Consequence of Double-Diffusion Convection and Partial Slip on Magneto-Oldroyd-4 Constants Nanofluids with Peristaltic Propulsion in an Asymmetric Channel

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The double-diffusive convection is a significant physical phenomenon that arises in fluid mechanics. It is primarily associated with a convection process in which two dissimilar density gradients with varying diffusion rates are considered. The primary goal of this study is to investigate the effects of double-diffusivity convection and partial slip with an inclined magnetic field on peristaltic propulsion in an asymmetric channel for Oldroyd-4 constants nanofluids. The flow of an Oldroyd-4 constant nanofluid is mathematically modeled in the presence of double-diffusivity convection and a tilted magnetic field. Lubrication methodology is applied to simplify the highly nonlinear system of partial differential equations (PDEs). The numerical scheme is used to calculate the solution of coupled nonlinear PDEs. Furthermore, the effect of changing the parameters associated with slip, thermophoresis, Brownian motion, Grashof number of nanoparticles, Hartmann number, pumping, and trapping are investigated in this article. It is noticed that the temperature rises as the Brownian motion and thermophoresis constraints increases. This is because the growth in the Brownian motion parameter indicates the increase in the kinetic energy of nanoparticles which results in warming up the nanofluid. Also, concentration falls as the Brownian motion and thermophoresis constraints increases.

1. Introduction

The peristaltic transportation of biological fluids has huge number of applications in biomedical industry and hence received huge attention from the researchers in last two decades. Transport phenomenon of biological fluids is based on peristaltic propulsion. This phenomenon can be internally observed in the gastrointestinal tract, urine flow, blood flow, male reproductive tract, esophageal swallowing, and ureter and externally as worm’s movement. This natural process is caused by periodic relaxation and contraction of muscles which produce a sinusoidal wave along the channel or tube walls. Some of the worthwhile theoretical and experimental studies considering the peristaltic propulsion for numerous basic fluid models are cited in references [1–6]. These studies have further been generalized for many non-Newtonian fluid models like the Carreau model [7], Maxwell model [8], Johnson–Segalman model [9], Williamson model [10], Casson model [11], six-constant Jeffreys model [12], Walter’s B model [13], hyperbolic tangent model [14], Herschel–Bulkley model [15], Jeffrey model [16], Sisko fluid [17], Oldroyd-4 model [18], Phan–Thien–Tanner model [19], and Burgers’ model [20]. The authors of these research studies extended the work to new findings which may widen...
the horizon of their usage to biomedical engineering and technology.

In the field of biomagnetic fluid dynamics (BFD), the study of peristaltic flow is based on magnetohydrodynamic (MHD) effects. The study of these biological fluids (examples of biofluids include blood, urine, and chyme) flows is important in bioengineering and medical sciences. These fluids are broadly found in living organisms and influence of magnetic field effects the flow greatly. Also, in peristaltic MHD compressor and blood pump machines, MHD effects on conductive physiological fluids become vital [21–25].

Nanotechnology is considered important in improving as well as revolutionizing information technology, industry sectors, homeland security, energy, food safety, medicine, transportation, and environmental discipline. In this century, many researchers and mathematicians are working on the development of mathematics and physics of nanofluid mechanics. This is due to the fact that nanotechnology is used in industry to get the optimum outputs in constrained environment. It is, therefore, an attractive research area in the modern fluid mechanics. Nanofluids are composed by dissemination of nanosized materials/particles in the base fluids (both viscous and inviscid liquids). It is known that the term nanofluid was introduced first ever by J. C. Maxwell, a Scottish scientist, in the late nineteenth century. Mostly, modern work is based on the analysis of nanofluids made by the Choi [26]. From this productive study, a vast number of applications of nanotechnology can be found in micro-channel cooling and reduction/enhancement in heat transfer. The bond of peristalsis with nanofluids has many utilizations in biomedical science (e.g., radiotherapy for cancer cure and drug delivery), chemical, and mechanical engineering (pumps and transportation of chemicals). Recently, such studies of nanoparticles along with different flow geometries have been examined extensively in physiological flows, such as references [27–35].

The double-diffusive convection is an important physical phenomenon arising in fluid mechanics. It is mainly associated with such a convection process where two dissimilar density gradients, having a diverse diffusion rate, are considered. The literature review tells us that none of the analysis has been considered for the double diffusion convection with assumptions of creeping phenomena and low Reynolds number. The double diffusion convection and peristaltic pumping have many applications of the innate mechanism in industrial and chemical engineering. Because of these applications, some authors have contributed to this area with various fluid models. Few are cited in references [36–42].

