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Research Article

Constructing Multiple-Objective Portfolio Selection for Green Innovation and Dominating Green Innovation Indexes

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Green innovation investments have rapidly grown since 2000. Green innovation indexes play important roles and are typically constructed by screening and indexing. However, Nobel Laureate Markowitz emphasizes portfolio selection instead of security selection and accentuates that "A good portfolio is more than a long list of good stocks." Moreover, the screening-indexing strategies ignore that investors can take green innovation as an additional objective and thus gain additional utility. We consequently construct 3-objective portfolio selection for green innovation in addition to variance and expected return. An efficient frontier of portfolio selection then extends to an efficient surface which is a panorama of the optimal variance, expected return, and expected green innovation. Investors thus fully envisage the trade-offs and enjoy the freedom of choosing preferred portfolios on the surface. In contrast, the screening-indexing strategies inflexibly leave investors with only one point (i.e., the green innovation index). As the originality, we prove in a theorem that there typically exists a curve on the efficient surface so all portfolios on the curve dominate the green innovation index. We test the dominance by component stocks of China Securities Index 300 and obtain affirmative results out of sample. The results still hold in robustness tests. At last, we classify green innovation into categories, further model the categories by general k-objective portfolio selection, and still illustrate the dominance. Consequently, investors can consider and control each category.

1. Introduction

1.1. Green Innovation Investments. As the COVID-19 pandemic unleashes crises of health and environment, environment and green are becoming more critical issues. Reference [1], p.310, defines green innovation as innovation mechanisms to improve environment. Green innovation investments have rapidly grown since 2000. As [2], p.631, contends sustainability as a driver of green innovation, the Global Sustainable Investment Review reports that global sustainable-investments assets reached $30.7 trillion in 2018 with a 34% increase from 2016.

Green innovation investments are typically fulfilled by screening-indexing strategies. Screening essentially means that investors exclude stocks of potentially socially questionable firms (e.g., gambling or tobacco). Indexing means that investors replicate a capital-market index (as documented by [3], p.341). For example, Morgan Stanley Capital International Inc. (MSCI) deploys the screening-indexing strategies for MSCI KLD 400 Social Index. The Forum of Sustainable and Responsible Investment and the Principles for Responsible Investment Association harness similar strategies.

On one side, the screening-indexing strategies are highly practical and thus substantially expedite green innovation

investments. However on the other side, the strategies suffer from the following weakness:

(1) The strategies build portfolios by a list of good stocks and thus ignore portfolio completeness, because [4], p.3, emphasizes portfolio selection rather than security selection and stresses portfolio wholeness.

(2) The strategies ignore that investors can take green innovation as an additional objective and thus gain additional utility. Reference [5], p.683, deduces that investors obtain utility from social responsibility. Reference [6], p.2, highlights society objectives and the urge to balance them with risk and return.

(3) The strategies inflexibly prescribe an index for most investors and disregard investor difference. Screening out some kinds of stocks is appropriate for some investors but inappropriate for others. References [7], p.5, and [8], pp.27&28, contend that investors are fairly different.

1.2. Portfolio Selection. Reference [4], p.6, discovers that investors mind both risk and return. Therefore, [9], p.83, formulates portfolio selection as the following 2-objective optimization:

$$\begin{aligned} &\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\}, \text{risk},\\ &\max\left\{z_2 = \mathbf{x}^T\boldsymbol{\mu}\right\}, \text{return},\\ &\text{s.t. } \mathbf{x} \in S, \text{feasible region}. \end{aligned} \tag{1}$$

For n stocks:

$\boldsymbol{\mu}$ is a vector of stock expected returns,

superscript T denotes transposition,

$\boldsymbol{\Sigma}$ is a covariance matrix of stock returns,

$\mathbf{x}$ is a portfolio-weight vector,

z_1 measures the variance of the return of portfolio $\mathbf{x}$,

z_2 measures the expectation of the return of portfolio $\mathbf{x}$,

S is a feasible region in $\mathbb{R}^n$,

$Z = \{(z_1, z_2)|\mathbf{x} \in S\}$ is the feasible region in (z_1, z_2) space.

Reference [9] calls the optimal solutions in (z_1, z_2) space as an efficient frontier. References [10, 11] fully describe multiple-objective optimization. For notations, normal symbols (e.g., n) denote scalars; bold-face symbols (e.g., $\mathbf{x}$ or $\boldsymbol{\Sigma}$) denote vectors or matrices.

Mathematically, (1) maps S to Z. For instance, we depict $\mathbf{x}_1 \ldots \mathbf{x}_3$ to $\mathbf{z}_1 \ldots \mathbf{z}_3$ by three arrows in the upper part of Figure 1. We also depict S and Z as shaded regions and depict the efficient frontier as a thick curve.

1.3. Multiple-Objective Portfolio Selection. Reference [12], pp.471&476, later discovers additional objectives (in addition to the variance and expectation). Reference [13] also realizes additional objectives and incorporates them into a utility function. References [14], pp.445–447, [15], pp.62–63, and [16], pp.1081–1082, emphasize multiple factors for asset pricing and further propose the factors' risks as objectives. Reference [17], p.182, advocates expressive benefits (e.g., social responsibility) as objectives.

References [14, 18–21] and [22] formulate additional objectives by multiple-objective portfolio selection as follows:

$$\begin{aligned} &\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\}, \text{risk}\\ &\max\left\{z_2 = \mathbf{x}^T\boldsymbol{\mu}\right\}, \text{return}\\ &\max\left\{z_3 = \mathbf{x}^T\boldsymbol{\mu}_3\right\}, \text{general objective 1}\\ &\vdots\\ &\max\left\{z_k = \mathbf{x}^T\boldsymbol{\mu}_k\right\}, \text{general objective } k-2\\ &\text{s.t. } \mathbf{x} \in S, \text{feasible region}, \end{aligned} \tag{2}$$

where

$\boldsymbol{\mu}_3 \ldots \boldsymbol{\mu}_k$ are vectors of general stock expected objectives (e.g., expected R&D and liquidity),

$z_3 \ldots z_k$ measure the general portfolio expected objectives,

$Z = \{(z_1, \ldots, z_k)|\mathbf{x} \in S\}$ is the feasible region in $(z_1, \ldots, z_k)$ space.

Mathematically, (2) maps S to Z. For instance, we depict $\mathbf{x}_1 \ldots \mathbf{x}_3$ to $\mathbf{z}_1 \ldots \mathbf{z}_3$ by three arrows in the lower part of Figure 1. We also depict S and Z as shaded regions. The S of (1) and of (2) remains the same. The Z of (1) extends to a high-dimensional set for (2). The efficient frontier of (1) also extends to a high-dimensional efficient surface for (2) (denoted as N). The surface is the optimal solutions of (2) in $(z_1, \ldots, z_k)$ space.

1.4. Originality for Green Innovation Investments

1.4.1. Formulation Originality: 3-Objective Portfolio Selection for Green Innovation. We originate the following 3-objective model for green innovation by taking $\boldsymbol{\mu}_3$ as a vector of stock expected green innovations and taking z_3 as the portfolio expected green innovation:

$$\begin{aligned} &\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\}, \text{risk}\\ &\max\left\{z_2 = \mathbf{x}^T\boldsymbol{\mu}\right\}, \text{return}\\ &\max\left\{z_3 = \mathbf{x}^T\boldsymbol{\mu}_3\right\}, \text{green innovation}\\ &\text{s.t. } \mathbf{A}^T\mathbf{x} = \mathbf{b}, \text{constraints}, \end{aligned} \tag{3}$$

where

$\mathbf{A}$ is an $n \times m$ constraint matrix,

$\mathbf{b}$ is an m-vector for the right hand side.

Equation (3) can overcome the weakness of the screening-indexing strategies as follows:

First (3) hinges on portfolio selection and thus enjoys portfolio completeness. Second, (3) explicitly operates green innovation as an additional objective and reserves rooms for the utility. Last, (3) prescribes a whole efficient surface for investors.

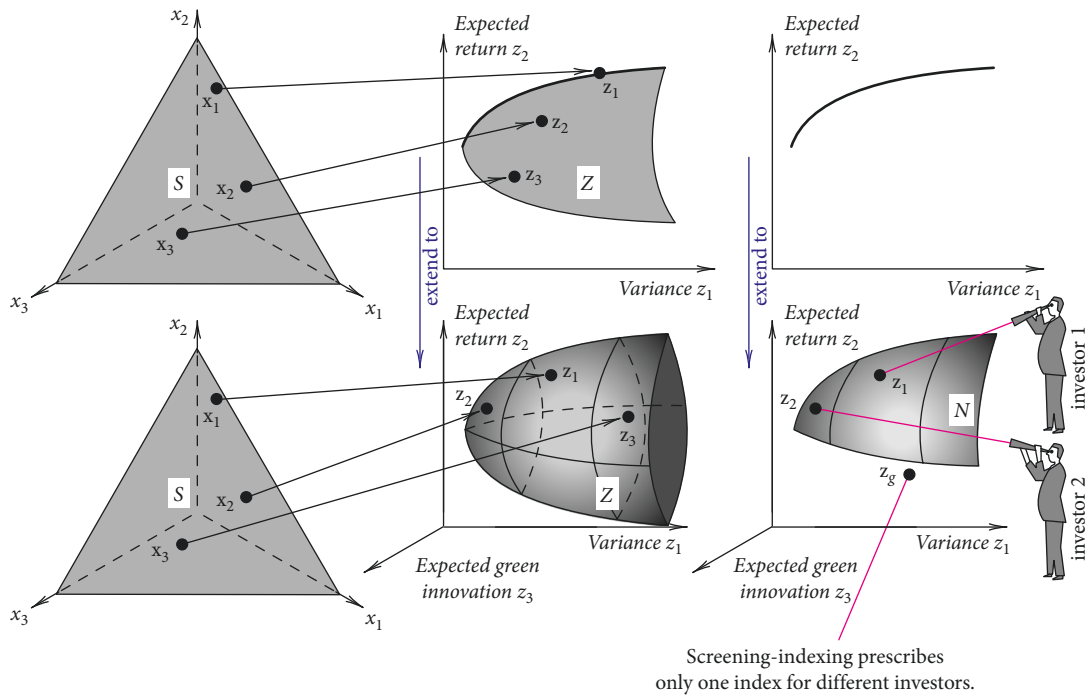

Figure 1: Portfolio selection (the upper part) and multiple-objective portfolio selection and its advantage (the lower part).

With the sheer amount of portfolios on the surface, different investors can pick distinctive portfolios. For instance, investor 1 prefers portfolio $\mathbf{z}_1$, while investor 2 prefers portfolio $\mathbf{z}_2$ (as depicted in the lower part of Figure 1). In contrast, the screening-indexing strategies invariably prescribe only one index for vastly different investors. The index is denoted as $\mathbf{z}_g$ (as depicted in the lower part of Figure 1).

1.4.2. Formulation Advantage: Dominating Green Innovation Indexes. Moreover, investors can exploit the efficient surface and harvest portfolios which dominate the green innovation index $\mathbf{z}_g$. As the originality, we will prove in a theorem that there exist portfolios which

(1) enjoy both better expected return and better expected green innovation than the green innovation index does;

(2) possess the same variance as that of the green innovation index.

The portfolios are depicted as the thickest curve from $\mathbf{z}_{d2}$ to $\mathbf{z}_{d3}$ in the right part of Figure 2. The figure will be deliberated later.

1.5. Paper Structure. The rest of this paper is organized as follows: We review theoretical background in the next section. Then in the next section, we prove the existence of dominating portfolios by a theorem and propose hypotheses for the out-of-sample dominance. Then in the next section, we measure green innovation on the basis of patent-citation numbers, set up (3), compute the dominance, test the hypotheses by component stocks of China Securities Index 300, and obtain affirmative results. Then in the next section, we divide green innovation into categories, further formulate the categories as objectives, extend (3) into general k-objective portfolio selection, and still demonstrate the dominance. We conclude this paper in the last section.

2. Literature Review: Green Innovation Investments and Multiple-Objective Portfolio Selection

We briefly review green innovation measurement and green innovation investments, summarize multiple-objective portfolio selection, and display quantitative results.

2.1. Overall Research for Green Innovation. A large body of green innovation research focuses on

(1) the driving force behind green innovation;

(2) the impact of green innovation on corporations or society.

For research line 1, for example, [23] proposes regulation and pressure as the driving force and verifies the proposition by environment-related patents. Reference [24] proposes political capital as the driving force. Reference [25] proposes green supplier involvement.

For research line 2, for example, [26] contends that green innovation is positively correlated to corporate competitive advantage. Reference [27] reports positive correlations between green innovation and green core competence. Reference [1] concludes that green innovation has positive effects on organizational performance.

