

## Research Article

# Adaptive Neural Tracking Control for a Two-Joint Robotic Manipulator with Unknown Time-Varying Delays

Jiayao Wang  and Yang Cui 

*School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan 114051, China*

Correspondence should be addressed to Yang Cui; [cuiyangfly@sina.com](mailto:cuiyangfly@sina.com)

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This paper presents an adaptive neural tracking control approach for a two-joint robotic manipulator with unknown time-varying delays. In order to work out the effect of unknown time-varying delays on the two-joint robotic manipulator, the appropriate Lyapunov–Krasovskii functionals (LKFs) and separation technology are chosen to settle this matter. The neural networks work as an approximator that has the advantage of estimating the unknown function in the system. In this paper, Lyapunov stability analysis can prove that all signals of the closed-loop system are semiglobal uniformly ultimately bounded and the tracking error can converge to a compact neighborhood with respect to zero. The simulation consequences demonstrate the availability of the feedforward control approach.

## 1. Introduction

Recently, the adaptive backstepping control method has been used in a wide range of applications in control field, such as marine vessels control [1], autonomous underwater vehicle system [2], and aircraft flight control system [3]. The adaptive control method is principally worked out for the nonlinear system with uncertain linear parameters. However, there are many nonlinear systems with uncertainty that cannot be linearly parameterized. For the sake of settling this difficulty, these uncertainties are always approximated by some universal approximators in the literature [4–9], such as fuzzy logic systems and neural networks. In [10, 11], the tracking controllers are designed based on adaptive neural/fuzzy backstepping control methods and the tracking error can converge to the accuracy defined a priori in probability. In the literature [12, 13], the adaptive neural/fuzzy control strategies are proposed that aim to approximate unknown function for the nonlinear multiple-input and multiple-output (MIMO) system. The traditional adaptive neural/fuzzy backstepping control methods have great development, but with the increase of system order, the structure of the controller is complex.

For the traditional backstepping controller design process of the system, the controller design in each step requires to take the derivative of the previous virtual controller in [14–16]; the controller design process will be more complicated when the order of the system is higher, and there will be appeared “explosion of complexity” phenomenon. To work out the “explosion of complexity” problem, the command filtering technique is used in the traditional backstepping process to eliminate the repeated differentiation of the virtual controller in [17–21]. In the literature [22, 23], the authors have proposed improved control methods by designing compensation signals, which can eliminate filter errors caused by command filters. In the meantime, the singularity problem is a possible problem in controller design process; however, this point was not considered in the abovementioned literature. Therefore, this paper will consider command filtering techniques and study the singularity problems that may occur in the controller design process.

Time delays are common in the practical application of control. The problem of time delays in the system will greatly reduce the control performance. However, the abovementioned articles do not consider the issue of time delays. In order to solve this problem, many scholars and experts have shown interest in solving the problem of time delays. LKFs are effective tools to

restrain the influence of time delays in nonlinear systems [24–28]. In the literature [24–27], the authors use LKFs and backstepping method to solve the problem of the nonlinear system with time-varying delay. In [28], an adaptive neural tracking controller is constructed by using LKFs and separation techniques for MIMO nonlinear systems with constant time delays. Inspired by the above literature, the method of combining LKFs and separation technology will be utilized to solve the time-varying delay.

Due to the industry being developed increasingly more rapidly, robot manipulators have been widely applied in the production. More recently, the robotic manipulator has been a research hotspot and some achievements have been made in [29–31]. In the literature [32, 33], based on the barrier Lyapunov functions, the adaptive neural/fuzzy control methods are proposed for the manipulator with full state constraints. By using auxiliary design system, an adaptive impedance controller is designed for the manipulator system with input saturation in [34]. The adaptive control methods proposed in [29–34] can effectively solve the control problem of the robotic manipulator without time-varying delays. When using a robotic manipulator to solve actual industrial problems, due to signal transmission and other reasons, time delay is inevitable. Therefore, it is of theoretical and practical significance to study the adaptive tracking control problem of a robotic manipulator with time-varying delays.

Enlightened by the above results, based on the backstepping control method, we will employ neural networks, command filtering technology, LKFs, and the separation techniques to achieve the tracking control objective of the robotic manipulator with unknown time-varying delays. In the meantime, this paper will take into account the singularity problem that may occur in the controller design process. The contributions of this paper are stated as follows:

- (1) For the traditional backstepping control method, there exists the repeated derivative phenomenon of the virtual controller in [14–16]. In order to avoid the derivation of the virtual controller, we choose the command filtering control method to solve this problem.
- (2) In the literature [35], the authors have adopted L'Hopital's rule to effectively avoid the occurrence of the singularity problem. However, L'Hopital's rule must meet the following conditions before it can be used: (1) The limits of the numerator and denominator are both equal to zero or infinity. (2) The numerator and denominator are, respectively, differentiable within the restricted region. However, since the situation of this paper is different from that in the literature [35], this paper further considers the case that the numerator and denominator are not zero at the same time when the singularity problem occurs. Then, we propose a piecewise function method to solve this singularity problem.
- (3) The adaptive neural controllers of the robot manipulator without time-varying delay were investigated in the literature [32, 34]. But, in fact, time-varying delays are often encountered in practical

engineering systems. In this paper, we consider studying a novel control method of the robot manipulator with time-varying delays and the unknown time-varying delays were solved by the separation technology and LKFs. The separation technique shows advantage of breaking up the time-varying delay functions to several continuous known functions, which are eliminated by making use of the LKFs.

The structure of this article is shown as follows: In Section 2, the two-joint robot manipulator with time-varying delays is introduced. The adaptive neural controller design and stability analysis are performed in Section 3. In Section 4, the availability of the proposed control method is proved by a simulation. Section 5 is the conclusion of this paper.

