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Research Article

A New Approach to Decision-Making Problem under Complex Pythagorean Fuzzy Information

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In daily life, decision-making (DM) problem is a complicated work related to uncertainties and vagueness. To overcome these imprecisions, many fuzzy sets and theories have been presented by different scholars. Probabilistic models are the communal models proposed for the management of uncertainties. On the other hand, if these uncertainties are not probabilistic in nature, then other models such as fuzzy linguistic and fuzzy logic are developed. Here, a new approach known as the complex Pythagorean fuzzy Maclaurin symmetric mean (CPFMSM) operator is used to handle these uncertainties in DM issues. This complex Pythagorean fuzzy set (CPFS) is a modified form of the Pythagorean fuzzy set (PFS) and of the complex intuitionistic fuzzy set (CIFS). The aggregation operators have the ability to combine different sources of information. Therefore, an aggregation operator known as the MSM operator is utilized under the complex Pythagorean fuzzy (CPF) environment to extend the theory and applications of traditional MSM. For this purpose, we devised new operators known as CPFMSM and CPF dual Maclaurin symmetric mean (CPFDMSM) to aggregate CPF data. To evaluate an emergency program, the MAGDM approach is used, which is based on the newly introduced operators. Furthermore, the viability and applicability of the propounded method are certified by a detailed analysis with the other approaches researched in the past.

1. Introduction

Decision-making is a task performed frequently in practical life. The selection of the best suitable and desirable alternative from a group of reasonable alternatives is the main goal of DM. Decision-making is mainly composed of the judgment and the preferred information of the DM experts. Due to the rapid changes in technology, emergency management, and society, the complexity in the DM environment increases day by day. Therefore, more DMs and experts are required for the evaluation of complicated DM problems. Therefore, such problems are called group decision-making problems. In this scenario, the problem under consideration is also a group decision-making problem. There have been many papers in the literature in which complicated group decision-making problems have been handled [1, 2]. Integration of the judgment information and the way to describe the solution are two main problems in the DM process. Many approaches and methods from various aspects are introduced for the selection process of the desired value. In the conventional methods, the decision-making experts had to describe their desired value in the accurate form of numerical terms. If decision-makers represent their data in numerical terms, it becomes simple and easy for calculation. Due to increasing intricacy and vagueness in the field of decision-making, we have to manage the different types of uncertain information, which are inaccurate, incomplete, and sometimes inconsistent in nature. Many sets and theories are presented for the satisfactory solution of DM issues. One of them is the complex fuzzy set (CFS) theory put forward by Ramot et al. [3]. He presented the concept of CFS by extending the range of membership degree (MD) from real to complex numbers within the unit disc. A mathematical analysis of the CFS was given by Yazdanbakhsh and Dick [4]. Afterwards, Alkouri and Salleh [5, 6] extended the concept of CFS to CIFS by the expression of the complex-valued nonmembership degree. They also presented the idea of CIFS relation and distance measures under the CIF environment. The CIFMSM operator introduced by Ali et al. [7] is applied successfully for the optimal selection of emergency management program. Atanassov [8] introduced the intuitionistic fuzzy theory, which is an extended form of the fuzzy set (FS) theory [9]. All the members of the IFS are expressed through an ordered pair, and all the ordered pairs are categorized through membership degree (MD) and nonmembership degree (NMD). The summation of the MD and NMD of each ordered pair must be equal to or less than 1. The IFS theory has gained much attention after the discovery, but the IFS theory fails in depicting the evaluation information, i.e., if a decision-maker judges his evaluation information as $0.5 + 0.7 > 1$.

So, Yager [10] first presented the PFS to handle the undefined opinion in the DM problems. Moreover, PFS is also an efficient tool for the depiction of ambiguity in multiple attribute decision-making (MADM) issues. The PFS model satisfies the condition $g + h \leq 1$ with $g^2 + h^2 \leq 1$ and hence has a better ability than the IFS to handle the complex uncertainty in DM issues. The PFS has the ability to solve those problems, which are not solved by the IFS. In short, we can say that the PFS is more universal and all the intuitionistic fuzzy degrees are considered to be a fragment of the Pythagorean fuzzy degrees and thus are included in it. The idea of the Pythagorean fuzzy number (PFN) was introduced by Zhang and Xu [11]. They also propounded the detailed mathematical form for PFS and the Pythagorean fuzzy TOPSIS, which is a technique for order preference similar to the best solution. This method is used to solve the MCDM issue within PFNs. Pythagorean fuzzy superiority and inferiority approach and the subtraction and division operation were presented by Peng and Yang [12] to solve the MAGDM issue by PFNs. The PFNs were used by Reformat and Yager [13] to handle the collaborative-based recommender system. The characteristics of the continuous Pythagorean fuzzy information were studied by Gou et al. [14]. Using Einstein's operations, Garg [15] propounded the new generalized Pythagorean fuzzy information aggregator. By utilizing an interval-valued Pythagorean fuzzy (IVPF) environment, Garg researched a novel accuracy function for the evaluation of multi-criteria decision-making issues. Similarly, Li et al. [16] integrated IVPF sets (IVPFSs) with the MULTIMOORA technique and suggested a new FMEA system. This approach was implemented for the evaluation of emergency risk assessment of the east route of the south-to-north water diversion project. Likewise, Huang et al. proposed a MCDM approach by the fusion of PFS with the MULTIMOORA approach based on new score function and distance measure and applied it for the evaluation of solid-state disk productions [17]. The Pythagorean fuzzy Einstein, a novel operator, is proposed for the solution of real-life DM problems [18]. Furthermore, Ashraf et al. [19] integrated the sine trigonometric (ST) operator with PF, introduced the sine trigonometric Pythagorean fuzzy operator, and implemented it for the evaluation of Internet finance soft power problem. For understanding the historical progress and current situation, Lin et al. [20] presented the comprehensive analysis of PFS from 2013 to 2020. Furthermore, Lin et al. [21] devised some directional correlation coefficients for the measurement of the interrelationship between the PFSs and implemented them for medical diagnosis and cluster analysis. Akram et al. [22] extended the concept of ELECTRE I and TOPSIS method under the PFS environment. The main goal of the proposed techniques is to select a small set of "failures," which have high-risk priorities [23]. Similarly, Khan et al. extended the concept of PFS for decision support system and in this regard suggested a novel operator Pythagorean fuzzy Dombi aggregation [24]. In addition, to opt ideal medicine from various medicines, particularly, for coronavirus disease, Batool et al. [25, 26] used the concept of probabilistic hesitant fuzzy and Pythagorean fuzzy set. Wu developed various new operators using the hesitant fuzzy set (HFS) and PFS and validated them by three numerical examples of decision-making problems [27]. Furthermore, based on the single-valued neutrosophic 2-tuple linguistic concept, some novel Hamacher aggregation operators were proposed for the evaluation of MAGDAM problems. For more precision computation, they extend single-valued neutrosophic linguistic sets (SVNLSs) to single-valued neutrosophic 2-tuple linguistic sets (SVN2TLSs) [28]. The PFS has the capability to handle undefined and ambiguous information but fails to present the fractional ignorance of the information and its variations in a particular time period throughout the execution. In practical life, the ambiguity and vagueness existing in the information change according to the change in the periodicity of the data. The current hypothesis fails to consider this information, and in this way, some data are lost in the execution process.

