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*Research Article*

# EFP-GA: An Extended Fuzzy Programming Model and a Genetic Algorithm for Management of the Integrated Hub Location and Revenue Model under Uncertainty

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The aviation industry is one of the most widely used applications in transportation. Due to the limited capacity of aircraft, revenue management in this industry is of high significance. On the other hand, the hub location problem has been considered to facilitate the demands assignment to hubs. This paper presents an integrated p-hub location and revenue management problem under uncertain demand to maximize net revenue and minimize total cost, including hub establishment and transportation costs. A fuzzy programming model and a genetic algorithm are developed to solve the proposed model with different sizes. The mining and petroleum industry is used for case studies. Results show that the proposed algorithm can obtain a suitable solution in a reasonable amount of time.

## 1. Introduction

Hub location and revenue management are two research issues in network design that have been considered recently. The hub location model designs the transportation network structure. In contrast, the revenue management model allocates network capacity to customers in various classes based on price sensitivity.

Revenue management determines which products sell to which customers and at what price [1]. On the other hand, it has been widely used in the aviation industry due to the limited number of aircraft seats. Seats are usually offered to various customer classes at different prices [2]. Airlines categorize customers and allocate different capacities according to price to obtain maximum revenue. Capacity control includes several models, algorithms, and policies that allocate seats to maximize expected profits [3]. Hub location problem is related to hub facilities placement and allocation to demand nodes to determine traffic routes between origin and destination pairs. Researchers attract this issue because it significantly reduces the number of network connections and system costs. In the star $p$-hub network, p nodes are selected. Each node is connected to only one hub, and all hubs are connected to a central hub. A central hub is predefined, while other hubs are determined by the model [4]. There are four types of hub location problems: median, center, covering, and fixed cost hub location. In the $p$-hub median problem, p nodes are located to minimize the total cost of flows in the network. A number of the hub is predefined in this problem. A $p$-hub center problem seeks the optimal location of $p$-hubs. It allocates nonhub nodes to hub nodes where the full path in-network is minimized. The number of hubs is not specified in the hub covering problem, and demands are covered within a certain distance. Minimizing the cost of installing facilities covered by hubs in such problems. In hub location problem with fixed costs number of hubs is not defined at first. Flow and installation hubs costs are minimized in this problem [5]. This research is structured as follows: the literature review is presented in Section 2. The credibility-based fuzzy theory is described in Section 3. Section 4 defines the problem statement along with model formulation. Section 5 describes the proposed solution method. In Section 6, computational results are presented, and finally, in Section 7, conclusions and recommendations for future research studies are presented.

## 2. Literature Review

Nowadays, the hub location problem is studied to maximize profit considering a revenue-cost trade-off. Revenue management has been considered in several forms in research. It is derived from the transportation flows [6–9]. In another category, it is derived from a combination of pricing and hub location [10]. Finally, integrated revenue management and hub location are considered another category [11, 12]. Hörhammer [13] studied a dynamic multiperiod hub location problem with multiple capacity levels. They considered a nonhub node can be a hub in the next period. Proposed a method that has four main steps called Distribution-Map-Transfer-Combination (DMTC). A quadratic mixed-integer programming model based on flow and route is developed. The aim is to minimize connection costs between a nonhub and hub nodes, transportation costs between a hub and other ones, and installation costs. He et al. [14] proposed a nonlinear mixed-integer programming model for hub location problems considering support hub. Lagrangian relaxation and branch and bound methods were applied to solve the proposed problem. Ebrahimnejad et al. [15] developed a particle swarm optimization algorithm for shortest path problems with mixed fuzzy arc weights. Adibi and Razmi [16] presented a two-stage stochastic model for multiple allocations in the hub location problem. It is assumed that demand and transportation costs are probabilistic. Damgacioglu et al. [17] developed a GA to solve the problem considering uncapacitated allocation. Alumur et al. [18] presented a multiperiod hub location problem for multiple allocations. Installing a new hub and available hub capacity expansion is allowed in the study. A MIP model is developed to minimize shipment, hubs connection, hub installation, and capacity expansion cost. Azizi et al. [19] presented a hub location model under hub failure risk. They considered that a support hub could be applied to supply demand when a hub goes out of order. Grauberger and Kimms [20] investigated an airline revenue management problem considering price competition and limited capacity. He [21] studied the revenue management effect on a hub-to-hub network.

Tikani et al. [3] studied an integrated hub location and revenue management considering several customer classes to maximize profit and minimize costs. To do this end, a two-stage stochastic model is developed to determine hub location. Furthermore, an efficient genetic algorithm is proposed to solve the problem on a large scale. Alumur et al. [22] investigated capacitated single and multiple hub location problems. A direct connection between two nonhubs is considered in this study. A MIP model is developed to minimize transportation and hub installation costs. Hou et al. [12] presented an integrated $p$-hub location and revenue management problem considering multiple capacities under disruptions. A two-stage stochastic model is developed to maximize net profit in which hub installation cost, shipment cost, and revenue obtained from ticket selling are considered. A robust integrated optimization and stochastic programming to maximize weighted total profit is presented to obtain reasonable solutions. Huo et al. [11] studied an integrated hub location and revenue management problem considering average and worst-case analysis. A $p$-hub is selected from $n$ nodes while uncertain data and some scenarios are considered in the study. Then, a two-stage stochastic programming model is developed to maximize profit. Ahmadi et al. [23] proposed a unique hybrid strategy for selecting users with Deep-Q-Reinforcement Learning with Federated Learning. Korani et al. [24] proposed a reliable multimodal hub location problem. They developed a Lagrangian method considering the strategic level that causes to achieve accurate solutions. Čvokić and Stanimirović [25] introduced a new uncapacitated single allocation hub location problem under a deterministic and robust approach to maximize net profit. A mixed-integer quadratic model is proposed. Furthermore, a two-phase meta-heuristic algorithm is developed. Rouzpeykar et al. [26] developed a robust optimization model for the integrated hub location and revenue management problem under uncertainty. They applied a case study in Iran to validate the proposed model.

