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Review Article

Pricing and Coordination of Remanufacturing Supply Chain with Government Participation considering Consumers' Preferences and Quality of Recycled Products

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Remanufacturing has become an important way to realize sustainable development strategy. For remanufacturing closed-loop supply chain under different circumstances, many factors are considered, such as consumers' different preferences for the purchase and payment of remanufactured products and the quality of recycled products. In this study, three models are presented for supply chain system, including a manufacturer lead and a retailer recycle. While nongovernment participation and non-supply-chain coordination are considered in model I, model II has government participation but non-supply-chain coordination, and model III has both government participation and supply chain coordination. The equilibrium decisions are derived for each model using the game-theoretic approach. The study found that consumers' preferences affect the pricing decisions of remanufactured products. When consumers' willingness to buy and pay for remanufactured products is close to that of new products, the total profit of the supply chain will significantly increase; the quality of recycled products will also influence the pricing decisions of new/remanufactured products, which in turn will affect the market demand for products. Government participation is conducive to encouraging manufacturer, retailer, and consumers to actively participate in the recycling and remanufacturing activities of used products. Supply chain coordination contracts can further enhance the role of the consumer market and promote the implementation of government policies. Under certain conditions, revenue sharing contract coordination can significantly increase the total profit of the supply chain system. The research conclusions have certain reference value for the coordination among the nodes of the remanufacturing supply chain and the formulation of policies with government participation.

1. Introduction

With the rapid development of global industry, the shortages of energy and resource are increasingly plagued by economic development. Meanwhile, the environmental pollution and ecological destruction also bring unpredictable problems to mankind [1]. How to achieve sustainable development has become the focus of global research. Remanufacturing has become an important way to realize sustainable development strategy [2, 3]. The remanufacturing supply chain can not only reduce resource consumption and environmental pollution caused by used products, but also save costs, increase profits, enhance corporate competitiveness [4], and realize comprehensive economic and environmental benefits [5]. Compared with new products, remanufactured products have less impact on the ecological environment [6]. The remanufacturing supply chain based on the closed-loop concept emphasizes the implementation of product management from the product life cycle. Enterprises have the responsibility to provide new products, as well as the obligation to recycle and remanufacture used products. Different consumers cause different levels of product loss, so the quality of recycled used products is different, which leads different recycling and remanufacturing costs [7]. At the same time, due to reason of personal habits, psychological factors, economic income, education level, etc., consumers have different willingness to buy and pay for new/remanufactured products. Such differences in consumption preferences will inevitably affect the market demand and pricing decision of corresponding products [8]. According to the UK carbon trust, consumers are willing to pay more for some green products. According to the report on the current situation of public green consumption in China (2019 Edition), 83.34% of the respondents expressed support for green consumption. Therefore, considering the influence of consumers' preferences and the quality of recycled products on the pricing decisions of remanufacturing supply chains has attracted widespread attention from the business community, academia, and the government.

In order to reduce the burden on the social environment and encourage enterprises to support sustainable development strategies, it is necessary for the government to promote the development of the remanufacturing industry through environmental regulations and fiscal interventions. Taking China as an example, in order to support the promotion and use of remanufactured products, promote the recycling of remanufactured cores, and expand the market share of remanufactured products, the National Development and Reform Commission, together with four other departments, has issued a notice to jointly print and distribute the notice on printing and distributing the pilot implementation plan of "old for remanufactured products" since 2013. When considering the role of consumer market and government participation, the "double marginal effect" still exists because the decision makers in the supply chain system pursue their own profit maximization. Remanufacturing supply chain is in a state of imbalance, which affects the overall benefits of the system. Therefore, how to formulate the internal contract coordination of the supply chain to promote the cooperation among the node enterprises has become one of the important contents of the supply chain management. The government is willing to provide various subsidies to improve the recovery rate of used products, increase the market share of remanufactured products, and guide consumers to buy remanufactured products. For different government subsidies, rewards, and punishment measures, consumers' preferences and the quality of recycled products will affect the operation decision of the supply chain.

In the light of the above considerations, we address the following research questions:

(1) Only the role of the consumer market is considered, that is, how to make decisions in remanufacturing supply chain system without government participation?

(2) How does government participation affect enterprise remanufacturing decision-making, product market demand, and supply chain efficiency?

(3) Under the joint action of the consumer market and government participation, what kind of coordination contract is designed to improve the efficiency of the supply chain system?

The remainder of the paper is organized as follows. Section 2 presents a literature review. Based on problem description and modeling assumptions in Section 3, Section 4 designs the different decision Models. Using a numerical example, a sensitivity analysis and discussion of the parameters are presented in Section 5. The final section provides conclusions and suggestions for future research.

2. Literature Review

In this section, the literature is reviewed regarding three research streams to highlight the contributions.

2.1. Consumers' Preferences. At present, some scholars have studied the supply chain decision-making of consumer preference behavior and made some breakthroughs. Cheng et al. [9] mentioned that consumers have different acceptance of remanufactured products because of different cognitive levels and consumption tendencies. Consumers always believe that the value of remanufactured products is lower than that of new products, and therefore, their willingness to pay is also lower than that of new products. Hazen et al. [10] showed that consumers tend to buy remanufactured products and are willing to pay higher prices when the remanufacturing process is transparent. Combined with consumer environmental preferences, Liu et al. [11] studied the decision-making behavior of manufacturers and retailers. By establishing a centralized and decentralized game model, Xiong et al. [12] deduced the optimal strategy combination of manufacturers and retailers in the two decision-making modes and analyzed the impact of consumer environmental awareness on retail prices. Yu et al. [13] analyzed the optimal production green policy effect with consumer environmental awareness. Using the centralized structure, decentralized structure, and repurchase contract structure, Zhang et al. [14] discussed the optimal decision of traditional products and green products. Ji et al. [15] considered consumer preferences in a dual-channel supply chain. By comparing the centralized and decentralized structures, Li et al. [16] found that the willingness to buy low-carbon products is significantly different in terms of values, age, income, and education level. Taking into account consumers' low-carbon awareness, Xia et al. [17] established a game model dominated by manufacturers. The research shows that it can effectively improve the enthusiasm of supply chain members to invest in low-carbon industries and increase their investment profits. By constructing three decision-making models, centralized, decentralized without altruistic preference, and decentralized with altruistic preference in the low-carbon supply chain, Wang et al. [18] claimed that the altruistic preference can help increase the small- and medium-sized manufacturer's profit and system efficiency but decrease the retailer's profit.

2.2. Government Participation. Government participation in remanufacturing decision-making is also a research hotspot in academic circles. Wang et al. [19] considered the government's carbon emission tax policy in the supply chain system. Huang et al. [20] discussed the impact of government subsidies on the electric vehicle industry and the environment. Yang et al. [21] researched the policy effects of government subsidies on consumers and corresponding supply chain member companies. Xia et al. [22] analyzed the impact of government subsidies on consumer perception and found that not all consumers expect government subsidies. In order to study the problem of energy conservation and emission reduction in the supply chain, Yi et al. [23, 24] established a Stackelberg model for retailers and manufacturers and found that government subsidies can increase the profits of supply chain members. Under the quota trading policy and low-carbon subsidy policy, Cao et al. [25] point out that the profits of production enterprises do not completely depend on low-carbon subsidy policies. Sana et al. [26, 27] concluded that government agencies encourage enterprises to adopt green technology and green marketing in different ways, including government subsidies and tax implementation. Among them, the government provides higher subsidies and lower taxes to green producers rather than lower subsidies and higher taxes to non-green producers.

2.3. The Quality of Recycled Products. In fact, compared with the traditional supply chain, the remanufacturing supply chain has a high degree of uncertainty, such as the quality of recycled products and the cost of remanufacturing. The literatures [28, 29] carried out research on the pricing strategy of closed-loop supply chain based on the assumption that the recycled products are all remanufactured products. Liu et al. [30] indicated that the remanufacturing of used products with different recycling qualities should consider the difference in remanufacturing process costs. Furthermore, a dual-channel competition model was constructed to study the impact of government subsidies and recycling quality restrictions on the development of the remanufacturing industry. Heydari et al. [31, 32] proposed that the recycler or retailer persuades consumers to take back their used products by providing incentives on the premise. Only items can be accepted and remanufactured only when they meet the minimum quality level. Aydin et al. [33] proposed that both the purchase price and the remanufacturing cost are highly dependent on the quality of recycled products, and consumer preferences are incorporated into the demand estimation of new/remanufactured products. Finally, the optimal recycling quantity and remanufacturing decision in multiple cycles are obtained. Aleizadeh et al. [34] considered the impact of the uncertainty of the recycled products quality on the repurchase price and the remanufacturing process cost. The modal interval algorithm was developed to analyze the dynamic pricing and recycling strategies of remanufacturing companies.

In summary, the decision-making problem of remanufacturing supply chain, which mainly focuses on consumers' preferences, the quality of recycled products, and the pricing decision-making of remanufacturing closed-loop supply chain under government incentive policies, has attracted widespread attention from scholars. Although the research results are rich, and the angles are diverse, there are still two deficiencies. (1) Although the above scholars have studied the influence of consumers' preferences on the pricing decisions of closed-loop supply chains, they are all limited to the determined quality of recycled products. They have not discussed in detail the changes in closed-loop supply chain pricing strategies when there are both quality differences of recycled products and consumers' preferences. This is not consistent with the uneven quality of used products in real life. (2) Despite the fact that the government separately provides recycling or remanufacturing subsidies in the above-mentioned articles, when the quality of recycled products is different, few literatures take into account government subsidies to consumers participating in recycling activities, as well as retailer recycling incentives. However, the difference between the selection of government's subsidies and the design of subsidy models on the efficiency of the closed-loop supply chain is significant. Based on the above-mentioned literature review, on the basis of the quality differences of recycled used products and consumers' different preferences for purchasing and payment of new/remanufactured products, it constructs a decision model for the role of the consumer market and government participation in the remanufacturing supply chain. In addition, internal coordination contracts in the supply chain based on revenue sharing are designed to improve the economic benefits of manufacturers and retailers.

In short, the main content of the paper includes the following:

(1) A Stackelberg game model I without government participation in non-supply-chain coordination is constructed. On the premise of considering consumers' preferences for remanufactured products and the quality difference of used products, the optimal pricing strategy and profit function of manufacturer and retailer are established.

(2) On the basis of model I, the government subsidy, reward, and punishment measures for supply chain are introduced, and the model II with government participation and non-supply-chain coordination is established. On the one hand, it is considered that the government will provide recovery price subsidies to consumers participating in recovery activities and ultimately affect the recovery rate and manufacturer's recovery cost function. On the other hand, considering the government's supervision and regulation function, the recovery rate of used products is required, and reward and punishment factors are introduced. The results of models II and I are compared and researched, and the effects of subsidy coefficient and reward and punishment factor on the optimal decision-making results are discussed through numerical discussion.

(3) On the basis of model II, the coordination contract of revenue sharing is introduced, and model III with government participation and supply chain coordination is established. By coordinating the profit

distribution of each member of the supply chain, the overall profit of the supply chain is finally improved. Moreover, the solution results of models III and II are compared and analyzed deeply.

3. Problem Description and Modeling Assumptions

3.1. The Framework of the Models. In a multilevel closed-loop supply chain system, manufacturer manufactures and sells new products to retailer at wholesale prices. Then, retailer retails them to consumers; retailer has the advantages of information channels and huge logistics network resources. Entrusted by manufacturer, retailer issues advertisements for recycled used products to consumers and assists them to return used products of different quality levels to manufacturers; manufacturer recycles used products for remanufacturing, and retailer is responsible for the retail of remanufactured products to consumers. As the leader of the remanufacturing supply chain system, the manufacturer first determines the wholesale price of the products, and then the retailer determines the retail price of the products.

Figure 1 schematically illustrates the proposed decision framework of the remanufacturing supply chain. Firstly, this paper discusses the model I of nongovernment participation and non-supply-chain coordination based on the role of consumer market, that is, the decision-making problem of remanufacturing supply chain without government participation; secondly, further consider model II where the government participates in the coordination without supply chain. In this model, the government subsidizes the recycling price of consumers who participate in recycling activities. Implement reward and punishment measures based on basic recovery rate for retailer. If the total recovery rate of used products is higher than the basic recovery rate, retailer will be rewarded; otherwise, they will be punished. Compare models I and II to analyze the impact of government participation on remanufacturing strategies and supply chain pricing decisions. Finally, model III with supply chain coordination is considered to improve the operation of remanufacturing supply chain based on government participation. In model III, the manufacturer shares part of the retailer's revenue. By comparing models II and III, the impact of the revenue sharing contract on the efficiency of business operations and the implementation of government policies is studied.

3.2. Symbol Description. The major indices, parameters, and variables are listed, respectively, in Tables 1–3.

3.3. Model Assumptions. Before presenting our model, we state and discuss key assumptions specify to our remanufacturing environment.

Assumption 1. The information of the manufacturer and the retailer is completely symmetrical, and the decision is made rationally.

Assumption 2. Based on a large number of similar studies such as Ferguson and Toktay [35], when the consumer market capacity is A, a_n^j is uniformly distributed from 0 to A, that is, $a_n^j \sim U[0, A]$, and $f(a_n^j) = 1$. If the consumers' preferences coefficient for remanufactured products is r, $a_r^j = r a_n^j$. The utilities are, respectively, expressed as $u_n^j = a_n^j - p_n^j, u_r^j = a_r^j - p_r^j = r a_n^j - p_r^j$. When $u_n^j \geq u_r^j$ and $u_n^j \geq 0$, that is, the psychological satisfaction that consumers can obtain when buying and paying for new products is greater than that of remanufactured products, consumers choose new products and obtain the condition $(p_n^j - p_r^j)/(1-r) \leq a_n^j \leq A$. Therefore, the market demand for new products is $D_n^j = \int_{(p_n^j - p_r^j)/(1-r)}^{A} f(a_n^j) \mathrm{d}a_n^j = A - p_n^j - p_r^j/(1-r)$. When $u_r^j > u_n^j$ and $u_r^j \geq 0$, that is, the psychological satisfaction that consumers can obtain when buying and paying for remanufactured products is greater than that of new products, consumers choose remanufactured products and obtain the condition $p_r^j/r < \alpha_n^j < (p_n^j - p_r^j)/(1-r)$. Therefore, the market demand for remanufactured products is $D_r^j = \int_{p_r^j/r}^{(p_n^j - p_r^j)/(1-r)} f(a_n^j) \mathrm{d}a_n^j = (p_n^j - p_r^j)/(1-r) - p_r^j/r$.