Hence, to fulfill the above-mentioned defects of the CIF and PFS, Ullah et al. [29] put forward a new conception of CPFS, which satisfies the limitations of phase term and amplitude. CPFS can translate the ambiguity and vagueness of the personal opinion in a logical and detailed manner than the CIF because of the prominent property that $MD^2 + NMD^2 \leq 1$. Thus, the space to express the ill-defined information is a wider range, and therefore, due to this important characteristic, the CPFS is better than the previous existing theories and models. Besides this, by the removal of the phase term, we got PFS, which solves the problem by the degree of membership function, whereas the phase term is entirely overlooked. This ignorance causes loss of information during execution. So, the PFS is considered to be the particular case of CPFS. The association of the CPFS with the previous theories and approaches is stated in [30]. Akram et al. presented a hybrid DM technique utilizing the concept of CPFS and N-soft sets, and it was further used for the selection of best laptop and the plant locations [31]. Currently, Akram et al. [32] used TOPSIS and ELECTRIC I technique under the CPFS environment to address the multi-criteria group decision-making issues. The competition graphs in the CPF environment were discovered by Akram and Sattar [33]. It is the modified version [34] of the

VIKOR methodology for the solution of MAGDM problems under the CPF environment. Furthermore, a technique known as CP Dombi fuzzy operator is proposed for the evaluation of two-dimensional phenomena [35].

From the aforementioned discussion, we can conclude that many of the Pythagorean fuzzy aggregation operators presented are based on the algebraic sum and algebraic product of PFSs to continue the aggregation process. So, currently researchers and scholars are using the aggregation operators under the CPFS environment for the satisfactory solution of ill-defined decision-making issues. The Bonferroni mean (BM) and Heronian mean (HM) operators are presented to take the significance of any two-dimensional data into account. So, for the computation of picture fuzzy number (PiFN), Lin et al. [36] suggested interactional operational laws (IOLs), and on the basis of these laws, they proposed some aggregation operators such as the picture fuzzy interactional partitioned HM (PiFIPHM) and geometric PiFIPHM (PiFIPGHM). In the same way, Lin et al. [37] integrated linguistic q-rung orthopair fuzzy sets (LqROFSs) with HM operator and developed LqROF interactional weighted Pythagorean geometric HM (LqROFIWPGHM) operator, but unfortunately, both the HM and BM operators are unable in considering the interrelation between the multi-input data arguments. Therefore, the Maclaurin symmetric mean operator was basically presented by Maclaurin [38] and was further studied by Detemple and Robertson [39]. The dominant feature of MSM is that it is able to seize the relationship between the multi-input data arguments. This operator is also able to create robustness and flexibility in the data integration process and thus makes it suitable for the solution of MADM issue, where attributes are considered to be independent. In addition, for a set of the arguments, the MSM operator decreases monotonically with the decrease in the value of parameter. This shows the risk preference of the decision-making experts in real-life problems. In recent years, MSM has received much attention from scholars in various fields. Therefore, various essential developments have been made in theory and application of MSM [7, 40, 41]. So, using the benefits of the MSM and CPF, the fuzziness and ambiguity of the complex system are solved in a better way and the potential of the suggested method is confirmed by a numerical example of the emergency management program. We proposed MSM operators under the CPF environment for decision-making experts to express their cognitions for the MAGDM issues. Emergency management is a suitable term for handling the hazards of disasters and accidents in short interval of time by limited information. In the emergency management, public sectors and government try to confirm the activities associated with the emergency management, safety of the public life, health, property, and promotion of healthy and safe society, through the establishment of essential response mechanism. It contains essential measures, utilization of technology, science, management techniques, and plans. In the past, weak emergency management planes caused significant loss and damage to the global economy and to human lives. Therefore, for the optimal selection of emergency management plan, several researchers have implemented different aggregation operators in the past. For the most optimal selection in this study, we have propounded a more suitable approach CPFMSM, which is more universal than PFMSM and CIFMSM aggregation operators. As CPF is the general form of PF and CIF, similarly MSM is the universal form of HM and BM. Up to now, no research is based on the MSM operator for the aggregation of data under the CPF environment; so, it is required to focus on this issue. Inspired by this notion, this study takes the membership and nonmembership degrees into consideration as the complex Pythagorean fuzzy elements.

The summary of aforementioned discussion is as follows: (1) in the past research studies, fuzzy sets failed in illustrating the ill-defined and uncertain information, because the decision-making problems are complicated to evaluate in one dimension. For the solution of this problem, we used CPFS, which is useful in handling decision-making problems in two dimensions, and it avoids the loss of information while dealing with linguistic information. The CPFS is more reliable and flexible to solve hesitation where PFS fails. Therefore, CPFS can be considered more general than the present fuzzy sets. Hence, we initially discussed the CPFS and its basic concepts to express the evaluation data. On the other hand, the operator under consideration is not perfect in all aspects. The propounded approach is capable for two-dimensional values. Therefore, someone can extend the proposed concept for q dimensions to solve real-world complex DM problems more perfectly than the suggested approach. This can be done by defining more hybrids operators, which work better than the existing operators in the literature. (2) The integration of data has an essential role in the fusion of the preferred information of DM experts. Besides this, a lot of practical problems require the association of the recognized attributes. So, due to the usefulness of MSM operator and CPFS, some complex Pythagorean fuzzy MSM operators are presented to handle 2-dimensional fuzzy data, to define more novel operational laws for CPFSs, and to handle the uncertainties and vagueness of the complex system in a lucrative way. (3) For effectively integrating different DM matrices, here we have proposed two operators, CPFMSM and CPFDMSM. Furthermore, on the basis of integrated matrix, a ranking method is suggested for the evaluation of the DM problems. (4) The presented techniques show the risk attitude of various decision-makers in the practical application by the parameter k. (5) The PFMSM operator opts the best alternative from a group of proper alternatives using the Pythagorean fuzzy MSM operator framework, but it losses some of data due to the lack of phase term. The complex Pythagorean fuzzy set makes it possible to represent the information at a time in two dimensions. Similarly, CPFS contains the characteristic of PFS and CFS at the same time. Therefore, CPFS is superior to CFS, CIFS, and PFS. (6) The drawbacks and shortcomings of the present operators are being addressed by the propounded operators; as these operators are more universal, they work excellently not only for CPF data but also for IF, CPF, and PF information. (7) Some of the particular DM problems in our daily life have irrational calculation values, in which the multiple input arguments are not related to each other. Neither the CP nor the MSM operator is able to solve the problem individually; so, it is required to select an extensive operator, which not only fades the influence of the unreasonable values but also takes into consideration the relationship among the multiple input data arguments. The CPFS is able to express the fuzzy data more efficiently than the PFS and CIFS; so, it is extensively used to deliberate the assessment data in the above-described DM problems. Now, it is necessary to integrate the MSM operator with CPFS to introduce the CPFMSM operator for the evaluation of decision-making issues. The CPFMSM can fully exploit the benefits of the CPFS and MSM operators, and at the same time, it is also able to consider the features of the CPFS in describing the fuzzy data. (8) The selection and evaluation of emergency program in management sciences are a serious and important topic for research. Because of the complexity and irregularity in the emergency plan, various assessment approaches are required to handle the problem of emergency program in a proper way.