Di Caprio et al. [27] developed an ant colony algorithm under uncertainty for the shortest path method problem. They assumed that the arc weights were fuzzy. The proposed algorithm is compared with GA, PSO, and the artificial bee colony algorithms. Ebrahimnejad et al. [28] developed an artificial bee colony algorithm under uncertainty for the shortest path method problem. They considered mixed interval fuzzy numbers for the arc weights. Sori et al. [29] studied the constrained shortest path problem in location-based online services to find a path with the lowest cost with fuzzy time and cost. The summary of the last works is presented in Table 1.

According to Table 1, many studies have addressed the hub location problem. In contrast, some of them have considered it with revenue management simultaneously. A few numbers types of research have examined this issue under uncertainty. Those research studies have applied stochastic or robust approaches to deal with uncertainty. In this study, a credibility-based fuzzy theory will be used to model uncertainty in an integrated hub location and revenue management for the first time. Through this method, managers can select different levels of confidence based on their experiences. A fuzzy mixed-integer programming model has been developed to deal with the proposed problem with uncertain parameters.

Table 1: Summary of previous research.

| Researcher (year) | Problem | | Objective | | Model | | | |
|---|---|---|---|---|---|---|---|---|
| | Location | Revenue management | Cost | Revenue | Deterministic | Stochastic | Robust | Fuzzy |
| Hörhammer [13] | * | | * | | * | | | |
| He et al. [14] | * | | * | | * | | | |
| Adibi and Razmi [16] | * | | * | | | * | | |
| Alumur et al. [18] | * | | * | | * | | | |
| Grauberger and Kimms [20] | | * | | * | * | | | |
| He [21] | | * | | * | * | | | |
| Tikani et al. [3] | * | * | * | * | | * | | |
| Alumur et al. [22] | * | | * | | * | | | |
| Hou et al. [12] | * | * | * | * | | * | * | |
| Huo et al. [11] | * | * | * | * | | * | | |
| Čvokić and Stanimirović [25] | * | | | * | * | | * | |
| Present study | * | * | * | * | | | | * |

## 3. Credibility-Based Fuzzy Theory

This study uses a fuzzy approach to consider uncertainty [30].

$$\widetilde{A} = \left\{\left(x, \mu_{\widetilde{A}}(x)\right) | x \in X\right\}, \tag{1}$$

where $\widetilde{A}$ is a fuzzy set and $\mu_{\widetilde{A}}(x)$ is calculated by the following equation:

$$\mu_{\widetilde{A}}(x)\colon X \longrightarrow [0,1]. \tag{2}$$

Different fuzzy numbers such as triangular fuzzy numbers or trapezoidal fuzzy numbers can be used in the fuzzy approach [31]. Due to the nature of the proposed problem, the trapezoidal fuzzy number has been used in this study. In a triangular fuzzy number, only one parameter value gives the maximum amount of confidence. In contrast, in the trapezoidal fuzzy number, the maximum value of a parameter is obtained. In this case, the risk-taking of decision-makers is reduced, and they can accept uncertainty in natural conditions with more confidence [32]. A membership function of a trapezoidal fuzzy number $\xi = (l, m_1, m_2, u)$ is as follows (Figure 1):

$$\mu(x) = \begin{cases} \dfrac{x-l}{m_1-l} & l \le x \le m_1, \\ 1, & m_1 \le x \le m_2, \\ \dfrac{u-x}{u-m_2}, & m_2 \le x \le u, \\ 0, & O.W. \end{cases} \tag{3}$$

A fuzzy credibility model is applied for the integrated proposed model. The credibility measure is defined as [33]

$$Cr\{\xi \le A\} = \frac{1}{2}\{\mathrm{Pos}\{\xi \le A\} + \mathrm{Nec}\{\xi \le A\}\}, \tag{4}$$

where $\xi$ and $A$ are fuzzy variables and real numbers, respectively. Possibility (Pos) and necessity (Nec) measures are defined as (5) and (6), respectively.

$$\mathrm{pos}\{\xi \le A\} = \sup \mu_x(x) = \begin{cases} 0, & x \le l, \\ \dfrac{x-l}{m_1-l}, & l \le x \le m_1, \\ 1, & x \ge m_1, \end{cases} \quad \mathrm{Nec}\{\xi \le A\} = 1 - \sup_{\xi \ge A} \mu_x(x) = \begin{cases} 0, & x \le m_2, \\ 1 - \dfrac{u-x}{u-m_2} = \dfrac{x-m_2}{u-m_2}, & m_2 \le x \le u, \\ 1, & x \ge u. \end{cases} \tag{5}$$

The possibility and necessity measures are also shown in Figure 2.