Assumption 3. The quality level of the disassembled used products is i. q_i is employed to represent the quality of them. The quality of recycled used products increases with the increase of q_i, which leads to higher recycling cost and lower remanufacturing cost [36].

Assumption 4. In the process of recycling used products, the manufacturer will incur recycling costs. Due to the different quality levels, the total recycling cost C_b^j is different. It is assumed that the recovery is the product of D_n^j and g_i^j. Therefore, the total recycling cost of used products of different quality levels is $C_b^j = \sum_{i=1}^{3} D_n^j g_i^j b_i^j$ [37]. The recovery subsidy price b_i^j is positively correlated with q_i in model I; the government provides undifferentiated subsidies to consumers who participate in the activities of recycling used products in models II and III. Then, $b_i^j = \begin{cases} q_i B, & j = \mathrm{I} \\ q_i B + k, & j = \mathrm{II}, \mathrm{III} \end{cases}$.

Assumption 5. According to the theory of consumer behavior, consumers are more willing to participate in recycling activities when the price of recycling subsidies reaches the level of consumers' psychological satisfaction. For used products with quality level i, the recovery rate g_i^j has a linear relationship with the recovery subsidy price b_i^j, that is, $g_i^j = g_0 + \lambda b_i^j$, and the total recovery rate $G^j = \sum_{i=1}^{3} g_i^j \leq 1$.

Assumption 6. Three types of costs incurred in the remanufacturing process.

(1) The purchase and replacement cost of new parts is m_i, and the functional form of m_i is as follows:

$$m_i = (1 - q_i) C_n. \tag{1}$$

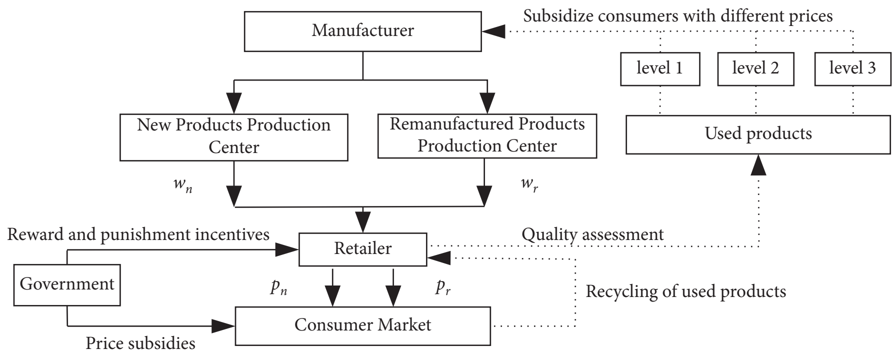


Figure 1: Decision framework for remanufacturing supply chain.

Table 1: Indices.

Index	Definitions
i	$i = 1, 2, 3$, respectively, indicate used products with quality levels 1, 2, 3;
j	j = I, II, III, respectively represent the nongovernment participation and non-supply-chain coordination model (Non-GP, Non-SC), the government participation and non-supply-chain coordination model (GP, Non-SC), and the government participation and supply chain coordination model (GP, SC).

Table 2: Parameters.

Parameters	Definitions
A	Consumer market capacity;
r	Consumers' preferences coefficient for remanufactured products;
a_n^j	Consumers' psychological willingness to pay when purchasing new products in model j;
a_r^j	Consumers' psychological willingness to pay when purchasing remanufactured products in model j;
u_n^j	Utility obtained when consumers purchase new products in model j;
u_r^j	Utility obtained when consumers purchase remanufactured products in model j;
D_n^j	Market demand for new products in model j;
D_r^j	Market demand for remanufactured products in model j;
q_i	Quality level coefficient of used products with quality level i, $q_i \in (0.3, 1)$;
m_i	Purchase and replacement costs of new parts are required when remanufacturing used products with quality level i;
β	Sensitivity coefficient of manual maintenance cost to material replacement cost;
e_i	Manual maintenance cost when remanufacturing used products with quality level i;
z	Recovery of transfer fees and other fixed costs;
c_{ri}	Cost of the remanufacturing process per unit of used products with quality level i;
L^j	Reward and punishment function of the government to the retailer in model j;
k	Government provides undifferentiated unit subsidy prices to consumers who participate in the activities of recycling used products;
G^j	Total recovery rate of the retailer's recycling of used products in model j;
g_a	Basic recovery rate set by the government;
θ	Reward and punishment coefficient set by the government;
λ	Sensitivity coefficient of consumers to the price of recovery subsidy;
B	Manufacturer's subsidy price for recycling to consumers;
C_n	Manufacturing cost per unit of new products;
g_0	When the recycling subsidy price is 0, the recycling rate of used products;
t	Proportion of retailer with revenue sharing, $0 < t < 1$;
C_r^j	Total remanufacturing cost of used products with quality differences in model j;
$\overline{C}_r^j$	Average remanufacturing cost per unit of used products in model j;
g_i^j	Recovery rate of used products with quality level i in model j;
b_i^j	Recycling subsidy price of used products with quality level i in model j;
C_b^j	Total recycling cost of used products with quality differences in model j.

Table 3: Variables (including objective functions).

Variables	Definitions
w_n^j	Unit wholesale price of the new products in model j;
w_r^j	Unit wholesale price of remanufactured products in model j;
p_n^j	Unit retail price of the new products in model j;
p_r^j	Unit retail price of the remanufactured products in model j;
W^j	Manufacturer profit in model j;
V^j	Retailer profit in model j;
F^j	Total profit of the remanufacturing supply chain system in model j.

(2) Manual maintenance cost is e_i which mainly refers to the cost incurred by the maintenance staff engaged in the replacement of parts and components. The more parts that are completely damaged and need to be replaced, the greater the cost of manual repairs. It is assumed that e_i and m_i have a linear relationship, that is, $e_i = \beta m_i$.

(3) The total amount of recovered transfer expenses and fixed link costs is z, which mainly include the logistics transfer cost of recycling and the cost incurred in a series of remanufacturing links (such as disassembly, cleaning, testing, and final packaging).

For a unit of used products with quality level i, the cost of the remanufacturing process is $c_{ri} = m_i + e_i + z = (1+\beta)(1-q_i)C_n + z$. The total remanufacturing cost is $C_r^j = \sum_{i=1}^{3}[(1+\beta)(1-q_i)C_n + z]D_n^j g_i^j$. Therefore, the average remanufacturing cost per unit of used products is $\overline{C}_r^j = \sum_{i=1}^{3}[(1+\beta)(1-q_i)C_n + z]g_i^j / \sum_{i=1}^{3} g_i^j$.

Assumption 7. The market conditions in each cycle are similar, so we only consider the situation in one cycle. Assuming that the market has sold a round of new products, it is considered that the used products can be directly recycled for remanufacturing in the current cycle, and then the new/remanufactured products can be sold together.

Assumption 8. This paper adopts the retailer-led recycling channel. The government sets a basic recycling rate g_a for the retailer and employs the function L^j to restrict the retailer's rewards and punishments, where $L^j = \theta(G - g_a)D_n^j$. When $g_a < G^j$, the government rewards the retailer for recycling; When $g_a > G^j$, the government punishes the retailer for recycling.

Assumption 9. The recycling and remanufacturing of used products have a certain economic feasibility, where the manufacturer is in a dominant position, and all members of the supply chain take the maximum profit as the decision-making goal. Therefore, variables and parameters satisfy the relations

$$\begin{aligned}(1+\beta)(1-q_i)C_n + z + q_i B \le C_n < p_n^j,\\ w_n^j < p_n^j,\\ w_r^j < p_r^j.\end{aligned} \tag{2}$$

4. The Different Decision Models

Based on the modeling notations and assumptions in Section 3, three decision models are developed, analyzed, and compared in this section.

4.1. Model I (Non-GP, Non-SC). In this model, the remanufacturing supply chain decision is only affected by the role of the consumer market. Manufacturer and retailer together constitute a Stackelberg game model led by the manufacturer and followed by the retailer. First, the manufacturer determines the wholesale price of the new/remanufactured products, and then the retailer determines the retail price of them. The profit functions could be written as follows:

$$\begin{aligned}W^{\mathrm{I}} &= D_n^{\mathrm{I}}\left(w_n^{\mathrm{I}} - C_n\right) + D_r^{\mathrm{I}}\left(w_r^{\mathrm{I}} - \overline{C}_r^{\mathrm{I}}\right) - C_b^{\mathrm{I}},\\ V^{\mathrm{I}} &= D_n^{\mathrm{I}}\left(p_n^{\mathrm{I}} - w_n^{\mathrm{I}}\right) + D_r^{\mathrm{I}}\left(p_r^{\mathrm{I}} - w_r^{\mathrm{I}}\right),\\ F^{1} &= W^{1} + V^{1}.\end{aligned} \tag{3}$$

Proposition 1. *In Model I, the equilibrium results of manufacturer's and retailer's decisions are as follows (the proof is provided in Appendix A).*

$$\begin{aligned}w_n^{\mathrm{I}*} &= x_1^{\mathrm{I}*} + \frac{A}{2},\\ w_r^{\mathrm{I}*} &= x_2^{\mathrm{I}*},\\ p_n^{\mathrm{I}*} &= \frac{x_1^{\mathrm{I}*}}{2} + \frac{3A}{4},\\ p_r^{\mathrm{I}*} &= \frac{x_2^{\mathrm{I}*} + Ar}{2}.\end{aligned} \tag{4}$$

Corresponding to

$$\begin{aligned}D_n^{\mathrm{I}*} &= A - \frac{2\left(x_1^{\mathrm{I}*} - x_2^{\mathrm{I}*} - Ar\right) + 3A}{4(1-r)},\\ D_r^{\mathrm{I}*} &= \frac{2\left(x_1^{\mathrm{I}*} - x_2^{\mathrm{I}*} - Ar\right) + 3A}{4(1-r)} - \frac{x_2^{\mathrm{I}*} + Ar}{2r}.\end{aligned} \tag{5}$$

The profits of manufacturer and retailer are given by

$$W^{I*}=\left[A-\frac{2\left(x_1^{I*}-x_2^{I*}-Ar\right)+3A}{4(1-r)}\right]\left(x_1^{I*}+\frac{A}{2}-C_n\right)$$
$$+\left[\frac{2\left(x_1^{I*}-x_2^{I*}-Ar\right)+3A}{4(1-r)}-\frac{x_2^{I*}+Ar}{2r}\;\frac{2\left(x_1^{I*}-x_2^{I*}-Ar\right)+3A}{4(1-r)}-\frac{x_2^{I*}+Ar}{2r}\right]\left(x_2^{I*}-\frac{\sum_{i=1}^{3}\left[(1+\beta)(1-q_i)C_n+z\right]}{\sum_{i=1}^{3}\left(g_0+\lambda Bq_i\right)}\right)$$
$$-\sum_{i=1}^{3}Bq_i\left(g_0+\lambda Bq_i\right)\left[A-\frac{2\left(x_1^{I*}-x_2^{I*}-Ar\right)+3A}{4(1-r)}\right],$$
$$V^{I*}=\left[A-\frac{2\left(x_1^{I*}-x_2^{I*}-Ar\right)+3A}{4(1-r)}\right]\left(\frac{A}{4}-\frac{x_1^{I*}}{2}\right)+\left[\frac{2\left(x_1^{I*}-x_2^{I*}-Ar\right)+3A}{4(1-r)}-\frac{x_2^{I*}+Ar}{2r}\right]\left(\frac{Ar}{2}-\frac{x_2^{I*}}{2}\right). \tag{6}$$

Respectively, the results of x_1^{I*} and x_2^{I*} are as follows:

$$x_2^{I*}=\frac{C_n g_0(1+\beta)\left(3-\sum_{i=1}^{3}q_i\right)+(z+Ar)\left(3g_0+\lambda B\sum_{i=1}^{3}q_i\right)+\lambda BC_n(1+\beta)\sum_{i-1}^{3}\left(q_i-q_i^2\right)}{6g_0+2B\lambda\sum_{i=1}^{3}q_i},$$
$$x_1^{I*}=\frac{\lambda B^2\sum_{i=1}^{2}q_i^2+Bg_0\sum_{i=1}^{3}\left(q_i+C_n\right)}{2}. \tag{7}$$

Further analysis of Proposition 1 leads to Lemma 1 and 2.

Lemma 1. *In Model I, w_r^{I*}, p_r^{I*} and D_r^{I*} are positively correlated with r; D_n^{I*} is negatively correlated with r; w_n^{I*} and p_n^{I*} are independent of r (the proof is provided in Appendix B).*

Lemma 1 shows that as consumers increase their willingness to buy and pay for remanufactured products, the encroachment of remanufactured products on new products increases, and the market demand for new products decreases. When r increases to a certain value, the production cost of the new products is higher, but the price has not changed. Even if the price of remanufactured products increases, it is still far below the price of new products, so more consumers tend to buy remanufactured products. Under the remanufacturing, supply chain decisions are only affected by the role of the consumer market, and manufacturer and retailer can profit by producing or selling remanufactured products.

