Research Article

Constructing Multiple-Objective Portfolio Selection for Green Innovation and Dominating Green Innovation Indexes

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Green innovation investments have rapidly grown since 2000. Green innovation indexes play important roles and are typically constructed by screening and indexing. However, Nobel Laureate Markowitz emphasizes portfolio selection instead of security selection and accentuates that “A good portfolio is more than a long list of good stocks.” Moreover, the screening-indexing strategies ignore that investors can take green innovation as an additional objective and thus gain additional utility. We consequently construct 3-objective portfolio selection for green innovation in addition to variance and expected return. An efficient frontier of portfolio selection then extends to an efficient surface which is a panorama of the optimal variance, expected return, and expected green innovation. Investors thus fully envisage the trade-offs and enjoy the freedom of choosing preferred portfolios on the surface. In contrast, the screening-indexing strategies inflexibly leave investors with only one point (i.e., the green innovation index). As the originality, we prove in a theorem that there typically exists a curve on the efficient surface so all portfolios on the curve dominate the green innovation index. We test the dominance by component stocks of China Securities Index 300 and obtain affirmative results out of sample. The results still hold in robustness tests. At last, we classify green innovation into categories, further model the categories by general k-objective portfolio selection, and still illustrate the dominance. Consequently, investors can consider and control each category.

1. Introduction

1.1. Green Innovation Investments. As the COVID-19 pandemic unleashes crises of health and environment, environment and green are becoming more critical issues. Reference [1], p.310, defines green innovation as innovation mechanisms to improve environment. Green innovation investments have rapidly grown since 2000. As [2], p.631, contends sustainability as a driver of green innovation, the Global Sustainable Investment Review reports that global sustainable-investments assets reached $30.7 trillion in 2018 with a 34% increase from 2016.

Green innovation investments are typically fulfilled by screening-indexing strategies. Screening essentially means that investors exclude stocks of potentially socially questionable firms (e.g., gambling or tobacco). Indexing means that investors replicate a capital-market index (as documented by [3], p.341). For example, Morgan Stanley Capital International Inc. (MSCI) deploys the screening-indexing strategies for MSCI KLD 400 Social Index. The Forum of Sustainable and Responsible Investment and the Principles for Responsible Investment Association harness similar strategies.

On one side, the screening-indexing strategies are highly practical and thus substantially expedite green innovation
investments. However on the other side, the strategies suffer from the following weakness:

(1) The strategies build portfolios by a list of good stocks and thus ignore portfolio completeness, because [4], p.3, emphasizes portfolio selection rather than security selection and stresses portfolio wholeness.

(2) The strategies ignore that investors can take green innovation as an additional objective and thus gain additional utility. Reference [5], p.683, deduces that investors obtain utility from social responsibility. Reference [6], p.2, highlights society objectives and the urge to balance them with risk and return.

(3) The strategies inflexibly prescribe an index for most investors and disregard investor difference. Screening out some kinds of stocks is appropriate for some investors but inappropriate for others. References [7], p.5, and [8], pp.27&28, contend that investors but inappropriate for others. Reference [9], pp.27&28, contend that investors but inappropriate for others. Reference [11], pp.545–547, [12], pp.62–63, and [16], pp.1081–1082, emphasize multiple factors for asset pricing and further propose the factors’ risks as objectives. Reference [17], p.182, advocates expressive benefits (e.g., social responsibility) as objectives.

References [14, 18–21] and [22] formulate additional objectives by multiple-objective portfolio selection as follows:

\[
\begin{align*}
\min & \{ z_1 = x^T \Sigma x \}, \text{risk} \\
\max & \{ z_2 = x^T \mu \}, \text{return} \\
& \text{s.t. } x \in S, \text{feasible region.}
\end{align*}
\]

For \( n \) stocks:

\( \mu \) is a vector of stock expected returns,

superscript \( T \) denotes transposition,

\( \Sigma \) is a covariance matrix of stock returns,

\( x \) is a portfolio-weight vector,

\( z_1 \) measures the variance of the return of portfolio \( x \),

\( z_2 \) measures the expectation of the return of portfolio \( x \),

\( S \) is a feasible region in \( \mathbb{R}^n \),

\( Z = \{(z_1, z_2) | x \in S \} \) is the feasible region in \( (z_1, z_2) \) space.

Reference [9] calls the optimal solutions in \( (z_1, z_2) \) space as an efficient frontier. References [10, 11] fully describe multiple-objective optimization. For notations, normal symbols (e.g., \( n \)) denote scalars; bold-face symbols (e.g., \( \mathbf{x} \) or \( \mathbf{\Sigma} \)) denote vectors or matrices.

Mathematically, (1) maps \( S \) to \( Z \). For instance, we depict \( x_1 \ldots x_i \) to \( z_1 \ldots z_2 \) by three arrows in the lower part of Figure 1. We also depict \( S \) and \( Z \) as shaded regions and depict the efficient frontier as a thick curve.

1.2. Portfolio Selection. Reference [4], p.6, discovers that investors mind both risk and return. Therefore, [9], p.83, formulates portfolio selection as the following 2-objective optimization:

\[
\begin{align*}
\min & \{ z_1 = x^T \Sigma x \}, \text{risk} \\
\max & \{ z_2 = x^T \mu \}, \text{return} \\
& \text{s.t. } x \in S, \text{feasible region.}
\end{align*}
\]

1.3. Multiple-Objective Portfolio Selection. Reference [12], pp.471&476, later discovers additional objectives (in addition to the variance and expectation). Reference [13] also realizes additional objectives and incorporates them into a utility function. References [14], pp.445–447, [15], pp.62–63,

and [16], pp.1081–1082, emphasize multiple factors for asset pricing and further propose the factors’ risks as objectives. Reference [17], p.182, advocates expressive benefits (e.g., social responsibility) as objectives.

References [14, 18–21] and [22] formulate additional objectives by multiple-objective portfolio selection as follows:

\[
\begin{align*}
\min & \{ z_1 = x^T \Sigma x \}, \text{risk} \\
\max & \{ z_2 = x^T \mu \}, \text{return} \\
\max & \{ z_3 = x^T \mu_k \}, \text{general objective 1} \\
& \vdots \\
\max & \{ z_k = x^T \mu_k \}, \text{general objective } k-2 \\
& \text{s.t. } x \in S, \text{feasible region,}
\end{align*}
\]

where

\( \mu_1 \ldots \mu_k \) are vectors of general stock expected objectives (e.g., expected R&D and liquidity),

\( z_1 \ldots z_k \) measure the general portfolio expected objectives,

\( Z = \{(z_1, \ldots, z_k) | x \in S \} \) is the feasible region in \( (z_1, \ldots, z_k) \) space.

Mathematically, (2) maps \( S \) to \( Z \). For instance, we depict \( x_1 \ldots x_i \) to \( z_1 \ldots z_k \) by three arrows in the lower part of Figure 1. We also depict \( S \) and \( Z \) as shaded regions. The \( S \) of (1) and of (2) remains the same. The \( Z \) of (1) extends to a high-dimensional set for (2). The efficient frontier of (1) also extends to a high-dimensional efficient surface for (2) (denoted as \( N \)). The surface is the optimal solutions of (2) in \( (z_1, \ldots, z_k) \) space.