Our basic contribution to this study is to devise a general technique, for the evaluation of MAGDM and MCDM problems by the synthesis of the CPFS with MSM and DMSM aggregation operators. The selection of the best alternative is a complicated task in the DM setting. When the evaluation data are demonstrated by CIFNs and PFNs, they lead to information mutilation. Therefore, we require a more general technique for improving the capacity of alternatives. To attain this goal, first we have to find an appropriate tool to express the information. Then, we have to develop a decision-making algorithm, which will be useful in various aspects. As CPFSs are a remarkable extension of CIFSs and PFSs, it enables the situations to be modeled more broadly than CIFSs and PFSs. Subsequently, these previous approaches fail to succeed in solving some practical situations. MSM operators make the decision results more accurate and precise when implemented in daily life MADM based on the CPF environment. At the last, we have to confirm that our proposed method is helpful and efficient in different aspects by comparison with other methods. So, the main goal of this manuscript was stated as follows:

(1) To propound complex Pythagorean fuzzy averaging (CPFA) operator and complex Pythagorean fuzzy geometric (CPFG) operator on the basis of CPFMSM and CPFDMSM
(2) To propose and explain various MSM operators such as complex Pythagorean fuzzy Maclaurin symmetric mean (CPFMSM) and complex Pythagorean fuzzy dual Maclaurin symmetric mean (CPFDMSM) operator
(3) Some basic characteristics of the proposed operators are discussed such as idempotency, monotonicity, and boundedness
(4) To establish a MAGDM method based on these new approaches
(5) To explain the performance and validation of the introduced approaches by a universal example for the evaluation of emergency plan

Inspired by this idea, in this study the MSM is extended for the implementation in MAGDM and for the accumulation of complex Pythagorean fuzzy data. For this, the structure of the study is designed as follows.

In Section 2, the basic definitions and concepts related to CPFMSM such as PFS, complex Pythagorean fuzzy number (CPFN), CPFS, and operational laws of CPFNs are discussed in detail; furthermore, methods for comparison, the basic operational rules, and some important theorems are discussed, and basic operators and the aggregation operators MSM and DMSM are presented. Section 3 proposes new operators such as CPFMSM and CPFDMSM and also includes the explanation of their suitable characteristics. In Section 4, the MAGDM approach is presented, founded on the CPFMSM and CPFDMSM operators, and the emergency program is evaluated to prove the effectiveness of the method. A relative study is also conducted to prove the practicability of the introduced method. At the last in Section 5, some concluding remarks are enlisted.

2. Preliminaries

The purpose of this part is to present succinctly the pre-existing basic definitions associated with PFS, CPFS, and some correlated concepts and notations.

Definition 1 (see [10]). Suppose U be a fixed set, and a PFS B on U is a set of order pair and defined as follows:

$$B = \{u, (\mu_B(u), \nu_B(u)) | u \epsilon U\}, \tag{1}$$

where the mapping μ_B: $U \longrightarrow [0, 1]$ signifies the membership degree and ν_B: $U \longrightarrow [0, 1]$ indicates the non-membership degree of the member $u \epsilon U$ to B, respectively, and satisfies the condition that $0 \leq (\mu_B(u))^2 + (\nu_B(u))^2 \leq 1$ for all $u \epsilon U$. For ease, Zhang and Xu [11] named the pair of these membership functions as PFN, which is signified by $\beta = \mu_\beta, \nu_\beta$.

Definition 2 (see [11]). Suppose that $\beta = \mu_\beta, \nu_\beta$ is a PFN with the restriction that $0 \leq \mu_\beta, \nu_\beta \leq 1$ and $0 \leq (\mu_\beta)^2 + (\nu_\beta)^2 \leq 1$, and the score index S of β is defined as follows:

$$S(\beta) = \mu_B^2 - \nu_B^2, \quad S(\beta) \epsilon [-1, 1]. \tag{2}$$

The accuracy index E is defined as follows:

$$E(\beta) = \mu_B^2 + \nu_B^2, \quad E(\beta) \epsilon [-1, 1]. \tag{3}$$

For the comparison of PFNs, the following laws are presented by Zhang and Xu [11].

Definition 3 (see [11, 12]). Suppose that β_1 and β_2 are PFNs and $S(\beta_t)$ and $E(\beta_t)$ are score function and accuracy function of $\beta_t\, (t = 1, 2)$ respectively, and then,

(1) If $S(\beta_1) > S(\beta_2)$, then $\beta_1 \succ \beta_2$
(2) If $S(\beta_1) = S(\beta_2)$, then
   (i) If $E(\beta_1) > E(\beta_2)$, then $\beta_1 \succ \beta_2$

(ii) If $E(\beta_1) = E(\beta_2)$, the same numbers are represented and are denoted as $\beta_1, \sim, \beta_2$

Definition 4 (see [29]). A complex Pythagorean fuzzy set L on universal set U' is an object of the form

$$L = \left\{\left(u, g_L(u)e^{i2\pi\alpha_L(u)}, h_L(u)e^{i2\pi\beta_L(u)}\right)|u\epsilon U'\right\}, \tag{4}$$

where $0 \leq g_L(u)$, $h_L(u) \leq 1$, $\alpha_{g_L}(u)$, $\beta_{h_L}(u)\epsilon[0, 2\pi]$, $g_L^2(u) + h_L^2(u)\epsilon[0, 1]$, $\alpha_{g_L}^2(u) + \beta_{h_L}^2(u)\epsilon[0, 2\pi]$, and $i = \sqrt{-1}$.

Definition 5 (see[29]). Let $L_1 = \left\{(u, g_{L_1}(u)e^{i2\pi\alpha_{L_1}(u)}, h_{L_1}(u)e^{i2\pi\beta_{L_1}(u)})|u\epsilon U'\right\}$, $L_2 = \left\{(u, g_{L_2}(u)e^{i2\pi\alpha_{L_2}(u)}, h_{L_2}(u)e^{i2\pi\beta_{L_2}(u)})|u\epsilon U'\right\}$, and $L_3 = \left\{(u, g_{L_3}(u)e^{i2\pi\alpha_{L_3}(u)}, h_{L_3}(u)e^{i2\pi\beta_{L_3}(u)})|u\epsilon U'\right\}$ be three CPFSs in $U\prime$; then,

(a) $L_1 \subseteq L_2 \leftrightarrow g_{L_1}(u) \leq g_{L_2}(u)$, $h_{L_1}(u) \geq h_{L_2}(u)$ for amplitude terms and for phase terms $\alpha_{L_1}(u) \leq \alpha_{L_2}(u)$, $\beta_{L_1}(u) \geq \beta_{L_2}(u)$, for each $u\epsilon U'$