## 2. System Descriptions and Preliminaries

The dynamic model for the two-joint robot manipulator with unknown time-varying delays is expressed as

$$\begin{aligned} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + T(q(t - \vartheta_1(t)), \dot{q}(t - \vartheta_2(t))) + G(q) \\ = u - J^T(q)p(t), \end{aligned} \quad (1)$$

where  $q = [q_1, q_2]^T$ ,  $\dot{q}$ , and  $\ddot{q} \in R^2$  are the angular location, speed, and acceleration vectors;  $\vartheta_1(t)$  and  $\vartheta_2(t)$  stand for the unknown time-varying delays;  $\vartheta_1(t)$  and  $\vartheta_2(t)$  satisfied  $\vartheta_i(t) \leq \vartheta_{\max}$  and  $\vartheta_i(t) \leq \vartheta \leq 1$ , where  $\vartheta_{\max}$  and  $\vartheta$  are the known constants;  $u$  presents the applied torque;  $p(t)$  stands for the restraining force and satisfies  $|p(t)| \leq \bar{p}$  for  $t > 0$ , where  $\bar{p}$  is a positive constant;  $M(q)$  represents the symmetric positive definite inertia matrix;  $C(q, \dot{q})\dot{q}$  stands for the unknown centripetal and Coriolis torque;  $T(\cdot)$  stands for the unknown time delay function;  $G(q)$  is the unknown gravitational force; and  $J^T(q)$  represents the unknown reversible Jacobian matrix.

For the states defined as  $x_1 = [q_1, q_2]^T$  and  $x_2 = [\dot{q}_1, \dot{q}_2]^T$ , respectively, the state-space model of the dynamic model for the two-joint robot manipulator system can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = M^{-1}(x_1)(-C(x_1, x_2)x_2 - G(x_1) - J^T(x_1)p(t) \\ - T(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))) + u), \\ y_1 = x_1. \end{cases} \quad (2)$$

In this paper, the control objective is to design an adaptive neural tracking controller based on backstepping technique for systems (1), so that the angular position  $y_1 = x_1 = [q_1, q_2]^T$  can track the reference signal  $y_r$  and all signals of the closed-loop system are semiglobal uniformly ultimately bounded.

*Notation 1* (see [34]). The matrix  $2C(q, \dot{q}) - \dot{M}(q)$  represents skew-symmetric when matrix  $C$  has an appropriate definition.

*Notation 2* (see [36]). The matrix  $M(q)$  stands for symmetric and positive definite.

*Assumption 1* (see [37]). The first and second derivatives of the reference signal  $y_r$  are continuous and bounded.

*Assumption 2* (see [38, 39]). For the unknown and smooth nonlinear function  $T(\cdot)$  satisfying  $|T(\cdot)| \leq \bar{T}(\cdot)$ , where  $\bar{T}(\cdot)$  is a positive and smooth function, we can obtain the following inequality:

$$\begin{aligned} & \|T(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t)))\| \\ & \leq \bar{T}(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))). \end{aligned} \quad (3)$$

*Assumption 3* (see [38, 39]). There exist positive and smooth functions  $q_1(\cdot)$  and  $q_2(\cdot)$  that are the upper bound of the time-varying delay function  $\bar{T}(\cdot)$ ; the following inequality holds:

$$\begin{aligned} \bar{T}(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))) & \leq q_1(x_1(t - \vartheta_1(t))) \|x_1(t - \vartheta_1(t))\| \\ & + q_2(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))) \|x_2(t - \vartheta_2(t))\|. \end{aligned} \quad (4)$$

*Remark 1.* It is noticed that Assumptions 1–3 are the standard assumptions for tracking control of the system with unknown time-varying delays and similar assumptions can be found in [37–39]. Without these assumptions, the proposed scheme cannot be realized.

*Remark 2.* How to eliminate the unknown function  $T(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t)))$  with the time-varying delays  $\vartheta_1(t)$  and  $\vartheta_2(t)$  is the key to solving the problem. To settle this matter, this paper adopts the separation technique that disintegrates the unknown time-varying delays function  $T(\cdot)$  to a number of positive and continuous functions and applies the appropriate LKFs to compensate that. In the following part, in order to convince the statement, the functions  $q_1(x_1(t - \vartheta_1(t)))$  and  $q_2(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t)))$  are simplified to  $q_1$  and  $q_2$ .

### 3. Controller Design and Stability Analysis

Denote the tracking error signals as

$$\begin{aligned} z_1 &= x_1 - y_r, \\ z_2 &= x_2 - s_2, \end{aligned} \quad (5)$$

where  $s_2$  stands for the filter output.

Denote the compensating tracking error as

$$\begin{aligned} \tilde{z}_1 &= z_1 - r_1, \\ \tilde{z}_2 &= z_2 - r_2, \end{aligned} \quad (6)$$

where  $r_1$  and  $r_2$  are the compensating signals.

Then, the derivative of  $\tilde{z}_1$  can be expressed as

$$\dot{\tilde{z}}_1 = x_2 - \dot{y}_r - \dot{r}_1. \quad (7)$$

We choose

$$\dot{r}_1 = -k_1 r_1 + r_2 + s_2 - s_2^0, \quad (8)$$

where  $k_1 = \text{diag}\{k_{11}, k_{12}\}$  and  $k_{1i}$ ,  $i = 1, 2$ , represent the positive constant.  $s_2^0$  stands for the virtual controller that will be defined later.

Substituting (8) into (7) gives

$$\dot{\tilde{z}}_1 = \tilde{z}_2 - \dot{y}_r + k_1 r_1 + s_2^0. \quad (9)$$

Design the virtual control law as

$$s_2^0 = -k_1 z_1 + \dot{y}_r. \quad (10)$$

From (10), (9) can be obtained as

$$\dot{\tilde{z}}_1 = -k_1 \tilde{z}_1 + \tilde{z}_2. \quad (11)$$

Choose a Lyapunov function candidate in the following:

$$V_1 = \frac{1}{2} \tilde{z}_1^T \tilde{z}_1. \quad (12)$$

Taking the derivative of  $V_1$  yields

$$\begin{aligned} \dot{V}_1 &= \tilde{z}_1^T \dot{\tilde{z}}_1 \\ &= -\sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 + \tilde{z}_1^T \tilde{z}_2. \end{aligned} \quad (13)$$