1.4. Originality for Green Innovation Investments

1.4.1. Formulation Originality: 3-Objective Portfolio Selection for Green Innovation. We originate the following 3-objective model for green innovation by taking \( \mu_3 \) as a vector of stock expected green innovations and taking \( z_3 \) as the portfolio expected green innovation:

\[
\begin{align*}
\min & \{ z_1 = x^T \Sigma x \}, \text{risk} \\
\max & \{ z_2 = x^T \mu \}, \text{return} \\
\max & \{ z_3 = x^T \mu_3 \}, \text{green innovation} \\
& \text{s.t. } A^T x = b, \text{constraints,}
\end{align*}
\]

where

\( A \) is an \( n \times m \) constraint matrix,

\( b \) is an \( m \)-vector for the right hand side.

Equation (3) can overcome the weakness of the screening-indexing strategies as follows:

First (3) hinges on portfolio selection and thus enjoys portfolio completeness. Second, (3) explicitly operates green innovation as an additional objective and reserves rooms for the utility. Last, (3) prescribes a whole efficient surface for investors.
With the sheer amount of portfolios on the surface, different investors can pick distinctive portfolios. For instance, investor 1 prefers portfolio $z_1$, while investor 2 prefers portfolio $z_2$ (as depicted in the lower part of Figure 1). In contrast, the screening-indexing strategies invariably prescribe only one index for vastly different investors. The index is denoted as $z_g$ (as depicted in the lower part of Figure 1).

1.4.2. Formulation Advantage: Dominating Green Innovation Indexes. Moreover, investors can exploit the efficient surface and harvest portfolios which dominate the green innovation index $z_g$. As the originality, we will prove in a theorem that there exist portfolios which

1. enjoy both better expected return and better expected green innovation than the green innovation index does;
2. possess the same variance as that of the green innovation index.

The portfolios are depicted as the thickest curve from $z_{d2}$ to $z_{d3}$ in the right part of Figure 2. The figure will be deliberated later.

1.5. Paper Structure. The rest of this paper is organized as follows: We review theoretical background in the next section. Then in the next section, we prove the existence of dominating portfolios by a theorem and propose hypotheses for the out-of-sample dominance. Then in the next section, we measure green innovation on the basis of patent-citation numbers, set up (3), compute the dominance, test the hypotheses by component stocks of China Securities Index 300, and obtain affirmative results. Then in the next section, we divide green innovation into categories, further formulate the categories as objectives, extend (3) into general $k$-objective portfolio selection, and still demonstrate the dominance. We conclude this paper in the last section.

2. Literature Review: Green Innovation Investments and Multiple-Objective Portfolio Selection

We briefly review green innovation measurement and green innovation investments, summarize multiple-objective portfolio selection, and display quantitative results.

2.1. Overall Research for Green Innovation. A large body of green innovation research focuses on

1. the driving force behind green innovation;
2. the impact of green innovation on corporations or society.

For research line 1, for example, [23] proposes regulation and pressure as the driving force and verifies the proposition by environment-related patents. Reference [24] proposes political capital as the driving force. Reference [25] proposes green supplier involvement.

For research line 2, for example, [26] contends that green innovation is positively correlated to corporate competitive advantage. Reference [27] reports positive correlations between green innovation and green core competence. Reference [1] concludes that green innovation has positive effects on organizational performance.
2.2. ESG Measurement and Green Innovation Measurement

2.2.1. ESG Measurement. Overall, ESG is a well-dissected but still evolving area. References [28, 29] review more than 2000 ESG studies, advocate future directions, and discover that the studies mainly focus on Europe and US. Reference [30] detects relatively slow ESG developments in Asia and deduces the reason as the lack of reliable ESG measurement. Reference [31] maneuvers rough-set approaches and forecasts ESG ratios by corporate financial performance variables. Several scholars further highlight the following weakness in ESG measurement.

First, scholars naturally instigate all kinds of ESG measurement. While financial measures are definite, environmental measures are relatively vague. To make things more diverse, there are more than 600 ESG-rating agencies with dissimilar measurement themes. References [32, 33] uncover inconsistent measurement results among key rating agencies (e.g., MSCI, Thomson Reuters, and GES).

Second, ESG measures heavily rely on corporate voluntary disclosures, but the disclosures are typically unaudited. Therefore, some corporations tend to manipulate their disclosures. References [34, 35] investigate such manipulation motivations. Reference [36] validates that some firms may exaggerate environmental performance for better images. Reference [37] further designs mechanisms to deter such exaggerations.

2.2.2. Green Innovation Measurement. In contrast, scholars sponsor relatively concentrated measurement for green innovation. Moreover, green innovation can be objectively measured by patent-citation numbers. The numbers are issued by independent third parties (e.g., World Intellectual Property Organization). Therefore, in contrast to the lack of reliable ESG measurement in Asia, green innovation can be relatively reliably measured in Asia. References [38, 39] survey green innovation development, recommend future directions, and endorse the patent-citation measurement.

In the early stage of green innovation development, scholars (e.g., [26, 40]) do not have sufficient patent-information support and thus typically utilize surveys to measure green innovation. Later, [23], pp.897–898, questions the potential insight of the surveys, emphasizes the potential survey bias, and proposes the total number of citations received by the patents as the measure. Similarly, [41], p.637, acknowledges patent-citation numbers and executes Citations/RD (RD as R&D). Reference [42], pp.2249–2250, also acknowledges patent-citation numbers and employs natural logarithms of one plus the numbers.

2.3. Green Innovation Investments. Reference [43] stresses the gigantic volume of green innovation investments, underscores the importance of portfolio selection, and inaugurates their models. Reference [29] surveys 463 articles and books for green innovation investments and reviews the following two major methods:

First, investors readily deploy the screening-indexing strategies. For example, [44] utilizes the strategies and reports some positive risk-adjusted performance. Reference [45] utilizes the strategies, compares green innovation investments against passive investments, and finds mixed results. Reference [46] utilizes the strategies and emphasizes opportunity costs for screening. Reference [47] picks the top 100 CSR companies in the world by Forbes, assembles market-capitalization-index portfolios, and locates significant abnormal returns. During the COVID-19 pandemic, [48] attests investor preference for ESG indexes and validates the outperformance. Reference [49] concentrates on the component stocks of S&P 500 Index, executes the screening-indexing strategies, and portrays confirmatory results. In contrast, [50] surveys main-stream investment organizations and finds the strategies relatively ineffective.