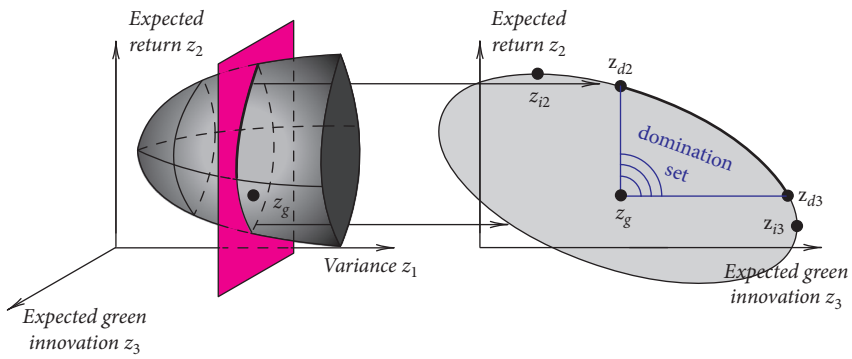

Figure 2: The plane passing through the green innovation index $\mathbf{z}_g$, intersection between the plane and feasible region Z, and portfolios which dominate the index (on the thickest curve from $\mathbf{z}_{d2}$ to $\mathbf{z}_{d3}$).

2.2. ESG Measurement and Green Innovation Measurement

2.2.1. ESG Measurement. Overall, ESG is a well-dissected but still evolving area. References [28, 29] review more than 2000 ESG studies, advocate future directions, and discover that the studies mainly focus on Europe and US. Reference [30] detects relatively slow ESG developments in Asia and deduces the reason as the lack of reliable ESG measurement. Reference [31] maneuvers rough-set approaches and forecasts ESG ratios by corporate financial performance variables. Several scholars further highlight the following weakness in ESG measurement.

First, scholars naturally instigate all kinds of ESG measurement. While financial measures are definite, environmental measures are relatively vague. To make things more diverse, there are more than 600 ESG-rating agencies with dissimilar measurement themes. References [32, 33] uncover inconsistent measurement results among key rating agencies (e.g., MSCI, Thomson Reuters, and GES).

Second, ESG measures heavily rely on corporate voluntary disclosures, but the disclosures are typically unaudited. Therefore, some corporations tend to manipulate their disclosures. References [34, 35] investigate such manipulation motivations. Reference [36] validates that some firms may exaggerate environmental performance for better images. Reference [37] further designs mechanisms to deter such exaggerations.

2.2.2. Green Innovation Measurement. In contrast, scholars sponsor relatively concentrated measurement for green innovation. Moreover, green innovation can be objectively measured by patent-citation numbers. The numbers are issued by independent third parties (e.g., World Intellectual Property Organization). Therefore, in contrast to the lack of reliable ESG measurement in Asia, green innovation can be relatively reliably measured in Asia. References [38, 39] survey green innovation development, recommend future directions, and endorse the patent-citation measurement.

In the early stage of green innovation development, scholars (e.g., [26, 40]) do not have sufficient patent-information support and thus typically utilize surveys to measure green innovation. Later, [23], pp.897–898, questions the potential insight of the surveys, emphasizes the potential survey bias, and proposes the total number of citations received by the patents as the measure. Similarly, [41], p.637, acknowledges patent-citation numbers and executes Citations/RD (RD as R&D). Reference [42], pp.2249–2250, also acknowledges patent-citation numbers and employs natural logarithms of one plus the numbers.

2.3. Green Innovation Investments. Reference[43] stresses the gigantic volume of green innovation investments, underscores the importance of portfolio selection, and inaugurates their models. Reference [29] surveys 463 articles and books for green innovation investments and reviews the following two major methods:

First, investors readily deploy the screening-indexing strategies. For example, [44] utilizes the strategies and reports some positive risk-adjusted performance. Reference [45] utilizes the strategies, compares green innovation investments against passive investments, and finds mixed results. Reference [46] utilizes the strategies and emphasizes opportunity costs for screening. Reference [47] picks the top 100 CSR companies in the world by Forbes, assembles market-capitalization-index portfolios, and locates significant abnormal returns. During the COVID-19 pandemic, [48] attests investor preference for ESG indexes and validates the outperformance. Reference [49] concentrates on the component stocks of S&P 500 Index, executes the screening-indexing strategies, and portrays confirmatory results. In contrast, [50] surveys main-stream investment organizations and finds the strategies relatively ineffective.

Second, mutual-fund companies structure funds by the screening-indexing strategies. For example, [51, 52] and [53] compare green innovation funds' returns against conventional mutual funds' returns but reveal the two groups of returns as typically identical. By further considering crisis and normal periods, [54] discovers that the two groups of returns are typically identical in crisis periods but the green innovation funds underperform the conventional mutual funds in normal periods. Reference [55] exposes that the risk-adjusted returns of funds with high green innovation measures equal those of funds with low green innovation measures. During the COVID-19 pandemic, [56] unveils that investors deliberate ESG risk and that low-ESG-risk funds outperform high-ESG-risk funds.

2.4. Multiple-Objective Portfolio Selection for Green Innovation and for Other Objectives. Reference[51], p.1723, concludes that SRI investors are willing to sacrifice financial performance for social objectives. The conclusion can be explained by multiple-objective portfolio selection for green innovation. That is, investors accept some optimal portfolios in (z_1, z_2, z_3) space of (3), although the portfolios are not optimal in (z_1, z_2) space of (1). Moreover, [57] considers the traditional financial goal and an ethical goal for utility functions and optimizes the 2-goal model. Reference [58] analyzes social responsibility by multiple-objective portfolio selection and studies the maximum-Sharpe-ratio portfolio and maximum-Delta-ratio portfolio.

For other objectives, [59] studies skewness. Reference [60], pp.119–120, reports that investors compute several risk measures in practice. Reference [61] studies two risk measures: variance and CVaR. Reference [62] analyzes tracking error. Reference [63] studies liquidity. Reference [64] studies R&D. References [65–69] and [70] offer surveys.

2.5. Multiple-Objective Portfolio Optimization with Equality Constraints Only

2.5.1. A Fundamental Model (4) for Portfolio Selection and Asset Pricing. References [71], pp.57–62, and [72] inspect the following model:

$$\begin{aligned} &\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\} \\ &\max\left\{z_2 = \mathbf{x}^T\boldsymbol{\mu}\right\} \\ &s.t. \mathbf{1}^T\mathbf{x} = 1. \end{aligned} \tag{4}$$

$\mathbf{1}$ is a vector of ones. Reference [72] analytically derives the minimum-variance frontier and proves the frontier as a parabola. The frontier is the boundary of the feasible region Z of (4) and is depicted as a curve in the upper part of Figure 1. Although unlimited portfolio weights are allowed, (4) enjoys analytical tractability and thus is widely deployed in portfolio selection and asset pricing (as acclaimed by [73], p.60).

2.5.2. Extending (4) by Adding an Objective. Reference [21] extends (4) by adding an objective as follows:

$$\begin{aligned} &\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\} \\ &\max\left\{z_2 = \mathbf{x}^T\boldsymbol{\mu}\right\} \\ &\max\left\{z_3 = \mathbf{x}^T\boldsymbol{\mu}_3\right\} \\ &s.t. \mathbf{1}^T\mathbf{x} = 1, \end{aligned} \tag{5}$$

where z_3 is a general objective. Reference [21] also analytically derives the result.

2.5.3. Further Extending (4) by Adding Objectives and Constraints. Reference [22] further extends (4) by adding objectives and constraints as follows:

$$\begin{aligned} &\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\} \\ &\max\left\{z_2 = \mathbf{x}^T\boldsymbol{\mu}\right\} \\ &\max\left\{z_3 = \mathbf{x}^T\boldsymbol{\mu}_3\right\} \\ &\vdots \\ &\max\left\{z_k = \mathbf{x}^T\boldsymbol{\mu}_k\right\} \\ &s.t. \mathbf{A}^T\mathbf{x} = \mathbf{b}. \end{aligned} \tag{6}$$

They embrace the following assumptions.

Assumption 1. Covariance matrix $\boldsymbol{\Sigma}$ is positive definite and thus invertible.

Assumption 2. Vectors $\boldsymbol{\mu}$ and $\boldsymbol{\mu}_3 \ldots \boldsymbol{\mu}_k$ and all columns of $\mathbf{A}$ (altogether $k-1+m$ vectors) are linearly independent.

Reference [22] commands an e-constraint method (as described by [10], pp.202–206) to (6) as follows:

$$\begin{aligned} &\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\} \\ &s.t.\ \mathbf{x}^T\boldsymbol{\mu} = e_2 \\ &\vdots \\ &\mathbf{x}^T\boldsymbol{\mu}_k = e_k \\ &\mathbf{A}^T\mathbf{x} = \mathbf{b}. \end{aligned} \tag{7}$$

$e_2 \ldots e_k$ are the method parameters. As $e_2 \ldots e_k$ vary, the set of all $(z_1, \ldots, z_k)$ vectors of the optimal solutions of (7) forms a minimum-variance surface of (6). As an extension of the minimum-variance frontier of (4), the surface is the boundary of the feasible region Z of (6) and is depicted as a surface in the lower part of Figure 1. Reference [22], pp.527–528, proves the surface as a paraboloid and analytically derives the surface as follows:

$$z_1 = \begin{bmatrix} z_2 & \ldots & z_k & \mathbf{b}^T \end{bmatrix}\left(\mathbf{M}^T\boldsymbol{\Sigma}^{-1}\mathbf{M}\right)^{-1}\begin{bmatrix} z_2 \\ \vdots \\ z_k \\ \mathbf{b} \end{bmatrix}, \tag{8}$$

where $\mathbf{M}$ is introduced as follows:

$$\mathbf{M} = \begin{bmatrix} \boldsymbol{\mu} & \boldsymbol{\mu}_3 & \ldots & \boldsymbol{\mu}_k & \mathbf{A} \end{bmatrix}_{n\times(k-1+m)}. \tag{9}$$

For a portfolio on the surface with given $z_2 \ldots z_k$, [22], p.526, computes the portfolio-weight vector $\mathbf{x}$ as follows:

$$\mathbf{x} = \boldsymbol{\Sigma}^{-1}\mathbf{M}\left(\mathbf{M}^T\boldsymbol{\Sigma}^{-1}\mathbf{M}\right)^{-1}\begin{bmatrix} z_2 \\ \vdots \\ z_k \\ \mathbf{b} \end{bmatrix}. \tag{10}$$

Moreover, [22], pp.529–531, derives the $\mathbf{x}$ set of the efficient surface of (6) as follows:

$$\left\{\mathbf{x} \in \mathbb{R}^n | \mathbf{x} = \mathbf{x}_{\mathrm{mv}} + \lambda_2\mathbf{g}_2 + \cdots + \lambda_k\mathbf{g}_k, \lambda_2 \ldots \lambda_k \geq 0\right\}, \tag{11}$$

where $\mathbf{x}_{\mathrm{mv}}$ and $\mathbf{g}_2 \ldots \mathbf{g}_k$ are computed as follows:

$$\mathbf{x}_{mv} = \boldsymbol{\Sigma}^{-1}\mathbf{A}\left(\mathbf{A}^T\boldsymbol{\Sigma}^{-1}\mathbf{A}\right)^{-1}\mathbf{b}$$

$$[\,\mathbf{g}_2 \ \cdots \ \mathbf{g}_k\,] = \frac{1}{2}\boldsymbol{\Sigma}^{-1}\left(\mathbf{I}_n - \mathbf{A}\left(\mathbf{A}^T\boldsymbol{\Sigma}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\boldsymbol{\Sigma}^{-1}\right) \cdot [\,\boldsymbol{\mu} \ \boldsymbol{\mu}_3 \ \cdots \ \boldsymbol{\mu}_k\,], \tag{12}$$

with $\mathbf{I}_n$ as an $n \times n$ identity matrix. The efficient surface is obtained by substituting (11) into (6).

We will compare methods for general multiple-objective portfolio optimization in the next section.

3. Dominating Green Innovation Indexes and Testing the Dominance by Hypotheses

As an advantage of (3), the efficient surface precisely and completely demonstrates the trade-offs among optimal variance, expected return, and expected green innovation. Therefore, investors can study the surface, envisage the trade-offs, and pinpoint preferred portfolios on the surface. Moreover, we prove that investors can obtain surface portfolios which dominate the green innovation index. We justify (3), present a graphical interpretation, validate the dominance by a theorem, and propose hypotheses for the out-of-sample dominance.

3.1. Justifying (3). We accentuate that (3) can overpower the weakness of the screening-indexing strategies as follows:

First, (3) hinges on portfolio selection and thus enjoys portfolio completeness. For an $n \times n$ covariance matrix $\boldsymbol{\Sigma}$, the screening-indexing strategies exploit only the n variance elements as individual stock risks, but portfolio selection and (3) additionally exploit the $n^2 - n/2$ overwhelmingly more covariance elements as stock corisks. Moreover, the screening-indexing strategies implement relatively simplistic indexing, while portfolio selection and (3) implement forceful optimization methodologies. Reference [74] features the scarcity of optimization methodologies in green innovation investments and endorses portfolio selection. Reference [75] also features the scarcity and commences all-underlying-functionality models. Reference [76] scans reasons of the screening-indexing strategies' relatively unsatisfactory performance and urges rudimentary linear regression models. Reference [77] engineers ESG portfolios by utility-based nonparametric models.

Second, (3) explicitly utilizes green innovation as an additional objective and reserves rooms for the utility. References [5], p.683, and [6], p.2, hint investors' such motives. Reference [78] implies that investors balance financial objectives versus ESG objectives and even relatively sacrifice financial objectives to enrich ESG objectives. Reference [79] surveys household preferences for ESG investments and also implies that investors relatively sacrifice financial objectives to enrich ESG objectives. Reference [80] substantiates that international institutional investors care about both financial objectives and social objectives.