The derivative of  $\tilde{z}_2$  can be described as

$$\dot{\tilde{z}}_2 = \dot{x}_2 - \dot{s}_2 - \dot{r}_2. \quad (14)$$

We choose

$$\dot{r}_2 = -k_2 r_2 - r_1. \quad (15)$$

Substituting (15) into (14), we can get

$$\dot{\tilde{z}}_2 = \dot{x}_2 - \dot{s}_2 + k_2 r_2 + r_1. \quad (16)$$

Consider the state-space model of system (2), (16) can be changed as

$$\begin{aligned} \dot{\tilde{z}}_2 &= M^{-1}(x_1) (-C(x_1, x_2) x_2 - G(x_1) - J^T(x_1) p(t) \\ & + u - T(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t)))) - \dot{s}_2 + k_2 r_2 + r_1. \end{aligned} \quad (17)$$

Choose a Lyapunov function candidate from the following:

$$V_2 = V_1 + \frac{1}{2} \tilde{z}_2^T M(x_1) \tilde{z}_2 + \sum_{i=1}^2 \frac{1}{2\beta} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i, \quad (18)$$

where  $\tilde{W}_i = W_i^* - \hat{W}_i$  represents the weight estimation error and  $\beta$  stands for the positive constant.

Then, the derivative of  $V_2$  can be expressed as

$$\dot{V}_2 = - \sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 + \tilde{z}_1^T \tilde{z}_2 + \tilde{z}_2^T M(x_1) \tilde{z}_2 - \sum_{i=1}^2 \frac{1}{\beta} \tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i. \quad (19)$$

Substituting (17) into (19) gives

$$\begin{aligned} \dot{V}_2 = & - \sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 + \tilde{z}_1^T \tilde{z}_2 + \tilde{z}_2^T (-C(x_1, x_2) x_2 - G(x_1) - J^T(x_1) p(t) + M(x_1) (-\dot{s}_2 + k_2 r_2 + r_1)) \\ & + \tilde{z}_2^T u - \sum_{i=1}^2 \frac{1}{\beta} \tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i - \tilde{z}_2^T T(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))). \end{aligned} \quad (20)$$

Combining Young's inequalities with Assumption 2, we can get

$$\begin{aligned} & - \tilde{z}_2^T T(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))) \\ & \leq \frac{1}{4\varrho} \tilde{z}_2^T \tilde{z}_2 + \varrho \bar{T}^2 (x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))), \end{aligned} \quad (21)$$

where  $\varrho = 1 - \vartheta$  is a positive constant.

Based on (21), (20) is rewritten as

$$\begin{aligned} \dot{V}_2 \leq & - \sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 + \tilde{z}_1^T \tilde{z}_2 + \tilde{z}_2^T \left( -C(x_1, x_2) x_2 - G(x_1) - J^T(x_1) p(t) + M(x_1) (-\dot{s}_2 + k_2 r_2 + r_1) \tilde{W}_i + \tilde{z}_2^T u - \sum_{i=1}^2 \frac{1}{\beta} \tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i \right) \\ & + \frac{1}{4\varrho} \tilde{z}_2^T \tilde{z}_2 + \varrho \bar{T}^2 (x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))). \end{aligned} \quad (22)$$

Denote the unknown function  $H(Z)$  as follows:

$$\begin{aligned} H(Z) = & -C(x_1, x_2) x_2 - G(x_1) + M(x_1) (-\dot{s}_2 + k_2 r_2 + r_1) \\ & - J^T(x_1) p(t) + \frac{1}{z_2} \sum_{k=1}^2 \exp(\vartheta_k(t)) \left( q_k^2 \|x_k(t)\|^2 \right). \end{aligned} \quad (23)$$

According to (23), (22) can be changed as

$$\begin{aligned} \dot{V}_2 \leq & - \sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 + \tilde{z}_1^T \tilde{z}_2 + \tilde{z}_2^T H(Z) + \frac{1}{4\varrho} \tilde{z}_2^T \tilde{z}_2 - \sum_{k=1}^2 \exp(\vartheta_k(t)) \left( q_k^2 \|x_k(t)\|^2 \right) \\ & + \tilde{z}_2^T u + \varrho \bar{T}^2 (x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))) - \sum_{i=1}^2 \frac{1}{\beta} \tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i. \end{aligned} \quad (24)$$

On the basis of the neural network approximation, the function  $H(Z)$  can be approximated as

$$H_i(Z) = W_i^* S_i(Z) + \varepsilon_i(Z), \quad (25)$$

where  $Z$  stands for the input vector;  $W_i^*$  represents the optimal weight vector of the neural networks;

$S_i(Z) = [S_{i1}(Z), \dots, S_{i l_i}(Z)]^T$  denotes Gaussian basis function vector; and  $l_i$  represents the node number of the neural networks. Let  $W^* = \text{diag}\{W_1^*, W_2^*\}$ ,  $\hat{W} = \text{diag}\{\hat{W}_1, \hat{W}_2\}$ , and  $S(Z) = \text{diag}\{S_1(Z), S_2(Z)\}$ ;  $\bar{\varepsilon}_i$  denotes a positive constant, and it represents the upper bound of approximation error  $\varepsilon_i(Z)$ , which satisfies  $|\varepsilon_i(Z)| \leq \bar{\varepsilon}_i$ .

Then, define the piecewise function as

$$Y(\tilde{z}_2) = \begin{cases} 0, & \tilde{z}_2 = [0, 0]^T, \\ 1, & \text{otherwise.} \end{cases} \quad (26)$$

In terms of application practice, the control performance of the system is optimal when  $\tilde{z}_2 = [0, 0]^T$ . According to (19), it can be expressed as  $\dot{V}_2 = -\sum_{i=1}^2 k_1 \tilde{z}_1^T \tilde{z}_1 - \sum_{i=1}^2 1/\beta \tilde{W}_i \Gamma_i^{-1} \dot{\tilde{W}}_i$ . The above equation is rewritten as  $\dot{V}_2 = -\sum_{i=1}^2 k_1 \tilde{z}_1^T \tilde{z}_1$  when the neural networks are left out. Employing Barbalat's lemma can testify the stability when  $\tilde{z}_2 = [0, 0]^T$ .