Second, mutual-fund companies structure funds by the screening-indexing strategies. For example, [51, 52] and [53] compare green innovation funds’ returns against conventional mutual funds’ returns but reveal the two groups of returns as typically identical. By further considering crisis and normal periods, [54] discovers that the two groups of returns are typically identical in crisis periods but the green innovation funds underperform the conventional mutual funds in normal periods. Reference [55] exposes that the risk-adjusted returns of funds with high green innovation measures equal those of funds with low green innovation measures. During the COVID-19 pandemic, [56] unveils that investors deliberate ESG risk and that low-ESG-risk funds outperform high-ESG-risk funds.
2.4. Multiple-Objective Portfolio Selection for Green Innovation and for Other Objectives. Reference [51], p.1723, concludes that SRI investors are willing to sacrifice financial performance for social objectives. The conclusion can be explained by multiple-objective portfolio selection for green innovation. That is, investors accept some optimal portfolios in \((z_1, z_2, z_3)\) space of (3), although the portfolios are not optimal in \((z_1, z_2)\) space of (1). Moreover, [57] considers the traditional financial goal and an ethical goal for utility functions and optimizes the 2-goal model. Reference [58] analyzes social responsibility by multiple-objective portfolio selection and studies the maximum-Sharpe-ratio portfolio and maximum-Delta-ratio portfolio.


2.5. Multiple-Objective Portfolio Optimization with Equality Constraints Only

2.5.1. A Fundamental Model (4) for Portfolio Selection and Asset Pricing. References [71], pp.57–62, and [72] inspect the following model:

\[
\begin{align*}
\min &\ z_1 = x^T \Sigma x \\
\max &\ z_2 = x^T \mu \\
\max &\ z_3 = x^T \mu_3 \\
s.t. &\ I^T x = 1.
\end{align*}
\]

(4)

\(I\) is a vector of ones. Reference [72] analytically derives the minimum-variance frontier and proves the frontier as a parabola. The frontier is the boundary of the feasible region \(Z\) of (4) and is depicted as a curve in the upper part of Figure 1. Although unlimited portfolio weights are allowed, (4) enjoys analytical tractability and thus is widely deployed in portfolio selection and asset pricing (as acclaimed by [73], p.60).

2.5.2. Extending (4) by Adding an Objective. Reference [21] extends (4) by adding an objective as follows:

\[
\begin{align*}
\min &\ z_1 = x^T \Sigma x \\
\max &\ z_2 = x^T \mu \\
\max &\ z_3 = x^T \mu_3 \\
s.t. &\ I^T x = 1,
\end{align*}
\]

(5)

where \(z_3\) is a general objective. Reference [21] also analytically derives the result.

2.5.3. Further Extending (4) by Adding Objectives and Constraints. Reference [22] further extends (4) by adding objectives and constraints as follows:

\[
\begin{align*}
\min &\ z_1 = x^T \Sigma x \\
\max &\ z_2 = x^T \mu \\
\max &\ z_3 = x^T \mu_3 \\
\vdots &\ \\
\max &\ z_k = x^T \mu_k \\
s.t. &\ A^T x = b.
\end{align*}
\]

(6)

They embrace the following assumptions.

Assumption 1. Covariance matrix \(\Sigma\) is positive definite and thus invertible.

Assumption 2. Vectors \(\mu\) and \(\mu_3, \ldots, \mu_k\) and all columns of \(A\) (altogether \(k−1+m\) vectors) are linearly independent.

Reference [22] commands an \(e\)-constraint method (as described by [10], pp.202–206) to (6) as follows:

\[
\begin{align*}
\min &\ z_1 = x^T \Sigma x \\
s.t. &\ x^T \mu = e_2 \\
&\ \vdots \\
&\ x^T \mu_k = e_k \\
&\ A^T x = b.
\end{align*}
\]

(7)

\(e_2\ldots e_k\) are the method parameters. As \(e_2\ldots e_k\) vary, the set of all \((z_1, \ldots, z_k)\) vectors of the optimal solutions of (7) forms a minimum-variance surface of (6). As an extension of the minimum-variance frontier of (4), the surface is the boundary of the feasible region \(Z\) of (6) and is depicted as a surface in the lower part of Figure 1. Reference [22], pp.527–528, proves the surface as a paraboloid and analytically derives the surface as follows:

\[
\begin{bmatrix}
z_1 \\
z_2 \\
\vdots \\
z_k
\end{bmatrix} = \begin{bmatrix}
z_2 \\
\vdots \\
z_k
\end{bmatrix}
\]

(8)

where \(M\) is introduced as follows:

\[
M = \begin{bmatrix}
\mu & \mu_3 & \cdots & \mu_k & A \\
\end{bmatrix}_{n \times (k−1+nm)}.
\]

(9)

For a portfolio on the surface with given \(z_2\ldots z_k\), [22], p.526, computes the portfolio-weight vector \(x\) as follows:

\[
\begin{bmatrix}
x \\
\vdots \\
x
\end{bmatrix} = \Sigma^{-1} M (M^T \Sigma^{-1} M)^{-1} \begin{bmatrix}
z_2 \\
\vdots \\
z_k
\end{bmatrix},
\]

(10)

Moreover, [22], pp.529–531, derives the \(x\) set of the efficient surface of (6) as follows:

\[
\{ x \in \mathbb{R}^n | x = x_m + \lambda_2 b_2 + \cdots + \lambda_k b_k, \lambda_2 \ldots \lambda_k \geq 0 \},
\]

(11)
where \( \mathbf{x}_{mv} \) and \( \mathbf{g}_2 \ldots \mathbf{g}_k \) are computed as follows:

\[
\mathbf{x}_{mv} = \mathbf{A}^T \Sigma^{-1} \mathbf{A}^{-1} \mathbf{b} \\
[\mathbf{g}_2 \ldots \mathbf{g}_k] = \frac{1}{2} \mathbf{X}^{-1} \left( \mathbf{I}_n - \mathbf{A} \mathbf{A}^T \right)^{-1} \mathbf{A}^T \Sigma^{-1} \left( \begin{array}{c} \mathbf{m} \\ \mathbf{m}_3 \\ \vdots \\ \mathbf{m}_k \end{array} \right),
\]

with \( \mathbf{I}_n \) as an \( n \times n \) identity matrix. The efficient surface is obtained by substituting (11) into (6).

We will compare methods for general multiple-objective portfolio optimization in the next section.

3. Dominating Green Innovation Indexes and Testing the Dominance by Hypotheses

As an advantage of (3), the efficient surface precisely and completely demonstrates the trade-offs among optimal variance, expected return, and expected green innovation. Therefore, investors can study the surface, envisage the trade-offs, and pinpoint preferred portfolios on the surface. Moreover, we prove that investors can obtain surface portfolios which dominate the green innovation index. We justify (3), present a graphical interpretation, validate the dominance by a theorem, and propose hypotheses for the out-of-sample dominance.

3.1. Justifying (3). We accentuate that (3) can overpower the weakness of the screening-indexing strategies as follows:

First, (3) hinges on portfolio selection and thus enjoys portfolio completeness. For an \( n \times n \) covariance matrix \( \Sigma \), the screening-indexing strategies exploit only the \( n \) variance elements as individual stock risks, but portfolio selection and (3) additionally exploit the \( n^2 - n/2 \) overwhelmingly more covariance elements as stock corisks. Moreover, the screening-indexing strategies implement relatively simplistic indexing, while portfolio selection and (3) implement forceful optimization methodologies. Reference [74] features the scarcity of optimization methodologies in green innovation investments and endorses portfolio selection. Reference [75] also features the scarcity and commences all-underlying-functionality models. Reference [76] scans reasons of the screening-indexing strategies’ relatively unsatisfactory performance and urges rudimentary linear regression models. Reference [77] engineers ESG portfolios by utility-based nonparametric models.

