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Research Article

A New Flexible Logarithmic-X Family of Distributions with Applications to Biological Systems

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Probability distributions play an essential role in modeling and predicting biomedical datasets. To have the best description and accurate prediction of the biomedical datasets, numerous probability distributions have been introduced and implemented. We investigate a novel family of lifetime probability distributions to represent biological datasets in this paper. The proposed family is called a new flexible logarithmic-X (NFLog-X) family. The suggested NFLog-X family is obtained by applying the T-X method together with the exponential model having the PDF $m(t) = e^{-t}$. Based on the NFLog-X approach, a three parameters probability distribution, namely, a new flexible logarithmic-Weibull (NFLog-Wei) distribution is introduced. The method of maximum likelihood estimation is adopted for estimating the parameters of the NFLog-X family. In the end, we examine three different biological datasets in order to give a thorough numerical research that illustrates the NFLog-Wei distribution. Comparisons are made between the analytical goodness-of-fit metrics of the suggested distribution. We made comparison with the (i) alpha power transformed Weibull, (ii) exponentiated Weibull, (iii) Weibull, (iv) flexible reduced logarithmic-Weibull, and (v) Marshall–Olkin Weibull distributions. After performing the analyses, we observe that the proposed method outclassed other competitive distributions.

1. Introduction

Probability distributions are frequently used to model the lifetime phenomena in applied sectors [1]. In the literature of distributions theory, the most frequently used distributions to model the lifetime phenomena are the exponential (Exp), Rayleigh (Ray), and Weibull (Wei) distributions. However, when the lifetime phenomena are complex, then these probability distributions are not suitable to model and predict the data accurately (Ahmad et al. [2] and Liao et al. [3]). For example, the Exp distribution is concerned with describing data that have a constant HF (hazard function). On the other hand, the Ray distribution is used to model data with an increasing HF. Similarly, the Wei distribution having the Exp and Ray as the special models is one of the popular probability distributions (Sarhan and Zaindin [4] and Huo et al. [5]). The Weibull model offers/provides the features of both the Exp and Ray probability distributions and has widely been used in modeling lifetime phenomena with monotone failure rates. However, when the lifetime phenomena have a monotone (increasing, decreasing, and constant) HF, then, the Weibull distribution is the best choice to use (Almalki and Yuan [6] and Liu et al. [7]).

In the field of biomedical sciences, authors have shown greater interest and published numerous papers to propose new distributions. In the past decade, researchers' efforts have been devoted to deriving new families of probability distributions. The new probability distributions have been constructed by adding one or more new additional parameters to the baseline models (El-Morshedy et al. [8]; Guerra et al. [9]; Reyad et al. [10]; Bantan et al. [11]; Eghwerido et al. [12]; Eghwerido and Agu [13]; Alzaatreh et al. [14]; Lahcene [15]; ElSherpieny and Almetwally [16]; Roozegar et al. [17]; Klakattawi et al. [18]; Hussein et al. [19]; and Kilai et al. [20]).

Recently, Ahmad et al. [21] studied a Z-family by adding a new parameter. We can write the distribution function (DF) $F(x;\beta,\lambda)$ of the Z-family through the following equation:

$$F(x;\beta,\boldsymbol{\lambda}) = 1 - \frac{1-K(x;\boldsymbol{\lambda})}{\beta^{K(x;\boldsymbol{\lambda})}}, \quad x\in\mathbb{R}, \tag{1}$$

such that $\beta>0$ can be considered as an extra parameter.

Wang et al. [22] developed another method called, a NG-X (new generalized-X) family by the following DF:

$$F(x;\theta,\boldsymbol{\lambda}) = 1 - \frac{[1-K(x;\boldsymbol{\lambda})]^{\theta}}{e^{K(x;\boldsymbol{\lambda})}}, \quad x\in\mathbb{R}, \tag{2}$$

where $\theta>0$.

Mohammed et al. [23] proposed another new approach to develop new probability distribution for modeling lifetime events. They named their proposed method, a NLT-X (new lifetime-X) distributions. The DF $F(x;\eta,\lambda)$ of the NLT-X distributions is given by the following equation:

$$F(x;\eta,\boldsymbol{\lambda}) = 1 - \left(\frac{1-K(x;\boldsymbol{\lambda})}{e^{K(x;\boldsymbol{\lambda})}}\right)^{\eta}, \quad x\in\mathbb{R}, \tag{3}$$

with an additional parameter $\eta>0$.

We additionally propose a new class of probability distribution in this paper by implementing the T-X method. The new class is called a NFLog-X family of distributions. Using the proposed NFLog-X approach, we can formulate an upgraded version of the Wei distribution whicnh can be presented and dubbed as NFLog-Wei distribution. The proposed NFLog-Wei distribution offers a close fit to the healthcare datasets.

2. The Proposed Method

Here, we propose a new method to introduce new updated and modified versions of the lifetime distributions. By incorporating the exponential model, having the PDF $m(t)=e^{-t}$ with the T-X method (Alzaatreh et al. [24]), the suggested approach is presented.

Let us assume that we have a RV (random variable), represented by T, considered as a baseline RV with PDF $m(t)$, where $T\in[\pi_1,\pi_2]$ for $-\infty<\pi_1<\pi_2<\infty$. Let X be another RV with DF $K(x;\lambda)$. Let suppose $G[K(x;\lambda)]$ considered as a function in the DF, meeting each of the three requirements outlined below:

(i) $G[K(x;\lambda)]\in[\pi_1,\pi_2]$.

(ii) $G[K(x;\lambda)]$ is a differentiable and IF (increasing function).

(iii) $G[K(x;\lambda)]\longrightarrow\pi_1$ as $x\longrightarrow-\infty$ and $G[K(x;\lambda)]\longrightarrow\pi_2$ as $x\longrightarrow\infty$.

According to Alzaatreh et al. [24], the DF $F(x)$ of the T-X family is as follows:

$$F(x) = \int_{\pi_1}^{G[K(x;\boldsymbol{\lambda})]} m(t)\,dt, \tag{4}$$

with PDF given by

$$f(x) = m(G[K(x;\boldsymbol{\lambda})])\frac{d}{dx}G[K(x;\boldsymbol{\lambda})]. \tag{5}$$

Now, setting $G[K(x;\lambda)] = -\log(1-(\delta^2K(x;\lambda)/[\delta-\log(K(x;\lambda))]^2)s)$ and using $m(t)=e^{-t}$, exists in (1), we can obtain easilythe DF $F(x;\delta,\lambda)$ of the NFLog-X distributions, represented as below

$$F(x;\delta,\boldsymbol{\lambda}) = \frac{\delta^2K(x;\boldsymbol{\lambda})}{[\delta-\log(K(x;\boldsymbol{\lambda}))]^2}, \quad x\in\mathbb{R}, \tag{6}$$

with

$$f(x;\delta,\boldsymbol{\lambda}) = \frac{\delta^2k(x;\boldsymbol{\lambda})}{[\delta-\log(K(x;\boldsymbol{\lambda}))]^3}[2+\delta-\log(K(x;\boldsymbol{\lambda}))], \tag{7}$$

where $(d/dx)K(x;\lambda)=k(x;\lambda)$.

Related to equations (2) and (3), the SF (survival function) $S(x;\lambda)=1-K(x;\lambda)$, HF (hazard function) $h(x;\lambda)=(k(x;\lambda)/S(x;\lambda))$, and cumulative HF $K(x;\lambda)=-\log[1-K(x;\lambda)]$ are represented by the equations in the preceding:

$$\begin{aligned}S(x;\delta,\boldsymbol{\lambda}) &= 1-\frac{\delta^2K(x;\boldsymbol{\lambda})}{[\delta-\log(K(x;\boldsymbol{\lambda}))]^2},\\ h(x;\delta,\boldsymbol{\lambda}) &= \frac{\delta^2k(x;\boldsymbol{\lambda})[2+\delta-\log(K(x;\boldsymbol{\lambda}))]}{[\delta-\log(K(x;\boldsymbol{\lambda}))]^2-\delta^2K(x;\boldsymbol{\lambda})[\delta-\log(K(x;\boldsymbol{\lambda}))]},\end{aligned} \tag{8}$$

and

$$H(x;\delta,\boldsymbol{\lambda}) = -\log\left(1-\frac{\delta^2K(x;\boldsymbol{\lambda})}{[\delta-\log(K(x;\boldsymbol{\lambda}))]^2}\right), \tag{9}$$

on the same order.

In this article, we implement the NFLog-X distributions approach and introduce the NFLog-Wei distribution. Section 4 offers the expression of the DF, PDF, SF, HF, and CHF of the NFLog-Wei distribution.