Lemma 2. *In Model I, w_n^{I*}, p_n^{I*}, D_r^{I*} and G^{I*} are positively correlated with q_i; w_r^{I*}, p_r^{I*} and D_n^{I*} are negatively correlated with q_i.*

Lemma 2 proves that, with the increase in the quality level coefficient q_i of recycled products, consumers have increased their enthusiasm for participating in recycling activities under the incentive of high recycling price subsidies, helping retailer to increase recycling rates and helping manufacturers to save more costs, thereby promoting the reduction of the price of remanufactured products and increasing the market share of remanufacturing products. The increase in the price of remanufactured products will also lead to an increase in the price of new products, which will increase the profit of new products, which has a positive effect on improving the overall profit of the supply chain. Therefore, retailer needs to encourage more consumers to participate in recycling activities. In order to recycle high-quality used products, measures such as increasing their recycling prices can be taken to stimulate consumers to participate in recycling activities.

4.2. Model II (GP, Non-SC). In order to guide and encourage enterprises to actively participate in remanufacturing activities, the government adopts certain incentive measures to influence the decisions of consumers and retailer and indirectly affect the pricing decisions of the supply chain system, so as to achieve the purpose of increasing the recycling rate of used products and improving environmental performance. Model II has set up two types of government incentives. On the one hand, the government grants recycling price subsidies to consumers who participate in recycling, to stimulate consumers to participate in recycling and remanufacturing activities, and to improve the recycling rate and quality level of used products; on the other hand, the government implements a reward and punishment mechanism for retailer by setting a basic recycling rate and urges retailer responsible for recycling used products to increase advertising and publicity to increase the recycling rate and recycling volume. Meanwhile, the profit functions in model II are expressed as follows:

$$
\begin{aligned}
W^{\mathrm{II}} &= D_n^{\mathrm{II}}\left(w_n^{\mathrm{II}} - C_n\right) + D_r^{\mathrm{II}}\left(w_r^{\mathrm{II}} - \overline{C}_r^{\mathrm{II}}\right) - C_b^{\mathrm{II}},\\
V^{\mathrm{II}} &= D_n^{\mathrm{II}}\left(p_n^{\mathrm{II}} - w_n^{\mathrm{II}}\right) + D_r^{\mathrm{II}}\left(p_r^{\mathrm{II}} - w_r^{\mathrm{II}}\right) + L^{\mathrm{II}},\\
F^{\mathrm{II}} &= W^{\mathrm{II}} + V^{\mathrm{II}}.
\end{aligned} \tag{8}
$$

Proposition 2. *In Model II, the equilibrium results of the manufacturer's and retailer's decisions are as follows:*

$$
\begin{aligned}
w_n^{\mathrm{II}*} &= x_1^{\mathrm{II}*} + \frac{3(k+\theta)(g_0+k\lambda) - \theta g_a}{2},\\
w_r^{\mathrm{II}*} &= x_2^{\mathrm{II}*} + \frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^{3} q_i\right) + 3k\lambda(z+Ar)}{6g_0 + 6k\lambda + 2B\lambda\sum_{i=1}^{3} q_i},\\
p_n^{\mathrm{II}*} &= \frac{x_1^{\mathrm{II}*} + A}{2} + \frac{3(k-\theta)(g_0+k\lambda) + \theta g_a}{4},\\
p_r^{\mathrm{II}*} &= \frac{x_2^{\mathrm{II}*} + Ar}{2} + \frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^{3} q_i\right) + 3k\lambda(z+Ar)}{4\left(3g_0 + 3k\lambda + B\lambda\sum_{i=1}^{3} q_i\right)}.
\end{aligned} \tag{9}
$$

Corresponding to

$$
\begin{aligned}
D_n^{\mathrm{II}*} &= \frac{A}{2} - \left(\frac{x_1^{\mathrm{II}*} - x_2^{\mathrm{II}*}}{2(1-r)} + \frac{3(k-\theta)(g_0+k\lambda) + \theta g_a}{4(1-r)} - \frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^{3} q_i\right) + 3k\lambda(z+Ar)}{4(1-r)\left(3g_0 + 3k\lambda + B\lambda\sum_{i=1}^{3} q_i\right)}\right),\\
D_r^{\mathrm{II}*} &= \frac{2\left(x_1^{\mathrm{II}*} - x_2^{\mathrm{II}*}\right) + 3(k-\theta)(g_0+k\lambda) + \theta g_a}{4(1-r)} - \frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^{3} q_i\right) + 3k\lambda(z+Ar)}{4(1-r)\left(3g_0 + 3k\lambda + B\lambda\sum_{i=1}^{3} q_i\right)}\\
&\quad - \frac{x_2^{\mathrm{II}*}}{2r} - \frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^{3} q_i\right) + 3k\lambda(z+Ar)}{4r\left(3g_0 + 3k\lambda + B\lambda\sum_{i=1}^{3} q_i\right)},
\end{aligned} \tag{10}
$$

The profits of manufacturer and retailer are given by

$$
\begin{aligned}
W^{\mathrm{II}*} &= \left[\frac{A}{2} + \frac{2\left(x_2^{\mathrm{II}*} - x_1^{\mathrm{II}*}\right) - 3(k-\theta)(g_0+k\lambda) - \theta g_a}{4(1-r)} + \frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^{3} q_i\right) + 3k\lambda(z+Ar)}{4(1-r)\left(3g_0 + 3k\lambda + B\lambda\sum_{i=1}^{3} q_i\right)}\right]\\
&\quad \cdot \left[x_1^{\mathrm{II}*} - C_n + \frac{3(k+\theta)(g_0+k\lambda) - \theta g_a}{2}\right]\\
&\quad + \left[\frac{2\left(x_1^{\mathrm{II}*} - x_2^{\mathrm{II}*}\right) + 3(k-\theta)(g_0+k\lambda) + \theta g_a}{4(1-r)} - \frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^{3} q_i\right) + 3k\lambda(z+Ar)}{4(1-r)\left(3g_0 + 3k\lambda + B\lambda\sum_{i=1}^{3} q_i\right)}\right.\\
&\quad \left. - \frac{x_2^{\mathrm{II}*}}{2r} - \frac{k\lambda C_n(1+\beta)\left(3 - 3g_0 + 3k\lambda + B\lambda\sum_{i=1}^{3} q_i\right) + 3k\lambda(z+Ar)}{4r\left(3g_0 + 3k\lambda + B\lambda 3g_0 + 3k\lambda + B\lambda\sum_{i=1}^{3} q_i\right)}\right]
\end{aligned}
$$

$$\left[x_2^{\mathrm{II}*}+\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{2\left(3g_0+3k\lambda+B\lambda 3g_0+3k\lambda+B\lambda\sum_{i=1}^3 q_i\right)}-\frac{\sum_{i=1}^3\{[(1+\beta)(1-q_i)C_n+z](g_0+\lambda Bq_i+\lambda k)\}}{\sum_{i=1}^3(g_0+\lambda Bq_i+\lambda k)}\right]$$
$$-\sum_{i=1}^3\left\{\left[\frac{A}{2}+\frac{2\left(x_2^{\mathrm{II}*}-x_1^{\mathrm{II}*}\right)-3(k-\theta)(g_0+k\lambda)-\theta g_a}{4(1-r)}+\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{4(1-r)\left(3g_0+3k\lambda+B\lambda\sum_{i=1}^3 q_i\right)}\right]\right.$$
$$\left.(g_0+\lambda Bq_i+\lambda k)(Bq_i+k)\right\}$$

$$V^{\mathrm{II}*}=\left[\frac{A}{2}+\frac{2\left(x_2^{\mathrm{II}*}-x_1^{\mathrm{II}*}\right)-3(k-\theta)(g_0+k\lambda)-\theta g_a}{4(1-r)}+\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{2(1-r)\left(6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}\right]$$
$$+\left[\frac{2\left(x_1^{\mathrm{II}*}-x_2^{\mathrm{II}*}\right)+3(k-\theta)(g_0+k\lambda)+\theta g_a}{4(1-r)}-\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{2(1-r)\left(6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}\right. \tag{11}$$
$$\left.-\frac{x_2^{\mathrm{II}*}}{2r}-\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{2r\left(6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}\right]\left[\frac{Ar}{2}-\frac{x_2^{\mathrm{II}*}}{2}-\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{2\left(6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}\right].$$

$$V^{\mathrm{II}*}=\left[\frac{A}{2}+\frac{2\left(x_2^{\mathrm{II}*}-x_1^{\mathrm{II}*}\right)-3(k-\theta)(g_0+k\lambda)-\theta g_a}{4(1-r)}+\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{2(1-r)\left(6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}\right]$$
$$+\left[\frac{2\left(x_1^{\mathrm{II}*}-x_2^{\mathrm{II}*}\right)+3(k-\theta)(g_0+k\lambda)+\theta g_a}{4(1-r)}-\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{2(1-r)\left(6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}\right.$$
$$\left.-\frac{x_2^{\mathrm{II}*}}{2r}-\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{2r\left(6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}\right]\left[\frac{Ar}{2}-\frac{x_2^{\mathrm{II}*}}{2}-\frac{k\lambda C_n(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+3k\lambda(z+Ar)}{2\left(6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}\right]$$

Respectively, the values of $x_1^{\mathrm{II}*}$ and $x_2^{\mathrm{II}*}$ are as follows:

$$x_2^{\mathrm{II}*}=\frac{x_2^{\mathrm{I}*}\left(6g_0+2B\lambda\sum_{i=1}^3 q_i\right)}{\left(6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}$$
$$=\frac{C_n g_0(1+\beta)\left(3-\sum_{i=1}^3 q_i\right)+(z+Ar)\left(3g_0+\lambda B\sum_{i=1}^3 q_i\right)+\lambda BC_n(1+\beta)\sum_{i=1}^3\left(q_i-q_i^2\right)}{\left(6g_0+6k\lambda+2B\lambda 6g_0+6k\lambda+2B\lambda\sum_{i=1}^3 q_i\right)}, \tag{12}$$
$$x_1^{\mathrm{II}*}=x_1^{\mathrm{I}*}+\frac{A}{2}+\frac{B\lambda\sum_{i=1}^3 q_i(\theta+2k)}{2}$$
$$=\frac{\lambda B^2\sum_{i=1}^3 q_i^2+Bg_0\sum_{i=1}^3 q_i+C_n+A+B\lambda\sum_{i=1}^3 q_i(\theta+2k)}{2}.$$

Further analysis of Proposition 2 leads to Lemma 3~7.

Lemma 3. *In Model II, w_r^{II*}, p_r^{II*} and D_r^{II*} are positively correlated with r; D_n^{II*} is negatively correlated with r, w_n^{II*} and p_n^{II*} are independent of r.*

Combining Lemmas 1 and 3, it addresses that whether the government participates in the remanufacturing supply chain or not, in decentralized decision-making, manufacturer and retailer take their own interests as the starting point. As consumers increase their willingness to purchase and pay for remanufactured products, the price of remanufactured products will increase accordingly.

Lemma 4. *In Model II, w_n^{II*}, p_n^{II*}, D_r^{II*} and G^{II*} are positively correlated with q_i; w_r^{II*}, p_r^{II*} and D_n^{II*} are negatively correlated with q_i.*

Combining Lemmas 2 and 4 can show that regardless of whether the government participates in the remanufacturing supply chain, the recycling of high-quality used products has a positive effect on increasing the market share of remanufactured products.

Lemma 5. *In Model II, w_n^{II*} is positively correlated with k; given that $3g_0 - 3\lambda\theta + 6k\lambda + 2B\lambda \sum_{i=1}^{3} q_i > 0$, p_n^{II*} is positively correlated with k.*

Lemma 5 indicates that, with the increase of the government's subsidy k for consumer recycling prices, the wholesale price of new products increases; under certain conditions, the retail price of new products will increase, which shows that when the government adopts a recycling price subsidy mechanism for consumers, companies will try to increase the price of new products to increase their sales revenue.

Lemma 6. *In Model II, w_r^{II*} and p_r^{II*} are independent of θ; given that $3g_0 + 3k\lambda + B\lambda \sum_{i=1}^{3} q_i - g_a > 0$, w_n^{II*} and D_n^{II*} are positively correlated with θ, and p_n^{II*} and D_r^{II*} are negatively correlated with θ.*

Lemma 7. *In Model II, p_n^{II*} and D_r^{II*} are positively correlated with g_a; w_n^{II*} and D_n^{II*} are negatively correlated with g_a; w_r^{II*} and p_r^{II*} are independent of g_a.*

Lemmas 6~7 shows that, in the early development of the recycling industry of remanufacturing, both companies and consumers are on the sidelines, and retailer needs to make greater efforts to achieve a higher level of g_a. Under the conditions of government participation, if retailer strives to increase the recycling rate of used products to obtain government incentives, then the wholesale price of new products is positively correlated with θ, the retail price is negatively correlated with θ, and the price of remanufactured products is not affected by θ. This shows that the government's recycling rewards and punishment measures for retailer cannot directly affect the price of remanufactured products but indirectly affect the market demand for products by affecting the price of new products, thereby encouraging manufacturer and retailer to participate in remanufacturing recycling activities.

Combining Propositions 1 and 2, the solution results of models I and II are compared and analyzed, from which we can get Lemmas 8~10.

Lemma 8. *About the total recovery rate of used products, we have $G^{II*} > G^{I*}$ (the proof is provided in Appendix C).*

Lemma 9. *Regarding product pricing decisions, if $3g_0 - 3\lambda\theta + 6k\lambda + 2B\lambda \sum_{i=1}^{3} q_i > 0$, we can get that*

$$w_n^{\mathrm{II}*} > w_n^{\mathrm{I}*}, w_r^{\mathrm{II}*} > w_r^{\mathrm{I}*}; p_n^{\mathrm{II}*} > p_n^{\mathrm{I}*}, p_r^{\mathrm{II}*} > p_r^{\mathrm{I}*} \tag{13}$$

Lemma 10. *About the market demand for remanufactured products, we have $D_r^{II*} > D_r^{I*}$.*

Lemmas 8~10 reveal that manufacturer and retailer have different strategies for remanufacturing recycling activities in an environment whether the government is involved. When the government participates, the total recovery rate of used products is effectively increased; while raising the price of remanufactured products, the consumer market for remanufactured products can still be expanded. This shows that both manufacturer and retailer can benefit from remanufacturing products, and government policies can effectively promote manufacturer and retailer to actively participate in remanufacturing recycling activities. As consumers' preferences for remanufactured products have changed, the profits of manufacturer and retailer will also change.

