mathbb{R}^{m\times n}$ represents the set of $m\times n$-dimensional real matrix. x^T is the transposing matrix of x. $\|x\|$ is the Euclidean norm of x. $W_\chi(x)$ denotes the projection of a vector x onto a closed convex set χ, i.e., $W_\chi(x)=\arg\min_{\overline{x}\in\chi}\|x-\overline{x}\|$.

2. Preliminaries

2.1. Graph Theory. $G(V,E,A)$ is a directed weighted graph representing the communication graphs among the agents, where $V=\{v_1,v_2,\dots v_n\}$ is the set of node representing the agents and $E\subseteq V\times V$ is the set of directed edges representing the communication between the agents. $A=\{a_{ij}\}\in R^{n\times n}$ is a weighted adjacent matrix. Laplacian is a very important matrix in graph theory, which is defined as $L=D-A$, where

$$D=\begin{cases}\sum_{j=1,i\neq j}^{n}a_{ij}, & i=j,\\ 0, & i\neq j.\end{cases}\tag{1}$$

Then, the Laplacian matrix can be expressed as follows:

$$L=\left(l_{ij}\right)_{n\times n},\quad l_{ij}=\begin{cases}\sum_{j=1,i\neq j}^{n}a_{ij}, & i=j,\\ -a_{ij}, & i\neq j.\end{cases}\tag{2}$$

For heterogeneous multiagent systems consisted of n first-order and m second-order agents, the adjacent agents of second-order agent i_1 can be denoted as $N_{i_1}=N_{i_1}^s\cup N_{i_1}^f$. Similarly, the adjacent agents of first-order agent i_2 can be denoted as $N_{i_2}=N_{i_2}^s\cup N_{i_2}^f$.$s$ and f represent the second-order and the first-order agents, respectively. Partition the matrix A and D as

$$A=\begin{Bmatrix}A_s & A_{sf}\\ A_{fs} & A_f\end{Bmatrix},\quad D=\begin{Bmatrix}D_s+D_{sf} & 0\\ 0 & D_f+D_{fs}\end{Bmatrix},\tag{3}$$

where $D_{sf}=\mathrm{diag}(\sum_{j\in N_{i_1}^f}a_{i_1j}, i_1=1,2\dots m)$ and A_{sf} represents the adjacent relationship of the second-order and the first-order agents. $D_{fs}=\mathrm{diag}(\sum_{j\in N_{i_2}^s}a_{i_2j}, i_2=1,2\dots n)$ and A_{fs} represents the adjacent relationship of the first-order and the second-order agents. Then, the Laplacian matrix can be expressed as follows:

$$L=D-A=\begin{Bmatrix}\overline{L}_s & -A_{sf}\\ -A_{fs} & \overline{L}_f\end{Bmatrix},\tag{4}$$

where $\overline{L}_s=D_{sf}+L_s$, $\overline{L}_f=D_{fs}+L_f$, and L_s and L_f are the Laplacian matrix of second-order agents and first-order agents, respectively.

2.2. Nonconvex Constraints. The control inputs are generally subject to nonconvex constraints in many practical cases. Hence, we have the following assumption.

Assumption 1 (see [7]). $E\in\mathbb{R}^r$ is a nonempty bounded closed set, $0\in E_i$ and $\max_{x\in E_i}\|Q_{E_i}(x)\|=\overline{\eta}$, $\min_{x\notin E_i}\|Q_{E_i}(x)\|=\underline{\eta}>0$. Q_{E_i} is a nonconvex constraints' operator and is defined as follows:

$$Q_{E_i}(x)=\begin{cases}\dfrac{x}{\|x\|}\max_{0\le\varepsilon\le\|x\|}\left\{\varepsilon\,|\,\dfrac{\delta\varepsilon x}{\|x\|}\in E_i,\forall 0\le\delta\le 1\right\}, & x\neq 0,\\ 0, & x=0.\end{cases}\tag{5}$$

In this study, the operator $Q_{E_i}(.)$ is used to convert x to $Q_{E_i}(x)$ which has the alignment with the direction of x and satisfies $\|Q_{E_i}(x)\|\le\|x\|$ and $\delta Q_{E_i}(x)\in E_i$, for all $0\le\delta\le 1$. $\max_{x\subseteq E_i}\|Q_{E_i}(x)\|=\overline{\eta}$ means that the value cannot be arbitrarily large. We do not have any requirement for E_i to be convex or nonconvex. What we require is that the distance from any point outside E_i to the origin is lower bounded by a positive constant.

3. System Modeling

Consider a continuous-time multiagent system consisted of l leaders, n first-order followers, and m second-order followers. The first-order agents have the following dynamic equation:

$$\dot{x}_f(t) = u_f(t). \tag{6}$$

The dynamic equations of the second-order agents can be denoted as follows:

$$\begin{aligned} \dot{x}_s(t) &= v_s(t), \\ \dot{v}_s(t) &= u_s(t), \end{aligned} \tag{7}$$

where $s, f \in \Gamma$ represent the second-order and the first-order agents, respectively, and $x_s(t)$, $v_s(t)$, and $u_s(t)$ represent the position information, speed information, and control input of the second-order agents separately. $x_f(t)$ and $u_f(t)$ represent the position information and control input of the first-order agents separately. The set of all followers is expressed by $\Gamma = \{1, 2, \ldots, n+m\}$. We assume all leaders are stationary and the position states are denoted by $Y = \{y_1, y_2, \ldots y_l\}$.

In this study, if all the followers can converge into the convex set of leaders under the control protocol for any initial value given, that is, $\lim_{t\longrightarrow\infty} \|x_i(t) - W_\chi(x_i(t))\| = 0$, the control protocol can realize the containment control.