*Remark 3.* Noticing equation (23) when  $\tilde{z}_2 = [0, 0]^T$ , the appearance of singularity problem leads to a major difficulty. L'Hopital's rule is not satisfied because the numerator and denominator are not guaranteed to be zero at the same time and neural networks cannot be used to approximate the function. In order to solve the singularity problem, the piecewise function method will be utilized to work out the difficulties in the process of controller design. In the meantime, it is worth noting that the tracking performance of the control system is better when  $\tilde{z}_2 = [0, 0]^T$ .

The neural networks can be employed to approximate the unknown function  $H(Z)$  when  $\tilde{z}_2 \neq [0, 0]^T$ . Then, adding (25) into (24), we gain

$$\begin{aligned} \dot{V}_2 \leq & -\sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 + \tilde{z}_1^T \tilde{z}_2 + \sum_{i=1}^2 \tilde{z}_{2i} W_i^* S_i(Z) + \sum_{i=1}^2 \tilde{z}_{2i} \varepsilon_i(Z) + \frac{1}{4q} \tilde{z}_2^T \tilde{z}_2 - \sum_{k=1}^2 \exp(\vartheta_k(t)) \left( q_k^2 \|x_k(t)\|^2 \right) \\ & + \tilde{z}_2^T u + q\bar{T}^2 (x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))) - \sum_{i=1}^2 \frac{1}{\beta} \tilde{W}_i \Gamma_i^{-1} \dot{\tilde{W}}_i. \end{aligned} \quad (27)$$

According to Young's inequality, we can get

$$\sum_{i=1}^2 \tilde{z}_{2i} \varepsilon_i(Z) \leq \frac{1}{2} \sum_{i=1}^2 \frac{1}{\eta} \tilde{z}_{2i}^2 + \frac{1}{2} \sum_{i=1}^2 \eta \bar{\varepsilon}_i^2, \quad (28)$$

where  $\eta$  represent positive constant.

Adding (28) into (27), we can obtain

$$\begin{aligned} \dot{V}_2 \leq & -\sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 + \tilde{z}_1^T \tilde{z}_2 + \sum_{i=1}^2 \tilde{z}_{2i} W_i^* S_i(Z) + \frac{1}{2} \sum_{i=1}^2 \frac{1}{\eta} \tilde{z}_{2i}^2 \\ & + \frac{1}{2} \sum_{i=1}^2 \eta_i \bar{\varepsilon}_i^2 - \sum_{k=1}^2 \exp(\vartheta_k(t)) \left( q_k^2 \|x_k(t)\|^2 \right) + \tilde{z}_2^T u \\ & + \frac{1}{4} \tilde{z}_2^T \tilde{z}_2 + q\bar{T}^2 (x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))) \\ & - \sum_{i=1}^2 \frac{1}{\beta} \tilde{W}_i \Gamma_i^{-1} \dot{\tilde{W}}_i. \end{aligned} \quad (29)$$

Design the actual controller  $u$  by the following:

$$u = Y(\tilde{z}_2) \left( -k_3 \tilde{z}_2 - \frac{\tilde{z}_2}{2\eta} - \frac{\tilde{z}_2}{4q} - \tilde{W}^T S(Z) \right), \quad (30)$$

where  $k_3 = \text{diag}\{k_{31}, k_{32}\}$ ,  $k_{3i}$ ,  $i = 1, 2$ , represent positive constant, and  $\eta = \text{diag}\{\eta_1, \eta_2\}$ .

Adding (30) into (29) when  $\tilde{z}_2 \neq [0, 0]^T$ , it is rewritten as

$$\begin{aligned} \dot{V}_2 \leq & -\sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 + \tilde{z}_1^T \tilde{z}_2 + \sum_{i=1}^2 \tilde{z}_{2i} \tilde{W}_i S_i(Z) + \frac{1}{2} \sum_{i=1}^2 \eta_i \bar{\varepsilon}_i^2 - \sum_{k=1}^2 \exp(\vartheta_k(t)) \left( q_k^2 \|x_k(t)\|^2 \right) \\ & - \sum_{i=1}^2 k_{3i} \tilde{z}_{2i}^2 + q\bar{T}^2 (x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))) - \sum_{i=1}^2 \frac{1}{\beta} \tilde{W}_i \Gamma_i^{-1} \dot{\tilde{W}}_i. \end{aligned} \quad (31)$$

The adaptation law is considered in the following:

$$\dot{\widehat{W}}_i = Y(\widetilde{z}_2)\beta\Gamma_i(\widetilde{z}_{2i}S_i(Z) - \sigma_i\widehat{W}_i), \quad (32)$$

where  $\sigma_i$  represents the positive constant.

Adding (32) into (31) when  $\widetilde{z}_2 \neq [0, 0]^T$ , (31) will become

$$\begin{aligned} \dot{V}_2 \leq & -\sum_{i=1}^2 k_{1i}\widetilde{z}_{1i}^2 + \widetilde{z}_1^T \widetilde{z}_2 - \sum_{i=1}^2 k_{3i}\widetilde{z}_{2i}^2 + \frac{1}{2} \sum_{i=1}^2 \eta \widetilde{\varepsilon}_i^2 - \sum_{k=1}^2 \exp(\vartheta_k(t)) \left( q_k^2 \|x_k(t)\|^2 \right) \\ & + \sum_{i=1}^2 \sigma_i \widetilde{W}_i \widehat{W}_i + \varrho \overline{T}^2 (x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))). \end{aligned} \quad (33)$$

On the basis of the term  $\sum_{i=1}^2 \sigma_i \widetilde{W}_i \widehat{W}_i$  and the equation  $\widehat{W}_i = W_i^* - \widetilde{W}_i$ , we can obtain the following:

$$\sum_{i=1}^2 \sigma_i \widetilde{W}_i \widehat{W}_i = \sum_{i=1}^2 \sigma_i \widetilde{W}_i W_i^* - \sum_{i=1}^2 \sigma_i \|\widetilde{W}_i\|^2. \quad (34)$$

According to Young's inequality, we get

$$\sum_{i=1}^2 \sigma_i \widetilde{W}_i W_i^* \leq \frac{1}{2} \sum_{i=1}^2 \sigma_i \|\widetilde{W}_i\|^2 + \frac{1}{2} \sum_{i=1}^2 \sigma_i \|W_i^*\|^2. \quad (35)$$

