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Research Article

A New Adaptive Controller for Nonlinear Systems with Uncertain Virtual Control Gains

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This paper addresses the adaptive asymptotic tracking control problem for nonlinear systems whose virtual control gains are unknown nonlinear functions of system states. Only in the first step, the Nussbaum gain technique is utilized to handle the uncertain virtual control gain. In the remaining steps, virtual control gains are dealt with by constructing novel control laws without the approximation of the uncertain nonlinear functions and external disturbances by neural networks or fuzzy logic. New adaptive laws are defined to compensate for unknown virtual control gains, uncertain parameters, and external disturbances. Finally, an adaptive tracking controller is designed and applied to the control of a 3-order robot system, which guarantees the boundedness of all the signals in the closed-loop system and asymptotic stability of the tracking error.

1. Introduction

The tracking control has received considerable attention for the purpose of ensuring the output of the system is tracking a desired trajectory. To deal with the uncertainties in nonlinear systems, one of the most popular methods, adaptive control, has been introduced for controller design. There have been many related results in this area [1–11]. An adaptive data-driven controller was designed for nonlinear systems using goal representation heuristic and dynamic programming [4]. In view of unknown nonlinear fractional-order systems, an adaptive control scheme was proposed [5]. For nonlinear systems with dead-zone and actuator failure, two novel finite-time adaptive tracking controllers were developed [7–9]. By introducing a new performance function, [10, 11] they constructed two adaptive tracking controllers via prescribed performance control and funnel control, respectively. It needs to be emphasized that the VCGs (virtual control gains) of systems in the above papers are assumed to be known.

However, the VCGs are uncertain for many actual systems. In response to this challenge, the Nussbaum gain technique was first proposed [12]. There have been many achievements for nonlinear systems, whose VCGs were unknown constants [13–16]. Two adaptive control strategies were given for unknown nonlinear SISO (single input, single output) systems and stochastic nonlinear systems based on state observers [13, 14]. In consideration of unmodeled dynamics and an unknown dead-zone, an improved control strategy was addressed [15]. In view of nonlinear systems with actuator faults and state/input constraints, a controller was designed based on dynamic surface and Nussbaum gain [16]. Taking into account uncertain time-varying VCGs, an improved adaptive control method was given [17], which was further improved [18] such that the Nussbaum gain technique was applied to nonlinear systems whose virtual control gains were unknown nonlinear functions of system states. To handle time-varying uncertain control gains, new Nussbaum functions were defined [19]. Two adaptive robust control schemes were proposed [20] for nonlinear systems with certain and uncertain signs of VCGs. Aiming at nonlinear systems with uncertain dead-zone output, an adaptive fuzzy control scheme was designed using a state observer [21]. To achieve full state constraints, adaptive tracking control was studied by the Barrier Lyapunov function [22]. Considering MIMO (multiple input, multiple output) nonlinear systems with input saturations and uncertain control gains, two adaptive NN (neural network) controllers were constructed [23, 24]. A dynamic surface control strategy was given for nonlinear systems with uncertain VCGs using fuzzy logic [25].

Fuzzy logic and neural networks were also usually applied to handle the unknown VCGs. The VCGs were supposed to have known upper and lower bounds and were approximated by fuzzy logic, and then the adaptive fuzzy tracking controllers were constructed for strict feedback [26] and switched nonlinear systems [27]. Two adaptive fuzzy control schemes were proposed for nonaffine [28] and nonstrict-feedback [29] nonlinear systems based on funnel control and dynamic surface control, respectively. Several adaptive NN tracking controllers were constructed for different-type uncertain nonlinear systems [30–32]. Besides, by utilizing $|z| - z^2/\sqrt{z^2+\delta^2} < \delta$ with $\delta > 0$ and z being any real number, two controllers were designed by invoking the lower bounds of unknown VCGs [33, 34]. By describing UVCCs in terms of their known and unknown parts, a new adaptive tracking controller was constructed and applied to the attitude control of quadrotors [35]. More research results on nonlinear systems with unknown VCGs can be seen in [36, 37].

In summary, only the boundedness of tracking error can be obtained in the aforementioned achievements. The better tracking performance of asymptotic stability was not researched for nonlinear systems with uncertain VCGs being functions of system states. Recently, an adaptive asymptotic tracking control method was presented in [38]. There were also some others that dealt with unknown nonlinear functions and external disturbances by neural networks. However, according to the literature review, there has been no work reported on adaptive asymptotic tracking controllers for nonlinear systems with VCGs being uncertain state functions without using fuzzy logic or neural networks.

Inspired by the mentioned research achievements, the paper is concerned with adaptive tracking control for nonlinear systems with external disturbances, whose VCGs are uncertain functions of system states. An adaptive tracking controller is designed in a stepwise strategy and applied to the control of a robot system, which ensures both the boundedness of all the signals in the closed-loop system and the asymptotic stability of the tracking error. The effectiveness and practicability of the developed control strategy are validated by both theoretical analysis and simulations.

The paper possesses the following features:

(1) Different from the control methods based on the Nussbaum gain technique [13–25], a Nussbaum function is only employed in the first step and an improved adaptive law for the Nussbaum variable is defined such that the asymptotic stability of the tracking error is achieved. This is the main improvement on [13–32], which can only guarantee that the tracking error is bounded. Without using fuzzy logic or neural networks [26–32], and the lemma of $|z| - z^2/\sqrt{z^2+\delta^2} < \delta$ [33, 34], novel control laws are constructed in the remaining steps without the bounds of the unknown VCGs.

(2) To compensate for unknown VCGs, unknown external disturbances, and parameter uncertainties, new adaptive laws are adopted so that the assumption of unknown VCGs can be relaxed as in Assumption 2, where unknown VCGs only have unknown lower and upper bounds [22, 25, 26, 29, 31, 34, 38]. Unknown VCGs are assumed to be bound by with known upper and lower bounds [14, 18], known lower bounds [27, 30], and known upper bounds [13, 16]. In [32], unknown VCGs are assumed to be strictly either positive or negative. All the above assumptions are more restrictive. Therefore, the proposed controller can be suitable for more nonlinear systems.

(3) The reference signal is only required to be differentiable and the assumption is written as Assumption 1, which is less restrictive than the related assumption that the reference signal and its time derivatives up to the n-th order are continuous and bounded [13, 17, 18, 20–22, 26, 28–31, 33, 38].

The outline of this paper is presented below. In Section 2, the preliminaries and problem formulation are described. The design of an adaptive control algorithm and stability analysis are given in Section 3. Sections 4 and 5 provide the simulation examples of a second-order nonlinear system and a robotic system, respectively, and the summary of the paper.

2. Problem Formulation and Preliminaries

The uncertain nonlinear system is considered as follows:

$$\begin{cases} \dot{x}_i = g_i(\overline{x})_i x_{i+1} + \theta^T \varphi_i(\overline{x})_i + d_i(t), & i = 1, \ldots, n-1, \\ \dot{x}_n = g_n(\overline{x}_n)u + \theta^T \varphi_n(\overline{x}_n) + d_n(t), \\ y = x_1, \end{cases} \tag{1}$$

where $\overline{x}_i = [x_1, \ldots, x_i]^T \in R^i$, $u \in R$, $y \in R$ are the state vector, input, and output of the system, respectively, $i = 1, \ldots, n$. $g_i(\overline{x}_i) \in R$ denotes the VCG, which is an unknown smooth non-zero nonlinear function. $\theta \in R^{s_1}$ is an unknown parameter vector, and $\varphi_i(\overline{x}_i) \in R^{s_1}$. denotes a nonlinear function, which is known and smooth, s_1 is a positive integer; $d_i(t) \in R$ is an unknown bounded external disturbance.

Assumption 1 (see [24]). The ideal output vector $y_d^1(t) = [y_d(t), \dot{y}_d(t)]^T$ is absolutely continuous and bounded.

Assumption 2 (see [22]). Let us assume that the sign of $g_i(\overline{x}_i)$. does not change for all x_i and $g_i(\overline{x}_i)$. satisfies

$$0 < \underline{g}_{im} \leq |g_i(\overline{x}_i)| \leq \overline{g}_{iM} < +\infty, \tag{2}$$

with $\underline{g}_{im}$ and $\overline{g}_{iM}$ being unknown positive constants.

It is reasonable that $g_i(\overline{x}_i)$. is bigger than a positive constant $\underline{g}_{im}$ due to the system controllable condition of $g_i(\overline{x}_i)$ being away from 0, which could eliminate the controller singularity problem and has been given in many control strategies [22, 25, 26, 29, 31, 34, 38] and the references therein. It is worth noting that the VCGs in the above references possess the following synthesis: the lower and upper bounds of the VCGs are only used in the procedures of the controller design and are not required in the controller, which means that $\underline{g}_{im}$ and $\overline{g}_{iM}$ can be unknown. Without loss of generality, we assume that $g_i(\overline{x}_i) > 0$.

Assumption 3. There exists an unknown positive constant $\overline{d}_i$ satisfying

$$|d_i(t)| \leq \overline{d}_i. \tag{3}$$

Definition 1 (see [12]). A continuous function $N(\varsigma)$ is named as Nussbaum function when the following conditions hold:

$$\begin{aligned} &\lim_{\varepsilon \longrightarrow E} \sup \frac{1}{\varepsilon} \int_0^{\varepsilon} N(\varsigma) d\varsigma = +\infty, \\ &\lim_{\varepsilon \longrightarrow \infty} \inf \frac{1}{\varepsilon} \int_0^{\varepsilon} N(\varsigma) d\varsigma = -\infty. \end{aligned} \tag{4}$$

There are many Nussbaum-type functions, such as $e^{\varsigma^2} \sin((\pi/2)\varsigma)$, $e^{\varsigma^2} \cos((\pi/2)\varsigma)$, $\varsigma^2 \sin(\varsigma)$, and $\varsigma^2 \cos(\varsigma)$.

Lemma 1 (see [18]). *Let $g(x) \in [g_l, g_u]$ with g_l, g_u being non-zero real numbers and $0 \notin [g_l, g_u]$, $N(\varsigma)$ is an even Nussbaum function, $z(t)$ is an absolutely continuous function, and $x(t)$ is another function. If there exist $l > 0$ and a real constant γ such that a function $V(t)$ should be subject to*

$$V(t) \geq e^{-lt} \int_0^t (g(x(\tau)) N(\zeta(\tau)) + 1) \dot{\zeta}(\tau) e^{l\tau} d\tau + \gamma, \quad \forall t \in [0, t_f], \tag{5}$$

then $\zeta(t)$, $\int_0^t (g(x(\tau)) N(\zeta(\tau)) + 1) \dot{\zeta}(\tau) e^{l\tau} d\tau$ and $V(t)$ are all bounded on $[0, t_f]$.

According to Proposition 2 in [39], t_f can be extended to $+\infty$, when the solution of the closed-loop system is bounded.

3. Design and Analysis of the Controller

The procedure of the adaptive controller design and stability analysis are presented.

Firstly, the common coordinate transformation is introduced as

$$\begin{aligned} z_1 &= x_1 - y_d, \\ z_i &= x_i - \alpha_{i-1}, \quad i = 2, \ldots, n \end{aligned} \tag{6}$$

with α_{i-1} being the intermediate control law\designed later.