4. Model Transformation

To analyze the containment control of continuous-time heterogeneous multiagent system, a control protocol is proposed in this study as follows:

$$\begin{aligned} u_s &= Q_{E_i}\left[-q_s v_s + \sum_{i\in\Gamma} a_{si}(t)(x_i(t) - x_s(t)) - g_s(t)(x_s(t) - W_{\chi_s(t)}(x_s(t)))\right], \\ u_f &= Q_{E_i}\left[\sum_{i\in\Gamma} a_{fi}(t)\left(x_i(t) - x_f(t)\right) - g_f(t)x_f(t) - W_{\chi_f(t)}\left(x_f(t)\right)\right], \end{aligned} \tag{8}$$

where $s, f \in \Gamma$ and q_s is speed decay factor of followers. $a_{si}(t)$ and $a_{fi}(t)$ are the weight of edge (i, s), (i, f) at time t. And, we assume the weight of edge is always positive and lower bounded by a constant ε_d. If follower i can get the information from one or more agent directly, then $g_i(t) = \overline{g}_i$, otherwise, $g_i(t) = 0$. Assume that $\chi(t)$ is the convex set of the leaders. The Laplacian matrix of the heterogeneous multiagent system can be denoted by the following matrix:

$$L = \begin{Bmatrix} \overline{L}_s & -A_{sf} \\ -A_{fs} & L_f \end{Bmatrix}. \tag{9}$$

Then, algorithm (8) can be converted into

$$\begin{aligned} u_s(t) &= Q_{E_i}\left[-q_s v_s(t) - \overline{L}_s x_s(t) + A_{fs} x_f(t) - g_s(t)(x_s(t) - W_{\chi_s(t)}(x_s(t)))\right], \\ u_f(t) &= Q_{E_i}\left[-\overline{L}_f x_f(t) + A_{sf} x_s(t) - g_f(t)(x_f(t) - W_{\chi_f(t)}(x_f(t)))\right]. \end{aligned} \tag{10}$$

The following model transformation is introduced for further derivation:

$$z_i(t) = \frac{\left\|Q_{E_i}[\Phi(t)]\right\|}{\|\Phi(t)\|}, \tag{11}$$

when $\Phi(t) = 0$, $z_i(t) = 1$. For the first-order agents, (11) can be denoted as follows:

$$z_f(t) = \frac{\left\|Q_{E_i}\left[\omega_f(t)\right]\right\|}{\left\|\omega_f(t)\right\|}, \tag{12}$$

where $\omega_f(t) = -\overline{L}_f x_f(t) + A_{sf} x_s(t) - g_f(t)(x_f(t) - W_{\chi_f(t)}(x_f(t)))$. For the second-order agents, (11) can be transformed as follows:

$$z_s(t) = \frac{\left\|Q_{E_i}[-q_s v_s(t) + \omega_s(t)]\right\|}{\left\|-q_s v_s(t) + \omega_s(t)\right\|}, \tag{13}$$

where $\omega_s(t) = -\overline{L}_s x_s(t) + A_{fs} x_f(t) - g_s(t)(x_s(t) - W_{\chi_s(t)}(x_s(t)))$. From the above definition, it is easy to know the range of $z_i(t)$, that is, $0 < z_i(t) < 1$. We assume that $z_i(t)$ is lower bounded by a positive constant $\pi_i = \min\{z_i(t)\} > 0$, for all i. According to the above definition, system (8) can be converted into the following form.

For the first-order agents,

$$\dot{x}_f(t) = z_f(t)\omega_f(t). \tag{14}$$

For the second-order agents,

$$\begin{aligned}\dot{x}_s(t) &= v_s(t),\\ \dot{v}_s(t) &= -q_s z_s(t) v_s(t) + z_s(t)\omega_s(t).\end{aligned} \tag{15}$$

Denote $d_s = \max\left\{\sum_{j\in\Gamma} a_{sj}(t)\right\}, \overline{v}_s(t) = x_s(t) + v_s(t)/c_s$, where $c_s = d_s + \overline{g}_s$ is a positive constant. Then, (15) can be expressed in the following form:

$$\begin{aligned}\dot{x}_s(t) &= c_s\overline{v}_s(t) - c_s x_s(t),\\ \dot{\overline{v}}_s(t) &= (c_s - q_s z_s(t))\overline{v}_s(t) + \left[q_s z_s(t) - c_s - \frac{z_s(t)\left(\overline{L}_s + g_s(t)\right)}{c_s}x_s(t) + \frac{z_s(t)A_{fs}}{c_s}x_f(t) + \frac{z_s(t)g_s(t)}{c_s}W_{\chi_s(t)}\left(x_s(t)\right)\right].\end{aligned} \tag{16}$$

5. Main Result

Prior to the main theorems, we need to give the definition of switching communication graphs. Denote an infinite sequence of nonempty bounded continuous intervals as $[t_s, t_{s+1})$, $s = 0, 1, 2\ldots$ and $0 \le t_{s+1} - t_s \le T$, where T is a positive constant. Divide the above interval into a series of subintervals which are represented as $[t_{s_0}, t_{s_1})t, n[qt_{s_1}, t_{s_2}h)_{\ldots} x[7t_{s_n}, t_{s_{n+1}})$ with $t_{s_0} = t_s$ and $t_{s_{n+1}} = t_{s+1}$. We assume there exists a constant $\rho \ge 0$ that makes $t_{s_{n+1}} - t_n \ge \rho$. The communication topologies change at $t = t_{s_n}$ and do not change in the subinterval $[t_{s_n}, t_{s_{n+1}})$.

Assumption 2. Suppose that there exist at least one path between leaders and followers in the interval $[t_s, t_{s+1})$ for every agent. In other words, each follower can receive information from leaders directly or indirectly in the every interval $[t_s, t_{s+1})$.

Assumption 3 (see [29]). $\|\sum_{j\in\Gamma} a_{ij}(t)(x_j(t) - x_i(t))\| \le N_i/2$ and $g_i(t)(x_i(t) - W_{\chi_i(t)}(x_i(t))) \le N_i/2$ for some constant N_i.