The following equation calculates the credibility measure shown in Figure 3 [34]:

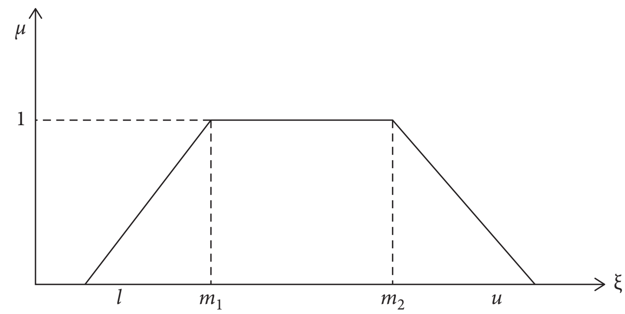


Figure 1: Trapezoidal fuzzy number.

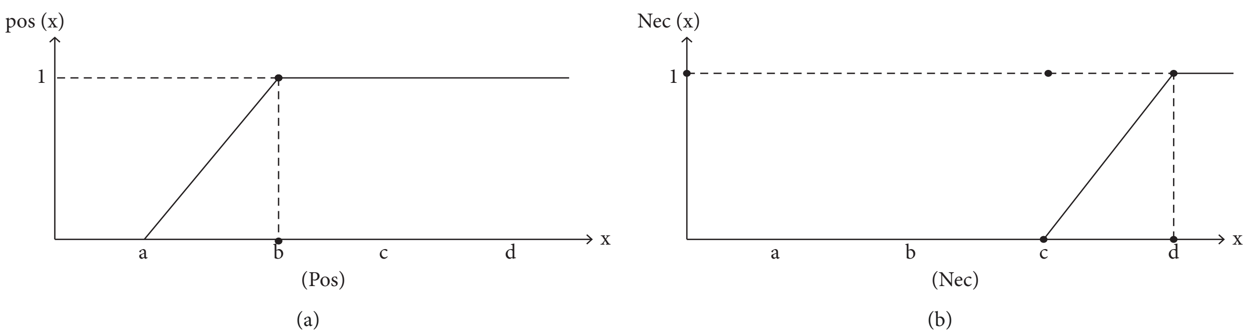


Figure 2: Possibility and necessity measures. (a) (Pos) and (b) (Nec).

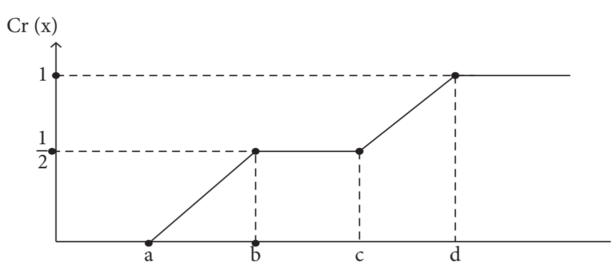


Figure 3: Credibility measures.

$$Cr\{\xi \leq A\} = \frac{1}{2}\{\text{pos}\{\xi \leq A\} + \text{Nec}\{\xi \leq A\}\}$$
$$= \begin{cases} 0, & x \leq l, \\ \dfrac{x-l}{2(m_1 - l)}, & l \leq x \leq m_1, \\ \dfrac{1}{2}, & m_1 \leq x \leq m_2, \\ \dfrac{1}{2}\left(1 + \dfrac{x - m_2}{u - m_2}\right), & m_2 \leq x \leq u, \\ 1, & x \geq u. \end{cases} \tag{6}$$

If $\xi$ is a trapezoidal fuzzy number and $\alpha > 0.5$, then [35]

$$\begin{gathered} Cr\{\xi \leq x\} \geq \alpha \Longrightarrow x \geq (2 - 2\alpha)m_2 + (2\alpha - 1)u, \\ Cr\{\xi \geq x\} \geq \alpha \Longrightarrow x \leq (2\alpha - 1)l + (2 - 2\alpha)m_1. \end{gathered} \tag{7}$$

## 4. Problem Statement

In this problem, a central hub is connected to some hub nodes. There are some candidate hub nodes that p of them should be selected. Then, other nonhub nodes are connected to hub ones so that total transportation and installation costs are minimized while the revenue obtained from selling tickets is maximized. Based on their capacity, aircraft determine their route from a hub node to nonhub ones where the maximum required demand is satisfied.

*4.1. Model Formulation.* The proposed model includes hub location problems and revenue management in the aircraft industry under uncertainty to maximize revenue from network transportation and minimize total cost. It is assumed that all nodes can be selected as a hub and $p$-hubs have been selected from a set of $n$ nodes connected to a central hub. Other assumptions are presented as follows.

*4.2. Assumptions*

(i) All origin and destination nodes are candidates to become a hub
(ii) The number of hubs is predefined
(iii) The central hub location is given

(iv) A node is only allocated to one hub
(v) Each two-node is not connected directly while they connected by a hub
(vi) There is not any direct shipment between the two hubs
(vii) The number of flights between nodes (between central and other hubs, as well as between hubs and nonhubs) is limited
(viii) Aircraft which traverse between hubs and nonhubs have different capacity
(ix) The ticket price at each class is predefined
(x) Extra cargo for passengers is allowed
(xi) Goods can be carried