Incorporation of partial slips in fluid flows is essential in the study of polymers and polishing the artificial heart valve. To the best of our knowledge, the slip effects were first used in the peristalsis by Chu and Fang [43]. Later, Akbar et al. [44] further added these effects to examine the influence on peristaltic flow of nanofluid. Also, these effects on hydro-magnetic driven peristaltic flow were investigated by Abbasi et al. in reference [45]. More recent works on these effects can be viewed in references [46–50].

From the abovementioned discussion, the heat convection impact and magnetic flux on double diffusion convection cannot be neglected. The study of the Oldroyd-4 constant nanofluid is studied in the literature but effects of double diffusion and partial slip with inclined MHD are not studied yet. So, the rationale of our current research is to show how magnetic field and slip boundaries affect peristaltic flow and heat transfer with double diffusion convection.

2. Flow Equations

The equations that describe flow in an incompressible fluid are as follows [36, 37]:

\[
\text{div} V = 0,
\]

\[
\rho_f \left( \frac{dV}{dt} \right) = \text{div} \tau + \rho f + g\left( 1 - \Theta_0 \right) \rho_f \beta_T (T - T_0) + \beta_C (C - C_0) - (\rho_p - \rho_f) (\Theta - \Theta_0),
\]

\[
(\rho c)_f \left( \frac{dT}{dt} \right) = k \nabla^2 T + (\rho c)_p \left[ D_b (\nabla \Theta \cdot \nabla T) + \left( \frac{D_T}{T_0} \right) \nabla T \cdot \nabla T \right] + D_{TC} \nabla^2 C + (\rho c)_p \left( D_b \nabla^2 \Omega + \left( \frac{D_T}{T_0} \right) \nabla^2 T \right).
\]

In the abovementioned equations, temperature is represented by \( T \), concentration, by \( \Theta \), particle volume fraction, by \( \Theta \), velocity, \( d/dt \) illustrates material time derivative, \( f \) stands for body force, \( \tau \) describes stress tensor, \( g \) is acceleration, \( \rho_f \) represents base fluid density, particles density is represented \( \rho_p \), \( \rho_f \) describes fluid density at \( T_0 \), \( (\rho c)_p \) denotes the thermal capacity of nanoparticles, \( (\rho c)_f \) refers fluid heat capacity, \( \beta_T \) describes fluid volumetrical thermal expansion coefficient, \( \beta_C \) stands for fluid volumetrical solutal expansion coefficient, \( D_{TC} \) denotes Dufour diffusively, Soret diffusively is denoted by \( D_{CT} \), \( D_b \) is coefficient of Brownian diffusion, \( D_T \) coefficient of thermodiophoretic diffusion, \( D_b \) stands for solutal diffusively, and thermal conductivity is represented by \( k \).

The Oldroyd-4 constant fluid stress tensor in [18] is defined by the following equations:
\[ \tau = -PI + S, \]
\[ S + \lambda_1 \frac{DS}{Dt} + \lambda_2 tr (S)A_1 = \mu \left( 1 + \lambda_3 \frac{D}{Dt} \right) A_1, \]
\[ A_1 = (VV) + (VV)^*, \]
\[ \frac{DS}{Dt} = \frac{dS}{dt} - (VV)S - S(VV)^*. \]

Where \((\lambda_1, \lambda_2)\) represents relaxation times, \(\lambda_3\) is retardation time, \(\mu\) is denoted by viscosity, \(*\) is used for transpose, and \(A_1\) is Rivlin–Ericksen tensors.

### 3. Mathematical Formulation

We assume that an incompressible peristaltic flow and electrically conducting of Oldroyd-4 constant nanofluid in 2 dimensional conduit having width equal to \(d_1 + d_2\), in the Cartesian coordinates system. The channel’s center is supposed to be along horizontal line, and cross-sectional area is assumed to be beside the vertical line. Conduit boundary is assumed to be moving with constant speed and shape like sinusoidal wave train. Temperatures, solvent concentrations, and nanoparticle concentrations of the lower and upper walls are \((T_1,T_0), (C_1,C_0)\), and \((\Theta_1,\Theta_0)\), respectively. Fixed magnetic field at an angle \(\beta\) is applied on the flow. It is assumed that electric field is zero and the Reynolds number is low so that it produces insignificant induced magnetic field as compared to applied magnetic field.