In summary, the government's participation in recycling activities for remanufacturing of used products can effectively promote the development of the remanufacturing industry, encourage consumers to participate in recycling activities, and guide retailer to increase the total recycling rate of used products and increase market demand for remanufactured products, which can contribute to the realization of sustainable development strategies. However, the overall benefits of companies and supply chains are still affected by consumes' preferences and the quality of recycled products.

4.3. Model III (GP, SC). When considering the role of the consumer market and government incentives, each decision-making member in the remanufacturing supply chain will produce a "double marginal effect" to maximize their own interests, leading to a state of imbalance in the supply chain, and ultimately affecting the overall efficiency of the system. In order to improve the performance level of the supply chain, this paper adopts a revenue-sharing contract to improve the remanufacturing supply chain operation based on government participation. The manufacturer first determines the wholesale price of new/remanufactured products, while sharing the retailer's sales revenue. Let $1 - t$ be the shared revenue ratio, and then the retailer's sales revenue ratio is t, $t \in (0, 1)$. The retailer determines the retail price of new/remanufactured products. The profit functions in model III are expressed as follows:

$$\begin{aligned} W^{\mathrm{III}} &= (1-t)\left(D_n^{\mathrm{III}} p_n^{\mathrm{III}} + D_r^{\mathrm{III}} p_r^{\mathrm{III}}\right) + D_n^{\mathrm{III}}\left(w_n^{\mathrm{III}} - C_n\right) + D_n^{\mathrm{III}}\left(w_r^{\mathrm{III}} - \overline{C}_r^{\mathrm{III}}\right) - C_b^{\mathrm{III}}, \\ V^{\mathrm{III}} &= t\left(D_n^{\mathrm{III}} p_n^{\mathrm{III}} + D_r^{\mathrm{III}} p_r^{\mathrm{III}}\right) - \left(D_n^{\mathrm{III}} w_n^{\mathrm{III}} + D_r^{\mathrm{III}} w_r^{\mathrm{III}}\right) + L^{\mathrm{III}}, \\ F^{\mathrm{III}} &= W^{\mathrm{III}} + V^{\mathrm{III}}. \end{aligned} \tag{14}$$

Proposition 3. *In Model III, the equilibrium results of the manufacturer's and retailer's decision are expressed as follows:*

$$
\begin{aligned}
w_n^{\mathrm{III}*} &= \frac{\lambda B^2 t\sum_{i=1}^3 q_i^2 + B\left(g_0 t + \theta\lambda + 2\lambda k t\right)\sum_{i=1}^3 q_i + 3\left(kt+\theta\right)\left(k\lambda + g_0\right) + t\left(C_n + At\right) - \theta g_a}{(t+1)},\\
w_r^{\mathrm{III}*} &= \frac{3tC_n\left(g_0 + k\lambda\right)(1+\beta) + 3t\left(z + \mathrm{Art}\right)\left(g_0 + k\lambda\right) + B\lambda t\left(z + \mathrm{Art}\right)\sum_{i=1}^3 q_i + tC_n(1+\beta)\left(\lambda B - g_0 - k\lambda\right)\sum_{i=1}^3 q_i - t\lambda BC_n(1+\beta)\sum_{i=1}^3 q_i^2}{\left(3g_0 + 3k\lambda + B\lambda\sum_{i=1}^3 q_i\right)(1+t)},\\
p_n^{\mathrm{III}*} &= \frac{At - 3\theta g_0 + \theta g_a - 3\theta k\lambda - \theta B\lambda\sum_{i=1}^3 q_i + w_n^{\mathrm{III}*}}{2t},\\
p_r^{\mathrm{III}*} &= \frac{\mathrm{Art} + w_r^{\mathrm{III}*}}{2t}.
\end{aligned}
\tag{15}
$$

Corresponding to

$$
\begin{aligned}
D_n^{\mathrm{III}*} &= \frac{A}{2} - \frac{-3\theta g_0 + \theta g_a - 3\theta k\lambda - \theta B\lambda\sum_{i=1}^3 q_i + w_n^{\mathrm{III}*} - w_r^{\mathrm{III}*}}{2t(1-r)},\\
D_r^{\mathrm{III}*} &= \frac{-3\theta g_0 + \theta g_a - 3\theta k\lambda - \theta B\lambda\sum_{i=1}^3 q_i + w_n^{\mathrm{III}*} - w_r^{\mathrm{III}*}}{2t(1-r)} - \frac{w_r^{\mathrm{III}*}}{2rt}.
\end{aligned}
\tag{16}
$$

The profit expressions are more complicated and will continue to be discussed in numerical analysis. By further comparing and analyzing Proposition 2 and 3, we obtain Lemmas 11~13.

Lemma 11. *About the product pricing decisions, we have* $w_n^{III*} < w_n^{II*}, w_r^{III*} < w_r^{II*}; p_n^{III*} < p_n^{II*}, p_r^{III*} < p_r^{II*}$.

Lemma 12. *About the market demand, we have* $D_r^{III*} > D_r^{II*}, D_n^{III*} > D_n^{II*}$.

Lemmas 11~12 reveal that supply chain members formulate revenue sharing contracts to coordinate system benefits. The price of new/remanufactured products has decreased, and the market demand for new/remanufactured products has increased to ensure the increase in the profits of manufacturer and retailer and promote the increase in the overall efficiency of the supply chain.

The decision variables in the three models are further analyzed, as shown in Lemma 13.

Lemma 13. $\partial w_r^{I*}/\partial r = \partial w_r^{II*}/\partial r \geq \partial w_r^{III*}/\partial r$; $\partial p_r^{I*}/\partial r = \partial p_r^{II*}/\partial r \geq \partial p_r^{III*}/\partial r$; $\partial p_r^{j}/\partial r > \partial w_r^{j}/\partial r$, $j = I, II, III$.

Combine Propositions 1–3 to obtain $\partial w_r^{\mathrm{II}*}/\partial r = \partial w_r^{\mathrm{I}*}/\partial r = A/2$; $\partial p_r^{\mathrm{II}*}/\partial r = \partial p_r^{\mathrm{I}*}/\partial r = 3A/4$; $\partial w_r^{\mathrm{III}*}/\partial r = At^2/(1+t)$; $\partial p_r^{\mathrm{III}*}/\partial r = A/2 + At^2/2(1+t)$. Since $t \in (0,1)$, we can obtain Lemma 13, which shows that when there is non-supply-chain coordination, whether the government participates in the remanufacturing supply chain or not, the price of remanufactured products responds to the consumers' preferences coefficient r at the same speed. The government's subsidies, rewards, and punishments for the remanufacturing industry cannot affect the sensitivity of the remanufactured products price to r; when there is supply chain coordination, the reaction speed of the price of remanufactured products to the consumers' preferences coefficient r is significantly reduced. In the three models, the retail price of remanufactured products responds faster to the consumers' preferences coefficient r than the wholesale price of remanufactured products. Retailer can make more profits by encouraging more consumers to buy and pay for remanufactured products.

5. Numerical Analysis and Discussion

By establishing three remanufacturing supply chain decision models, the equilibrium decisions of manufacturer and retailer can be obtained. In view of the complexity of the partial solution results, the relationship between variables cannot be seen intuitively. Based on the conclusions obtained in the previous section, this section uses numerical analysis methods to further analyze state variables and supply chain profit changes. This section refers to a number of related documents and assumes a series of reasonable parameters. Specific data are shown in Table 4.

The equilibrium results are obtained and presented in Table 5. Compared with model I, the total recycling volume of used products increased significantly because the

Table 4: The model parameters.

A	r	β	z	λ	g_0	q_1	q_2
5000	0.5	0.2	10	0.0003	0.03	0.9	0.6
B	C_n	k	g_a	θ	t	q_3	
1000	2000	100	0.5	120	0.82	0.3	

government subsidized consumers participating in recycling activities and urged retailer responsible for recycling to strengthen recycling through reward and punishment measures. It shows that consumers are more willing to participate in recycling activities when they have recycling subsidies; the retailer will strive to improve the recycling level in order to obtain higher recycling rewards. With the increase in wholesale and retail prices of remanufactured products, the market demand for remanufactured products has increased. It reveals that the government's subsidies and intervention measures for remanufacturing are conducive to the development of remanufacturing industry; however, the profits of manufacturer, retailer, and the total profits of remanufacturing supply chain system have decreased. In order to optimize the benefits of each member of the system and maximize the total profit of the supply chain system, it is necessary to coordinate the supply chain. With government participation and supply chain coordination in model III, manufacturer and retailer no longer blindly pursue the maximization of their own interests. Manufacturer greatly reduced the wholesale price of new/remanufactured products and gave profits to retailer; the retailer has significantly reduced the retail price of new/remanufactured products to give profits to consumers. It has greatly promoted the market demand for new/remanufactured products and significantly increased the total profit of remanufactured supply chain system. From a macro perspective, the contract coordination of revenue sharing can be used for reference, which is conducive to the development of remanufacturing industry.

5.1. The Effects of q_i on Supply Chain in Different Models. In this section, different (q_1, q_2, q_3) are presented to illustrate the effects of q_i on supply chain in the proposed models and obtain managerial insights. Table 6 depicts that, in model j ($j=$ I, II, III), the greater the quality level coefficient of recycled products, the higher the price of new products, and the lower the price of remanufactured products. This shows that the manufacturer recycles high-quality used products, and the remanufacturing cost is also reduced, which encourages the manufacturer to reduce the wholesale price of remanufactured products. Accordingly, the benefiting retailer also reduces the retail price of remanufactured products, and consumers become the ultimate beneficiaries, which greatly increases the market demand of remanufactured products. Meanwhile, consumers can also obtain higher subsidies for recycled products by providing high-quality recycled products, so as to further stimulate them to participate in recycling activities. Recycling high-quality used products plays a positive role in promoting the development of remanufacturing industry.

5.2. The Effects of r on Supply Chain in Different Models. This section involves the effects of r on prices of the products, demand for the products, and supply chain profits for the three models. The effects of r on wholesale (w_n^{j*} and w_r^{j*}) and retail prices (p_n^{j*} and p_r^{j*}) are shown in Figure 2. As can be observed in Figure 2, although, by growing r, w_n^{j*} and p_n^{j*} are constant, w_r^{j*} and p_r^{j*} are improved under all decision-making models. r is linear with w_r^{j*} and p_r^{j*}. It indicates that the stronger consumers' willingness to buy and pay for remanufactured products, the higher the price of remanufactured products, until they get closer and closer to the price of new products. However, the price of remanufactured products will always be lower than the price of new products. Further, as r increases, $w_n^{\mathrm{I}*}$ and $w_n^{\mathrm{II}*}$ are not much different, and $w_r^{\mathrm{I}*}$ and $w_r^{\mathrm{II}*}$ are also not much different. But after the contract coordination of revenue sharing, $w_n^{\mathrm{III}*}$ and $w_r^{\mathrm{III}*}$ decrease significantly. It shows that after the retailer shares part of the sales revenue with the manufacturer, the manufacturer obviously gives profit to the retailer in the wholesale price for compensating the retailer. As a result, internal coordination and incentives in the supply chain are more effective in reducing wholesale price than external factors such as consumer market and government participation. The impact of government subsidies and incentives on retail price is not obvious. After the coordination of revenue sharing contract, $p_n^{\mathrm{III}*}$ and $p_r^{\mathrm{III}*}$ are reduced by a part, which benefits consumers.

Figure 3 indicates the changes on market demand (D_n^{j*} and D_r^{j*}) under different values of r. It can be seen from Figures 3(a)–3(c) that D_n^{j*} decreases, but D_r^{j*} increases in three models as r rises. On the one hand, existing $r_0^j \in (0.85, 0.95)$, when $r \in (0, r_0^j)$, D_n^{j*} slowly decreases, and D_r^{j*} changes from rapid growth to gentle growth; when $r \in [r_0^j, 1)$, D_n^{j*} decreases rapidly, while D_r^{j*} increases significantly. We can conclude that when consumers' willingness to buy and pay for remanufactured products is very close to that of new products, because remanufactured products have price advantages, consumers will be more inclined to buy remanufactured products rather than new products with more expensive prices. The market for new products will be eroded by the market for remanufactured products, which is conducive to environmental protection and sustainable development. On the other hand, existing $r_1^j \in (0.4, 0.6)$, when $r = r_1^j$, D_n^{j*} and D_r^{j*} are flat, then the gap of them gradually is widened. This shows that even when r is not very large, consumers are more willing to buy remanufactured products for the price is lower. Furthermore, from Figure 3(d), it is found that $D_r^{\mathrm{III}*} > D_r^{\mathrm{II}*} > D_r^{\mathrm{I}*}$ in the growth rate. As a result, when r remains unchanged, model III is more conducive to the rapid expansion of the consumer market for remanufactured products.

Table 5: Equilibrium results of the three proposed models.

Model	w_n^{j*}	w_r^{j*}	p_n^{j*}	p_r^{j*}	D_n^{j*}	D_r^{j*}	$D_n^{j*}G^{j*}$	W^{j*}	V^{j*}	F^{j*}
I	3716	1632.1	4358	2066.1	416.1	451.7	262.1	926350.9	463175.4	1389526.3
II	3792.2	1645	4382.9	2072.5	379	475.8	272.9	874817.6	437408.8	1312226.4
III	3014.3	1279.6	4321.9	2030.2	416.7	522.9	300	961339.1	433130.3	1394468.4

Table 6: Equilibrium results under different q_i in the three models.