*4.3. Notation*

*4.3.1. Sets and Indices*

$N$: node number

$P$: hub number

$K$: flight class number

$i, m$: node indices $i, m = 1, 2, \ldots, N$

$j$: hub indices $j = 1, 2, \ldots, P$

$k$: flight class indices $k = 1, 2, \ldots, K$

*4.3.2. Parameters*

$dis_{j0}$: distance from the central hub to hub $j$

$\text{dis}_{ij}$: distance from hub $j$ to nonhub $i$

$c_{j0k}$: unit transfer cost between the central hub and hub $j$ for class $k$

$c_{ijk}$: unit transfer cost between hub $j$ and nonhub $i$ for class $k$

$\text{cap}_{j0}$: available flight number from the central hub to hub $j$

$\text{cap}_{ij}$: available flight number from hub $j$ and nonhub $i$

$p_{imk}$: ticket price from node $i$ to node $m$ for class $k$

$ph_{imk}$: extra cargo price per unit from $i$ to $m$ for class $k$

$pg_{\text{im}}$: goods carrying price per unit from $i$ to $m$

$d_{imk}$: demand between $i$ and $m$ for class $k$

$vh_{imk}$: amount of extra cargo from $i$ to $m$ for class $k$

$vg_{im}$: amount of goods carrying from $i$ to $m$

$cl_1$: the capacity of the link between the central hub and other hubs

$cl_2$: the capacity of the link between hubs and nonhubs

$fc_0$: fixed cost for establishing a central hub

$fc_j$: fixed cost for establishing hub $j$

$A$: a huge integer

$\lambda_{\text{im}}$: the confidence level of the decision-maker for the link between $i$ and $m$

*4.3.3. Decision Variables*

$x_{imk}$: number of tickets sold between nodes $i$ and $m$ for class $k$

$y_{imk}$: protection level between nodes $i$ and $m$ for class $k$

$z_{ij}$ 1: if nonhub $i$ is connected to hub $j$, and 0 otherwise

$z_{jj}$ 1: if node $i$ is selected as a hub, and 0 otherwise

$o_{\text{im}}$ 1: if a flight was done between nodes $i$ and $m$, and 0 otherwise

*4.3.4. Mathematical Model.* The proposed biobjective model is formulated as follows:

$$\begin{aligned} \max z_1 = & \sum_{i=1}^{N}\sum_{m=1}^{N}\sum_{k=1}^{K} p_{imk} \times x_{imk} \\ & + \sum_{i=1}^{N}\sum_{m=1}^{N}\sum_{k=1}^{K} ph_{imk} \times vh_{imk} \times o_{\text{im}} \\ & + \sum_{i=1}^{N}\sum_{m=1}^{N} pg_{\text{im}} \times vg_{\text{im}} \times o_{\text{im}}, \end{aligned} \tag{8}$$

$$\begin{aligned} \min z_2 = & \sum_{j=1}^{P}\sum_{k=1}^{K} c_{j0k} \left[ \text{dis}_{j0} \left( \sum_{i=1}^{N} \sum_{m=1, m\neq i}^{N} \left( \frac{x_{imk}}{cl_2} \right) z_{ij} \left( 1 - z_{mj} \right) \right) + \text{dis}_{0j} \left( \sum_{i=1}^{N} \sum_{m=1, m\neq i}^{N} \left( \frac{x_{imk}}{cl_2} \right) z_{ij} \left( 1 - z_{mj} \right) \right) \right] \\ & + \sum_{i=1}^{N}\sum_{j=1}^{P}\sum_{k=1}^{K} c_{ijk} \left[ \text{dis}_{ij} \left( \sum_{m=1, m\neq i}^{N} \frac{x_{imk}}{cl_1} \right) + \text{dis}_{ji} \left( \sum_{m=1, m\neq i}^{N} \frac{x_{imk}}{cl_1} \right) \right] z_{ij} \\ & + \sum_{j=1}^{P} fc_j z_{jj} + fc_0, \end{aligned} \tag{9}$$

st

$$\sum_{j=1}^{P} z_{ij} \leq 1 \quad \forall i = 1, 2, \ldots, N, \tag{10}$$

$$\sum_{j=1}^{N} z_{jj} = P, \tag{11}$$

$$z_{ij} \leq z_{jj} \quad \forall i = 1, 2, \ldots, N, \forall j = 1, 2, \ldots, P, \tag{12}$$

$$x_{imk} \leq \widetilde{d}_{imk} \quad \forall i, m = 1, 2, \ldots, N, \forall k = 1, 2, \ldots, K, \tag{13}$$

$$x_{imk} \leq y_{imk} \quad \forall i, m = 1, 2, \ldots, N, \forall k = 1, 2, \ldots, K, \tag{14}$$

$$\sum_{k=1}^{K} x_{imk} \geq o_{\mathrm{im}} \quad \forall i, m = 1, 2, \ldots, N, \tag{15}$$

$$\sum_{k=1}^{K} x_{imk} \leq A \times o_{\mathrm{im}} \quad \forall i, m = 1, 2, \ldots, N, \tag{16}$$

$$\begin{aligned} &\sum_{i=1}^{N} \sum_{j=1}^{P} \sum_{k=1}^{K} (y_{imk}/cl_2) z_{ij} \left(1 - z_{mj}\right) \\ &\quad + \sum_{i=1}^{N} \sum_{j=1}^{P} \sum_{k=1}^{K} (y_{\mathrm{mik}}/cl_2) z_{ij} \left(1 - z_{mj}\right) \leq \mathrm{cap}_{j0} \times z_{jj} \\ &\quad \forall j = 1, 2, \ldots, P, \end{aligned} \tag{17}$$

$$\begin{aligned} &\sum_{m=1}^{N} \sum_{k=1}^{K} (y_{imk}/cl_1) + \sum_{m=1}^{N} \sum_{k=1}^{K} (y_{mik}/cl_1) \leq \sum_{j=1}^{P} \mathrm{cap}_{ij} \times z_{ij} + A \\ &\quad \times z_{ii} \\ &\quad \forall i = 1, 2, \ldots, N, \end{aligned} \tag{18}$$

$$z_{ij}, o_{\mathrm{im}} \in \{0, 1\} \quad \forall i, m = 1, 2, \ldots, N, \forall j = 1, 2, \ldots, P, \tag{19}$$

$$x_{imk}, y_{imk} \geq 0 \quad \forall i, m = 1, 2, \ldots, N, \forall k = 1, 2, \ldots, K. \tag{20}$$