3. The Identifiability Property

The identifiability property is a very useful statistical property that ensures precise inferences. Here, we derive the identifiability property of the NFLog-X distributions. Let δ_1 and δ_2 be the two parameters having DFs $F(x;\delta_1,\lambda)$ and $F(x;\delta_2,\lambda)$, respectively. The parameter δ is identifiable, if $\delta_1=\delta_2$. Mathematically, we have

$$F(x;\delta_1,\boldsymbol{\lambda}) = F(x;\delta_2,\boldsymbol{\lambda}). \tag{10}$$

Incorporating equation (2) in equation (4), we get

$$\frac{\delta_1^2 K(x;\boldsymbol{\lambda})}{[\delta_1 - \log(K(x;\boldsymbol{\lambda}))]^2} = \frac{\delta_2^2 K(x;\boldsymbol{\lambda})}{[\delta_2 - \log(K(x;\boldsymbol{\lambda}))]^2}. \tag{11}$$

Taking square root of equation (5), we get

$$\begin{aligned}
\frac{\delta_1\sqrt{K(x;\boldsymbol{\lambda})}}{[\delta_1 - \log(K(x;\boldsymbol{\lambda}))]} &= \frac{\delta_2\sqrt{K(x;\boldsymbol{\lambda})}}{[\delta_2 - \log(K(x;\boldsymbol{\lambda}))]},\\
\delta_1\sqrt{K(x;\boldsymbol{\lambda})}[\delta_2 - \log(K(x;\boldsymbol{\lambda}))] &= \delta_2\sqrt{K(x;\boldsymbol{\lambda})}[\delta_1 - \log(K(x;\boldsymbol{\lambda}))],\\
\delta_1\delta_2\sqrt{K(x;\boldsymbol{\lambda})} - \delta_1\sqrt{K(x;\boldsymbol{\lambda})}\log(K(x;\boldsymbol{\lambda})) &= \delta_1\delta_2\sqrt{K(x;\boldsymbol{\lambda})} - \delta_2\sqrt{K(x;\boldsymbol{\lambda})}\log(K(x;\boldsymbol{\lambda})),\\
-\delta_1\sqrt{K(x;\boldsymbol{\lambda})}\log(K(x;\boldsymbol{\lambda})) &= -\delta_2\sqrt{K(x;\boldsymbol{\lambda})}\log(K(x;\boldsymbol{\lambda})),\\
\delta_1\sqrt{K(x;\boldsymbol{\lambda})}\log(K(x;\boldsymbol{\lambda})) &= \delta_2\sqrt{K(x;\boldsymbol{\lambda})}\log(K(x;\boldsymbol{\lambda})),\\
\delta_1 &= \delta_2.
\end{aligned} \tag{12}$$

From equation (6), we can see that $\delta_1=\delta_2$. Therefore, the parameter δ is identifiable.

4. The NFLog-Wei Distribution

Consider the DF $K(x;\lambda)$ and PDF $k(x;\lambda)$ of the two parameters $(\alpha>0,\beta>0)$ traditional Wei model are given, respectively, by

$$K(x;\boldsymbol{\lambda}) = 1 - e^{-\beta x^{\alpha}}, \tag{13}$$

and

$$k(x;\boldsymbol{\lambda}) = \alpha\beta x^{\alpha-1}e^{-\beta x^{\alpha}}, \tag{14}$$

where $\lambda=(\alpha,\beta)$.

Using equation (8) in equation (2), we get the DF $F(x;\delta,\lambda)$ of the NFLog-Wei distribution given by

$$F(x;\delta,\boldsymbol{\lambda}) = \frac{\delta^2\left(1-e^{-\beta x^{\alpha}}\right)}{\left[\delta - \log\left(1-e^{-\beta x^{\alpha}}\right)\right]^2}, \tag{15}$$

with SF $S(x;\delta,\lambda)$

$$S(x;\delta,\boldsymbol{\lambda}) = 1 - \frac{\delta^2\left(1-e^{-\beta x^{\alpha}}\right)}{\left[\delta - \log\left(1-e^{-\beta x^{\alpha}}\right)\right]^2}. \tag{16}$$

Some plots of $F(x;\delta,\lambda)$ and $S(x;\delta,\lambda)$ of the NFLog-Wei model are provided in Figure 1. The plots of $F(x;\delta,\lambda)$ and $S(x;\delta,\lambda)$ are obtained for (i) $\delta=2.8,\beta=1.0,\alpha=1.2$ (red curve), (ii) $\delta=1.2,\beta=1.0,\alpha=1.8$ (green curve), and (iii) $\delta=0.8,\beta=0.5,\alpha=2.5$ (blue curve).

Corresponding to $F(x;\delta,\lambda)$ in equation (9), the PDF $f(x;\delta,\lambda)$ is as follows:

$$f(x;\delta,\boldsymbol{\lambda}) = \frac{\delta^2\alpha\beta x^{\alpha-1}e^{-\beta x^{\alpha}}}{\left[\delta - \log\left(1-e^{-\beta x^{\alpha}}\right)\right]^3}\left[2+\delta-\log\left(1-e^{-\beta x^{\alpha}}\right)\right], \quad x>0. \tag{17}$$

Different plots for the PDF $f(x;\delta,\lambda)$ of the NFLog-Wei model are shown in Figure 2. The plots of $f(x;\delta,\lambda)$ are obtained for (i) $\delta=1.4,\beta=1.0,\alpha=0.4$ (red curve), (ii) $\delta=7.3,\beta=1.2,\alpha=2.6$ (green curve), (iii) $\delta=0.1,\beta=0.3,\alpha=3.2$ (black curve), and (iv) $\delta=0.4,\beta=0.8,\alpha=2.4$ (blue curve).

From Figure 2, we can see that the PDF $f(x;\delta,\lambda)$ of the NFLog-Wei model has four different patterns, including (i) decreasing or reverse in the form of J-shaped but reversed shown in (red curve), (ii) left-skewed (green curve), (iii) right-skewed (black curve), and (iv) symmetrical (blue curve).

Furthermore, the HF $h(x;\delta,\lambda)$ and CHF $H(x;\delta,\lambda)$ of the NFLog-Wei distribution are given by the following equations:

$$h(x;\delta,\boldsymbol{\lambda}) = \frac{\delta^2\alpha\beta x^{\alpha-1}e^{-\beta x^{\alpha}}\left[2+\delta-\log\left(1-e^{-\beta x^{\alpha}}\right)\right]}{\left(\left[\delta - \log\left(1-e^{-\beta x^{\alpha}}\right)\right]^2 - \delta^2\left(1-e^{-\beta x^{\alpha}}\right)\right)\left[\delta - \log\left(1-e^{-\beta x^{\alpha}}\right)\right]}, \quad x>0, \tag{18}$$

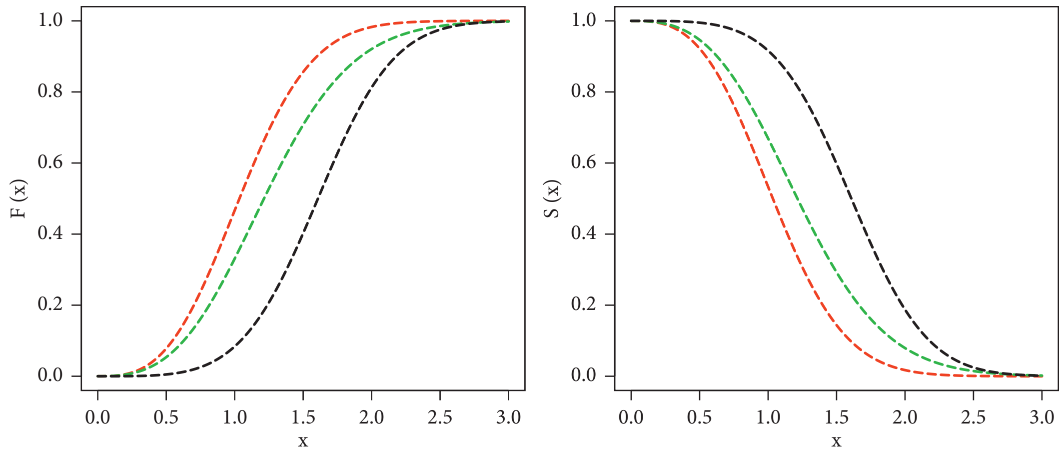

Figure 1: Graphical representation of $F(x;\delta,\lambda)$ and $S(x;\delta,\lambda)$.

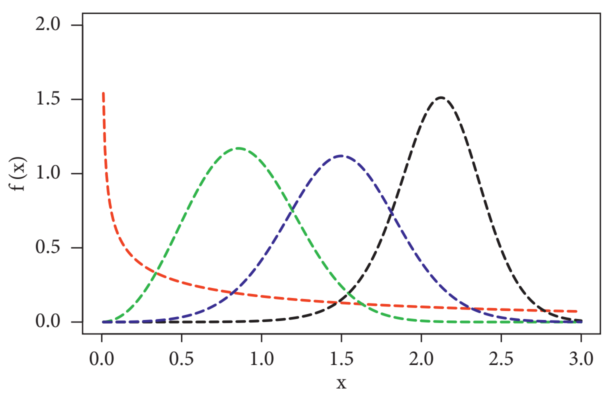

Figure 2: Visual display of $f(x;\delta,\lambda)$.

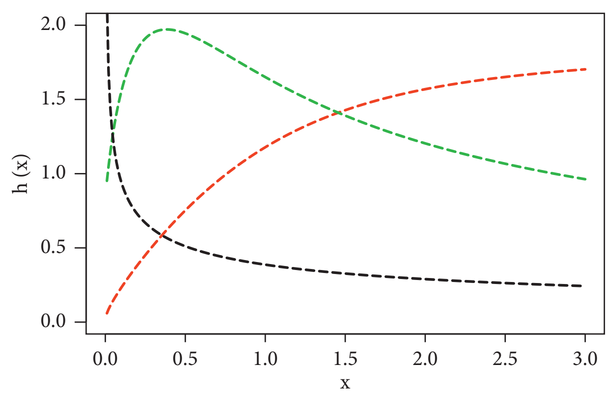

Figure 3: Visual display of $h(x;\delta,\lambda)$.

and

$$H(x;\delta,\boldsymbol{\lambda}) = -\log\left(1 - \frac{\delta^2\left(1-e^{-\beta x^{\alpha}}\right)}{\left[\delta - \log\left(1-e^{-\beta x^{\alpha}}\right)\right]^2}\right), \quad x>0, \tag{19}$$

respectively.

Different plots for the HF $h(x;\delta,\lambda)$ of the NFLog-Wei distribution are provided in Figure 3. The plots of $h(x;\delta,\lambda)$ are obtained for (i) $\delta=1.3, \beta=1.4, \alpha=1.1$ (red curve), (ii) $\delta=0.1, \beta=4.7, \alpha=0.4$ (green curve), and (iii) $\delta=4.1, \beta=0.9, \alpha=0.5$ (black curve).

