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Research Article

Stability Analysis for Differential Equations of the General Conformable Type

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Fractional calculus is nowadays an efficient tool in modelling many interesting nonlinear phenomena. This study investigates, in a novel way, the Ulam–Hyers (HU) and Ulam–Hyers–Rassias (HUR) stability of differential equations with general conformable derivative (GCD). In our analysis, we employ some version of Banach fixed-point theory (FPT). In this way, we generalize several earlier interesting results. Two examples are given at the end to illustrate our results.

1. Introduction

The stability issue gained a considerable attention in various research fields through applications. There are many kinds of stability, among them is the stability introduced by S. M. Ulam, in his famous talk at a conference held in Wisconsin University in 1940. Since then, it is known as HU stability or simply Ulam stability. Its applications for various types of differential equations have been investigated by many researchers. The readers can see the interesting results in 1–7, for more details. The stability problem of Ulam can be rewritten in the following form.

Consider a group G^* and a metric group (G^{**}, χ_1). Is it true that given $\varepsilon > 0$, there exist $\delta > 0$ such that if $\Lambda: G^* \longrightarrow G^{**}$ satisfies

$$\chi_1(\Lambda(x_1 x_2), \Lambda(x_1)\Lambda(x_2)) < \delta. \tag{1}$$

For all $x_1, x_2 \in G^*$, then a homomorphism $\Xi: G^* \longrightarrow G^{**}$ exists such that

$$\chi_1(\Lambda(x_1), \Xi(x_1)) < \varepsilon, \tag{2}$$

for every $x_1 \in G^*$?

The problem of Ulam has been extended in many directions for various interesting settings. In particular, Rassias (see [8]) generalized Ulam's result for Banach spaces.

Initial and boundary value problems with fractional-order derivatives are natural generalization of the classical initial and boundary value problems. It is much more complicated to investigate stability issues of fractional-order problems than their classical analogues; this is because of the singularity and nonlocality in the kernel of fractional differential operators. Fractional derivatives, in general, play negligible roles in a number of fields of science and engineering (see, e.g., [9–13] and the references there in).

In particular, during the last few decades, the area of fractional calculus has been investigated qualitatively by using different tools of functional analysis. These tools include but are not limited to *Gronwall Lemma*, see, e.g., [14], *Pachpatte's inequality*, see, e.g., [15], *Schaefer's FPT*, see, e.g., [16], *Schauder's FPT*, see, e.g., [17], *Banach FPT*, see, e.g., [18], and *Picard operator*, see, e.g., [15]. Various approaches have been used to define fractional derivatives (see, e.g., 19–28, for more details).

It should be remarked that generalized conformable derivative plays an essential role in many applications. For instance, the authors in [29] utilized it to examine some nonlinear evolution equations. Generalized conformable derivative also has been used in [30] to investigate some nonlinear evolution equations. A new generalized version of conformable derivative is given and in [31] with some applications in biological population. In the present study, we generalize several recent interesting works as follows. We use Theorem 2 to generalize the interesting results in [32, 33] by dropping some of the basic assumptions that have been used there. We also use Theorem 3 to generalize the work in [34].

The organization of the study is as follows. In Section 2, we present some preliminaries and some basic definitions. In Section 3, we introduce our stability results in the sense of HU and HUR. In Section 4, two examples are written to show the validity of our results, and in Section 5, we conclude our work.

2. Preliminaries

In this section, some definitions, lemmas, and theorems are given [35–39].

Definition 1. Let us consider a function ϕ defined on $[c,d]$; then, the GCD starting from the real c of a function ϕ is defined by

$$T_c^{\upsilon,\psi_c}\phi(z) = \lim_{\sigma\longrightarrow 0}\frac{\phi(z+\sigma\psi_c(z,\upsilon))-\phi(z)}{\sigma}. \tag{3}$$

For all $z>c$, $\upsilon\in(0,1)$ and $\psi_c(z,\upsilon)$ is a nonnegative continuous function that satisfies

$$\psi_c(z,1)=1,$$
$$\psi_c(.,\upsilon_1)\neq\psi_c(.,\upsilon_2), \text{where } \upsilon_1\neq\upsilon_2 \text{ and } \upsilon_1,\upsilon_2\in(0,1]. \tag{4}$$