Based on inequation (35), (34) can be rewritten as

$$\sum_{i=1}^2 \sigma_i \widetilde{W}_i \widehat{W}_i \leq -\frac{1}{2} \sum_{i=1}^2 \sigma_i \|\widetilde{W}_i\|^2 + \frac{1}{2} \sum_{i=1}^2 \sigma_i \|W_i^*\|^2. \quad (36)$$

Adding (36) into (33), we can further obtain

$$\begin{aligned} \dot{V}_2 \leq & -\sum_{i=1}^2 k_{1i}\widetilde{z}_{1i}^2 + \widetilde{z}_1^T \widetilde{z}_2 - \sum_{i=1}^2 k_{3i}\widetilde{z}_{2i}^2 - \frac{1}{2} \sum_{i=1}^2 \sigma_i \|\widetilde{W}_i\|^2 - \sum_{k=1}^2 \exp(\vartheta_k(t)) \left( q_k^2 \|x_k(t)\|^2 \right) + \frac{1}{2} \sum_{i=1}^2 \sigma_i \|W_i^*\|^2 \\ & + \frac{1}{2} \sum_{i=1}^2 \eta \widetilde{\varepsilon}_i^2 + \varrho \overline{T}^2 (x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))). \end{aligned} \quad (37)$$

On the basis of Assumption 3, we can rewrite the time-varying delay function  $\varrho \overline{T}^2(\cdot)$  as

$$\begin{aligned} \varrho \overline{T}^2(x_1(t - \vartheta_1(t)), x_2(t - \vartheta_2(t))) \leq & \varrho q_1^2 \|x_1(t - \vartheta_1(t))\|^2 \\ & + \varrho q_2^2 \|x_2(t - \vartheta_2(t))\|^2. \end{aligned} \quad (38)$$

Substituting (38) into (37), we obtain

$$\begin{aligned} \dot{V}_2 \leq & -\sum_{i=1}^2 k_{1i}\widetilde{z}_{1i}^2 + \widetilde{z}_1^T \widetilde{z}_2 - \sum_{i=1}^2 k_{3i}\widetilde{z}_{2i}^2 - \frac{1}{2} \sum_{i=1}^2 \sigma_i \|\widetilde{W}_i\|^2 + \varrho q_1^2 \|x_1(t - \vartheta_1(t))\|^2 + \varrho q_2^2 \|x_2(t - \vartheta_2(t))\|^2 \\ & + \frac{1}{2} \sum_{i=1}^2 \sigma_i \|W_i^*\|^2 + \frac{1}{2} \sum_{i=1}^2 \eta \widetilde{\varepsilon}_i^2 - \sum_{k=1}^2 \exp(\vartheta_k(t)) q_k^2 \|x_k(t)\|^2. \end{aligned} \quad (39)$$

Choose the LKF candidate as follows:

$$V_l = Y(\tilde{z}_2) \sum_{k=1}^2 \left( \exp(-(t - \vartheta_k(t))) \times \int_{t-\vartheta_k(t)}^t \exp(s) q_k^2 \|x_k(s)\|^2 ds \right). \quad (40)$$

*Remark 4.* For  $\tilde{z}_2 \neq [0, 0]^T$ , we think about Remark 3 and the piecewise function  $Y(\tilde{z}_2)$  at the same time and use LKFs to fully eliminate the positive continuous functions. The LKFs

are very effective way to settle the question when there are unknown time-varying delays in the system.

Then,  $\dot{V}_l$  can be expressed as

$$\begin{aligned} \dot{V}_l = & \sum_{k=1}^2 -\left(1 - \dot{\vartheta}_k(t)\right) \left( \exp(-(t - \vartheta_k(t))) \times \int_{t-\vartheta_k(t)}^t \exp(s) q_k^2 \|x_k(s)\|^2 ds \right) \\ & - \sum_{k=1}^2 \left(1 - \dot{\vartheta}_k(t)\right) (q_k \|x_k(t - \vartheta_k(t))\|)^2 + \sum_{k=1}^2 \exp(\vartheta(t)) q_k^2 \|x_k(t)\|^2. \end{aligned} \quad (41)$$

On the basis of  $\dot{\vartheta}_k(t) \leq \vartheta \leq 1$  and  $\varrho = 1 - \vartheta$ , we get  $-(1 - \dot{\vartheta}_k(t)) \leq -(1 - \vartheta) = -\varrho$ . Furthermore, (41) can be redescrbed as

$$\begin{aligned} \dot{V}_l \leq & -\varrho \left( q_1^2 \|x_1(t - \vartheta_1(t))\|^2 + q_2^2 \|x_2(t - \vartheta_2(t))\|^2 \right) - \varrho V_l \\ & + \sum_{k=1}^2 \exp(\vartheta_k(t)) q_k^2 \|x_k(t)\|^2. \end{aligned} \quad (42)$$

Construct the Lyapunov function candidate as follows:

$$V = V_2 + V_l. \quad (43)$$

According to (39), (42), and (43), the derivative of  $V$  is redescrbed as

$$\begin{aligned} \dot{V} \leq & -\sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 + \tilde{z}_1^T \tilde{z}_2 - \sum_{i=1}^2 k_{3i} \tilde{z}_{2i}^2 + \frac{1}{2} \sum_{i=1}^2 \eta \tilde{e}_i^2 - \varrho V_l \\ & - \frac{1}{2} \sum_{i=1}^2 \sigma_i \|\tilde{W}_i\|^2 + \frac{1}{2} \sum_{i=1}^2 \sigma_i \|W_i^*\|^2. \end{aligned} \quad (44)$$

On the basis of Young's inequality, we can easily obtain

$$\tilde{z}_1^T \tilde{z}_2 \leq \frac{1-\varrho}{2} \tilde{z}_1^2 + \frac{1-\varrho}{2} \tilde{z}_2^2. \quad (45)$$

Substituting (45) into (44), we get

$$\dot{V} \leq -\sum_{i=1}^2 \tilde{k}_{1i} \tilde{z}_{1i}^2 - \sum_{i=1}^2 \tilde{k}_{2i} \tilde{z}_{2i}^2 - \varrho V_l - \frac{1}{2} \sum_{i=1}^2 \sigma_i \|\tilde{W}_i\|^2 + \frac{1}{2} \sum_{i=1}^2 \sigma_i \|W_i^*\|^2 + \frac{1}{2} \sum_{i=1}^2 \eta \tilde{e}_i^2, \quad (46)$$

where  $\tilde{k}_{1i}$  and  $\tilde{k}_{2i}$ ,  $i=1, 2$ , represent positive constant,  $k_{1i} = k_{1i} - (1/2)$ , and  $k_{2i} = k_{3i} - (1/2)$ .