Model	(q_1, q_2, q_3)	w_n^{j*}	w_r^{j*}	p_n^{j*}	p_r^{j*}	D_n^{j*}	D_r^{j*}	G^{j*}
I	(0.6, 0.4, 0.3)	3611	1900	4305.5	2200	789	—	0.48
	(0.9, 0.7, 0.5)	3764	1575	4382	2037.5	311	614	0.72
II	(0.6, 0.4, 0.3)	3663.2	1905.5	4327.4	2202.8	750.7	—	0.57
	(0.9, 0.7, 0.5)	3854.6	1579.4	4408.7	2039.7	262	658.5	0.81
III	(0.6, 0.4, 0.3)	2896	1514.3	4260.9	2173.4	825	—	0.57
	(0.9, 0.7, 0.5)	3071.6	1220.5	4350.2	1994.2	288	723.6	0.81

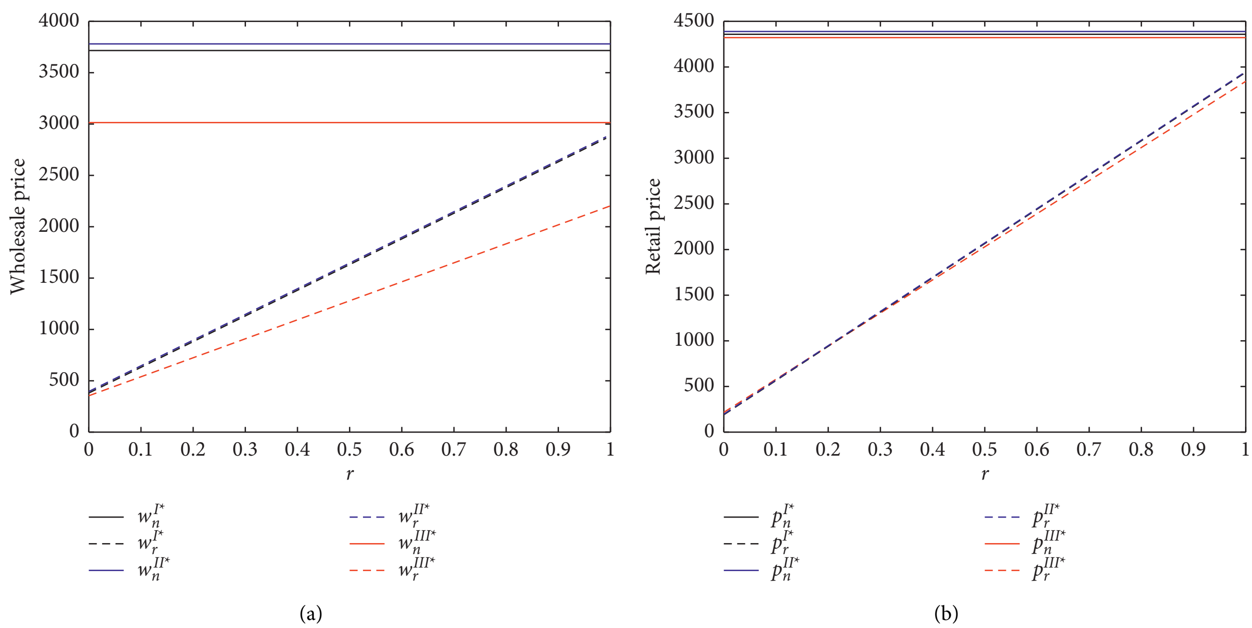


Figure 2: The effects of r on prices. (a) Wholesale prices. (b) Retail prices.

The effects of r on supply chain profits (W^{j*}, V^{j*} and F^{j*}) are shown in Figure 4. There are two special values $r_2^j \in (0, 0.1)$ and $r_3^j \in (0.9, 1)$. When $r \in (0, r_2^j)$, the profits decrease with the increase of r; when $r \in [r_2^j, r_3^j)$, the profits increase slowly with the increase of r; when $r \in [r_3^j, 1)$, the profits increase rapidly compared with the previous. Only when consumers treat remanufactured products equally with new products, and the degree of acceptance and recognition is very similar, the profit of the supply chain will increase significantly. Therefore, it is necessary to strengthen consumers' awareness of remanufactured products and improve the public's recognition and support for remanufactured products through advertising and environmental education. As shown in Figure 4(b), with the increase of r, $F^{\mathrm{III}*}$ has been improved remarkably, and its growth rate is significantly better than $F^{\mathrm{I}*}$ and $F^{\mathrm{II}*}$, which illustrates that the internal coordination contract of revenue sharing can optimize the total profit of the supply chain.

5.3. The Effects of k and θ on the Supply Chain in Different Models. In this section, we examine the impact of k and θ as two important parameters. The effects of k on wholesale price (w_n^{j*} and w_r^{j*}) and retail price (p_n^{j*} and p_r^{j*}) are shown in Figure 5. As k increases, the prices of new/remanufactured products will both increase, but the rate of increase in the price of new products is significantly higher than that of remanufactured products. As a result, it can be concluded that the larger the government subsidy price for recycled products can raise the price gap between new

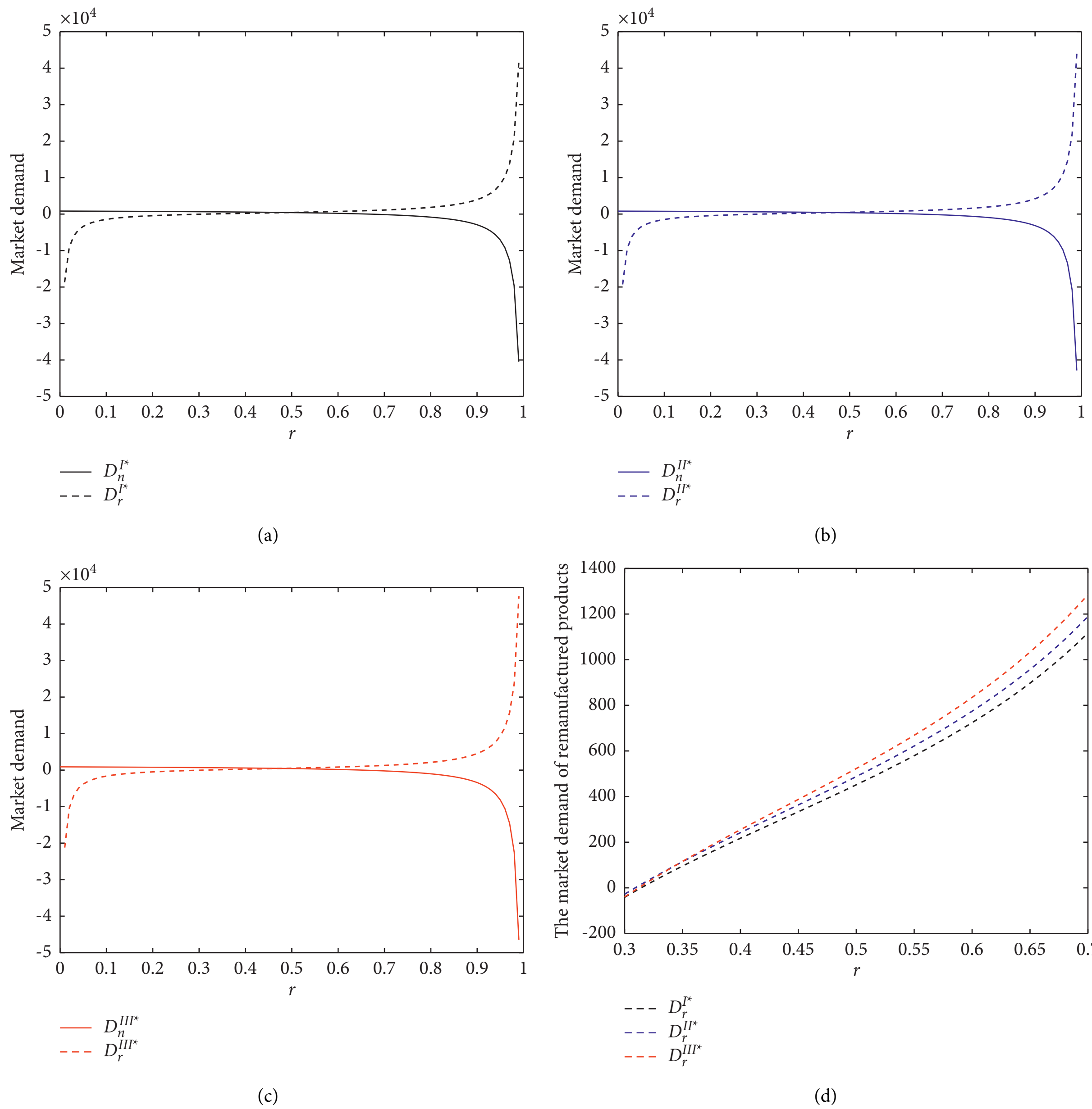


Figure 3: The effects of r on market demands. (a) Model I. (b) Model II. (c) Model III. (d) Remanufactured products market.

products and remanufactured products. For the same k, $w_n^{\mathrm{III}*}$ and $w_r^{\mathrm{III}*}$ are greatly reduced, which indicates that, after the contract coordination of revenue sharing, the manufacturer, in order to compensate the retailer, obviously gave the retailer a profit in the wholesale price.

The influences of k on market demands (D_n^{j*} and D_r^{j*}) and profits (W^{j*}, V^{j*} and F^{j*}) are revealed in Figures 6 and 7. It can be clearly seen that D_n^{j*} will decrease, but D_r^{j*} will increase by growing k. The government subsidy for the recycling price of used products is conducive to increase the market demand for remanufactured products. In addition, we can see that, for the same k, after supply chain coordination, $D_n^{\mathrm{III}*}$ and $D_r^{\mathrm{III}*}$ have a certain degree of improvement, which shows that the coordination contract of revenue sharing can increase the market demand for new/remanufactured products. There exists $k_0^j \in (550, 650)$, when $k \in (0, k_0^j)$, W^{j*}, V^{j*} and F^{j*} all gradually decrease with the increase of k; when $k \in (k_0^j, +\infty)$, W^{j*}, V^{j*} and F^{j*} all gradually increase with the increase of k. This shows that, in the early stage of remanufacturing, the government subsidized the unit's high prices for recycled products, which stimulated the development of the remanufacturing industry. The market demand for remanufactured products increased rapidly, and the market demand for new products decreased rapidly. But the price of remanufactured products is much lower than that of new products, so the profits of manufacturer and retailer are reduced, and the loss of profits in the closed-loop supply chain becomes greater. With the further development of the remanufacturing industry, the demand and price of remanufactured products have

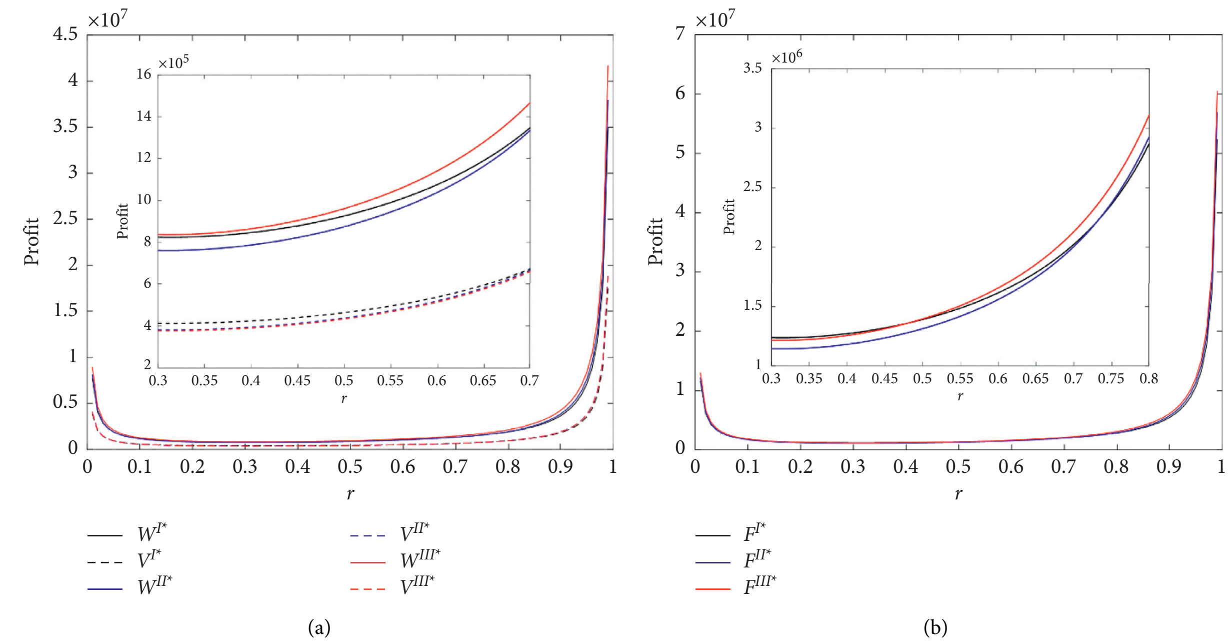


Figure 4: The effects of r on profits. (a) Supply chain members. (b) Supply chain system.

