Hindawi
Complexity
Volume 2022, Article ID 7466780, 15 pages
https://doi.org/10.1155/2022/7466780

Research Article

Adaptive Event-Triggered Finite-Time Tracking of Output-Constrained High-Order Nonlinear Systems with Time-Varying Powers

Fan Liu and You Wu

Institute of Automation, Qufu Normal University, Qufu, China

Correspondence should be addressed to You Wu; youwutom@126.com

Received 2 November 2021; Revised 21 December 2021; Accepted 30 December 2021; Published 7 July 2022

Academic Editor: Zidong Wang

This paper studies the adaptive event-triggered finite-time tracking of output-constrained high-order nonlinear systems with time-varying powers. Due to the presence of multiple unknown powers and the consideration of event-triggered control, all the existing control methods of output-constrained nonlinear systems are inapplicable. By introducing nonlinear mappings, finite-time performance functions, and low-power and high-power terms into adding a power integrator technique and the relative threshold strategy, an adaptive state-feedback controller is designed to eliminate the effects caused by the output constraint and time-varying powers. It is proved that all the closed-loop signals are bounded, the asymmetric time-varying output constraint is not violated, and the tracking error converges to a prescribed arbitrarily small region around zero in a preassigned finite time. Furthermore, the Zeno phenomenon can be avoided. Two simulation examples demonstrate the effectiveness of this control scheme.

1. Introduction

Due to hardware limitations, performance requirements, or safety specifications, state/output constraints are always involved in many nonlinear systems. For example, the speed and acceleration of motor vehicles should be restricted to prevent possible accidents. In the process of operation, the violation of state/output constraints may decrease system performance or even make the system unstable. Therefore, the research of constrained control for nonlinear systems becomes extremely important and urgent. In the past decade, barrier Lyapunov function (BLF) and nonlinear mapping (NM), which were firstly put forward in <u>[1, 2] </u>, respectively, have become two valid tools for handling state/output constraints of nonlinear systems. The value of BLF/NM will tend to infinity when the state/output closes to some constraints. As long as BLF/NM is bounded, the state/output does not exceed these constraints. Compared with the BLF-based constrained control method, NM-based counterpart can directly deal with the original state/output constraints, and thus, the undesirable feasibility conditions in [3] can be removed. Based on these two methods, fruitful results were obtained; see [4–19] and other papers.

Compared with traditional feedback linearized nonlinear systems, due to the presence of higher powers, the Jacobian linearization of more general high-order nonlinear systems (also known as p-normal nonlinear systems) may be neither feedback linearized nor controllable. Such inherent obstacles make the control design more challenging and difficult. Fortunately, with the development of a power integrator technique [20], these obstacles can be delicately overcome. By combining such a technique with different types of BLF/NM, some interesting results on the stabilization and practical tracking of constrained high-order nonlinear systems have been obtained in [21–28]. However, the powers of considered systems are constants and precisely known.

Just like the boiler-turbine unit in [29] and the underactuated, weakly coupled, and unstable mechanical system in [30], due to various operating conditions and the potential aging of the hardening spring, the powers of these two systems are unknown and variable. In view of these applications, the study of nonlinear systems with unknown powers is of vital importance. Recently, the authors in [31] studied nonlinear systems with unknown constant powers, and the stabilization of nonlinear systems with a single unknown time-varying power was discussed in [32]. In the presence of multiple unknown time-varying powers, Cui, Xie, and Lie <u>[33, 34]</u> deeply investigated finite-time stabilization and adaptive stabilization. Subsequently, [35–40] proposed the constrained control methods for nonlinear systems with multiple unknown powers.

Different from the traditional continuous sampling control in all the aforementioned works, the event-triggered control mechanism is a useful data scheduling method in networked control systems and only requires control signals to be discontinuously sent to the actuator rather than periodically sampled. Such a feature can effectively diminish the communication overload and computational cost. In view of these benefits, a number of efforts have been made for constructing the event-triggered controllers. Particularly, Xing et al. [41] proposed three original adaptive event-triggered control strategies for strict-feedback nonlinear systems without state/output constraints. Further in-depth studies on the event-triggered control were discussed in some latest results [42–44]. Specially, Zhang et al. and Liu and Li <u>[43, 44]</u> constructed different kinds of event-triggered controllers for two state-/output-constrained nonlinear systems. However, they are unavailable for high-order nonlinear systems with unknown powers. Besides, in view of faster convergence rates, higher accuracies, and better disturbance rejection properties [45], the finite-time stability needs to be further studied.

Based on these discussions, an interesting problem arises: is it possible to design an adaptive event-triggered finite-time tracking controller for output-constrained high-order nonlinear systems with time-varying powers?

In this paper, we will substantially solve this problem. Main contributions and difficulties are emphasized as follows:

(1) This is the first paper to study the event-triggered control of output-constrained nonlinear systems with time-varying powers. Due to the presence of multiple unknown powers and the consideration of event-triggered control, more complex nonlinear terms will inevitably produce in control design, and the constrained controllers in the existing results are all inapplicable. To overcome these essential difficulties, some NMs are first adopted to convert the original output-constrained system into a new one. Then, by introducing sign function, finite-time performance functions, and low-power and high-power terms into adding a power integrator technique and combining the relative threshold strategy, an innovative adaptive event-triggered state-feedback controller is designed to guarantee the performances of the closed-loop system. Furthermore, the Zeno phenomenon does not occur.

(2) Compared with <u>[1–19, 22–28, 35–40, 43, 44]</u> on nonlinear constrained control, the considered system is more general since it possesses parametric uncertainties, multiple unknown time-varying powers, and asymmetric time-varying output constraint simultaneously.

(3) Compared with <u>[20–28, 31–40]</u> on the continuous sampling control of nonlinear systems, more attractive adaptive event-triggered control is firstly adopted to reduce redundant data transmissions and consume less communication resources in the constrained control framework; see Example 2 for the detailed discussion.

The rest of this paper is organized as follows. Section 2 gives a motivation example and preliminaries. Section 3 presents the main result of this paper, following two simulation examples in Section 4. Section 5 concludes this paper.

Notations: $\mathbb{Z}^+$, $\mathbb{R}^+$, $\mathbb{R}$, and $\mathbb{R}^n$ denote the set of all positive integers, the set of all nonnegative real numbers, the set of real numbers, and the real n-dimensional space, respectively. $\mathscr{C}^1$ is the set of all functions with continuous partial derivatives. For a real vector $x = [x_1, \ldots, x_n]^\top \in \mathbb{R}^n$, the norm $\|x\|$ is defined by $\|x\| = (\sum_{i=1}^n x_i^2)^{1/2}$. For any $s \in \mathbb{R}$, sgn(s) denotes its sign function, which satisfies sgn$(s) = 1$ if $s > 0$, sgn$(s) = 0$ if $s = 0$, and sgn$(s) = -1$ if $s < 0$, and $\lceil s \rfloor^p \triangleq \mathrm{sgn}(s)|s|^p$ for positive p. For a $\mathscr{C}^1$ function $V(x)$: $\mathbb{R}^n \longrightarrow \mathbb{R}^+$, it is positive definite if $V(x) \geq 0$, and $V(x) = 0$ if and only if $x = 0$. The arguments of functions are sometimes simplified, for example, a function $f(x(t))$ can be written as $f(x)$, $f(\cdot)$, or f.

2. Motivation Example and Preliminaries

2.1. Motivation Example. Consider the underactuated, weakly coupled, and unstable system in Figure 1 [30]. This mechanical system contains a mass m_1 on a horizontal smooth surface and an inverted pendulum m_2 supported by a massless rod. The mass m_1 is interconnected to the wall by a linear spring and to the inverted pendulum by a nonlinear spring. Let x be the displacement of mass m_1 and θ be the angle of the pendulum from the vertical such that at $x = 0$ and $\theta = 0$. The springs are unstretched. A control force u acts on mass m_1. The equation of motion for this system is described as

$$\begin{aligned} \ddot{\theta} &= \frac{g}{l}\sin(\theta) + \frac{k_s}{m_2 l}\lceil x - l\sin(\theta)\rfloor^{p(t)}\cos(\theta), \\ \ddot{x} &= -\frac{k}{m_1}x - \frac{k_s}{m_1}\lceil x - l\sin(\theta)\rfloor^{p(t)} + \frac{u}{m_1}, \end{aligned} \tag{1}$$

where l is the length of the rod, g is the acceleration of gravity, k and k_s are spring coefficients, and $y = x - l\sin(\theta)$ in Figure 1. Assume that m_1, m_2, l, and k_s are unknown constant parameters which belong to a known interval $[\underline{c}, \overline{c}]$ with $\overline{c} \geq \underline{c} > 0$. Suppose that θ is small; equation (1) becomes

$$\begin{aligned} \ddot{\theta} &= \frac{g}{l}\theta + \frac{k_s}{m_2 l}\lceil x - l\theta\rfloor^{p(t)}, \\ \ddot{x} &= -\frac{k}{m_1}x - \frac{k_s}{m_1}\lceil x - l\theta\rfloor^{p(t)} + \frac{u}{m_1}. \end{aligned} \tag{2}$$

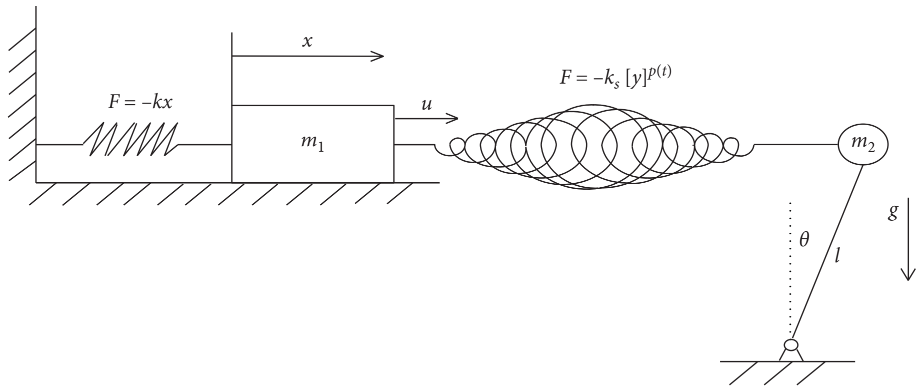

FIGURE 1: The underactuated, weakly coupled, and unstable system.

Then, by using the change of coordinates

$$x_1 = \theta,\ x_2 = \dot{\theta},\ x_3 = x - l\theta,\ x_4 = \dot{x}_3, \tag{3}$$

system (2) becomes the following system:

$$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= \frac{k_s}{m_2 l}\lceil x_3\rceil^{p(t)} + \frac{g}{l}x_1,\\
\dot{x}_3 &= x_4,\\
\dot{x}_4 &= \frac{u}{m_1} - \frac{k}{m_1}(\mathrm{l}x_1 + x_3) + \left(\frac{k_s}{m_1} + \frac{k_s}{m_2}\right)\lceil x_3\rceil^{p(t)} - \mathrm{g}x_1.
\end{aligned} \tag{4}$$

The unknown time-varying power $p(t)$ reflects the potential aging of the hardening spring, and the output $y = x_1$ of system (4) needs to be constrained in $(-\pi/2, \pi/2)$ to fit actual demands, which clearly illustrates the research motivation of output-constrained nonlinear systems with unknown powers.