The system (1) is changed into

$$\begin{cases} \dot{z}_1 = g_1(x_1) x_2 + \theta^T \varphi_1(x_1) + d_1(t) - \dot{y}_d, \\ \dot{z}_i = g_i(\overline{x}_i) x_{i+1} + \theta^T \varphi_i(\overline{x})_i + d_2(t) - \dot{\alpha}_{i-1}, \\ i = 2, \ldots, n-1, \\ \dot{z}_n = g_n(\overline{x}_n) u + \theta^T \varphi_n(\overline{x}_n) + d_n(t) - \dot{\alpha}_{n-1}, \\ y = x_1. \end{cases} \tag{7}$$

In order to estimate the unknown VCGs $g_i(\overline{x}_i)$, unknown parameter vector θ, and uncertain external disturbances $d_i(t)$, define

$$\varpi_i^* = \|\Gamma_i\|, \quad i = 1, 2, \ldots, n, \tag{8}$$

$$\tilde{\varpi}_i = \varpi_i^* - \varpi_i, \tag{9}$$

where $\|\cdot\|$ is the 2-norm of vectors, Γ_i is an unknown bounded vector given later; $\tilde{\varpi}_i$ is the approximation error with ϖ_i being the estimation of ϖ_i^*.

In what follows, the controller will be constructed in a stepwise strategy via backstepping. For convenience, let $g_i = g_i(\overline{x}_i)$, $\varphi_i = \varphi_i(\overline{x}_i)$, and $d_i = d_i(t)$.

Step 1. We choose the first Lyapunov function candidate as

$$V_1 = \frac{1}{2\underline{g}_{1m}} z_1^2 + \frac{1}{2q_1} \tilde{\varpi}_1^2, \tag{10}$$

where q_1 is a positive constant.

Differentiating V_1 and invoking (9) produce

$$\dot{V}_1 = \frac{1}{\underline{g}_{1m}} z_1 \dot{z}_1 - \frac{1}{q_1} \tilde{\varpi}_1 \dot{\varpi}_1^*. \tag{11}$$

Substituting $\dot{z}_1$ in (7) into (11) and considering (6) yield

$$\dot{V}_1 = \frac{z_1}{\underline{g}_{1m}} \left(g_1 z_2 + g_1 \alpha_1 + \theta^T \varphi_1 + d_1 - \dot{y}_d \right) - \frac{1}{q_1} \tilde{\varpi}_1 \dot{\varpi}_1. \tag{12}$$

Based on Young inequality, we have

$$\begin{aligned} &\frac{g_1}{\underline{g}_{1m}} z_1 z_2 \leq \frac{1}{2} \frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_1^2 + \frac{1}{2} \frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2, \\ &\frac{1}{\underline{g}_{1m}} \theta^T z_1 \varphi_1 \leq \frac{1}{2a_1} \frac{\theta^T \theta}{\underline{g}_{1m}} \varphi_1^T \varphi_1 z_1^2 + \frac{a_1}{2\underline{g}_{1m}}, \\ &\frac{1}{\underline{g}_{1m}} z_1 d_1 \leq \frac{1}{2b_1} \frac{1}{\underline{g}_{1m}} d_1^2 z_1^2 + \frac{b_1}{2\underline{g}_{1m}} \leq \frac{1}{2b_1} \frac{1}{\underline{g}_{1m}} \overline{d}_1^2 z_1^2 + \frac{b_1}{2\underline{g}_{1m}}, \\ &-\frac{1}{\underline{g}_{1m}} z_1 \dot{y}_d \leq \frac{1}{2c_1} \frac{1}{\underline{g}_{1m}} \dot{y}_d^2 z_1^2 + \frac{c_1}{2\underline{g}_{1m}} \leq \frac{1}{2c_1} \frac{1}{\underline{g}_{1m}} \dot{y}_{dM}^2 z_1^2 + \frac{c_1}{2\underline{g}_{1m}}, \end{aligned} \tag{13}$$

where a_1, b_1, and c_1 are positive constants and $\dot{y}_{dM}$ is the maximum of $|\dot{y}_d|$ and unknown.

By invoking (13), $\dot{V}_1$ is described as

$$\begin{aligned}\dot{V}_1 \le{}& \frac{z_1}{\underline{g}_{1m}} g_1 \alpha_1 + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_1^2 + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2 + \frac{1}{2a_1}\frac{\theta^T\theta}{\underline{g}_{1m}}\varphi_1^T\varphi_1 z_1^2 \\ &+ \frac{1}{2b_1}\frac{1}{\underline{g}_{1m}}\overline{d}_1^2 z_1^2 + \frac{a_1}{2\underline{g}_{1m}} + \frac{b_1}{2\underline{g}_{1m}} + \frac{1}{2c_1}\frac{1}{\underline{g}_{1m}}\dot{y}_{dM}^2 z_1^2 + \frac{c_1}{2\underline{g}_{1m}} - \frac{1}{q_1}\tilde{\varpi}_1\dot{\varpi}_1.\end{aligned} \tag{14}$$

Let us define

$$\Gamma_1 = \left[\frac{\overline{g}_{1M}}{\underline{g}_{1m}}, \frac{\theta^T\theta}{\underline{g}_{1m}}, \frac{1}{\underline{g}_{1m}}\overline{d}_1^2, \frac{\dot{y}_{dM}^2}{\underline{g}_{1m}}\right], \tag{15}$$

$$\Upsilon_1 = \left[\frac{1}{2}, \frac{1}{2a_1}\varphi_1^T\varphi_1, \frac{1}{2b_1}, \frac{1}{2c_1}\right]. \tag{16}$$

Substituting (15) and (16) into (14) results in

$$\begin{aligned}\dot{V}_1 \le{}& \frac{g_1}{\underline{g}_{1m}} z_1\alpha_1 + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2 + \Gamma_1\Upsilon_1^T z_1^2 + \frac{a_1}{2\underline{g}_{1m}} + \frac{b_1}{2\underline{g}_{1m}} + \frac{c_1}{2\underline{g}_{1m}} - \frac{1}{q_1}\tilde{\varpi}_1\dot{\varpi}_1 \\ ={}& \frac{g_1}{\underline{g}_{1m}} z_1\alpha_1 + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2 + \varpi_1^*\left\|\Upsilon_1\right\| z_1^2 + \frac{a_1}{2\underline{g}_{1m}} + \frac{b_1}{2\underline{g}_{1m}} + \frac{c_1}{2\underline{g}_{1m}} - \frac{1}{q_1}\tilde{\varpi}_1\dot{\varpi}_1.\end{aligned} \tag{17}$$

In the following, the virtual control signal α_1 is defined as

$$\alpha_1 = N(\varsigma)\overline{\alpha}_1, \tag{18}$$

$$\overline{\alpha}_1 = k_1 z_1 + \varpi_1\left\|\Upsilon_1\right\| z_1, \tag{19}$$

where $N(\varsigma)$ is a Nussbaum-type even function and k_1 is a positive design parameter.

ς is adjusted according to the following law:

$$\dot{\varsigma} = \lambda z_1 \overline{\alpha}_1, \tag{20}$$

where $\lambda > 0$ is a design parameter.

Remark 1. From (19) and the definition of $\overline{\alpha}_1$, ϖ_1, it can be known that all the terms of $\dot{\varsigma}$ are nonnegative, which is the key to proving the asymptotic stability of the tracking error later.

By invoking (18) and (20), $\dot{V}_1$ is shown as

$$\begin{aligned}\dot{V}_1 \le{}& \frac{g_1}{\underline{g}_{1m}} z_1 N(\varsigma)\overline{\alpha}_1 + z_1\overline{\alpha}_1 - z_1\overline{\alpha}_1 + \varpi_1^*\left\|\Upsilon_1\right\| z_1^2 + \frac{a_1}{2\underline{g}_{1m}} + \frac{b_1}{2\underline{g}_{1m}} + \frac{c_1}{2\underline{g}_{1m}} + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2 - \frac{1}{q_1}\tilde{\varpi}_1\dot{\varpi}_1 \\ ={}& -k_1 z_1^2 + \frac{g_1}{\lambda\underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{\dot{\varsigma}}{\lambda} - \frac{\tilde{\varpi}_1}{q_1}\left(\dot{\varpi}_1 - q_1\left\|\Upsilon_1\right\| z_1^2\right) + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2 + \frac{a_1}{2\underline{g}_{1m}} + \frac{b_1}{2\underline{g}_{1m}} + \frac{c_1}{2\underline{g}_{1m}}.\end{aligned} \tag{21}$$

The adaptive law $\dot{\varpi}_1$ is defined as

$$\dot{\varpi}_1 = q_1\left\|\Upsilon_1\right\| z_1^2 - \sigma_1\varpi_1. \tag{22}$$

With $\sigma_1 > 0$ being a design parameter.

Replacing $\dot{\varpi}_1$ in (21) by (22), $\dot{V}_1$ can be given as

$$\dot{V}_1 \le -k_1 z_1^2 + \frac{g_1}{\lambda\underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \frac{\sigma_1}{q_1}\tilde{\varpi}_1\varpi_1 + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2 + \frac{a_1}{2\underline{g}_{1m}} + \frac{b_1}{2\underline{g}_{1m}} + \frac{c_1}{2\underline{g}_{1m}}. \tag{23}$$

Step 2. A Lyapunov function candidate is given as

$$V_2 = V_1 + \frac{1}{2\underline{g}_{2m}} z_2^2 + \frac{1}{2q_2}\tilde{\varpi}_2^2, \tag{24}$$

where $q_2 > 0$ is a constant.