Third, (3) prescribes a whole efficient surface for investors. Consequently, investors perceive the efficient surface as a panorama of the optimal variance, expected return, and expected green innovation and thus enjoy the freedom of choosing preferred portfolios on the surface. Different investors can pick distinctive surface portfolios. References [7], p.5, and [8], pp.27&28, feature investor difference. Reference [50] globally surveys ESG information and recounts information-usage difference in motivation, demand, product strategy, and ethical considerations. Reference [81] probes investors' political ideology and fashions portfolio-selection models for a unique group of investors. Reference [43] further explores three types of investors: ESG-unaware investors, ESG-aware investors, and ESG-motivated investors.

Last, we scrutinize green innovation (instead of ESG) in China for (3), because green innovation can be objectively measured by patent-citation numbers.

3.2. Graphical Interpretation. For (3), let $\mathbf{z}_g = (z_{g1}, z_{g2}, z_{g3})$ denote the green innovation index in (z_1, z_2, z_3) space with z_{g1}, z_{g2}, and z_{g3} as the variance, expected return, and expected green innovation of the index, respectively. Let $\mathbf{x}_g$ denote the portfolio-weight vector of the index. By screening, $\mathbf{x}_g$ bears some zero elements and thus is typically inefficient, because efficient portfolio-weight vectors (11) basically bear few zero elements. Therefore, $\mathbf{z}_g$ is typically not on the efficient surface.

In the left part of Figure 2, we draw a plane which passes through $\mathbf{z}_g$ and is parallel to (z_2, z_3) subspace. The plane is as follows:

$$z_1 = z_{g1}. \tag{13}$$

The plane intersects the feasible region Z of (3). Because every portfolio on the intersection shares the same z_1 (i.e., $z_1 = z_{g1}$), we need to focus on just (z_2, z_3) elements of the intersection in the right part of Figure 2. The intersection is depicted as shaded in the right part. Because [22] proves the minimum-variance surface as a paraboloid (8) and the surface is the boundary of the feasible region Z, the boundary of the intersection is an ellipse. By (8) with $k = 3$ and (13), the ellipse is as follows:

$$\left[\,z_2 \ \ldots \ z_k \ \ \mathbf{b}^T\,\right]\left(\mathbf{M}^T\boldsymbol{\Sigma}^{-1}\mathbf{M}\right)^{-1}\begin{bmatrix} z_2 \\ \vdots \\ z_k \\ \mathbf{b} \end{bmatrix} = z_{g1}. \tag{14}$$

Specifically, the plane intersects the efficient surface at a thicker curve in the right part of Figure 2. The intersection is depicted as a thicker curve from $\mathbf{z}_{i2}$ to $\mathbf{z}_{i3}$.

We utilize the domination-set method of [10], pp.150–154, and draw a domination set at $\mathbf{z}_g$ in blue color. Every portfolio in the set (except $\mathbf{z}_g$ itself) dominates $\mathbf{z}_g$. The set finally touches the ellipse at the thickest curve from $\mathbf{z}_{d2}$ to $\mathbf{z}_{d3}$; $\mathbf{z}_{d2}$ and $\mathbf{z}_{d3}$ are depicted in the right part of Figure 2. By the method, all the portfolios on the thickest curve dominate $\mathbf{z}_g$ the most.

3.3. Proving the Dominance by a Theorem. We reiterate the graphical interpretation in the following theorem.

Theorem 1. *In (z_1, z_2, z_3) space of (3), there exists a plane passing through $\mathbf{z}_g$ and being parallel to (z_2, z_3) subspace unless $\mathbf{z}_g$ is efficient. The plane intersects the efficient surface and feasible region Z of (3). On the intersection of the plane and efficient surface, there exists a whole range of portfolios which dominate $\mathbf{z}_g$. That is,*

the portfolios' variances are identical to the variance of $\mathbf{z}_g$,

the portfolios' expected returns are greater than or equal to the expected return of $\mathbf{z}_g$,

the portfolios' expected green innovations are greater than or equal to the expected green innovation of $\mathbf{z}_g$,

at least one of the two "greater than or equal to" relationships is "greater than."

Proof. The plane (13) intersects the Z of (3). Because all portfolios on the intersection share the same z_1 (i.e., $z_1 = z_{g1}$) and the efficient surface is the optimal portfolios of (3), the optimal solutions of the intersection are the thicker curve from $\mathbf{z}_{i2} = (z_{g1}, z_{i2,2}, z_{i2,3})$ to $\mathbf{z}_{i3} = (z_{g1}, z_{i3,2}, z_{i3,3})$ in the right part of Figure 2. Investors calculate $\mathbf{z}_{i2}$ and $\mathbf{z}_{i3}$ by (8) and (11).

If $\mathbf{z}_g$ is to the right of $\mathbf{z}_{i2}$ (i.e., $z_{g3} \geq z_{i2,3}$ as depicted in the right part of Figure 2), investors compute $\mathbf{z}_{d2} = (z_{g1}, z_{d2,2}, z_{g3})$ by substituting $z_3 = z_{g3}$ into (14), calculating the two roots, and taking $z_{d2,2}$ as the bigger root. Otherwise (i.e., to the left of $\mathbf{z}_{i2}$, $z_{g3} < z_{i2,3}$ as depicted in the upper left part of Figure 3), investors just take $\mathbf{z}_{d2} = \mathbf{z}_{i2}$.

Similarly, if $\mathbf{z}_g$ is above $\mathbf{z}_{i3}$ (i.e., $z_{g2} \geq z_{i3,2}$ as depicted in the right part of Figure 2), investors compute $\mathbf{z}_{d3} = (z_{g1}, z_{g2}, z_{d3,3})$ by substituting $z_2 = z_{g2}$ into (14), calculating the two roots, and taking $z_{d3,3}$ as the bigger root. Otherwise (i.e., below $\mathbf{z}_{i3}$, $z_{g2} < z_{i3,2}$ as depicted in the upper right part of Figure 3), investors just take $\mathbf{z}_{d3} = \mathbf{z}_{i3}$.

In summary, there are four exhaustively exclusive cases as follows:

(1) $z_{g3} \geq z_{i2,3}$ and $z_{g2} \geq z_{i3,2}$ (right part of Figure 2),

(2) $z_{g3} \geq z_{i2,3}$ and $z_{g2} < z_{i3,2}$ (upper right part of Figure 3),

(3) $z_{g3} < z_{i2,3}$ and $z_{g2} \geq z_{i3,2}$ (upper left part of Figure 3),

(4) $z_{g3} < z_{i2,3}$ and $z_{g2} < z_{i3,2}$ (lower left part of Figure 3).

For the four cases, investors exploit the domination-set method at $\mathbf{z}_g$. Every portfolio in the set dominates $\mathbf{z}_g$. Specifically, all the portfolios on the thickest curve from $\mathbf{z}_{d2}$ to $\mathbf{z}_{d3}$ dominate $\mathbf{z}_g$ the most. The portfolio-weight vectors can be computed on the basis of (10). □

3.4. Testing Hypotheses for the Out-of-Sample Dominance. By Theorem 1, we can choose a portfolio on the thickest curve from $\mathbf{z}_{d2}$ to $\mathbf{z}_{d3}$, collect out-of-sample data, compute the returns and green innovations of the portfolio and of the green innovation index $\mathbf{z}_g$, and test the following hypotheses:

$$H_0\text{: variance of the portfolio = variance of } \mathbf{z}_g, \tag{15}$$

$$\begin{aligned} &H_a\text{: variance of the portfolio} \neq \text{variance of } \mathbf{z}_g, \\ &H_0\text{: expected return of the portfolio} \\ &\quad = \text{expected return of } \mathbf{z}_g, \end{aligned} \tag{16}$$

$$\begin{aligned} &H_a\text{: expected return of the portfolio > expected return of } \mathbf{z}_g \\ &H_0\text{: expected green innovation of the portfolio =} \\ &\text{expected green innovation of } \mathbf{z}_g \\ &H_a\text{: expected green innovation of the portfolio >} \\ &\text{expected green innovation of } \mathbf{z}_g. \end{aligned} \tag{17}$$

Reference [82], pp.497–499 & 454–457, describes standard test statistics.

3.5. Portfolio Selection and Its Out-of-Sample Performance. Several researchers (e.g., [83–85] and [86]) criticize portfolio selection for its weak out-of-sample performance. However, the researchers' arguments can be reconciled in the following aspects.

First, [9] seminally quantitatively formulates portfolio risk and return rather than focusing on out-of-sample performance (as proclaimed by [87], p.1041). Therefore, portfolio selection is universally described in classic textbooks (e.g., those of [3, 88–90] and [91]).

Second, strong out-of-sample performance implicitly requires that the out-of-sample is preferably large and comes from the same independent and identical distribution as the in-sample distribution. However in reality, financial markets instantly react to all kinds of information; corporations prosper and decline over time. Therefore, the requirement is rarely satisfied. Consequently, weak out-of-sample performance can be caused by the sample distribution.

Last, investors subjectively hope to consistently beat markets without extra cost. However, objectively, such financial perpetual-motion machines violate fundamental finance-theory laws (e.g., efficient-market hypotheses) and thus barely exist.

In this paper, in sample, we extend portfolio selection, instigate 3-objective portfolio selection for green innovation, and prove the existence of dominating portfolios in a theorem. We will launch both theoretical and practical implications in the conclusion.

3.6. Contrasting Methods of Multiple-Objective Portfolio Optimization. We briefly contrast different methods for general multiple-objective portfolio optimization.

3.6.1. Analytical Methods. Analytical methods are conducted in the form of formulae on the basis of calculus and linear algebra and are thus understandable. We have already

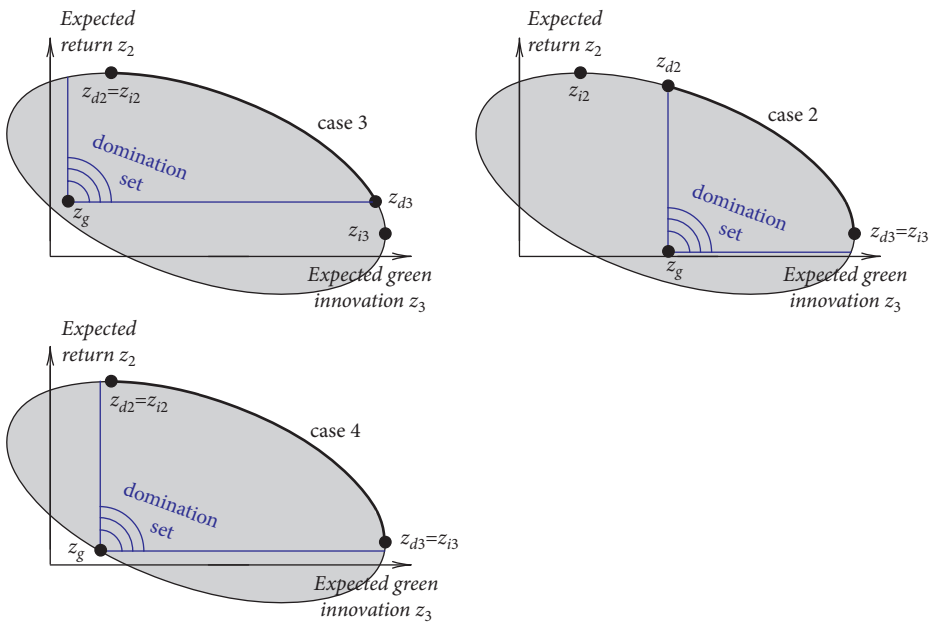

FIGURE 3: The three other cases of the relationship between $\mathbf{z}_g$ and $\mathbf{z}_{i2}$ and between $\mathbf{z}_g$ and $\mathbf{z}_{i3}$ for Theorem 1.

reviewed analytical methods in the previous section. We optimize (20) by analytical methods (8)–(11). Analytical methods' computational advantage is to readily reckon the efficient surface and dominating portfolios.

However, analytical methods decipher models with equality constraints only. Nevertheless with almost all results in formulae, analytical methods liberate researchers from mathematical programming. Therefore, researchers can enjoy the analytical tractability and empirical implications (as highlighted by [73], p.60).

3.6.2. Parametric Quadratic Programming. Parametric quadratic programming is an advanced form of quadratic programming with parameters in the formulation. References [92, 93] describe the topic in detail. Parametric quadratic programming can handle both equality constraints and inequality constraints. Particularly, inequality constraints can prescribe the following conditions:

some lower bound (floor) of $\mathbf{x}$ (e.g., $\mathbf{x} \geq 0$ to restrict short sales),

some lower bound and upper bound (ceiling) of an industry to regulate the industry exposure,

transaction cost.

To solve (1), [94, 95] deploy parametric quadratic programming and propose a *critical-line algorithm*. Reference [95], p.176, proves that the efficient frontier is piece-wisely made up by connected parabolic segments.