The corresponding networked control system including the actuator, plant (4), sensor, controller, and two communication networks is depicted in Figure 2.

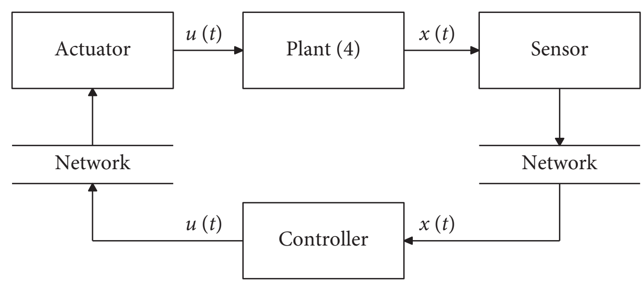

FIGURE 2: Sketch of the networked control system for plant (4).

2.2. Preliminaries. Lemmas 1–6 are used to enlarge inequalities in the following state-feedback control design and analysis.

Lemma 1 (see [46]). *Let $r_1(t)$, $r_2(t)$, and $\alpha(x, y)$ be some positive continuous real-valued functions. For any ,*

$$\begin{aligned}
|x|^{r_1(t)}|y|^{r_2(t)} \le{}& \frac{r_1(t)}{r_1(t) + r_2(t)}\alpha(x, y)|x|^{r_1(t)+r_2(t)}\\
&+ \frac{r_2(t)}{r_1(t) + r_2(t)}\alpha^{-(r_1(t)/r_2(t))}(x, y)|y|^{r_1(t)+r_2(t)}.
\end{aligned} \tag{5}$$

Lemma 2 (see [46]). *Let $r(t) \ge 1$ be a continuous real-valued function. For any $x_i \in \mathbb{R}$, $i = 1, \ldots, n$,*

$$\sum_{i=1}^{n} |x_i|^{r(t)} \le \left(\sum_{i=1}^{n} |x_i|\right)^{r(t)} \le n^{r(t)-1}\sum_{i=1}^{n} |x_i|^{r(t)}. \tag{6}$$

Lemma 3 (see [46]). *If $r(t)$ is a continuous real-valued function and satisfies $\underline{r}(t) \le r(t) \le \overline{r}(t)$, where $\underline{r}(t)$ and $\overline{r}(t)$ are positive real-valued functions, for any $x \in \mathbb{R}$,*

$$|x|^{r(t)} \le |x|^{\underline{r}(t)} + |x|^{\overline{r}(t)}. \tag{7}$$

Lemma 4 (see [47]). *Let $r(t) \ge 1$ be a continuous real-valued function. For any ,*

$$\begin{aligned}
\left|\lceil x\rceil^{r(t)} - \lceil y\rceil^{r(t)}\right| \le{}& r(t)\left(1 + 2^{r(t)-2}\right)\left(|x - y|^{r(t)}\right.\\
&\left.+ |x - y|\,|y|^{r(t)-1}\right),
\end{aligned} \tag{8}$$

where $|y|^{r(t)-1} \triangleq 0$ if $y = 0$ and $r(t) = 1$.

Lemma 5 (see [48]). *For given continuous function $f(x, y)$ with $x \in \mathbb{R}^m$ and $y \in \mathbb{R}^n$, there exist smooth functions $a(x) \ge 0$, $b(y) \ge 0$, $c(x) \ge 1$, and $d(y) \ge 1$ such that*

$$|f(x, y)| \le a(x) + b(y),\ |f(x, y)| \le c(x)d(y). \tag{9}$$

Lemma 6 (see [49]). *For any $\varepsilon > 0$ and $\nu \in \mathbb{R}$, there hold*

$$0 \le |\nu| - \nu\tanh\left(\frac{\nu}{\varepsilon}\right) \le 0.2875\varepsilon,\ -\nu\tanh\left(\frac{\nu}{\varepsilon}\right) \le 0. \tag{10}$$

3. Main Result

3.1. Problem Formulation and Assumptions. Inspired by (4), we consider more general nonlinear systems

$$\begin{aligned} \dot{x}_i &= g_i(\overline{x}_i)\lceil x_{i+1}\rceil^{p_i(t)} + f_i(\overline{x}_i, d), \quad i = 1, \ldots, n-1, \\ \dot{x}_n &= g_n(x)\lceil u\rceil^{p_n(t)} + f_n(x, u, d), \\ y &= x_1, \end{aligned} \tag{11}$$

with the asymmetric time-varying output constraint

$$y_d(t) - F_1(t) < y(t) < y_d(t) + F_2(t), \quad \forall t \geq 0, \tag{12}$$

where $\overline{x}_i = [x_1, \ldots, x_i]^\top \in \mathbb{R}^i$, $i = 1, \ldots, n$, $x = x_n$, $u \in \mathbb{R}$, and $y \in \mathbb{R}$ are measurable states, control input, and output, respectively, $d \in \mathbb{R}^r$ is an unknown constant vector, system powers $p_i(t)$: $\mathbb{R}^+ \longrightarrow \mathbb{R}^+$, $i = 1, \ldots, n$, are unknown time-varying functions with $p_n(t) = 1$, f_i: $\mathbb{R}^i \times \mathbb{R}^r \longrightarrow \mathbb{R}$, $i = 1, \ldots, n-1$, f_n: $\mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^r \longrightarrow \mathbb{R}$ are unknown locally Lipschitz continuous functions, g_i: $\mathbb{R}^i \longrightarrow \mathbb{R}$, $i = 1, \ldots, n$, are known $\mathscr{C}^1$ function, and $y_d(t)$: $\mathbb{R}^+ \longrightarrow \mathbb{R}$ denotes the desired trajectory; finite-time performance functions $F_1(t)$: $\mathbb{R}^+ \longrightarrow \mathbb{R}^+$ and $F_2(t)$: $\mathbb{R}^+ \longrightarrow \mathbb{R}^+$ are defined as

$$F_j(t) = \begin{cases} \left(F_{j0} - \dfrac{t}{t_s}\right) e^{\left(1 - t_s/t_s - t\right)} + F_{js}, \ t \in [0, t_s), \\ F_{js}, \quad t \in [t_s, \infty), \end{cases} \quad j = 1, 2, \tag{13}$$

with $F_{j0} > 5/4$ and $F_{js} > 0$ being design parameters. System (11) is called as the high-order nonlinear system if there exists at least one $p_i(t) > 1$. From Remark 2 in [50], it is easy to prove that $F_j(t) > 0$ is decreasing and smooth, and there exist known positive constants ρ_{j1}, ρ_{j2}, and ρ_{j3} such that $|F_j(t)| \leq \rho_{j1}$, $|\dot{F}_j(t)| \leq \rho_{j2}$, and $|\ddot{F}_j(t)| \leq \rho_{j3}$, $\forall t \geq 0$. It is worth mentioning that (13) plays a crucial role on the achievement of finite-time tracking, and the preassigned finite time $t_s > 0$ can be chosen prior to control implementation, which is independent of initial conditions and design parameters.

The control objective is to design a $\mathscr{C}^1$ adaptive event-triggered state-feedback controller

$$u = u\left(x, y_d, \hat{\Theta}, F, \dot{F}\right), \ \dot{\hat{\Theta}}_i = \zeta\left(\overline{x}_i, y_d, \overline{\hat{\Theta}}_i, F, \dot{F}\right), \tag{14}$$

where $\overline{\hat{\Theta}}_i = [\hat{\Theta}_1, \ldots, \hat{\Theta}_i]^\top$, $\hat{\Theta}_i \in \mathbb{R}$, $i = 1, \ldots, n$, are auxiliary variables used to handle parametric uncertainties, $\hat{\Theta} = \overline{\hat{\Theta}}_n$, $F = [F_1, F_2]^\top$, and $\zeta(\cdot)$ is a continuous function such that the following properties hold:

- (O1) All the closed-loop signals are bounded on $[0, \infty)$
- (O2) Asymmetric time-varying output constraint (12) is not violated
- (O3) The practical finite-time tracking can be fulfilled, i.e., for any small constants $F_{1s} > 0$ and $F_{2s} > 0$, there exists a preassigned finite time $t_s > 0$ such that the tracking error $e(t) = y(t) - y_d(t)$ satisfies $e(t) \in \Omega_e \triangleq \{e(t) \in \mathbb{R}: \ -F_{1s} \leq e(t) \leq F_{2s}\}$, $\forall t \geq t_s$

Remark 1. Although asymptotic tracking has better steady-state performance, practical tracking has a larger applicable scope than asymptotic one since it requires less restrictions on the desired trajectory $y_d(t)$ and the considered system. Actually, no continuous controller exists to achieve the global or even local asymptotic tracking for the considered system (5). This can be verified by a two-dimensional high-order nonlinear system in [51]. Hence, this paper focuses on the practical tracking problem rather than the asymptotic case.

To achieve (O1)–(O3), the following assumptions are needed.

Assumption 1. Unknown time-varying powers $p_i(t)$, $i = 1, \ldots, n$, satisfy $p \geq p_i(t) \geq 1$, where p is a known constant.

Assumption 2. For $i = 1, \ldots, n$, there exist unknown constants $\vartheta_i > 0$ and known $\mathscr{C}^1$ nonnegative functions $\overline{f}_i(\overline{x}_i)$ such that

$$|f_i| \leq \vartheta_i \overline{f}_i(\overline{x}_i), \quad i = 1, \ldots, n. \tag{15}$$

Assumption 3. There exists an unknown constant $M > 0$ such that $|y_d(t)| + |\dot{y}_d(t)| \leq M$, $\forall t \geq 0$.

Remark 2. Assumption 1 means that $p_1(t), \cdots, p_n(t)$ have a common upper bound p. In addition, the restrictive relationship $p_1(t) \geq p_2(t) \geq \cdots \geq p_n(t)$ is no longer needed in this paper.

Inequality (15) in Assumption 2 indicates the restriction on system nonlinearities $f_1, \ldots, f_n$, which allow the existence of parametric uncertainties $\vartheta_1, \ldots, \vartheta_n$ and do not necessarily vanish in the origin. Hence, $f_1, \ldots, f_n$ require less prior knowledge to implement the output tracking.

Assumption 3 describes the boundedness of $y_d(t)$ and $\dot{y}_d(t)$. Compared with the existing works on state-/output-constrained tracking control, no any bounded restriction is imposed on high-order derivatives of $y_d(t)$ in this paper.