Based on (23) and $\dot{z}_2$ in (7), we have

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \frac{1}{\underline{g}_{2m}} z_2\dot{z}_2 - \frac{1}{q_2}\tilde{\varpi}_2\dot{\varpi}_2 \\ &\le -k_1 z_1^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \frac{\sigma_1}{q_1}\tilde{\varpi}_1\varpi_1 + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2 \\ &\quad + \frac{a_1}{2\underline{g}_{1m}} + \frac{b_1}{2\underline{g}_{1m}} + \frac{1}{\underline{g}_{2m}} z_2\left(g_2 x_3 + \theta^T\varphi_2 + d_2 - \dot{\alpha}_1\right) + \frac{c_1}{2\underline{g}_{1m}} - \frac{1}{q_2}\tilde{\varpi}_2\dot{\varpi}_2.\end{aligned} \tag{25}$$

From (18), $\dot{\alpha}_1$ is expressed as

$$\dot{\alpha}_1 = \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d + \frac{\partial\alpha_1}{\partial\varsigma}\dot{\varsigma} + \frac{\partial\alpha_1}{\partial\varpi_1}\dot{\varpi}_1. \tag{26}$$

Substituting (26) into (25) leads to

$$\begin{aligned}\dot{V}_2 &\le -k_1 z_1^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \frac{\sigma_1}{q_1}\tilde{\varpi}_1\varpi_1 + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2 + \frac{a_1}{2\underline{g}_{1m}} \\ &\quad + \frac{b_1}{2\underline{g}_{1m}} + \frac{c_1}{2\underline{g}_{1m}} + \frac{1}{\underline{g}_{2m}} z_2\left(g_2(z_3 + \alpha_2) + \theta^T\varphi_2 + d_2\right) \\ &\quad - \frac{1}{q_2}\tilde{\varpi}_2\dot{\varpi}_2 - \frac{1}{\underline{g}_{2m}} z_2 \frac{\partial\alpha_1}{\partial x_1}\left(g_1 x_2 + \theta^T\varphi_1\right) - \frac{1}{\underline{g}_{2m}} z_2\left(\frac{\partial\alpha_1}{\partial y_d}\dot{y}_d + \frac{\partial\alpha_1}{\partial\varsigma}\dot{\varsigma} + \frac{\partial\alpha_1}{\partial\varpi_1}\dot{\varpi}_1\right).\end{aligned} \tag{27}$$

According to Young inequality, the following inequalities hold

$$\begin{aligned}\frac{g_2}{\underline{g}_{2m}} z_2 z_3 &\le \frac{1}{2}\frac{\overline{g}_{2M}}{\underline{g}_{2m}} z_2^2 + \frac{1}{2}\frac{\overline{g}_{2M}}{\underline{g}_{2m}} z_3^2, \\ \frac{1}{\underline{g}_{2m}} z_2 d_2 &\le \frac{1}{2b_2}\frac{1}{\underline{g}_{2m}} \bar{d}_2^2 z_2^2 + \frac{b_2}{2\underline{g}_{2m}}, \\ \frac{1}{\underline{g}_{2m}}\theta^T z_2\left(\varphi_2 - \frac{\partial\alpha_1}{\partial x_1}\varphi_1\right) &\le \frac{1}{2a_2}\frac{\theta^T\theta}{\underline{g}_{2m}}\left\|\varphi_2 - \frac{\partial\alpha_1}{\partial x_1}\varphi_1\right\|^2 z_2^2 + \frac{a_2}{2\underline{g}_{2m}}, \\ -\frac{1}{\underline{g}_{2m}} z_2\frac{\partial\alpha_1}{\partial y_d}\dot{y}_d &\le \frac{1}{2c_2}\frac{1}{\underline{g}_{2m}}\dot{y}_{dM}^2\left(\frac{\partial\alpha_1}{\partial y_d}\right)^2 z_2^2 + \frac{c_2}{2\underline{g}_{2m}}, \\ -\frac{1}{\underline{g}_{2m}} z_2\frac{\partial\alpha_1}{\partial x_1} g_1 x_2 &\le \frac{1}{2l_2}\frac{1}{\underline{g}_{2m}}\overline{g}_{1M}^2 x_2^2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2^2 + \frac{l_2}{2\underline{g}_{2m}}, \\ -\frac{z_2}{\underline{g}_{2m}}\left(\frac{\partial\alpha_1}{\partial\varsigma}\dot{\varsigma} + \frac{\partial\alpha_1}{\partial\varpi_1}\dot{\varpi}_1\right) &\le \frac{1}{2m_2}\frac{1}{\underline{g}_{2m}}\left(\frac{\partial\alpha_1}{\partial\varsigma}\dot{\varsigma} + \frac{\partial\alpha_1}{\partial\varpi_1}\dot{\varpi}_1\right)^2 z_2^2 + \frac{m_2}{2\underline{g}_{2m}},\end{aligned} \tag{28}$$

where a_2, b_2, c_2, l_2, and m_2 are positive constants.

Substituting (28) into (27) leads to

$$\dot{V}_2 \leq -k_1 z_1^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \frac{\sigma_1}{q_1}\tilde{\varpi}_1\varpi_1 + \frac{1}{2}\frac{\overline{g}_{1M}}{\underline{g}_{1m}} z_2^2 + \sum_{i=1}^{2}\frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \frac{g_2}{\underline{g}_{2m}} z_2\alpha_2$$
$$+ \frac{1}{2}\frac{\overline{g}_{2M}}{\underline{g}_{2m}} z_2^2 + \frac{1}{2}\frac{\overline{g}_{2M}}{\underline{g}_{2m}} z_3^2 - \frac{1}{q_2}\tilde{\varpi}_2\dot{\varpi}_2 + \frac{1}{2a_2}\frac{\theta^T\theta}{\underline{g}_{2m}}\left\|\varphi_2 - \frac{\partial\alpha_1}{\partial x_1}\varphi_1\right\|^2 z_2^2 + \frac{1}{2b_2}\frac{1}{\underline{g}_{2m}}\overline{d}_2^2 z_2^2 \tag{29}$$
$$+ \frac{1}{2c_2}\frac{1}{\underline{g}_{2m}}\dot{y}_{dM}^2\left(\frac{\partial\alpha_1}{\partial y_d}\right)^2 z_2^2 + \frac{1}{2l_2}\frac{\overline{g}_{1M}^2}{\underline{g}_{2m}} x_2^2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2^2 + \frac{l_2}{2\underline{g}_{2m}} + \frac{1}{2m_2}\frac{z_2^2}{\underline{g}_{2m}}\left(\frac{\partial\alpha_1}{\partial\varsigma}\dot{\varsigma} + \frac{\partial\alpha_1}{\partial\varpi_1}\dot{\varpi}_1\right)^2 + \frac{m_2}{2\underline{g}_{2m}}.$$

Γ_2 and Υ_2 are written as

$$\Gamma_2 = \left[\frac{\overline{g}_{1M}}{\underline{g}_{1m}} + \frac{\overline{g}_{2M}}{\underline{g}_{2m}}, \frac{\theta^T\theta}{\underline{g}_{2m}}, \frac{\overline{d}_2^2}{\underline{g}_{2m}}, \frac{\dot{y}_{dM}^2}{\underline{g}_{2m}}, \frac{\overline{g}_{1M}^2}{\underline{g}_{2m}}, \frac{1}{\underline{g}_{2m}}\right], \tag{30}$$

$$\Upsilon_2 = \left[\frac{1}{2}, \frac{\left\|\varphi_2 - \partial\alpha_1/\partial x_1\varphi_1\right\|^2}{2a_2}, \frac{1}{2b_2}, \frac{(\partial\alpha_1/\partial y_d)^2}{2c_2}, \Upsilon_2^*\right],$$
$$\Upsilon_2^* = \left[\frac{(\partial\alpha_1/\partial x_1 x_2)^2}{2l_2}, \frac{(\partial\alpha_1/\partial\varsigma\dot{\varsigma} + \partial\alpha_1/\partial\varpi_1\dot{\varpi}_1)^2}{2m_2}\right]. \tag{31}$$

Invoking (30) and (31) produces

$$\dot{V}_2 \leq -k_1 z_1^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \frac{\sigma_1}{q_1}\tilde{\varpi}_1\varpi_1 + \sum_{i=1}^{2}\frac{a_i + b_i + c_i}{2\underline{g}_{im}},$$
$$+ \frac{l_2}{2\underline{g}_{2m}} + \frac{m_2}{2\underline{g}_{2m}} + \frac{g_2}{\underline{g}_{2m}} z_2\alpha_2 + \varpi_2^*\left\|\Upsilon_2\right\| z_2^2 + \frac{1}{2}\frac{\overline{g}_{2M}}{\underline{g}_{2m}} z_3^2 - \frac{1}{q_2}\tilde{\varpi}_2\dot{\varpi}_2. \tag{32}$$

Designing the following virtual control signal as

$$\alpha_2 = -k_2 z_2 - \varpi_2\left\|\Upsilon_2\right\| z_2. \tag{33}$$

and substituting it into (32) give

$$\dot{V}_2 \leq -k_1 z_1^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \frac{\sigma_1}{q_1}\tilde{\varpi}_1\varpi_1 - \frac{g_2}{\underline{g}_{2m}}\left(k_2 z_2^2 + \varpi_2\left\|\Upsilon_2\right\| z_2^2\right)$$
$$+ \varpi_2^*\left\|\Upsilon_2\right\| z_2^2 + \sum_{i=1}^{2}\frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \frac{l_2}{2\underline{g}_{2m}} + \frac{m_2}{2\underline{g}_{2m}} + \frac{1}{2}\frac{\overline{g}_{2M}}{\underline{g}_{2m}} z_3^2 - \frac{1}{q_2}\tilde{\varpi}_2\dot{\varpi}_2, \tag{34}$$

where $k_2 > 0$ is a design parameter.

Remark 2. . Based on (32), Assumption 2, and the definition of ϖ_2, it is easy to check that all the items of $-(g_2/\underline{g}_{2m})(k_2 z_2^2 + \varpi_2\|\Upsilon_2\| z_2^2)$ are nonpositive, and $(g_2/\underline{g}_{2m}) \geq 1$, which means that $(g_2/\underline{g}_{2m})$ can be replaced by 1 in the controller design.

$\dot{V}_2$ is changed to

$$\dot{V}_2 \leq -\sum_{i=1}^{2} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \frac{\sigma_1}{q_1}\tilde{\varpi}_1\varpi_1 + \sum_{i=1}^{2}\frac{a_i + b_i + c_i}{2\underline{g}_{im}}$$
$$+ \frac{l_2}{2\underline{g}_{2m}} + \frac{m_2}{2\underline{g}_{2m}} - \frac{1}{q_2}\tilde{\varpi}_2\left(\dot{\varpi}_2 + q_2\left\|\Upsilon_2\right\| z_2^2\right) + \frac{1}{2}\frac{\overline{g}_{2M}}{\underline{g}_{2m}} z_3^2. \tag{35}$$

We construct an adaptive law as

$$\dot{\varpi}_2 = q_2\|\Upsilon_2\|z_2^2 - \sigma_2\varpi_2, \tag{36}$$

where σ_2 is a positive constant.

Substituting (36) into (35) produces

$$\dot{V}_2 \le -\sum_{i=1}^{2} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{2}\frac{\sigma_i}{q_i}\widetilde{\varpi}_i\varpi_i + \sum_{i=1}^{2}\frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \frac{l_2}{2\underline{g}_{2m}} + \frac{m_2}{2\underline{g}_{2m}} + \frac{1}{2}\frac{\overline{g}_{2M}}{\underline{g}_{2m}} z_3^2. \tag{37}$$

Step j $(3 \le j \le n-1)$: For the j th subsystem of (6), a Lyapunov function candidate is iteratively chosen from the previous step.

$$V_j = V_{j-1} + \frac{1}{2\underline{g}_{jm}} z_j^2 + \frac{1}{2q_j}\widetilde{\varpi}_j^2, \tag{38}$$

where $q_j > 0$ is a design parameter.

Differentiating V_j produces

$$\dot{V}_j = \dot{V}_{j-1} + \frac{1}{\underline{g}_{jm}} z_j \dot{z}_j - \frac{1}{q_j}\widetilde{\varpi}_j \dot{\varpi}_j. \tag{39}$$

By an induction argument, $\dot{V}_{j-1}$ is deduced as

$$\dot{V}_{j-1} \le -\sum_{i=1}^{j-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{j-1}\frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=1}^{j-1}\frac{\sigma_i}{q_i}\widetilde{\varpi}_i\varpi_i + \sum_{i=2}^{j-1}\frac{l_i + m_i}{2\underline{g}_{im}} + \frac{1}{2}\frac{\overline{g}_{j-1,M}}{\underline{g}_{j-1,m}} z_j^2, \tag{40}$$

where a_i, b_i, c_i, l_i, m_i, σ_i, q_i are all positive constants.