Reference[96] calculates the whole efficient surface of (2) with $k = 3$ and suggests that the surface is piece-wisely made up by connected paraboloidal segments. For example, a surface consists of two segments in the left part of Figure 4. The upper segment is a portion of paraboloid $z_1 = z_2^2 + 2z_3^2 + 3z_2z_3 + 4z_2 + 4z_3 + 9$; the lower segment is a portion of paraboloid $z_1 = z_2^2 + 5z_3^2 + 6z_2z_3 + 8z_2 + 9z_3 + 8$. The two segments connect at a thick curve. Reference [97] also exploits parametric quadratic programming and resolves (2).

However, parametric quadratic programming is much more convoluted than quadratic programming and thus rarely commanded. Moreover, parametric quadratic programming suffers from the following obstacles.

First, neither the shape nor the property of the feasible region $Z = \{(z_1, \ldots, z_k) | \mathbf{x} \in S\}$ of (2) is comprehensible. Investors can follow the method depicted in Figure 2 but can not envision the intersection between the plane $z_1 = z_{g1}$ (13) and Z. For example, the boundary of the intersection is unknown and depicted as broken curves in the right part of Figure 4. Therefore, the intersection is unknown either. With a green innovation index $\mathbf{z}_g$ near a corner, investors can obtain eccentric results by operating the domination-set method. In contrast, for (3), investors fully comprehend that the Z is a paraboloidal set by (8). Therefore, investors fully comprehend that the intersection between the plane (13) and Z is an elliptical area (as depicted in Figures 2 and 3).

Second, [96], p.181, proclaims that efficient surfaces of (2) with $k = 3$ and with $n = 102$ can be made up by 7994 connected paraboloidal segments. For the simplest only-1-segment efficient surface of (3), investors encounter 4 cases of the relationship between $\mathbf{z}_g$ and $\mathbf{z}_{i2}$ and between $\mathbf{z}_g$ and $\mathbf{z}_{i3}$ (as depicted in Figures 2 and 3). For the simplistic 2-segment efficient surface depicted in Figure 4, investors can encounter $2 \times 4 = 8$ cases of the relationship between $\mathbf{z}_g$ and $\mathbf{z}_{i2}$ and between $\mathbf{z}_g$ and $\mathbf{z}_{i3}$ and between $\mathbf{z}_g$ and $\mathbf{z}_{i4}$. For the 7994-segment efficient surfaces, investors will be perplexed by the case number.

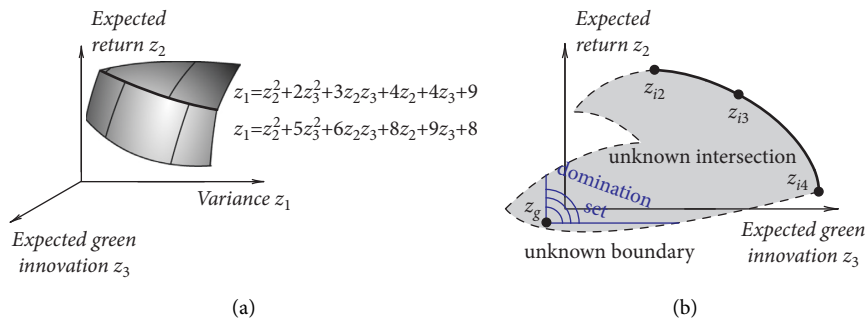

Figure 4: For (2) by parametric quadratic programming, an efficient surface with two segments (a) and the intersection between the plane $z_1 = z_{g1}$ (13) and the unknown feasible region Z (b).

Last, there is no public-domain software for (2) yet. Although [96] inaugurates and codes their algorithm, they do not publicly release their software.

3.6.3. Repetitive Quadratic Programming. Repetitive quadratic programming can handle both equality constraints and inequality constraints. Reference [10], pp.165–183&202–206, elucidates weighted-sums methods and e-constraint methods to transform multiple-objective optimization into (ordinary) 1-objective optimization. Investors can channel the methods for (2) as follows:

$$\begin{aligned} &\min\left\{\mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x} - \lambda_2\mathbf{x}^T\boldsymbol{\mu} - \lambda_3\mathbf{x}^T\boldsymbol{\mu}_3 - \cdots - \lambda_k\mathbf{x}^T\boldsymbol{\mu}_k\right\} \\ &s.t.\mathbf{x} \in S, \text{feasible region.} \end{aligned} \tag{18}$$

$$\begin{aligned} &\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\}, \\ &s.t.\mathbf{x}^T\boldsymbol{\mu} = e_2, \\ &\mathbf{x}^T\boldsymbol{\mu}_3 = e_3, \\ &\vdots \\ &\mathbf{x}^T\boldsymbol{\mu}_k = e_k, \\ &\mathbf{x} \in S, \end{aligned} \tag{19}$$

where

$\lambda_2 \ldots \lambda_k$ are coefficients for weighted-sums methods,

$e_2 \ldots e_k$ are coefficients for e-constraint methods.

Investors preset a group of $\lambda_2 \ldots \lambda_k$ and a group of $e_2 \ldots e_k$, repetitively solve (18) for each $\lambda_2 \ldots \lambda_k$, and repetitively solve (19) for each $e_2 \ldots e_k$. The optimal solutions are actually discrete versions of the efficient surface of (2). We depict some optimal solutions as isolated points in the left part of Figure 5. The isolated points are much less informative than the whole efficient surface depicted in the left part of Figure 4.

Repetitive quadratic programming is easy to understand and thus described in classic textbooks (e.g., Chapter 7 of [3]). Several computational software (e.g., Microsoft Excel and Matlab) can offer quadratic-programming solvers. However, repetitive quadratic programming suffers from the following obstacles:

First, the isolated points can cluster in small parts of the efficient surface instead of uniformly dispersing around the surface (as underlined by [98]).

Second, repetitive quadratic programming can not reveal the paraboloidal-segment structure. For instance, investors perceive just a bunch of isolated points in Figure 5.

Last, investors obtain also isolated points on the intersection and hardly execute the domination-set method for the points. We depict the situation in the right part of Figure 5.

3.6.4. Genetic Algorithms and Other Heuristic Methods. As important and popular tools, genetic algorithms and other heuristic methods can handle equality constraints, inequality constraints, and even special conditions (e.g., integer variables). Reference [99] systematically exhibits genetic algorithms. References [100, 101] and [102] employ genetic algorithms for multiple-objective portfolio selection. Reference [103] offers surveys for genetic algorithms.

However, heuristic methods inherently provide suboptimal solutions. For example, we depict some isolated points as suboptimal solutions in the left part of Figure 6. The points are below the efficient surface depicted in the left part of Figure 4. Moreover, the methods also suffer from the obstacles of repetitive quadratic programming. We depict some obstacles in the right part of Figure 6.

4. Empirical Tests by Component Stocks of China Securities Index 300

We sample the 300 component stocks of China Securities Index 300 from 2009 to 2018, measure the green innovation, gauge the dominating portfolios, test hypotheses for the out-of-sample dominance, implement robustness tests, and obtain supportive results. We report key computations and omit long calculations (e.g., 152×152 matrices). We have deposited all the data, codes, and results in Mendeley Data https://data.mendeley.com/. Please see [104] in the references.

4.1. Modeling. Specially, we develop the following model on the basis of (3):

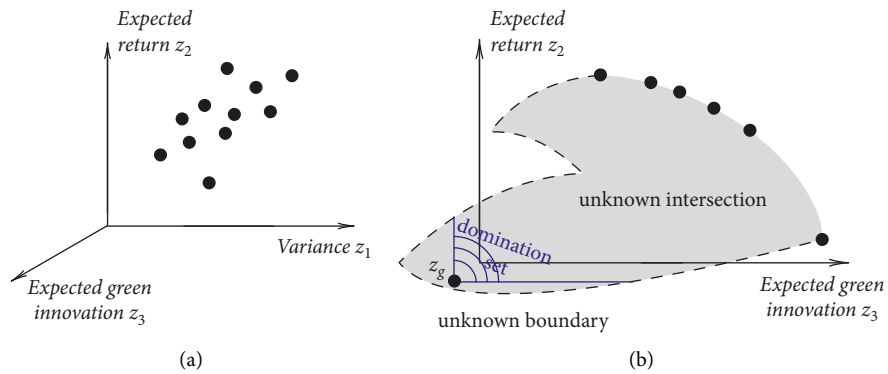

Figure 5: For (2) by repetitive quadratic programming, isolated points as discrete versions of the efficient surface (a) and the intersection between the plane $z_1 = z_{g1}$ (13) and the unknown feasible region Z (b).

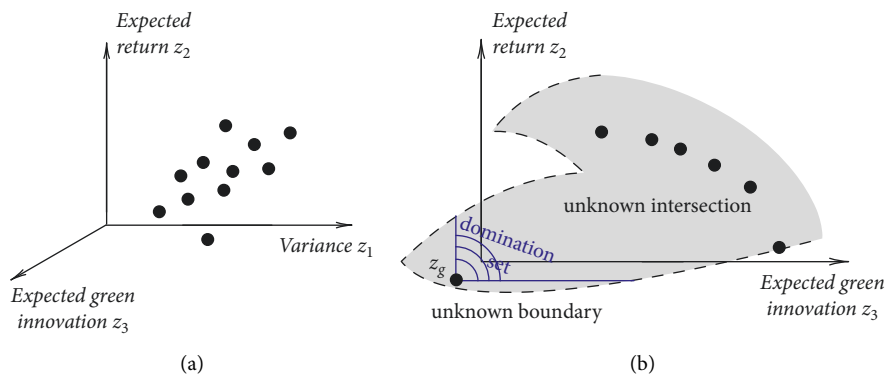

Figure 6: For (2) by genetic algorithms, isolated points to approximate the efficient surface (a) and the intersection between the plane $z_1 = z_{g1}$ (13) and the unknown feasible region Z (b).

$$\begin{aligned}
&\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\},\\
&\max\left\{z_2 = \mathbf{x}^T\boldsymbol{\mu}\right\},\\
&\max\left\{z_3 = \mathbf{x}^T\boldsymbol{\mu}_3\right\},\\
&s.t.\mathbf{1}^T\mathbf{x} = 1,\\
&\mathbf{d}^T\mathbf{x} = \overline{d},
\end{aligned} \tag{20}$$

$\mathbf{d}$ is a vector of stock expected dividend yields (dividend/price); $\overline{d}$ is a target expected dividend yield and taken as the average of $\mathbf{d}$.

Dividend stands as an important constraint. In theory, [105] proposes a theory of dividend by tax clienteles and concludes that corporations attract institutions by paying dividend. Reference [106] suggests a dividend-catering theory and contends that corporations cater to investors by paying dividend. Reference [107] contemplates discrete dividends and solves the corresponding portfolio-optimization model. Reference [108] demonstrates new evidence of dividend-catering theory in the emerging market. In practice, [109], p.71, assumes dividend as an important intrinsic-value factor.

4.2. Measuring Green Innovation of the Component Stocks by Patent-Citation Numbers. We follow [41, 42] and explicitly measure green innovation in the following steps:

(1) We also acknowledge patent-citation numbers.

(2) We sample all the 300 component stocks of China Securities Index 300 (data source: http://www.csindex.com.cn/en/indices/index-detail/000300, May 24, 2020). We follow empirical-test traditions to disregard finance-industry stocks and stocks with incomplete observations. We then obtain $n = 152$ component stocks with complete monthly returns from January 1, 2009, to December 31, 2018.

(3) For the 152 component stocks, we obtain the patent information (including international patent classification number) from 2009 to 2018 from State Intellectual Property Office of China (SIPO) (data source: http://pss-system.cnipa.gov.cn/sipopublicsearch/portal/uiIndex.shtml, May 29, 2020).

(4) We check the patent information for green innovation of World Intellectual Property Organization (WIPO) (data source: https://www.wipo.int/classifications/ipc/en/green_inventory/, June 5, 2020).

(5) We match the patent information of SIPO with that of WIPO by international patent classification number and finally obtain 6154 patents.

(6) We check Google Patent and obtain 39839 patent citations of the 6154 patents (data source: https://patents.google.com/patent/, June 21, 2020).

(7) Because [110], p.394, and [41], p.637, suggest considering patent-citation numbers in the previous years when they treat patent-citation numbers in this year, we accordingly measure green innovation of stock i in year t as follows:

$$G_{i,t} = \sum_{j=0}^{5} \sum_{f_{t-j}=1}^{F_{t-j}} C_{i,n_{t-j}}, \tag{21}$$

where F_{t-j} is total number of patents granted to stock i in year $t-j$ which are cited in year t; f_{t-j} is a counter with $f_{t-j} = 1 \ldots F_{t-j}$; and $C_{i,n_{t-j}}$ is the patent-citation number in year t by patent f_{t-j} which is granted to stock i in year $t-j$.

(8) Because researchers (e.g., [42], pp.2249–2250) demonstrate that patent-citation numbers are typically large and skewed, we accordingly take the natural logarithm as follows:

$$G_{i,t} = \log\left(1 + \sum_{j=0}^{5} \sum_{f_{t-j}=1}^{F_{t-j}} C_{i,n_{t-j}}\right). \tag{22}$$

We will describe summary statistics in the next subsection.

We do try to update the component stocks' green innovations to year 2019, but basically all the major Internet search or communication engines (e.g., Google, Yahoo, YouTube, Facebook, Instagram, and Twitter) are unavailable in our location. Moreover, numerous websites are implicitly linked to the engines and thus unreachable. However, we try to sufficiently exploit the existing sample (from 2009 to 2018) by varyingly partitioning the sample period for the robustness tests. We will report the result in a later subsection.