(b) $L_1 = L_2 \leftrightarrow g_{L_1}(u) = g_{L_2}(u)$, $h_{L_1}(u) = h_{L_2}(u)$ for amplitude terms and for phase terms $\alpha_{L_1}(u) = \alpha_{L_2}(u)$, $\beta_{L_1}(u) = \beta_{L_2}(u)$, for each $u\epsilon U'$

(c) $L^c = \left\{(u, h_L(u)e^{i2\pi\beta_L(u)}, g_L(u)e^{i2\pi\alpha_L(u)})|u\epsilon U\prime\right\}$

For convenience, the complex Pythagorean fuzzy number (CPFN) is represented by $(ge^{i2\pi\alpha}, he^{i2\pi\beta})$ as $L = ((g, \alpha), (h, \beta))$, where $0 \leq g, h \leq 1$ such that $g^2 + h^2 \epsilon [0, 1]$ and $0 \leq \alpha, \beta \leq 2\pi$ such that $\alpha^2 + \beta^2 \epsilon [0, 2\pi]$.

2.1. Complex Pythagorean Fuzzy Set. In this subsection, an usable expansion of CPFS is shown. Furthermore, the basic procedures, score index, accuracy index, and fundamental operators of CPFS are discussed in detail as follows.

2.1.1. Operational Laws of CPFNs

Definition 6 (see [42]). Let $\rho = (ge^{i2\pi\alpha}, he^{i2\pi\beta})$ be a CPFN, and the score of CPFN can be defined as follows:

$$S(\rho) = \left(g^2 - h^2\right) + \frac{1}{4\pi^2}\left(\alpha^2 - \beta^2\right), \tag{5}$$

where S indicates the score function of ρ and $S(\rho)$ obviously lies inside $[-2, 2]$.

For the comparison of CPFNs, a quite useful and relevant operator is introduced in the next definitions as follows.

Definition 7 (see [42]). The accuracy of a CPFN $\rho = (ge^{i2\pi\alpha}, he^{i2\pi\beta})$ may be defined as follows:

$$E(\rho) = \left(g^2 + h^2\right) + \frac{1}{4\pi^2}\left(\alpha^2 + \beta^2\right), \tag{6}$$

where E represents the accuracy function of ρ and $E(\rho)$ is obviously lies in $[0, 2]$.

Definition 8 (see [42]). Let $\rho = (g_\rho e^{i2\pi\alpha_\rho}, h_\rho e^{i2\pi\beta_\rho})$ and $\sigma = (g_\sigma e^{i2\pi\alpha_\sigma}, h_\sigma e^{i2\pi\beta_\sigma})$ be any CPFNs, and then, their comparison is stated as follows:

(i) If $S(\rho) > S(\sigma)$, then $\rho \succ \sigma$ (ρ is superior than σ)

(ii) If $S(\rho) = S(\sigma)$, then

(1) If $E(\rho) > E(\sigma)$, then $\rho \succ \sigma$ (ρ is superior to σ)

(2) If $E(\rho) = E(\sigma)$, then $\rho \sim \sigma$ (ρ is equivalent to σ)

Some basic operational laws on CPFNs will be essential.

Definition 9 (see [42]). For any three complex Pythagorean fuzzy numbers $L_1 = (g_{L_1}e^{i2\pi\alpha_{L_1}}, h_{L_1}e^{i2\pi\beta_{L_1}})$, $L_2 = (g_{L_2}e^{i2\pi\alpha_{L_2}}, h_{L_2}e^{i2\pi\beta_{L_2}})$, and $L_3 = (g_{L_3}e^{i2\pi\alpha_{L_3}}, h_{L_3}e^{i2\pi\beta_{L_3}})$, the basic operations on complex Pythagorean fuzzy numbers are given as follows:

$$\begin{aligned}
L_1 \oplus L_2 &= \left[\left(g_{L_1}^2 + g_{L_2}^2 - g_{L_1}^2 g_{L_2}^2\right)^{(1/2)} e^{i2\pi\left(\left(\alpha_{L_1}/2\pi\right)^2 + \left(\alpha_{L_2}/2\pi\right)^2 - \left(\alpha_{L_1}/2\pi\right)^2\left(\alpha_{L_2}/2\pi\right)^2\right)^{(1/2)}}, h_{L_1}h_{L_2}e^{i2\pi\left(\beta_{L_1}/2\pi\right)\left(\beta_{L_2}/2\pi\right)}\right],\\
L_1 \otimes L_2 &= \left[g_{L_1}g_{L_2}e^{i2\pi\left(\alpha_{L_1}/2\pi\right)\left(\alpha_{L_2}/2\pi\right)}, \left(h_{L_1}^2 + h_{L_2}^2 - h_{L_1}^2 h_{L_2}^2\right)^{(1/2)} e^{i2\pi\left(\left(\beta_{L_1}/2\pi\right)^2 + \left(\beta_{L_2}/2\pi\right)^2 - \left(\beta_{L_1}/2\pi\right)^2\left(\beta_{L_2}/2\pi\right)^2\right)^{(1/2)}}\right],\\
\gamma L &= \left[\left(1 - \left(1 - g_L^2\right)^\gamma\right)^{(1/2)} e^{i2\pi\left(1 - \left(1 - \left(\alpha_L/2\pi\right)^2\right)^\gamma\right)^{(1/2)}}, h_L^\gamma e^{i2\pi\left(\beta_L/2\pi\right)^\gamma}\right],\\
L^\gamma &= \left[g_L^\gamma e^{i2\pi\left(\alpha_L/2\pi\right)^\gamma}, \left(1 - \left(1 - h_L^2\right)^\gamma\right)^{(1/2)} e^{i2\pi\left(1 - \left(1 - \left(\beta_L/2\pi\right)^2\right)^\gamma\right)^{(1/2)}}\right].
\end{aligned} \tag{7}$$

2.2. Maclaurin Symmetric Mean. Maclaurin [38] initially presented the idea of Maclaurin symmetric mean, and it was further developed by Robertson and Detemple. Now, the MSM is defined as follows.

Definition 10. [38] Suppose s_r $(r = 1, 2, \ldots, m)$ is a set such that $s_r \geq 1$ and $k = 1, 2, \ldots, m$. If

$$\mathrm{MSM}^{(k)}(s_1, s_2, \ldots, s_m) = \left(\sum_{1 \le j_1 < \cdots < j_k \le m} \left(\frac{\prod_{r=1}^{k} s_{j_r}}{C_m^k} \right) \right)^{(1/k)}. \quad (8)$$

Then, (8) is known as the MSM operator, where C_m^k is the binomial coefficient and $(j_1, j_2, \ldots, j_k)$ traverses all the k-tuple combinations of $(1, 2, \ldots, m)$.