Equation (8) shows the revenue obtained from selling tickets in various classes and carrying extra cargo and goods. Equation (9) calculates the total cost of the network, including the total transportation cost between nodes and the total installation cost of hubs. Total network profit is obtained from the difference between the two objective functions ($z_1 - z_2$). Equation (10) ensures that each nonhub node should be allocated to only one hub. Equation (11) states that there are precisely $p$-hubs in the network. Equation (12) enforces that a nonhub node has been allocated to a hub node if this node had been selected as a hub. Equations (13) and (14) show that the maximum number of sold tickets equals demand and the protection level, respectively. Equations (15) and (16) indicate that a flight between nodes $i$ and $m$ if tickets had sold for that route. Equation (17) states that the protection level should not exceed the physical capacity between the central hub and other hubs. Equation (18) indicates that the protection level should not exceed the physical capacity between hob and nonhub nodes. Finally, variables of the model are introduced in equations (19) and (20).

*4.3.5. Credibility-Based Fuzzy Approach.* Generally, the credibility-based chance-constrained programming [31, 36] is a computationally efficient fuzzy mathematical programming approach that relies on solid mathematical concepts and can support different kinds of fuzzy numbers such as triangular and trapezoidal forms as well as enabling the decision-maker to satisfy some chance constraints in at least some given confidence levels. According to equation (18), a trapezoidal fuzzy number is considered for demand between nodes. Based on the credibility approach, equation (13) is reformulated in equation (21), which is equivalent to equation (24).

$$Cr\left\{x_{imk} \leq \widetilde{d}_{imk}\right\} \geq \lambda_{\mathrm{im}} \quad \forall i, m = 1, 2, \ldots, N, \forall k = 1, 2, \ldots, K, \tag{21}$$

$$\begin{aligned} x_{imk} \leq {} & (2\lambda_{\mathrm{im}} - 1)\widetilde{d}_{imk(1)} \\ & + (2 - 2\lambda_{\mathrm{im}})\widetilde{d}_{imk(2)} \\ & \forall i, m = 1, 2, \ldots, N, \quad \forall k = 1, 2, \ldots, K. \end{aligned} \tag{22}$$

## 5. Solution Methodology

The proposed model is NP-hard, and its complexity increases by increasing the number of hubs [3]. Thus hub selection and its assignment to other nonhub nodes would be more complex. Therefore, large-sized problems cannot be solved by the exact method in a reasonable time. To deal with this problem, a genetic algorithm, a population-based metaheuristic, is employed in this paper.

*5.1. Genetic Algorithm.* Few works applied metaheuristic algorithms to integrate revenue management and hub location problem. However, the genetic algorithm is used for this type of problem. The main reason to apply this approach is that it is easier to design the problem by GA. The pseudocode of the proposed genetic algorithm is depicted in Figure 4, where algorithm parameters are firstly set by the Taguchi parameter setting method. Then, initial solutions are created where infeasible solutions are revised until a feasible one is generated. If the feasible solution is not achieved, we use the death penalty as the infeasible solution. After that fitness function of each solution is calculated. After creating initial solutions, a repetitive process involving crossover and mutation operators to generate offsprings and mutated solutions and calculation of fitness functions of solutions and selection of the best solution is made until a predetermined stop condition is satisfied.

1- *Parameter Setting*
2- *Create initial solutions*
3- *Calculate fitness function*
4- *Select the best solution*
5- *Create offspring using:*
6- *Crossover*
7- *Mutation*
8- *Calculate fitness function*
9- *Select the best solution*
10- *Check stop condition*
11- *If it is true, go to the next step while going to step 5*
12- *End*

Figure 4: Pseudocode.

*5.1.1. Solution Representation.* The solution representation of this algorithm is illustrated in Figure 5 and described as follows: consider a central hub, two hubs, and three nonhubs nodes. Firstly, a random matrix is created equal to the node's number (5 in this example). Then, the maximum values in the diagonal are selected as a hub while exactly $p$-hubs are obtained. Then, the highest number in each row related to nonhubs nodes at the intersection of columns assigned as hubs determines its allocation.

As shown in Figure 5, nodes 1 and 4 are selected as a hub. In the next step, the maximum value at the intersection of columns selected as a hub in each nonhub row is allocated. According to the allocation structure, the network design is depicted in Figure 6.

*5.1.2. Crossover Operator.* This study applies a one-cut point crossover to create offspring chromosomes from two randomly selected parents. Two new offspring are obtained using the following equation:

$$\begin{aligned} p_1 &= b\, pfn + (1-b) psn, \\ p_2 &= (1-b) pfn + b\, psn. \end{aligned} \tag{23}$$

Here, $b$ is a matrix with parents size, and *pfn* and *psn* are $n$ dimension matrices of the first and the second parent, respectively.