Substituting (40) into (39) gives

$$\begin{aligned}\dot{V}_j \le{}& -\sum_{i=1}^{j-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{j-1}\frac{\sigma_i}{q_i}\widetilde{\varpi}_i\varpi_i + \sum_{i=1}^{j-1}\frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{j-1}\frac{l_i + m_i}{2\underline{g}_{im}} \\ &+ \frac{1}{2}\frac{\overline{g}_{j-1,M}}{\underline{g}_{j-1,m}} z_j^2 + \frac{z_j}{\underline{g}_{jm}}\left(g_j z_{j+1} + g_j\alpha_j + \theta^T\varphi_j + d_j - \dot{\alpha}_{j-1}\right) - \frac{1}{q_j}\widetilde{\varpi}_j\dot{\varpi}_j,\end{aligned} \tag{41}$$

where

$$\begin{aligned}\dot{\alpha}_{j-1} &= \sum_{i=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_i}\dot{x}_i + \frac{\partial\alpha_{j-1}}{\partial y_d}\dot{y}_d + \frac{\partial\alpha_{j-1}}{\partial\varsigma}\dot{\varsigma} + \frac{\partial\alpha_{j-1}}{\partial\varpi_{j-1}}\dot{\varpi}_{j-1}, \\ \dot{\varpi}_{j-1} &= q_{j-1}\|\Upsilon_{j-1}\|z_{j-1}^2 - \sigma_{j-1}\varpi_{j-1}.\end{aligned} \tag{42}$$

Furthermore, $\dot{V}_j$ is expressed as

$$\begin{aligned}\dot{V}_j \le{}& -\sum_{i=1}^{j-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{j-1}\frac{\sigma_i}{q_i}\widetilde{\varpi}_i\varpi_i + \sum_{i=1}^{j-1}\frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{j-1}\frac{l_i + m_i}{2\underline{g}_{im}} + \frac{1}{2}\frac{\overline{g}_{j-1,M}}{\underline{g}_{j-1,m}} z_j^2 \\ &+ \frac{g_j}{\underline{g}_{jm}} z_j z_{j+1} + \frac{g_j}{\underline{g}_{jm}} z_j\alpha_j + \frac{z_j}{\underline{g}_{jm}} d_j - \frac{z_j}{\underline{g}_{jm}}\sum_{i=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_i} g_i x_{i+1} - \frac{z_j}{\underline{g}_{jm}}\frac{\partial\alpha_{j-1}}{\partial y_d}\dot{y}_d \\ &- \frac{z_j}{\underline{g}_{jm}}\left(\frac{\partial\alpha_{j-1}}{\partial\varsigma}\dot{\varsigma} + \frac{\partial\alpha_{j-1}}{\partial\varpi_{j-1}}\dot{\varpi}_{j-1}\right) + \frac{z_j}{\underline{g}_{jm}}\theta^T\left(\varphi_j - \sum_{i=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_i}\varphi_i\right) - \frac{1}{q_j}\widetilde{\varpi}_j\dot{\varpi}_j.\end{aligned} \tag{43}$$

Based on the completion of squares, one has

$$
\begin{aligned}
\frac{g_j}{\underline{g}_{jm}} z_j z_{j+1} &\le \frac{1}{2}\frac{\overline{g}_{jM}}{\underline{g}_{jm}} z_j^2 + \frac{1}{2}\frac{\overline{g}_{jM}}{\underline{g}_{jm}} z_{j+1}^2,\\
\frac{1}{\underline{g}_{jm}} z_j d_j &\le \frac{1}{2b_j}\frac{1}{\underline{g}_{jm}} \overline{d}_j^2 z_j^2 + \frac{b_j}{2\underline{g}_{jm}},\\
\frac{\theta^T}{\underline{g}_{jm}} z_j \left(\varphi_j - \sum_{i=1}^{j-1}\frac{\partial \alpha_{j-1}}{\partial x_i}\varphi_i\right) &\le \frac{1}{2a_j}\frac{\theta^T\theta}{\underline{g}_{jm}} \left\|\varphi_j - \sum_{i=1}^{j-1}\frac{\partial \alpha_{j-1}}{\partial x_i}\varphi_i\right\|^2 z_j^2 + \frac{a_j}{2\underline{g}_{jm}},\\
-\frac{1}{\underline{g}_{jm}} z_j \frac{\partial \alpha_{j-1}}{\partial y_d}\dot{y}_d &\le \frac{1}{2c_j}\frac{1}{\underline{g}_{jm}} \dot{y}_{dM}^2 \left(\frac{\partial \alpha_{j-1}}{\partial y_d}\right)^2 z_j^2 + \frac{c_j}{2\underline{g}_{jm}} - \frac{1}{\underline{g}_{jm}} z_j \sum_{i=1}^{j-1}\frac{\partial \alpha_{j-1}}{\partial x_i} g_i x_{i+1},\\
&\le \frac{1}{2l_j}\frac{1}{\underline{g}_{jm}} \overline{g}_{jM}^* \left(\sum_{i=1}^{j-1}\frac{\partial \alpha_{j-1}}{\partial x_i} x_{i+1}\right)^2 z_j^2 + \frac{l_j}{2\underline{g}_{jm}} - \frac{z_j}{\underline{g}_{jm}}\left(\frac{\partial \alpha_{j-1}}{\partial \varsigma}\dot{\varsigma} + \frac{\partial \alpha_{j-1}}{\partial \varpi_{j-1}}\dot{\varpi}_{j-1}\right),\\
&\le \frac{1}{2m_j}\frac{z_j^2}{\underline{g}_{jm}} \left(\frac{\partial \alpha_{j-1}}{\partial \varsigma}\dot{\varsigma} + \frac{\partial \alpha_{j-1}}{\partial \varpi_{j-1}}\dot{\varpi}_{j-1}\right)^2 + \frac{m_j}{2\underline{g}_{jm}},
\end{aligned}
\tag{44}
$$

where a_j, b_j, c_j, l_j and m_j are positive design parameters and $\overline{g}_{jM}^* = \max\{\overline{g}_{1M}^2, \ldots, \overline{g}_{j-1,M}^2\}$.

Invoking (44), $\dot{V}_j$ is rewritten as

$$
\begin{aligned}
\dot{V}_j \le & -\sum_{i=1}^{j-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{j-1}\frac{\sigma_i}{q_i}\tilde{\varpi}_i\varpi_i + \sum_{i=1}^{j}\frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{j}\frac{l_i + m_i}{2\underline{g}_{im}}\\
& + \frac{1}{2}\frac{\overline{g}_{j-1,M}}{\underline{g}_{j-1,m}} z_j^2 + \frac{1}{2}\frac{\overline{g}_{jM}}{\underline{g}_{jm}} z_j^2 + \frac{1}{2}\frac{\overline{g}_{jM}}{\underline{g}_{jm}} z_{j+1}^2 + \frac{g_j}{\underline{g}_{jm}} z_j \alpha_j + \frac{1}{2b_j}\frac{1}{\underline{g}_{jm}} \overline{d}_j^2 z_j^2 + \frac{1}{2a_j}\frac{\theta^T\theta}{\underline{g}_{jm}} \left\|\varphi_j - \sum_{i=1}^{j-1}\frac{\partial \alpha_{j-1}}{\partial x_i}\varphi_i\right\|^2 z_j^2\\
& + \frac{1}{2c_j}\frac{1}{\underline{g}_{jm}} \dot{y}_{dM}^2 \left(\frac{\partial \alpha_{j-1}}{\partial y_d}\right)^2 z_j^2 + \frac{1}{2l_j}\frac{1}{\underline{g}_{jm}} \overline{g}_{jM}^* \left(\sum_{i=1}^{j-1}\frac{\partial \alpha_{j-1}}{\partial x_i} x_{i+1}\right)^2 z_j^2 - \frac{1}{q_j}\tilde{\varpi}_j\dot{\varpi}_j + \frac{1}{2m_j}\frac{1}{\underline{g}_{jm}}\left(\frac{\partial \alpha_{j-1}}{\partial \varsigma}\dot{\varsigma} + \frac{\partial \alpha_{j-1}}{\partial \varpi_1}\dot{\varpi}_1\right)^2 z_j^2.
\end{aligned}
\tag{45}
$$

Similarly, we define

$$
\Gamma_j = \left[\frac{\overline{g}_{j-1,M}}{\underline{g}_{j-1,m}} + \frac{\overline{g}_{jM}}{\underline{g}_{jm}}, \frac{\theta^T\theta}{\underline{g}_{jm}}, \frac{\overline{d}_j^2}{\underline{g}_{jm}}, \frac{\dot{y}_{dM}^2}{\underline{g}_{jm}}, \frac{\overline{g}_{jM}^*}{\underline{g}_{jm}}, \frac{1}{\underline{g}_{jm}}\right], \tag{46}
$$

$$
\begin{aligned}
\Upsilon_j &= \left[\frac{1}{2}, \frac{\left\|\varphi_j - \sum_{i=1}^{j-1}\left(\partial \alpha_{j-1}/\partial x_i\right)\varphi_i\right\|^2}{2a_j}, \frac{1}{2b_j}, \Upsilon_{j1}^*, \Upsilon_{j2}^*\right],\\
\Upsilon_{j1}^* &= \left[\frac{\left(\left(\partial \alpha_{j-1}/\partial y_d\right)\right)^2}{2c_j}, \frac{\left(\sum_{i=1}^{j-1}\left(\partial \alpha_{j-1}/\partial x_i\right) x_{i+1}\right)^2}{2l_j}\right],\\
\Upsilon_{j2}^* &= \left[\frac{\left(\left(\partial \alpha_{j-1}/\partial \varsigma\right)\dot{\varsigma} + \left(\partial \alpha_{j-1}/\partial \varpi_{j-1}\right)\dot{\varpi}_{j-1}\right)^2}{2m_j}\right].
\end{aligned}
\tag{47}
$$

By invoking (46) and (47), $\dot{V}_j$ is shown as

$$
\begin{aligned}
\dot{V}_j \le & -\sum_{i=1}^{j-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{j-1}\frac{\sigma_i}{q_i}\tilde{\varpi}_i\varpi_i + \sum_{i=1}^{j}\frac{a_i + b_i + c_i}{2\underline{g}_{im}}\\
& + \sum_{i=2}^{j}\frac{l_i + m_i}{2\underline{g}_{im}} + \frac{1}{2}\frac{\overline{g}_{jM}}{\underline{g}_{jm}} z_{j+1}^2 + \frac{g_j}{\underline{g}_{jm}} z_j \alpha_j + \varpi_i^* \left\|\Upsilon_j\right\| z_j^2 - \frac{1}{q_j}\tilde{\varpi}_j\dot{\varpi}_j.
\end{aligned}
\tag{48}
$$

Defining the j th virtual control law as

$$
\alpha_j = -k_j z_j - \varpi_j \left\|\Upsilon_j\right\| z_j. \tag{49}
$$

and substituting it into (48) to yield

$$\dot{V}_j \leq -\sum_{i=1}^{j-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{j-1} \frac{\sigma_i}{q_i}\tilde{\varpi}_i \varpi_i + \sum_{i=1}^{j} \frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{j} \frac{l_i + m_i}{2\underline{g}_{im}} + \frac{1}{2}\frac{\overline{g}_{jM}}{\underline{g}_{jm}} z_{j+1}^2 + \tilde{\varpi}_j \left\| \Upsilon_j \right\| z_j^2 - \frac{1}{q_j}\tilde{\varpi}_j \dot{\varpi}_j, \quad (50)$$

where $k_j > 0$ is a constant.