If $T_c^{\upsilon,\psi_c}\phi(z)$ exists, for every $z\in(c,a)$; for some $a>c$, $\lim_{t\longrightarrow c^+}T_c^{\upsilon,\psi_c}\phi(z)$ exists; then, by definition,

$$T_c^{\upsilon,\psi_c}\phi(c) = \lim_{t\longrightarrow c^+}T_c^{\upsilon,\psi_c}\phi(z). \tag{5}$$

Remark 1. To further study the properties of GCD, we suppose that $\psi_c(z,\upsilon)>0$, for all $z>c$, and $1/\psi_c(.,\upsilon)$ is locally integrable.

Definition 2. Let $0<\upsilon<1$. The conformable fractional integral starting from c of a function ϕ is defined by

$$I_c^{\upsilon,\psi_c}\phi(z) = \int_c^z\frac{\phi(x)}{\psi_c(x,\upsilon)}dx\ . \tag{6}$$

Lemma 1. *Suppose that $\phi\in C([c,d])$. Thus,*

$$T_c^{\upsilon,\psi}I_c^{\upsilon,\psi_c}\phi(z)=\phi(z),\quad \forall z\geq c. \tag{7}$$

Lemma 2. *Suppose that $\phi\in AC^1([c,d])$. Thus,*

$$I_c^{\upsilon,\psi_c}T_c^{\upsilon,\psi}\phi(z)=\phi(z)-\phi(c),\quad \forall z\geq c. \tag{8}$$

Remark 2. Assume that $\vartheta\in\mathbb{R}^*$. If $g(z):=\mathbb{E}_\upsilon^{\psi_c}(\vartheta,z,c)=e^{\vartheta\int_c^z 1/\psi_c(x,\upsilon)dx}$, then $T_c^{\upsilon,\psi_c}g(z)=\vartheta g(z)$ and $I_c^{\upsilon,\psi_c}g(z)=1/\vartheta(g(z)-1)$.

The following is the notion of a generalized metric on some set $\mathcal{S}_1$.

Definition 3 (see 40). Consider a mapping $\varrho\colon \mathcal{S}_1\times\mathcal{S}_1\longrightarrow[0,\infty]$. The mapping ϱ is called a generalized metric on set $\mathcal{S}_1$ iff ϱ satisfies:

M1 $\varrho(o_1,o_2)=0$ if and only if $o_1=o_2$

M2 $\varrho(o_1,o_2)=\varrho(o_2,o_1)$, for all $o_1,o_2\in\mathcal{S}_1$

M3 $\varrho(o_1,o_3)\leq\varrho(o_1,o_2)+\varrho(o_2,o_3)$, for all $o_i\in\mathcal{S}_1, i=1,2,3$

The following theorem (see [40]) represents one of the interesting results of FPT. This theorem plays a fundamental role in our study.

Theorem 1. *Suppose that (Q,F) is a metric space that is generalized complete. Let $B\colon Q\longrightarrow Q$ be a strictly contractive operator. If there is an integer $t\geq 0$ with $F(\Gamma^{t+1}c,\Gamma^t c)<\infty$ for some $c\in Q$, thus*

(a) $\lim_{s\longrightarrow+\infty}B^s c=c^*$, where c^* is the unique fixed point of Γ in $Q^*:=\{c_1\in Q\colon F(B^t c,c_1)<\infty\}$

(b) If $c_1\in Q^*$, then $F(c_1,c^*)\leq 1/1-LF(Bc_1,c_1)$

Define the space X as $X:=C(I,\mathbb{R})$, with $I=[a,a+T]$ (a is some real number).

Lemma 3. *Define a metric $\eta\colon X\times X\longrightarrow[0,\infty]$ in such a way that*

$$\eta(\beta_1,\beta_2)=\inf\left\{A\in[0,\infty]\colon\frac{|\beta_1(z)-\beta_2(z)|}{\mathbb{E}_\theta^{\psi_a}(\vartheta,z,a)}\leq A\lambda(z), z\in I\right\}, \tag{9}$$

where $\vartheta>0$, $\theta\in(0,1)$, and λ is positive and continuous. Thus, (X,η) is a generalized complete metric space.