The geometric shape of wall is specified in Figure 1 and mathematical expression is defined as follows[3]:

\[ H_1 = Y = d_1 + d_2 \cos \left[ \frac{2\pi}{\lambda} (X - ct) \right], \]
\[ H_2 = Y = -d_2 - d_1 \cos \left[ \frac{2\pi}{\lambda} (X - ct) + \varphi \right], \]

where \((d_1,d_2)\) illustrates wave amplitudes, \(d_1 + d_2\) is channel width, \(\lambda\) illustrates wavelength, \(t\) indicates time, and \(c\) is wave velocity. The range of phase difference \((\varphi)\) is \(0 \leq \varphi \leq \pi\). When \(\varphi = 0\), the channel is symmetric without a phase wave and at \(\varphi = \pi\), the channel with a phase wave. Furthermore, the constraints \(\varphi, d_1, d_2, d_3, \) and \(d_4\) satisfy the condition \(d_1^2 + d_2^2 + 2d_3d_4 \sin \varphi \leq (d_1 + d_2)^2\) The velocity field in \(2-\) directional and dimensional flow is.

\[ V = (U(X,Y,t)), \]

The motion equations in \(2-\) directional and dimensional flow comprising Oldroyd-4 constant incompressible are as follows:

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \]
\[ \rho_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) U = -\frac{\partial P}{\partial X} + \frac{\partial}{\partial X} (S_{XX}) + \frac{\partial}{\partial Y} (S_{XY}) - \sigma \beta^3 \cos \beta (U \cos \beta - V \sin \beta) \]
\[ + \beta \left\{ (1 - \Theta_0) \rho_f \left[ \beta \left( \frac{T}{T_0} \right) + \beta \left( C - C_0 \right) \right] - (\rho_p - \rho_f) \left( \Theta - \Theta_0 \right) \right\}, \]
\[ \rho_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) V = \frac{\partial P}{\partial Y} + \frac{\partial}{\partial X} (S_{XY}) + \frac{\partial}{\partial Y} (S_{YY}) \]
\[ + \sigma \beta^3 \sin \beta (U \cos \beta - V \sin \beta), \]
\[ (\rho c_f) \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) T = k \left[ \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right] + (\rho c)_f \left[ D_B \left( \frac{\partial T}{\partial X} \frac{\partial}{\partial X} + \frac{\partial T}{\partial Y} \frac{\partial}{\partial Y} \right) \right] \]
\[ \left( \frac{D_T}{T_0} \right) \left[ \left( \frac{\partial T}{\partial X} \right)^2 + \left( \frac{\partial T}{\partial Y} \right)^2 \right] + D_{RC} \left( \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right), \]
\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) C = D_c \left( \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) + D_{RC} \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right), \]
\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) \Theta = D_B \left( \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right) + \left( \frac{D_B}{T_0} \right) \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right), \]

Using the Galilean transformation between fixed and wave frames as follows:
\( v = V, \)  
\( y = Y, \)  
\( u = U - c, \)  
\( x = X - ct, \)  
\( p(x, y) = P(X, Y, t), \)

Defining the dimensionless quantities as follows:

\[
\begin{align*}
\bar{x} &= \frac{x}{\lambda}, \bar{u} = \frac{u}{c}, \quad \bar{y} = \frac{y}{d_1}, \delta = \frac{d_1}{\lambda}, d = \frac{d_2}{\lambda}, \bar{h}_2 &= \frac{H_2}{d_2}, h_1 = \frac{H_1}{d_1}, b = \frac{d_4}{d_1}, \bar{p} = \frac{d^2 p}{\mu c \lambda}, \\
\lambda_1 &= \frac{\lambda}{c_1}, \lambda_2 = \frac{\lambda}{c_0}, \lambda_3 = \frac{\lambda}{d_1}, S = \frac{\mu c}{d_1}, N_{TC} = \frac{D_{CT} (C_1 - C_0)}{k (T_1 - T_0)}, \\
G_{rt} &= \frac{g d^2 (1 - \Theta_0) (T_1 - T_0) \rho_f \beta_T}{\mu c}, G_{rc} = \frac{g (1 - \Theta_0) \rho_f \beta_c (C_1 - C_0) d_1^2}{\mu c}, \Omega = \frac{\Theta - \Theta_0}{\Theta_1 - \Theta_0}, \lambda_1 = \frac{\lambda_2}{d_1}, \\
G_{rF} &= \frac{g (\rho_f - \rho_c) (\Theta_1 - \Theta_0)}{\mu c} d_1, N_1 = \frac{(\rho_c) p D_T (T_1 - T_0)}{T_0 k}, N_2 = \frac{(\rho_c) p D_B (\Theta_1 - \Theta_0)}{k}, \\
N_{CT} &= \frac{D_{CT} (T_1 - T_0)}{C_1 - C_0} D_s.
\end{align*}
\]