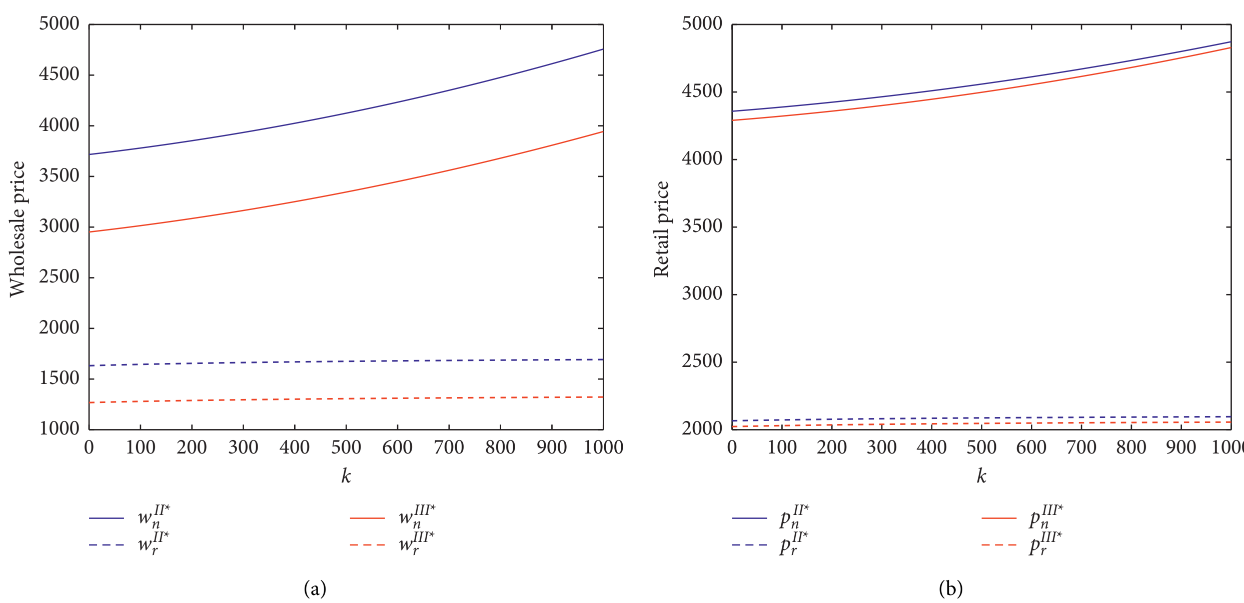


Figure 5: The effects of k on prices. (a) Wholesale prices. (b) Retail prices.

continued to increase. Manufacturer and retailer mainly rely on remanufactured products to make profits. The price of new products is increasing at a faster rate, which is used to increase the profit of new products. The interaction between the two leads to an increase in profits. After supply chain coordination, $W^{\mathrm{III}*}$ and $V^{\mathrm{III}*}$ have been significantly improved, and finally, $F^{\mathrm{III}*}$ has been optimized. At the same time, it can be seen that, compared with model II, the profit response speed of model III is more sensitive to k. It shows that internal coordination and incentives in the supply chain can promote the improvement of corporate profits more than external factors such as government participation.

Figure 8 represents the changes of wholesale price (w_n^{j*} and w_r^{j*}) and retail price (p_n^{j*} and p_r^{j*}) by increasing θ. As θ increases, w_r^{j*} and p_r^{j*} do not change, w_n^{j*} increases, and p_n^{j*} decreases.

Figures 9 and 10 demonstrate the trends of market demands (D_n^{j*} and D_r^{j*}) and profits (W^{j*}, V^{j*} and F^{j*}) by growing θ in different models. It can clearly be seen that as θ increases, both models II and III lead to an increase in the

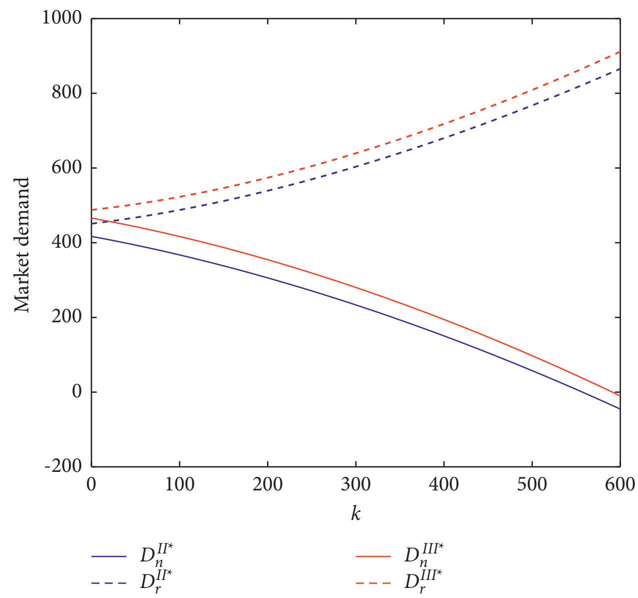


FIGURE 6: The effects of k on market demands.

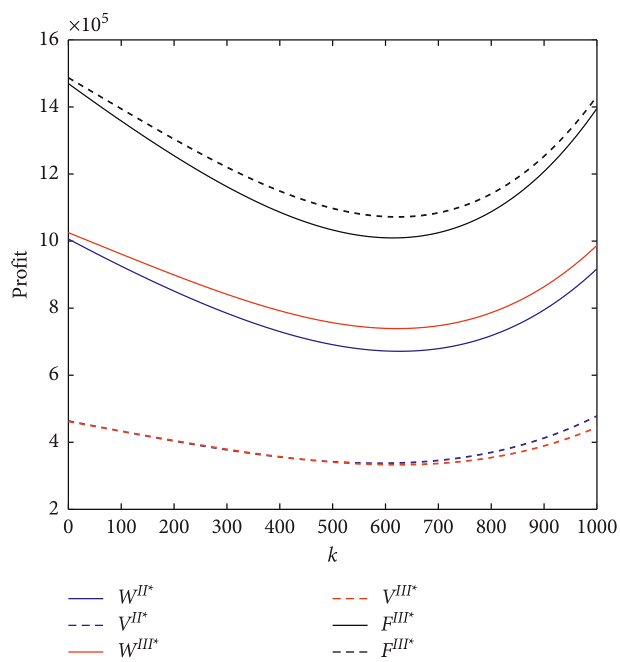


FIGURE 7: The effects of k on profits.

D_n^{j*} and profits but a decrease in D_r^{j*}. This is because, with the increase of the government's reward and punishment coefficient for retailer, without changing the recovery rate of used products, manufacturer and retailer will make more efforts to produce and sell new products and improve the recovery of used products, so as to obtain higher profits. No matter in which model, there is a special value $\theta_0^j \in (500, 600)$, making $D_n^{j*} = D_r^{j*}$; when $\theta \in (0, \theta_0^j)$, $D_n^{j*} < D_r^{j*}$; when $\theta \in (\theta_0^j, +\infty)$, $D_n^{j*} > D_r^{j*}$. In model III, the reaction of profits to θ is more sensitive than that of model II. It shows that when the value of k is not large, the internal coordination and incentives of the supply chain can promote the increase of corporate profits more than external factors such as government participation.

On the other hand, from Figures 11 and 12, we find that when $k = 1000$, $\theta = 1000$, manufacturer and retailer get less profit, but when $k = 0$, $\theta = 1000$, manufacturer and retailer get more profit. This shows that the government's recycling price subsidy to consumers and the retailer's recycling reward and punishment coefficient should be set appropriately, which should not be too high or too low at the same time. Moreover, compared with the government's subsidy measures for consumers' recycling prices, the government's recycling reward and punishment mechanism for retailer can better promote the profit. When θ is relatively small, the profits of manufacturer and retailer decrease first and increase as k increases; when θ is relatively large, the profits of manufacturer and retailer decrease as k increases. When k is relatively small, the profits of manufacturer and retailer increase with the raise of θ; when k is relatively large, the profits of manufacturer and retailer decrease with the increase of θ; the profit of manufacturer under supply chain coordination is much higher than that under the no-revenue-sharing contract, and the profit of retailer is not much different, indicating that the internal coordination of supply chain members can effectively promote the implementation of government measures and improve corporate profits.

5.4. The Effects of θ and g_a on Profits in Different Models. In this section, we examine the effects of θ and g_a on profits of the supply chain. In models II and III, when g_a is relatively small, the profits of members increase by growing θ, but when g_a is relatively large, the profits of members decrease with the increase of θ, according to Figure 13. This is mainly because when $G^j > g_a$, the government adopts incentive measures, and the bigger the θ, the more the retailer benefit; on the contrary, when $G^j < g_a$, the government takes

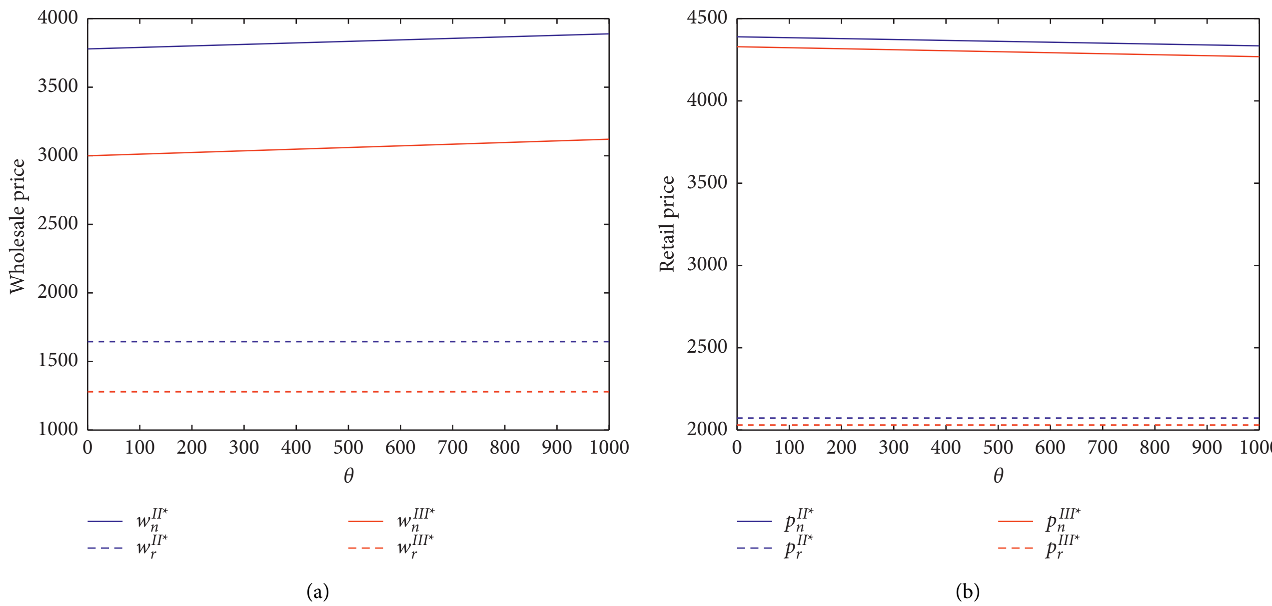


Figure 8: The effects of θ on prices. (a) Wholesale prices. (b) Retail prices.

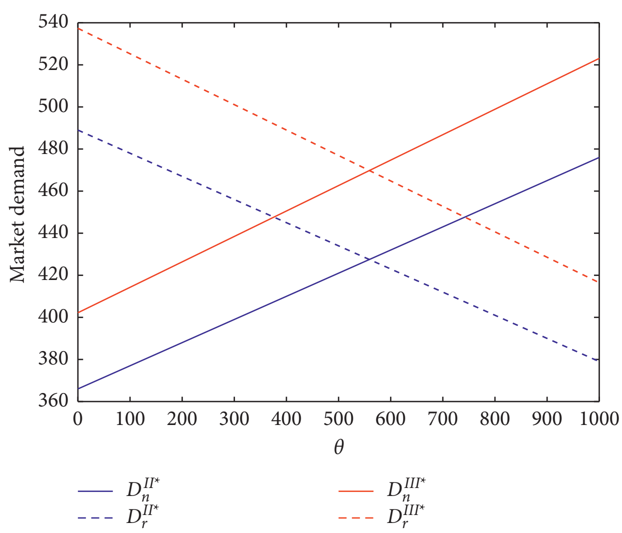


Figure 9: The effects of θ on market demands.

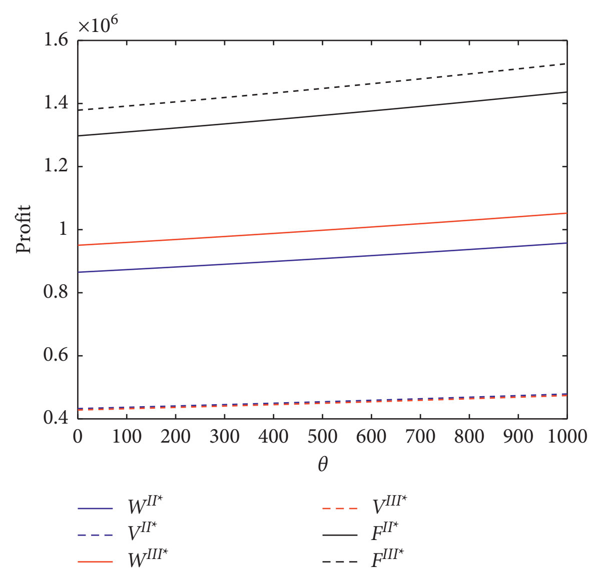
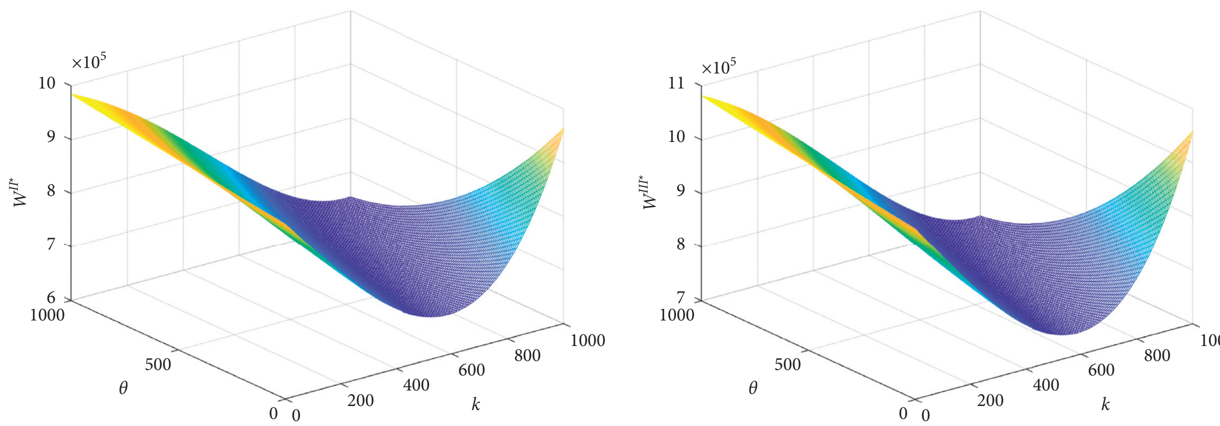


Figure 10: The effects of θ on profits.