From Figure 3, we can see that the HF $h(x;\delta,\lambda)$ of the NFLog-Wei distribution has three different patterns, including (i) increasing (red curve), (ii) unimodal (green curve), and (iii) reverse J-shaped (black curve).

Despite the prominent advantages of the NFLog-Wei distribution over the other distributions, the NFLog-Wei model has also disadvantages, for example

(i) The NFLog-Wei distribution is a continuous distribution used to evaluate continuous datasets. Consequently, the suggested NFLog-Wei distribution cannot be utilized to assess discrete data sets.

(ii) Because of the NFLog-Weidistribution PDF's complicated structure, the expressions of its estimators cannot be reduced to a simple, closed form easily represented. Therefore, the numerical estimates of the estimators can be obtained with the help of computer software.

(iii) Due to the complexity of the PDF of the NFLog-Wei distribution, additional computing work is necessary to determine its mathematical features.

5. Estimation and Simulation

Here, we obtain the MLEs $(\widehat{\delta}_{MLE}, \widehat{\lambda}_{MLE})$ of the NFLog-X distributions. In addition, we do provide a comprehensive Monte–Carlo simulation study (MCSS) for assessing the performances of $\widehat{\delta}_{MLE}$ and $\widehat{\lambda}_{MLE}$.

5.1. Estimation. In the research that has been conducted on the topic, a number of different strategies and procedures for estimating the parameters of probability models have been proposed and put into practice. Among them, the MLE is one of the most usually adopted methods. Here, we implement this method to obtain the $\widehat{\delta}_{MLE}$ and $\widehat{\lambda}_{MLE}$.

Let $x_1, x_2, \ldots, x_n$ be a set of observed values of size n taken from the PDF $f(x;\delta,\lambda)$. Then, the corresponding likelihood function (LF) $\varphi(x;\delta,\lambda)$ is obtained as follows:

$$\varphi(x;\delta,\boldsymbol{\lambda}) = \prod_{i=1}^{n} \frac{\delta^2 k(x_i;\boldsymbol{\lambda})}{[\delta - \log(K(x_i;\boldsymbol{\lambda}))]^3}[2 + \delta - \log(K(x_i;\boldsymbol{\lambda}))]. \quad (20)$$

Corresponding to $\varphi(x;\delta,\lambda)$, the log LF $\ell(\Phi)$ is obtained as follows:

$$\begin{aligned}\ell(\boldsymbol{\Phi}) &= 2n \log \delta + \sum_{i=1}^{n} \log k(x_i;\boldsymbol{\lambda}) \\ &+ \sum_{i=1}^{n} \log[2 + \delta - \log(K(x_i;\boldsymbol{\lambda}))] \\ &- 3\sum_{i=1}^{n} \log[\delta - \log(K(x_i;\boldsymbol{\lambda}))].\end{aligned} \quad (21)$$

Using $\ell(\Phi)$, the following equation will provide the partial derivatives:

$$\begin{aligned}\frac{\partial}{\partial\delta}\ell(\boldsymbol{\Phi}) &= \frac{2n}{\delta} + \sum_{i=1}^{n} \frac{1}{[2 + \delta - \log(K(x_i;\boldsymbol{\lambda}))]} \\ &- 3\sum_{i=1}^{n} \frac{1}{[\delta - \log(K(x_i;\boldsymbol{\lambda}))]},\end{aligned} \quad (22)$$

and

$$\begin{aligned}\frac{\partial}{\partial\boldsymbol{\lambda}}\ell(\boldsymbol{\Phi}) &= \sum_{i=1}^{n} \frac{\partial/\partial\boldsymbol{\lambda} k(x_i;\boldsymbol{\lambda})}{k(x_i;\boldsymbol{\lambda})} - \sum_{i=1}^{n} \frac{[K(x_i;\boldsymbol{\lambda})]^{-1}\partial/\partial\boldsymbol{\lambda} K(x_i;\boldsymbol{\lambda})}{[2 + \delta - \log(K(x_i;\boldsymbol{\lambda}))]} \\ &+ 3\sum_{i=1}^{n} \frac{[K(x_i;\boldsymbol{\lambda})]^{-1}\partial/\partial\boldsymbol{\lambda} K(x_i;\boldsymbol{\lambda})}{[\delta - \log(K(x_i;\boldsymbol{\lambda}))]},\end{aligned} \quad (23)$$

respectively, where $\Phi = (\delta, \lambda)$.

On solving $(\partial/\partial\delta)\ell(\Phi) = 0$ and $\partial/\partial\lambda\ell(\Phi) = 0$, we obtain $\widehat{\delta}_{MLE}$ and $\widehat{\lambda}_{MLE}$, respectively.

5.2. Simulation. In this second subsection, a comprehensive MCSS is conducted to assess the behaviors of $\widehat{\delta}_{MLE}$ and $\widehat{\lambda}_{MLE}$ of the NFLog-Wei distribution. The RNs (random numbers) are successfully generated from the PDF $f(x;\delta,\lambda)$ via the inverse DF method. The outcome of the simulation are acquired for a total of four groups and sets (Set I, Set II, Set III, and Set IV) of parameters values, given by Set I: $\alpha = 0.6$, $\beta = 2.3$, and $\delta = 3.5$, Set II: $\alpha = 1.3$, $\beta = 3.8$, and $\delta = 4.6$, Set III: $\alpha = 3.0$, $\beta = 4.0$, and $\delta = 4.5$, and Set IV: $\alpha = 3.4$, $\beta = 2.5$, and $\delta = 3.5$.

To check performances of $\widehat{\delta}_{MLE}$ and $\widehat{\lambda}_{MLE}$, two statistical measures are considered. These measures include the (i) mean square error (MSE) and (ii) bias. The numerical values of the MSE and bias are, respectively, computed as follows:

$$\frac{1}{n}\sum_{i=1}^{n} (\widehat{\delta} - \delta)^2, \quad (24)$$

and

$$\frac{1}{n}\sum_{i=1}^{n} (\widehat{\delta} - \delta). \quad (25)$$

The values of the MSE and bias are also computed for λ.

Corresponding to Set I: $\alpha = 0.6$, $\beta = 2.3$, and $\delta = 3.5$ and Set II: $\alpha = 1.3$, $\beta = 3.8$, and $\delta = 4.6$, we can easily see the outcomes resulted from doing the simulation in Table 1. Whereas, in link to Set III: $\alpha = 3.0$, $\beta = 4.0$, $\delta = 4.5$ and Set IV: $\alpha = 3.4$, $\beta = 2.5$, $\delta = 3.5$, the outcomes of the simulation are shown in Table 2.

Based on the findings of the simulation, which are shown in Tables 1 and 2, we can see that as the size of n increases.

(i) The estimated values of $\widehat{\lambda}_{MLE}$ and $\widehat{\delta}_{MLE}$ tend to be stable.

(ii) The MSEs of $\widehat{\lambda}_{MLE}$ and $\widehat{\delta}_{MLE}$ decrease.

(iii) The biases of $\widehat{\lambda}_{MLE}$ and $\widehat{\delta}_{MLE}$ decay to zero.

6. Applications

By doing an analysis on three different biomedical datasets, the purpose of this article is to demonstrate the utility of the NFLog-Wei distribution (Table 3).

We compare the NFLog-Wei distribution with the Wei model and three other traditional and new probability distributions, as an example of these distributions, the Marshall–Olkin Weibull (MO-Wei) studied by Marshall and Olkin [25], APT-Wei (alpha power transformed Weibull) proposed by Dey et al. [26], and a flexible reduced logarithmic-Weibull (FRLog-Wei) distribution, introduced by Liu et al. [7]. The DFs of the competitive probability distributions are outlined below:

(i) The APT-Wei distribution is obtained by the following equation:

$$K(x;\alpha_1,\boldsymbol{\lambda}) = \frac{\alpha_1^{\left(1-e^{-\beta x^\alpha}\right)} - 1}{\alpha_1 - 1}, \quad (26)$$

where $\alpha_1 \neq 1, \alpha_1 > 0$.

(ii) The MO-Wei distribution is represented by the equation that is as follows:

$$K(x;\gamma,\boldsymbol{\lambda}) = \frac{\left(1 - e^{-\beta x^\alpha}\right)}{\gamma + (1-\gamma)\left(1 - e^{-\beta x^\alpha}\right)}, \quad (27)$$

where $\gamma > 0$.

(iii) The FRLog-Wei distribution is represented by the equation that is as follows:

$$K(x;\sigma,\boldsymbol{\lambda}) = 1 - \frac{\log\left(1 + \sigma - \sigma\left(1 - e^{-\beta x^\alpha}\right)\right)}{\log(1+\sigma)}, \quad (28)$$

where $\sigma > 0$.