Second, (3) explicitly utilizes green innovation as an additional objective and reserves rooms for the utility. References [5, pp.683, and [6], p.2, hint investors’ such motives. Reference [78] implies that investors balance financial objectives versus ESG objectives and even relatively sacrifice financial objectives to enrich ESG objectives. Reference [79] surveys household preferences for ESG investments and also implies that investors relatively sacrifice financial objectives to enrich ESG objectives. Reference [80] substantiates that international institutional investors care about both financial objectives and social objectives.

Third, (3) prescribes a whole efficient surface for investors. Consequently, investors perceive the efficient surface as a panorama of the optimal variance, expected return, and expected green innovation and thus enjoy the freedom of choosing preferred portfolios on the surface. Different investors can pick distinctive surface portfolios. References [7], pp.5, and [8], pp.278–28, feature investor difference. Reference [50] globally surveys ESG information and recounts information-usage difference in motivation, demand, product strategy, and ethical considerations. Reference [81] probes investors’ political ideology and fashions portfolio-selection models for a unique group of investors. Reference [43] further explores three types of investors: ESG-unaware investors, ESG-aware investors, and ESG-motivated investors.

Last, we scrutinize green innovation (instead of ESG) in China for (3), because green innovation can be objectively measured by patent-citation numbers.

3.2. Graphical Interpretation. For (3), let \( \mathbf{z}_g = (z_{g1}, z_{g2}, z_{g3}) \) denote the green innovation index in \( z_{g1}, z_{g2}, z_{g3} \) as the variance, expected return, and expected green innovation of the index, respectively. Let \( \mathbf{x}_g \) denote the portfolio-weight vector of the index. By screening, \( \mathbf{x}_g \) bears some zero elements and thus is typically inefficient, because efficient portfolio-weight vectors (11) basically bear few zero elements. Therefore, \( \mathbf{z}_g \) is typically not on the efficient surface.

In the left part of Figure 2, we draw a plane which passes through \( \mathbf{z}_g \) and is parallel to \( (z_2, z_3) \) subspace. The plane is as follows:

\[
z_1 = z_{g1}.
\]

The plane intercepts the feasible region \( Z \) of (3). Because every portfolio on the intersection shares the same \( z_1 \) (i.e., \( z_1 = z_{g1} \)), we need to focus on just \( (z_2, z_3) \) elements of the intersection in the right part of Figure 2. The intersection is depicted as shaded in the right part. Because [22] proves the minimum-variance surface as a paraboloid (8) and the surface is the boundary of the feasible region \( Z \), the boundary of the intersection is an ellipse. By (8) with \( k = 3 \) and (13), the ellipse is as follows:

\[
\begin{bmatrix}
z_2 \\
\vdots \\
z_k \\
b
\end{bmatrix}
\left(M^T \Sigma^{-1} M\right)^{-1}
= z_{g1}.
\]

Specifically, the plane intersects the efficient surface at a thicker curve in the right part of Figure 2. The intersection is depicted as a thicker curve from \( z_{g2} \) to \( z_{g3} \).

We utilize the domination-set method of [10], pp.150–154, and draw a domination set at \( \mathbf{z}_g \) in blue color. Every portfolio in the set (except \( \mathbf{z}_g \) itself) dominates \( \mathbf{z}_g \). The set finally touches the ellipse at the thickest curve from \( \mathbf{z}_{g2} \) to \( \mathbf{z}_{g3} \). \( \mathbf{z}_{g2} \) and \( \mathbf{z}_{g3} \) are depicted in the right part of Figure 2. By the method, all the portfolios on the thickest curve dominate \( \mathbf{z}_g \) the most.
3.3. Proving the Dominance by a Theorem. We reiterate the graphical interpretation in the following theorem.

**Theorem 1.** In \((z_1, z_2, z_3)\) space of (3), there exists a plane passing through \(z_y\) and being parallel to \((z_2, z_3)\) subspace unless \(z_y\) is efficient. The plane intersects the efficient surface and feasible region \(Z\) of (3). On the intersection of the plane and efficient surface, there exists a whole range of portfolios which dominate \(z_y\). That is,

- the portfolios’ variances are identical to the variance of \(z_y\),
- the portfolios’ expected returns are greater than or equal to the expected return of \(z_y\),
- the portfolios’ expected green innovations are greater than or equal to the expected green innovation of \(z_y\),

at least one of the two “greater than or equal to” relationships is “greater than.”

**Proof.** The plane (13) intersects the \(Z\) of (3). Because all portfolios on the intersection share the same \(z_1\) (i.e., \(z_1 = z_1\)) and the efficient surface is the optimal portfolios of (3), the optimal solutions of the intersection are the thicker curve from \(z_4 = (z_{g1}, z_{d2,2}, z_{d2,3})\) to \(z_3 = (z_{g1}, z_{d2,2}', z_{d2,3}')\) in the right part of Figure 2. Investors calculate \(z_4\) and \(z_3\) by (8) and (11).

If \(z_4\) is to the right of \(z_2\) (i.e., \(z_2 \geq z_{d2,3}\) as depicted in the right part of Figure 2), investors compute \(z_{d2} = (z_{g1}, z_{d2,2}', z_{d2,3}')\) by substituting \(z_3 = z_{g3}\) into (14), calculating the two roots, and taking \(z_{d2,2}'\) as the bigger root. Otherwise (i.e., to the left of \(z_2\), \(z_2 < z_{d2,3}\) as depicted in the upper left part of Figure 3), investors just take \(z_{d2} = z_2\).

Similarly, if \(z_y\) is above \(z_3\) (i.e., \(z_2 \geq z_{d2,3}'\) as depicted in the right part of Figure 2), investors compute \(z_{d3} = (z_{g1}, z_{d2,2}', z_{d2,3}')\) by substituting \(z_2 = z_{g2}\) into (14), calculating the two roots, and taking \(z_{d2,3}'\) as the bigger root. Otherwise (i.e., below \(z_3\), \(z_2 < z_{d2,3}'\) as depicted in the upper right part of Figure 3), investors just take \(z_{d3} = z_3\).

In summary, there are four exhaustively exclusive cases as follows:

1. \(z_{g3} \geq z_{d2,3}\) and \(z_{g2} \geq z_{d2,3}'\) (right part of Figure 2),
2. \(z_{g3} \geq z_{d2,3}\) and \(z_{g2} < z_{d2,3}'\) (upper right part of Figure 3),
3. \(z_{g3} < z_{d2,3}\) and \(z_{g2} \geq z_{d2,3}'\) (upper left part of Figure 3),
4. \(z_{g3} < z_{d2,3}\) and \(z_{g2} < z_{d2,3}'\) (lower left part of Figure 3).