Similar to (36), $\dot{\varpi}_j$ can be specified as

$$\dot{\varpi}_j = q_j \left\| \Upsilon_j \right\| z_j^2 - \sigma_j \varpi_j. \quad (51)$$

with $\sigma_j > 0$ being a design parameter.

Then, $\dot{V}_j$ is given as

$$\dot{V}_j \leq -\sum_{i=1}^{j-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{\dot{\varsigma}}{\lambda} + \sum_{i=1}^{j} \frac{\sigma_i}{q_i}\tilde{\varpi}_i \varpi_i + \sum_{i=1}^{j} \frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{j} \frac{l_i + m_i}{2\underline{g}_{im}} + \frac{\overline{g}_{jM}}{2\underline{g}_{jm}} z_{j+1}^2. \quad (52)$$

Step n: following the similar procedure for (38) in *Step j*, the Lyapunov function candidate for the last step is designed as

$$V_n = V_{n-1} + \frac{1}{2\underline{g}_{nm}} z_n^2 + \frac{1}{2q_n}\tilde{\varpi}_n^2. \quad (53)$$

and $\dot{V}_n$ is presented as

$$\dot{V}_n = \frac{1}{\underline{g}_{nm}} z_n \dot{z}_n + \dot{V}_{n-1} - \frac{1}{q_n}\tilde{\varpi}_n \dot{\varpi}_n, \quad (54)$$

where $q_n > 0$ is a constant.

It follows from (52) that

$$\dot{V}_{n-1} \leq -\sum_{i=1}^{n-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{n-1} \frac{\sigma_i}{q_i}\tilde{\varpi}_i \varpi_i + \sum_{i=1}^{n-1} \frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{n-1} \frac{l_i + m_i}{2\underline{g}_{im}} + \frac{1}{2}\frac{\overline{g}_{n-1,M}}{\underline{g}_{n-1,m}} z_n^2, \quad (55)$$

where a_i, b_i, c_i, l_i, m_i, σ_i, and q_i are all positive constants.

$\dot{V}_n$ is given as

$$\dot{V}_n \leq -\sum_{i=1}^{n-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{n-1} \frac{\sigma_i}{q_i}\tilde{\varpi}_i \varpi_i + \sum_{i=1}^{n-1} \frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{n-1} \frac{l_i + m_i}{2\underline{g}_{im}} + \frac{1}{\underline{g}_{nm}} z_n \left(g_n u + \theta^T \varphi_n + d_n - \dot{\alpha}_{n-1} \right) + \frac{1}{2}\frac{\overline{g}_{n-1,M}}{\underline{g}_{n-1,m}} z_n^2 - \frac{1}{q_n}\tilde{\varpi}_n \dot{\varpi}_n, \quad (56)$$

where

$$\dot{\alpha}_{n-1} = \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i}\dot{x}_i + \frac{\partial \alpha_{n-1}}{\partial y_d}\dot{y}_d + \frac{\partial \alpha_{n-1}}{\partial \varsigma}\dot{\varsigma} + \frac{\partial \alpha_{n-1}}{\partial \varpi_{n-1}}\dot{\varpi}_{n-1}, \quad (57)$$
$$\dot{\varpi}_{n-1} = q_{n-1} \left\| \Upsilon_{n-1} \right\| z_{n-1}^2 - \sigma_{n-1}\varpi_{n-1}.$$

Furthermore, $\dot{V}_n$ is reformulated as

$$\dot{V}_n \leq -\sum_{i=1}^{n-1} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{n-1} \frac{\sigma_i}{q_i}\tilde{\varpi}_i \varpi_i + \sum_{i=1}^{n-1} \frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{n-1} \frac{l_i + m_i}{2\underline{g}_{im}} + \frac{1}{2}\frac{\overline{g}_{n-1,M}}{\underline{g}_{n-1,m}} z_n^2 + \frac{g_n}{\underline{g}_{nm}} z_n u + z_n d_n - \frac{z_n}{\underline{g}_{nm}} \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i} g_i x_{i+1} - \frac{z_n}{\underline{g}_{nm}} \left(\frac{\partial \alpha_{n-1}}{\partial \varsigma}\dot{\varsigma} + \frac{\partial \alpha_{n-1}}{\partial \varpi_{n-1}}\dot{\varpi}_{n-1} \right) - \frac{z_n}{\underline{g}_{nm}} \frac{\partial \alpha_{n-1}}{\partial y_d}\dot{y}_d + \frac{z_n}{\underline{g}_{nm}} \theta^T \left(\varphi_n - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i}\varphi_i \right) - \frac{1}{q_n}\tilde{\varpi}_n \dot{\varpi}_n. \quad (58)$$

Repeating the similar procedures in *Step j*, we have

$$\frac{\theta^T}{\underline{g}_{nm}} z_n\left(\varphi_n - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}\varphi_i\right) \le \frac{1}{2a_n}\frac{\theta^T\theta}{\underline{g}_{nm}}\left\|\varphi_n - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}\varphi_i\right\|^2 z_n^2 + \frac{a_n}{2\underline{g}_{nm}},$$
$$\frac{1}{\underline{g}_{nm}} z_n d_n \le \frac{1}{2b_n}\frac{1}{\underline{g}_{nm}}\overline{d}_n^2 z_n^2 + \frac{b_n}{2\underline{g}_{nm}},$$
$$-\frac{1}{\underline{g}_{nm}} z_n\frac{\partial\alpha_{n-1}}{\partial y_d}\dot{y}_d \le \frac{1}{2c_n}\frac{1}{\underline{g}_{nm}}\dot{y}_{dM}^2\left(\frac{\partial\alpha_{n-1}}{\partial y_d}\right)^2 z_n^2 + \frac{c_n}{2\underline{g}_{nm}}, \tag{59}$$
$$-\frac{1}{\underline{g}_{nm}} z_n\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i} g_i x_{i+1} \le \frac{1}{2l_n}\frac{1}{\underline{g}_{nm}}\overline{g}_{nM}^*\left(\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}x_{i+1}\right)^2 z_n^2 + \frac{l_n}{2\underline{g}_{nm}},$$
$$-\frac{z_n}{\underline{g}_{nm}}\left(\frac{\partial\alpha_{n-1}}{\partial\varsigma}\dot{\varsigma} + \frac{\partial\alpha_{n-1}}{\partial\varpi_{n-1}}\dot{\varpi}_{n-1}\right) \le \frac{z_n^2}{2\underline{g}_{nm} m_n}\left(\frac{\partial\alpha_{n-1}}{\partial\varsigma}\dot{\varsigma} + \frac{\partial\alpha_{n-1}}{\partial\varpi_{n-1}}\dot{\varpi}_{n-1}\right)^2 + \frac{m_n}{2\underline{g}_{nm}},$$

$$\Gamma_n = \left[\frac{\overline{g}_{n-1,M}}{\underline{g}_{n-1,m}}, \frac{\theta^T\theta}{\underline{g}_{nm}}, \frac{\overline{d}_n^2}{\underline{g}_{nm}}, \frac{\dot{y}_{dM}^2}{\underline{g}_{nm}}, \frac{\overline{g}_{nM}^*}{\underline{g}_{nm}}, \frac{1}{\underline{g}_{nm}}\right], \tag{60}$$

$$\Upsilon_n = \left[\frac{1}{2}, \frac{\left\|\varphi_n - \sum_{i=1}^{n-1}(\partial\alpha_{n-1}/\partial x_i)\varphi_i\right\|^2}{2a_n}, \frac{1}{2b_n}, \Upsilon_{n1}^*, \Upsilon_{n2}^*\right],$$
$$\Upsilon_{n1}^* = \left[\frac{(\partial\alpha_{n-1}/\partial y_d)^2}{2c_n}, \frac{\left(\sum_{i=1}^{n-1}(\partial\alpha_{n-1}/\partial x_i)x_{i+1}\right)^2}{2l_n}\right], \tag{61}$$
$$\Upsilon_{n2}^* = \left[\frac{((\partial\alpha_{n-1}/\partial\varsigma)\dot{\varsigma} + (\partial\alpha_{n-1}/\partial\varpi_{n-1})\dot{\varpi}_{n-1})^2}{2m_n}\right],$$

where a_n, b_n, c_n, l_n, and m_n are positive design parameters and $\overline{g}_{nM}^* = \max\{\overline{g}_{1M}^2, \ldots, \overline{g}_{n-1,M}^2\}$.

Substituting (60) and (61) into (58) results in

$$\dot{V}_n \le -\sum_{i=1}^{n-1} k_i z_i^2 + \frac{g_1}{\lambda\underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{n-1}\frac{\sigma_i}{q_i}\tilde{\varpi}_i\varpi_i$$
$$+ \sum_{i=1}^{n}\frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{n}\frac{l_i + m_i}{2\underline{g}_{im}} + \frac{g_n}{\underline{g}_{nm}} z_n u + \varpi_n^*\|\Upsilon_n\| z_n^2 - \frac{1}{q_n}\tilde{\varpi}_n\dot{\varpi}_n. \tag{62}$$

Similar to (49) and (51), the real control law and n th adaptive law can be defined, respectively, as follows:

$$u = -k_n z_n - \varpi_n\|\Upsilon_n\| z_n, \tag{63}$$

and

$$\dot{\varpi}_n = q_n\|\Upsilon_n\| z_n^2 - \sigma_n\varpi_n, \tag{64}$$

where k_n and σ_n are positive design parameters.

By some direct calculations, V_n can be described as

$$\dot{V}_n \le -\sum_{i=1}^{n} k_i z_i^2 + \frac{g_1}{\lambda\underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{n}\frac{\sigma_i}{q_i}\tilde{\varpi}_i\varpi_i$$
$$+ \sum_{i=1}^{n}\frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{n}\frac{l_i + m_i}{2\underline{g}_{im}}. \tag{65}$$

According to the above controller design, the main conclusion is summarized as

Theorem 1. For system (1) with Assumptions 1–3, there exists an adaptive controller with the adaptive laws in (21), (35), (51), (65), and real control signal u in (64), which can guarantee the boundedness of all the signals in the closed-loop system for $t \in [0, +\infty)$ and asymptotic convergence of the tracking error.