3.2. System Transformation. To begin with, we infer from (12) that the constraint on $y = x_1$ is equivalent to $-F_1(t) < e(t) < F_2(t)$, $\forall t \geq 0$. To ensure output constraint (12) and then achieve finite-time tracking, inspired by [2] and (13), we introduce NMs

$$\xi_1 = T_1(e) = \frac{e}{h_1(e)}, \xi_i = T_i(x_i) = \frac{x_i}{h_i}, \quad i = 2, \ldots, n, \tag{16}$$

where $h_1(e) = (F_1 + e)(F_2 - e)$, $h_i = 1$, $i = 2, \ldots, n$. It is easy to show that $T_1(e)$ is strictly increasing, smooth, and diffeomorphic with respect to e. Besides, $\xi_1 \longrightarrow \infty$ when $e \longrightarrow -F_1$ and $e \longrightarrow F_2$. In other words, (6) is not violated when the proper initial condition and the boundedness of the transformed state ξ_1 can be guaranteed in the closed-loop system. From (5) and (10), we obtain a new transformed system:

$$\begin{aligned}\dot{\xi}_1 &= H_1(x_1, y_d, F)\left(g_1\lceil\xi_2\rceil^{p_1(t)} + f_1 - \dot{y}_d\right)\\&\quad + G_1\left(x_1, y_d, F, \dot{F}\right),\\\dot{\xi}_i &= g_i\lceil\xi_{i+1}\rceil^{p_i(t)} + f_i, \quad i = 2, \ldots, n-1,\\\dot{\xi}_n &= g_n u + f_n,\end{aligned} \tag{17}$$

where $H_1 = (F_1F_2 + e^2)/h_1^2$ and $G_1 = -(\dot{F}_1F_2 + F_1\dot{F}_2 + (\dot{F}_2 - \dot{F}_1)e)e/h_1^2$.

3.3. State-Feedback Control Design. In view of, we let $X_0 = \beta_0 = \alpha_0 = 0$ and specify the coordinate transformations

$$\begin{aligned}z_i &= \xi_i - \alpha_{i-1}(X_{i-1}), \alpha_i(X_i)\\&= -\beta_i(X_i)\left(z_i + \lceil z_i\rceil^p\right), \quad i = 1, \ldots, n,\end{aligned} \tag{18}$$

where $X_1 = [x_1, y_d, \hat{\Theta}_1, F, \dot{F}]$, $X_i = [\overline{x}_i, y_d, \overline{\hat{\Theta}}_i, F, \dot{F}]$, $i = 2, \ldots, n$, and the estimations $\hat{\Theta}_i \geq 0$ of are generated by the adaptive laws:

$$\dot{\hat{\Theta}}_i = \tau_i(X_i) - \sigma_i\hat{\Theta}_i, \ \hat{\Theta}_i(0) \geq 0, \quad i = 1, \ldots, n, \tag{19}$$

with $\sigma_i > 0$ being design parameters, and nonnegative $\mathscr{C}^1$ functions $\beta_i(\cdot)$ and $\tau_i(\cdot)$ will be determined later.

Remark 3. Compared with [34–37, 39, 40] on the adaptive control of nonlinear systems with time-varying powers, we relax the requirement $\hat{\Theta}_i \geq 1$ to $\hat{\Theta}_i \geq 0$ in (13). Moreover, the simultaneous introduction of low-power term z_i and high-power term $\lceil z_i\rceil^p$ in (12) is the key to deal with unknown time-varying powers $p_1(t), \ldots, p_n(t)$.

Based on (11)–(13), we provide the detailed procedure of control design in a recursive manner.

Step 1. Take the Lyapunov function $V_1 = z_1^2/2 + \tilde{\Theta}_1^2/2$ with $\tilde{\Theta}_1 = \Theta_1 - \hat{\Theta}_1$. By (11)–(13), we have

$$\begin{aligned}\dot{V}_1 &= z_1H_1\left(g_1\lceil\xi_2\rceil^{p_1(t)} + f_1 - \dot{y}_d\right) + z_1G_1 - \tilde{\Theta}_1\dot{\hat{\Theta}}_1\\&= z_1H_1g_1\lceil\alpha_1\rceil^{p_1(t)} + z_1H_1g_1\left(\lceil\xi_2\rceil^{p_1(t)} - \lceil\alpha_1\rceil^{p_1(t)}\right)\\&\quad + z_1H_1f_1 - z_1H_1\dot{y}_d + z_1G_1 - \tilde{\Theta}_1\tau_1 + \sigma_1\tilde{\Theta}_1\hat{\Theta}_1.\end{aligned} \tag{20}$$

For the sake of consistency, we specify $H_2 = \cdots = H_n = 1$. According to Assumptions 1–3, Lemmas 1–3, and $\Theta_1 = \max\left\{1, \vartheta_1^{1+p}, M^{1+p}\right\}$, one gets

$$z_1H_1f_1 \leq |z_1|H_1\vartheta_1\overline{f}_1 \leq \Theta_1\phi_{11}(X_1)|z_1|^{1+p} + M_0, \tag{21}$$

$$-z_1H_1\dot{y}_d \leq |z_1|H_1M \leq \Theta_1\phi_{12}(X_1)|z_1|^{1+p} + M_0, \tag{22}$$

$$z_1G_1 \leq |z_1 \parallel G_1| \leq \Theta_1\phi_{13}(X_1)|z_1|^{1+p} + M_0, \tag{23}$$

$$\begin{aligned}&H_ig_i\left(\left(H_ig_i\right)^{-1} + \left(H_ig_i\right)^{-1/p}\right)^{p_i(t)}\\&\quad \geq H_ig_i\left(\left(H_ig_i\right)^{-p_i(t)} + \left(H_ig_i\right)^{-p_i(t)/p}\right) \geq 1,\end{aligned} \tag{24}$$

$$\begin{aligned}&|z_i|\left(|z_i| + |z_i|^p\right)^{p_i(t)}\\&\quad \geq |z_i|\left(|z_i|^{p_i(t)} + |z_i|^{pp_i(t)}\right)\\&\quad = |z_i|^{1+p_i(t)} + |z_i|^{1+pp_i(t)} \geq |z_i|^{1+p},\end{aligned} \tag{25}$$

$$\sigma_i\tilde{\Theta}_i\hat{\Theta}_i = \sigma_i\tilde{\Theta}_i\left(\Theta_i - \tilde{\Theta}_i\right) \leq -\frac{\sigma_i\tilde{\Theta}_i^2}{2} + \frac{\sigma_i\Theta_i^2}{2}, \tag{26}$$

for each $i = 1, \ldots, n$, where $M_0 = p(1/(1+p))^{1/p}/(1+p) > 0$ is a constant and $\phi_{1j} \geq 0$, $j = 1, 2$, and 3, are known $\mathscr{C}^1$ functions independent of $p_1(t), \ldots, p_n(t)$. Since $p_1(t) \geq 1$ and $\hat{\Theta}_1(t) \geq 0$, one has $(1 + \hat{\Theta}_1\phi_1)^{p_1(t)} \geq 1 + \hat{\Theta}_1\phi_1$ with $\phi_1 = \phi_{11} + \phi_{12} + \phi_{13}$. From this, by choosing $\beta_1 = ((H_1g_1)^{-1} + (H_1g_1)^{-1/p})(1 + \hat{\Theta}_1\phi_1)$ and noting $\alpha_1 = -\beta_1(z_1 + \lceil z_1\rceil^p)$, (18), and (19) when $i = 1$, it yields

$$\begin{aligned}z_1H_1g_1\lceil\alpha_1\rceil^{p_1(t)} &= z_1H_1g_1\operatorname{sgn}(\alpha_1)|\alpha_1|^{p_1(t)}\\&= -|z_1|H_1g_1\beta_1^{p_1(t)}\left(|z_1| + |z_1|^p\right)^{p_1(t)}\\&\leq -\left(1 + \hat{\Theta}_1\phi_1\right)|z_1|^{1+p}.\end{aligned} \tag{27}$$

Substituting (21)–(23), (27), and $\tau_1 \triangleq \phi_1|z_1|^{1+p}$ into (20) and using (26) when $i = 1$, it yields

$$\begin{aligned}\dot{V}_1 &\leq -\left(1 + \hat{\Theta}_1\phi_1\right)|z_1|^{1+p} + z_1H_1g_1\left(\lceil\xi_2\rceil^{p_1(t)} - \lceil\alpha_1\rceil^{p_1(t)}\right)\\&\quad + \Theta_1\phi_1|z_1|^{1+p} + 3M_0 - \tilde{\Theta}_1\phi_1|z_1|^{1+p} - \frac{\sigma_1\tilde{\Theta}_1^2}{2} + \frac{\sigma_1\Theta_1^2}{2}\\&\quad - |z_1|^{1+p} - \frac{\sigma_1\tilde{\Theta}_1^2}{2} + z_1H_1g_1\left(\lceil\xi_2\rceil^{p_1(t)} - \lceil\alpha_1\rceil^{p_1(t)}\right) + M_1,\end{aligned} \tag{28}$$

where $M_1 = 3M_0 + \sigma_1\Theta_1^2/2 > 0$.

Step$i$ ($i = 2, \ldots, n$): for the sake of consistency, we specify $H_j = 1$, $j = 2, \ldots, i$. Suppose that there exists a positive definite and $\mathscr{C}^1$ function V_{i-1} such that

$$\begin{aligned}\dot{V}_{i-1} &\leq -\sum_{j=1}^{i-1}|z_j|^{1+p} - \sum_{j=1}^{i-1}\frac{\sigma_j}{2}\tilde{\Theta}_j^2 + z_{i-1}H_{i-1}g_{i-1}\\&\quad \cdot\left(\lceil\xi_i\rceil^{p_{i-1}(t)} - \lceil\alpha_{i-1}\rceil^{p_{i-1}(t)}\right) + M_{i-1},\end{aligned} \tag{29}$$

where $M_{i-1} > 0$ is an unknown constant. In the following, we prove that (29) still holds at this step. Take the Lyapunov function $V_i = V_{i-1} + z_i^2/2 + \tilde{\Theta}_i^2/2$ with $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$. By (11)–(13) and (29), we have

$$
\begin{aligned}
\dot{V}_i \le{}& \sum_{j=1}^{i-1}\left|z_j\right|^{1+p} - \sum_{j=1}^{i-1}\frac{\sigma_j}{2}\widetilde{\Theta}_j^2 + z_i g_i\lceil\alpha_i\rceil^{p_i(t)} + z_{i-1}H_{i-1}g_{i-1}\left(\lceil\xi_i\rceil^{p_{i-1}(t)} - \lceil\alpha_{i-1}\rceil^{p_{i-1}(t)}\right)\\
&+ z_i H_i g_i\left(\lceil\xi_{i+1}\rceil^{p_i(t)} - \lceil\alpha_i\rceil^{p_i(t)}\right)\\
&+ z_i\left(f_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}f_j\right) - z_i\frac{\partial\alpha_{i-1}}{\partial y_d}\dot{y}_d - z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}g_j\lceil\xi_{j+1}\rceil^{p_j(t)} - z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}_j}\dot{\hat{\Theta}}_j - z_i\sum_{l=0}^{1}\frac{\partial\alpha_{i-1}}{\partial F^{(l)}}F^{(l+1)}\\
&- \widetilde{\Theta}_i\tau_i + \sigma_i\widetilde{\Theta}_i\hat{\Theta}_i + M_{i-1}.
\end{aligned}
\tag{30}
$$