To determine the optimum model among the fitted distributions, we consider different goodness-of-fit measures (analytical measures). It is generally agreed that

Table 1: The results of conducting a simulation using the NFLog-Wei distribution utilizing sets I and II.

n	Est.	Set I: $\alpha = 0.6$, $\beta = 2.3$, and $\delta = 3.5$			Set II: $\alpha = 1.3$, $\beta = 3.8$, and $\delta = 4.6$		
		MLE	MSE	Bias	MLE	MSE	Bias
25	$\hat{\alpha}$	1.359714	3.352142	0.959714	1.470587	1.916907	0.070586
	$\hat{\beta}$	2.558740	2.272587	0.258740	4.107370	1.019913	0.507369
	$\hat{\delta}$	3.932950	1.830509	0.432950	4.536813	0.659791	−0.263187
50	$\hat{\alpha}$	1.261625	2.610044	0.861625	1.324459	1.355837	−0.075540
	$\hat{\beta}$	2.527184	2.103823	0.227184	4.074657	0.827990	0.474657
	$\hat{\delta}$	3.854091	1.742594	0.354091	4.428739	0.809154	−0.371261
75	$\hat{\alpha}$	1.249921	2.380179	0.849921	1.304567	1.105549	−0.095432
	$\hat{\beta}$	2.373155	1.660890	0.073155	4.027904	0.720244	0.427903
	$\hat{\delta}$	3.903062	1.633956	0.403062	4.434167	0.722466	−0.365833
100	$\hat{\alpha}$	1.121601	1.880341	0.721601	1.311440	1.055909	−0.088560
	$\hat{\beta}$	2.409936	1.568806	0.109935	3.990797	0.645796	0.390796
	$\hat{\delta}$	3.827443	1.488893	0.327443	4.464805	0.678206	−0.335194
200	$\hat{\alpha}$	0.937715	1.208062	0.537715	1.268340	0.643048	−0.131660
	$\hat{\beta}$	2.367964	1.106299	0.067963	3.910828	0.469170	0.310828
	$\hat{\delta}$	3.755479	1.181334	0.255478	4.493781	0.563858	−0.306218
300	$\hat{\alpha}$	0.846743	0.925916	0.446743	1.229159	0.517821	−0.170841
	$\hat{\beta}$	2.330760	0.853094	0.030759	3.896567	0.414911	0.296567
	$\hat{\delta}$	3.718240	0.963105	0.218240	4.481787	0.556714	−0.318213
400	$\hat{\alpha}$	0.820652	0.786768	0.420652	1.249808	0.489246	−0.150192
	$\hat{\beta}$	2.268908	0.698790	−0.031091	3.869472	0.370287	0.269471
	$\hat{\delta}$	3.744079	0.849120	0.244079	4.507406	0.504602	−0.292593
500	$\hat{\alpha}$	0.726767	0.635959	0.326767	1.240931	0.413434	−0.159069
	$\hat{\beta}$	2.321649	0.575961	0.021649	3.834327	0.293431	0.234327
	$\hat{\delta}$	3.652565	0.701084	0.152564	4.531517	0.424718	−0.268482
600	$\hat{\alpha}$	0.666051	0.491945	0.266051	1.280216	0.402240	−0.119784
	$\hat{\beta}$	2.309838	0.453986	0.009838	3.803557	0.258676	0.203557
	$\hat{\delta}$	3.629989	0.587730	0.129989	4.559654	0.384078	−0.240346
700	$\hat{\alpha}$	0.701362	0.504666	0.301362	1.299725	0.378063	−0.100275
	$\hat{\beta}$	2.256093	0.430943	−0.043907	3.790054	0.250775	0.190053
	$\hat{\delta}$	3.694393	0.588829	0.194392	4.590190	0.345656	−0.209809
800	$\hat{\alpha}$	0.642816	0.378790	0.242816	1.270926	0.336466	−0.129073
	$\hat{\beta}$	2.255541	0.350091	−0.044459	3.789066	0.208206	0.189066
	$\hat{\delta}$	3.658857	0.464875	0.158856	4.586876	0.324176	−0.213124
900	$\hat{\alpha}$	0.652121	0.396530	0.252121	1.307186	0.327404	−0.092814
	$\hat{\beta}$	2.257956	0.353270	−0.042043	3.755260	0.186083	0.155260
	$\hat{\delta}$	3.668265	0.508969	0.168265	4.618437	0.274431	−0.181562
1000	$\hat{\alpha}$	0.597530	0.304498	0.197530	1.296956	0.316957	−0.123043
	$\hat{\beta}$	2.292465	0.316204	−0.017534	3.791939	0.188118	0.171938
	$\hat{\delta}$	3.518328	0.415432	0.118328	4.600692	0.302786	−0.199307

the best competing model is the probability model that achieves the lowest values of these analytical metrics. The numerical values of the analytical measure are computed as follows:

(i) The CM test statistic is computed as follows:

$$\sum_{i=1}^{n}\left(\frac{2i-1}{2n}-K\left(x_i;\boldsymbol{\lambda}\right)\right)^2+\frac{1}{12n}. \tag{29}$$

(ii) The AD test statistic is calculated as follows:

$$-n-\frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\log K\left(x_i;\boldsymbol{\lambda}\right)+\log\left(1-K\left(x_i;\boldsymbol{\lambda}\right)\right)\right]. \tag{30}$$

(iii) The KS test statistic is derived as follows:

$$\sup_x\left[K_n\left(x;\boldsymbol{\lambda}\right)-K\left(x;\boldsymbol{\lambda}\right)\right]. \tag{31}$$

(iv) The AIC is obtained as follows:

$$2m-2\ell(\boldsymbol{\Phi}). \tag{32}$$

(v) The BIC is calculated as follows:

$$m\log(n)-2\ell(\boldsymbol{\Phi}). \tag{33}$$

(vi) The HQIC is obtained as follows:

Table 2: The results of conducting a simulation using the NFLog-Wei distribution with sets III and IV.

n	Est.	Set III: $\alpha = 3.0$, $\beta = 4.0$, and $\delta = 4.5$			Set IV: $\alpha = 3.4$, $\beta = 2.5$, and $\delta = 3.5$		
		MLE	MSE	Bias	MLE	MSE	Bias
25	$\widehat{\alpha}$	2.381875	2.794807	−0.118124	3.410283	4.557224	−0.489717
	$\widehat{\beta}$	4.296184	0.720145	0.396183	3.035897	1.458385	0.6358970
	$\widehat{\delta}$	4.518220	0.628748	−0.281780	3.427576	0.913335	−0.172423
50	$\widehat{\alpha}$	2.319224	2.574264	−0.180776	3.413811	4.507704	−0.486189
	$\widehat{\beta}$	4.275190	0.585435	0.375189	2.935012	1.212232	0.535011
	$\widehat{\delta}$	4.516319	0.584158	−0.283680	3.348358	0.754218	−0.251642
75	$\widehat{\alpha}$	2.343670	2.636672	−0.156330	3.337658	4.571121	−0.562342
	$\widehat{\beta}$	4.249329	0.527632	0.3493285	2.898243	1.057149	0.498242
	$\widehat{\delta}$	4.486865	0.607032	−0.313134	3.294626	0.669432	−0.305374
100	$\widehat{\alpha}$	2.337194	2.565672	−0.162806	3.206369	4.830113	−0.693630
	$\widehat{\beta}$	4.208815	0.470883	0.308815	2.913587	1.014642	0.513587
	$\widehat{\delta}$	4.486939	0.573087	−0.313060	3.261691	0.674156	−0.338309
200	$\widehat{\alpha}$	2.345233	2.217673	−0.154767	3.317303	4.219805	−0.582696
	$\widehat{\beta}$	4.155143	0.343410	0.255143	2.768109	0.635353	0.368108
	$\widehat{\delta}$	4.498184	0.533703	−0.301816	3.319766	0.477188	−0.280234
300	$\widehat{\alpha}$	2.256714	2.014241	−0.243285	3.294189	3.997942	−0.605811
	$\widehat{\beta}$	4.173869	0.327524	0.273868	2.718230	0.445176	0.318229
	$\widehat{\delta}$	4.490608	0.520666	−0.309391	3.331029	0.382241	−0.268971
400	$\widehat{\alpha}$	2.296481	1.781173	−0.203519	3.377208	3.613586	−0.522792
	$\widehat{\beta}$	4.124638	0.263154	0.224638	2.656047	0.327071	0.256046
	$\widehat{\delta}$	4.531144	0.433862	−0.268856	3.369713	0.300703	−0.230287
500	$\widehat{\alpha}$	2.279038	1.641563	−0.220962	3.427285	3.272158	−0.472715
	$\widehat{\beta}$	4.109448	0.235986	0.209448	2.610146	0.231249	0.210146
	$\widehat{\delta}$	4.541954	0.404543	−0.258045	3.406040	0.236117	−0.193959
600	$\widehat{\alpha}$	2.290175	1.499904	−0.209825	3.427226	3.213429	−0.472773
	$\widehat{\beta}$	4.081241	0.204391	0.181240	2.600575	0.209351	0.200574
	$\widehat{\delta}$	4.554415	0.362888	−0.245585	3.412186	0.215645	−0.187813
700	$\widehat{\alpha}$	2.326864	1.380761	−0.173135	3.430730	3.088489	−0.469269
	$\widehat{\beta}$	4.066115	0.175150	0.166115	2.584588	0.177430	0.184588
	$\widehat{\delta}$	4.590552	0.327957	−0.209447	3.421499	0.197229	−0.178501
800	$\widehat{\alpha}$	2.323243	1.338615	−0.176757	3.447615	3.004778	−0.452385
	$\widehat{\beta}$	4.058958	0.160689	0.158958	2.578249	0.163990	0.178248
	$\widehat{\delta}$	4.593206	0.306266	−0.206794	3.427515	0.180315	−0.172485
900	$\widehat{\alpha}$	2.332042	1.424302	−0.167957	3.597858	2.660379	−0.302141
	$\widehat{\beta}$	4.059840	0.169700	0.159839	2.541229	0.131273	0.141229
	$\widehat{\delta}$	4.583009	0.321603	−0.216991	3.462147	0.151573	−0.137853
1000	$\widehat{\alpha}$	2.296988	0.802130	−0.203012	3.415560	1.019690	−0.444440
	$\widehat{\beta}$	4.054148	0.095211	0.154147	2.512916	0.084999	0.092915
	$\widehat{\delta}$	4.519912	0.088924	−0.210088	3.490555	0.089363	−0.159445

Table 3: The medical datasets.