Lemma 1 (see [29]). *Suppose that $U \subseteq \mathbb{R}^r$ is a nonempty closed set. For any vector $x_i \in \mathbb{R}^r, i \in \{1, 2\ldots, n\}$, and $\sum_{i=1}^n a_i = 1$, we have*

$$\left\|\sum_{i=1}^n a_i x_i - W_U\left(\sum_{i=1}^n a_i x_i\right)\right\| \le \sum_{i=1}^n a_i\left\|x_i - W_U(x_i)\right\|. \tag{17}$$

To analyze the stability and convergence of the system, we construct a Lyapunov function as follows:

$$F(t) = \max_k\left\|\zeta_k(t) - W_{\chi_k(t)}\left(\zeta_k(t)\right)\right\|, \tag{18}$$

where $t \ge T_0$, $k \in \{1, 2\ldots 2(n+m)\}$.

Theorem 1. *If $q_s \ge (c_s + 1)/\pi_i$ and $\eta/2 < N_i < \eta$, under assumption 1 and 2, $F(t)$ is a nonincreasing function relative to time t and $F(t) \ge 0$, namely, the limit of $F(t)$ exits. And the following inequalities hold:*

$$\frac{\mathrm{d}\left\|x_s(t) - W_{\chi_s(t)}\left(x_s(t)\right)\right\|}{\mathrm{d}t} \le c_s\left\|\overline{v}_s(t) - W_{\chi_s(t)}\left(\overline{v}_s(t)\right)\right\| - c_s\left\|x_s(t) - W_{\chi_s(t)}\left(x_s(t)\right)\right\|, \tag{19}$$

$$\begin{aligned}\frac{\mathrm{d}\left\|\overline{v}_s(t) - W_{\chi_s(t)}\left(\overline{v}_s(t)\right)\right\|}{\mathrm{d}t} &\le \left(c_s - q_s z_s(t)\right)\left\|\overline{v}_s(t) - W_{\chi_s(t)}\left(\overline{v}_s(t)\right)\right\|\\ &+\left(q_s z_s(t) - c_s - \frac{z_s(t)\left(\overline{L}_s + g_s(t)\right)}{c_s}\right)\left\|x_s(t) - W_{\chi_s(t)}\left(x_s(t)\right)\right\|\\ &+\left(\frac{z_s(t)A_{sf}}{c_s}\right)\left\|x_f(t) - W_{\chi_s(t)}\left(x_f(t)\right)\right\|,\end{aligned} \tag{20}$$

$$\begin{aligned}\frac{\mathrm{d}\left\|x_f(t) - W_{\chi_f(t)}\left(x_f(t)\right)\right\|}{\mathrm{d}t} &\le \left(-\left(\overline{L}_f z_f(t) + g_f(t)\right)\right)\left\|x_f(t) - W_{\chi_f(t)}\left(x_f(t)\right)\right\|\\ &+\left(A_{sf} z_f(t)\right)\left\|x_s(t) - W_{\chi_s(t)}\left(x_s(t)\right)\right\|.\end{aligned} \tag{21}$$

Proof. For the second-order agents, we can deduce

$$\lim_{\Delta t\longrightarrow 0} x_s(t+\Delta t) = \lim_{\Delta t\longrightarrow 0}(1-c_s\Delta t)x_s(t) + \lim_{\Delta t\longrightarrow 0}(c_s\Delta t)\overline{v}_s(t), \quad (22)$$

where Δt is time increment and $\Delta t \longrightarrow 0$. c_s is a positive constant. Then, it is evident that $c_s\Delta t > 0$, $1 - c_s\Delta t > 0$, and $1 - c_s\Delta t + c_s\Delta t = 1$. According to Lemma 1, (22) is converted to

$$\begin{aligned}\left\|x_s(t+\Delta t) - W_{\chi_s(t)}(x_s(t+\Delta t))\right\| &\leq (1-c_s\Delta t)\left\|x_s(t+\Delta t) - W_{\chi_s(t)}(x_s(t+\Delta t))\right\| \\ &\quad + c_s\Delta t\left\|\overline{v}_s(t+\Delta t) - W_{\chi_s(t)}(\overline{v}_s(t+\Delta t))\right\|.\end{aligned} \quad (23)$$

Analogously, we can do the same conversion for $\overline{v}_s(t)$:

$$\begin{aligned}\overline{v}_s(t+\Delta t) &= (1+(c_s - q_s z_s(t))\Delta t)\overline{v}_s(t) + \left(q_s z_s(t) - c_s - \frac{z_s(t)(\overline{L}_s + g_s(t))}{c_s}\right)\Delta t x_s(t) + \frac{z_s(t)A_{sf}}{c_s}\Delta t x_f(t) \\ &\quad + \frac{z_s(t)g_s(t)}{c_s}\Delta t W_{\chi_s(t)}(x_s(t)).\end{aligned} \quad (24)$$

To facilitate later calculations, we have simplified the upper formula to the following form:

$$\overline{v}_s(t+\Delta t) = \beta_{1i}\overline{v}_s(t) + \beta_{2i}x_s(t) + \beta_{3i}x_f(t) + \beta_{4i}W_{\chi_s(t)}(x_s(t)), \quad (25)$$

where

$$\begin{aligned}\beta_{1i} &= 1 + (c_s - q_s z_s(t))\Delta t, \\ \beta_{2i} &= \left(q_s z_s(t) - c_s - \frac{z_s(t)(\overline{L}_s + g_s(t))}{c_s}\right)\Delta t, \\ \beta_{3i} &= \frac{z_s(t)A_{sf}}{c_s}\Delta t, \\ \beta_{4i} &= \frac{z_s(t)g_s(t)}{c_s}\Delta t.\end{aligned} \quad (26)$$

For all $t \geq T_0$, we have

$$\beta_{1i} > 0, \beta_{3i} \geq 0, \beta_{4i} \geq 0. \quad (27)$$

When $q_s \geq (c_s + 1)/\pi_i$, we obtain

$$\beta_{2i} = \left(q_s z_s(t) - c_s - \frac{z_s(t)(\overline{L}_s + g_s(t))}{c_s}\right) \geq q_s\pi_i - c_s - 1 \geq 0. \quad (28)$$