*5.1.3. Mutation Operator.* In this operator, a hub node exchanges with a nonhub node. One of the genes representing a hub node is selected randomly. Then, it is changed to a nonhub node so that the nonhub node with a higher random value is selected as a hub based on Figure 5. The following equation is used to mutate each gene of a solution:

$$\text{gen}_j^{\text{new}} = 1 - \text{gen}_j^{\text{old}}. \tag{24}$$

Figure 7 shows the mechanism of the mutation.

## 6. Computational Results

The proposed model validity is assessed using two problem instances based on Tikani et al. [3]. In the following, sample problems are designed, and the algorithm parameters are tuned.

*6.1. Data Generation.* Sample problems are presented in various scales, as shown in Table 2. It should be noted that input parameters for medium and large instances are randomly generated using uniform distributions. Also, Table 3 states the problem dimension.

*6.2. Parameters Tuning.* Taguchi experimental design in MINITAB software is applied to tune the parameters of the proposed GA, including population size, number of iteration, mutation, and crossover rates. Their values are assessed at three levels shown in Table 4.

RPD (relative percentage deviation) shown in the following equation is used as a GAP criterion to analyze the performance of the proposed GA:

$$\text{GAP} = \left( \frac{\text{alg}_{\text{sol}} - \text{best}_{\text{sol}}}{\text{best}_{\text{sol}}} \right) \times 100, \tag{25}$$

where $\text{alg}_{\text{sol}}$ and $\text{best}_{\text{sol}}$ are the objective function value and the best value of them obtained by the algorithm execution, respectively. An instance (No. 1) is randomly selected to execute for each of the combinations listed in Table 5 and then the GAP measure is calculated and plotted as shown in Figure 5.

Figure 8 indicates the Taguchi method analysis to tune the proposed genetic algorithm parameters. As can be seen, the best values for population size, number of iteration, mutation, and crossover rate are 200, 300, 0.01, and 0.85, respectively.

*6.3. Solution Results.* As mentioned above, a fuzzy MIP model is proposed for the integrated hub location and revenue management problem under fuzzy demand. A genetic algorithm is developed to solve large-sized problems. To validate the proposed genetic algorithm, small-sized problems are solved by both the GAMS optimization package and the proposed GA. Then, large-sized problems are solved, and the results of the two methods are compared in Table 6.

The comparative results obtained from the two methods revealed that the proposed GA could achieve the same solution as the exact method of GAMS [37]. This indicates the validity of the proposed genetic algorithm. Moreover, the proposed GA can solve the proposed problem on a large scale. The computational time of solving sample problems demonstrates that the problem has high complexity. The average time is increased by increasing the size of the problems, as shown in Figure 9.

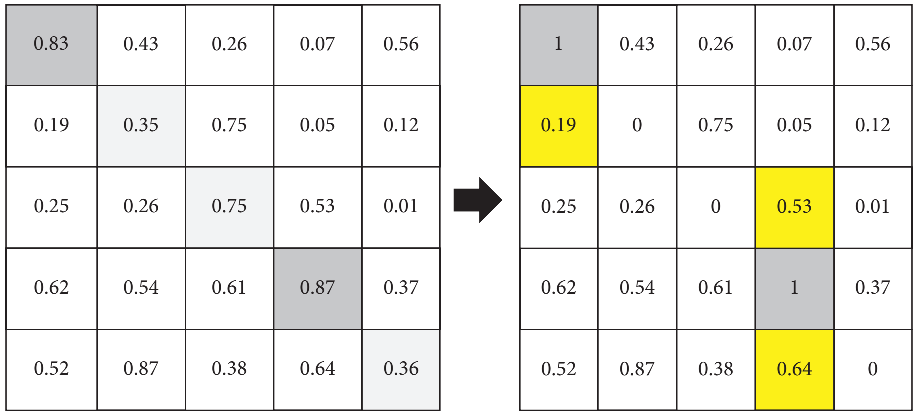


Figure 5: Solution representation.

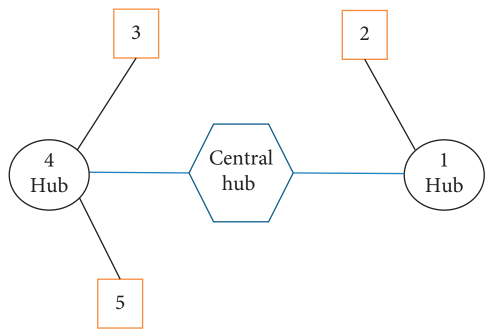


Figure 6: Network design [26].

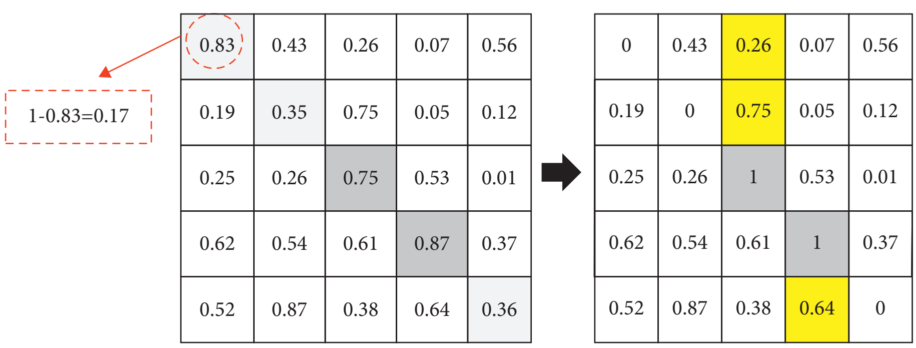


Figure 7: Mutation operator.