No.	Observations of the datasets	References
Data 1	12.20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36, 63.47, 68.46, 78.26, 74.47, 81.43, 84, 92, 94, 110, 112, 119, 127, 130, 133, 140, 146, 155, 159, 173, 179, 194, 195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725, 817, 1776	Ceren et al. [29]
Data 2	10, 33, 44, 56, 59, 72, 74, 77, 92, 93, 96, 100, 100, 102, 105, 107, 107, 108, 108, 108, 109, 112, 113, 115, 116, 120, 121, 122, 122, 124, 130, 134, 136, 139, 144, 146, 153, 159, 160, 163, 163, 168, 171, 172, 176, 183, 195, 196, 197, 202, 213, 215, 216, 222, 230, 231, 240, 245, 251, 253, 254, 255, 278, 293, 327, 342, 347, 361, 402, 432, 458, 555	Bjerkedal [28]
Data 3	0.080, 0.200, 0.400, 0.500, 0.510, 0.810, 0.900, 1.050, 1.190, 1.260, 1.350, 1.400, 1.460, 1.760, 2.020, 2.020, 2.070, 2.090, 2.230, 2.260, 2.460, 2.540, 2.620, 2.640, 2.690, 2.690, 2.750, 2.830, 2.870, 3.020, 3.250, 3.310, 3.360, 3.360, 3.480, 3.520, 3.570, 3.640, 3.700, 3.820, 3.880, 4.180, 4.230, 4.260, 4.330, 4.340, 4.400, 4.500, 4.510, 4.870, 4.980, 5.060, 5.090, 5.170, 5.320, 5.320, 5.340, 5.410, 5.410, 5.490, 5.620, 5.710, 5.850, 6.250, 6.540, 6.760, 6.930, 6.940, 6.970, 7.090, 7.260, 7.280, 7.320, 7.390, 7.590, 7.620, 7.630, 7.660, 7.870, 7.930, 8.260, 8.370, 8.530, 8.650, 8.660, 9.020, 9.220, 9.470, 9.740, 10.06, 10.34, 10.66, 10.75, 11.25, 11.64, 11.79, 11.98, 12.02, 12.03, 12.07, 12.63, 13.11, 13.29, 13.80, 14.24, 14.76, 14.77, 14.83, 15.96, 16.62, 17.12, 17.14, 17.36, 18.10, 19.13, 20.28, 21.73, 22.69, 23.63, 25.74, 25.82, 26.31, 32.15, 34.26, 36.66, 43.01, 46.12, 79.05	Aldeni et al. [27]

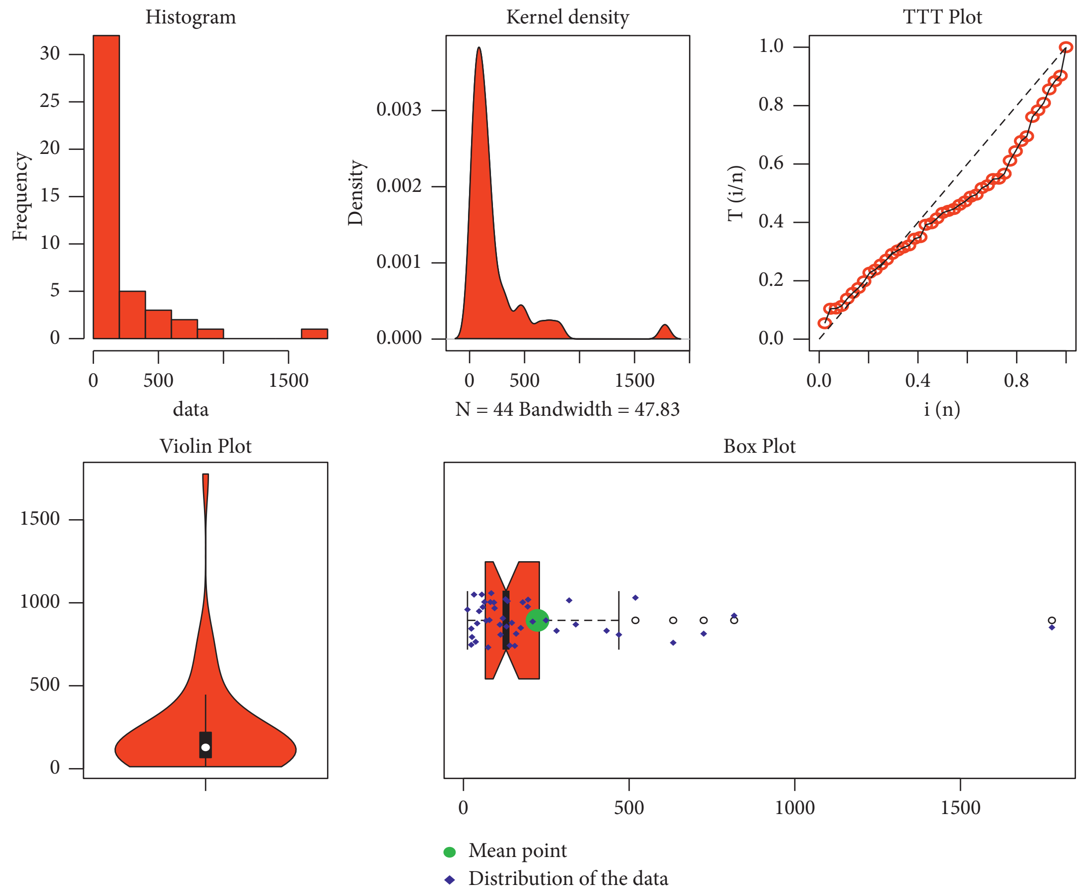

Figure 4: Basic plots of Data 1.

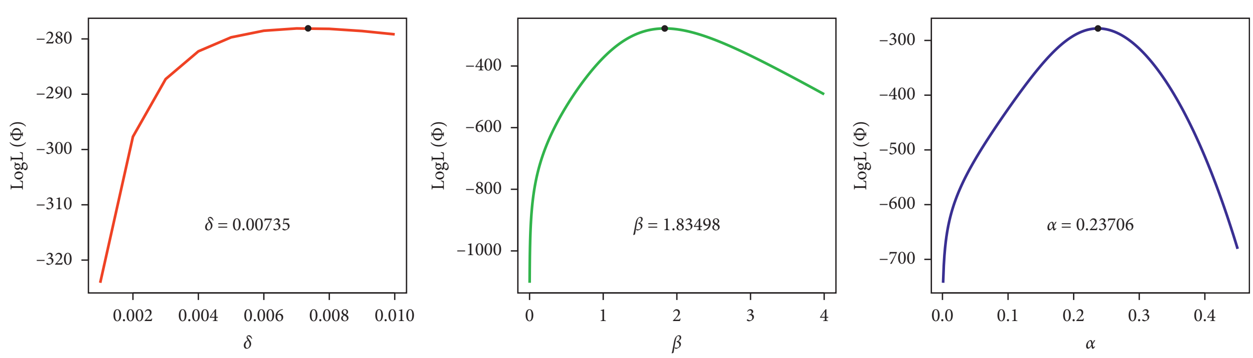

Figure 5: The profiles of the log LF of the NFLog-Wei distribution using Data 1.

Table 4: The values of $\widehat{\alpha}_{MLE}, \widehat{\beta}_{MLE}, \widehat{\delta}_{MLE}, \widehat{\alpha}_{1MLE}, \widehat{\gamma}_{MLE}$, and $\widehat{\sigma}_{MLE}$ of the fitted models using Data 1.

Models	$\widehat{\alpha}_{MLE}$	$\widehat{\beta}_{MLE}$	$\widehat{\delta}_{MLE}$	$\widehat{\alpha}_{1MLE}$	$\widehat{\gamma}_{MLE}$	$\widehat{\sigma}_{MLE}$
NFLog-Wei	1.83498	0.23706	0.00735	—	—	—
APT-Wei	0.99270	0.00326	—	0.24503	—	—
FRLog-Wei	0.76184	0.02859	—	—	—	5.72175
Weibull	0.93131	0.00677	—	—	—	—
MO-Wei	2.41759	0.00303	—	—	0.50752	—

Table 5: The values of CM, AD, KS, and P value of the fitted models for Data 1.

Models	CM	AD	KS	P value
NFLog-Wei	0.02815	0.18922	0.06204	0.9919
APT-Wei	0.09338	0.55387	0.10551	0.6723
FRLog-Wei	0.19103	1.09553	0.13355	0.3789
Weibull	0.13983	0.81427	0.12612	0.4494
MO-Wei	0.09492	0.56181	0.11255	0.5933

Table 6: The values of AIC, BIC, CAIC, and HQIC of the fitted models for Data 1.