According to Lemmas 1–3, Assumptions1–3, (13) and (19), $\Theta_i = \max\left\{1, \vartheta_1^{1+p}, \ldots, \vartheta_i^{1+p}, M^{1+p}\right\}$, and $\hat{\Theta}_j \ge 0$, one gets

$$
\begin{aligned}
z_{i-1}H_{i-1}g_{i-1}\left(\lceil\xi_i\rceil^{p_{i-1}(t)} - \lceil\alpha_{i-1}\rceil^{p_{i-1}(t)}\right) &\le p_{i-1}(t)\left(1 + 2^{p_{i-1}(t)-2}\right)\left|z_{i-1}\right|\left|H_{i-1}\right|\left|g_{i-1}\right|\left(\left|z_i\right|^{p_{i-1}(t)-1} + \alpha_{i-1}^{p_{i-1}(t)-1}\right)\left|z_i\right|\\
&\le \Theta_i\phi_{i1}(X_i)\left|z_i\right|^{1+p} + M_0,
\end{aligned}
\tag{31}
$$

$$
z_i\left(f_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}f_j\right) \le \left|z_i\right|\left(\vartheta_i\overline{f}_i + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial\xi_j}\right|\vartheta_j\overline{f}_j\right) \le \Theta_i\phi_{i2}(X_i)\left|z_i\right|^{1+p} + \mathrm{i}\mathrm{M}_0,
\tag{32}
$$

$$
-z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}g_j\lceil\xi_{j+1}\rceil^{p_j(t)} \le \left|z_i\right|\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial\xi_j}\right| g_i \parallel \xi_{j+1}|^{p_j(t)} \le \Theta_i\phi_{i3}(X_i)\left|z_i\right|^{1+p} + (i-1)M_0,
\tag{33}
$$

$$
-z_i\frac{\partial\alpha_{i-1}}{\partial y_d}\dot{y}_d \le \left|z_i\right|\left|\frac{\partial\alpha_{i-1}}{\partial y_d}\right| M \le \Theta_i\phi_{i4}(X_i)\left|z_i\right|^{1+p} + M_0,
\tag{34}
$$

$$
-z_i\frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}_j}\dot{\hat{\Theta}}_j \le \left|z_i\right|\left(\left|\frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}_j}\right|\left(\tau_j + \sigma_j\hat{\Theta}_j\right)\right) \le \Theta_i\phi_{i5}(X_i)\left|z_i\right|^{1+p} + (i-1)M_0,
\tag{35}
$$

$$
-z_i\sum_{l=0}^{1}\frac{\partial\alpha_{i-1}}{\partial F^{(l)}}F^{(l+1)} \le \left|z_i\right|\sum_{j=1}^{2}\sum_{l=0}^{1}\left|\frac{\partial\alpha_{i-1}}{\partial F^{l}}\right|\rho_{j,l+2} \le \Theta_i\phi_{i6}(X_i)\left|z_i\right|^{1+p} + 4M_0,
\tag{36}
$$

where $\phi_{ij}(\cdot) \ge 0$, $j = 1, \ldots, 6$, are known $\mathscr{C}^1$ functions independent of $p_1(t), \ldots, p_n(t)$. Substituting (25)–(30) and $\tau_i \triangleq \phi_i|z_i|^{1+p}$ into (30) and using (26), it yields

$$
\begin{aligned}
\dot{V}_i \le{}& -\sum_{j=1}^{i-1}\left|z_j\right|^{1+P} - \sum_{j=1}^{j=1}\frac{\sigma_j}{2}\widetilde{\Theta}_j^2 + z_i g_i\lceil\alpha_i\rceil^{p_i(t)} + z_i H_i g_i\left(\lceil\xi_{i+1}\rceil^{p_i(t)} - \lceil\alpha_i\rceil^{p_i(t)}\right) + \Theta_i\phi_i\left|z_i\right|^{1+p}\\
&+ (4+3i)M_0 - \widetilde{\Theta}_i\phi_i\left|z_i\right|^{1+p} - \frac{\sigma_i\widetilde{\Theta}_i^2}{2} + \frac{\sigma_i\Theta_i^2}{2} + M_{i-1}\\
\le{}& -\sum_{j=1}^{i-1}\left|z_j\right|^{1+p} - \sum_{j=1}^{i}\frac{\sigma_j}{2}\widetilde{\Theta}_j^2 + z_i g_i\lceil\alpha_i\rceil^{p_i(t)} + z_i H_i g_i\left(\lceil\xi_{i+1}\rceil^{p_i(t)} - \lceil\alpha_i\rceil^{p_i(t)}\right) + \hat{\Theta}_i\phi_i\left|z_i\right|^{1+p} + M_i,
\end{aligned}
\tag{37}
$$

where $M_i = M_{i-1} + (4+3i)M_0 + \sigma_i \Theta_i^2/2 > 0$. Similar to (27), by choosing $\beta_i = (g_i^{-1} + g_i^{-1/p})(1 + \hat{\Theta}_i\phi_i)$ and employing (18) and (19), $H_i = 1$, and $(1 + \hat{\Theta}_i\phi_i)^{p_i(t)} \geq 1 + \hat{\Theta}_i\phi_i$ with $\phi_i = \sum_{j=1}^{6} \phi_{ij}$, the virtual controller $\alpha_i = -\beta_i (z_i + \lceil z_i \rceil^p)$ leads to

$$z_i g_i \lceil \alpha_i \rceil^{p_i(t)} \leq -\left(1 + \hat{\Theta}_i\phi_i\right)\left|z_i\right|^{1+p}. \tag{38}$$

By virtue of (37), (38) becomes

$$\begin{aligned}\dot{V}_i \leq & -\sum_{j=1}^{i}\left|z_j\right|^{1+p} - \sum_{j=1}^{i}\frac{\sigma_j}{2}\tilde{\Theta}_j^2 + z_i H_i g_i \\ & \cdot \left(\lceil \xi_{i+1} \rceil^{p_i(t)} - \lceil \alpha_i \rceil^{p_i(t)}\right) + M_i.\end{aligned} \tag{39}$$

At Step n, inspired by the relative threshold strategy in [41], we adopt the following event-triggered controller:

$$\begin{aligned} & u(t) = \omega(t_k), \quad \forall t \in t_k, (t_{k+1}), \\ & t_{k+1} = \inf\{t > t_k \parallel e_1(t) > \delta|u(t)| + m\}, \\ & \omega(t) = -(1+\delta)\left(\alpha_n \tanh\left(\frac{g_n z_n \alpha_n}{\varepsilon}\right) + \overline{m}\tanh\left(\frac{g_n z_n \overline{m}}{\varepsilon}\right)\right),\end{aligned} \tag{40}$$

where $e_1(t) = \omega(t) - u(t)$ denotes the measurement error, $0 < \delta < 1$, $m > 0$, $\varepsilon > 0$, and $\overline{m} > m/(1-\delta)$ are all design parameters, and t_k, $k \in \mathbb{Z}^+$, is the triggered time. It is shown that the control signal $u(t)$ will not change during $t \in [t_k, t_{k+1})$ and be updated from $\omega(t_k)$ to $\omega(t_{k+1})$ at $t = t_{k+1}$. For $t \in [t_k, t_{k+1})$, it follows from [34] that $|\omega(t) - u(t)| \leq \delta| u(t)| + m$, and thus, there exist two continuous functions $\lambda_1(t)$ and $\lambda_2(t)$ satisfying $|\lambda_1(t)| \leq 1$ and $|\lambda_2(t)| \leq 1$ such that $\omega(t) = (1 + \lambda_1\delta)u(t) + \lambda_2 m$, i.e.,

$$u(t) = \frac{\omega(t) - \lambda_2(t)m}{1 + \lambda_1(t)\delta}. \tag{41}$$

Since for any $a \in \mathbb{R}$ and $\varepsilon > 0$, $-a\tanh(a/\varepsilon) \leq 0$ holds from Lemma 6, by [34], we get $z_n g_n \omega \leq 0$. As $\lambda_1(t) \in [-1, 1]$ and $\lambda_2(t) \in [-1, 1]$, we have $z_n g_n \omega / (1 + \lambda_1(t)\delta) \leq z_n g_n \omega / (1+\delta)$ and $|\lambda_2(t)m/(1 + \lambda_1(t)\delta)| \leq m/(1-\delta) < \overline{m}$. Therefore, by considering (37) when $i = n$, $\xi_{n+1} \triangleq u$, $p_n = 1$, $z_n g_n \alpha_n \leq -(1 + \hat{\Theta}_n\phi_n)|z_n|^{1+p}$ from (32), (34), and (35), and Lemma 6, one obtains

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n-1}\left|z_j\right|^{1+p} - \sum_{j=1}^{n}\frac{\sigma_j}{2}\tilde{\Theta}_j^2 + z_n g_n \frac{\omega(t) - \lambda_2(t)m}{1 + \lambda_1(t)\delta} + \hat{\Theta}_n\phi_n\left|z_n\right|^{1+p} + M_n \\ \leq & -\sum_{j=1}^{n}\left|z_j\right|^{1+p} - \sum_{j=1}^{n}\frac{\sigma_j}{2}\tilde{\Theta}_j^2 \\ & + z_n g_n\left(-\alpha_n\tanh\left(\frac{z_n g_n \alpha_n}{\varepsilon}\right) - \alpha_n + \alpha_n - \overline{m}\tanh\left(\frac{z_n g_n \overline{m}}{\varepsilon}\right) - \overline{m} + \overline{m}\right) \\ & + \hat{\Theta}_n\phi_n\left|z_n\right|^{1+p} + M_n \\ \leq & -\sum_{j=1}^{n}\left|z_j\right|^{1+p} - \sum_{j=1}^{n}\frac{\sigma_j}{2}\tilde{\Theta}_j^2 + \left|z_n g_n \overline{m}\right| + \left|\frac{z_n g_n m}{1-\delta}\right| + M_n + 0.557\varepsilon \\ \leq & -\sum_{j=1}^{n}\left|z_j\right|^{1+p} - \sum_{j=1}^{n}\frac{\sigma_j}{2}\tilde{\Theta}_j^2 + M_n + 0.557\varepsilon.\end{aligned} \tag{42}$$

3.4. Stability and Constraint Analysis. We state the main result of this paper.

Theorem 1. *Consider the closed-loop system consisting of (5), (13), and (34) under Assumptions 1–3. For i = 1, . . ., n, if the initial condition satisfies*

$$-F_1(0) < e(0) < F_2(0), \hat{\Theta}_i(0) \geq 0, \tag{43}$$

then properties (O1)–(O3) in Section 3.1 hold, and the Zeno phenomenon does not occur, i.e., there exists an interexecution time $t^* > 0$ such that $\{t_{k+1} - t_k\} \geq t^*$, $\forall k \in \mathbb{Z}^+$.

Proof. The closed-loop system is rewritten as $\dot{\varsigma}(t) = h(\varsigma(t), u(t))$, where $\varsigma(t) = [x(t), \hat{\Theta}(t)]$. It is clear that $h(\cdot)$ is locally Lipschitz in $\varsigma(t)$. Hence, $\varsigma(t)$ is well defined on the maximal interval $[0, t_f)$ with $0 < t_f < \infty$.