And $\beta_{1i} + \beta_{2i} + \beta_{3i} + \beta_{4i} = 1$. Then, (24) can be converted into the following forms:

$$\begin{aligned}\left\|\overline{v}_s(t+\Delta t) - W_{\chi_s(t)}(\overline{v}_s(t+\Delta t))\right\| &\leq \beta_{1i}\left\|\overline{v}_s(t) - W_{\chi_s(t)}(\overline{v}_s(t))\right\| + \beta_{2i}\left\|x_s(t) - W_{\chi_s(t)}(x_s(t))\right\| + \beta_{3i}\left\|x_f(t) - W_{\chi_f(t)}(x_f(t))\right\| \\ &\quad + \beta_{4i}\left\|W_{\chi_s(t)}(x_s(t)) - W_{\chi_s(t)}(W_{\chi_s(t)}(x_s(t)))\right\|.\end{aligned} \quad (29)$$

Combined with the following conditions, $\chi_s(t) \in \chi$, $\chi_f(t) \in \chi$, and $W_{\chi_s(t)}(W_{\chi_s(t)}(x_s(t))) = W_{\chi_s(t)}(x_s(t))$; it follows that

$$\left\|\overline{v}_s(t+\Delta t) - W_{\chi_s(t)}(\overline{v}_s(t+\Delta t))\right\| \leq \beta_{1i}\left\|\overline{v}_s(t) - W_{\chi_s(t)}(\overline{v}_s(t))\right\| + \beta_{2i}\left\|x_s(t) - W_{\chi_s(t)}(x_s(t))\right\| + \beta_{3i}\left\|x_f(t) - W_{\chi_f(t)}\left(x_f(t)\right)\right\|. \tag{30}$$

Similarly, for the first-order agents, we can deduce

$$\lim_{\Delta t \longrightarrow 0} x_f(t+\Delta t) = \lim_{\Delta t \longrightarrow 0} \dot{x}_f(t)\Delta t + \lim_{\Delta t \longrightarrow 0} x_f(t)\Delta t, \tag{31}$$

where Δt is time increment and $\Delta t \longrightarrow 0$. According to (31) and Lemma 1, we have

$$x_f(t+\Delta t) = \left[1 - z_f(t)\left(\overline{L}_f + g_f(t)\right)\right]\Delta t x_f(t) + A_{sf} z_f(t)\Delta t x_s(t) + g_f(t) z_f(t)\Delta t W_{\chi_f(t)}\left(\overline{v}_f(t)\right). \tag{32}$$

To facilitate later calculations, we have simplified the upper formula to the following form:

$$x_f(t+\Delta t) = \alpha_{1i} x_f(t) + \alpha_{2i} x_s(t) + \alpha_{3i} W_{\chi_f(t)}\left(x_f(t)\right), \tag{33}$$

where $\alpha_{1i} = [1 - z_f(t)(\overline{L}_f + g_f(t))]\Delta t$, $\alpha_{2i} = A_{\mathrm{sf}} z_f(t)\Delta t$, and $\alpha_{3i} = g_f(t) z_f(t)\Delta t$. We can deduce from (33)

$$\left\|x_f(t+\Delta t) - W_{\chi_f(t)}\left(x_f(t+\Delta t)\right)\right\| \leq \alpha_{1i}\left\|x_f(t) - W_{\chi_f(t)}\left(x_f(t)\right)\right\| + \alpha_{2i}\left\|x_s(t) - W_{\chi_s(t)}(x_s(t))\right\| + \alpha_{3i}\left\|W_{\chi_f(t)}\left(x_f(t)\right) - W_{\chi_f(t)}(W_{\chi_f(t)}(x_f(t)))\right\|. \tag{34}$$

Similar to the second-order agents, according to the condition $\chi_f(t) \in \chi$ and $W_{\chi_s(t)}(W_{\chi_s(t)}(x_s(t))) = W_{\chi_s(t)}(x_s(t))$, we have $W_{\chi_f(t)}(W_{\chi_f(t)}(x_f(t))) = W_{\chi_f(t)}(x_f(t))$. Then,

$$\left\|x_f(t) - W_{\chi_f(t)}\left(x_f(t)\right)\right\| \leq \alpha_{1i}\left\|x_f(t) - W_{\chi_f(t)}\left(x_f(t)\right)\right\| + \alpha_{2i}\left\|x_s(t) - W_{\chi_s(t)}(x_s(t))\right\|. \tag{35}$$

It is evident that every $\|\zeta_k(t) - W_\chi(\zeta_k(t))\|$ is a convex hull that is composed of $\|\zeta_j(t) - W_\chi(\zeta_j(t))\|$. From (23), (30), and (35), we can obtain that

$$\left\|\zeta_j(t+\Delta t) - W_{\chi_j(t)}\left(\zeta_j(t+\Delta t)\right)\right\| \leq \max_j\left\|\zeta_j(t) - W_{\chi_j(t)}\left(\zeta_j(t)\right)\right\|. \tag{36}$$

Thus, $F(t+\Delta t) \leq F(t)$ can be deduced which indicate that $F(t)$ is a nonincreasing function relative to time t. It is obvious that $F(t) \geq 0$. Thus, it is easy to know that the limit of $F(t)$ exists.