Models	AIC	BIC	CAIC	HQIC
NFLog-Wei	562.1700	567.5226	562.7700	564.1550
APT-Wei	567.7712	573.1238	568.3712	569.7562
FRLog-Wei	572.8833	578.2359	573.4833	574.8683
Weibull	567.6941	571.2625	567.9868	569.0175
MO-Wei	568.2084	573.5610	568.8084	570.1934

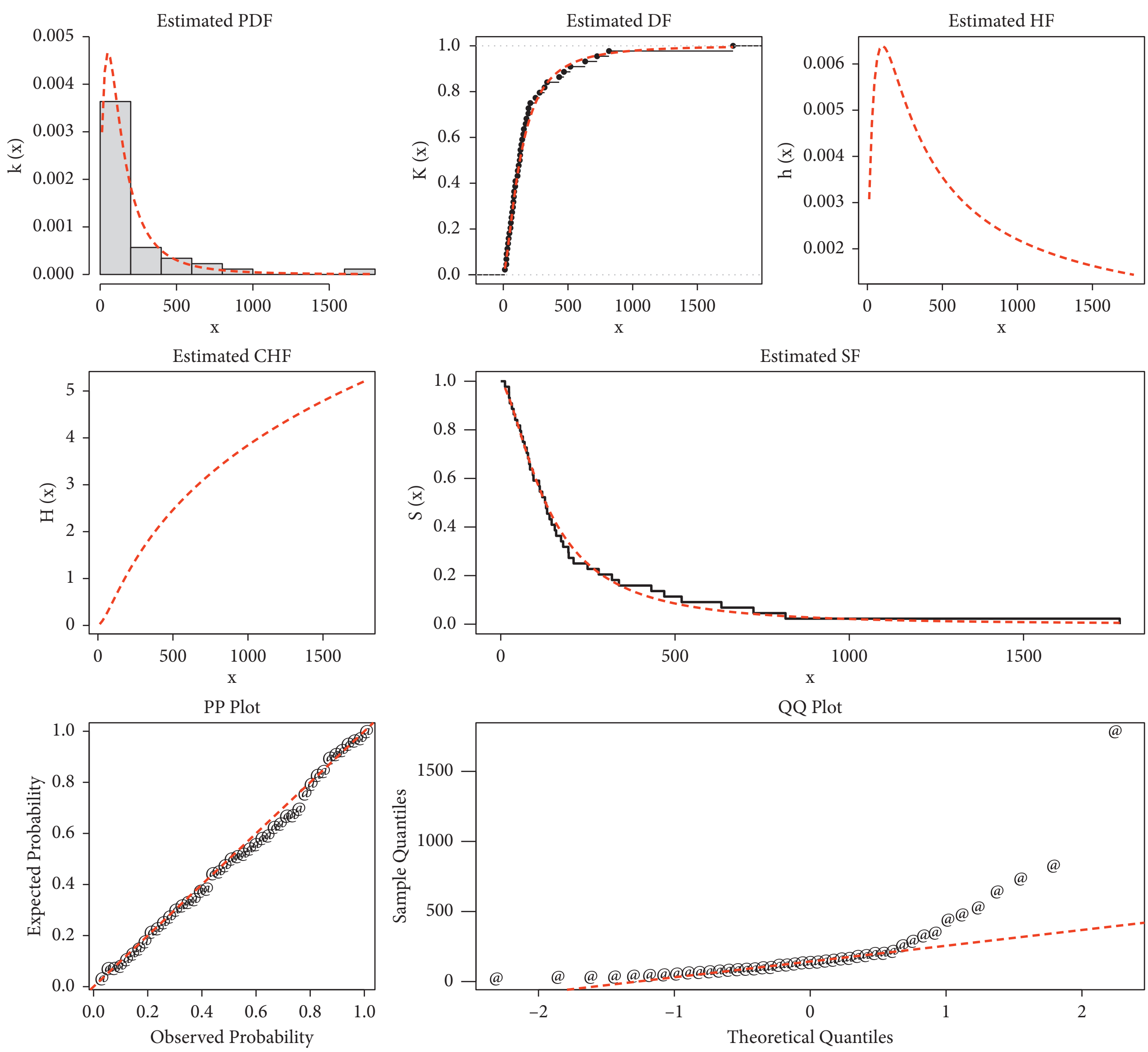

Figure 6: Visual illustration of the NFLog-Wei model using Data 1.

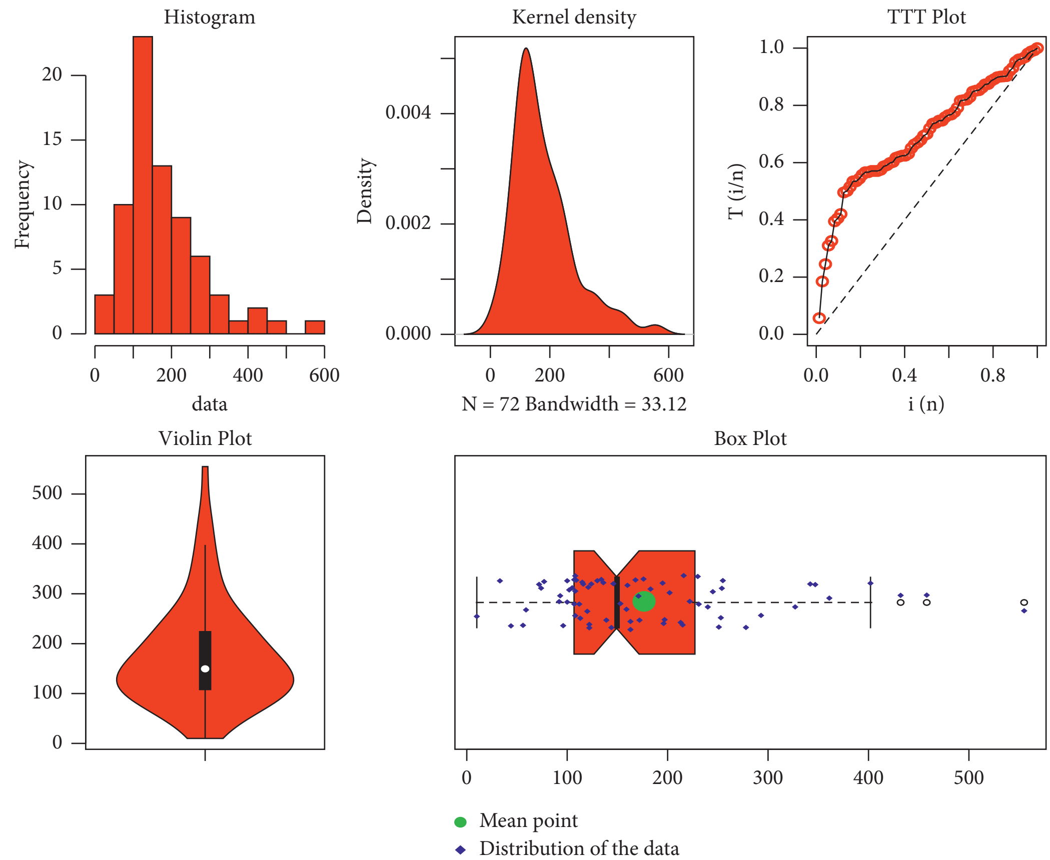

Figure 7: Basic plots of Data 2.

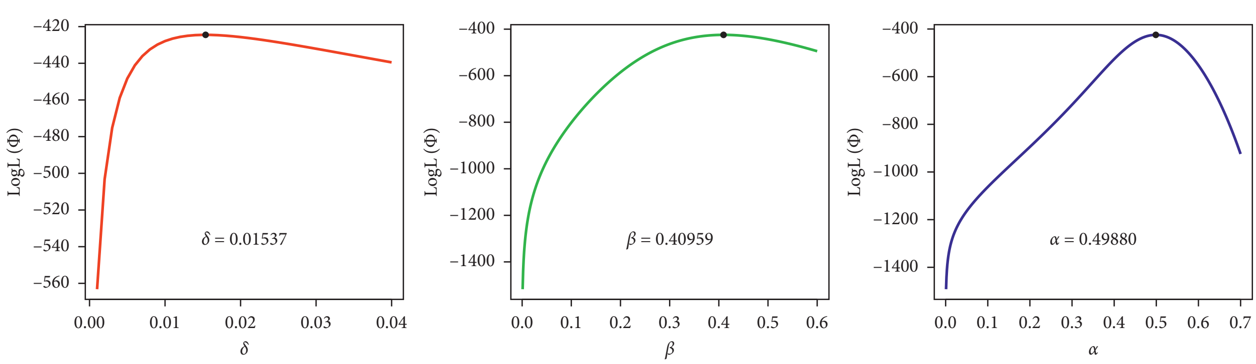

Figure 8: The profiles of the log LF of the NFLog-Wei distribution using Data 2.

Table 7: The values of $\widehat{\alpha}_{MLE}, \widehat{\beta}_{MLE}, \widehat{\delta}_{MLE}, \widehat{\alpha}_{1MLE}, \widehat{\gamma}_{MLE}$, and $\widehat{\sigma}_{MLE}$ of the fitted models using Data 2.

Models	$\widehat{\alpha}_{MLE}$	$\widehat{\beta}_{MLE}$	$\widehat{\delta}_{MLE}$	$\widehat{\alpha}_{1MLE}$	$\widehat{\gamma}_{MLE}$	$\widehat{\sigma}_{MLE}$
NFLog-Wei	0.01537	0.40959	0.49880	—	—	—
APT-Wei	1.19003	0.00325	—	6.81765	—	—
FRLog-Wei	1.20746	0.00354	—	—	—	11.84593
Weibull	1.19177	0.00208	—	—	—	—
MO-Wei	1.24252	0.00240	—	—	2.37148	—

Table 8: The values of CM, AD, KS, and P value of the fitted models for Data 2.

Models	CM	AD	KS	P value
NFLog-Wei	0.06315	0.37584	0.07504	0.81220
APT-Wei	0.13802	0.80373	0.17147	0.20986
FRLog-Wei	0.20630	1.20443	0.17744	0.19768
Weibull	0.10908	0.65947	0.25498	0.17134
MO-Wei	0.16136	0.94136	0.17256	0.20469

Table 9: The values of AIC, BIC, CAIC, and HQIC of the fitted probability models for Data 2.