Where \( G_{rt}, G_{rc}, N_{CT}, G_{rc}, PrN_{TC}, MN_1, \lambda_1, \lambda, \mu, k, \lambda, \rho_0, \) and \( \beta \) stand for thermal Grashof number, Reynolds number, nanoparticle Grashof number, Soret parameter, solutal (species) concentration, Hartmann number, thermophoresis parameter, nanoparticle fraction, Lewis

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**Figure 1:** Geometry of the problem.
number, temperature, Brownian motion, Lewis number of nanofluid, and wave number, respectively.

Now, using (10) and (11), (4) is identically satisfied and equations (12)–(16) (after omitting bars) in the wave frame becomes as follows:

\[
\text{Re} \delta(\Psi_{x y} - \Psi_{x y}) = \frac{\partial p}{\partial x} + \frac{\partial S_{x x}}{\partial x} + \frac{\partial S_{x y}}{\partial y} - M^2 \cos^2 \beta(\Psi_y + 1),
\]

\[
+ G_r \theta + G_r y - G_r \Omega,
\]

\[
\text{Re} \delta^3(\Psi_{x y} - \Psi_{x y}) = \frac{\partial p}{\partial x} + \frac{\partial S_{x y}}{\partial y} + \frac{\partial S_{y y}}{\partial x} + M^2 \delta \sin \beta(\Psi_y + 1) \cos \beta + \Psi_y \sin \beta, \tag{13}
\]

\[
\text{RePr} \delta(\Psi_{y y} - \Psi_{y y}) = (\chi_{y y} + \delta \chi_{x y} + N_{TC}(\delta^2 \chi_{x x} + \chi_{y y}) + N_b(\delta^2 \chi_{x x} + \chi_{y y}), \tag{14}
\]

\[
\text{ReL} \delta(\Psi_{x x} - \Psi_{y y}) = (\delta^2 \chi_{x x} + \chi_{y y}) + N_{CT}(\delta^2 \chi_{x x} + \chi_{y y}), \tag{15}
\]

\[
\text{ReL} \delta(\Psi_{x x} - \Psi_{y y}) = (\delta^2 \chi_{x x} + \chi_{y y}) + N_{CT}(\delta^2 \chi_{x x} + \chi_{y y}), \tag{16}
\]

Now, imposing constraints of Re \(\rightarrow 0\) (low Reynolds number) and \(\delta \ll 1\) (long wavelength), the equations. (19)–(23) are now reduced as follows:

\[
0 = \frac{\partial p}{\partial x} + \frac{\partial S_{x y}}{\partial y} - M^2 \cos^2 \beta(\Psi_y + 1)
\]

\[
+ G_r \theta + G_r y - G_r \Omega, \tag{17}
\]

\[
0 = \frac{\partial p}{\partial y}, \tag{18}
\]

\[
\frac{\partial^2 \theta}{\partial y^2} + N_{TC} \frac{\partial^2 \chi}{\partial y^2} + N_b \left(\frac{\partial^2 \chi}{\partial y^2}\right)^2 + N_i \left(\frac{\partial \Omega}{\partial y}\right)^2 = 0, \tag{19}
\]

\[
\frac{\partial^2 \chi}{\partial y^2} + N_{CT} \frac{\partial^2 \theta}{\partial y^2} = 0, \tag{20}
\]

\[
\frac{\partial^2 \chi}{\partial y^2} + N_{CT} \frac{\partial^2 \theta}{\partial y^2} = 0, \tag{21}
\]

Now, taking pressure out of (17) and (18) yields the following expression as follows:

\[
\frac{\partial^2 S_{x y}}{\partial y^2} - M^2 \cos^2 \beta \frac{\partial^2 \Psi}{\partial y^2} + G_r \frac{\partial \theta}{\partial y} + G_r \frac{\partial y}{\partial y} - G_r \frac{\partial \Omega}{\partial y} = 0, \tag{22}
\]

where the dimensionless equation for \(S_{x y}\) is obtained from equation (7) and expressed as follows:

\[
S_{x y} = \left[1 + 2 \eta_1 \left(\frac{\partial^2 \chi}{\partial y^2}\right)^2 \right] \left(\frac{\partial^2 \chi}{\partial y^2}\right), \tag{23}
\]

in which \(\eta_1 = \lambda_1 \lambda_2 \) and \(\eta_2 = \lambda_1 \lambda_2 \). Now, if \(\eta_1 = \eta_2 \) the model of Oldroyd-4 constant fluid reduces to viscous fluid.