It is promising that [31] maneuvers rough-set approaches and forecasts ESG ratios by corporate financial performance variables. We will try to duplicate their research methods for green innovation.

4.3. Sampling Returns and Dividend Yields and Summary Statistics from 2009 to 2018. For the 152 component stocks of China Securities Index 300, we sample the stocks' monthly returns, annual dividend yields, and annual market capitalizations from January 1, 2009, to December 31, 2018 (data source: China Stock Market & Accounting Research Database, http://www.gtarsc.com, July 6, 2021). For each component stock, we compute its time-series sample means of annual green innovations, monthly returns, and annual dividend yields. For the 152 component stocks, we document the summary statisics of the cross section sample means in Table 1. For example, for the sample means of green innovations in the second column, the standard deviation is 0.7772. Particularly, the 25th percentile and median are both 0, but the 75th percentile is 0.1637. Therefore, the sample means of green innovations are skewed. Many corporations barely participate in green innovations. We will further dissect the issue and illuminate the number of participating stocks in a later subsection.

We separate January 1, 2009, to December 31, 2018, into the following subperiods:

(1) From January 1, 2009, to December 31, 2013, as an in-sample subperiod, we sample the data, estimate $\mathbf{\Sigma}$ and $\boldsymbol{\mu}$ and $\boldsymbol{\mu}_3$ for (20), set up (20), compute $\mathbf{z}_g$, and calculate the thickest curve from $\mathbf{z}_{d2}$ to $\mathbf{z}_{d3}$ of Theorem 1.

(2) From January 1, 2014, to December 31, 2018, as an out-of-sample subperiod, we select some portfolios on the thickest curve and test hypotheses (15)-(17).

(3) Reference [111], pp.103–104&142–143&149–150, frequently explicates designs of 5-year in-sample subperiods and next-5-year out-of-sample subperiods. References [112], p.1680, and [113], p.439, accordingly implement the designs. Consequently, we similarly implement the designs. Moreover, we will unequally partition the sample period (January 1, 2009, to December 31, 2018) for robustness tests in a later subsection.

4.4. Estimating $\mathbf{\Sigma}$, $\boldsymbol{\mu}$, $\boldsymbol{\mu}_3$, and $\mathbf{d}$ by the Data from 2009 to 2013. In order to estimate portfolio-selection parameters, we follow [111], p.87, and compute the sample covariance matrix and sample mean vector of the 152 component stocks' monthly returns from January 1, 2009, to December 31, 2013. We next annualize the matrix and vector by following [3], pp.123&133. Because the sample covariance matrix is singular but invertible covariance matrices are required for this paper, we add 0.1 to the diagonal elements. We then assume the matrix and vector as $\mathbf{\Sigma}$ and $\boldsymbol{\mu}$, respectively.

Similarly, we compute the sample mean vectors of the annual green innovations and annual dividend yields from 2009 to 2013 and then assume them as $\boldsymbol{\mu}_3$ and $\mathbf{d}$, respectively.

4.5. Optimizing (20) and Dominating the Index by the Data from 2009 to 2013. Among the 152 component stocks, we follow the screening strategies and drop the stocks whose green innovations from 2009 to 2018 are continually zero. Among the remaining component stocks, we follow [111], p.21, and calculate the market-capitalization-weighted index $\mathbf{x}_g$.

We substitute $\mathbf{x}_g$ into (20) and obtain the green innovation index in (z_1, z_2, z_3) space as follows:

$$\mathbf{z}_g = \begin{bmatrix} z_{g1} \\ z_{g2} \\ z_{g3} \end{bmatrix} = \begin{bmatrix} (\mathbf{x}_g)^T \mathbf{\Sigma} \mathbf{x}_g \\ (\mathbf{x}_g)^T \boldsymbol{\mu} \\ (\mathbf{x}_g)^T \boldsymbol{\mu}_3 \end{bmatrix} = \begin{bmatrix} 0.0456 \\ 0.0634 \\ 2.6962 \end{bmatrix}. \tag{23}$$

With $z_{g1} = 0.0456$, the plane passing through $\mathbf{z}_g$ and being parallel to (z_2, z_3) subspace is as follows:

$$z_1 = 0.0456. \tag{24}$$

Table 1: Descriptive statistics.

	Green innovation	Return (in %)	Dividend yield (in %)
Mean	0.2887	0.2267	1.3393
Standard deviation	0.7772	0.1795	1.0861
25th percentile	0	0.0918	0.5577
Median	0	0.2179	1.081
75th percentile	0.1637	0.3418	1.7864
Minimum	0	−0.2024	0
Maximum	5.9491	0.716	5.1259

We compute the minimum-variance surface (8) as follows:

$$\begin{aligned} z_1 = {} & 0.0394z_2^2 + 0.0014z_3^2 + 0.0006z_2z_3 \\ & - 0.0121z_2 - 0.0009z_3 + 0.0107. \end{aligned} \tag{25}$$

The plane intersects the feasible region Z of (20). The intersection is depicted as shaded in Figure 7. By (24)–(25), we compute the boundary of the intersection as an ellipse as follows:

$$\begin{aligned} 0.0456 = {} & 0.0394z_2^2 + 0.0014z_3^2 + 0.0006z_2z_3 \\ & - 0.0121z_2 - 0.0009z_3 + 0.0107. \end{aligned} \tag{26}$$

By solving (11) and (26), we locate $\mathbf{z}_{i2}$ and $\mathbf{z}_{i3}$ as follows:

$$\begin{aligned} \mathbf{z}_{i2} &= (0.0456, 1.1022, 0.6407). \\ \mathbf{z}_{i3} &= (0.0456, 0.1254, 5.2810). \end{aligned} \tag{27}$$

Although Figure 7 depicts (z_2, z_3) subspace, we still label points in (z_1, z_2, z_3) format (e.g., $\mathbf{z}_{i2} = (0.0456, 1.1022, 0.6407)$). The plane (24) intersects the efficient surface of (20) at a thicker curve from $\mathbf{z}_{i2}$ to $\mathbf{z}_{i3}$ in Figure 7. By comparing $\mathbf{z}_g$ vs. both $\mathbf{z}_{i2}$ and $\mathbf{z}_{i3}$, we know that Case 2 of the four cases of Theorem 1 happens. We accordingly compute $\mathbf{z}_{d2}$ and $\mathbf{z}_{d3}$ as follows:

$$\begin{aligned} \mathbf{z}_{d2} &= (0.0456, 0.9712, 2.6962), \\ \mathbf{z}_{d3} &= (0.0456, 0.1254, 5.2810). \end{aligned} \tag{28}$$

The thickest curve from $\mathbf{z}_{d2}$ to $\mathbf{z}_{d3}$ in Figure 7 dominates the green innovation index $\mathbf{z}_g$ the most. We choose $\mathbf{z}_{d4} \ldots \mathbf{z}_{d7}$ which are evenly spaced on the curve. We label $\mathbf{z}_{d2} \ldots \mathbf{z}_{d7}$ in Figure 7, substitute them into (10), and obtain their portfolio-weight vectors $\mathbf{x}_{d2} \ldots \mathbf{x}_{d7}$.

4.6. Testing Hypotheses for the Out-of-Sample Dominance by the Data from 2014 to 2018. By the historical returns and green innovations from 2014 to 2018 and $\mathbf{x}_{d2} \ldots \mathbf{x}_{d7}$ and $\mathbf{x}_g$, we compute the historical returns and green innovations of the portfolios $\mathbf{z}_{d2} \ldots \mathbf{z}_{d7}$ and $\mathbf{z}_g$ from 2014 to 2018.

We then take $\mathbf{z}_{d2}$ vs. $\mathbf{z}_g$, …, $\mathbf{z}_{d7}$ vs. $\mathbf{z}_g$ for hypotheses (15)–(17) and report the result in Table 2. The first row lists $\mathbf{z}_{d2}$ vs. $\mathbf{z}_g$, …, $\mathbf{z}_{d7}$ vs. $\mathbf{z}_g$. For hypothesis (15), the second to fourth rows list the test statistic (as described by [82], pp.497–499), p-value, and decision. For hypothesis (16), the 5th to 7th rows list the test statistic (as described by [82], pp.454–457), p-value, and decision. For hypothesis (17), the 8th to 10th rows list the test statistic (as described by [82], pp.454–457), p-value, and decision. The last row lists whether the dominance holds. We universally set the level of significance as 0.10. For example for $\mathbf{z}_{d2}$ vs. $\mathbf{z}_g$ in the second column, the statistic, p-value, and decision for (15) are 0.0562, $p > 0.10$, and "accept H_0". On the basis of accepting H_0 of (15), accepting H_0 of (16), and accepting H_0 of (17), we conclude that the dominance does not hold (i.e., portfolio $\mathbf{z}_{d2}$ does not dominate the green innovation index $\mathbf{z}_g$). In the same format, portfolios $\mathbf{z}_{d3} \ldots \mathbf{z}_{d7}$ dominate the green innovation index $\mathbf{z}_g$.

4.7. Detecting the Number of Stocks of the Dominating Portfolios $\mathbf{z}_{d2}$ to $\mathbf{z}_{d7}$. On the basis of (20) and $n = 152$, we further operationalize (10) as follows:

$$\begin{aligned} \mathbf{x} = {} & \boldsymbol{\Sigma}_{152\times152}^{-1} \left[\boldsymbol{\mu}_2 \;\; \boldsymbol{\mu}_3 \;\; \mathbf{1} \;\; \mathbf{d} \right]_{152\times4} \\ & \cdot \left(\begin{bmatrix} \boldsymbol{\mu}_2^T \\ \boldsymbol{\mu}_3^T \\ \mathbf{1}^T \\ \mathbf{d}^T \end{bmatrix} \boldsymbol{\Sigma}^{-1} \left[\boldsymbol{\mu}_2 \;\; \boldsymbol{\mu}_3 \;\; \mathbf{1} \;\; \mathbf{d} \right] \right)_{4\times4}^{-1} \begin{bmatrix} z_2 \\ z_3 \\ 1 \\ \overline{d} \end{bmatrix}_{4\times1}. \end{aligned} \tag{29}$$

We filter the nonzero elements of $\mathbf{x}$ (29) and count the nonzero element number as a diversification measure. We notice that $\mathbf{z}_{d2} \ldots \mathbf{z}_{d7}$ and $\mathbf{z}_{d8} \ldots \mathbf{z}_{d11}$ all consist of 152 stocks (i.e., complete diversification).

We also filter the nonzero elements of $\boldsymbol{\mu}_3$ due to its skewness (as reported in Table 1). We match the nonzero elements of $\mathbf{x}$ and $\boldsymbol{\mu}_3$, count the number, and thus obtain the number of stocks which are engaged in green innovation. For example, we list the following $\mathbf{x}$ and $\boldsymbol{\mu}_3$ of $n = 4$ for simplicity:

$$\begin{aligned} \mathbf{x} &= \left[0.1 \;\; 0 \;\; 0.3 \;\; 0.6 \right]^T, \\ \boldsymbol{\mu}_3 &= \left[1.0 \;\; 2.0 \;\; 0 \;\; 4.0 \right]^T. \end{aligned} \tag{30}$$

The number of engaged stocks is two.

For the sample with $n = 152$, the numbers of engaged stocks of $\mathbf{z}_{d2} \ldots \mathbf{z}_{d7}$ are all 45. For the next subsection, the numbers of engaged stocks of $\mathbf{z}_{d8} \ldots \mathbf{z}_{d11}$ are all 45 as well.

4.8. Robustness Tests for $\mathbf{z}_{d4}$ to $\mathbf{z}_{d7}$. To check the robustness of the research methodology, we choose $\mathbf{z}_{d8} \ldots \mathbf{z}_{d11}$ instead of $\mathbf{z}_{d4} \ldots \mathbf{z}_{d7}$ on the curve from $\mathbf{z}_{d2}$ to $\mathbf{z}_{d3}$. We express $\mathbf{z}_{d8} \ldots \mathbf{z}_{d11}$ in Table 3 (e.g., $\mathbf{z}_{d8} = (0.0456, 0.9048, 3.2131)$ in the second column).

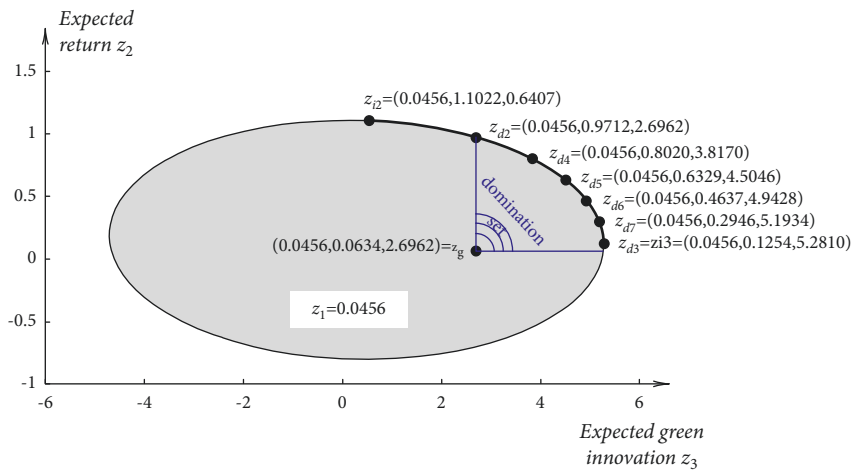

Figure 7: The green innovation index $\mathbf{z}_g$ and dominating portfolios $\mathbf{z}_{d2}$ to $\mathbf{z}_{d7}$ on the intersection for the 152 component stocks of China Securities Index 300.