Proof: . since $\tilde{\varpi}_i \varpi_i = \tilde{\varpi}_i(\varpi_i^* - \tilde{\varpi}_i) \le (1/2)\varpi_i^{*2} - (1/2)\tilde{\varpi}_i^2$, it is easy to check that

$$\sum_{i=1}^{n} \frac{\sigma_i}{q_i}\tilde{\varpi}_i\varpi_i = \sum_{i=1}^{n} \frac{\sigma_i}{q_i}\tilde{\varpi}_i(\varpi_i^* - \tilde{\varpi}_i) \le \sum_{i=1}^{n} \frac{\sigma_i}{2q_i}\varpi_i^{*2} - \sum_{i=1}^{n} \frac{\sigma_i}{2q_i}\tilde{\varpi}_i^2. \tag{66}$$

Substituting (66) into (65) leads to

$$\begin{aligned} \dot{V}_n &\le -\sum_{i=1}^{n} k_i z_i^2 + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma} + \sum_{i=1}^{n} \frac{\sigma_i}{2q_i}\varpi_i^{*2} \\ &\quad - \sum_{i=1}^{n} \frac{\sigma_i}{2q_i}\tilde{\varpi}_i^2 + \sum_{i=1}^{n} \frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{n} \frac{l_i + m_i}{2\underline{g}_{im}} \\ &\le -\kappa V_n + \iota + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma} + \frac{1}{\lambda}\dot{\varsigma}. \end{aligned} \tag{67}$$

with

$$\begin{aligned} \kappa &= \min\{2k_1, \ldots, 2k_n, \sigma_1, \ldots, \sigma_n\}, \\ \iota &= \sum_{i=1}^{n} \frac{a_i + b_i + c_i}{2\underline{g}_{im}} + \sum_{i=2}^{n} \frac{l_i + m_i}{2\underline{g}_{im}} + \sum_{i=1}^{n}\left(\frac{\sigma_i}{2q_i}\varpi_i^{*2}\right). \end{aligned} \tag{68}$$

Moving $-\kappa V_n$ to the left-hand side and then multiplying both sides of (67) by $e^{\kappa t}$ produce

$$\frac{d\left(V_n e^{\kappa t}\right)}{dt} \le \iota e^{\kappa t} + \frac{g_1}{\lambda \underline{g}_{1m}} N(\varsigma)\dot{\varsigma}e^{\kappa t} + \frac{1}{\lambda}\dot{\varsigma}e^{\kappa t}. \tag{69}$$

Integrating (69) gives

$$\begin{aligned} V_n(t)e^{\kappa t} - V_n(0) &\le \int_0^t \frac{g_1(x_1)}{\lambda \underline{g}_{1m}} N((\varsigma(\tau))\dot{\varsigma}(\tau))e^{\kappa\tau} d\tau \\ &\quad + \int_0^t \frac{\dot{\varsigma}(\tau)}{\lambda} e^{\kappa\tau} d\tau + \frac{\iota}{\kappa}\left(e^{\kappa t} - 1\right). \end{aligned} \tag{70}$$

By some direct calculations, (70) is described as

$$V_n(t) \le \frac{e^{-\kappa t}}{\lambda} \int_0^t \left(\frac{g_1(x_1)}{\underline{g}_{1m}} N(\varsigma(\tau)) + 1\right)\dot{\varsigma}(\tau)e^{\kappa\tau} d\tau + \gamma. \tag{71}$$

where $\gamma = V_n(0) + (\iota/\kappa)$ is a positive constant.

By virtue of Lemma 1, it is shown that $V_n(t)$., $N(\varsigma(\tau))$., $\varsigma(\tau)$ are bound for $t \in [0, +\infty)$. Therefore, the boundedness of all the signals in the closed-loop system is certified for $t \in [0, +\infty)$.

Furthermore, using the definition of $\dot{\varsigma}$, it can be seen that

$$\dot{\varsigma} = \lambda k_1 z_1^2 + \lambda \varpi_1 \|\Upsilon_1\| z_1^2 \ge \lambda k_1 z_1^2. \tag{72}$$

Based on the boundedness of $\varsigma(t)$, the following inequality holds.

$$\lim_{t \longrightarrow +\infty} \int_0^t \lambda k_1 z_1^2 d\tau \le \lim_{t \longrightarrow +\infty} (\varsigma(t) - \varsigma(0)) < +\infty. \tag{73}$$

From the definition of $\dot{z}_1$ in (6) and the boundedness of x_1, x_2, and $\dot{y}_d$, it is easily shown that $\dot{z}_1$ is bounded.

Thus, by utilizing Barbalat's Lemma, one has

$$\lim_{t \longrightarrow \infty} z_1(t) = 0. \tag{74}$$

which means that the tracking error is asymptotically stable.

Control parameters selection. The selection ranges of σ_i and k_i are wide. The bigger the parameters σ_i and k_i, the faster the asymptotic convergence and response. Besides, λ is the other key parameter, which affects the asymptotic convergence of the tracking error. According to the simulation tests (see Figure 1), the bigger λ the faster the transient process and the smaller the steady-state errors. But it cannot be too big. In addition, the choices of a_i, b_i, c_i, and q_i are flexible. □

4. Simulation Results

To illustrate the feasibility of the developed adaptive control approach, an uncertain nonlinear system and a single-link robot are considered.

4.1. Simulation of a Second-Order Nonlinear System. Let us consider the strict-feedback nonlinear system as

$$\begin{cases} \dot{x}_1 = g_1(x_1)x_2 + \theta^T \varphi_1(x_1) + d_1(t), \\ \dot{x}_2 = g_2(\overline{x}_2)u + \theta^T \varphi_2(\overline{x}_2) + d_2(t), \\ y = x_1, \end{cases} \tag{75}$$

where $g_1 = 1 + 0.1 \sin x_1$ with $g_{1m} = 0.9$ and $\overline{g}_{1M} = 1.1$, $g_2 = 1 + 0.1 \cos x_1 \sin x_2$ with $g_{2m} = 0.9$, and $\overline{g}_{2M} = 1.1$, $\theta = [1, 1]^T$ $\varphi_1 = \cos x_1$, $\varphi_2 = \sin x_1 \cos x_2$, $d_1 = e^{-t} \sin t$, $d_2 = e^{-t} \cos t$.

The desired trajectories are selected as $y_{d1}(t) = 4 \sin(0.2\pi t)$. and $y_{d2}(t) = 2.5(\sin(0.5t) + \sin t)$. In order to see the controller performance under different initial conditions for the system states, the initial value of x_1 is given as 1.5 and 3, respectively; $\varsigma(0) = 1$; and other initial conditions are chosen to be 0.

The intermediate control signal α_1, Nussbaum variable $\dot{\varsigma}$, and actual control law u of the developed control strategy are defined as (17), (19), and (64), where $n = 2$, $k_1 = 5$, and $k_2 = 10$, $a_1 = a_2 = b_1 = b_2 = c_1 = c_2 = l_2 = m_2 = 10$, $\lambda = 0.1$, 0.5, 1, and 3. It is worth noting that the upper limit of λ is 3.5. Otherwise, the controller is easy to be ill-defined such that the states in the system are divergent.

$\dot{\varpi}_1$ and $\dot{\varpi}_2$ are designed as (21) and (65), with $n = 2$, $q_1 = q_2 = 1$, and $\sigma_1 = \sigma_2 = 0.1$.

The proposed controller (PC) is compared with the adaptive fuzzy (AFC) [28] and adaptive controllers (AC) [34], respectively.

The AFC is presented as

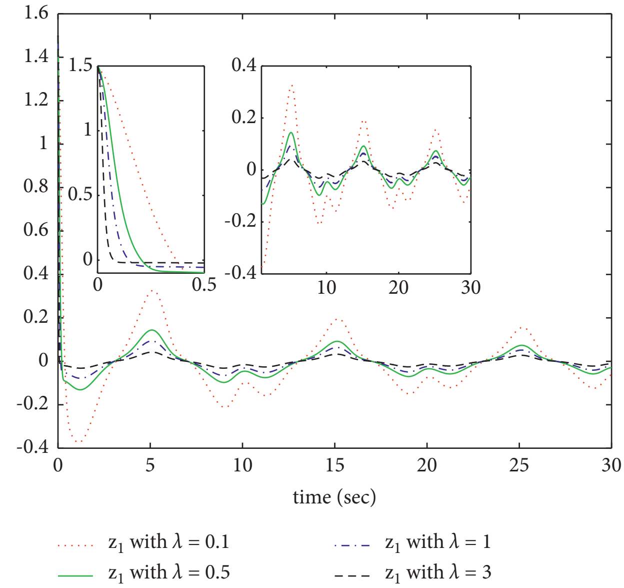

Figure 1: The tracking errors with different λ.

$$\begin{aligned}
\alpha_1' &= -\left(k_1 z_1 + 0.5 z_1 + \frac{1}{2a_1^2} z_1 \theta_1 R_1^T R_1\right),\\
u\prime &= -\left(k_2 z_2 + 0.5 z_2 + \frac{1}{2a_2^2} z_2 \theta_2 R_2^T R_2\right),\\
\dot{\theta}_1 &= \frac{q_1}{2a_1^2} z_1^2 R_1^T R_1 - \sigma_1 \theta_1,\\
\dot{\theta}_2 &= \frac{q_2}{2a_2^2} z_2^2 R_2^T R_2 - \sigma_2 \theta_2.
\end{aligned} \tag{76}$$

where θ_i and R_i are defined in [28], k_i, a_i, q_i, and σ_i are taken the same values as the PC, $i = 1\ ,\ 2$.

The AC is given as

$$\begin{aligned}
\alpha_1 &= -\frac{z_1 \overline{\alpha}_1^2}{\sqrt{z_1^2 \overline{\alpha}_1^2 + \delta_1^2}}, \quad \overline{\alpha}_1 = k_1 z_1 + \varpi_1 \left\|\Upsilon_1\right\| z_1,\\
u &= -\frac{z_2 \overline{\alpha}_2^2}{\sqrt{z_2^2 \overline{\alpha}_2^2 + \delta_2^2}}, \quad \overline{\alpha}_2 = k_2 z_2 + \varpi_2 \left\|\Upsilon_2\right\| z_2,\\
\dot{\varpi}_1 &= q_1 \left\|\Upsilon_1\right\| z_1^2 - \sigma_1 \varpi_1,\\
\dot{\varpi}_2 &= q_2 \left\|\Upsilon_2\right\| z_2^2 - \sigma_2 \varpi_2,
\end{aligned} \tag{77}$$

where k_i, a_i, b_i, c_i, l_2, m_2, q_i, and σ_i are as same as the PC, $i = 1, 2$. δ_1, δ_2 are positive constants, $\delta_1 = 0.1$, $\delta_2 = 1$.

Figures. 1–11 display the simulation results. In Figure 1, the tracking errors are drawn under different λ. It can be seen that the transient processes of the tracking errors are faster and the steady-state errors decrease as λ increase. Under different initial values $x_1(0) = 1.5$ and 3, the comparisons of tracking performance for the desired trajectory y_{d1} between the AFC, AC, and PC are displayed in Figures 2–4. It is shown that the tracking performance with the PC is better and the tracking error is much smaller. Furthermore, the AFC can only ensure the boundedness of tracking errors, and the PC guarantees the asymptotic convergence of tracking errors, which is obvious in Figure 2. In addition, the absolute mean and root mean square values of tracking errors with the PC are smaller than the values with AFC and AC. Figures 5–7 draw the comparisons of tracking errors and tracking performance for the ideal trajectory y_{d2}, and the similar conclusion can be obtained as Figures 2–4. The inputs u are given in Figure 8, which are large at the initial stage and converge rapidly to a range defined by ± 10. The states x_2 and adaptive parameters ϖ_1^*, ϖ_2^* are presented in Figures 9 and 10, which are all bounded. Finally, Figure 11 shows the Nussbaum variables ς, which tends to positive constants from the initial values 1 as time goes on.