The goal of this study is to investigate the stability of the following initial value problem:

$$T_a^{\theta,\psi_a}y(z)=\xi(z,y(z)), y(a)=y_a, \tag{10}$$

in the sense of HU and HUR. Notice that the solution of the initial value problem (10) is the solution of

$$y(z)=\int_a^z\frac{\xi(p,y(p))}{\psi_a(p,\theta)}dp+y_a,\ z\in I. \tag{11}$$

3. Ulam–Hyers–Rassias Stability Results

We use this section to present our main results. The theorem below represents the stability of (10) in the sense of HU.

Theorem 2. *Suppose ξ is continuous and satisfies*

$$|\xi(z,\gamma_1)-\xi(z,\gamma_2)|\le P|\gamma_1-\gamma_2|,\quad \forall z\in I,\gamma_i\in\mathbb{R}, i=1,2. \tag{12}$$

If an absolutely continuous function $x: I \longrightarrow \mathbb{R}$ satisfies

$$|T_a^{\theta,\psi_a}x(z)-\xi(z,x(z))|\le\epsilon, \tag{13}$$

for some $\epsilon>0$, therefore, there is a unique solution x^* of (10) with

$$|x(z)-x^*(z)|\le\epsilon\frac{P+\varrho}{\varrho}M\mathbb{E}_\theta^{\psi_a}((P+\varrho),a+T,a), \tag{14}$$

for every $z\in I$, where ϱ is any positive constant and $M=\sup_{s\in[a,a+T]}(I_a^{\theta,\psi_a}(1)(s)/\mathbb{E}_\theta^{\psi_a}((P+\varrho),s,a))$.

Proof. For any $B_1,B_2\in X$, we define the metric d in this way:

$$d(B_1,B_2)=\inf\left\{V\in[0,\infty): \frac{|B_1(z)-B_2(z)|}{\mathbb{E}_\theta^{\psi_a}((P+\varrho),z,a)}\le V, z\in I\right\}. \tag{15}$$

Define the operator $\mathcal{G}: X\longrightarrow X$ such that

$$(\mathcal{G}y)(z):=x(a)+\int_a^z\frac{\xi(s,y(s))}{\psi_a(s,\theta)}ds,\quad \forall y\in X. \tag{16}$$

Since

$$\frac{|(\mathcal{G}y_0)(z)-y_0(z)|}{\mathbb{E}_\theta^{\psi_a}((P+\varrho),z,a)}<\infty,\quad \forall y_0\in X, z\in I, \tag{17}$$

so that it is clear that $d(\mathcal{G}y_0,y_0)<\infty$, in addition, we get $\{y\in X: d(y_0,y)<\infty\}=X$.

Now, we prove that $\mathcal{G}$ is strictly contractive:

$$\begin{aligned}|(\mathcal{G}y_1)(z)-(\mathcal{G}y_2)(z)|&\le\left|\int_a^z\frac{(\xi(s,y_1(s))-\xi(s,y_2(s)))}{\psi_a(s,\theta)}ds\right|\\&\le\int_a^z\frac{|\xi(s,y_1(s))-\xi(s,y_2(s))|}{\psi_a(s,\theta)}ds\\&\le P\int_a^z\frac{|y_1(s)-y_2(s)|}{\psi_a(s,\theta)}ds\\&\le P\int_a^z\frac{1}{\psi_a(s,\theta)}\frac{|y_1(s)-y_2(s)|}{\mathbb{E}_\theta^{\psi_a}((P+\varrho),s,a)}\mathbb{E}_\theta^{\psi_a}((P+\varrho),s,a)ds\\&\le\frac{P\,d(y_1,y_2)}{P+\varrho}\left(\mathbb{E}_\theta^{\psi_a}((P+\varrho),z,a)-1\right)\\&\le\frac{P\,d(y_1,y_2)}{P+\varrho}\mathbb{E}_\theta^{\psi_a}((P+\varrho),z,a)\text{for all } z\in I.\end{aligned} \tag{18}$$

So, it is clear that

$$\frac{|(\mathcal{G}y_1)(z)-(\mathcal{G}y_2)(z)|}{\mathbb{E}_\theta^{\psi_a}((P+\varrho),z,a)}\le\frac{P}{P+\varrho}d(y_1,y_2), \tag{19}$$

which implies that

$$d(\mathcal{G}y_1,\mathcal{G}y_2)\le\frac{P}{P+\varrho}d(y_1,y_2), \tag{20}$$

which prove that the operator $\mathcal{G}$ is a strictly contractive one.