Table 2: Testing hypotheses (15)–(17) for $\mathbf{z}_{d2}$ vs. $\mathbf{z}_g$, . . ., $\mathbf{z}_{d7}$ vs. $\mathbf{z}_g$.

	$\mathbf{z}_{d2}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d4}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d5}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d6}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d7}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d3}$ vs. $\mathbf{z}_g$
For (15), statistic	0.0562	0.0549	0.0470	0.0443	0.0470	0.0679
For (15), p-value	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$
For (15), decision	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0
For (16), statistic	−0.1862	−0.0152	−0.0684	−0.1163	−0.1573	0.0425
For (16), p-value	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$
For (16), decision	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0
For (17), statistic	3.9421	7.2135	6.8907	6.3193	5.4176	7.3204
For (17), p-value	$p > 0.10$	$p < 0.10$	$p < 0.10$	$p < 0.10$	$p < 0.10$	$p < 0.10$
For (17), decision	Accept H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0
Dominate?	No	Yes	Yes	Yes	Yes	Yes

Table 3: Choosing $\mathbf{z}_{d8} \ldots \mathbf{z}_{d11}$ instead of $\mathbf{z}_{d4} \ldots \mathbf{z}_{d7}$.

	$\mathbf{z}_{d8}$	$\mathbf{z}_{d9}$	$\mathbf{z}_{d10}$	$\mathbf{z}_{d11}$
z_1	0.0456	0.0456	0.0456	0.0456
z_2	0.9048	0.8189	0.7052	0.5427
z_3	3.2131	3.7301	4.2470	4.7640

Table 4: Testing hypotheses (15)–(17) for $\mathbf{z}_{d8}$ vs. $\mathbf{z}_g$, . . ., $\mathbf{z}_{d11}$ vs. $\mathbf{z}_g$ for robustness tests.

	$\mathbf{z}_{d2}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d8}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d9}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d10}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d11}$ vs. $\mathbf{z}_g$	$\mathbf{z}_{d3}$ vs. $\mathbf{z}_g$
For (15), statistic	0.0562	0.0517	0.0476	0.0448	0.0451	0.0679
For (15), p-value	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$
For (15), decision	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0
For (16), statistic	−0.1862	−0.1770	−0.1608	−0.1348	−0.0915	0.0425
For (16), p-value	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$	$p > 0.10$
For (16), decision	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0
For (17), statistic	3.9421	4.6233	5.3035	5.9821	6.6581	7.3204
For (17), p-value	$p > 0.10$	$p < 0.10$	$p < 0.10$	$p < 0.10$	$p < 0.10$	$p < 0.10$
For (17), decision	Accept H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0
Dominate?	No	Yes	Yes	Yes	Yes	Yes

We then take $\mathbf{z}_{d8}$ vs. $\mathbf{z}_g$, . . ., $\mathbf{z}_{d11}$ vs. $\mathbf{z}_g$ for hypotheses (15)–(17) and report the result in Table 4. Portfolios $\mathbf{z}_{d3}$ and $\mathbf{z}_{d8} \ldots \mathbf{z}_{d11}$ dominate the green innovation index $\mathbf{z}_g$. Overall, the results of Tables 2 and 4 are similar. The robustness of the research methodology can be verified.

As a comment, scholars typically obtain insignificant results in testing portfolios' returns against indexes' returns due to the volatile nature of stock returns. For example, [114], p.1936, [58], p.1181, [115], p.257, and [116], p.509, report similar insignificant results.

4.9. Robustness Tests with Different Sample Partitions. We also unequally partition the sample period (January 1, 2009, to December 31, 2018) as follows:

from January 1, 2009, to December 31, 2014, as an in-sample subperiod and from January 1, 2015, to December 31, 2018, as an out-of-sample subperiod;

from January 1, 2009, to December 31, 2015, as an in-sample subperiod and from January 1, 2016, to December 31, 2018, as an out-of-sample subperiod.

For both partitions, we obtain results similar to those reported in Tables 2–4. For all the equal and unequal partitions, we have deposited all the data, codes, and results in Mendeley Data https://data.mendeley.com/. Please see [104] in the references.

5. Further Formulating Green Innovation by General k-Objective Portfolio Selection

In this section, we divide green innovation into categories, further formulate each category as an objective, construct general k-objective portfolio selection, and demonstrate the existence of portfolios which dominate green innovation indexes. The advantage of this further formulation is that investors can consider and control each category.

5.1. Dividing Green Innovation into Categories (Green Technology Innovation and Green Management Innovation) and Formulating

5.1.1. Formulating the Categories by a 4-Objective Model. Reference [117], p.463, divides green innovation into two categories: green technology innovation and green management innovation. Therefore, we follow the division and further formulate each category as an objective in the following model:

$$\begin{aligned}
&\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\}, \text{risk}\\
&\max\left\{z_2 = \mathbf{x}^T\boldsymbol{\mu}\right\}, \text{return}\\
&\max\left\{z_3 = \mathbf{x}^T\boldsymbol{\mu}_3\right\}, \text{green technology innovation}\\
&\max\left\{z_4 = \mathbf{x}^T\boldsymbol{\mu}_4\right\}, \text{green management innovation}\\
&s.t. \mathbf{A}^T\mathbf{x} = \mathbf{b},
\end{aligned} \tag{31}$$

where

$\boldsymbol{\mu}_3$ is a vector of stock expected green technology innovations,

$\boldsymbol{\mu}_4$ is a vector of stock expected green management innovations,

z_3 measures the portfolio expected green technology innovation,

z_4 measures the portfolio expected green management innovation.

5.1.2. Still Dominating the Green Innovation Index by (31). By the analytical result for (6), we can extend the computations of (13)–(17) and extend Theorem 1 for (31). To save space, the detailed computations are skipped; we just present the key computation and extend Figure 2 to Figure 8.In $(z_1, \ldots, z_4)$ space, we suppose the minimum-variance surface of (31) as follows:

$$\begin{aligned}
z_1 = {} & p_1 z_2^2 + p_2 z_3^2 + p_3 z_4^2 + p_4 z_2 z_3 + p_5 z_2 z_4 + p_6 z_3 z_4\\
& + p_7 z_2 + p_8 z_3 + p_9 z_4 + p_{10},
\end{aligned} \tag{32}$$

where $p_1 \ldots p_{10}$ are the coefficients. We draw a plane passing through $\mathbf{z}_g$ and being parallel to (z_2, z_3, z_4) subspace. The plane is as follows:

$$z_1 = z_{g1}. \tag{33}$$

The plane intersects the feasible region Z of (31). Because every portfolio on the intersection shares the same z_1 (i.e., $z_1 = z_{g1}$), we need to focus on just (z_2, z_3, z_4) elements of the intersection. The intersection is depicted in the lower left part of Figure 8. Because [22] proves the minimum-variance surface as a paraboloid and the surface is the boundary of the feasible region, the boundary of the intersection is an ellipsoid as follows:

$$\begin{aligned}
z_{g1} = {} & p_1 z_2^2 + p_2 z_3^2 + p_3 z_4^2 + p_4 z_2 z_3 + p_5 z_2 z_4 + p_6 z_3 z_4\\
& + p_7 z_2 + p_8 z_3 + p_9 z_4 + p_{10}.
\end{aligned} \tag{34}$$

Moreover, we can select a portion of the ellipsoid, so every portfolio on the portion dominates the green innovation index. The portion is depicted in the lower right part of Figure 8.

5.2. Further Dividing Green Innovation and Formulating

5.2. Further Dividing Green Innovation and Formulating. World Intellectual Property Organization divides green innovation into seven categories (data source: https://www.wipo.int/classifications/ipc/en/green_inventory/, July 8, 2020). We formulate them in the following model:

$$\begin{aligned}
&\min\left\{z_1 = \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right\}, \text{risk}\\
&\max\left\{z_2 = \mathbf{x}^T\boldsymbol{\mu}\right\}, \text{return}\\
&\max\left\{z_3 = \mathbf{x}^T\boldsymbol{\mu}_3\right\}, \text{regulatory or design aspects}\\
&\max\left\{z_4 = \mathbf{x}^T\boldsymbol{\mu}_4\right\}, \text{agriculture or forestry}\\
&\max\left\{z_5 = \mathbf{x}^T\boldsymbol{\mu}_5\right\}, \text{alternative energy production}\\
&\max\left\{z_6 = \mathbf{x}^T\boldsymbol{\mu}_6\right\}, \text{energy conservation}\\
&\max\left\{z_7 = \mathbf{x}^T\boldsymbol{\mu}_7\right\}, \text{nuclear power generation}\\
&\max\left\{z_8 = \mathbf{x}^T\boldsymbol{\mu}_8\right\}, \text{transportation}\\
&\max\left\{z_9 = \mathbf{x}^T\boldsymbol{\mu}_9\right\}, \text{waste management}\\
&s.t. \mathbf{A}^T\mathbf{x} = \mathbf{b},
\end{aligned} \tag{35}$$

where

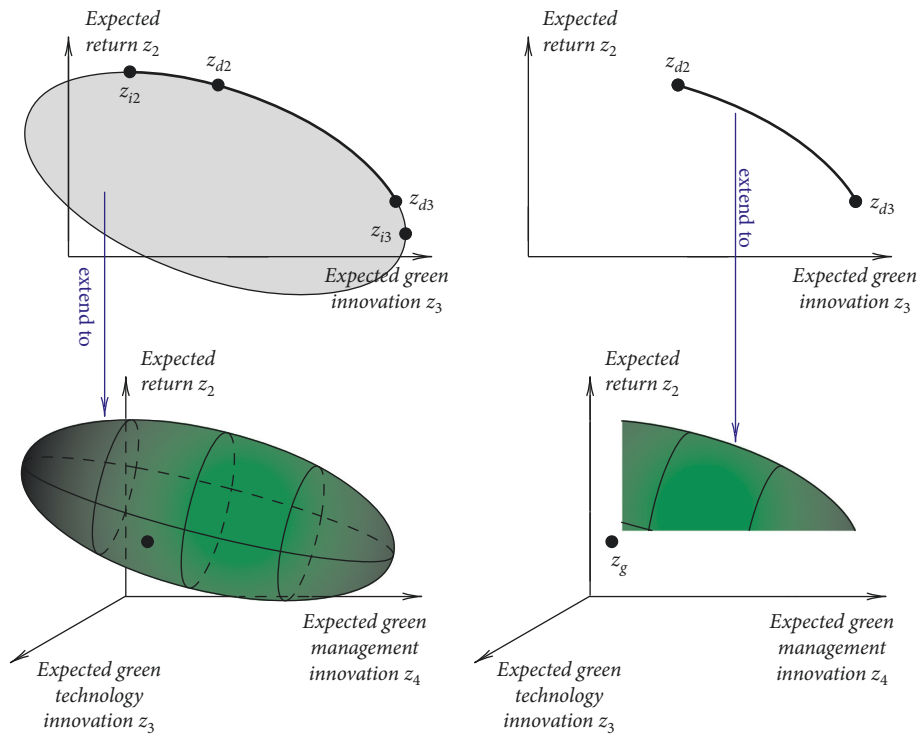

Figure 8: Extending (3) in the upper part to (31) in the lower part, the intersection between the plane and feasible region, and the ellipsoid's portion which dominates the index $\mathbf{z}_g$.

$\boldsymbol{\mu}_3 \ldots \boldsymbol{\mu}_9$ are vectors of stock expected categories,

$z_3 \ldots z_9$ measure the portfolio expected categories.

Furthermore, researchers propose different divisions. Reference [1], p.318, divides green innovation into green product innovation and green process innovation. Reference [118], pp.1829–1830, extracts and classifies green patents on the basis of the division of World Intellectual Property Organization. Overall, we can follow the divisions, treat them as objectives, formulate them by multiple-objective portfolio selection, and compute the dominating portfolios by (6).

6. Conclusions

6.1. Positive Methodology vs. Normative Methodology. On the basis of (4), Nobel laureate [119], pp.433–434, assumes investor homogeneity and proposes his profound CAPM (capital asset pricing model). Although (4) and investor homogeneity are too simplistic and even unrealistic, [119], p.434, justifies his methodology by arguing for the CAPM's insights. References [119], p.425, and [120], p.99, call this methodology of simplistic assumptions but profound insights as positive methodology.

In contrast, [94, 95] propose a general portfolio-selection model (1) which incorporates kinds of practical constraints. They deploy parametric quadratic programming to solve (1). References [119], p.426, and [120], p.99, call this general-model methodology as normative methodology. It could be a pity that [9, 94] may totally concentrate on the model and optimization rather than contemplating simplistic but profound insights.

We principally pursue positive methodology by assuming simplistic constraints but trying to unveil dominating-portfolio strategies.