4.2. Simulation of a 3-Order Robot. In order to demonstrate the practicability, the developed controller is applied to the following 3-order robot system [40].

$$\begin{cases}
\dot{x}_1 = x_2,\\
\dot{x}_2 = \dfrac{K_m}{M} x_3 + \dfrac{-N \sin x_1 - B x_2}{M} + \dfrac{\tau_e}{M}, \dot{x}_3 = \dfrac{1}{L} V + \dfrac{-R x_3 - K_e x_2}{L},
\end{cases} \tag{78}$$

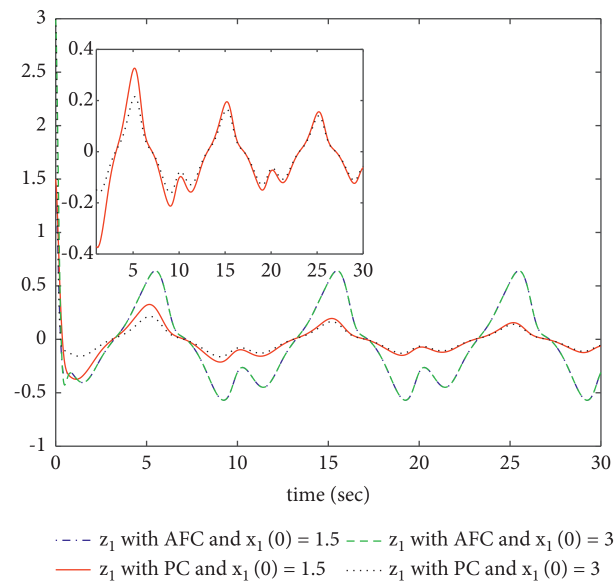

FIGURE 2: The comparison of tracking errors between PC and AFC with y_{d1}.

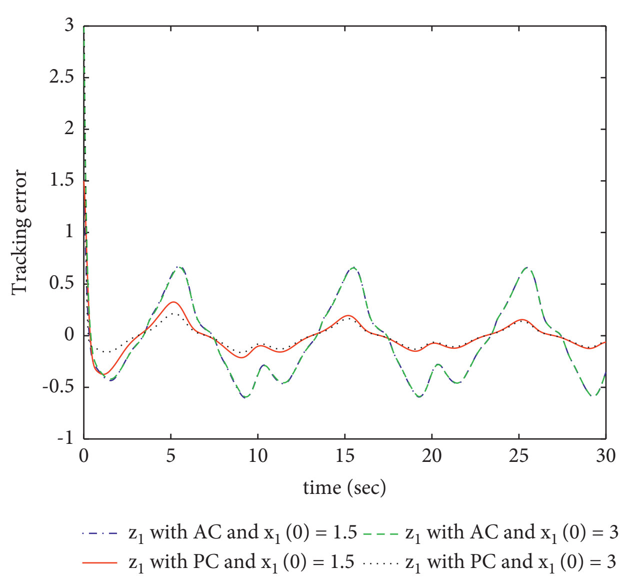

FIGURE 3: The comparison of tracking errors PC and AC with y_{d1}.

where x_1, x_2, and x_3 denote the angle, angular velocity, and motor current of the manipulator, respectively. $\overline{M}$ is the system inertia, $K_m = KN_c$ represents the coefficient between the current and moment with K being the constant torque, and N_c expressing the joint reduction ratio. Furthermore, τ_e represents the uncertain disturbance of torque by the external environment, L is the armature inductance and unknown, K_e denotes the back electromotive force constant, R displays the circuitous resistance, and V is the input voltage. The formulae of B, N, and M are introduced as follows:

$$\begin{aligned} B &= \frac{B_0}{K_\tau} \\ N &= \frac{m_1 dg}{2K_\tau} + \frac{m_2 dg}{K_\tau} \\ M &= \frac{J}{K_\tau} + \frac{m_1 d^2}{3K_\tau} + \frac{m_2 d^2}{K_\tau} + \frac{2m_2 \delta^2}{5K_\tau} \end{aligned} \tag{79}$$

where K_τ expresses the coefficient of the electromechanical conversion of armature current to torque and B_0 shows the coefficient of viscous friction at the joint. g is the gravity

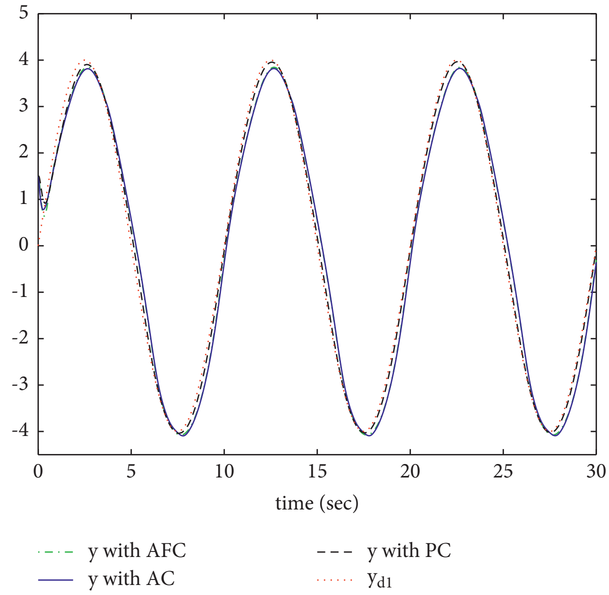

Figure 4: The trajectories with $x_1(0) = 1.5$.

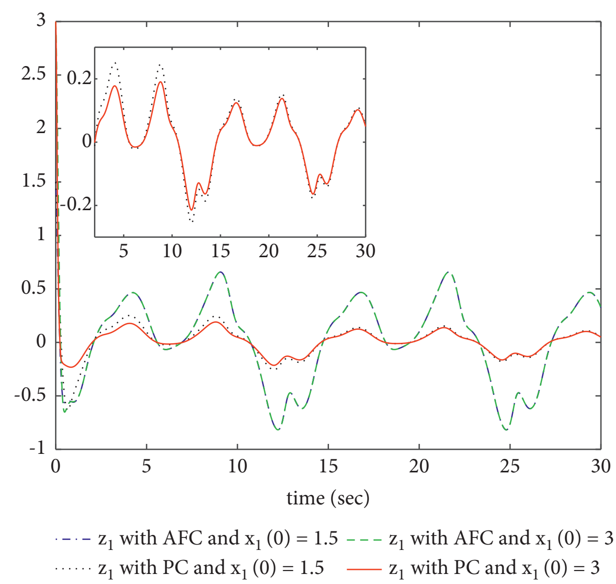

Figure 5: The comparison of tracking errors between PC and AFC with y_{d2}.

coefficient, J denotes the rotor inertia, d represents the link length, δ is the radius of the load, and m_1 and m_2 display the mass of the link and load. Moreover, the parameter values are as

$$\begin{aligned}
J &= 1.625 \times 10^{-3} kg \cdot m^2, \\
d &= 0.305m, \\
m_1 &= 0.506\, kg, \\
m_2 &= 0.434kg, \\
K_e &= 0.9V \frac{s}{rad}, \\
B_0 &= 16.25 \times 10^{-3} \frac{N.m.s}{rad}, \\
\delta &= 0.023m, \\
K_\tau &= 0.9N \frac{m}{A}, \\
L &= 25 \times 10^{-3} H.
\end{aligned} \tag{80}$$

The external disturbance τ_e is defined as the Gaussian white noise written as $wgn(1, L, 0.2)$ with L being the time series.

The initial condition vector is selected as $[x_1(0),\ x_2(0), x_3(0), \varpi_1(0), \varpi_2(0),$

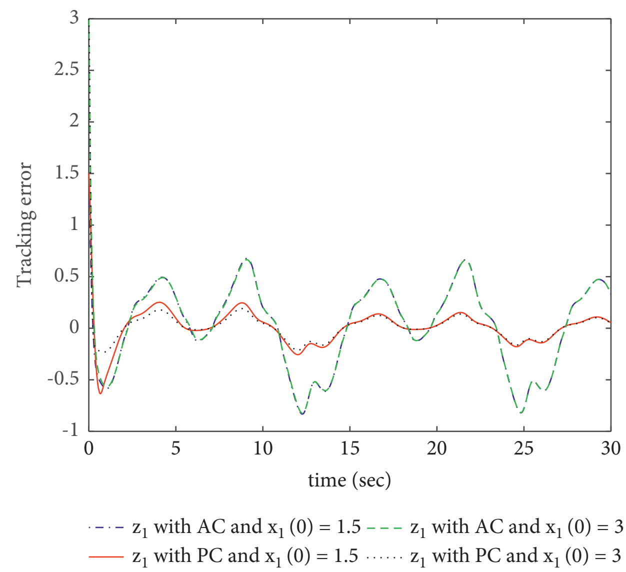

FIGURE 6: The comparison of tracking errors between PC and AC with y_{d2}.

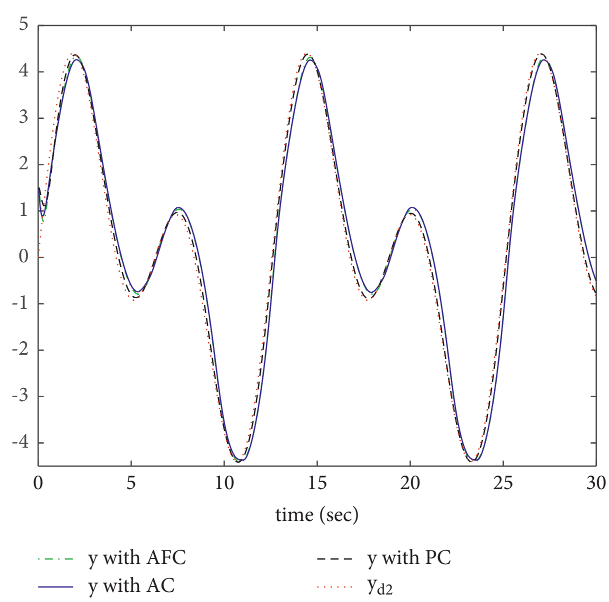

FIGURE 7: The trajectories with $x_1(0) = 1.5$.

$\varpi_3(0)\varsigma] = [0.1\,\text{rad},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0.7]$. The reference trajectory is chosen to be a step signal. To satisfy the first order differentiability, it is written as $y_d = 0.5(\tanh(p(t-5)) + t1)$ with $p = 2$.

The Nussbaum variable $\dot{\varsigma}$, control laws α_1, $\alpha_2\,u$, and adaptive laws $\dot{\varpi}_1$, $\dot{\varpi}_2$, $\dot{\varpi}_3$ are defined as (19), (17), (32), (64), (21), (35), and (65) with $k_1 = 10$, $k_2 = 12$, $k_3 = 20$, $q_1 = 0.02$, $q_2 = 0.01$, $q_3 = 0.025$, $\sigma_1 = 15$, $\sigma_2 = 12$, $\sigma_3 = 28$, and $\lambda = 10$. The other parameters are equal to 1.