The expression for \(Q\) (mean flow) is computed in the dimensionless form as follows:

\[
Q = F + 1 + d, \tag{24}
\]

\[
F = \int_{h_1(x)}^{h_2(x)} \frac{\partial \Psi}{\partial y} \, dy = \Psi(h_1(x) - h_2(x)), \tag{25}
\]

\[
h_1(x) = 1 + \cos 2\pi x, \quad h_2(x) = -d - \cos (2\pi x + \phi).
\]

For the problem under investigation, the slip boundary conditions in dimensionless forms are defined as follows:

\[
\Psi = \frac{F}{2} \frac{\partial \Psi}{\partial y} + \epsilon_1 S_{x y} = -1 \text{ on } y = h_1(x),
\]

\[
\Psi = -\frac{F}{2} \frac{\partial \Psi}{\partial y} - \epsilon_1 S_{x y} = -1 \text{ on } y = h_2(x),
\]

\[
\theta + \epsilon_2 \frac{\partial \theta}{\partial y} = 0, \text{ on } y = h_1,
\]

\[
\theta - \epsilon_2 \frac{\partial \theta}{\partial y} = 1, \text{ on } y = h_2, \tag{26}
\]

\[
\gamma + \epsilon_3 \frac{\partial \gamma}{\partial y} = 0, \text{ on } y = h_1,
\]

\[
\gamma - \epsilon_3 \frac{\partial \gamma}{\partial y} = 1, \text{ on } y = h_2,
\]

\[
\Omega + \epsilon_4 \frac{\partial \Omega}{\partial y} = 0, \text{ on } y = h_1,
\]

\[
\Omega - \epsilon_4 \frac{\partial \Omega}{\partial y} = 1, \text{ on } y = h_2.
\]

If \(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 = 0\) in the parameters, then no slip conditions exist.

3.1. Special Cases. In the absence of slip conditions \((\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0)\), \(M = 0, G_r = 0, G_r = 0, G_r F = 0,\) and \(\eta_1 = \eta_2,\) the findings of reference [4] can also be recovered as a limited case of existing problem.
4. Numerical Solution and Graphical Outcomes

Numerical simulations become essential as analytical solutions can only be found for a limited number of cases. Numerical simulations provide us an alternative mean to understand the problem and its solution without wasting the real resources that are involved in the study. We can develop a comprehensive understanding of the flow situations using modern tools available in the form of software’s like: MATLAB, MATHEMATICA, and ANSYS. The main objective of the present work is to evaluate the consequence of double-diffusion convection and partial slip on magneto-Oldroyd-4 constants nanofluids with peristaltic propulsion in an asymmetric channel. The exact solutions of the equations (24)-(29) are not easy to evaluate due to coupled and highly nonlinear characteristics. Therefore, the regressive equations are numerically solved in MATHEMATICA by using built-in command ND-Solve. Solutions are utilized in obtaining pressure gradient, pressure rise, and streamlines for different flow parameters. The graphical results are also established employing numerical solutions to verify that numerical data are accurate and to examine the impact of a variety of flow parameters.

4.1. Effects of the Hartmann Number. The Hartmann number is described as ratio of electromagnetic force to viscous force. It is a regular occurrence in fluid flows passing via magnetic fields. Figures 2(a) to 2(c) explain the impact of velocity profile, pressure rise, and pressure gradient on the Hartmann number $M$. As Hartmann number enhances, the magnitude of the velocity profile...
Figure 3: Continued.
significantly reduces in channel’s center and tends to increase near peristaltic walls (see Figure 2(a)). Furthermore, the profile of velocity has a parabolic shape. In fact, enhancing magnetic number causes the Lorentz force to increase, which tends as a retarding force and slowing fluid motion. Figure 2(b) depicts the response to pressure rise. It is illustrated in Figure 2(b) that increasing $M$ causes pressure rise to increase in peristaltic ($Q > 0, \Delta p > 0$), retrograde ($Q < 0, \Delta p > 0$), and free ($\Delta p = 0$) pumping zones. Moreover, it reduces in the augmented ($Q > 0, \Delta p < 0$) pumping zone due to higher values of $M$. The pressure gradient continues to decrease as the Hartmann number rises (see Figure 2(c)).