For the second-order agents, according to the definition of derivative and (23), we have

$$\lim_{\Delta t\longrightarrow 0}\frac{\left(\left\|x_s(t+\Delta t)-W_{\chi_s(t)}(x_s(t+\Delta t))\right\|-\left\|x_s(t)-W_{\chi_s(t)}(x_s(t))\right\|\right)}{\Delta t}\le c_s\left\|\overline{v}_s(t)-W_{\chi_s(t)}(\overline{v}_s(t))\right\|-c_s\left\|x_s(t)-W_{\chi_s(t)}(x_s(t))\right\|. \tag{37}$$

That is,

$$\frac{\mathrm{d}\left\|x_s(t)-W_{\chi_s(t)}(x_s(t))\right\|}{\mathrm{d}t}\le c_s\left\|\overline{v}_s(t)-W_{\chi_s(t)}(\overline{v}_s(t))\right\|-c_s\left\|x_s(t)-W_{\chi_s(t)}(x_s(t))\right\|. \tag{38}$$

Similarly, according to (30), we have

$$\begin{aligned}\lim_{\Delta t\longrightarrow 0}&\frac{\left(\left\|\overline{v}_s(t+\Delta t)-W_{\chi_s(t)}(\overline{v}_s(t+\Delta t))\right\|-\left\|\overline{v}_s(t)-W_{\chi_s(t)}(\overline{v}_s(t))\right\|\right)}{\Delta t}\le\lambda_{1i}\left\|\overline{v}_s(t)-W_{\chi_s(t)}(\overline{v}_s(t))\right\|+\lambda_{2i}\left\|x_s(t)-W_{\chi_s(t)}(x_s(t))\right\|\\&+\lambda_{3i}\left\|x_f(t)-W_{\chi_f(t)}(x_f(t))\right\|.\end{aligned} \tag{39}$$

That is,

$$\begin{aligned}\frac{\mathrm{d}\left\|\overline{v}_s(t)-W_{\chi_s(t)}(\overline{v}_s(t))\right\|}{\mathrm{d}t}&\le\lambda_{1i}\left\|\overline{v}_s(t)-W_{\chi_s(t)}(\overline{v}_s(t))\right\|+\lambda_{2i}\left\|x_s(t)-W_{\chi_s(t)}(x_s(t))\right\|\\&+\lambda_{3i}\left\|x_f(t)-W_{\chi_f(t)}(x_f(t))\right\|,\end{aligned} \tag{40}$$

where

$$\begin{aligned}\lambda_{1i}&=c_s-q_s z_s(\mathrm{t}),\\ \lambda_{3i}&=\frac{z_s(t)A_{\mathrm{sf}}}{c_s},\\ \lambda_{2i}&=q_s z_s(t)-c_s-\frac{z_s(t)\left(\overline{L}_s+g_s(t)\right)}{c_s}.\end{aligned} \tag{41}$$

For the first-order agents, similarly, according to (35), we have

$$\lim_{\Delta t\longrightarrow 0}\frac{\left(\left\|x_f(t+\Delta t)-W_{\chi_f(t)}(x_f(t+\Delta t))\right\|-\left\|x_f(t)-W_{\chi_f(t)}(x_f(t))\right\|\right)}{\Delta t}\le\mu_{1i}\left\|x_f(t)-W_{\chi_f(t)}(x_f(t))\right\|+\mu_{2i}\left\|x_s(t)-W_{\chi_s(t)}(x_s(t))\right\|. \tag{42}$$

That is,

$$\frac{\mathrm{d}\left\|x_f(t)-W_{\chi_f(t)}(x_f(t))\right\|}{\mathrm{d}t}\le\mu_{1i}\left\|x_f(t)-W_{\chi_f(t)}(x_f(t))\right\|+\mu_{2i}\left\|x_s(t)-W_{\chi_s(t)}(x_s(t))\right\|, \tag{43}$$

where $\mu_{1i} = -z_f(t)(\overline{L}_f + g_f(t))$, $\mu_{2i} = A_{sf} z_f(t)$. □

Theorem 2. *Under assumption 1 and 2, all the followers can move into the convex hull spanned by static leaders with algorithm (3), i.e.,* $\lim_{t\longrightarrow\infty}\|x_i(t) - W_{\chi_i(t)}(x_i(t))\| = 0$.

Proof. We will discuss the convergence of the heterogeneous multiagent system with switching graphs and nonconvex control input constraints at intervals in three cases.

Case 1: suppose that there exists a agent i_n can receive the information from leaders at time $t = t_{s_n}$. At this point, $g_i(t) = \overline{g}_i$. First of all, we assume i_n is a second-order agent. From (40), we have

$$\begin{aligned}\frac{\mathrm{d}\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\|}{\mathrm{d}t} &\leq \lambda_{1i_n}\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| + \left(-\lambda_{1i_n} - \frac{z_{si_n}(t)\overline{g}_{si_n}}{c_{si_n}}\right)F\left(t_{s_n}\right)\\ &\leq \left(c_{si_n} - q_{si_n}\right)\left\|x_{si_n}(t) - W_{\chi_{si_n}(t)}\left(x_{si_n}(t)\right)\right\| + \left(q_{si_n} - c_{si_n} - \frac{\pi_{i_n}\overline{g}_{si_n}}{c_{si_n}}\right)F\left(t_{s_n}\right).\end{aligned} \tag{44}$$

According to the definition of calculus, for all $t \in [t_{s_n}, t_{s_{n+1}})$, we have

$$\begin{aligned}\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| &\leq e^{\left(c_{si_n} - q_{si_n}\right)\left(t - t_{s_n}\right)}F\left(t_{s_n}\right) + \int_{t_{s_n}}^{t} e^{\left(c_{si_n} - q_{si_n}\right)(t-\tau)}\left(q_{si_n} - c_{si_n} - \frac{\pi_{i_n}\overline{g}_{si_n}}{c_{si_n}}\right)F\left(t_{s_n}\right)\\ &\leq e^{\left(c_{si_n} - q_{si_n}\right)\left(t - t_{s_n}\right)}F\left(t_{s_n}\right) + \frac{\left(q_{si_n} - c_{si_n} - \pi_{i_n}\overline{g}_{si_n}/c_{si_n}\right)F\left(t_{s_n}\right)}{q_{si_n} - c_{si_n}}\left[1 - e^{\left(c_{si_n} - q_{si_n}\right)\left(t - t_{s_n}\right)}\right] \leq \left[1 + \frac{\pi_{i_n}\overline{g}_{si_n}\left(e^{\left(c_{si_n} - q_{si_n}\right)\left(t - t_{s_n}\right)} - 1\right)}{q_{si_n}\left(q_{si_n} - c_{si_n}\right)}\right]F\left(t_{s_n}\right).\end{aligned} \tag{45}$$