6.2. Theoretical Implications. Armed with multiple-objective portfolio selection for green innovation, investors can substantially enrich green innovation investments. Particularly, investors can upgrade security selection to portfolio selection, take green innovation as objectives, obtain the efficient surface, and select portfolios which dominate green innovation indexes.

In the sample, $\mathbf{z}_{d2} \ldots \mathbf{z}_{d7}$ and $\mathbf{z}_{d8} \ldots \mathbf{z}_{d11}$ all consist of 152 stocks, enjoy complete diversification, and contain 45 stocks which are engaged in green innovation. Investors can naturally increase sample size to check this status of being both efficient and diversified.

Reference [73], p.61, clarifies the consistency between portfolio selection and maximizing-expected-utility approaches. Consequently for (3) , investors can naturally extend the traditional utility function and probe the extended utility function for green innovation.

6.3. Practical Implications. Instead of the traditionally passive screening-indexing tool, investors perceive a new instrument to actively construct portfolios for risk, return, and green innovation.

Because portfolio selection carries mixed results of out-of-sample performances, investors should not be too fascinated by the formulation (3) and should put (3) into more rigorous tests.

In the more rigorous tests, investors can install kinds of actual constraints (e.g., bounds of **x**, industry restrictions, and transaction costs). For the optimization to charter the unknown territory depicted in Figures 4–6, investors can channel genetic algorithms and even fuzzy-number methods (as commenced by [101, 102]).

6.4. Concluding Remarks. Instigating green innovation as multiple objectives opens new opportunities for portfolio selection and green innovation investments. New areas which may not be previously contemplated are now brought to the theoretical and practical forefront.

Data Availability

The authors have deposited all the data, codes, and results in Mendeley Data https://doi.org/10.17632/rcwggcsxnh.1. Please see [104] in the references.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] J.-W. Huang and Y.-H. Li, "Green innovation and performance: the view of organizational capability and social reciprocity," *Journal of Business Ethics*, vol. 145, no. 2, pp. 309–324, 2017.

[2] M. Saunila, J. Ukko, and T. Rantala, "Sustainability as a driver of green innovation investment and exploitation," *Journal of Cleaner Production*, vol. 179, no. 1, pp. 631–641, 2018.

[3] Z. Bodie, A. Kane, and A. J. Marcus, *Investments*, McGraw-Hill Education, New York, New York, USA, 11th edition, 2018.

[4] H. M. Markowitz, *Portfolio Selection: Efficient Diversification In Investments*, John Wiley & Sons, New York, New York, USA, 1st edition, 1959.

[5] N. P. B. Bollen, "Mutual fund attributes and investor behavior," *Journal of Financial and Quantitative Analysis*, vol. 42, no. 3, pp. 683–708, 2007.

[6] C. Louche and S. Lydenberg, *Dilemmas in Responsible Investment*, Greenleaf Publishing Limited, Sheffield, UK, 2011.

[7] M. Hood, J. Nofsinger, and A. Varma, "Conservation, discrimination, and salvation: investors' social concerns in the stock market," *Journal of Financial Services Research*, vol. 45, no. 1, pp. 5–37, 2014.

[8] A. C. T. Borgers and R. A. J. Pownall, "Attitudes towards socially and environmentally responsible investment," *Journal of Behavioral and Experimental Finance*, vol. 1, pp. 27–44, 2014.

[9] H. M. Markowitz, "Portfolio selection," *The Journal of Finance*, vol. 7, no. 1, pp. 77–91, 1952.

[10] R. E. Steuer, *Multiple Criteria Optimization: Theory, Computation, and Application*, John Wiley & Sons, New York, New York, USA, 1986.

[11] G. W. Evans, *Multiple Criteria Decision Analysis for Industrial Engineering: Methodology and Applications*, CRC Press, Boca Raton, Florida, USA, 2017.

[12] H. M. Markowitz, "Foundations of portfolio theory," *The Journal of Finance*, vol. 46, no. 2, pp. 469–477, 1991.

[13] W. F. Sharpe, "Optimal portfolios without bounds on holdings," *Graduate School of Business*, https://web.stanford.edu/~wfsharpe/mia/opt/mia_opt2.htm, 2001.

[14] E. F. Fama, "Multifactor portfolio efficiency and multifactor asset pricing," *Journal of Financial and Quantitative Analysis*, vol. 31, no. 4, pp. 441–465, 1996.

[15] J. H. Cochrane, "Portfolio advice for a multifactor world," *Economic Perspectives*, vol. 233, pp. 59–78, 3rd edition, 1999.

[16] J. H. Cochrane, "Presidential address: discount rates," *The Journal of Finance*, vol. 66, no. 4, pp. 1047–1108, 2011.

[17] M. Statman, *Finance for Normal People: How Investors and Markets Behave*, Oxford University Press, New York, New York, USA, 2017.

[18] R. E. Steuer, Y. Qi, and M. Hirschberger, "Suitable-portfolio investors, nondominated frontier sensitivity, and the effect of multiple objectives on standard portfolio selection," *Annals of Operations Research*, vol. 152, no. 1, pp. 297–317, 2007.

[19] G. Dorfleitner, M. Leidl, and J. Reeder, "Theory of social returns in portfolio choice with application to microfinance," *Journal of Asset Management*, vol. 13, no. 6, pp. 384–400, 2012.

[20] S. Utz, M. Wimmer, and R. E. Steuer, "Tri-criterion modeling for constructing more-sustainable mutual funds," *European Journal of Operational Research*, vol. 246, no. 1, pp. 331–338, 2015.

[21] Y. Qi, R. E. Steuer, and M. Wimmer, "An analytical derivation of the efficient surface in portfolio selection with three criteria," *Annals of Operations Research*, vol. 251, no. 1-2, pp. 161–177, 2017.

[22] Y. Qi and R. E. Steuer, "On the analytical derivation of efficient sets in quad-and-higher criterion portfolio selection," *Annals of Operations Research*, vol. 293, no. 2, pp. 521–538, 2020.

[23] B. Pascual, A. Fosfuri, L. Gelabert, and L. R. Gomez-Mejia, "Necessity as the mother of 'green' inventions: institutional pressures and environmental innovations," *Strategic Management Journal*, vol. 34, no. 8, pp. 891–909, 2013.

[24] L. He, S. Zeng, H. Ma, G. Qi, and V. W. Y. Tam, "Can political capital drive corporate green innovation? Lessons from China," *Journal of Cleaner Production*, vol. 64, no. 2, pp. 63–72, 2014.

[25] C. C. J. Cheng, "Sustainability orientation, green supplier involvement, and green innovation performance: evidence from diversifying green entrants," *Journal of Business Ethics*, vol. 161, no. 2, pp. 1–22, 2020.

[26] Y.-S. Chen, S.-B. Lai, and C.-T. Wen, "The influence of green innovation performance on corporate advantage in Taiwan," *Journal of Business Ethics*, vol. 67, no. 4, pp. 331–339, 2006.

[27] Y.-S. Chen, "The driver of green innovation and green image - green core competence," *Journal of Business Ethics*, vol. 81, no. 3, pp. 531–543, 2008.

[28] G. Friede, T. Busch, and A. Bassen, "ESG and financial performance: aggregated evidence from more than 2000 empirical studies," *Journal of Sustainable Finance & Investment*, vol. 5, no. 4, pp. 210–233, 2015.

[29] D. Daugaard, "Emerging new themes in environmental, social and governance investing: a systematic literature review," *Accounting and Finance*, vol. 60, no. 2, pp. 1501–1530, 2020.

[30] M. F. Yen, Y. M. Shiu, and C. F. Wang, "Socially responsible investment returns and news: evidence from Asia," *Corporate Social Responsibility and Environmental Management*, vol. 26, no. 6, pp. 1565–1578, 2019.

[31] F. García, J. González-Bueno, F. Guijarro, and J. Oliver, "Forecasting the environmental, social, and governance rating of firms by using corporate financial performance variables: a rough set approach," *Sustainability*, vol. 12, no. 8, pp. 1–18, Article ID 3324, 2020.

[32] N. Semenova and L. G. Hassel, "On the validity of environmental performance metrics," *Journal of Business Ethics*, vol. 132, no. 2, pp. 249–258, 2015.

[33] A. K. Chatterji, R. Durand, D. I. Levine, and S. Touboul, "Do ratings of firms converge? Implications for managers, investors and strategy researchers," *Strategic Management Journal*, vol. 37, no. 8, pp. 1597–1614, 2016.

[34] C. Marquis, M. W. Toffel, and Y. Zhou, "Scrutiny, norms, and selective disclosure: a global study of greenwashing," *Organization Science*, vol. 27, no. 2, pp. 483–504, 2017.

[35] B. Pascual, A. Fosfuri, and L. Gelabert, "Does greenwashing pay off? Understanding the relationship between environmental actions and environmental legitimacy," *Journal of Business Ethics*, vol. 144, no. 2, pp. 363–379, 2017.

[36] C. Radu and C. Francoeur, "Does innovation drive environmental disclosure? A new insight into sustainable development," *Business Strategy and the Environment*, vol. 26, no. 7, pp. 893–911, 2017.

[37] E. Pei-yi Yu, B. Van Luu, and C. H. Chen, "Greenwashing in environmental, social and governance disclosures," *Research in International Business and Finance*, vol. 52, no. April, pp. 1–23, Article ID 101192, 2020.

[38] J. Yin, L. Gong, and S. Wang, "Large-scale assessment of global green innovation research trends from 1981 to 2016: a bibliometric study," *Journal of Cleaner Production*, vol. 197, no. 1, pp. 827–841, 2018.

[39] S. Karimi Takalo, H. Sayyadi Tooranloo, and Z. Shahabaldini Parizi, "Green innovation: a systematic literature review," *Journal of Cleaner Production*, vol. 279, no. 1, pp. 1–22, Article ID 122474, 2021.

[40] C.-H. Chang, "The influence of corporate environmental ethics on competitive advantage: the mediation role of green innovation," *Journal of Business Ethics*, vol. 104, no. 3, pp. 361–370, 2011.

[41] D. Hirshleifer, P.-H. Hsu, and D. Li, "Innovative efficiency and stock returns," *Journal of Financial Economics*, vol. 107, no. 3, pp. 632–654, 2013.

[42] Y. Chu, X. Tian, and W. Wang, "Corporate innovation along the supply chain," *Management Science*, vol. 65, no. 6, pp. 2445–2466, 2019.

[43] L. H. Pedersen, S. Fitzgibbons, and L. Pomorski, "Responsible investing: the ESG-efficient frontier," *Journal of Financial Economics*, vol. 142, no. 2, pp. 572–597, 2021.

[44] P. Bertrand and V. Lapointe, "How performance of risk-based strategies is modified by socially responsible investment universe?" *International Review of Financial Analysis*, vol. 38, no. 3, pp. 175–190, 2015.

[45] B. R. Auer, "Do socially responsible investment policies add or destroy European stock portfolio value?" *Journal of Business Ethics*, vol. 135, no. 2, pp. 381–397, 2016.

[46] P. Jan Trinks and B. Scholtens, "The opportunity cost of negative screening in socially responsible investing," *Journal of Business Ethics*, vol. 140, no. 2, pp. 193–208, 2017.

[47] Z. Li, D. B. Minor, J. Wang, and Y. Chong, "A learning curve of the market: chasing alpha of socially responsible firms," *Journal of Economic Dynamics and Control*, vol. 109, pp. 1–17, Article ID 103772, 2019.

[48] A. Singh, "COVID-19 and safer investment bets," *Finance Research Letters*, vol. 36, pp. 1–7, Article ID 101729, 2020.

[49] V. Díaz, D. Ibrushi, and J. Zhao, "Reconsidering systematic factors during the COVID-19 pandemic-the rising importance of ESG," *Finance Research Letters*, vol. 38, pp. 1–6, Article ID 101870, 2021.

[50] A. Amel-Zadeh and G. Serafeim, "Why and how investors use ESG information: evidence from a global survey," *Financial Analysts Journal*, vol. 74, no. 3, pp. 87–103, 2018.

[51] L. Renneboog, J. Ter Horst, and J. Ter Horst, "Socially responsible investments: institutional aspects, performance, and investor behavior," *Journal of Banking & Finance*, vol. 32, no. 9, pp. 1723–1742, 2008.

[52] F. Climent and P. Soriano, "Green and good? The investment performance of US environmental mutual funds," *Journal of Business Ethics*, vol. 103, no. 2, pp. 275–287, 2011.

[53] Gunther Capelle-Blancard and S. Monjon, "The performance of socially responsible funds: does the screening process matter?" *European Financial Management*, vol. 20, no. 3, pp. 494–520, 2014.

[54] F. Muñoz, M. Vargas, and I. Marco, "Environmental mutual funds: financial performance and managerial abilities," *Journal of Business Ethics*, vol. 124, no. 4, pp. 551–569, 2014.

[55] S. Dolvin, J. Fulkerson, and A. Krukover, "Do "good guys" finish last? The relationship between Morningstar sustainability ratings and mutual fund performance," *Journal of Investing*, vol. 28, no. 2, pp. 77–91, 2019.

[56] F. Ferriani and F. Natoli, "ESG risks in times of COVID-19," *Applied Economics Letters*, vol. 28, no. 18, pp. 1537–1541, 2021.