Some properties of the MSM are presented as follows:

(i) $\mathrm{MSM}^{(k)}(0,0, \ldots, 0) = 0$
(ii) $\mathrm{MSM}^{(k)}(s, s, \ldots, s) = s$
(iii) $\mathrm{MSM}^{(k)}(s_1, s_2, \ldots, s_m) \le \mathrm{MSM}^{(k)}(h_1, h_2, \ldots, h_m)$, if $s_j \le h_j$ for all j
(iv) $\min_j \{s_j\} \le \mathrm{MSM}^{(k)}(s_1, s_2, \ldots, s_m) \le \max_j \{s_j\}$

2.3. Dual Maclaurin Symmetric Mean. In [43], Qin and Liu propounded the idea of DMSM based on the MSM operator, which is presented in the next definition.

Definition 11. Let $s_r\,(r = 1, 2, \ldots, m)$ be a collection such that $s_r \ge 1$ and $k = 1, 2, \ldots, m$. If

$$\mathrm{DMSM}^{(k)}(s_1, s_2, \ldots, s_m) = \frac{1}{k} \left(\prod_{1 \le j_1 < \cdots < j_k \le m} \left(\sum_{j=1}^{k} s_{j_r} \right)^{(1/C_m^k)} \right), \quad (9)$$

where$\mathrm{DMSM}^{(k)}$ is known as the DMSM, $(j_1, j_2, \ldots, j_k)$ traverses all the k-tuple combinations of $(1, 2, \ldots, m)$, and $C_m^k = m!/k!(m-k)!$ is the binomial coefficient.

The properties of DMSM are omitted as same as the properties of MSM.

3. Complex Pythagorean Fuzzy Maclaurin Symmetric Mean Operator for CPFNs

Based on the aforementioned discussion, MSM is important, especially for the evaluation of MAGDM problems. The MSM operator is extended in the CPF environment to propose CPFMSM and CPFDMSM operators. In addition, some basic characteristics of CPFMSM and CPFDMSM are discussed in detail.

Definition 12. Let $Q_j = (g_j e^{i2\pi\alpha_{g_j}}, h_j e^{i2\pi\beta_{h_j}})\,(j = 1, 2, \ldots, m)$ be a collection of CPFNs and $(j_1, j_2, \ldots, j_k)$ traverses all the k-tuple combinations $(1, 2, \ldots, m)$. A function: $\aleph \longrightarrow \aleph$ is termed CPFMSM operator and stated as follows:

$$\mathrm{CPFMSM}^{(k)}(Q_1, Q_2, \ldots Q_m) = \left(\frac{\overset{\oplus}{1 \le l_1 < \cdots < l_k \le m} \left(\otimes_{j=1}^{k} Q_{l_j} \right)}{C_m^k} \right)^{(1/k)}, \quad (10)$$

where $\aleph$ is the family of CPFNs.

Theorem 1. *Suppose $Q_i = (g_i e^{i2\pi\alpha_{g_i}}, h_i e^{i2\pi\beta_{h_i}})\,(i = 1, 2, \ldots, m)$ is a collection of CPFNs, and the synthesis result of $\{Q_1, Q_2, \ldots, Q_m\}$ using the CPFMSM operator is stated as follows:*

$$\begin{aligned}
&\mathrm{CPFMSM}^{(k)}(Q_1, Q_2, \ldots Q_m) \\
&= \left[\begin{array}{c}
\left(\left(1 - \left(\prod_{\sqsupset} \left(1 - \left(\otimes_{i=1}^{k} g_{j_i} \right)^2 \right) \right)^{(1/C_m^k)} \right)^{(1/2)} \right)^{(1/k)} e^{i2\pi \left(\left(1 - \left(\prod_{\sqsupset} \left(1 - \left(\otimes_{i=1}^{k} \left(\alpha_{j_i}/2\pi \right) \right)^2 \right) \right)^{(1/C_m^k)} \right)^{(1/2)} \right)^{(1/k)}}, \\
\left(1 - \left(1 - \left(\prod_{\sqsupset} \left(1 - \otimes_{i=1}^{k} \left(1 - \left(h_{j_i} \right)^2 \right) \right) \right)^{(1/C_m^k)} \right)^{(1/k)} \right)^{(1/2)} e^{i2\pi \left(1 - \left(1 - \left(\prod_{\sqsupset} \left(1 - \otimes_{i=1}^{k} \left(1 - \left(\beta_{j_i}/2\pi \right)^2 \right) \right) \right)^{(1/C_m^k)} \right)^{(1/k)} \right)^{(1/2)}}
\end{array} \right],
\end{aligned} \quad (11)$$

where $\sqsupset$ used for ease represents the subscript $(1 \le j_1 < \cdots < j_k \le m)$.

Proof. In view of the basic operations of CPFMSMNs, we get

$$\otimes_{i=1}^{k} Q_{j_i} = \left[\left(\otimes_{i=1}^{k} g_{j_i} e^{i2\pi\left(\otimes_{i=1}^{k}\left(\alpha_{j_i}/2\pi\right)\right)}\right), \left(\left(1-\otimes_{i=1}^{k}\left(1-\left(h_{j_i}\right)^2\right)\right)^{(1/2)} e^{i2\pi\left(1-\otimes_{i=1}^{k}\left(1-\left(\left(\beta_{j_i}/2\pi\right)\right)^2\right)\right)^{(1/2)}}\right)\right]$$

$$1 \le j_1 < \overset{\oplus}{\cdots} < j_k \le m \left(\otimes_{i=1}^{k} Q_{j_i}\right) = \left[\begin{array}{c} \left(1-\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} g_{j_i}\right)^2\right)\right)^{(1/2)} e^{i2\pi\left(1-\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(\alpha_{j_i}/2\pi\right)\right)^2\right)\right)^{(1/2)}}, \\ \prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-\left(h_{j_i}\right)^2\right)\right)^{(1/2)} e^{i2\pi\left(\prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-\left(\beta_{j_i}/2\pi\right)^2\right)\right)^{(1/2)}\right)} \end{array}\right]. \tag{12}$$

Then,

$$\frac{1}{C_m^k}\left(1 \le j_1 < \overset{\oplus}{\cdots} < j_k \le m \left(\otimes_{i=1}^{k} Q_{j_i}\right)\right) = \left[\begin{array}{c} \left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} g_{j_i}\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)} e^{i2\pi\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(\alpha_{j_i}/2\pi\right)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}}, \\ \left(\prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-\left(h_{j_i}\right)^2\right)\right)^{(1/2)}\right)^{\left(1/C_m^k\right)} e^{i2\pi\left(\prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-\left(\beta_{j_i}/2\pi\right)^2\right)\right)^{(1/2)}\right)^{\left(1/C_m^k\right)}} \end{array}\right]. \tag{13}$$

Hence,

$$\left(\frac{1}{C_m^k}\left(1 \le j_1 < \overset{\oplus}{\cdots} < j_k \le m \left(\otimes_{i=1}^{k} Q_{j_i}\right)\right)\right)^{(1/k)} = \left[\begin{array}{c} \left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} g_{j_i}\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)} e^{i2\pi\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(\alpha_{j_i}/2\pi\right)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)}}, \\ \left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-\left(h_{j_i}\right)^2\right)\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)} e^{i2\pi\left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-\left(\beta_{j_i}/2\pi\right)^2\right)\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)}} \end{array}\right]. \tag{14}$$

3.1. Properties of CPFMSM Operator. Let $\grave{Q}_i = (g_i e^{i2\pi\alpha_i}, h_i e^{i2\pi\beta_i})$ and $Q_i = (g_i\ e^{i2\pi\alpha_i}, h_i\ e^{i2\pi\beta_i})$ be two groups of CPFNs, where ($i = 1, 2, \ldots, m$), and then, the following properties are present in the CPFMSM operator.