For the four cases, investors exploit the domination-set method at \(z_y\). Every portfolio in the set dominates \(z_y\). Specifically, all the portfolios on the thickest curve from \(z_{d2}\) to \(z_{d3}\) dominate \(z_y\) the most. The portfolio-weight vectors can be computed on the basis of (10).

3.4. Testing Hypotheses for the Out-of-Sample Dominance. By Theorem 1, we can choose a portfolio on the thickest curve from \(z_{d2}\) to \(z_{d3}\), collect out-of-sample data, compute the returns and green innovations of the portfolio and of the green innovation index \(z_g\), and test the following hypotheses:

1. \(H_0:\) variance of the portfolio = variance of \(z_g\),
2. \(H_a:\) variance of the portfolio ≠ variance of \(z_g\),
3. \(H_0:\) expected return of the portfolio = expected return of \(z_y\),
4. \(H_a:\) expected return of the portfolio > expected return of \(z_y\),
5. \(H_0:\) expected green innovation of the portfolio = expected green innovation of \(z_g\),
6. \(H_a:\) expected green innovation of the portfolio > expected green innovation of \(z_g\).


3.5. Portfolio Selection and Its Out-of-Sample Performance. Several researchers (e.g., [83–85] and [86]) criticize portfolio selection for its weak out-of-sample performance. However, the researchers’ arguments can be reconciled in the following aspects.

First, [9] seminally quantitatively formulates portfolio risk and return rather than focusing on out-of-sample performance (as proclaimed by [87], p.1041). Therefore, portfolio selection is universally described in classic textbooks (e.g., those of [3, 88–90] and [91]).

Second, strong out-of-sample performance implicitly requires that the out-of-sample is preferably large and comes from the same independent and identical distribution as the in-sample distribution. However in reality, financial markets instantly react to all kinds of information; corporations prosper and decline over time. Therefore, the requirement is rarely satisfied. Consequently, weak out-of-sample performance can be caused by the sample distribution.

Last, investors subjectively hope to consistently beat markets without extra cost. However, objectively, such financial perpetual-motion machines violate fundamental finance-theory laws (e.g., efficient-market hypotheses) and thus barely exist.

In this paper, in sample, we extend portfolio selection, instigate 3-objective portfolio selection for green innovation, and prove the existence of dominating portfolios in a theorem. We will launch both theoretical and practical implications in the conclusion.

3.6. Contrasting Methods of Multiple-Objective Portfolio Optimization. We briefly contrast different methods for general multiple-objective portfolio optimization.

3.6.1. Analytical Methods. Analytical methods are conducted in the form of formulae on the basis of calculus and linear algebra and are thus understandable. We have already
reviewed analytical methods in the previous section. We optimize (20) by analytical methods (8)–(11). Analytical methods’ computational advantage is to readily reckon the efficient surface and dominating portfolios.

However, analytical methods decipher models with equality constraints only. Nevertheless with almost all results in formulae, analytical methods liberate researchers from mathematical programming. Therefore, researchers can enjoy the analytical tractability and empirical implications (as highlighted by [73], p.60).

3.6.2. Parametric Quadratic Programming. Parametric quadratic programming is an advanced form of quadratic programming with parameters in the formulation. References [92, 93] describe the topic in detail. Parametric quadratic programming can handle both equality constraints and inequality constraints. Particularly, inequality constraints can prescribe the following conditions:

- some lower bound (floor) of \( x \) (e.g., \( x \geq 0 \) to restrict short sales),
- some lower bound and upper bound (ceiling) of an industry to regulate the industry exposure,
- transaction cost.

To solve (1), [94, 95] deploy parametric quadratic programming and propose a critical-line algorithm. Reference [95], p.176, proves that the efficient frontier is piece-wisely made up by connected parabolic segments.

Reference [96] calculates the whole efficient surface of (2) with \( k = 3 \) and suggests that the surface is piece-wisely made up by connected paraboloidal segments. For example, a surface consists of two segments in the left part of Figure 4. The upper segment is a portion of paraboloid 
\[
z_1 = z_2^2 + 2z_3^2 + 3z_2z_3 + 4z_2 + 4z_3 + 9; \]
the lower segment is a portion of paraboloid 
\[
z_1 = z_2^2 + 5z_2^2 + 6z_2z_3 + 8z_2 + 9z_3 + 8. \]
The two segments connect at a thick curve. Reference [97] also exploits parametric quadratic programming and resolves (2).

However, parametric quadratic programming is much more convoluted than quadratic programming and thus rarely commanded. Moreover, parametric quadratic programming suffers from the following obstacles.

First, neither the shape nor the property of the feasible region \( Z = \{(x_j, \ldots, x_n) | x \in S\} \) of (2) is comprehensible. Investors can follow the method depicted in Figure 2 but can not envision the intersection between the plane \( z_1 = z_{gl} \) (13) and \( Z \). For example, the boundary of the intersection is unknown and depicted as broken curves in the right part of Figure 4. Therefore, the intersection is unknown either. With a green innovation index \( z_g \) near a corner, investors can obtain eccentric results by operating the domination-set method. In contrast, for (3), investors fully comprehend that the \( Z \) is a paraboloidal set by (8). Therefore, investors fully comprehend that the intersection between the plane (13) and \( Z \) is an elliptical area (as depicted in Figures 2 and 3).

Second, [96], p.181, proclaims that efficient surfaces of (2) with \( k = 3 \) and with \( n = 102 \) can be made up by 7994 connected paraboloidal segments. For the simplest only-1-segment efficient surface of (3), investors encounter 4 cases of the relationship between \( z_g \) and \( z_2 \) and between \( z_g \) and \( z_3 \) (as depicted in Figures 2 and 3). For the simplistic 2-segment efficient surface depicted in Figure 4, investors can encounter \( 2 \times 4 = 8 \) cases of the relationship between \( z_g \) and \( z_2 \) and between \( z_g \) and \( z_3 \). For the 7994-segment efficient surfaces, investors will be perplexed by the case number.

![Figure 3: The three other cases of the relationship between \( z_g \) and \( z_2 \) and between \( z_g \) and \( z_3 \) for Theorem 1.](image-url)
3.6.3. Repetitive Quadratic Programming. Repetitive quadratic programming can handle both equality constraints and inequality constraints. Reference [10], pp.165–183 & 202–206, elucidates weighted-sums methods and e-constraint methods to transform multiple-objective optimization into (ordinary) 1-objective optimization. Investors can channel the methods for (2) as follows:

$$\min \left\{ x^T \Sigma x - \lambda_1 x^T \mu - \lambda_2 x^T \mu_3 - \cdots - \lambda_k x^T \mu_k \right\}$$

s.t. \( x \in S, \) feasible region.

$$\min \left\{ z_1 = x^T \Sigma x \right\},$$

s.t. \( x^T \mu = e_2, \)

$$x^T \mu_3 = e_3,$$

$$\vdots$$

$$x^T \mu_k = e_k,$$

$$x \in S,$$

where

- \( \lambda_1 \ldots \lambda_k \) are coefficients for weighted-sums methods,
- \( e_2 \ldots e_k \) are coefficients for e-constraint methods.