Models	AIC	BIC	CAIC	HQIC
NFLog-Wei	854.9775	861.8075	855.3304	857.6965
APT-Wei	864.1267	870.9567	864.4797	866.8458
FRLog-Wei	868.2720	875.1020	868.6249	870.9910
Weibull	877.7535	882.3068	877.9274	879.5662
MO-Wei	865.7111	872.5411	866.0641	868.4302

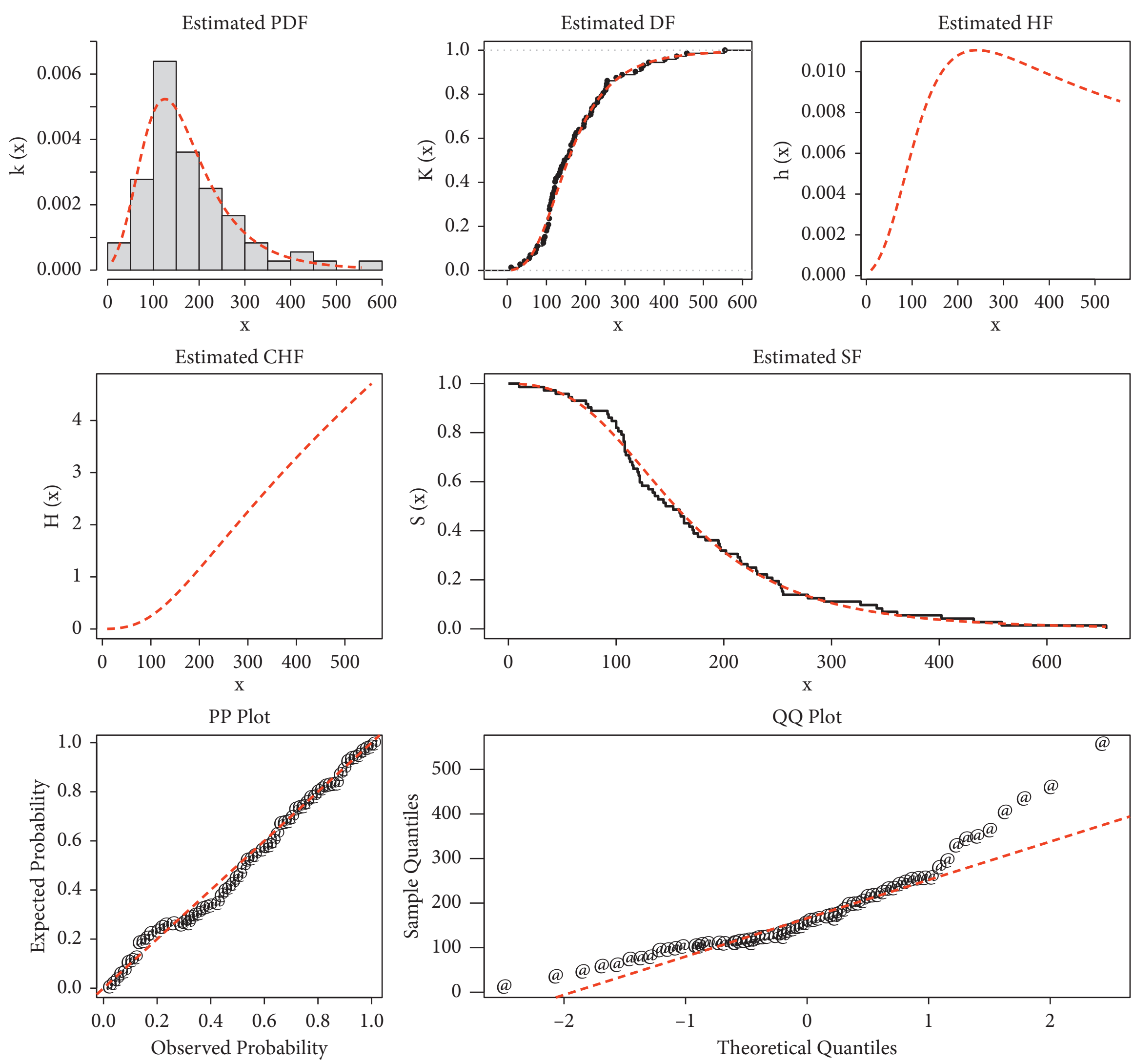

Figure 9: Visual illustration of the NFLog-Wei distribution using Data 2.

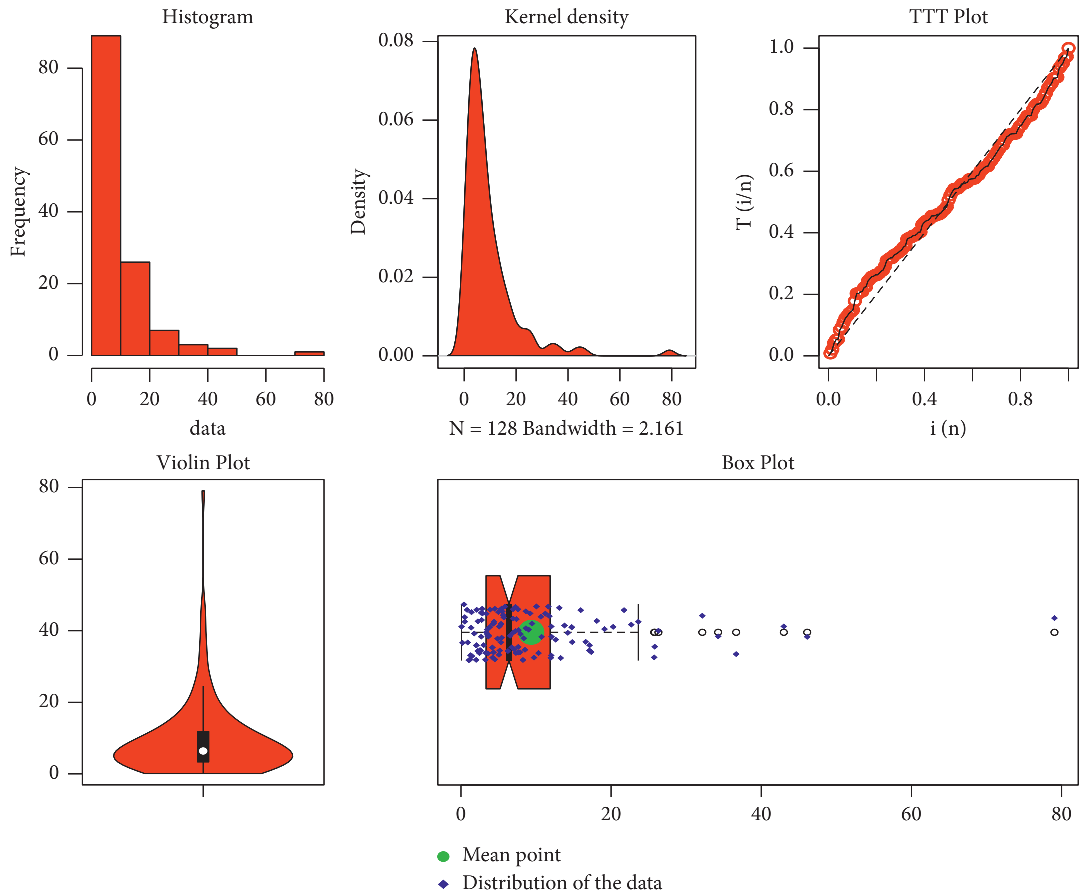

Figure 10: Basic plots of Data 3.

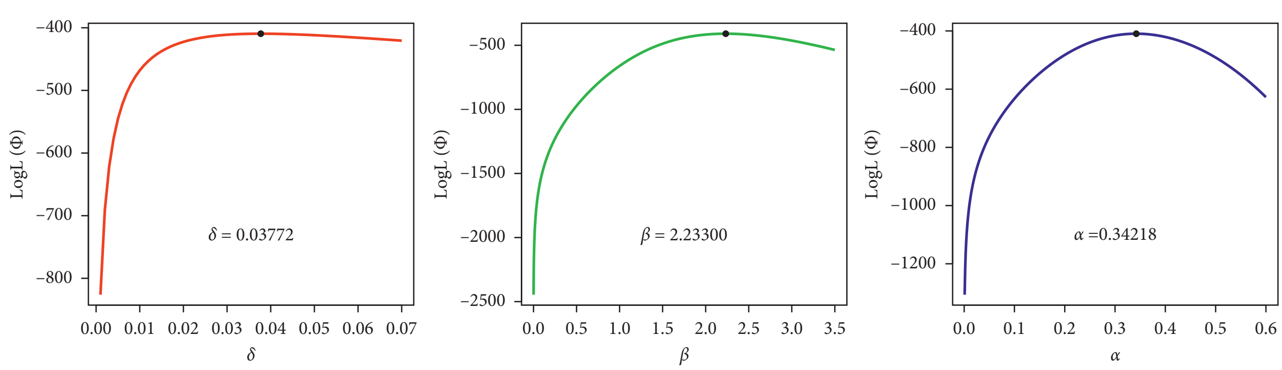

Figure 11: The profiles of the log LF of the NFLog-Wei distribution using Data 3.

Table 10: The values of $\widehat{\alpha}_{MLE}, \widehat{\beta}_{MLE}, \widehat{\delta}_{MLE}, \widehat{\alpha}_{1MLE}, \widehat{\gamma}_{MLE}$, and $\widehat{\sigma}_{MLE}$ of the fitted models using Data 3.

Models	$\widehat{\alpha}_{MLE}$	$\widehat{\beta}_{MLE}$	$\widehat{\delta}_{MLE}$	$\widehat{\alpha}_{1MLE}$	$\widehat{\gamma}_{MLE}$	$\widehat{\sigma}_{MLE}$
NFLog-Wei	0.03770	2.23388	0.34211	—	—	—
APT-Wei	1.26652	0.01661	—	0.01490	—	—
FRLog-Wei	1.27348	0.03210	—	—	—	0.81243
Weibull	1.04777	0.09390	—	—	—	—
MO-Wei	1.50855	0.00673	—	—	0.11186	—

Table 11: The values of CM, AD, KS, and P value of the fitted models for Data 3.

Models	CM	AD	KS	P value
NFLog-Wei	0.01562	0.10017	0.03054	0.9998
APT-Wei	0.04227	0.25411	0.04660	0.9437
FRLog-Wei	0.09868	0.61203	0.06859	0.5836
Weibull	0.13135	0.78639	0.06999	0.5573
MO-Wei	0.03150	0.22134	0.04008	0.9863

Table 12: The values of AIC, BIC, CAIC, and HQIC of the fitted models for Data 3.