Table 2: Sample problems parameters value.

| Parameter | Value | | Value |
|---|---|---|---|
| $\mathrm{dis}_{j0}$ | Uniform (3500, 50000) | $\mathrm{dis}_{ij}$ | Uniform (1000, 3500) |
| $c_{j0k}$ | Uniform (200, 500) | $c_{ijk}$ | Uniform (100, 200) |
| $\mathrm{cap}_{j0}$ | Uniform (10, 20) | $\mathrm{cap}_{ij}$ | Uniform (15, 30) |
| $fc_0$ | Uniform (100000, 200000) | $fc_j$ | Uniform (50000, 80000) |
| $cl_1$ | Uniform (5000, 10000) | $cl_2$ | Uniform (10000, 15000) |
| $p_{imk}$ | Uniform (500, 1500) | $d_{imk}$ | Uniform (400, 800) |
| $ph_{imk}$ | Uniform (20, 100) | $pg_{\mathrm{im}}$ | Uniform (50, 150) |
| $vh_{imk}$ | Uniform (50, 500) | $vg_{\mathrm{im}}$ | Uniform (1000, 2000) |
| $A$ | 1000000 | $\lambda_{\mathrm{im}}$ | Uniform (0.5, 0.8) |

Table 3: Sample problems dimension.

| Size | Prob. no. | Nodes no. | Hubs no. | Classes no. |
|---|---|---|---|---|
| Small | 1 | 5 | 1 | 3 |
| | 2 | 6 | 1 | 3 |
| Medium | 3 | 10 | 2 | 4 |
| | 4 | 12 | 2 | 4 |
| Large | 5 | 18 | 4 | 5 |
| | 6 | 20 | 4 | 5 |

Table 4: GA parameters value.

| Population size | Crossover rate | Mutation rate | No. of iterations |
|---|---|---|---|
| 70 | 0.75 | 0.006 | 150 |
| 150 | 0.85 | 0.009 | 300 |
| 200 | 0.95 | 0.01 | 500 |

Table 5: Genetic algorithm orthogonal.

| State no. | Population size | Crossover rate | Mutation rate | No. of iterations | Value GAP |
|---|---|---|---|---|---|
| 1 | 70 | 0.75 | 0.006 | 150 | 0.5032 |
| 2 | 70 | 0.85 | 0.009 | 300 | 0.1259 |
| 3 | 70 | 0.95 | 0.01 | 500 | 0.7419 |
| 4 | 150 | 0.75 | 0.009 | 500 | 0.6635 |
| 5 | 150 | 0.85 | 0.01 | 150 | 0.4917 |
| 6 | 150 | 0.95 | 0.006 | 300 | 0.0045 |
| 7 | 200 | 0.75 | 0.01 | 300 | 0.7124 |
| 8 | 200 | 0.85 | 0.006 | 500 | 0.7280 |
| 9 | 200 | 0.95 | 0.009 | 300 | 0.2942 |

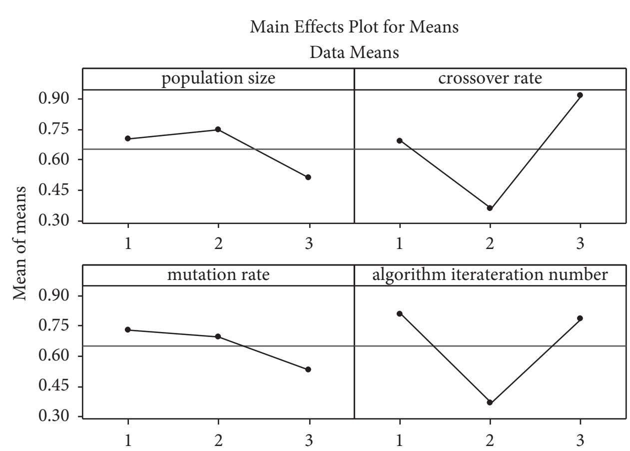


Figure 8: The mean effect of GA.

*6.4. Sensitivity Analysis.* In this section, the effectiveness of essential parameters is analyzed. Firstly, the impact of demand on objective functions is investigated. As shown in Figure 10, objective function values increased by increasing the amount of demand. Network revenue and total costs increase when the number of passengers increases.

Furthermore, link capacity's effect on objective function is assessed. To this end, we change this capacity from −10% to +10%, as shown in Figure 11. As expected, the capacity link is only affected by the total cost.

Finally, the impact of confidence level (fuzzy membership) on revenue and cost is analyzed. This parameter shows the confidence level of decision-makers for demand. Thus, $\lambda_{im}$ it is changed from 0.5 to 1 and depicted in Figure 12. It indicates that the higher the confidence level decision-makers adopt, the total cost fluctuation will decrease based.