4.2. Effects of Nanoparticle ($G_{rF}$), Solutal ($G_{rc}$), and Thermal ($G_{rt}$) Grashof Numbers. In fluid dynamics and heat transfer, the Grashof numbers are described by ratio of buoyancy to viscous force that acts on a fluid. The consequence of nanoparticle, solutal, and thermal Grashof numbers on velocity profile is illustrated in Figures 3(a) to 3(c). The magnitudes of the flow velocity rises when $y \in [-0.2, 0.35]$ by enhancing $G_{rF}$, while opposite phenomenon occurs when $y \in [0.35, 0.87]$. Here magnitude of velocity field drops (see Fig. 3(a)). This happens because nanoparticles viscosity drops, causing the velocity to decrease. In Figure 3(b) and 3(c), it is noted that magnitudes of fluid velocity drops when $y \in [-0.2, 0.35]$ but it increases when
Figure 4: Continued.
$y \in [0.35, 0.87]$ by enhancing $G_{rc}$ and $G_{rt}$. In most instances, thermal buoyancy serves to slow down the flow in the regime. The behavior of pressure rise for $G_{rt}$, $G_{rc}$, and $G_{rt}$ is demonstrated in Figures 3(d) to 3(f). It is noted in Figure 3(d) that in all peristaltic regions (augmented ($Q > 0, \Delta p < 0$), free ($\Delta p = 0$), augmented ($Q > 0, \Delta p < 0$), and retrograde ($Q < 0, \Delta p > 0$)) the pressure rise drops by rising $G_{rt}$ values. On the other hand, $G_{rc}$ and $G_{rt}$ exhibit the opposite trend. Here, pressure rises in all peristaltic zones are increased by boosting $G_{rc}$ and $G_{rt}$ values (see Figure 3(e) and 3(f)). The roll of pressure gradient for $G_{rc}$ and $G_{rt}$ are explained in Figure 3(g) and 3(h). It is indicated in Figure 3(g) that the pressure gradient significantly increases when values of $G_{rc}$ increases. The pressure gradient tends to reduce when $G_{rt}$ increases (see Figure 3(h)).

4.3. Effects of Soret ($N_{CT}$) and Dufour ($N_{TC}$) Parameters. The outcomes of Soret and Dufour constraints are shown in Figures 4(a) to 4(h). It is shown in Figures 4(a) and 4(b) that $N_{CT}$ and $N_{TC}$ have a similar behavior on the velocity profile, as already explained in Figure 3(a). Figures 4(c) and 4(d) depict the impact of $N_{CT}$ and $N_{TC}$ on temperature profile. The temperature increases by increasing $N_{CT}$ and $N_{TC}$. It is only because temperature has a direct connection with the constraints of Soret and Dufour. The concentration and
nanoparticle fraction profiles decrease by enhancing $N_{CT}$ and $N_{TC}$ values (see Figures 4(e) to 4(h)). Its because random motion reacts with micromixing and random collision tendency of solid nanoparticle, spreading the solid nanoparticles and lessening solute concentration.

4.4. **Effects of the Brownian Motion ($N_b$) Parameter.** Figures 5(a) to 5(d) look at the impact of the Brownian motion on $dp/dx$, $\theta$, $\Omega$, and $c$. It is clear from Figure 5(a) that by increasing $N_b$ values pressure gradient drops. Furthermore, nanoparticle volume fraction has a direct relationship with $N_b$. The adverse trends are noted for the case of concentration. Here, by rising $N_b$ values, the concentration drops (see Figure 5(d)). In nature, the nanofluid is just a two-phase fluid, and stochastic mobility of isolated nanoparticles increases energy exchange rates while dropping concentrations in the fluid flow.

4.5. **Effects of the Thermophoresis ($N_t$) Parameter.** The roll of velocity, concentration, temperature, nanoparticle fraction, and pressure gradient on $N_t$ are indicated in Figures 6(a)–
Figure 6: Continued.
6(e). The outcomes of altering the thermophoresis coefficient on the flow velocity are seen in Figure 6(a). When value of thermophoresis parameters rises the magnitude of flow velocity, it increases in the zone \( y \in [-0.2, 0.3] \). Furthermore, it tends to fall when \( y \in [0.3, 0.86] \). The velocity of fluid is maximum near the channel’s center. The role of \( N_t \) on temperature is shown in Figure 6(b). It shows that \( N_t \) have an identical behavior of fluid temperature, as already shown in Figure 5(b). The slightly different effects are noted for the case of concentration and nanoparticle fraction (see Figures 6(c) and 6(d)). Here, due to the rising tendency of \( N_t \), the concentration and nanoparticle fraction decreases. The occurrence of pressure gradient is elaborated in Figure 6(e). It is noted in Figure 6(e) that pressure gradient falls as thermophoresis parameter rises.