[57] E. Ballestero, M. Bravo, B. Pérez-Gladish, M. Arenas-Parra, and D. Plà-Santamaria, "Socially responsible investment: a multicriteria approach to portfolio selection combining ethical and financial objectives," *European Journal of Operational Research*, vol. 216, no. 2, pp. 487–494, 2012.

[58] M. Stephan, S. M. Gasser, M. Rammerstorfer, and S. K. Weinmayer, "Markowitz revisited: social portfolio engineering," *European Journal of Operational Research*, vol. 258, no. 3, pp. 1181–1190, 2017.

[59] R. H. Campbell, M. W. Liechty, and P. Müller, "Portfolio selection with higher moments," *Quantitative Finance*, vol. 10, no. 5, pp. 469–485, 2010.

[60] F. J. Fabozzi, S. Focardi, and C. Jonas, "Trends in quantitative equity management: survey results," *Quantitative Finance*, vol. 7, no. 2, pp. 115–122, 2007.

[61] D. Roman, K. Darby-Dowman, and G. Mitra, "Mean-risk models using two risk measures: a multi-objective approach," *Quantitative Finance*, vol. 7, no. 4, pp. 443–458, 2007.

[62] P. Jorion, "Portfolio optimization with tracking-error constraints," *Financial Analysts Journal*, vol. 59, no. 5, pp. 70–82, 2003.

[63] A. W. Lo, C. Petrov, and M. Wierzbicki, "It's 11pm – do you know where your liquidity is? The mean-variance-liquidity Frontier," *Journal of Investment Management*, vol. 1, no. 1, pp. 55–93, 2003.

[64] J. B. Guerard and A. Mark, "The optimization of efficient portfolios: the case for an R&D quadratic term," *Research in Finance*, vol. 20, pp. 217–247, 2003.

[65] J. Spronk and W. G. Hallerbach, "Financial modelling: where to go? With an illustration for portfolio management," *European Journal of Operational Research*, vol. 99, no. 1, pp. 113–127, 1997.

[66] C. A. B. e Costa and J. O. Soares, "Multicriteria approaches for portfolio selection: an overview," *Review of Financial Markets*, vol. 4, no. 1, pp. 19–26, 2001.

[67] R. E. Steuer and N. Paul, "Multiple criteria decision making combined with finance: a categorized bibliography," *European Journal of Operational Research*, vol. 150, no. 3, pp. 496–515, 2003.

[68] C. Zopounidis, E. Galariotis, M. Doumpos, S. Sarri, and K. Andriosopoulos, "Multiple criteria decision aiding for finance: an updated bibliographic survey," *European Journal of Operational Research*, vol. 247, no. 2, pp. 339–348, 2015.

[69] M. Masmoudi and F. Ben Abdelaziz, "Portfolio selection problem: a review of deterministic and stochastic multiple objective programming models," *Annals of Operations Research*, vol. 267, no. 1-2, pp. 335–352, 2018.

[70] B. Aouni, Michael Doumpos, B. Pérez-Gladish, and R. E. Steuer, "On the increasing importance of multiple criteria decision aid methods for portfolio selection," *Journal of the Operational Research Society*, vol. 69, no. 10, pp. 1525–1542, 2018.

[71] F. W. Sharpe, *Portfolio Theory And Capital Markets*, McGraw-Hill, New York, New York, USA, 1st edition, 1970.

[72] R. C. Merton, "An analytical derivation of the efficient portfolio Frontier," *Journal of Financial and Quantitative Analysis*, vol. 7, no. 4, pp. 1851–1872, 1972.

[73] C.-F. Huang and R. H. Litzenberger, *Foundations for Financial Economics*, Prentice-Hall, Englewood Cliffs, New Jersey, USA, 1988.

[74] I. Oikonomou, E. Platanakis, and C. Sutcliffe, "Socially responsible investment portfolios: does the optimization process matter?" *The British Accounting Review*, vol. 50, no. 4, pp. 379–401, 2018.

[75] K. Liagkouras, K. Metaxiotis, and G. Tsihrintzis, "Incorporating environmental and social considerations into the portfolio optimization process," *Annals of Operations Research*, vol. 2021, 2021.

[76] B. H. Taljaard and E. Maré, "Why has the equal weight portfolio underperformed and what can we do about it?" *Quantitative Finance*, vol. 21, no. 11, pp. 1855–1868, 2021.

[77] T. K. Do, "Socially responsible investing portfolio: an almost stochastic dominance approach," *International Journal of Finance & Economics*, vol. 26, no. 1, pp. 1122–1132, 2021.

[78] A. Riedl and Paul Smeets, "Why do investors hold socially responsible mutual funds?" *The Journal of Finance*, vol. 72, no. 6, pp. 2505–2550, 2017.

[79] M. Rossi, D. Sansone, Arthur van Soest, and C. Torricelli, "Household preferences for socially responsible investments," *Journal of Banking & Finance*, vol. 105, no. 8, pp. 107–120, 2019.

[80] Alexander Dyck, K. V. Lins, L. Roth, and H. F. Wagner, "Do institutional investors drive corporate social responsibility? International evidence," *Journal of Financial Economics*, vol. 131, no. 3, pp. 693–714, 2019.

[81] A. L. Aiken, J. A. Ellis, and M. Kang, "Do politicians "put their money where their mouth is?" ideology and portfolio choice," *Management Science*, vol. 66, no. 1, pp. 376–396, 2019.

[82] D. R. Anderson, D. J. Sweeney, T. A. Williams, J. D. Camm, and J. J. Cochran, *Statistics for Business and Economics*, Cengage Learning, Boston, Massachusetts, USA, 13th edition, 2018.

[83] J. D. Jobson and R. M. Korkie, "Putting Markowitz theory to work," *Journal of Portfolio Management*, vol. 7, no. 4, pp. 70–74, 1981.

[84] R. O. Michaud, "The Markowitz optimization enigma: is 'optimized' optimal?" *Financial Analysts Journal*, vol. 45, no. 1, pp. 31–42, 1989.

[85] R. Jagannathan and T. Ma, "Risk reduction in large portfolios: why imposing the wrong constraints helps," *The Journal of Finance*, vol. 58, no. 4, pp. 1651–1684, 2003.

[86] V. DeMiguel, L. Garlappi, F. J. Nogales, and U. Raman, "A generalized approach to portfolio optimization: improving performance by constraining portfolio norms," *Management Science*, vol. 55, no. 5, pp. 798–812, 2009.

[87] M. Rubinstein, "Markowitz's "portfolio selection": a fifty-year retrospective," *The Journal of Finance*, vol. 57, no. 3, pp. 1041–1045, 2002.

[88] R. W. Ross, R. W. Westerfield, J. Jaffe, and B. Jordan, *Corporate Finance*, McGraw-Hill Education, New York, New York, USA, 11th edition, 2016.

[89] K. E. Back, *Asset Pricing And Portfolio Choice Theory*, Oxford University Press, New York, New York, USA, 2nd edition, 2017.

[90] J. Y. Campbell, *Financial Decisions and Markets: A Course in Asset Pricing*, Princeton University Press, Princeton, New Jersey, USA, 1st edition, 2018.

[91] R. A. Brealey, S. C. Myers, and F. Allen, *Principles of Corporate Finance*, McGraw-Hill Education, New York, New York, USA, 13th edition, 2020.

[92] B. Bank, J. Guddat, D. Klatte, B. Kummer, and K. Tammer, *Non-Linear Parametric Optimization*, Birkhäuser Verlag, Basel, 1983.

[93] E. N. Pistikopoulos, N. A. Diangelakis, and R. Oberdieck, *Multi-parametric Optimization and Control*, John Wiley & Sons, Hoboken, New Jersey, USA, 1st edition, 2021.

[94] H. M. Markowitz, "The optimization of a quadratic function subject to linear constraints," *Naval Research Logistics Quarterly*, vol. 3, no. 1-2, pp. 111–133, 1956.

[95] H. M. Markowitz and G. Peter Todd, *Mean-Variance Analysis in Portfolio Choice and Capital Markets*, Frank J. Fabozzi Associates, New Hope, Pennsylvania, USA, 2000.

[96] M. Hirschberger, R. E. Steuer, S. Utz, M. Wimmer, and Y. Qi, "Computing the nondominated surface in tri-criterion portfolio selection," *Operations Research*, vol. 61, no. 1, pp. 169–183, 2013.

[97] P. L. W. Jayasekara, N. Adelgren, and W. W. Wiecek, "On convex multiobjective programs with application to portfolio optimization," *Journal of Multi-Criteria Decision Analysis*, vol. 27, no. 3-4, pp. 189–202, 2019.

[98] Y. Qi, M. Hirschberger, and R. E. Steuer, "Dotted representations of mean-variance efficient frontiers and their computation," *INFOR: Information Systems and Operational Research*, vol. 47, no. 1, pp. 15–22, 2009.

[99] K. Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*, John Wiley & Sons, New York, New York, USA, 2001.

[100] M. Ehrgott, K. Klamroth, and C. Schwehm, "An MCDM approach to portfolio optimization," *European Journal of Operational Research*, vol. 155, no. 3, pp. 752–770, 2004.

[101] F. García, J. González-Bueno, F. Guijarro, and J. Oliver, "A multiobjective credibilistic portfolio selection model. empirical study in the Latin American integrated market," *Enterpreneurship and Sustainability Issues*, vol. 8, no. 2, pp. 1027–1046, 2020.

[102] F. García, J. González-Bueno, F. Guijarro, J. Oliver, and R. Tamošiūnienė, "Multiobjective approach to portfolio optimization in the light of the credibility theory," *Technological and Economic Development of Economy*, vol. 26, no. 6, pp. 1165–1186, 2020.

[103] K. Metaxiotis and K. Liagkouras, "Multiobjective evolutionary algorithms for portfolio management: a comprehensive literature review," *Expert Systems with Applications*, vol. 39, no. 14, pp. 11685–11698, 2012.

[104] Y. Qi, "Data for "constructing multiple-objective portfolio selection for green innovation and dominating green-innovation indexes"," *Mendeley Data*, 2020.

[105] F. Allen, A. E. Bernardo, and I. Welch, "A theory of dividends based on tax clienteles," *The Journal of Finance*, vol. 55, no. 6, pp. 2499–2536, 2000.

[106] M. Baker and J. Wurgler, "A catering theory of dividends," *The Journal of Finance*, vol. 59, no. 3, pp. 1125–1165, 2004.

[107] S. Desmettre, S. Grün, and R. Korn, "Portfolio optimization with early announced discrete dividends," *Operations Research Letters*, vol. 46, no. 5, pp. 548–552, 2018.

[108] H. Bilel and K. Mondher, "What can explain catering of dividend? environment information and investor sentiment," *Journal of Economics and Finance*, vol. 45, pp. 428–450, 2021.

[109] B. Graham and D. L. Dodd, *Security Analysis*, McGraw-Hill, New York, New York, USA, 6th edition, 2009.

[110] F. Gu, "Innovation, future earnings, and market efficiency," *Journal of Accounting, Auditing and Finance*, vol. 20, no. 4, pp. 385–418, 2005.

[111] E. J. Elton, M. J. Gruber, S. J. Brown, and W. N. Goetzmann, *Modern Portfolio Theory and Investment Analysis*, John Wiley & Sons, New York, New York, USA, 9th edition, 2014.

[112] Y. Qi, "Parametrically computing efficient frontiers of portfolio selection and reporting and utilizing the piecewise-segment structure," *Journal of the Operational Research Society*, vol. 71, no. 10, pp. 1675–1690, 2020.

[113] Y. Qi, Y. Zhang, and S. Ma, "Parametrically computing efficient frontiers and reanalyzing efficiency-diversification discrepancies and naive diversification," *INFOR: Information Systems and Operational Research*, vol. 57, no. 3, pp. 430–453, 2019.

[114] V. DeMiguel, L. Garlappi, and U. Raman, "Optimal versus naive diversification: how inefficient is the 1/N portfolio strategy?" *Review of Financial Studies*, vol. 22, no. 5, pp. 1915–1953, 2009.

[115] P.-H. Hsu, Q. Han, W. Wu, and Z. Cao, "Asset allocation strategies, data snooping, and the 1/N rule," *Journal of Banking & Finance*, vol. 97, no. 12, pp. 257–269, 2018.

[116] Y. Qi, "On outperforming social-screening-indexing by multiple-objective portfolio selection," *Annals of Operations Research*, vol. 267, no. 1-2, pp. 493–513, 2018.

[117] D. Li, Y. Zhao, L. Zhang, X. Chen, and C. Cao, "Impact of quality management on green innovation," *Journal of Cleaner Production*, vol. 170, no. 2, pp. 462–470, 2018.

[118] Grazia Cecere, N. Corrocher, C. Gossart, and M. Ozman, "Technological pervasiveness and variety of innovators in green ICT: a patent-based analysis," *Research Policy*, vol. 43, no. 10, pp. 1827–1839, 2014.

[119] W. F. Sharpe, "Capital asset prices: a theory of market equilibrium," *The Journal of Finance*, vol. 19, no. 3, pp. 425–442, 1964.

[120] H. M. Markowitz, "Normative portfolio analysis: past, present, and future," *Journal of Economics and Business*, vol. 42, pp. 99–103, 1990.