For all $t \in [t_{s_n}, t_{s_{n+1}})$, thus, we have

$$\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| \leq m_1 F\left(t_{s_n}\right), \tag{46}$$

where $m_1 = 1 + \pi_{i_n}\overline{g}_{si_n}(e^{(c_{si_n} - q_{si_n})(t - t_{s_n})} - 1)/q_{si_n}(q_{si_n} - c_{si_n})$. It is obvious that $0 < m_1 < 1$. According to (38) and (46), we obtain

$$\left\|x_{si_n}(t) - W_{\chi_{si_n}(t)}\left(x_{si_n}(t)\right)\right\| \leq m_1 F\left(t_{s_n}\right) + (1 - m_1)e^{-q_{si_n}\left(t - t_{s_n}\right)}F\left(t_{s_n}\right). \tag{47}$$

For all $t \in [t_{s_n}, t_{s_{n+1}})$, we have

$$\left\|x_{si_n}(t) - W_{\chi_{si_n}}\left(x_{si_n}(t)\right)\right\| \leq m_2 F\left(t_{s_n}\right), \tag{48}$$

where $m_2 = 1 + (1 - m_1)e^{q_{si_n}(t - t_{s_n})}$. Obviously, $0 < m_2 < 1$.

If i_n is a first-order agent, we have

$$\frac{\mathrm{d}\left\|x_{fi_n}(t) - W_{\chi_{fi_n}(t)}\left(x_{fi_n}(t)\right)\right\|}{\mathrm{d}t} \leq -\overline{g}_{fi_n}\left\|x_{fi_n}(t) - W_{\chi_{fi_n}(t)}\left(x_{fi_n}(t)\right)\right\| \leq -\overline{g}_{fi_n}F\left(t_{s_n}\right) \leq \mathrm{e}^{-\overline{g}_{fi_n}\left(t - t_{s_n}\right)}F\left(t_{s_n}\right). \tag{49}$$

Then,

$$\left\|x_{fi_n}(t) - W_{\chi_{fi_n}(t)}\left(x_{fi_n}(t)\right)\right\| \leq n_1 F\left(t_{s_n}\right), \tag{50}$$

where $n_1 = e^{-\overline{g}_{fi_n}(t-t_{s_n})}$. Obviously, $0 < n_1 < 1$.

Case 2: If there exists a follower i_n such that $\|x_{i_n}(t) - W_{\chi_{i_n}(t)}(x_{i_n}(t))\| \leq b_{i_n} F(t_{s_n})$, for the second-order agents, from (23), we obtain

$$\left\|x_{si_n}(t) - W_{\chi_{si_n}(t)}\left(x_{si_n}(t)\right)\right\| \leq e^{-c_{si_n}\left(t-t_{s_n}\right)} b_{i_n} F\left(t_{s_n}\right) + \left(1 - e^{-c_{si_n}\left(t-t_{s_n}\right)}\right) F\left(t_{s_n}\right). \tag{51}$$

For all $t \in [t_{s_n}, t_{s_{n+1}})$, we have

$$\left\|x_{si_n}(t) - W_{\chi_{si_n}(t)}\left(x_{si_n}(t)\right)\right\| \leq m_3 F\left(t_{s_n}\right), \tag{52}$$

where $m_3 = 1 + (b_{i_n} - 1) e^{-c_{si_n}(t-t_{s_n})}$. Obviously, $0 < m_3 < 1$. Then, we can obtain that

$$\begin{aligned} \frac{\mathrm{d}\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\|}{\mathrm{d}t} &\leq \left[(q_{si_n} z_{si_n}(t) - c_{si_n}) m_3 + 1 - m_3 \right]\left(F\left(t_{s_n}\right) - \left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| \right) \\ &+ \left[\left(1 - m_3\right)\left(c_{si_n} - q_{si_n} z_{si_n}(t) + 1\right) \times \left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| \right] \leq \left[\left(q_{si_n} - c_s\right) m_3 + (1 - m_3)\left(q_{si_n} - c_s \pi_{i_n}\right) \right] \\ &\times \left\|\overline{v}_s(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| + \left[\left(q_{si_n} - c_s\right) m_3 + 1 - m_3 \right] F\left(t_{s_n}\right). \end{aligned} \tag{53}$$

And

$$\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| \leq \left[1 + (1 - m_3)\left(c_s \pi_{i_n} - q_{si_n} - 1\right) \times \left(\frac{e^{-B_s\left(t-t_{s_n}\right)} - 1}{B_s} \right) \right] F\left(t_{s_n}\right), \tag{54}$$

where

$$B_s = 1 + \left(1 - m_3\right)\left(c_{si_n} \pi_{i_n} - q_{si_n}\right). \tag{55}$$

Hence,

$$\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| \leq m_4 F\left(t_{s_n}\right), \tag{56}$$

where

$$m_4 = \left[1 + (1 - m_3)\left(c_{si_n} \pi_{i_n} - q_{si_n} - 1\right) \times \left(\frac{e^{-B_s\left(t-t_{s_n}\right)} - 1}{B_s} \right) \right], \quad 0 < m_4 < 1. \tag{57}$$

For the first-second agents, similarly, we have

$$\frac{\mathrm{d}\left\|x_{fi_n}(t) - W_{\chi_{fi_n}(t)}\left(x_{fi_n}(t)\right)\right\|}{\mathrm{d}t} \leq \overline{g}_{i_n} b_{i_n} F\left(t_{s_n}\right). \tag{58}$$

Then,

$$\left\|x_{fi_n}(t) - W_{\chi_{fi_n}(t)}\left(x_f(t)\right)\right\| \leq n_2 F\left(t_{s_n}\right), \tag{59}$$

where $n_2 = e^{-\overline{g}_{i_n}(t-t_{s_n})} b_{i_n}, 0 < n_2 < 1$.