(1) *Idempotency*. If $Q_1 = Q_2 = \ldots = Q_m = Q = (ge^{i2\pi\alpha}, he^{i2\pi\beta})$, then for all i $CPFMSM(Q_1, Q_2, \ldots, Q_m) = Q$.

(2) *Monotonicity*. For Q_i and $\grave{Q}_i$ ($i = 1, 2, \ldots, m$), if $Q_i \le \grave{Q}_i\ \forall i$, then $CPFMSM(Q_1, Q_2, \ldots, Q_m) \le CPFMSM(\grave{Q}_1, \grave{Q}_2, \ldots, \grave{Q}_m)$.

(3) *Boundedness*. Suppose Q_i ($i = 1, 2, \ldots, m$) be a family of CPFNs, $\check{Q} = \min_i\{Q_i\}$ and $\widehat{Q} = \max_i\{Q_i\}$. Then, $\check{Q} \le CPFMSM(Q_1, Q_2, \ldots, Q_m) \le \widehat{Q}$.

Proof. (Property 1) Let $\text{CPFMSM}^{(k)}(Q, Q, \ldots, Q)$

$$=\left[\begin{array}{c}\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} g_{j_i}\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)} e^{i2\pi\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(\alpha_{j_i}/2\pi\right)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{1/k}},\\ \left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-\left(h_{j_i}\right)^2\right)\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)} e^{i2\pi\left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-\left(\beta_{j_i}/2\pi\right)^2\right)\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)}}\end{array}\right]$$

$$=\left[\begin{array}{c}\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} g\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)} e^{i2\pi\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}(\alpha/2\pi)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)}},\\ \left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-(h)^2\right)\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)} e^{i2\pi\left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\otimes_{i=1}^{k}\left(1-(\beta/2\pi)^2\right)\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)}}\end{array}\right], \tag{15}$$

$$=\left[\begin{array}{c}\left(\left(1-\left(\left(1-(g)^{2\times k}\right)^{C_m^k}\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)} e^{i2\pi\left(\left(1-\left(\left(1-(\alpha/2\pi)^{2\times k}\right)^{C_m^k}\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)}},\\ \left(1-\left(1-\left(\left(1-\left(1-h^2\right)^k\right)^{C_m^k}\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)} e^{i2\pi\left(1-\left(1-\left(\left(1-\left(1-(\beta/2\pi)^2\right)^k\right)^{C_m^k}\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)}}\end{array}\right]$$

$$=\left[\begin{array}{c}\left(\left(1-\left(1-(g)^{2\times k}\right)\right)^{(1/2)}\right)^{(1/k)} e^{i2\pi\left(\left(1-\left(1-(\alpha/2\pi)^{2\times k}\right)\right)^{(1/2)}\right)^{(1/k)}}\\ \left(h^2\right)^{(1/2)} e^{i2\pi\left(\left(\beta/2\pi\right)^2\right)^{(1/2)}}\end{array}\right]=\left(ge^{i\alpha}, he^{i\beta}\right).$$

Proof. (Property (2) As $Q_i \le Q_i'$, then we have $g_i \le g_i'$, $\alpha_i \le \alpha_i'$ and $h_i \ge h_i$, $\beta_i \ge \beta_i'$ for $\forall i$. Then, for the real valued membership functions of CPFMSMNs, as $g_i \le g_i'$,one has

$$\left(\otimes_{i=1}^{k} g_i\right)^2 \le \left(\otimes_{i=1}^{k} \check{g}_i\right)^2 \Rightarrow \prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} g_i\right)^2\right) \ge \prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} \check{g}_i\right)^2\right)$$
$$\Rightarrow 1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} g_i\right)^2\right)\right)^{\left(1/C_m^k\right)} \le 1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} \check{g}_i\right)^2\right)\right)^{\left(1/C_m^k\right)}$$
$$\Rightarrow \left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} g_i\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)} \le \left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} \check{g}_i\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)}. \quad (16)$$

Correspondingly, for the imaginary valued membership functions of CPFMSMNs, we get

$$\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} \alpha_i\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)} \le \left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} \check{\alpha}_i\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)}. \quad (17)$$

Therefore, we obtain

$$\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} g_i\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)} e^{i2\pi\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(\alpha_i/2\pi\right)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)}}$$
$$\le \left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k} \check{g}_i\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)} e^{i2\pi\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(\check{\alpha}_i/2\pi\right)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/2)}\right)^{(1/k)}}. \quad (18)$$

Also, for the nonmembership degree, we can get

$$\left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(1-\left(h_i\right)^2\right)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)} e^{i2\pi\left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(1-\left(\beta_i/2\pi\right)\right)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)}}$$
$$\ge \left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(1-\left(\check{h}_i\right)^2\right)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)} e^{i2\pi\left(1-\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(1-\left(\check{\beta}_i/2\pi\right)\right)\right)^2\right)\right)^{\left(1/C_m^k\right)}\right)^{(1/k)}\right)^{(1/2)}}. \quad (19)$$

Proof. (Property 3) Because of the idempotency and monotonicity of the proposed CPFMSM operator, we can get results stated as follows.

For $Q_i \ge \check{Q} = \min_i\{Q_i\}$, one has $CPFMSM(Q_1, Q_2, \ldots, Q_m) \ge CPFMSM(\check{Q}, \check{Q}, \ldots, \check{Q}) = \check{Q}$.

For $Q_i \le \widehat{Q} = \max_i\{Q_i\}$, one has $CPFMSM(Q_1, Q_2, \ldots, Q_m) \le CPFMSM(\widehat{Q}, \widehat{Q}, \ldots, \widehat{Q}) = \widehat{Q}$.

Therefore, $\min_i\{Q_i\} \le CPFMSM(Q_1, Q_2, \ldots, Q_m) \le \max_i\{Q_i\}$.

Now, various novel operators will be attained by allocating diver values to parameter k.