Then, inequation (46) can be rewritten as

$$\dot{V} \leq -\kappa V + \phi, \quad (47)$$

where

$$\kappa = \min \left\{ 2\tilde{k}_{11}, 2\tilde{k}_{12}, 2\tilde{k}_{21}, 2\tilde{k}_{22}, \varrho, \sigma_i \varsigma_{\min}(\Gamma_i), i = 1, 2 \right\}, \quad (48)$$

$$\phi = \frac{1}{2} \sum_{i=1}^2 \sigma_i \|W_i^*\|^2 + \frac{1}{2} \sum_{i=1}^2 \eta \tilde{e}_i^2.$$

The parameter selection of the above controller design process directly affects the control performance. And, the control performance can be improved by increasing the

design parameter  $k_1$  and decreasing the design parameters  $\eta$ ,  $\sigma_1$ , and  $\sigma_2$ .

**Theorem 1.** *Assumptions 1–3, the virtual control law (10), the actual controller (30), and the adaptive law (32) are considered for the 2-joint rigid manipulator with unknown time-varying delays; by selecting appropriate parameters, the tracking errors can converge to a compact neighborhood with respect to zero, and all signals of the closed-loop system are semiglobal uniformly ultimately bounded.*

*Proof.* We will discuss two cases when  $\tilde{z}_2 \neq [0, 0]^T$  and  $\tilde{z}_2 = [0, 0]^T$ .

Case 1: for  $\tilde{z}_2 \neq [0, 0]^T$ , we can obtain

$$V = \frac{1}{2} \tilde{z}_1^T \tilde{z}_1 + \frac{1}{2} \tilde{z}_2^T M(x_1) \tilde{z}_2 + \frac{1}{2\beta} \sum_{i=1}^2 \tilde{W}_i \Gamma_i^{-1} \tilde{W}_i + \sum_{k=1}^2 \left( \exp(-(t - \vartheta_k(t))) \times \int_{t-\vartheta_k(t)}^t \exp(s) q_k^2 \|x_k(s)\|^2 ds \right). \quad (49)$$

Let us multiply both sides of (47) by  $e^{\kappa t}$ ; we can obtain

$$\frac{d(V(t)e^{\kappa t})}{dt} \leq \phi e^{\kappa t}. \quad (50)$$

Then, integrating (50) from  $[0, t]$ , (50) will become

$$V(t) \leq \left( V(0) - \frac{\phi}{\kappa} \right) e^{-\kappa t} + \frac{\phi}{\kappa} \leq V(0) e^{-\kappa t} + \frac{\phi}{\kappa}. \quad (51)$$

Obviously, each of the terms of (49) is greater than zero, so we can get the following inequality:

$$\frac{1}{2} \tilde{z}_1^T \tilde{z}_1 \leq V(t) \leq V(0) e^{-\kappa t} + \frac{\phi}{\kappa}. \quad (52)$$

On the basis of (49), we easily obtain

$$V(0) = \frac{1}{2} \tilde{z}_1^T(0) \tilde{z}_1(0) + \frac{1}{2} \tilde{z}_2^T(0) \tilde{z}_2(0) + \frac{1}{2\beta} \sum_{i=1}^2 \varsigma_{\max}(\Gamma_i^{-1}) \|\tilde{W}_i(0) - W_i^*\|^2 + \sum_{k=1}^2 \left( \exp(-(0 - \vartheta_k(0))) \times \int_{0-\vartheta_k(0)}^0 \exp(s) (q_k^2 \|x_k(s)\|^2) ds \right). \quad (53)$$

Next, we can obtain

$$-D_1 \leq \tilde{z}_{1i} \leq D_1, \quad (54)$$

where  $D_1 = \sqrt{2(V(0)e^{-\kappa t} + \phi/\kappa)}$ .

Then, we can get the compact set of  $\tilde{z}_1$ :

$$\Omega_1 = \left\{ \tilde{z}_1 \in R^2 \mid \|\tilde{z}_{1i}\| \leq \sqrt{2\left(V(0)e^{-\kappa t} + \frac{\phi}{\kappa}\right)}, i = 1, 2 \right\}. \quad (55)$$

At the same time, we can obtain

$$\frac{1}{2} \tilde{z}_2^T M(x_1) \tilde{z}_2 \leq V(0) e^{-\kappa t} + \frac{\phi}{\kappa}. \quad (56)$$

Next, we have

$$-D_2 \leq \tilde{z}_{2i} \leq D_2, \quad (57)$$

where  $D_2 = \sqrt{V(0)e^{-\kappa t} + (\phi/\kappa)/\varsigma_{\min}(M(x_1))}$ .

The compact set of  $\tilde{z}_2$  can be obtained as

$$\Omega_2 = \left\{ \tilde{z}_2 \in R^2 \mid \|\tilde{z}_{2i}\| \leq \sqrt{\frac{V(0)e^{-\kappa t} + (\phi/\kappa)}{\varsigma_{\min}(M(x_1))}}, i = 1, 2 \right\}. \quad (58)$$

Case 2: for  $\tilde{z}_2 = [0, 0]^T$ , we can get controller  $u = 0$ , adaptive law  $\dot{W}_i = 0$ , and LKFs  $V_i = 0$ , so (49) will be rewritten as

$$V = \frac{1}{2} \tilde{z}_1^T \tilde{z}_1. \quad (59)$$

The derivative of  $V$  is expressed in the following:

$$\dot{V} = \tilde{z}_1^T \dot{\tilde{z}}_1. \quad (60)$$

On the basis of (11), we can obtain

$$\dot{V} = - \sum_{i=1}^2 k_{1i} \tilde{z}_{1i}^2 \leq 0. \quad (61)$$

According to functional monotony, we can easily get

$$V(t) \leq V(0). \quad (62)$$

Then, (62) can be expressed as

$$\frac{1}{2} \tilde{z}_1^T \tilde{z}_1 \leq V(0). \quad (63)$$

On the basis of (59), we can get

$$V(0) = \frac{1}{2} \tilde{z}_1^T(0) \tilde{z}_1(0). \quad (64)$$

Then, we can further have

$$-D_3 \leq \tilde{z}_{1i} \leq D_3, \quad (65)$$

where  $D_3 = \sqrt{2V(0)}$ .