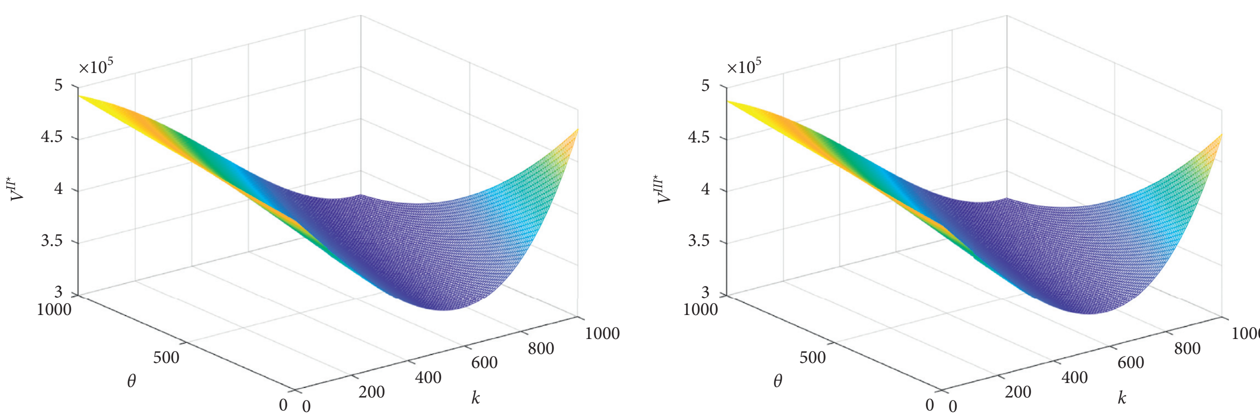
punitive measures, and the bigger the θ is, the more the retailer will lose. Meanwhile, the profits of members decrease as g_a raises in the two models. With improving θ, profits respond more quickly to g_a. The profits of manufacturer and retailer decrease more slowly in model III than other models, which indicates that the internal coordination contract of the supply chain is beneficial to improving the loss of corporate benefits under the participation of government remanufacturing activities. In addition, the profits of members are always higher than the profits with government participation.

5.5. The Effects of t on Profits in Different Models. The influence of t on profit of supply chain is shown in Figure 14. It can clearly be seen that as t increases, the profit of the manufacturer decreases, and the profit of the retailer increases. However, the rate and amount of change of the manufacturer are greater than those of the retailer, resulting in a decrease in the total profit of the supply chain. When $t \in (0, 0.9)$, we have $W^{\mathrm{III}*} > W^{\mathrm{I}*} > W^{\mathrm{II}*}$; when $t \in (0.8, 1)$, $V^{\mathrm{III}*}$, $V^{\mathrm{II}*}$ and $V^{\mathrm{I}*}$ are almost equal; when $t \in (0, 0.83)$, we have $F^{\mathrm{III}*} > F^{\mathrm{I}*} > F^{\mathrm{II}*}$. From here, we see that when the proportion of sales revenue sharing is too low or too high, it is not conducive to the improvement of corporate profits. It leads to the failure of supply chain coordination contract and hinders the improvement of the overall efficiency of the supply chain, and the system profit decreases significantly. Therefore, when an enterprise enters into a sales revenue sharing contract, it is necessary to set a reasonable revenue



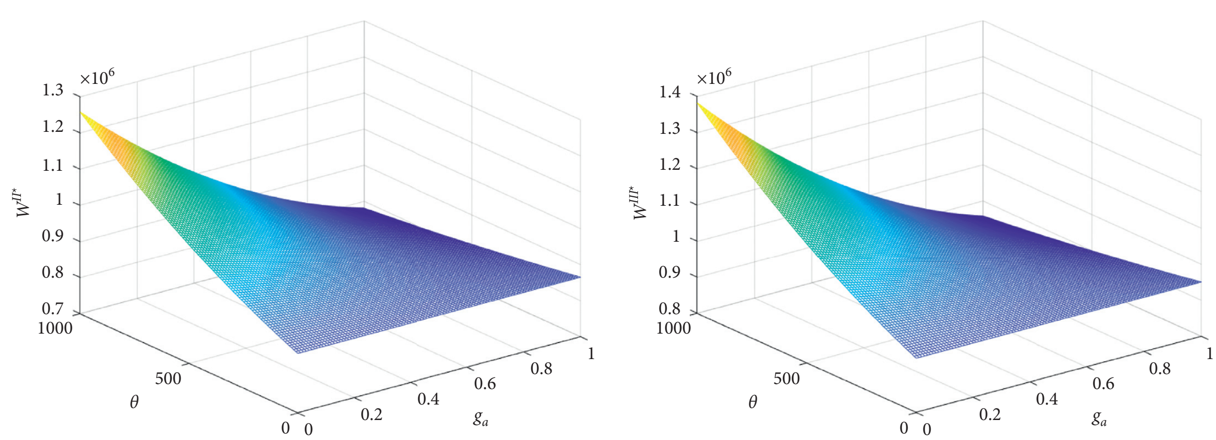
(a) (b)

Figure 11: The effects of θ and k on profit of manufacturer. (a) $W^{\mathrm{II}*}$ (b) $W^{\mathrm{III}*}$.



(a) (b)

Figure 12: The effects of θ and k on profit of retailer. (a) $V^{\mathrm{II}*}$ (b) $V^{\mathrm{III}*}$.



(a) (b)

Figure 13: Continued.

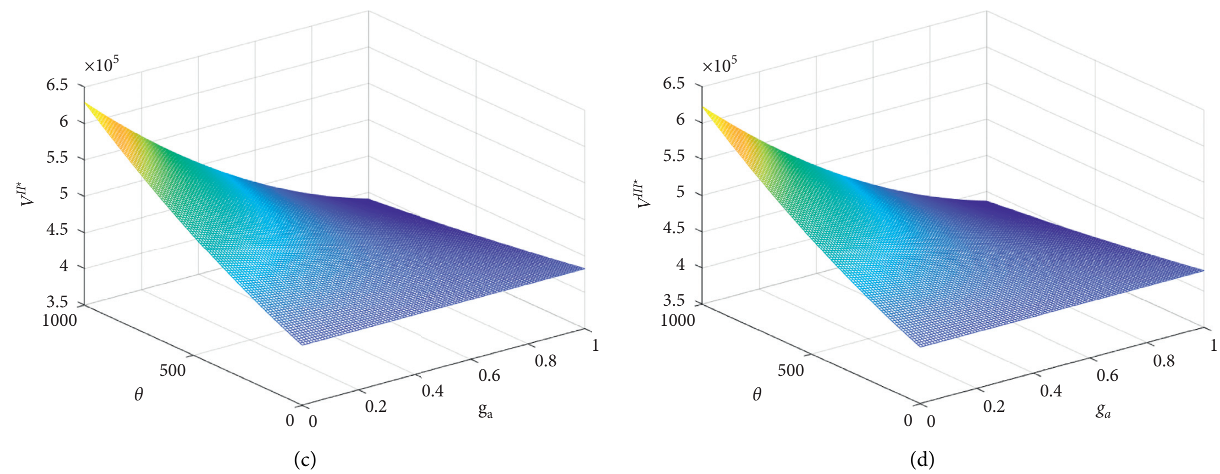


FIGURE 13: The effects of θ and g_a on profits of members. (a) W^{II*} (b) W^{III*} (c) V^{II*}.(d) V^{III*}.

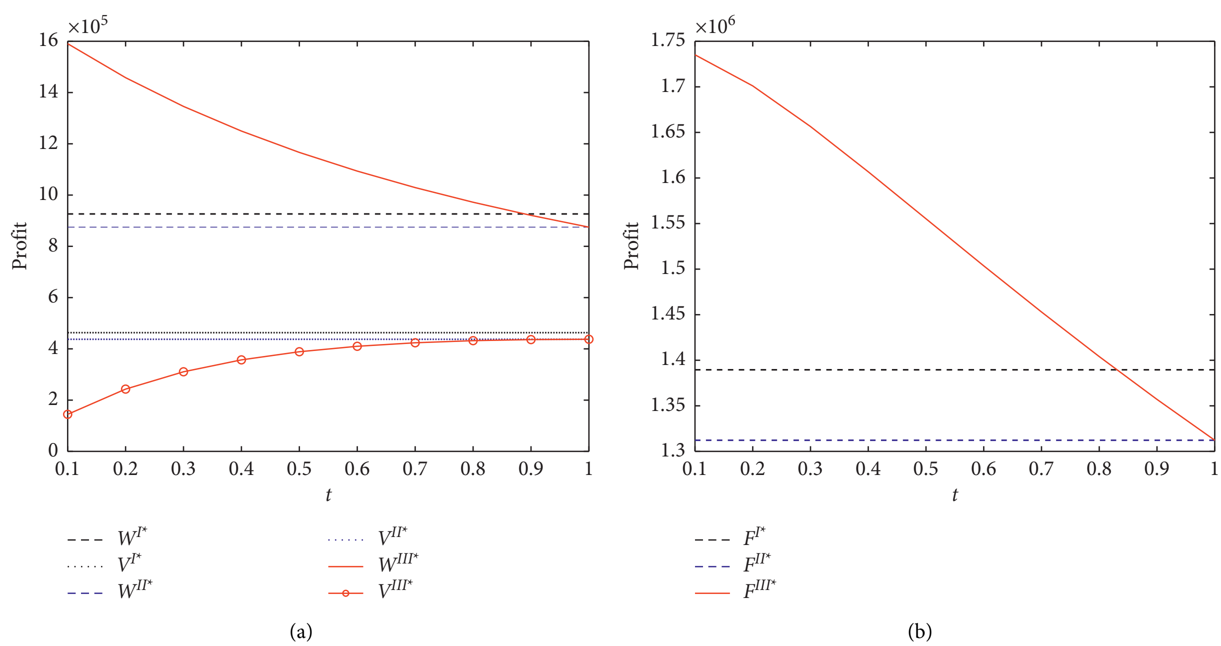


FIGURE 14: The effects of t on profits. (a) The profits of manufacturer and retailer. (b) The profit of supply chain system.

sharing ratio. Under certain conditions, the contract can effectively improve the benefits of manufacturer, retailer, and the entire supply chain system.

6. Conclusion

6.1. Conclusions and Managerial Implications. In order to satisfy consumers' increasing environmental awareness and the requirements of government policies and regulations, companies implement recycling and remanufacturing of used products. This paper constructs a manufacturer-led Stackelberg game model with government participation, supply chain coordination, and the retailer recycling. Through comparative analysis of the quality differences of recycled used products, consumers' preferences for remanufactured products, product pricing, and market demand are analyzed in three models, that is, without government participation and non-supply-chain coordination, with government participation but non-supply-chain coordination, and with government participation and supply chain coordination. The influence of the consumer market and government participation parameters on the decision-making and profit of the remanufacturing supply chain is obtained. The main findings of this study are as follows:

(1) Consumers' preferences for remanufactured products will affect the pricing decisions of all members of the supply chain regarding remanufactured products but will not affect the price of new products. The

stronger the consumers' desire to buy and pay for remanufactured products, the higher the price of remanufactured products until it is closer and closer to the price of new products, but it is still slightly lower than it. When consumers' willingness to buy and pay for remanufactured products is very close to that of new products, for remanufactured products have price advantages, most consumers will buy remanufactured products. At this time, the profits of the supply chain will increase significantly. Therefore, it is necessary to strengthen consumers' awareness of environmentally-friendly products and raise the public's recognition and support for remanufactured products through advertising, environmental protection education, and other means.

(2) Recycling high-quality used products can increase consumer recycling subsidy prices and effectively reduce the price of remanufactured products. Therefore, consumers become the biggest beneficiaries, thereby promoting the expansion of the demand market for remanufactured products, which has a positive effect on the development of the remanufacturing industry.

(3) The role of the consumer market and the internal coordination contract of the supply chain has limited incentives for the remanufacturing activities of the enterprise. The independent behavior of the enterprise alone cannot achieve the increase in the recycling rate of used products and the market demand for remanufactured products. However, the government's financial subsidies, rewards, and punishment mechanisms are conducive to consumers and retailer to actively participate in recycling and remanufacturing activities, increasing the recycling rate of used products and the market demand for remanufactured products.

(4) The government's excessively high basic recycling level has led to a decline in the profits of manufacturers and retailers, which is adverse to the improvement of the overall efficiency of the supply chain. The reward and punishment mechanism is more conducive to encouraging manufacturer and retailer to actively participate in remanufacturing activities than subsidy measures.

(5) In the model that lacks supply chain coordination, government subsidies, rewards, and punishments directly affect consumers and retailer but have little impact on manufacturer. In the supply chain coordination model, the incentives of government measures to the manufacturer are partially transferred to manufacturer through revenue sharing contracts. When the revenue sharing ratio is within a certain range, the contract can significantly increase the manufacturer's profit and effectively improve the benefits of the supply chain system.

(6) Compared with the role of external consumer market and the influence of government participation, the internal coordination contract of the supply chain has a more significant effect on the improvement of enterprise and system benefits. Manufacturer and retailer no longer blindly pursue the maximization of their own interests. Manufacturer has drastically reduced the unit wholesale price of new/remanufactured products, obviously giving up profits to retailer. Meanwhile, retailer has also significantly lowered retail prices, giving consumers preferential treatment. The recycling volume of used products, market demand for remanufactured products, and supply chain profit are all optimized in the supply chain coordination model. Coordination among supply chain members can effectively enhance the role of the consumer market and promote the implementation of government measures. From a macro perspective, supply chain coordination with revenue sharing is a very beneficial measure for market development.