We get, from (27),

$$|x(z)-\mathcal{G}x(z)|\le\epsilon\int_a^t\frac{1}{\psi_a(s,\theta)}ds\ \le\epsilon I_a^{\theta,\psi_a}(1)(z). \tag{21}$$

Therefore,

$$d(x,\mathcal{G}x)\le\epsilon M. \tag{22}$$

Now, according to Theorem 1, there is some solution x^* of (10) satisfying

$$d(x,x^*)\le\epsilon\frac{P+\varrho}{\varrho}M, \tag{23}$$

so that

$$\frac{|x(z)-x^*(z)|}{\mathbb{E}_\theta^{\psi_a}((P+\varrho),z,a)}\le\epsilon\frac{P+\varrho}{\varrho}M, \tag{24}$$

which implies that

$$|x(z)-x^*(z)|\le\epsilon\frac{P+\varrho}{\varrho}M\mathbb{E}_\theta^{\psi_a}((P+\varrho),a+T,a). \tag{25}$$

□

Remark 3. It is clear that our findings in the sense of HU are some generalized version of the results obtained in [32, 33] as follows. In our analysis, we do not impose any constrains on P unlike equation 5 in Theorem 2 in [32]. In [33], the authors assumed conditions on the function φ which is not the case in our study.

The following theorem represents the stability of (10) in HUR sense.

Theorem 3. *Suppose ξ is continuous and satisfies*

$$|\xi(z,\gamma_1)-\xi(z,\gamma_2)|\le P|\gamma_1-\gamma_2|,\quad \forall z\in I,\gamma_i\in\mathbb{R}, i=1,2. \tag{26}$$

If an absolutely continuous function $x: I\longrightarrow\mathbb{R}$ satisfies

$$|T_a^{\theta,\psi_a}x(z)-\xi(z,x(z))|\le\kappa(z), \tag{27}$$

where $\kappa(z)$ is a nondecreasing, continuous function, therefore, there is a unique solution x^* of (10) with

$$|x(z)-x^*(z)|\le\frac{P+\varrho}{\varrho}M\mathbb{E}_\theta^{\psi_a}((P+\varrho),a+T,a)\kappa(z), \tag{28}$$

for every $z\in I$, where ϱ is any positive constant and $M=\sup_{s\in[a,a+T]}(I_a^{\theta,\psi_a}(1)(s)/\mathbb{E}_\theta^{\psi_a}((P+\varrho),s,a))$.

Proof. For any $B_1,B_2\in X$, we define the metric d as follows:

$$d(B_1,B_2)=\inf\left\{V\in[0,\infty): \frac{|B_1(z)-B_2(z)|}{\mathbb{E}_\theta^{\psi_a}((P+\varrho),z,a)}\le V\kappa(z), z\in I\right\}. \tag{29}$$

Define the operator $\mathscr{G}: X \longrightarrow X$ such that

$$(\mathscr{G}y)(z) := x(a) + \int_a^z \frac{\xi(s, y(s))}{\psi_a(s,\theta)} ds, \quad \forall y \in X. \tag{30}$$

We have $d(\mathscr{G}y_0, y_0) < \infty$, for all y_0. In addition, we get $\{y \in X: d(y_0, y) < \infty\} = X$.