The following proof is divided into four parts. □

Part 1. By Lemma 3 and Assumption 1, we have $z_j^2 \leq 1 + |z_j|^{1+p}$, i.e., $-z_j^{1+p} \leq -z_j^2 + 1$, $j = 1, \ldots, n$. Hence, (36) becomes

$$\dot{V}_n(t) \leq -\lambda_1 V_n(t) + \lambda_2, \forall t \in \left[0, t_f\right), \tag{44}$$

where $\lambda_1 = \min_{1\le j\le n}\{2, \sigma_j\}$ and $\lambda_2 = M_n + 0.557\varepsilon + n$. By [37], we deduce that

$$V_n(t) \le e^{-\lambda_1 t} V_n(0) + \int_0^t e^{-\lambda_1(t-\tau)}\lambda_2 d\tau \le \frac{\lambda_2}{\lambda_1} + \left(V_n(0) - \frac{\lambda_2}{\lambda_1}\right)e^{-\lambda_1 t} \le \frac{\lambda_2}{\lambda_1} + V_n(0). \quad (45)$$

Therefore, the boundedness of $V_n(t)$ is ensured on $[0, t_f)$, so are $z_i(t)$ and $\hat{\Theta}_i(t)$, $i = 1, \ldots, n$. In view of the boundedness of $\xi_1(t) = z_1(t)$ on $[0, t_f)$ and (10), for $-F_1(0) < e(0) < F_2(0)$, $y_d(t) - F_1(t) < x_1(t) < y_d(t) + F_2(t)$ is obtained for all $t \in [0, t_f)$. The continuous virtual controller $\alpha_1(X_1(t))$ is bounded because of the boundedness of for all $t \in [0, t_f)$, and $x_2(t) = \xi_2(t) = z_2(t) + \alpha_1(t)$ is also bounded on $t \in [0, t_f)$. Similarly, we can recursively prove that $\alpha_{i-1}(X_{i-1}(t))$ and $x_i(t) = \xi_i(t) = z_i(t) + \alpha_{i-1}(t)$, $i = 3, \ldots, n$, are all bounded on $[0, t_f)$. Hence, we conclude that the actual controller $u(X_n(t))$ is also bounded on $[0, t_f)$.

Part 2. Next, we prove that $t_f = \infty$. If $t_f < \infty$, at least one signal in the closed-loop system will tend to ∞ when $t = t_f$, which is a contradiction to the boundedness of $[x(t), \hat{\Theta}(t), u(t)]$ on $[0, t_f)$. Therefore, $t_f = \infty$ is proved, and ξ_1 is bounded on $[0, \infty)$, which means that there exist positive constants υ_1, υ_2 and positive functions $\breve{F}_1(t)$, $\breve{F}_2(t)$ satisfying $F_j(t) - \breve{F}_j(t) \ge \upsilon_j > 0$, $j = 1, 2$, such that

$$-F_1(t) < -\breve{F}_1(t) \le e(t) \le \breve{F}_2(t) < F_2(t), \quad (46)$$

holds for all $t \ge 0$. By choosing the proper initial condition $-F_1(0) < e(0) < F_2(0)$ and $\hat{\Theta}_i(0) \ge 0$ in (43) and repeating the previous control design and analysis process, $-\breve{F}_1(t) \le e(t) \le \breve{F}_2(t)$ and $y_d(t) - F_1(t) < y(t) < y_d(t) + F_2(t)$, $\forall t \ge 0$, can be ensured. Hence, all the closed-loop signals are bounded on $[0, \infty)$, and asymmetric time-varying output constraint (6) is not violated. It is worth mentioning that inequality (37) prevents all the terms with $h_1(e(t)) = (F_1(t) + e(t))\ (F_2(t) - e(t))$ in the denominator from becoming unbounded, and thus, the zero division does not occur.

Part 3. From (40), $-F_1(t) < e(t) < F_2(t)$ holds for all $t \ge 0$, which, together with (7), implies that $e(t) \in \Omega_e$, $\forall t \ge t_s$, with Ω_e being defined in (O3). Hence, the practical finite-time tracking can be fulfilled.

Part 4. Finally, we prove that the Zeno phenomenon does not occur. Since $e(t) = \omega(t) - u(t)$, $\forall t \in [t_k, t_{k+1})$, and $u(t)$ remains $u(k)$ in $[t_k, t_{k+1})$, we have $d(|e_1|)/dt = d(e_1 \times e_1)^{1/2}/dt = \mathrm{sgn}(e_1)\dot{e}_1 \le |\dot{\omega}|$. From (34), we obtain

$$\dot{\omega}(t) = -(1+\delta)\left(\dot{\alpha}_n \tanh\left(\frac{z_n g_n \alpha_n}{\varepsilon}\right) + \frac{\alpha_n(z_n \dot{g}_n \alpha_n + z_n g_n \dot{\alpha}_n + \dot{z}_n g_n \alpha_n)}{\cos h^2(z_n g_n \alpha_n/\varepsilon)\varepsilon} + \frac{\overline{m}(z_n \dot{g}_n \overline{m} + g_n \dot{z}_n \overline{m})}{\cos h^2(z_n g_n \overline{m}/\varepsilon)\varepsilon}\right). \quad (47)$$

Since $\dot{z}_n(t)$, $\dot{\alpha}_n(t)$, and $g_n(t)$ are continuous, we deduce that $\dot{\omega}(t)$ is a continuous function. In view of the boundedness of all the closed-loop signals, there exists a constant $\overline{\omega} > 0$ such that $|\dot{\omega}(t)| \le \overline{\omega}$, $\forall t \ge 0$. Due to $e_1(t_k) = 0$ and $\lim_{t \longrightarrow t_{k+1}} e_1(t) = \delta|u(t)| + m$, the lower bound of the interexecution time t^* satisfies $t^* \ge (\delta|u(t)| + m)/\overline{\omega} \ge 0$, and the Zeno phenomenon can be avoided.

4. Two Simulation Examples

Example 1. Consider a reduced-order model of the boiler-turbine unit:

$$\begin{aligned} \dot{x}_1 &= \lceil x_2 \rceil^{p_1(t)}, \\ x_2 &= \left(1 + x_1^2\right)\lceil u \rceil^{p_2(t)} + \frac{\vartheta_2 x_1^2}{1 + x_2^2}, \\ y &= x_1, \end{aligned} \quad (48)$$

where x_1, x_2 are the drum and reheater pressures, u is the position of the control valve, $1 \le p_1(t) \le 11/9$, $p_2(t) = 1$ are time-varying powers, and $\vartheta_2 > 0$ is an unknown constant. The desired trajectory is chosen as $y_d(t) = 0.1\sin(t)$. Clearly, system (48) is a special case of system (13), and Assumptions1–3 are satisfied with $p = 11/9$, $g_1 = 1$, $g_2 = 1 + x_1^2$, $\overline{f}_1 = 0$, and $\overline{f}_2 = x_1^2$. In order to achieve the practical finite-time tracking and satisfy the actual demands, the time-varying output constraint $y_d(t) - F_1(t) < y_d(t) < y_d(t) + F_2(t)$, $\forall t \ge 0$, needs to be guaranteed, where $F_1(t)$ and $F_2(t)$ are defined in (13) with $t_s = 20$s, $F_{10} = F_{20} = 1.2$, and $F_{1s} = F_{2s} = 0.06$.

According to control design in Section 3.3, by setting $e = -y_d$, $h_1 = (F_1 + e)\ (F_2 - e)$, $\xi_1 = e/h_1$, $\xi_2 = x_2$, $H_1 = (F_1 F_2 + e^2)/h_1^2$, $G_1 = -(\dot{F}_1 F_2 + F_1 \dot{F}_2 + (\dot{F}_2 - \dot{F}_1)e)e/h_1^2$, $z_1 = \xi_1$, and $z_2 = \xi_2 - \alpha_1$, the event-triggered controller and adaptive laws are obtained as

$$\begin{aligned} u(t) &= \omega(t_k), \quad \forall t \in [t_k, t_{k+1}), \\ t_{k+1} &= \inf\{t > t_k \,\|\, e_1(t) > \delta|u(t)| + m\}, \\ \omega(t) &= -(1+\delta)\left(\alpha_2 \tanh\left(\frac{z_2 \alpha_2}{\varepsilon}\right) + \overline{m}\tanh\left(\frac{z_2 \overline{m}}{\varepsilon}\right)\right), \\ \dot{\hat{\Theta}}_i &= \phi_i z_i^{20/9} - \sigma_i \hat{\Theta}_i, \quad i = 1, 2, \end{aligned} \quad (49)$$

where

$$
\begin{aligned}
e_1 &= \omega - u,\\
\alpha_i &= -\beta_i\left(z_i + \lceil z_i \rceil^{11/9}\right), \quad i = 1,2\\
\beta_1 &= \left(H_1^{-1} + H_1^{-9/11}\right)\left(1 + \widehat{\Theta}_1\phi_1\right),\\
\beta_2 &= \left(g_2^{-1} + g_2^{-9/11}\right)\left(1 + \widehat{\Theta}_2\phi_2\right),\\
\phi_1 &= H_1^{20/9} + G_1^{20/9},\\
\phi_2 &= \sum_{j=1}^{6} \phi_{2j},\\
\phi_{21} &= \varrho_1 H_1^{20/9} z_1^{20/9}\left(2 + \beta_1\left(s_{z_1} + \left|z_1\right|^{11/9}\right) + s_{z_2}\right),\\
\phi_{22} &= \overline{f}_2^{20/9},\\
\phi_{23} &= d_{x_1}^{20/9}\left(1 + x_2^{220/81}\right),\\
\phi_{24} &= d_{x_1}^{20/9},\\
\phi_{25} &= d_{x_2}^{20/9}\left(\tau_1 + \sigma_1\widehat{\Theta}_1\right)^{20/9},\\
\phi_{26} &= \left(\frac{25}{20}d_{x_3} + \frac{1}{500}d_{x_4}\right)^{20/9},\\
\varrho_1 &= \left(\frac{9 \times 2^{2/9}}{10}\right)\left(\frac{11}{20}\right)^{11/9}\left(\frac{11}{9} \times \left(1 + 2^{-7/9}\right)\right)^{20/9},\\
d_{x_1} &= \left(H_1^{-2} + \frac{9}{11}H_1^{-20/11}\right)\frac{\partial H_1}{\partial x_1}\left(1 + \widehat{\Theta}_1\phi_1\right)\left(s_{z1} + \left|z_1\right|^{11/9}\right) + \left(H_1^{-1} + H_1^{-9/11}\right)\\
&\quad \widehat{\Theta}_1\left(\frac{20}{9}\left(H_1^{11/9}\frac{\partial H_1}{\partial x_1} + G_1^{11/9}\frac{\partial G_1}{\partial x_1}\right)\left(s_{z1} + \left|z_1\right|^{11/9}\right) + \beta_1\left(\frac{e^2 + F_1^2}{h_1^2}\right)\left(1 + z_1^{2/9}\right)\right),\\
d_{x_2} &= \left(H_1^{-1} + H_1^{-9/11}\right)\phi_1\left(s_{z1} + \left|z_1\right|^{11/9}\right),\\
d_{x_3} &= \left(H_1^{-2} + \frac{9}{11}H_1^{-20/11}\right)\frac{\partial H_1}{\partial F_1}\left(1 + \widehat{\Theta}_1\phi_1\right)\left(s_{z1} + \left|z_1\right|^{11/9}\right)\\
&\quad + \left(H_1^{-1} + H_1^{-9/11}\right)\widehat{\Theta}_1\left(\frac{20}{9}\left(H_1^{11/9}\frac{\partial H_1}{\partial F_1} + G_1^{11/9}\frac{\partial G_1}{\partial F_1}\right)\right)\left(s_{z1} + \left|z_1\right|^{11/9}\right) + \beta_1\left(\frac{2\left(1 + e^2\right)^{1/2}}{h_1^2}\right)\left(1 + z_1^{2/9}\right),\\
d_{x_4} &= \left(H_1^{-1} + H_1^{-9/11}\right)\widehat{\Theta}_1\left(\frac{\partial G_1}{\partial \dot{F}_1}\right)\left(s_{z1} + \left|z_1\right|^{11/9}\right).
\end{aligned}
$$

Figures 3 and 4 provide the responses of the closed-loop system and the time interval of each triggered event. From Figure 3, we know that the closed-loop signals x_1, x_2, $\widehat{\Theta}_1$, $\widehat{\Theta}_2$, and u are bounded, the asymmetric time-varying output constraint is not violated, and then the practical finite-time tracking can be fulfilled.