Investors preset a group of \( \lambda_1 \ldots \lambda_k \) and a group of \( e_2 \ldots e_k \), repetitively solve (18) for each \( \lambda_1 \ldots \lambda_k \), and repetitively solve (19) for each \( e_2 \ldots e_k \). The optimal solutions are actually discrete versions of the efficient surface of (2). We depict some optimal solutions as isolated points in the left part of Figure 4. The isolated points are much less informative than the whole efficient surface depicted in the left part of Figure 4.

Repetitive quadratic programming is easy to understand and thus described in classic textbooks (e.g., Chapter 7 of [3]). Several computational software (e.g., Microsoft Excel and Matlab) can offer quadratic-programming solvers. However, repetitive quadratic programming suffers from the following obstacles:

First, the isolated points can cluster in small parts of the efficient surface instead of uniformly dispersing around the surface (as underlined by [98]).

Second, repetitive quadratic programming cannot reveal the paraboloidal-segment structure. For instance, investors perceive just a bunch of isolated points in Figure 5.

Last, investors obtain also isolated points on the intersection and hardly execute the domination-set method for the points. We depict the situation in the right part of Figure 5.

3.6.4. Genetic Algorithms and Other Heuristic Methods. As important and popular tools, genetic algorithms and other heuristic methods can handle equality constraints, inequality constraints, and even special conditions (e.g., integer variables). Reference [99] systematically exhibits genetic algorithms. References [100,101] and [102] employ genetic algorithms for multiple-objective portfolio selection. Reference [103] offers surveys for genetic algorithms.

However, heuristic methods inherently provide suboptimal solutions. For example, we depict some isolated points as suboptimal solutions in the left part of Figure 6. The points are below the efficient surface depicted in the left part of Figure 4. Moreover, the methods also suffer from the obstacles of repetitive quadratic programming. We depict some obstacles in the right part of Figure 6.

4. Empirical Tests by Component Stocks of China Securities Index 300

We sample the 300 component stocks of China Securities Index 300 from 2009 to 2018, measure the green innovation, gauge the dominating portfolios, test hypotheses for the out-of-sample dominance, implement robustness tests, and obtain supportive results. We report key computations and omit long calculations (e.g., 152 × 152 matrices). We have deposited all the data, codes, and results in Mendeley Data https://data.mendeley.com/. Please see [104] in the references.

4.1. Modeling. Specially, we develop the following model on the basis of (3):
\[
\begin{align*}
\min \{z_1 &= x^T \Sigma x\}, \\
\max \{z_2 &= x^T \mu\}, \\
\max \{z_3 &= x^T \mu_3\}, \\
\text{s.t.} & \quad t^T x = 1, \\
& \quad d^T x = d,
\end{align*}
\]  \tag{20}

\(d\) is a vector of stock expected dividend yields (dividend/price); \(d\) is a target expected dividend yield and taken as the average of \(d\).


4.2. Measuring Green Innovation of the Component Stocks by Patent-Citation Numbers. We follow [41, 42] and explicitly measure green innovation in the following steps:

(1) We also acknowledge patent-citation numbers.

(2) We sample all the 300 component stocks of China Securities Index 300 (data source: http://www.csindex.com.cn/en/indices/index-detail/000300, May 24, 2020). We follow empirical-test traditions to disregard finance-industry stocks and stocks with incomplete observations. We then obtain \(n = 152\) component stocks with complete monthly returns from January 1, 2009, to December 31, 2018.

(3) For the 152 component stocks, we obtain the patent information (including international patent classification number) from 2009 to 2018 from State Intellectual Property Office of China (SIPO) (data source: http://pss-system.cnipa.gov.cn/sipopublicsearch/portal/uiIndex.shtml, May 29, 2020).


(5) We match the patent information of SIPO with that of WIPO by international patent classification number and finally obtain 6154 patents.

further dissect the issue and illuminate the number of participating stocks in a later subsection.

We separate January 1, 2009, to December 31, 2018, into the following subperiods:

1. From January 1, 2009, to December 31, 2013, as an in-sample subperiod, we sample the data, estimate $\Sigma$ and $\mu$ and $\mu_1$ for (20), set up (20), compute $z_{g_2}$ and calculate the thickest curve from $z_{d2}$ to $z_{d3}$ of Theorem 1.

2. From January 1, 2014, to December 31, 2018, as an out-of-sample subperiod, we select some portfolios on the thickest curve and test hypotheses (15)-(17).

3. Reference [111], pp.103–104&142–143&149–150, frequently explicates designs of 5-year in-sample subperiods and next-5-year out-of-sample subperiods. References [112], p.1680, and [113], p.439, accordingly implement the designs. Consequently, we similarly implement the designs. Moreover, we will unequally partition the sample period (January 1, 2009, to December 31, 2018) for robustness tests in a later subsection.

4.4. Estimating $\Sigma$, $\mu$, $\mu_1$, and $\mathbf{d}$ by the Data from 2009 to 2013.

In order to estimate portfolio-selection parameters, we follow [111], p.87, and compute the sample covariance matrix and sample mean vector of the 152 component stocks’ monthly returns from January 1, 2009, to December 31, 2013. We next annualize the matrix and vector by following [3], pp.123&133. Because the sample covariance matrix is singular but invertible covariance matrices are required for this paper, we add 0.1 to the diagonal elements. We then assume the matrix and vector as $\Sigma$ and $\mu$, respectively.

Similarly, we compute the sample mean vectors of the annual green innovations and annual dividend yields from 2009 to 2013 and then assume them as $\mu_1$ and $\mathbf{d}$, respectively.

4.5. Optimizing (20) and Dominating the Index by the Data from 2009 to 2013. Among the 152 component stocks, we follow the screening strategies and drop the stocks whose green innovations from 2009 to 2018 are continually zero. Among the remaining component stocks, we follow [111], p.21, and calculate the market-capitalization-weighted index $\mathbf{x}_{g^*}$. We substitute $\mathbf{x}_{g^*}$ into (20) and obtain the green innovation index in $(z_1, z_2, z_3)$ space as follows:

$$z_g = \begin{bmatrix} z_{g1} \\ z_{g2} \\ z_{g3} \end{bmatrix} = \begin{bmatrix} (x_g)^T \Sigma x_g \\ (x_g)^T \mu \\ (x_g)^T \mu_1 \end{bmatrix} = \begin{bmatrix} 0.0456 \\ 0.0634 \\ 2.6962 \end{bmatrix}.$$  
(23)

With $z_{g1} = 0.0456$, the plane passing through $z_g$ and being parallel to $(z_2, z_3)$ subspace is as follows:

$$z_1 = 0.0456.$$  
(24)
We compute the minimum-variance surface (8) as follows:

\[ z_1 = 0.0394z_2^2 + 0.0014z_3^2 + 0.0006z_2z_3 - 0.0121z_2z_3 - 0.0009z_3 + 0.0107. \] (25)