The compact set of  $\tilde{z}_2$  can be obtained as

$$\Omega_3 = \left\{ \tilde{z}_1 \in R^2 \mid \|\tilde{z}_{1i}\| \leq \sqrt{2V(0)}, i = 1, 2 \right\}. \quad (66)$$

#### 4. Simulation Example

The dynamic model for the two-joint robotic manipulator with unknown time-varying delay is expressed as

$$M(q)\dot{q} + C(q, \dot{q})\dot{q} + T(q(t - \vartheta_1(t)), \dot{q}(t - \vartheta_2(t))) + G(q) = u - J^T(q)p(t). \quad (67)$$

Choose the gravitational force as follows:



$$\begin{aligned}
G(q) &= [G_{11}, G_{21}]^T, \\
G_{11} &= (m_a L_{cb} + m_b L_a)g \cos q_1 + m_b L_{cb} g \cos(q_1 + q_2), \\
G_{21} &= m_b L_{cb} g \cos(q_1 + q_2).
\end{aligned} \quad (68)$$

Consider the inertia matrix as

$$\begin{aligned}
M(q) &= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^T, \\
M_{11} &= m_b(L_a^2 + L_{cb}^2 + 2L_a L_{cb} \cos q_2) + m_a L_{ca}^2 + I_a + I_b, \\
M_{12} &= m_b(L_{cb}^2 + L_a L_{cb} \cos q_2) + I_b, \\
M_{21} &= m_b(L_{cb}^2 + L_a L_{cb} \cos q_2) + I_b, \\
M_{22} &= m_b L_{cb}^2 + I_b.
\end{aligned} \quad (69)$$

The centripetal and Coriolis torques are selected from the following:

$$\begin{aligned}
C(q, \dot{q}) &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T, \\
C_{11} &= -m_b L_a L_{cb} \dot{q}_2 \sin q_2, \\
C_{12} &= -m_b L_a L_{cb} (\dot{q}_1 + \dot{q}_2) \sin q_2, \\
C_{21} &= m_b L_a L_{cb} \dot{q}_2 \sin q_2, \\
C_{22} &= 0.
\end{aligned} \quad (70)$$

Choose the reversible Jacobian matrix as

$$\begin{aligned}
J(q) &= \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}^T, \\
J_{11} &= -L_a \sin q_1 + L_b \sin(q_1 + q_2), \\
J_{12} &= -L_b \sin(q_1 + q_2), \\
J_{21} &= L_a \cos q_1 + L_b \cos(q_1 + q_2), \\
J_{22} &= L_b \cos(q_1 + q_2).
\end{aligned} \quad (71)$$

The time delay function is chosen as

$$\begin{aligned}
T(q, \dot{q}) &= [T_1, T_2]^T, \\
T_1 &= \sin(q_1(t - \vartheta_1(t)))q_2(t - \vartheta_2(t)) \\
&\quad + e^{(q_1(t - \vartheta_1(t)) + q_2(t - \vartheta_2(t)))}, \\
T_2 &= \cos(q_1(t - \vartheta_1(t)) + q_2(t - \vartheta_2(t))) \\
&\quad + q_1(t - \vartheta_1(t))q_2(t - \vartheta_2(t)).
\end{aligned} \quad (72)$$

The virtual control law is considered as

$$s_2^0 = -k_1 z_1 + \dot{y}_r. \quad (73)$$

Consider the controller  $u$  as follows:

$$u = Y(\tilde{z}_2) \left( -k_3 \tilde{z}_2 - \frac{\tilde{z}_2}{2\eta} - \frac{\tilde{z}_2}{4\varrho} - \hat{W}^T S(Z) \right). \quad (74)$$

The NNs adaptation law is chosen as

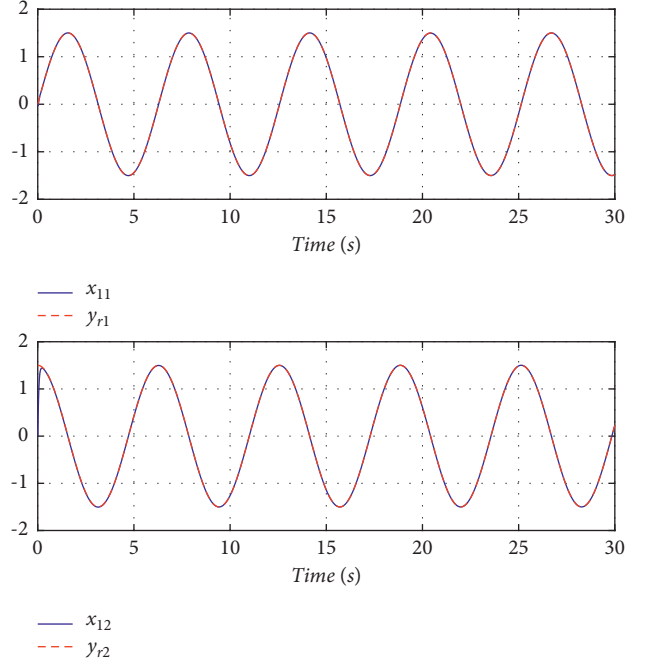


FIGURE 1: Trajectories of  $x_1$  and reference  $y_r$ .

$$\hat{W}_i = Y(\tilde{z}_2) \beta \Gamma_i (\tilde{z}_2^T S_i(Z) - \sigma_i \hat{W}_i). \quad (75)$$

The first-order filter is selected as

$$F(t) \dot{s}_2 + s_2 = s_2^0, \quad (76)$$

where  $s_2^0$  is the input and  $s_2$  is the output of the first-order filter;  $s_2(0) = 0$  is the initial condition;  $F(t)$  is settled as  $F(t) = ae^{-\omega t} + c$ , and we have the parameters  $a > 0$ ,  $\omega > 0$ , and  $c > 0$ .