4.6. Effects of Slip Parameters (\( \varepsilon_1 - \varepsilon_4 \)). The outcomes of parameters of slip-on velocity, concentration, temperature, nanoparticle fraction pressure gradient, and pressure rise are shown in Figures 7(a) to 7(g). It is noted in Figure 7(a) that nature of velocity curve is parabolic. Moreover, by increasing parameter of velocity slip, the magnitude of flow velocity tends to fall when \( y \in [-0.2, 0.1] \) and \( y \in [0.65, 0.86] \) but reverse effects are noted when \( y \in [0.1, 0.65] \), here magnitude of flow velocity increases. The impact of temperature slip \( \varepsilon_3 \) is shown in Figure 7(b). It is indicated in Figure 7(b) that by enhancing temperature slip \( \varepsilon_3 \), temperature drops in the region \( y \in [-0.2, -0.1] \) but it tends to rise in region \( y \in [-0.1, 0.65] \). The kinetic energy of particles of fluid increases due to slip which rises the fluid temperature. Figures 7(c) and 7(d) depict the influence of parameters of slip concentration \( \varepsilon_3 \) and slip nanoparticle fraction \( \varepsilon_4 \). There is a fall in concentration and nanoparticle fraction in the region \( y \in [-0.2, 0.65] \) due to the rising values of slip parameter of concentration and slip factor of nanoparticle fraction. Furthermore, opposite effects are noted in the region \( y \in [0.65, 0.85] \). The particle of fluid is interrupted less by the walls of channel, so concentration drops as the value of \( \varepsilon_3 \) increases. Hence, the rate of mass transfer of nanoparticles is slowed. The role of pressure rise on velocity slip \( \varepsilon_3 \) is drawn in Figure 7(e). It is noted in Figure 7(e) that pressure rise reduces in augmented and retrograde pumping zones but increases in augmented region by increasing velocity slip constraints. The pressure gradient increases due to the increasing behaviour of velocity slip constraints \( \varepsilon_4 \) and nanoparticle slip factor \( \varepsilon_4 \) (see Figures 7(f) and 7(g)).

4.7. Effects of Non-Newtonian Parameters (\( \eta_1 \) and \( \eta_2 \)). To discuss the roll of non-Newtonian parameters \( \eta_1 \) and \( \eta_2 \) on pressure rise, pressure gradient, and velocity, Figures 8(a) to 8(e) are displayed. It is exhibited in Figures 8(a) and 8(b) that \( \eta_1 \) and \( \eta_2 \) shows similar behavior on pressure rise. It is illustrated in Figures 8(a) and 8(b) that by increasing \( \eta_1 \) and \( \eta_2 \), pressure rise increases in peristaltic and free and retrograde pumping areas but reduces in the augmented region. The pressure gradient is maximum at channel’s center, but near the channel wall’s pressure gradient drops due to the increasing values of non-Newtonian parameters \( \eta_1 \) and \( \eta_2 \) (see (Figures 8(c) and 8(d)). It is shown in Figure 8(e) that \( M \) and \( (\eta_1, \eta_2) \) have an identical behavior on fluid velocity, as already explain in Figure 2(a).

4.8. Trapping Phenomenon. Trapping is an unusual occurrence in peristaltic propelling flows. It is begun by the development of a fluid mass that internally moves and is enclosed by streamlines of peristaltic wave. Streamlines capture the mass bolus of fluid and move it forward using waves of peristaltic at high flow rates and significant
Figure 7: Effects of slip parameters on $u, \theta, \gamma, \Omega, \Delta p$, and $dp/dx$. 
Figure 8: Effects of non-Newtonian parameters ($\eta_1, \eta_2$) on $\Delta p, dp/dx$, and $u$. 

(a) 

(b) 

(c) 

(d) 

(e)
Figure 9: Impact of streamlines on $\omega_1$.

Figure 10: Impact of streamlines on $\eta_2$.

Figure 11: Impact of streamlines on $G_{rt}$.