6.2. Future Directions. This paper takes into account such things as the role of consumer market, government participation, and supply chain coordination, providing reference for the long-term decision-making of remanufacturing supply chain, improving the overall efficiency of supply chain, and realizing system optimization. The proposed model has some limitations and can be extended from several viewpoints. For instance, the situation in complex environments (such as uncertain market demand, uncertain recycling quantity of used products, uncertain the amount of government subsidies, asymmetry of information, etc.) is not considered. Thus, we will conduct more in-depth research in the follow-up. Moreover, in the case of government participation, only the price subsidy for consumers and the reward and punishment measures for retailer are discussed. However, there is almost no attention to constraints such as government incentives and penalties for manufacturers' remanufacturing capacity and the balance of government revenue and expenditure, which are also among the issues worth studying in the future.

Appendix

A. Proof of Proposition 1

Proof: To obtain the equilibrium strategy of the Stackelberg game, we use the concept of backward induction.

(1) After substituting the market demand function, the following results are obtained as follows:

$$V^{\mathrm{I}} = \left(A - \frac{p_n^{\mathrm{I}} - p_r^{\mathrm{I}}}{1-r}\right)\left(p_n^{\mathrm{I}} - w_n^{\mathrm{I}}\right) + \left(\frac{p_n^{\mathrm{I}} - p_r^{\mathrm{I}}}{1-r} - \frac{p_r^{\mathrm{I}}}{r}\right)\left(p_r^{\mathrm{I}} - w_r^{\mathrm{I}}\right). \tag{A.1}$$

Hessian matrix of $V^{\mathrm{I}}(p_n^{\mathrm{I}}, p_r^{\mathrm{I}})$ is H_1^{I},

$$H_1^{\mathrm{I}} = \begin{bmatrix} \dfrac{2}{r-1} & \dfrac{-2}{r-1} \\ \dfrac{-2}{r-1} & \dfrac{2}{r-1} - \dfrac{2}{r} \end{bmatrix}. \quad (A.2)$$

Since $r \in (0,1)$, the first-order principal subformula of H_1^{I} is $2/r-1<0$, and the second-order principal subformula is $4/r(1-r)>0$. So, H_1^{I} is a negative definite matrix. When the first partial derivative of V^{I} with respect to the decision variables p_n^{I} and p_r^{I} is 0, that is,

$$\begin{cases} \dfrac{\partial V^{\mathrm{I}}}{\partial p_n^{\mathrm{I}}} = A + \dfrac{p_n^{\mathrm{I}} - p_r^{\mathrm{I}}}{r-1} + \dfrac{p_n^{\mathrm{I}} - w_n^{\mathrm{I}}}{r-1} - \dfrac{p_r^{\mathrm{I}} - w_r^{\mathrm{I}}}{r-1} = 0, \\ \dfrac{\partial V^{\mathrm{I}}}{\partial p_r^{\mathrm{I}}} = \left(p_r^{\mathrm{I}} - w_r^{\mathrm{I}}\right)\left(\dfrac{1}{r-1} - \dfrac{1}{r}\right) - \dfrac{p_n^{\mathrm{I}} - w_n^{\mathrm{I}}}{r-1} - \dfrac{p_r^{\mathrm{I}}}{r} - \dfrac{p_n^{\mathrm{I}} - p_r^{\mathrm{I}}}{r-1} = 0. \end{cases} \quad (A.3)$$

The optimal solution of p_n^{I} and p_r^{I} can be obtained.

$$\begin{cases} p_n^{\mathrm{I}} = \dfrac{A}{2} + \dfrac{w_n^{\mathrm{I}}}{2}, \\ p_r^{\mathrm{I}} = \dfrac{w_r^{\mathrm{I}}}{2} + \dfrac{Ar}{2}. \end{cases} \quad (A.4)$$

(2) Substituting the optimal retail prices obtained in the previous step, we get

$$W^{\mathrm{I}} = \frac{A - w_n^{\mathrm{I}} + w_r^{\mathrm{I}} - Ar}{2(1-r)} \left(w_n^{\mathrm{I}} - C_n - \sum_{i=1}^{e} (g_0 + \lambda B q_i) B q_i \right) + \frac{w_r^{\mathrm{I}} - r w_n^{\mathrm{I}}}{2r(r-1)} \left(w_r^{\mathrm{I}} - \frac{\sum_{i=1}^{3} \left((1+\beta)(1-q_i)C_n + z\right)(g_0 + \lambda B q_i)}{\sum_{i=1}^{3} (g_0 + \lambda B q_i)} \right). \quad (A.5)$$

Hessian matrix of $W^{\mathrm{I}}(w_n^{\mathrm{I}}, w_r^{\mathrm{I}})$ is H_2^{I},

$$H_2^{\mathrm{I}} = \begin{bmatrix} \dfrac{1}{r-1} & \dfrac{1}{1-r} \\ \dfrac{1}{1-r} & \dfrac{1}{r(r-1)} \end{bmatrix}. \quad (A.6)$$

Since $r \in (0,1)$, the first-order principal subformula of H_2^{I} is $1/(r-1)<0$, and the second-order principal subformula is $1/r(1-r)>0$. So, H_2^{I} is a negative definite matrix.

When the first partial derivative of W^{I} with respect to the decision variables w_n^{I} and w_r^{I} is 0, the optimal solution of w_n^{I} and w_r^{I} can be obtained.

$$\begin{cases} w_n^{\mathrm{I}*} = x_1^{\mathrm{I}*} + \dfrac{A}{2}, \\ w_r^{\mathrm{I}*} = x_2^{\mathrm{I}*}. \end{cases} \quad (A.7)$$

The results of $x_1^{\mathrm{I}*}$ and $x_2^{\mathrm{I}*}$ are as follows:

$$\begin{aligned} x_1^{\mathrm{I}*} &= \frac{\lambda B^2 \sum_{i=1}^{n} q_i^2 + B g_0 \sum_{i=1}^{n} (q_i + C_n)}{2}, \\ x_2^{\mathrm{I}*} &= \frac{C_n g_0 (1+\beta)\left(3 - \sum_{i=1}^{n} q_i\right) + (z + Ar)\left(3g_0 + \lambda B \sum_{i=1}^{n} q_i\right) + \lambda \mathrm{B} C_n (1+\beta) \sum_{i-1}^{n} \left(q_i - q_i^2\right)}{\left(6g_0 + 2B\lambda \sum_{i=1}^{n} q_i\right)}. \end{aligned} \quad (A.8)$$

Therefore, the equilibrium results are as follows:

$$\begin{aligned} w_n^{I*} &= x_1^{I*} + \frac{A}{2}, \\ w_r^{I*} &= x_2^{I*}, \\ p_n^{I*} &= \frac{x_1^{I*}}{2} + \frac{3A}{4}, \\ p_r^{I*} &= \frac{x_2^{I*} + Ar}{2}. \end{aligned} \tag{A.9}$$

Corresponding to

$$\begin{aligned} D_n^{I*} &= A - \frac{2\left(x_1^{I*} - x_2^{I*} - Ar\right) + 3A}{4(1-r)}, \\ D_r^{I*} &= \frac{2\left(x_1^{I*} - x_2^{I*} - Ar\right) + 3A}{4(1-r)} - \frac{x_2^{I*} + Ar}{2r}, \end{aligned} \tag{A.10}$$

The maximum profits of the corresponding manufacturer and retailer are as follows:

$$\begin{aligned} W^{I*} &= \left[A - \frac{2\left(x_1^{I*} - x_2^{I*} - Ar\right) + 3A}{4(1-r)}\right]\left(x_1^{I*} + \frac{A}{2} - C_n\right) + \left[\frac{2\left(x_1^{I*} - x_2^{I*} - Ar\right) + 3A}{4(1-r)} - \frac{x_2^{I*} + Ar}{2r}\right], \\ &\left(x_2^{I*} - \frac{\sum_{i=1}^{3}\left(g_0 + \lambda B q_i\right)\left[(1+\beta)(1-q_i)C_n + z\right]}{\sum_{i=1}^{3}\left(g_0 + \lambda B q_i\right)}\right) - \sum_{i=1}^{3} B q_i \left(g_0 + \lambda B q_i\right)\left[A - \frac{2\left(x_1^{I*} - x_2^{I*} - Ar\right) + 3A}{4(1-r)}\right], \\ V^{I*} &= \left[A - \frac{2\left(x_1^{I*} - x_2^{I*} - Ar\right) + 3A}{4(1-r)}\right]\left(\frac{A}{4} - \frac{x_1^{I*}}{2}\right) + \left[\frac{2\left(x_1^{I*} - x_2^{I*} - Ar\right) + 3A}{4(1-r)} - \frac{x_2^{I*} + Ar}{2r}\right]\left(\frac{Ar}{2} - \frac{x_2^{I*}}{2}\right). \end{aligned} \tag{A.11}$$

This ends the proof.

Owing to the fact that the solving process of Propositions 2 and 3 is similar to that of Proposition 1, we will not repeat it here. □

B. Proof of Lemma 1

Proof: it can be obtained that $\partial w_n^{I*}/\partial r = \partial p_n^{I*}/\partial r = 0$, $\partial w_r^{I*}/\partial r = A/2 > 0, \partial p_r^{I*}/\partial r = 3A/4 > 0$ from Proposition 1. Furthermore, according to Assumption 2, we can obtain

$$\begin{aligned} \frac{\partial D_n^{I*}}{\partial r} &= \frac{\partial\left(A - p_n^{I*} - p_r^{I*}/(1-r)\right)}{\partial r} \\ &= -\frac{\lambda B^2 \sum_{i=1}^{3} q_i^2 + B g_0 \sum_{i=1}^{3} q_i + C_n - z}{4(1-r)^2} + \frac{C_n g_0 (1+\beta)\left(3 - \sum_{i=1}^{3} q_i\right) + \lambda B C_n (1+\beta) \sum_{i=1}^{3}\left(q_i - q_i^2\right)}{4(1-r)^2\left(3g_0 + B\lambda \sum_{i=1}^{3} q_i\right)}, \end{aligned} \tag{B.1}$$

To prove $\partial D_n^{I*}/\partial r < 0$, that is to say,

$$C_n g_0 (1+\beta)\left(3 - \sum_{i=1}^{3} q_i\right) + \lambda B C_n (1+\beta) \sum_{i=1}^{3}\left(q_i - q_i^2\right) < \left(3g_0 + B\lambda \sum_{i=1}^{3} q_i\right)\left(\lambda B^2 \sum_{i=1}^{3} q_i^2 + B g_0 \sum_{i=1}^{3} q_i + C_n - z\right), \tag{B.2}$$

This is to certify that

$$(\beta C_n + z)\left(3g_0 + B\lambda \sum_{i=1}^{3} q_i\right) < \left[B\left(3g_0 + B\lambda \sum_{i=1}^{3} q_i\right) + C_n(1+\beta)\right]\left(g_0 \sum_{i=1}^{3} q_i + B\lambda \sum_{i=1}^{3} q_i^2\right), \tag{B.3}$$

From Assumption 7, we get

$$3\beta C_n + 3z + B\sum_{i=1}^{3} q_i < (1+\beta)C_n \sum_{i=1}^{3} q_i, \quad \text{that is } \beta C_n + z < \frac{\left[(1+\beta)C_n - B\right]\sum_{i=1}^{3} q_i}{3}, \tag{B.4}$$

So,

$$(\beta C_n + z)\left(3g_0 + B\lambda\sum_{i=1}^{3} q_i\right) < \left[C_n(1+\beta) - B\right]\left[g_0\sum_{i=1}^{3} q_i + \frac{B\lambda\left(\sum_{i=1}^{3} q_i\right)^2}{3}\right]. \tag{B.5}$$

For

$$\left[C_n(1+\beta) - B\right]\left[g_0\sum_{i=1}^{3} q_i + \frac{B\lambda\left(\sum_{i=1}^{3} q_i^2\right)}{3}\right] < \left[B\left(3g_0 + B\lambda\sum_{i=1}^{3} q_i\right) + C_n(1+\beta)\right]\left(g_0\sum_{i=1}^{3} q_i + B\lambda\sum_{i=1}^{3} q_i^2\right), \tag{B.6}$$

This is obviously true; therefore, we get $\partial D_n^{\mathrm{I}*}/\partial r < 0$. For

$$\frac{\partial D_r^{\mathrm{I}*}}{\partial r} = -\frac{\partial D_n^{\mathrm{I}*}}{\partial r} + \frac{C_n g_0(1+\beta)\left(3 - \sum_{i=1}^{3} q_i\right) + z\left(3g_0 + \lambda B\sum_{i=1}^{3} q_i\right) + \lambda \mathrm{B} C_n(1+\beta)\sum_{i=1}^{3}\left(q_i - q_i^2\right)}{r^2} > 0, \tag{B.7}$$

Therefore, $w_r^{\mathrm{I}*}$, $p_r^{\mathrm{I}*}$ and $D_r^{\mathrm{I}*}$ are positively correlated with r, $D_n^{\mathrm{I}*}$ is negatively correlated with r, and $w_n^{\mathrm{I}*}$ and $p_n^{\mathrm{I}*}$ are independent of r.

This ends the proof.

The proof process of Lemmas 2~5 is similar to Lemma 1, so it will not be repeated here. □

C. Proof of Lemma 6

Proof: According to Propositions 1 and 2, we can obtain

$$\begin{aligned} G^{\mathrm{II}*} - G^{\mathrm{I}*} &= \left(3g_0 + 3k\lambda + \lambda B\sum_{i=1}^{3} q_i\right) - \left(3g_0 + \lambda B\sum_{i=1}^{3} q_i\right) \\ &= 3k\lambda > 0, \end{aligned} \tag{C.1}$$

This ends the proof.

The proof process of Lemmas 7~12 is similar to Lemma 6, so it will not be repeated here. □

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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