Case (1). When $k = 1$, the CPFMSM operator is changed into the CPF arithmetic averaging (CPFAA) operator, which is stated as follows:

$$CPFMSM^{(1)}(Q_1,Q_2,\ldots,Q_m)=\frac{1}{m}\left(\underset{1\le j_1\le m}{\oplus}Q_{j_i}\right)=\frac{1}{m}\left(\otimes_{j=1}^{n}Q_j\right)(\text{let} j_1=j)$$
$$=\begin{bmatrix}\left(1-\otimes_{j=1}^{m}\left(1-g_j^2\right)^{1/m}\right)^{1/2}e^{i2\pi\left(1-\otimes_{j=1}^{m}\left(1-\left(\alpha_j/2\pi\right)^2\right)^{1/m}\right)^{1/2}}\\ \otimes_{j=1}^{m}h_j^{1/m}e^{i2\pi\left(\otimes_{i=1}^{m}\left(\beta_j/2\pi\right)^{1/m}\right)}\end{bmatrix}$$
$$=CPFAA^{(1)}(Q_1,Q_2,\ldots,Q_m)\frac{1}{n(n-1)}\oplus^{k}\underset{j_1\neq j_2}{j_1,j_2=1}\cdot \tag{20}$$

Case (2). If $k=2$, then the CPFMSM operator is reduced to a specific operator CPF Bonferroni mean (CPFBM) operator, stated as follows:

$$\text{CPFMSM}^{(2)}=(Q_1,Q_2,\ldots,Q_m)$$
$$=\left(\frac{\underset{1\le j_1<j_2\le m}{\oplus}\left(\otimes_{i=1}^{2}Q_{j_i}\right)}{C_m^2}\right)^{(1/2)}=\left(\frac{1}{n(n-1)}\oplus^{k}\underset{j_1\neq j_2}{j_1,j_2=1}\left(Q_{j_1}^1\otimes Q_{j_2}^1\right)\right)^{(1/2)}$$
$$=\begin{bmatrix}\left(\left(1-\left(\otimes^{k}\underset{j_1\neq j_2}{j_1,j_2=1}\left(1-g_{j_1}^2g_{j_2}^2\right)\right)^{(2/n(n-1))}\right)^{(1/2)}\right)^{(1/2)}e^{i2\pi\left(\left(1-\left(\otimes^{k}\underset{j_1\neq j_2}{j_1,j_2=1}\left(1-\left(\alpha_{j_1}/2\pi\right)^2\left(\alpha_{j_2}/2\pi\right)^2\right)\right)^{(2/n(n-1))}\right)^{(1/2)}\right)^{(1/2)}},\\ \left(1-\left(1-\left(\otimes^{k}\underset{j_1\neq j_2}{j_1,j_2=1}\left(1-\left(1-h_{j_1}^2\right)\left(1-h_{j_2}^2\right)\right)\right)^{(2/n(n-1))}\right)^{(1/2)}\right)^{(1/2)}e^{i2\pi\left(1-\left(1-\left(\otimes^{k}\underset{j_1\neq j_2}{j_1,j_2=1}\left(1-\left(1-\left(\beta_{j_1}/2\pi\right)^2\right)\left(1-\left(\beta_{j_2}/2\pi\right)^2\right)\right)\right)^{(2/n(n-1))}\right)^{(1/2)}\right)^{(1/2)}}\end{bmatrix}$$
$$=\text{CPFBM}^{(1,1)}(Q_1,Q_2,\ldots,Q_m). \tag{21}$$

Case (3). If $k=3$, then the CPFMSM operator is reduced to a specific operator CPF generalized Bonferroni mean (CPFGBM) operator, stated as follows:

$$\mathrm{CPFMSM}^{(3)}\left(Q_1,Q_2,\ldots,Q_m\right)=\left(\frac{\overset{\oplus}{1\le j_1<j_2<j_3\le m}\left(\otimes_{i=1}^{3}Q_{j_i}\right)}{C_m^3}\right)^{(1/3)}$$

$$=\left(\frac{1}{n(n-1)(n-2)}\underset{j_1\ne j_2\ne j_3}{\oplus^{k}_{j_1,j_2,j_3=1}}\left(Q_{j_1}^1\otimes Q_{j_2}^1\otimes Q_{j_3}^1\right)\right)^{(1/3)} \tag{22}$$

$$=\mathrm{CPFGBM}^{(1,1,1)}=\left(Q_1,Q_2,\ldots,Q_m\right).$$

Case (4). If $k=m$, then the CPFMSM operator is reduced to a specific operator CPF geometric average mean (CPFGAM) operator, stated as follows:

$$\mathrm{CPFMSM}^{(m)}\left(Q_1,Q_2,\ldots,Q_m\right)=\left(\otimes_{i=1}^{m}Q_i\right)^{(1/m)}$$

$$=\begin{bmatrix}\left(\otimes_{i=1}^{m}g_i\right)^{(1/m)}e^{i2\pi\left(\otimes_{i=1}^{m}\left(\alpha_i/2\pi\right)\right)^{(1/m)}},\\ \left(1-\left(\otimes_{i=1}^{m}\left(1-h_i^2\right)\right)^{(1/m)}\right)^{(1/2)}e^{i2\pi\left(1-\left(\otimes_{i=1}^{m}\left(1-\left(\beta_i/2\pi\right)^2\right)\right)^{(1/m)}\right)^{(1/2)}}\end{bmatrix} \tag{23}$$

$$=\mathrm{CPFGAM}^{(m)}\left(Q_1,Q_2,\ldots,Q_m\right).$$

3.2. Complex Pythagorean Fuzzy Dual Maclaurin Symmetric Mean. Next, the dual form of CPFMSM operator called CPFDMSM operator is propounded based on the fusion of the CPFS and DMSM operators.

Definition 13. Suppose $Q_i=\left(g_ie^{i2\pi(\alpha_i/2\pi)},\ h_ie^{i2\pi(\beta_i/2\pi)}\right)(i=1,2\ldots,m)$ be a set of CPFNs and k is a parameter, and its values are taken from the collection $\{1,2,\ldots,m\}$. A function: $\aleph\longrightarrow\aleph$ is called CPFDMSM operator and is defined as follows:

$$CPF\ DM\ SM^{(k)}\left(Q_1,Q_2,\ldots,Q_m\right)\frac{1}{k}\left(\underset{1\le j_1<\cdots<j_k\le m}{\otimes}\left(\oplus_{i=1}^{k}Q_{j_i}\right)^{1/C_m^k}\right), \tag{24}$$

where $\aleph$ denotes the collection of CPFNs.

Theorem 2. *Suppose* $Q_i=\left(g_ie^{i2\pi(\alpha_i/2\pi)},h_ie^{i2\pi(\beta_i/2\pi)}\right)(i=1,2,\ldots,m)$ *be a set of CPFNs, and then, the combination result of* $\left(Q_1,Q_2,\ldots,Q_m\right)$ *using the CPFMSM operator is presented as follows:*

$$CPF\ DM\ SM^{(k)}(Q_1, Q_2, \ldots, Q_m)$$

$$= \left[\begin{array}{c} \left(1 - \left(1 - \left(\prod_{\sqsupset}\left(1 - \otimes_{i=1}^{k}\left(1 - g_{j_i}^2\right)\right)\right)^{1/C_m^k}\right)^{1/k}\right)^{1/2} e^{i2\pi\left(1-\left(1-\left(\prod_{\sqsupset}\left(1 - \otimes_{i=1}^{k}\left(1-\left(\alpha_{j_i}/2\pi\right)^2\right)\right)\right)^{1/C_m^k}\right)^{1/k}\right)^{1/2}} \\ \left(\left(1 - \left(\prod_{\sqsupset}\left(1 - \left(\otimes_{i=1}^{k} h_{j_i}\right)^2\right)\right)^{1/C_m^k}\right)^{1/2}\right)^{1/k} e^{i2\pi\left(\left(1-\left(\prod_{\sqsupset}\left(1-\left(\otimes_{i=1}^{k}\left(\beta_{j_i}/2\pi\right)\right)^2\right)\right)^{1/C_m^k}\right)^{1/2}\right)^{1/k}} \end{array} \right], \tag{25}$$

where $\sqsupset$ *stands for the subscript* $(1 \leq j_1 < \cdots < j_k \leq m)$.