Figure 12: Impact of streamlines on M.
occlusions. It is indicated in Figure 9 that due to the increasing behavior of velocity slip factor $\varepsilon_1$, size of trapped bolus increases and amount of trapped bolus reduces in both upper and lower portion of channel. By increasing non-Newtonian parameters, $\eta_2$ streamlines show that the trapped bolus size enhances (see Figure 10). In Figure 11, it is illustrated that the trapped bolus volume grows as the thermal Grashof number $Gr_T$ rises. The reverse behavior is noted for the Hartmann number $M$ case. Here, the size reduces by rising values of $M$ (see Figure 12).

Table 1 shows the comparison with the existing literature.

<table>
<thead>
<tr>
<th>$y = h(x)$</th>
<th>Present work (with partial slip)</th>
<th>Velocity profile $u(x)$</th>
</tr>
</thead>
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<tr>
<td>0.919093</td>
<td>-0.973134</td>
<td>-1</td>
</tr>
<tr>
<td>0.80702</td>
<td>-0.958414</td>
<td>-0.960643</td>
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<td>0.694947</td>
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<td>0.582873</td>
<td>-0.933854</td>
<td>-0.914322</td>
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<tr>
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</tr>
</tbody>
</table>

5. Conclusion

The main objectives of this paper are to examine the effect of double-diffusion convection and partial slip with a tilted magnetic field on peristaltic movement in an asymmetric channel for the Oldroyd-4 constants nanofluids. A numerical methodology is used to solve nonlinear system of PDEs. The impact of several physiological parameters on flow quantities is visually depicted. Based on our analysis, it was found that the nanoparticle fraction and concentration profile decrease by boosting slip factors of concentration and nanoparticles. Also, the velocity profiles can be controlled by adjusting the parameters under observation (like slip parameter, non-Newtonian parameter, and diffusivity parameters). Throughout the velocity profiles, it is observed that velocity tends to increase, reach its maximum value, and then decreases to meet the boundary condition while moving from one end to the other. It was also found that the profile of temperature rises as the Brownian motion and thermophoresis constraints increases. This means that by making electromagnetic forces dominant as compared to viscous forces can be effective in drug delivery. Lastly, the confined bolus size grows as the thermal Grashof number rises since buoyancy force becomes dominant as compared to viscous forces.

**Nomenclature**

- $C$: Solutal concentration
- $M$: Hartmann number
- $T$: Temperature
- $Pr$: Prandtl number
- $Gr_F$: Grashof number of nanoparticles
- $Le$: Lewis number
- $Q$: Nanoparticle volume fraction
- $Nb$: Brownian motion parameter
- $D_B$: Brownian diffusion coefficient
- $D_T$: Thermophoretic diffusion coefficient
- $Re$: Reynolds number
- $Gr_T$: Thermal Grashof number
- $D_s$: Solutal diffusively
- $N_{TC}$: Dufour parameter
- $N_i$: Thermophoresis parameter
- $Ln$: Nanofluid Lewis number
- $(pc)_f$: Heat capacity of fluid
- $(pc)_p$: Heat capacity of nanoparticle
- $N_{CT}$: Soret parameter
- $D_{TC}$: Dufour diffusively
- $Gr_c$: Solutal Grashof number
- $D_{CT}$: Soret diffusively

**Small alphabets**

- $u$: Axial velocity
- $v$: Transverse velocity
- $g$: Acceleration due to gravity
- $b$: Wave amplitude
- $d_1, d_3$: Channel width
- $p$: Pressure
- $k$: Thermal conductivity
- $t$: Time
- $d_2, d_4$: Wave amplitudes
- $c$: Propagation of velocity
Greek symbols

- \( \delta \): Wavelength
- \( \Psi \): Stream function
- \( \rho_f \): Nanoparticle mass density
- \( \overline{(pc)} \): Nanoparticle heat capacity
- \( \gamma \): Solutal concentration
- \( \omega \): Magnetic field inclination angle
- \( \beta_T \): Volumetric coefficient of thermal expansion
- \( \beta_C \): Volumetric coefficient of solutal expansion
- \( \varepsilon_1 \): Concentration slip parameter
- \( \varepsilon_2 \): Nanoparticles slip parameter
- \( \varepsilon_3 \): Temperature slip parameter
- \( \Omega \): Wave number
- \( \rho_f \): Fluid density
- \( \rho_f(0) \): Fluid density at \( T_0 \).

Data Availability

No data were used to support this study.

Conflicts of Interest

All authors declare that there are no conflicts of interest for this manuscript.

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References