In the process of simulation, the initial conditions are  $q_1(0) = q_2(0) = 0$  and  $\dot{q}_1(0) = \dot{q}_2(0) = 0$ ; the link mass is  $m_a = 1$  kg and  $m_b = 0.7$  kg; the link length is  $L_a = 0.36$  m and  $L_b = 0.32$  m;  $L_{ca}$  and  $L_{cb}$  represent the midpoint of link; the link inertia is  $I_a = 56.12 \times 10^{-3}$  kgm<sup>2</sup> and  $I_b = 19.78 \times 10^{-3}$  kgm<sup>2</sup>; the gravitational acceleration is  $g = 9.81$  m/s<sup>2</sup>; the external disturbance is selected as  $p = [0.5 \sin(t) + 1.5, 1.5 \cos(t) + 0.5]^T$ ; the reference signal stands for  $y_r = [1.5 \sin(t), 1.5 \cos(t)]^T$ ; the unknown time-varying delays are  $\vartheta_1(t) = 1.5 \sin(0.8t)$  and  $\vartheta_2(t) = 0.7 \cos(1.2t)$ ; the design parameters are selected as  $k_1 = \text{diag}\{19, 19\}$ ,  $k_3 = \text{diag}\{37, 37\}$ ,  $\eta = \text{diag}\{0.01, 0.02\}$ ,  $\varrho = 0.02$ ,  $\sigma_1 = 0.005$ , and  $\sigma_2 = 0.001$ . In the process of simulation, it can be discovered that  $k_1$  is selected appropriately larger and the design parameter  $\eta$ ,  $\sigma_1$ , and  $\sigma_2$  are decreased and there will be good control performance. Neural network nodes are  $l_1 = l_2 = 20$ ,  $\beta = 10$ ,  $\Gamma_1 = 0.5 \text{diag}\{\text{ones}(1, 20)\}$ , and  $\Gamma_2 = 0.3 \text{diag}\{\text{ones}(1, 20)\}$ . The first-order filter design parameters are chosen as  $a = 0.001$ ,  $\omega = 0.001$ , and  $c = 0.001$ .

According to the design of the control method, the simulation consequences are shown by Figures 1–3. From Figure 1, we see that the output signal can follow the desired reference signal  $y_r$ . Figure 2 presents the trajectories of  $x_{11}$ ,

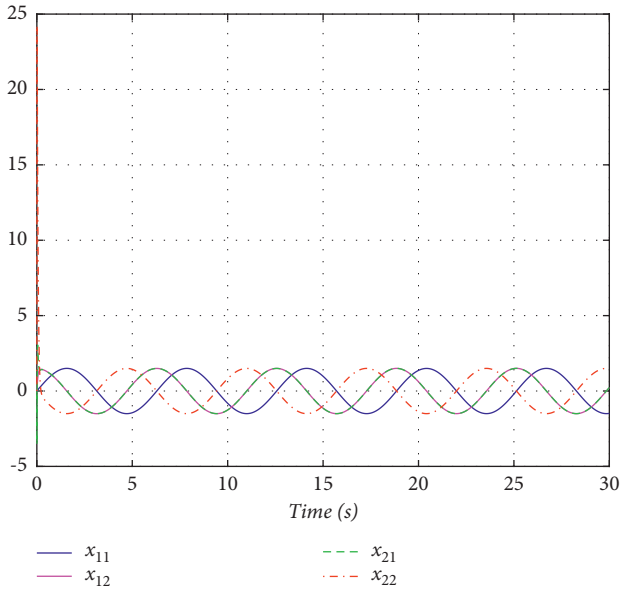


FIGURE 2: Trajectories of  $x_{11}$ ,  $x_{12}$ ,  $x_{21}$ , and  $x_{22}$ .

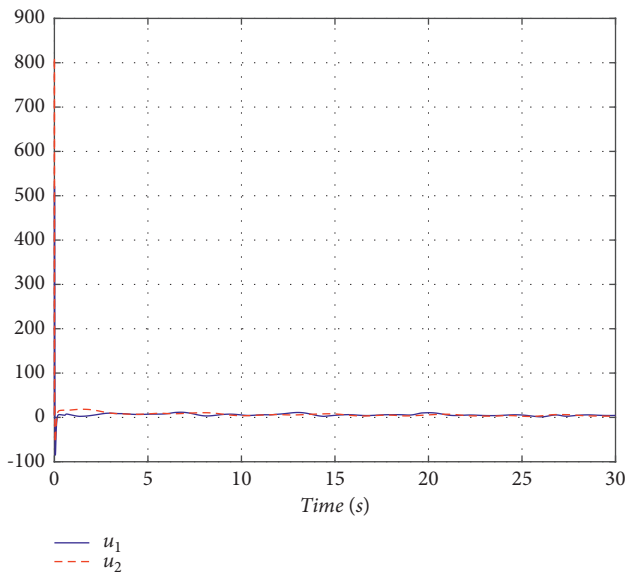


FIGURE 3: Trajectories of  $u$ .

$x_{12}$ ,  $x_{21}$ , and  $x_{22}$ , which illustrate distinctly that all the signals are bounded stable. Figure 3 shows the actual controller. From Figures 1–3, the proposed control method is verified to be effective.

## 5. Conclusion

In this paper, an adaptive neural tracking controller has been investigated for a two-joint rigid manipulator with unknown time-varying delays. The command filter technology is adopted in the traditional backstepping process to avoid repeated derivation of the virtual controller. In order to work out the unknown time-varying delay issue in two-joint rigid manipulators, a method that combines separation technology with LKFs is proposed. The piecewise function is constructed with the aim to settle the singularity issue in controller design process. By utilizing Lyapunov analysis, it has been proved that the adaptive neural tracking method can guarantee that all signals in the closed-loop system are bounded, and the tracking error can converge to a compact neighborhood with respect to zero. Eventually, the simulation example is verified with the effectiveness of the control approach. In the future, we will study the control method of the robotic manipulator with prespecified tracking accuracy.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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