Figures 12–15 show the simulation results of the 3-order single-link robot. The angle tracking error is displayed in Figure 12, which converges to the initial value $0.1ra\,d$ rapidly and oscillates at the step time $t = 5s$. In Figure 13, the output can track the reference trajectory well. This is a step signal. The input voltage u is presented in Figure 14. It is easily seen that the oscillation also occurs at the step time. The angular velocity x_2 and motor current x_3 are given in Figure 15, respectively, both of which are bounded.

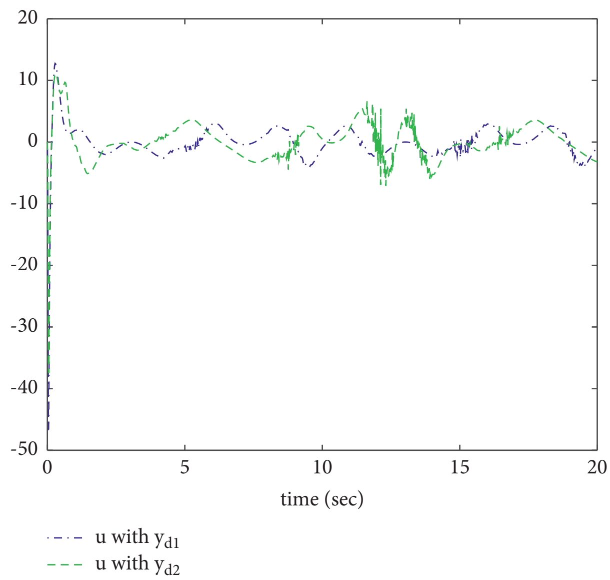

FIGURE 8: The control inputs u with $x_1(0) = 1.5$.

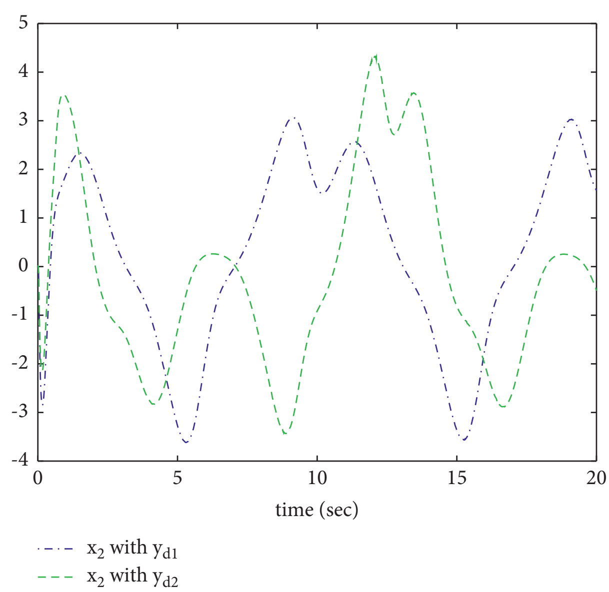

FIGURE 9: The state variables x_2 with $x_1(0) = 1.5$.

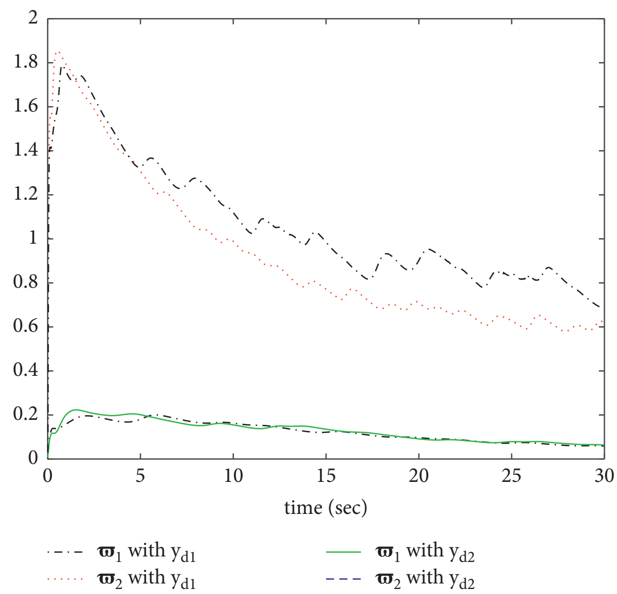

FIGURE 10: The adaptive parameters ϖ_1, ϖ_2 with $x_1(0) = 1.5$.

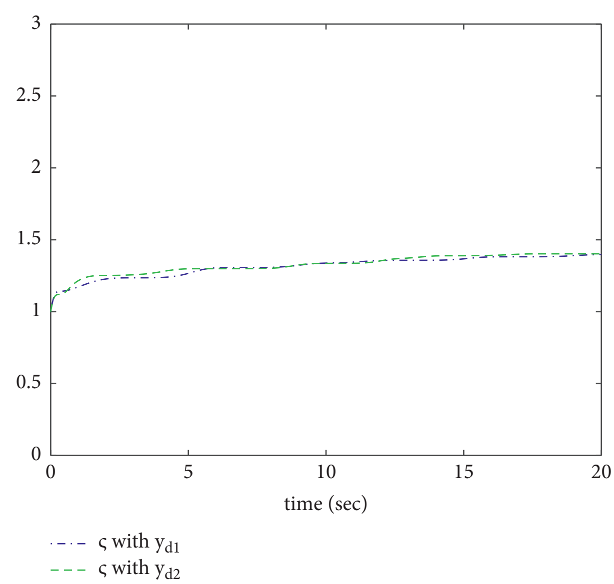

FIGURE 11: The Nussbaum variables ς with $x_1(0) = 1.5$.

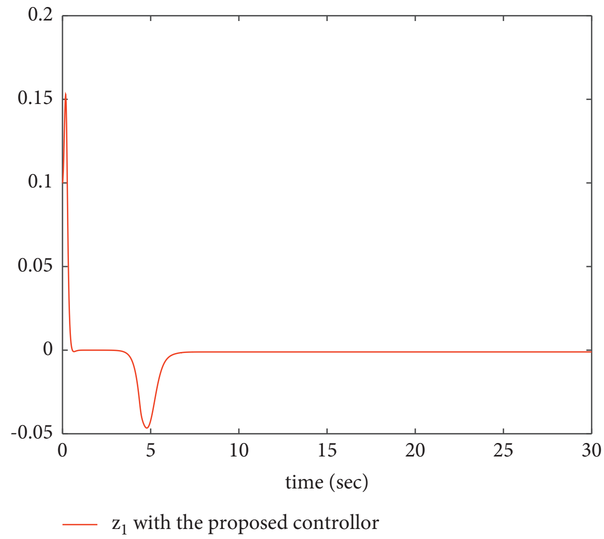

Figure 12: The angle error.

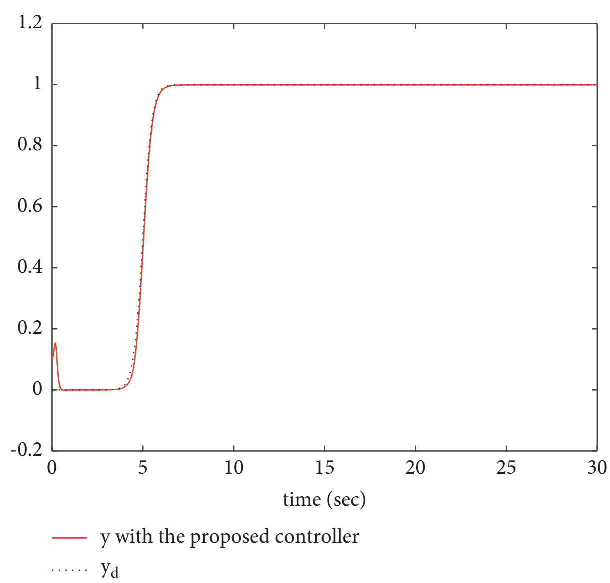

Figure 13: The output and desired trajectories.

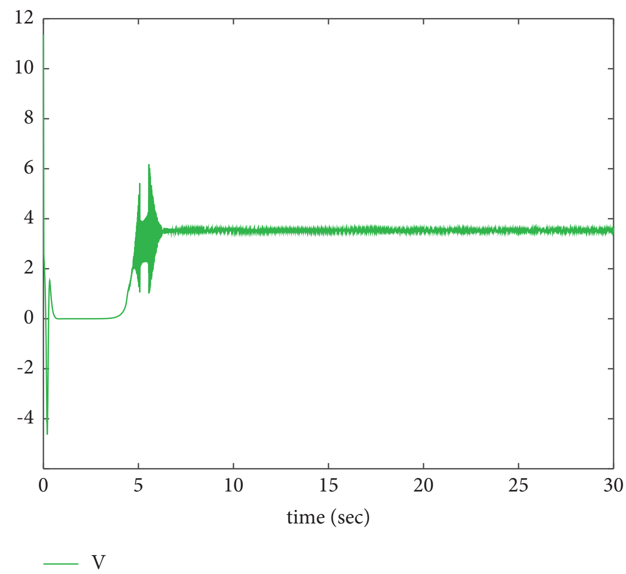

Figure 14: The input voltage.

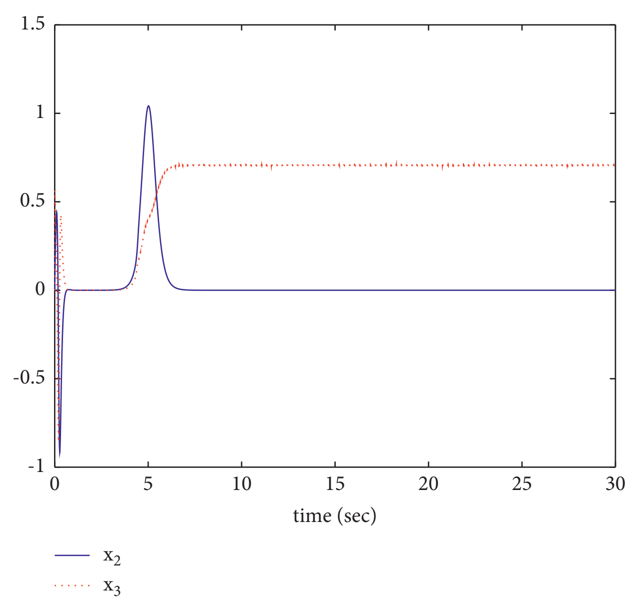

Figure 15: The angular velocity and motor current.

5. Conclusion

In the article, an adaptive asymptotic tracking controller has been designed for nonlinear systems with uncertain VCGs. Compared with these existing achievements, the developed control method could not only handle the VCGs in the form of unknown nonlinear functions but also achieve the asymptotic stability of the tracking error, which was carried out without the approximation by fuzzy logic or neural network and repetitive use of Nussbaum-type functions. New adaptive laws were defined to compensate for unknown virtual control gains, uncertain parameters, and external disturbances. Finally, the proposed control scheme was designed and applied to the control of a robot system. Both theoretical analysis and simulation were used to validate the effectiveness and practicability of the developed control strategy. In the future, we plan to apply the proposed controller to MIMO nonlinear systems by combining dead zone, saturation, hysteresis, and so on.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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