Example 2. Consider a numerical example

$$
\begin{aligned}
\dot{x}_1 &= \lceil x_2 \rceil^{p_1(t)} + \vartheta_1 x_1,\\
\dot{x}_2 &= \lceil u \rceil^{p_2(t)} + \vartheta_2\left(x_1^2 + \sin x_2\right),\\
y &= x_1,
\end{aligned}
\tag{50}
$$

where $1 \leq p_1(t) \leq 21/19$, $p_2(t) = 1$ are time-varying powers and ϑ_1 and ϑ_2 are unknown positive constants. The desired trajectory is chosen as $y_d(t) = 0.4\sin(t)$. Clearly, system (48) is a special case of system (11), and Assumptions1–3 are satisfied with $p = 21/19$, $g_1 = g_2 = 1$, $\overline{f}_1 = (1 + x_1^2)^{1/2}$, and $\overline{f}_2 = 1 + x_1^2$. In order to achieve the practical finite-time tracking and satisfy the actual demands, the time-varying output constraint $y_d(t) - F_1(t) < y(t) < y_d(t) + F_2(t)$, $\forall t \geq 0$, needs to be guaranteed, where $F_1(t)$ and $F_2(t)$ are defined in (13) with $t_s = 40s$, $F_{10} = F_{20} = 2.4$, and $F_{1s} = F_{2s} = 0.05$.

Similar to Example 1, the event-triggered controller and adaptive laws are obtained as

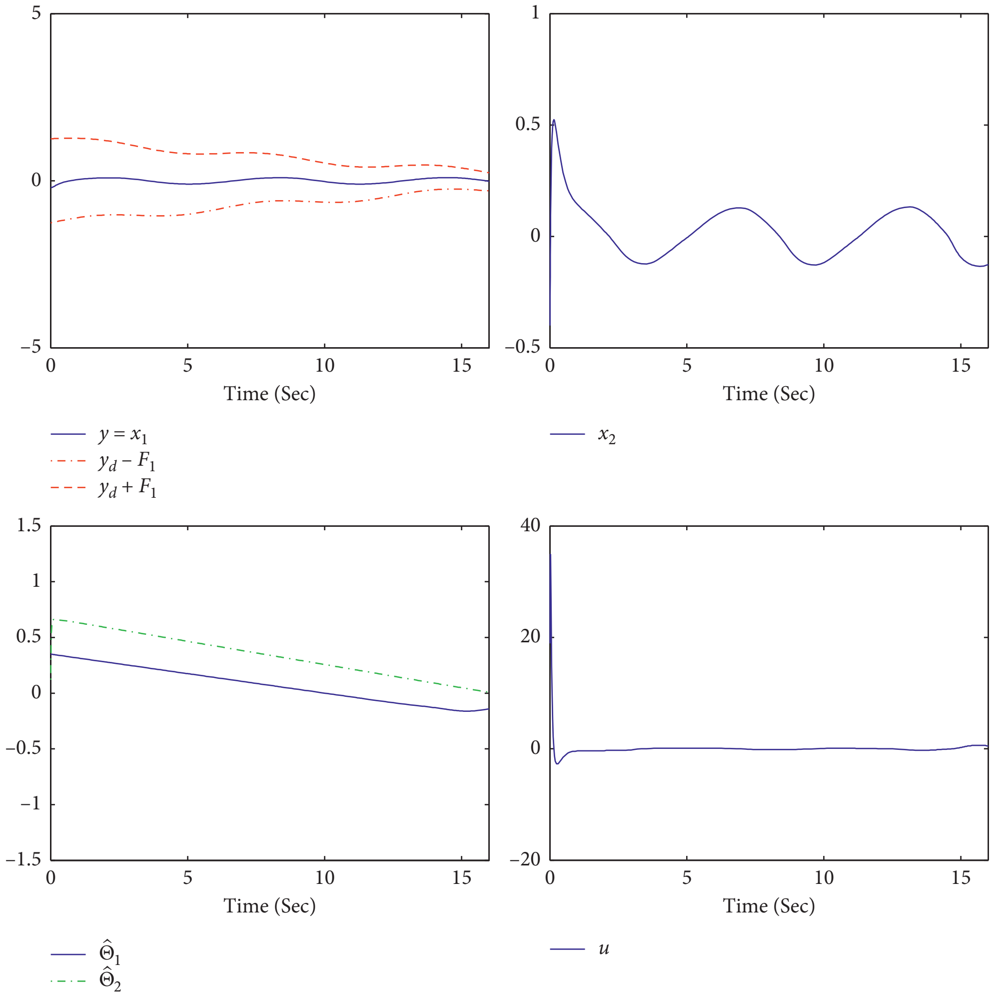

Figure 3: Responses of the closed-loop system (42) and (43).

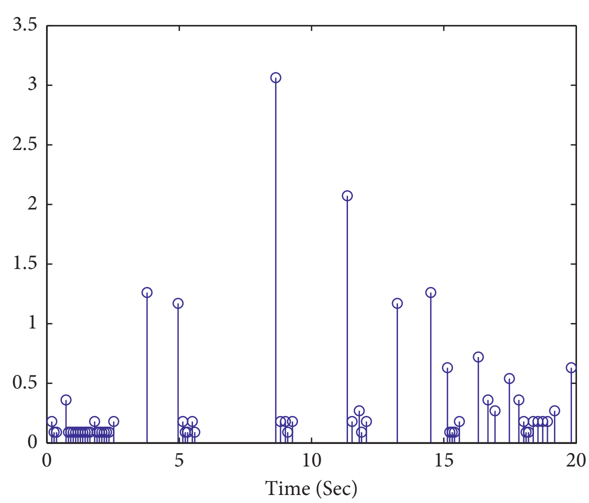

Figure 4: Time interval of each triggered event.

$$
\begin{aligned}
u(t) &= \omega(t_k, \quad \forall t \in [t_k, t_{k+1}), \\
t_{k+1} &= \inf\{t > t_k \parallel e_1(t) > \delta|u(t)| + m\}, \\
\omega(t) &= -(1+\delta)\left(\alpha_2 \tanh\left(\frac{z_2\alpha_2}{\varepsilon}\right) + \overline{m}\tanh\left(\frac{z_2\overline{m}}{\varepsilon}\right)\right), \\
\dot{\hat{\Theta}}_i &= \phi_i z_i^{40/19} - \sigma_i \hat{\Theta}_i, \quad i = 1, 2,
\end{aligned}
\tag{51}
$$

where

$$
\begin{aligned}
e_1 &= \omega - u, \\
\alpha_i &= -\beta_i\left(z_i + \lceil z_i \rceil^{12/19}\right)\oplus, \quad i = 1, 2, \\
\beta_1 &= \left(H_1^{-1} + H_1^{-19/21}\right)\left(1 + \hat{\Theta}_1\phi_1\right), \\
\beta_2 &= 2\left(1 + \phi_2\hat{\Theta}_2\right), \\
\phi_1 &= \left(H_1\overline{f}_1\right)^{40/19} + H_1^{40/19} + G_1^{40/19}, \\
\phi_2 &= \sum_{j=1}^{6}\phi_{2j}, \\
\phi_{21} &= \varrho_1 H_1^{40/19} z_1^{40/19}\left(2 + \beta_1\left(s_{z_1} + |z_1|^{21/19}\right) + s_{z_2}\right), \\
\phi_{22} &= \overline{f}_2^{40/19} + d_{x_1}\overline{f}_1^{40/19}, \\
\phi_{23} &= d_{x_1}^{40/19}\left(1 + x_2^{840/361}\right), \\
\phi_{24} &= d_{x_1}^{40/19}, \\
\phi_{25} &= d_{x_2}^{40/19}\left(\tau_1 + \sigma_1\hat{\Theta}_1\right)^{40/19}, \\
\phi_{26} &= \frac{17}{40}^{40/19} d_{x_3}^{40/19} + \frac{1}{4000}^{40/19} d_{x_4}^{40/19}, \\
\varrho_1 &= 2^{21/19}\left(\frac{19}{40}\right)\left(\frac{21}{40}\right)^{21/19}\left(21\left(\frac{1 + 2^{-17/19}}{19}\right)\right)^{40/19}, \\
s_{z_i} &= \left(1 + z_i^2\right)^{1/2}, \quad i = 1, 2, \\
d_{x_1} &= \beta_1 H_1\left(\frac{40}{19} + \frac{21 s_{z_1}}{19}\right) + \left(\frac{40}{19}\right)\hat{\Theta}_1\left(s_{z_1} + z_1^{40/19}\right) \\
&\quad \cdot \left(H_1^{-1} + H_1^{-19/21}\right)\left(\frac{\partial H_1}{\partial x_1}\left(1 + \overline{f}_1^{40/19}\right) + H_1^{40/19}\overline{f}_1^{21/19}\frac{\partial \overline{f}_1}{\partial x_1} + G_1^{40/19}\left(\frac{\partial G_1}{\partial x_1}\right)\right) \\
&\quad + \left(1 + \hat{\Theta}_1\phi_1\right)\left(s_{z_1} + z_1^{40/19}\right)\left(H_1^{-2} + H_1^{-40/21}\right)\left(\frac{\partial H_1}{\partial x_1}\right), \\
d_{x_2} &= \phi_1\left(H_1^{-1} + H_1^{-19/21}\right)\left(s_{z_1} + |z_1|^{21/19}\right), \\
d_{x_3} &= \beta_1\left(\frac{40}{19} + \frac{21 s_{z_1}}{19}\right)\left(\frac{2\dot{F}_1}{\left(1 + e^2\right)^{1/2}}\right) \\
&\quad + \frac{40}{19}\hat{\Theta}_1\left(s_{z_1} + z_1^{40/19}\right)H_1^{21/19}\overline{f}_1^{40/19}\frac{\partial H_1}{\partial F_1} + H_1^{21/19}\frac{\partial H_1}{\partial F_1} \\
&\quad + G_1^{21/19}\frac{\partial G_1}{\partial F_1} + \left(1 + \hat{\Theta}_1\phi_1\right)\left(s_{z_1} + z_1^{40/19}\right)\left(H_1^{-2} + H_1^{-40/21}\right)\frac{\partial H_1}{\partial F_1}, \\
d_{x_4} &= \frac{40}{21}\hat{\Theta}_1\left(H_1^{-1} + H_1^{-19/21}\right)G_1^{21/19}\left(\frac{2F_1\left(1 + e^2\right)^{1/2}}{h_1^2}\right)\left(s_{z_1} + |z_1|^{21/19}\right).
\end{aligned}
$$