Table 6: Solution results of sample problems.

| Size | Prob. no. | GAMS | | | GA | | |
|---|---|---|---|---|---|---|---|
| | | $Z_1$ | $Z_2$ | CPU time (s) | $Z_1$ | $Z_2$ | CPU time (s) |
| Small | 1 | 640.000 | 425.000 | 74.6 | 640.000 | 425.000 | 43.6 |
| | 2 | 673.500 | 447.000 | 91.4 | 673.500 | 447.000 | 76.4 |
| Medium | 3 | 1.098.000 | 921.500 | 753.6 | 1.095.500 | 923.000 | 136.1 |
| | 4 | — | — | >1000 | 1.154.000 | 963.500 | 160.8 |
| Large | 5 | — | — | >1000 | 2.041.500 | 1.714.500 | 273.7 |
| | 6 | — | — | >1000 | 2.327.000 | 1.908.000 | 301.2 |

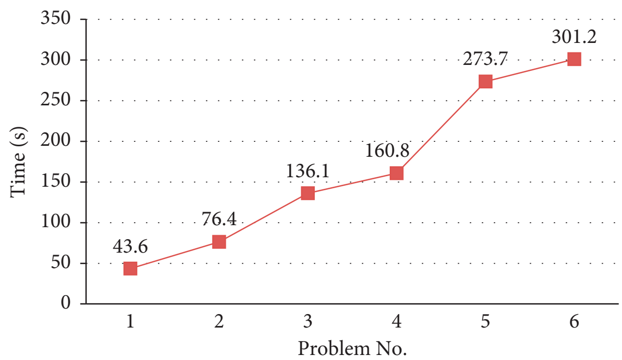


Figure 9: Computational time comparison.

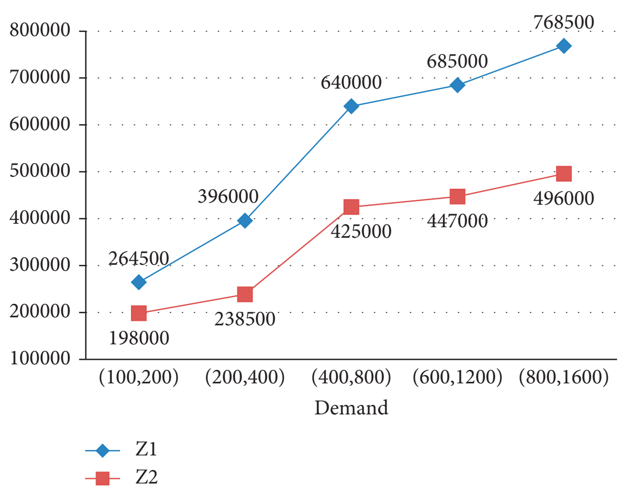


Figure 10: Effect of demand on the objective functions.

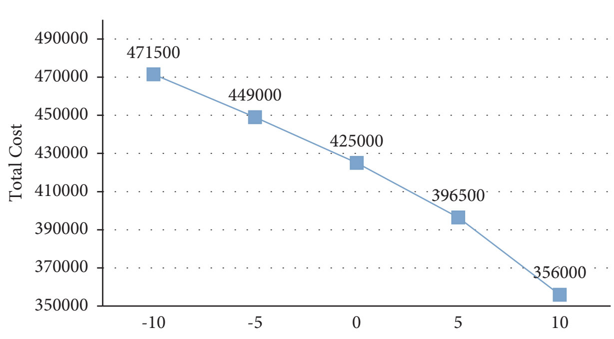


Figure 11: Effect link capacity on the total cost.

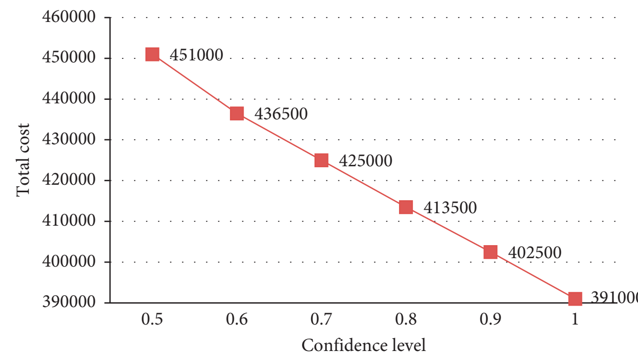


Figure 12: Confidence level effect on total cost.

## 7. Conclusions and Future Directions

This paper has developed a fuzzy MIP model for the integrated revenue management and $p$-hub location problem. The objectives are maximizing network revenue as well as minimizing total costs. A credibility-based fuzzy theory has been used to deal with uncertainty in an integrated problem. In order to evaluate the proposed mathematical model, some problem instances have been used and solved using the CPLEX solver of GAMS software. Furthermore, a genetic algorithm has been developed for large-sized problems. Then, a sensitivity analysis has been performed on crucial inputs of the problem, including demand, link capacity, and confidence level. The revenue will be increased by decreasing the number of hub nodes. Thus, the number of hubs has an essential effect on revenue earned from the network. Therefore, managers should make connections among nonhub nodes so that the minimum number of hub nodes is opened and all nonhub nodes are allocated to the hub ones.

As mentioned above, only the demand for flight is considered uncertain in this study. However, there are different parameters, such as parameters related to cost, that can be considered uncertain, too. A fuzzy approach is applied in this study. At the same time, other methods such as robust optimization and stochastic programming could be used in future works. Moreover, a credibility-based fuzzy approach is applied in this study; however, there are other approaches such as equivalent auxiliary crisp and $\alpha$-cut level concept. Moreover, artificial intelligence-based algorithms can solve large-sized problems in reasonable run times [38, 39].

## Data Availability

Data are available from the corresponding author (r.soltani@khatam.ac.ir) upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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