Now, we prove that $\mathscr{G}$ is strictly contractive:

$$\begin{aligned}
\left|(\mathscr{G}y_1)(z) - (\mathscr{G}y_2)(z)\right| &\leq \left|\int_a^z \frac{(\xi(s, y_1(s)) - \xi(s, y_2(s)))}{\psi_a(s,\theta)} \mathrm{d}s\right|, \\
&\leq P \int_a^z \frac{\left|y_1(s) - y_2(s)\right|}{\psi_a(s,\theta)} \mathrm{d}s \\
&\leq P\, d(y_1, y_2)\kappa(z) \int_a^z \frac{\mathbb{E}_\theta^{\psi_a}((P+\varrho), s, a)}{\psi_a(s,\theta)} \mathrm{d}s \\
&\leq \frac{P\, d(y_1, y_2)}{P+\varrho} \mathbb{E}_\theta^{\psi_a}((P+\varrho), z, a)\kappa(z) \text{for all } z \in I.
\end{aligned} \tag{31}$$

So,

$$d(\mathscr{G}y_1, \mathscr{G}y_2) \leq \frac{P}{P+\varrho} d(y_1, y_2), \tag{32}$$

which prove that the operator $\mathscr{G}$ is a strictly contractive one.

We get, from (13),

$$|x(z) - \mathscr{G}x(z)| \leq \int_a^t \frac{\kappa(s)}{\psi_a(s,\theta)} ds \leq \kappa(z) I_a^{\theta,\psi_a}(1)(z). \tag{33}$$

Hence,

$$d(x, \mathscr{G}x) \leq M. \tag{34}$$

Using Theorem 1, there is a solution x^* of (10) with

$$d(x, x^*) \leq \frac{P+\varrho}{\varrho} M. \tag{35}$$

Thus,

$$|x(z) - x^*(z)| \leq \frac{P+\varrho}{\varrho} M \mathbb{E}_\theta^{\psi_a}((P+\varrho), a+T, a)\kappa(z). \tag{36}$$

□

Remark 4. Notice that the authors in [34] used conformable fractional Laplace transform to study the HUR stability of several kinds of differential equations. They had to assume some specific conditions, see, e.g., condition 12 in Theorem 3.6 is given in [34].

Remark 5. The authors in [41] obtained stability results for differential equations with integer-order derivatives $\psi_a = 1$, while in our study it is for GCD. In this sense, we introduce a generalized version of the interesting results [41].

4. Examples

This section uses two examples to show the validity of results.

Example 1. Consider equation (10) for $\psi_a(z,\theta) = (z-a)^{1-\theta}$, $a = 0$, $\theta = 0.6$, $T = 2$, and $\xi(z, \nu) = z^4 \sin(\nu)$.

We have

$$|z^4 \sin(\nu_1) - z^4 \sin(\nu_2)| \leq 16|\nu_1 - \nu_2|, \quad \forall z \in [0, 9], \nu_1, \nu_2 \in \mathbb{R}. \tag{37}$$

Then, $L = 16$.

Suppose that ν satisfies

$$|T_0^{0.6,\psi_0} \nu(z) - \xi(z, \nu(z))| \leq 0.01, \tag{38}$$

for all $z \in [0, 2]$.

Here, $\epsilon = 0.01$. Using Theorem 2, there is ν^* and $M > 0$ such that

$$|\nu(z) - \nu^*(z)| \leq 0.01M, \quad \forall z \in [0, 2]. \tag{39}$$

Example 2. Consider equation (10) for $\psi_a(z,\theta) = (z-a)^{1-\theta}$, $a = 3$, $\theta = 0.3$, $T = 3$, and $\xi(z, \nu) = z \cos(\nu)$.

We have

$$|z \cos(\nu_1) - z \cos(\nu_2)| \leq 6|\nu_1 - \nu_2|, \quad \forall z \in [3, 6], \nu_1, \nu_2 \in \mathbb{R}. \tag{40}$$

Then, $L = 6$.

Suppose that ν satisfies

$$|T_3^{0.3,\psi_3} \nu(z) - \xi(z, \nu(z))| \leq (z+2), \tag{41}$$

for all $z \in [3, 6]$.

Here, $\kappa(z) = z + 2$. Using Theorem 3, there is ν^* and $M > 0$ such that

$$|\nu(z) - \nu^*(z)| \leq M(z+2), \quad \forall z \in [0, 2]. \tag{42}$$

5. Conclusion

We managed to utilize a version of Banach FPT to present stability results in the sense of HU and HUR for some differential equations involving GCDs. In our analysis, we generalized some interesting results by dropping some of the basic assumptions that have been used in recent known investigations. We used two examples to show the validity of our main findings. We believe that the methodology used in this study can be further applied to many other fractional differential equations.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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