Case 3: suppose that there exists a follower i_n such that $a_{i_n j_0}(t_{s_n}) > 0$, $\|x_{j_0}(t_{s_n}) - W_{\chi_{j0}(t)}(x_{j_0}(t))\| \leq e_{j0} F(t_{s_n})$, where $0 < e_{j0} < 1$. According to the above assumption, for the second-order agents, we have

$$\left\|x_{si_n}\left(t_{s_n}\right) - W_{\chi_{si_n}(t)}\left(x_{si_n}\left(t_{s_n}\right)\right)\right\| \leq \left(1 + \left(e_{j_0} - 1\right) e^{-\overline{g}_{si_n}\left(t-t_{s_n}\right)}\right) F\left(t_{s_n}\right). \tag{60}$$

Hence,

$$\frac{\mathrm{d}\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\|}{\mathrm{d}t} \leq \left(c_{si_n} - q_{si_n}\right)\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| + \left[q_{si_n} - c_{si_n} + \frac{\pi_{i_n} c_{si_n}}{q_{si_n}}\left(e_{j_0} - 1\right) e^{-c_{si_n} T} \right] F\left(t_{s_n}\right). \tag{61}$$

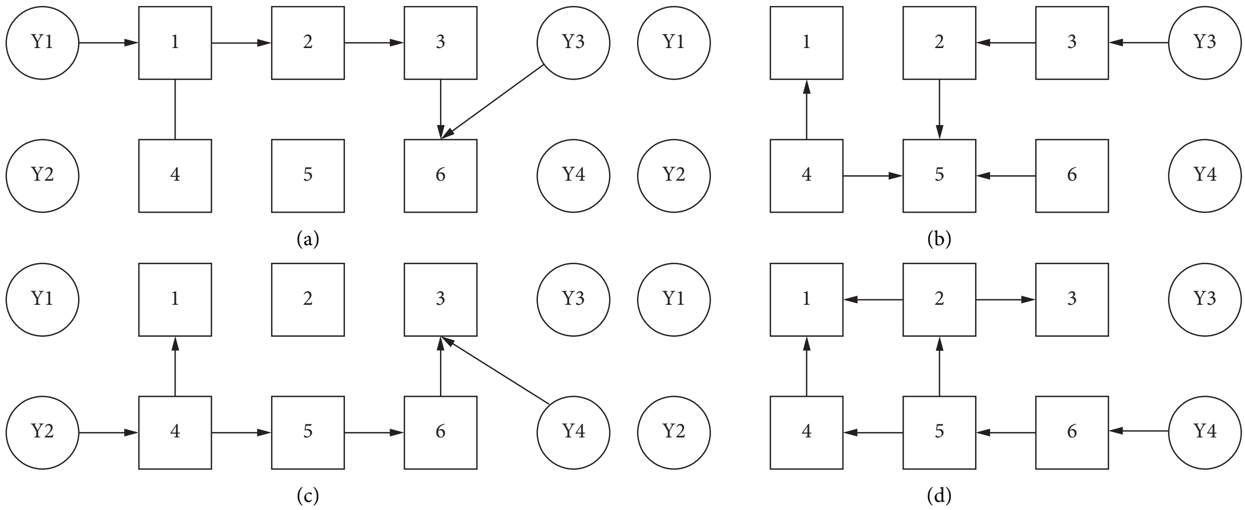

Figure 1: Four directed graphs.

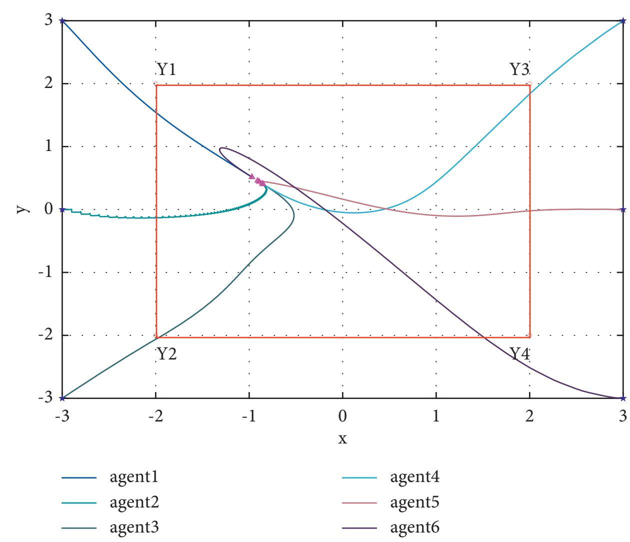

Figure 2: Position of followers.

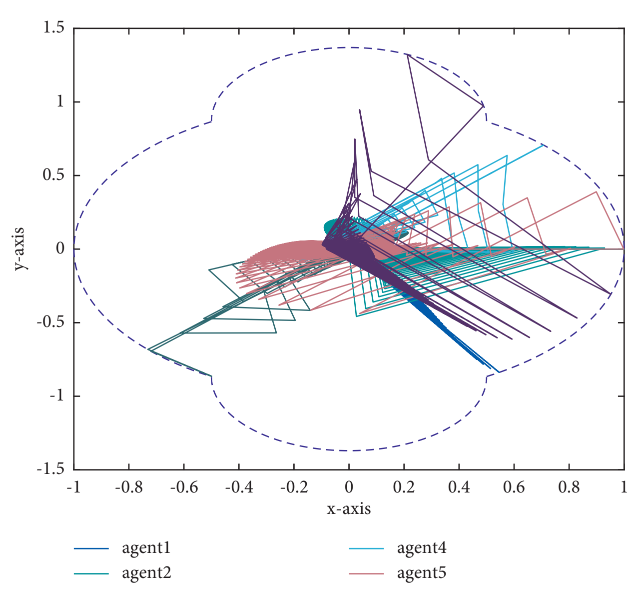

Figure 3: Nonconvex constraints of control input.