Figures 5 and 6 provide the responses of the closed-loop system (44) and (45) and the time interval of each triggered event. By calculation, the number of triggered control signal transmissions is 207, and the amount of continuous time control sampling is 2072. Since the control signal can be discontinuously sent to the actuator rather than periodically

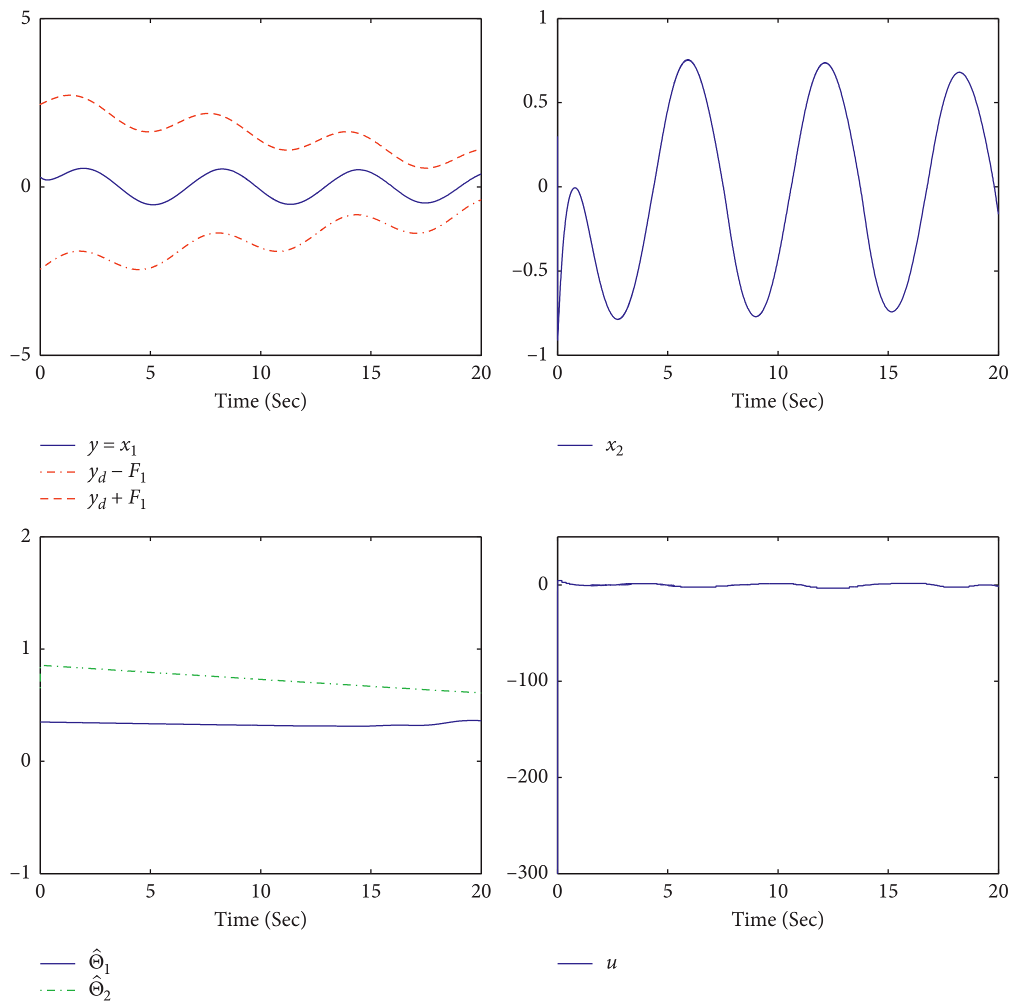

Figure 5: Responses of the closed-loop system (44) and (45).

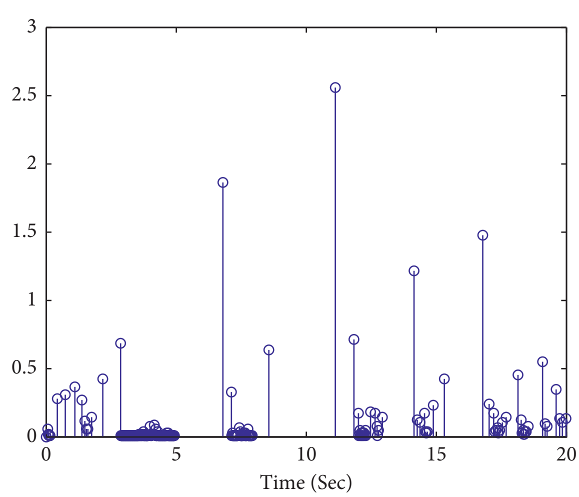

Figure 6: Time interval of each triggered event.

sampled in the traditional continuous sampling control, the designed event-triggered controller reduces redundant data transmissions and consumes less communication resources in the constrained control framework.

5. Conclusions

For more general output-constrained high-order nonlinear systems with unknown time-varying powers, this paper investigated the adaptive event-triggered finite-time tracking problem.

Some challenging problems are still unsolved:

(1) In this paper, $p_i(t) \geq 1$ in Assumption 1 is assumed. When $p_i(t) < 1$, $i = 1, \ldots, n$, system (11) is called as a low-order nonlinear system. Recently, Cui and Xie [33] constructed a state-feedback controller for low-order nonlinear systems with unknown time-varying powers, but it does not consider the output constraint and the event-triggered control. Hence, a more challenging problem is can we design an adaptive event-triggered tracking controller for output-constrained low-order nonlinear systems with unknown powers?

(2) Recently, some important results on systems with stochastic noise, incomplete measurements, coding-decoding mechanisms, and protocol scheduling have been achieved; see [52–67] and other papers.

However, they do not consider the effects of the output constraint. Hence, our subsequent works are to consider stochastic systems with incomplete measurements and coding-decoding mechanisms and solve the output-constrained tracking problem with protocol scheduling.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Taishan Scholar Project of Shandong Province of China under grant ts201712040, the National Natural Science Foundation of China under grant 62073186, the National Key R&D Program of China under grant 2018YFC2001700, and the Natural Science Foundation of Shandong Province under grant 501100007129.

References

[1] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier Lyapunov functions for the control of output-constrained nonlinear systems," *Automatica*, vol. 45, no. 4, pp. 918–927, 2009.

[2] T. Guo and X. Wu, "Backstepping control for output-constrained nonlinear systems based on nonlinear mapping," *Neural Computing & Applications*, vol. 25, no. 7-8, pp. 1665–1674, 2014.

[3] K. P. Tee and S. S. Ge, "Control of nonlinear systems with partial state constraints using a barrier Lyapunov function," *International Journal of Control*, vol. 84, no. 12, pp. 2008–2023, 2011.

[4] W. Sun, Y. Wu, and L. Wang, "Trajectory tracking of constrained robotic systems via a hybrid control strategy," *Neurocomputing*, vol. 330, pp. 188–195, 2019.

[5] W. Sun, L. Wang, and Y. Wu, "Adaptive dynamic surface fuzzy control for state constrained time-delay nonlinear nonstrict feedback systems with unknown control directions," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 12, pp. 7423–7434, 2021.

[6] W. Sun, Y. Wu, and X. Lv, "Adaptive neural network control for full-state constrained robotic manipulator with actuator saturation and time-varying delays," *IEEE Transactions on Neural Networks and Learning Systems*, pp. 1–12, 2021.

[7] L. Liu, Y. J. Liu, A. Chen, S. Tong, and C. L. P. Chen, "Integral barrier Lyapunov function-based adaptive control for switched nonlinear systems," *Science China Information Sciences*, vol. 63, Article ID 132203, 2020.

[8] Y.-J. Liu, W. Zhao, L. Liu, D. Li, S. Tong, and C. L. P. Chen, "Adaptive neural network control for a class of nonlinear systems with function constraints on states," *IEEE Transactions on Neural Networks and Learning Systems*, pp. 1–10, 2021.

[9] L. Liu, T. Gao, Y. J. Liu, S. Tong, C. L. P. Chen, and L. Ma, "Time-varying IBLFs-based adaptive control of uncertain nonlinear systems with full state constraints," *Automatica*, vol. 129, Article ID 109595, 2021.

[10] L. Kong, W. He, C. Yang, Z. Li, and C. Sun, "Adaptive fuzzy control for coordinated multiple robots with constraint using impedance learning," *IEEE Transactions on Cybernetics*, vol. 49, no. 8, pp. 3052–3063, 2019.

[11] L. Kong, W. He, C. Yang, G. Li, and Z. Zhang, "Adaptive fuzzy control for a marine vessel with time-varying constraints," *IET Control Theory & Applications*, vol. 12, no. 10, pp. 1448–1455, 2018.

[12] C.-X. Wang, X.-F. Liu, J.-B. Yu, and J.-L. Yu, "State transformation-based adaptive tracking control for a class of nonlinear systems with time-varying state constraints," *ISA Transactions*, vol. 110, pp. 129–137, 2021.

[13] C.-X. Wang, Y.-Q. Wu, Y. Zhao, and J.-L. Yu, "Asymptotic tracking control for time-delay nonlinear systems with parametric uncertainties and full state constraints," *ISA Transactions*, vol. 98, pp. 101–109, 2020.

[14] C. Wang, L. Qi, X. Yu, and J. Yu, "Adaptive cross backstepping control for a class of nonstrict feedback nonlinear systems with partial state constraints," *International Journal of Adaptive Control and Signal Processing*, vol. 35, no. 6, pp. 915–940, 2021.

[15] C. Wang, Y. Wu, F. Wang, and Y. Zhao, "TABLF-based adaptive control for uncertain nonlinear systems with time-varying asymmetric full-state constraints," *International Journal of Control*, vol. 94, no. 5, pp. 1238–1246, 2021.

[16] Y. Wu and X.-J. Xie, "Robust adaptive control for state-constrained nonlinear systems with input saturation and unknown control direction," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 2, pp. 1192–1202, 2021.

[17] H. Min, S. Xu, and Z. Zhang, "Adaptive finite-time stabilization of stochastic nonlinear systems subject to full-state constraints and input saturation," *IEEE Transactions on Automatic Control*, vol. 66, no. 3, pp. 1306–1313, 2021.

[18] Z. Zhang and Y. Wu, "Adaptive fuzzy tracking control of autonomous underwater vehicles with output constraints," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 5, pp. 1311–1319, 2021.

[19] Z. Zhang and S. Zhang, "Adaptive stabilization of state-constrained uncertain nonholonomic system via dynamic surface control," *International Journal of Adaptive Control and Signal Processing*, vol. 35, no. 8, pp. 1649–1661, 2021.

[20] W. Lin and C. Qian, "Adding one power integrator: a tool for global stabilization of high-order lower-triangular systems," *Systems & Control Letters*, vol. 39, no. 5, pp. 339–351, 2000.