The plane intersects the feasible region \( Z \) of (20). The intersection is depicted as shaded in Figure 7. By (24)–(25), we compute the boundary of the intersection as an ellipse as follows:

\[ 0.0456 = 0.0394z_2^2 + 0.0014z_3^2 + 0.0006z_2z_3 - 0.0121z_2z_3 - 0.0009z_3 + 0.0107. \] (26)

By solving (11) and (26), we locate \( z_{i2} \) and \( z_{i3} \) as follows:

\[ z_{i2} = (0.0456, 1.1022, 0.6407). \] (27)

\[ z_{i3} = (0.0456, 0.1254, 5.2810). \] (28)

Although Figure 7 depicts \((z_2, z_3)\) subspace, we still label points in \((z_1, z_2, z_3)\) format (e.g., \( z_{i2} = (0.0456, 1.1022, 0.6407) \)). The plane (24) intersects the efficient surface of (20) at a thicker curve from \( z_{i2} \) to \( z_{i3} \) in Figure 7. By comparing \( z_g \) vs. both \( z_{i2} \) and \( z_{i3} \), we know that Case 2 of the four cases of Theorem 1 happens. We accordingly compute \( z_{d2} \) and \( z_{d3} \) as follows:

\[ z_{d2} = (0.0456, 0.9712, 2.6962), \]
\[ z_{d3} = (0.0456, 0.1254, 5.2810). \] (29)

The thickest curve from \( z_{d2} \) to \( z_{d3} \) in Figure 7 dominates the green innovation index \( z_g \) the most. We choose \( z_{d1} \ldots z_{d7} \) which are evenly spaced on the curve. We label \( z_{d2} \ldots z_{d7} \) in Figure 7, substitute them into (10), and obtain their portfolio-weight vectors \( x_{d2} \ldots x_{d7} \).

4.6. Testing Hypotheses for the Out-of-Sample Dominance by the Data from 2014 to 2018. By the historical returns and green innovations from 2014 to 2018 and \( x_{d2} \ldots x_{d7} \) and \( x_g \), we compute the historical returns and green innovations of the portfolios \( z_{d2} \ldots z_{d7} \) and \( z_g \) from 2014 to 2018.

We then take \( z_{d2} \) vs. \( z_g \), \ldots, \( z_{d7} \) vs. \( z_g \) for hypotheses (15)–(17) and report the result in Table 2. The first row lists \( z_{d2} \) vs. \( z_g \), \ldots, \( z_{d7} \) vs. \( z_g \). For hypothesis (15), the second to fourth rows list the test statistic (as described by [82], pp.497–499), \( p \)-value, and decision. For hypothesis (16), the 5th to 7th rows list the test statistic (as described by [82], pp.454–457), \( p \)-value, and decision. For hypothesis (17), the 8th to 10th rows list the test statistic (as described by [82], pp.454–457), \( p \)-value, and decision. The last row lists whether the dominance holds. We universally set the level of significance as 0.10. For example for \( z_{d2} \) vs. \( z_g \) in the second column, the statistic, \( p \)-value, and decision for (15) are 0.0562, \( p \)-value, and the dominance does not hold (i.e., portfolio \( z_{d2} \) does not dominate the green innovation index \( z_g \)). In the same format, portfolios \( z_{d3} \ldots z_{d7} \) dominate the green innovation index \( z_g \).

4.7. Detecting the Number of Stocks of the Dominating Portfolios \( z_{d2} \) to \( z_{d7} \). On the basis of (20) and \( n = 152 \), we further operationalize (10) as follows:

\[ x = \Sigma_{i=152}^{-1} \begin{bmatrix} \mu_2 & \mu_3 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ \end{bmatrix} \begin{bmatrix} z_2 \\ z_3 \\ z_4 \\ z_5 \\ \end{bmatrix}. \] (29)

We filter the nonzero elements of \( x \) (29) and count the nonzero element number as a diversification measure. We notice that \( z_{d2} \ldots z_{d7} \) and \( z_{d8} \ldots z_{d11} \) all consist of 152 stocks (i.e., complete diversification).

We also filter the nonzero elements of \( \mu_i \) due to its skewness (as reported in Table 1). We match the nonzero elements of \( x \) and \( \mu_i \), count the number, and thus obtain the number of stocks which are engaged in green innovation. For example, we list the following \( x \) and \( \mu_3 \) of \( n = 4 \) for simplicity:

\[ x = \begin{bmatrix} 0.1 & 0.3 & 0.6 \end{bmatrix}, \]
\[ \mu_3 = \begin{bmatrix} 1.0 & 2.0 & 0 & 4.0 \end{bmatrix}. \] (30)

The number of engaged stocks is two.

For the sample with \( n = 152 \), the numbers of engaged stocks of \( z_{d2} \ldots z_{d7} \) are all 45. For the next subsection, the numbers of engaged stocks of \( z_{d8} \ldots z_{d11} \) are all 45 as well.

4.8. Robustness Tests for \( z_{d8} \) to \( z_{d7} \). To check the robustness of the research methodology, we choose \( z_{d8} \ldots z_{d11} \) instead of \( z_{d4} \ldots z_{d7} \) on the curve from \( z_{d2} \) to \( z_{d3} \). We express \( z_{d8} \ldots z_{d11} \) in Table 3 (e.g., \( z_{d8} = (0.0456, 0.9048, 3.2131) \) in the second column).
We then take $z_{d6}$ vs. $z_{g'}$, ..., $z_{d11}$ vs. $z_{g}$ for hypotheses (15)–(17) and report the result in Table 4. Portfolios $z_{d3}$ and $z_{d8} \ldots z_{d11}$ dominate the green innovation index $z_{g}$. Overall, the results of Tables 2 and 4 are similar. The robustness of the research methodology can be verified.

As a comment, scholars typically obtain insignificant results in testing portfolios’ returns against indexes’ returns due to the volatile nature of stock returns. For example, [114], p.1936, [58], p.1181, [115], p.257, and [116], p.509, report similar insignificant results.
4.9. Robustness Tests with Different Sample Partitions. We also unequally partition the sample period (January 1, 2009, to December 31, 2018) as follows:

- from January 1, 2009, to December 31, 2014, as an in-sample subperiod and from January 1, 2015, to December 31, 2018, as an out-of-sample subperiod;
- from January 1, 2009, to December 31, 2015, as an in-sample subperiod and from January 1, 2016, to December 31, 2018, as an out-of-sample subperiod.

For both partitions, we obtain results similar to those reported in Tables 2–4. For all the equal and unequal partitions, we have deposited all the data, codes, and results in Mendeley Data https://data.mendeley.com/. Please see [104] in the references.

5. Further Formulating Green Innovation by General $k$-Objective Portfolio Selection

In this section, we divide green innovation into categories, further formulate each category as an objective, construct general $k$-objective portfolio selection, and demonstrate the existence of portfolios which dominate green innovation indexes. The advantage of this further formulation is that investors can consider and control each category.