For all $t \in [t_{s_n}, t_{s_{n+1}})$, we have

$$\left\|\overline{v}_{si_n}(t) - W_{\chi_{si_n}(t)}\left(\overline{v}_{si_n}(t)\right)\right\| \leq m_5 F\left(t_{s_n}\right), \tag{62}$$

where $m_5 = 1 + \pi_{i_n} c_{si_n}/q_{si_n}(1 - e_{j_0})\ e^{-c_{si_n}(\mathrm{t}-t_{s_n})}(\mathrm{e}^{(c_{si_n}-q_{si_n})(\mathrm{t}-t_{s_n})} - 1)/q_{si_n}(c_{si_n} - q_{si_n})$, $0 \leq m_5 \leq 1$. Furthermore,

$$\left\|x_{si_n}(t) - W_{\chi_{si_n}(t)}\left(x_{si_n}(t)\right)\right\| \leq m_6 F\left(t_{s_n}\right), \tag{63}$$

where $m_6 = 1 + (m_5 - 1)(1 - e^{c_{si_n}(\mathrm{t}-t_{s_n})})$, $0 < m_6 < 1$. And similarly, for the first-order agents, there exists $0 < n_3 < 1$ that makes

$$\left\|x_{fi_n}(t) - W_{\chi_{fi_n}(t)}\left(x_{fi_n}(t)\right)\right\| \leq n_3 F\left(t_{s_n}\right). \tag{64}$$

Under assumption 2, we know that there exists at least one follower which can receive the information directly from leaders. From the analysis of the above cases, in the time interval $[t_s, t_{s+1}]$, we have the following conclusions for all agents:

$$\begin{aligned}
&\left\|x_{i_{n1}}(t_{s+1}) - W_\chi\left(x_{i_{n1}}(t_{s+1})\right)\right\| \leq \gamma_1 F(t_s), \\
&\left\|\overline{v}_{i_{n1}}(t_{s+1}) - W_\chi\left(\overline{v}_{i_{n1}}(t_{s+1})\right)\right\| \leq \gamma_2 F(t_s),
\end{aligned} \tag{65}$$

where the constant $\gamma_1, \gamma_2 \in (0, 1)$. And there exists another follower $i_{n2} \neq i_{n1}$ which can receive the information from leaders or i_{n1} in the interval $[t_{s+1}, t_{s+2}]$. Hence,

$$\begin{aligned}
&\left\|x_{i_{n1}}(t_{s+2}) - W_\chi\left(x_{i_{n1}}(t_{s+2})\right)\right\| \leq \gamma_3 F(t_{s+1}), \\
&\left\|\overline{v}_{i_{n1}}(t_{s+2}) - W_\chi\left(\overline{v}_{i_{n1}}(t_{s+2})\right)\right\| \leq \gamma_4 F(t_{s+1}), \\
&\left\|x_{i_{n2}}(t_{s+2}) - W_\chi\left(x_{i_{n2}}(t_{s+2})\right)\right\| \leq \gamma_3 F(t_{s+1}), \\
&\left\|\overline{v}_{i_{n2}}(t_{s+2}) - W_\chi\left(\overline{v}_{i_{n2}}(t_{s+2})\right)\right\| \leq \gamma_4 F(t_{s+1}),
\end{aligned} \tag{66}$$

where the constant $\gamma_3, \gamma_4 \in (0, 1)$. Then, we can draw the following conclusions:

$$\begin{aligned}
&\left\|x_i(t_{s+n}) - W_\chi(x_i(t_{s+n}))\right\| \leq \gamma F(t_s), \\
&\left\|\overline{v}_i(t_{s+n}) - W_\chi(\overline{v}_i(t_{s+n}))\right\| \leq \gamma F(t_s),
\end{aligned} \tag{67}$$

where $\gamma \in (0, 1)$. Hence, there exists a constant $\varepsilon \in (0, 1)$ that makes $F(t_{s+nr}) \leq \varepsilon^r F(t_s)$. Then, we can know the limit of $F(t)$ exists and $\lim_{t \longrightarrow \infty} \max\|\zeta_k(t) - W_\chi(\zeta_k(t))\| = 0$. Therefore, we can draw the conclusion that the control protocol can realize the containment control. □

6. Simulation

Consider a continuous-time heterogeneous multiagent system with 6 followers and 4 leaders. According to the topology assumptions, we design four different communication topologies which are shown in Figure 1. And they switch every 1.5s. The control inputs are limited in the following set: $\chi = \{x \| \|x\| \leq 1\} \cup \left\{x \| \|x - [0, \sqrt{3}/2]^T\| \leq 0.5\right\} \cup \left\{x \| \|x - [0, -\sqrt{3}/2]^T\| \leq 0.5\right\}$. Suppose the weight of every edge is 0.6. Initial position values of followers are $x_1(0) = [-3, 3]^T$, $x_2(0) = [-3, 0]^T$, $x_3(0) = [-3, -3]^T$, $x_4(0) = [3, 3]^{\mathrm{T}}$, $x_5(0) = [3, 0]^{\mathrm{T}}$, and $x_6(0) = [3, -3]^T$. Initial position values of leaders are $y_1(0) = [-2, 2]^T$, $y_2(0) = [2, 2]^T$, $y_3(0) = [-2, -2]^T$, and $y_4(0) = [2, -2]^T$. Initial velocity of agents are $v_1(0) = v_3(0) = [1, 1]^T$, $v_2(0) = [1, 0]^T$, $v_4(0) = [-1, -1]^T$, $v_5(0) = [-1, 0]^T$, and $v_6(0) = [-1, 0]^T$. We suppose each parameter q_i is taken as $q_i = 10$. Figure 2 shows the trajectory of followers. As you can see in Figures 2 and 3, all the followers finally converge to the set composed of multiple stationary leaders while the control input of each followers remains in their corresponding constraint set. The simulation result is consistent with theorem.

7. Conclusion

This paper focuses on distributed containment control with nonconvex control input constraints of heterogeneous multiagent systems. For this system, a protocol with nonconvex control input is proposed. Based on the Lyapunov function and the definition of the derivative, we can theoretically infer the distance between each follower and the set consisted of leaders in each time interval. Then, we can obtain that each follower is driven to converge to a convex set composed of multiple stationary leaders. Future work could be directed to the containment control of the system which consisted of more different order multiagent.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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