[21] Y. Wu, R. Xie, and X.-J. Xie, "Adaptive finite-time fuzzy control of full-state constrained high-order nonlinear systems without feasibility conditions and its application," *Neurocomputing*, vol. 399, pp. 86–95, 2020.

[22] Y. Wu and X.-J. Xie, "Adaptive fuzzy control for high-order nonlinear time-delay systems with full-state constraints and input saturation," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 8, pp. 1652–1663, 2020.

[23] Y. Wu and X.-J. Xie, "Stabilisation of high-order nonlinear systems with full-state constraints and input saturation," *International Journal of Control*, vol. 94, no. 4, pp. 999–1009, 2021.

[24] R. Xie, Y. Wu, and X.-J. Xie, "Removing feasibility conditions on practical preassigned finite-time tracking for state-constrained high-order nonlinear systems," *International Journal of Control*, pp. 1–9, 2020.

[25] Y. Wu, X.-J. Xie, and Z.-G. Hou, "Adaptive fuzzy asymptotic tracking control of state-constrained high-order nonlinear

time-delay systems and its applications," *IEEE Transactions on Cybernetics*, pp. 1–10, 2020.

[26] X.-J. Xie, Y. Wu, and Z.-G. Hou, "Further results on adaptive practical tracking for high-order nonlinear systems with full-state constraints," *IEEE Transactions on Cybernetics*, pp. 1–8, 2021.

[27] R. H. Cui and X. J. Xie, "Adaptie state-feedback stabilization of state-constrained stochastic high-order nonlinear systems," *Science China Information Sciences*, vol. 64, no. 10, Article ID 200203, 2021.

[28] R. H. Cui and X. J. Xie, "Finite-time stabilization of output-constrained stochastic high-order nonlinear systems with high-order and low-order nonlinearities," *Automatica*, vol. 136, Article ID 110085, 2022.

[29] J.-Z. Liu, S. Yan, D.-L. Zeng, Y. Hu, and Y. Lv, "A dynamic model used for controller design of a coal fired once-through boiler-turbine unit," *Energy*, vol. 93, no. 2, pp. 2069–2078, 2015.

[30] C. Rui, M. Reyhanoglu, I. Kolmanovsky, S. Cho, and N. H. McClamroch, "Nonsmooth stabilization of an under-actuated unstable two degrees of freedom mechanical system," in *Proceedings of the 36th IEEE Conference on Decision and Control*, pp. 3998–4003, IEEE, San Diego, CA, USA, 12 December 1997.

[31] M. Wang, Y. Liu, and Y. Man, "Switching adaptive controller for the nonlinear systems with uncertainties from unknown powers," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 7, pp. 2375–2385, 2020.

[32] Z. Su, C. Qian, and J. Shen, "Interval homogeneity-based control for a class of nonlinear systems with unknown power drifts," *IEEE Transactions on Automatic Control*, vol. 62, no. 3, pp. 1445–1450, 2017.

[33] R.-H. Cui and X.-J. Xie, "Finite-time stabilization of stochastic low-order nonlinear systems with time-varying orders and FT-SISS inverse dynamics," *Automatica*, vol. 125, Article ID 109418, 2021.

[34] G.-J. Li and X.-J. Xie, "Adaptive state-feedback stabilization of stochastic high-order nonlinear systems with time-varying powers and stochastic inverse dynamics," *IEEE Transactions on Automatic Control*, vol. 65, no. 12, pp. 5360–5367, 2020.

[35] C. Guo, R. Xie, and X.-J. Xie, "Adaptive control of full-state constrained high-order nonlinear systems with time-varying powers," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 8, pp. 5189–5197, 2021.

[36] X.-J. Xie, C. Guo, and R.-H. Cui, "Removing feasibility conditions on tracking control of full-state constrained nonlinear systems with time-varying powers," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 10, pp. 6535–6543, 2021.

[37] C. Guo, X.-J. Xie, and Z.-G. Hou, "Removing feasibility conditions on adaptive neural tracking control of nonlinear time-delay systems with time-varying powers, input, and full-state constraints," *IEEE Transactions on Cybernetics*, pp. 1–12, 2020.

[38] R. Xie, C. Guo, and X.-J. Xie, "Asymptotic tracking control of state-constrained nonlinear systems with time-varying powers," *IEEE Transactions on Cybernetics*, pp. 1–7, 2020.

[39] Y. Wu and X. J. Xie, "Adaptive preassigned finite-time stability of nonlinear systems with time-varying powers and full-state constraints," *International Journal of Robust and Nonlinear Control*, vol. 32, no. 4, 2022.

[40] W. Hou, Y. Wu, and X. J. Xie, "Adaptive finite-time stabilization of output-constrained low-order uncertain nonlinear systems with time-varying powers," *International Journal of Control*.

[41] L. Xing, C. Wen, Z. Liu, H. Su, and J. Cai, "Event-triggered adaptive control for a class of uncertain nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 2071–2076, 2017.

[42] L. Liu, X. Li, Y. J. Liu, and S. Tong, "Neural network based adaptive event trigger control for a class of electromagnetic suspension systems," *Control Engineering Practice*, vol. 106, Article ID 104675, 2021.

[43] J. Zhang, B. Niu, P. Duan, P. Zhao, and H. Wang, "Event-triggered adaptive tracking control for nonlinear systems with predefined time-varying output constraints based on Nussbaum design," *International Journal of Robust and Nonlinear Control*, vol. 31, no. 15, pp. 7513–7527, 2021.

[44] L. Liu and X. Li, "Event-triggered tracking control for active seat suspension systems with time-varying full-state constraints," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 1, pp. 582–590, 2022.

[45] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM Journal on Control and Optimization*, vol. 38, no. 3, pp. 751–766, 2000.

[46] C. Qian and W. Lin, "Non-Lipschitz continuous stabilizers for nonlinear systems with uncontrollable unstable linearization," *Systems & Control Letters*, vol. 42, no. 3, pp. 185–200, 2001.

[47] Z.-Y. Sun, X.-H. Zhang, and X.-J. Xie, "Global continuous output-feedback stabilization for a class of high-order nonlinear systems with multiple time delays," *Journal of the Franklin Institute*, vol. 351, no. 8, pp. 4334–4356, 2014.

[48] W. Wei Lin and C. Chunjiang Qian, "Adaptive control of nonlinearly parameterized systems: the smooth feedback case," *IEEE Transactions on Automatic Control*, vol. 47, no. 8, pp. 1249–1266, 2002.

[49] B. Beibei Ren, S. S. Shuzhi Sam Ge, K. P. Keng Peng Tee, and T. H. Tong Heng Lee, "Adaptive neural control for output feedback nonlinear systems using a barrier Lyapunov function," *IEEE Transactions on Neural Networks*, vol. 21, no. 8, pp. 1339–1345, 2010.

[50] Y. Liu, X. Liu, and Y. Jing, "Adaptive fuzzy finite-time stability of uncertain nonlinear systems based on prescribed performance," *Fuzzy Sets and Systems*, vol. 374, no. 1, pp. 23–39, 2019.

[51] C. Chunjiang Qian and W. Wei Lin, "Practical output tracking of nonlinear systems with uncontrollable unstable linearization," *IEEE Transactions on Automatic Control*, vol. 47, no. 1, pp. 21–36, 2002.

[52] T. Zhang, F. Deng, Y. Sun, and P. Shi, "Fault estimation and fault-tolerant control for linear discrete time-varying stochastic systems," *Science China Information Sciences*, vol. 64, no. 10, Article ID 200201, 2021.

[53] H. Mukaidani and H. Xu, "Robust SOF Stackelberg game for stochastic LPV systems," *Science China Information Sciences*, vol. 64, no. 10, Article ID 200202, 2021.

[54] X. Jiang and D. Zhao, "Event-triggered fault detection for nonlinear discrete-time switched stochastic systems: a convex function method," *Science China Information Sciences*, vol. 64, no. 10, Article ID 200204, 2021.

[55] X.-J. Xie and M. Jiang, "Dynamic state feedback stabilization of stochastic cascade nonlinear time-delay systems with SISS inverse dynamics," *IEEE Transactions on Automatic Control*, vol. 64, no. 12, pp. 5132–5139, 2019.

[56] M.-M. Jiang, X.-J. Xie, and K. Zhang, "Finite-time stabilization of stochastic high-order nonlinear systems with FT-

SISS inverse dynamics," *IEEE Transactions on Automatic Control*, vol. 64, no. 1, pp. 313–320, 2019.

[57] L. Liu and X.-J. Xie, "State feedback stabilization for stochastic feedforward nonlinear systems with time-varying delay," *Automatica*, vol. 49, no. 4, pp. 936–942, 2013.

[58] X. Xin Yu and X. J. Xue-Jun Xie, "Output feedback regulation of stochastic nonlinear systems with stochastic iISS inverse dynamics," *IEEE Transactions on Automatic Control*, vol. 55, no. 2, pp. 304–320, 2010.

[59] X.-J. Xie and L. Liu, "Further results on output feedback stabilization for stochastic high-order nonlinear systems with time-varying delay," *Automatica*, vol. 48, no. 10, pp. 2577–2586, 2012.

[60] W. Sun, X. Lv, and M. Qiu, "Distributed estimation for stochastic Hamiltonian systems with fading wireless channels," *IEEE Transactions on Cybernetics*, pp. 1–10, 2020.

[61] W. He, J. Li, Z. Yan, and F. Chen, "Bidirectional human-robot bimanual handover of big planar object with vertical posture," *IEEE Transactions on Automation Science and Engineering*, pp. 1–12, 2021.

[62] Y. Chen, Z. Wang, L. Wang, and W. Sheng, "Finite-horizon $H\infty$ state estimation for stochastic coupled networks with random inner couplings using round-robin protocol," *IEEE Transactions on Cybernetics*, vol. 51, no. 3, pp. 1204–1215, 2021.

[63] L. Zou, Z. Wang, Q.-L. Han, and D. Zhou, "Full information estimation for time-varying systems subject to Round-Robin scheduling: a recursive filter approach," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 3, pp. 1904–1916, 2021.

[64] Y. Luo, Z. Wang, Y. Chen, and X. Yi, "$H_{\infty }$ state estimation for coupled stochastic complex networks with periodical communication protocol and intermittent nonlinearity switching," *IEEE Transactions on Network Science and Engineering*, vol. 8, no. 2, pp. 1414–1425, 2021.

[65] B. Shen, Z. Wang, and H. Qiao, "Event-triggered state estimation for discrete-time multidelayed neural networks with stochastic parameters and incomplete measurements," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 5, pp. 1152–1163, 2017.

[66] K. Zhu, Z. Wang, Y. Chen, and G. Wei, "Neural-network-based set-membership fault estimation for 2-D systems under encoding-decoding mechanism," *IEEE Transactions on Neural Networks and Learning Systems*, pp. 1–13, 2021.

[67] J. Hu, Z. Wang, G. P. Liu, C. Jia, and J. Williams, "Event-triggered recursive state estimation for dynamical networks under randomly switching topologies and multiple missing measurements," *Automatica*, vol. 115, Article ID 108908, 2020.