Models	AIC	BIC	CAIC	HQIC
NFLog-Wei	824.9405	833.4966	825.1340	828.4169
APT-Wei	826.3801	834.9362	826.5737	829.8565
FRLog-Wei	832.5411	841.0972	832.7346	836.0175
Weibull	832.1738	837.8778	832.2698	834.4913
MO-Wei	826.5740	835.1301	826.7675	830.0504

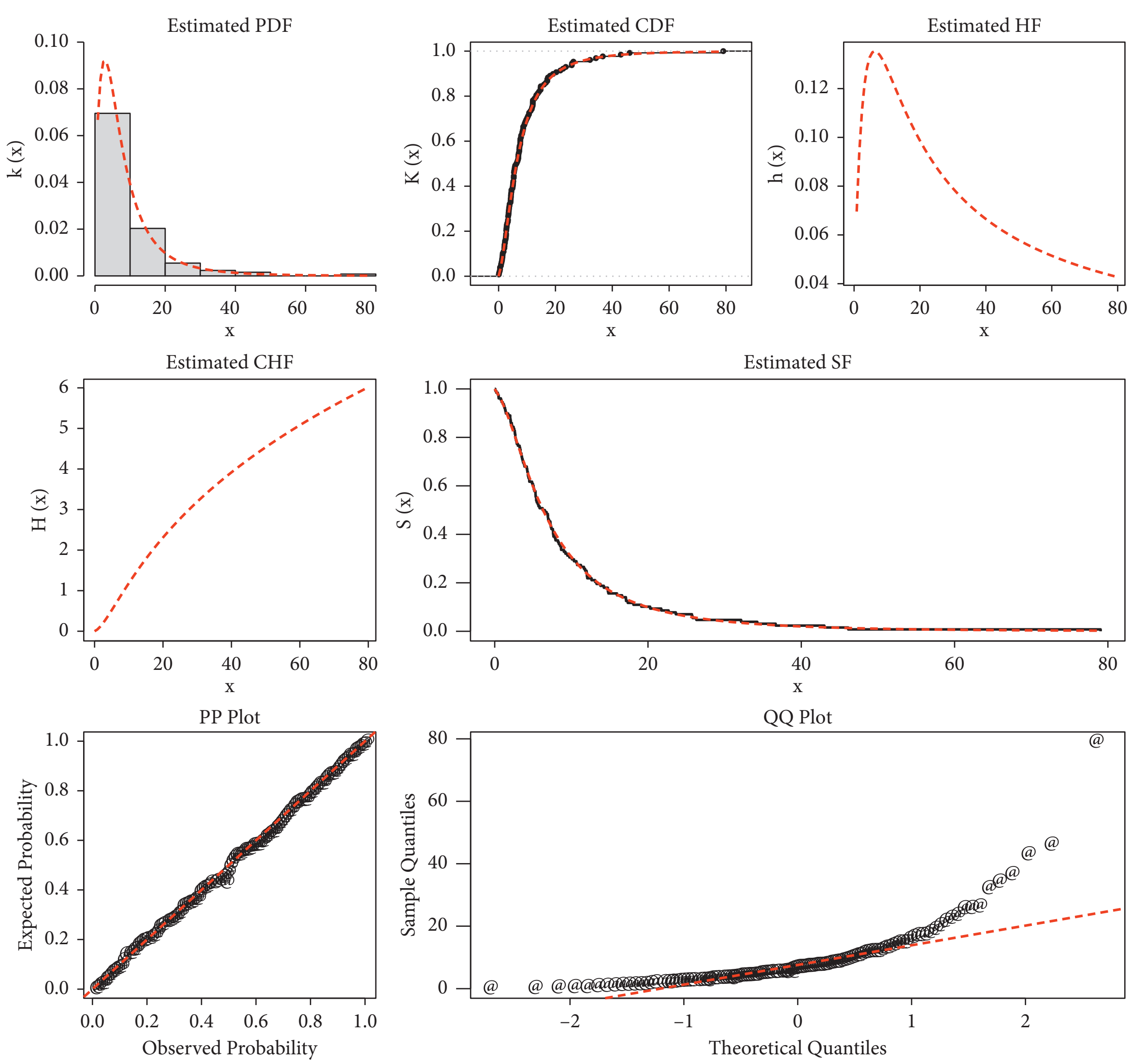

Figure 12: Visual illustration of the NFLog-Wei distribution using Data 3.

$$2m\log[\log(n)] - 2\ell(\Phi). \quad (34)$$

(vii) The CAIC is derived as follows:

$$\frac{2mn}{n-m-1} - 2\ell(\Phi). \quad (35)$$

6.1. Data 1. The first dataset (Data 1) consists of forty-four observations. The lengths of time that patients managed to stay alive may be seen here in this dataset. The key measures (summary measures) of Data 1 are as follows: skewness = 3.38382, minimum = 12.20, 1st quartile = 67.21, median = 128.50, mean = 223.48, 3rd quartile = 219.00, maximum = 1776.00, kurtosis = 16.5596, variance = 93286.41, and range = 1763.8. Corresponding to Data 1, some basic plots and the profiles of the MLEs of the NFLog-Wei distribution are presented in Figures 4 and 5, respectively.

Furthermore, for Data 1, the numerical values of the NFLog-Wei distribution and other competing models are presented in Table 4. The numerical values of the analytical measures of the competing probability models are provided in Tables 5 and 6. In justification of the numerical depiction that may be seen in Tables 5 and 6, confirm the best fitting of the NFLog-Wei model to Data 1. Figure 6 presents a graphic representationof the NFLog-Wei model. We can see that the plots shown in Figure 6 also confirm the close-fitting (best fitting) of the NFLog-Wei model to Data 1.

6.2. Data 2. The second dataset (Data 2) consists of seventy-two observations. The following are the important indicators of Data 2: minimum = 10.0, 1st quartile = 108.0, median = 149.5, mean = 176.8, 3rd quartile = 224.0, maximum = 555.0, skewness = 1.341284, kurtosis = 4.988524, variance = 10705.1, and range = 545. As in Figure 7, we can provide some summary graphs in relation to Data 2. Using Data 2, the profiles of the MLEs of the NFLog-Wei distribution are shown in Figure 8.

We also applied the NFLog-Wei distribution and other competing probability models to Data 2. Corresponding to this dataset, the numerical values of the competing probability distributions can be easily shown in Table 7. Also, the metrics of the analysis measures of the distributions that were "fitted" are already provided in Tables 8 and 9. The numerical results, in Tables 8 and 9, demonstrate that the NFLog-Wei distribution has the least results of the analytical metrics. This fact supports the best fitting power of the NFLog-Wei distribution to the guinea pigs infected dataset. In addition, Figure 9 illustrates the NFLog-Wei distribution graphically. The plots in Figure 9 support the close fit (best fit) of the NFLog-Wei distribution to the guinea pig-infected dataset.

6.3. Data 3. The third dataset (Data 3) consists of one hundred and twenty-eight observations. The key measures of Data 3 are as follows: skewness = 0.634064, median = 5.320, minimum = 0.080, 1st quartile = 2.830, mean = 6.017, 3rd quartile = 8.370, maximum = 15.960, kurtosis = 2.5349, variance = 15.66289, and range = 15.88. Figure 10 depicts many major charts that correspond to Data 3. In link to this dataset, the profiles of the MLEs of the NFLog-Wei distribution are displayed in Figure 11.

Again, we applied the NFLog-Wei distribution and the competing distributions to Data 3. Corresponding to Data 3, the numerical results of the fitted probability models can be found in Table 10. The numerical values of the statistical tests of the competing probability models are given in Tables 11 and 12. In light of the numerical findings presented in the Tables 11 and 12, we can observe that the NFLog-Wei is the best competing probability model for Data 3. To support the best fitting power of the NFLog-Wei distribution to Data 3, a graphical illustration is also presented in Figure 12. The visual illustration provided in Figure 12 supports the best fit capability of the NFLog-Wei distribution to Data 3.

7. Conclusion

In this study, a novel family of probability models was presented. The proposed family was named a new flexible logarithmic-X family. A subcase of the NFLog-X family was studied in detail. The unknown parameters of the NFLog-X family of distributions were computed using the maximum likelihood method. Furthermore, a MCSS was carried out to assess the performances of $\widehat{\delta}_{MLE}$ and $\widehat{\lambda}_{MLE}$ of the NFLog-X family. Finally, three applications (real-life datasets) to the biomedical datasets were presented to illustrate the potentiality and flexibility of the NFLog-X method. The comparison of the NFLog-X method was made with the Wei distribution and its three other well-known distributions including the APT-Wei, FRLog-Wei, and MO-Wei distributions. On the basis of eight analytic metrics, it is demonstrated that the NFLog-Wei distribution is the optimal probability distribution for modeling the medical datasets.

In the future, we are motivated to introduce further flexible forms of the NFLog-X distributions for data modeling in various sectors. We are also motivated to study the bivariate and multivariate extensions of the NFLog-X distributions [27–29] and also in the upcoming stage of this research, we will use the newly invented family of distributions in addition to the suggested distribution to analyse the censored sample technique. In order to create randomised censored samples based on the new distribution, we will carry out research on a variety of censoring techniques, including the type-I and type-II censored sample. The scope of our analysis might be increased to encompass the implementation of the suggested model to various accelerated life testing scenarios, such as constant and partially constant tests, and perhaps even outcomes of progressive load accelerated life tests.

Data Availability

In the paper, the datasets are listed.

Conflicts of Interest

It is stated by the authors that they have no competing interests.

Acknowledgments

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