5.1. Dividing Green Innovation into Categories (Green Technology Innovation and Green Management Innovation) and Formulating

5.1.1. Formulating the Categories by a 4-Objective Model. Reference [117], p.463, divides green innovation into two categories: green technology innovation and green management innovation. Therefore, we follow the division and further formulate each category as an objective in the following model:

$$
\begin{aligned}
\min z_1 &= \mathbf{x}^T \mathbf{\Sigma} \mathbf{x}, \text{ risk} \\
\max z_2 &= \mathbf{x}^T \mathbf{\mu}, \text{ return} \\
\max z_3 &= \mathbf{x}^T \mathbf{\mu}_3, \text{ green technology innovation} \\
\max z_4 &= \mathbf{x}^T \mathbf{\mu}_4, \text{ green management innovation} \\
\end{aligned}
$$

where

- $\mathbf{\mu}$ is a vector of stock expected green technology innovations,
- $\mathbf{\mu}_3$ is a vector of stock expected green management innovations,
- $z_1$ measures the portfolio expected green technology innovation,
- $z_4$ measures the portfolio expected green management innovation.

5.1.2. Still Dominating the Green Innovation Index by (31). By the analytical result for (6), we can extend the computations of (13)–(17) and extend Theorem 1 for (31). To save space, the detailed computations are skipped; we just present the key computation and extend Figure 2 to Figure 8.

For both partitions, we obtain results similar to those reported in Tables 2–4. For all the equal and unequal partitions, we have deposited all the data, codes, and results in Mendeley Data https://data.mendeley.com/. Please see [104] in the references.

The plane intersects the feasible region $Z$ of (31). Because every portfolio on the intersection shares the same $z_1$ (i.e., $z_1 = z_{g1}$), we need to focus on just $(z_2, z_3, z_4)$ elements of the intersection. The intersection is depicted in the lower left part of Figure 8. Because [22] proves the minimum-variance surface as a paraboloid and the surface is the boundary of the feasible region, the boundary of the intersection is an ellipsoid as follows:

$$
\begin{aligned}
z_{g1} &= p_1 z_1^2 + p_2 z_2^2 + p_3 z_3^2 + p_4 z_2 z_3 + p_5 z_2 z_4 + p_6 z_3 z_4 \\
&+ p_7 z_2 + p_8 z_3 + p_9 z_4 + p_{10}.
\end{aligned}
$$

Moreover, we can select a portion of the ellipsoid, so every portfolio on the portion dominates the green innovation index. The portion is depicted in the lower right part of Figure 8.

5.2. Further Dividing Green Innovation and Formulating. World Intellectual Property Organization divides green innovation into seven categories (data source: https://www.wipo.int/classifications/ipc/en/green_inventory/, July 8, 2020). We formulate them in the following model:

$$
\begin{aligned}
\min z_1 &= \mathbf{x}^T \mathbf{\Sigma} \mathbf{x}, \text{ risk} \\
\max z_2 &= \mathbf{x}^T \mathbf{\mu}, \text{ return} \\
\max z_3 &= \mathbf{x}^T \mathbf{\mu}_3, \text{ regulatory or design aspects} \\
\max z_4 &= \mathbf{x}^T \mathbf{\mu}_4, \text{ agriculture or forestry} \\
\max z_5 &= \mathbf{x}^T \mathbf{\mu}_5, \text{ alternative energy production} \\
\max z_6 &= \mathbf{x}^T \mathbf{\mu}_6, \text{ energy conservation} \\
\max z_7 &= \mathbf{x}^T \mathbf{\mu}_7, \text{ nuclear power generation} \\
\max z_8 &= \mathbf{x}^T \mathbf{\mu}_8, \text{ transportation} \\
\max z_9 &= \mathbf{x}^T \mathbf{\mu}_9, \text{ waste management} \\
s.t. A^T \mathbf{x} &= \mathbf{b},
\end{aligned}
$$

where
\[ \mathbf{\mu}_1 \ldots \mathbf{\mu}_9 \] are vectors of stock expected categories, 
\[ z_3 \ldots z_9 \] measure the portfolio expected categories.

Furthermore, researchers propose different divisions. Reference [1], p.318, divides green innovation into green product innovation and green process innovation. Reference [118], pp.1829–1830, extracts and classifies green patents on the basis of the division of World Intellectual Property Organization. Overall, we can follow the divisions, treat them as objectives, formulate them by multiple-objective portfolio selection, and compute the dominating portfolios by (6).

6. Conclusions

6.1. Positive Methodology vs. Normative Methodology. On the basis of (4), Nobel laureate [119], pp.433–434, assumes investor homogeneity and proposes his profound CAPM (capital asset pricing model). Although (4) and investor homogeneity are too simplistic and even unrealistic, [119], p.434, justifies his methodology by arguing for the CAPM’s insights. References [119], p.425, and [120], p.99, call this methodology of simplistic assumptions but profound insights as positive methodology.

In contrast, [94, 95] propose a general portfolio-selection model (1) which incorporates kinds of practical constraints. They deploy parametric quadratic programming to solve (1). References [119], p.426, and [120], p.99, call this general-model methodology as normative methodology. It could be a pity that [9, 94] may totally concentrate on the model and optimization rather than contemplating simplistic but profound insights.

We principally pursue positive methodology by assuming simplistic constraints but trying to unveil dominating-portfolio strategies.

6.2. Theoretical Implications. Armed with multiple-objective portfolio selection for green innovation, investors can substantially enrich green innovation investments. Particularly, investors can upgrade security selection to portfolio selection, take green innovation as objectives, obtain the efficient surface, and select portfolios which dominate green innovation indexes.

In the sample, \[ z_{d2} \ldots z_{d7} \] and \[ z_{d8} \ldots z_{d11} \] all consist of 152 stocks, enjoy complete diversification, and contain 45 stocks which are engaged in green innovation. Investors can naturally increase sample size to check this status of being both efficient and diversified.

Reference [73], p.61, clarifies the consistency between portfolio selection and maximizing-expected-utility approaches. Consequently for (3), investors can naturally extend the traditional utility function and probe the extended utility function for green innovation.

6.3. Practical Implications. Instead of the traditionally passive screening-indexing tool, investors perceive a new instrument to actively construct portfolios for risk, return, and green innovation.

Because portfolio selection carries mixed results of out-of-sample performances, investors should not be too fascinated by the formulation (3) and should put (3) into more rigorous tests.
In the more rigorous tests, investors can install kinds of actual constraints (e.g., bounds of $x$, industry restrictions, and transaction costs). For the optimization to charter the unknown territory depicted in Figures 4–6, investors can channel genetic algorithms and even fuzzy-number methods (as commenced by [101, 102]).

6.4. Concluding Remarks. Instigating green innovation as multiple objectives opens new opportunities for portfolio selection and green innovation investments. New areas which may not be previously contemplated are now brought to the theoretical and practical forefront.

Data Availability

The authors have deposited all the data, codes, and results in Mendeley Data https://doi.org/10.17632/rcwggc5xnh.1. Please see [104] in the references.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References