The proof of this theorem is omitted due to its resemblance to Theorem 1.

3.3. Properties of CPFDMSM Operator. Suppose $Q_i = (g_i e^{i2\pi(\alpha_i/2\pi)}, h_i e^{i2\pi(\beta_i/2\pi)})$ and $\grave{Q}_i = (\grave{g}_i e^{i2\pi(\grave{\alpha}_i/2\pi)}, \grave{h}_i e^{i2\pi(\grave{\beta}_i/2\pi)})$ $(i = 1, 2, \ldots, m)$ be two families of CPFNs, and then, the CPFDMSM operator holds the following features:

(1) *Idempotency.* If $Q_1 = Q_2 = \ldots = Q_m = Q = (ge^{i2\pi(\alpha/2\pi)}, he^{i2\pi(\beta/2\pi)})$, then $CPFDMSM(Q_1, Q_2, \ldots, Q_m) = Q$.

(2) *Monotonicity.* If $g_i \leq \grave{g}_i$, $h_i \geq \grave{h}_i$, $\alpha_i \leq \grave{\alpha}_i$, and $\beta_i \geq \grave{\beta}_i$, then $CPFDMSM(Q_1, Q_2, \ldots, Q_m) \geq CPFDMSM(\grave{Q}_1, \grave{Q}_2, \ldots, \grave{Q}_m)$.

(3) *Boundedness.* Let $Q_i\ (i = 1, 2, \ldots, m)$ be a set of CPFNs, $\check{Q} = \min\{Q_i\}$ and $\widehat{Q} = \max\{Q_i\}$, then $\check{Q} \leq CPFDMSM(Q_1, Q_2, \ldots, Q_m) \leq \widehat{Q}$.

The proof of these properties is omitted due to similarity with the properties of CPFMSM. Furthermore, some particular operators of CPFDMSM operator will be attained by taking diverse values of the parameter k.

Case (1). When $k = 1$, the CPFDMSM operator is reduced to the CPF arithmetic averaging (CPFAA) operator, defined as follows:

$$\begin{aligned} &CPF\ DM\ SM^{(1)}(Q_1, Q_2, \ldots, Q_m)\left(\otimes_{i=1}^{m} Q_i\right)^{1/m} \\ &= \left[\begin{array}{c} \otimes_{i=1}^{m} g_i^{1/m} e^{i2\pi\left(\otimes_{i=1}^{m}\left(\alpha_i/2\pi\right)^{1/m}\right)}, \\ \left(1 - \left(\otimes_{i=1}^{m}\left(1 - h_i^2\right)\right)^{1/m}\right)^{1/2} e^{i2\pi\left(1-\left(\otimes_{i=1}^{m}\left(1-\left(\beta_i/2\pi\right)^2\right)\right)^{1/m}\right)^{1/2}} \end{array} \right] \\ &= CPFAA^{(1)}(Q_1, Q_2, \ldots, Q_m). \end{aligned} \tag{26}$$

Case (2). If $k = 2$, then the CPFDMSM operator is reduced to a specific operator CPF geometric Bonferroni mean (CPFGBM) operator, presented as follows:

$$
\begin{aligned}
&CPF\ DM\ SM^{(2)}(Q_1, Q_2, \ldots, Q_m)\\
&= \frac{1}{2}\left(\otimes^{k}_{\substack{j_1, j_2 = 1\\ j_1 \neq j_2}} \left(Q_{j_1} \oplus Q_{j_2}\right)^{2/n(n-1)} \right)\\
&= \left[\begin{array}{c}
\left(1-\left(1-\left(\otimes^{k}_{\substack{j_1, j_2 = 1\\ j_1\neq j_2}}\left(1-\left(1-g_{j_1}^2\right)\left(1-g_{j_2}^2\right)\right)\right)^{2/n(n-1)}\right)^{1/2}\right)^{1/2} e^{i2\pi\left(1-\left(1-\left(\otimes^{k}_{\substack{j_1, j_2 = 1\\ j_1\neq j_2}}\left(1-\left(1-\left(\alpha_{j_1}/2\pi\right)^2\right)\left(1-\left(\alpha_{j_2}/2\pi\right)^2\right)\right)\right)^{2/n(n-1)}\right)^{1/2}\right)^{1/2}},\\
\left(\left(1-\left(\otimes^{k}_{\substack{j_1, j_2 = 1\\ j_1\neq j_2}}\left(1-h_{j_1}^2 h_{j_2}^2\right)\right)^{2/n(n-1)}\right)^{1/2}\right)^{1/2} e^{i2\pi\left(\left(1-\left(\otimes^{k}_{\substack{j_1, j_2 = 1\\ j_1\neq j_2}}\left(1-\left(\beta_{j_1}/2\pi\right)^2\left(\beta_{j_2}/2\pi\right)^2\right)\right)^{2/n(n-1)}\right)^{1/2}\right)^{1/2}}
\end{array} \right]\\
&CPFGBM^{(1,1)}(Q_1, Q_2, \ldots, Q_m).
\end{aligned} \tag{27}
$$

Case (3). If $k=3$, then the CPFDMSM is reduced to a specific operator CPF generalized geometric Bonferroni mean (CPFGGBM) operator, shown as follows:

$$
\begin{aligned}
CPF\ DM\ SM^{(1,1,1)}(Q_1, Q_2, \ldots, Q_m) &= \frac{1}{k}\left(\otimes_{1 \le j_1 < \cdots < j_k \le m} \left(\oplus_{i=1}^{k} Q_{j_i}\right)^{1/C_m^k} \right)\\
&= \frac{1}{3}\left(\otimes^{k}_{\substack{j_1, j_2, j_3 = 1\\ j_1\neq j_2 \neq j_3}} \left(Q_{j_1} \oplus Q_{j_2} \oplus Q_{j_3}\right)^{3/n(n-1)(n-2)} \right)\\
&= CPFGGBM^{(1,1,1)}(Q_1, Q_2, \ldots, Q_m).
\end{aligned} \tag{28}
$$

Case (4). If $k=m$, then the CPFDMSM operator is reduced to a specific operator CPF geometric average mean (CPFGAM) operator, shown as follows:

$$
\begin{aligned}
CPF\ DM\ SM^{(m)}(Q_1, Q_2, \ldots, Q_m) &= \frac{1}{m}\left(\oplus_{1 \le j_1 \le m} Q_{j_i} \right) = \frac{1}{m}\left(\oplus_{j=1}^{m} Q_j\right)\left(\text{set}\, j_1 = j\right)\\
&= \left[\begin{array}{c}
\left(1-\otimes_{j=1}^{m}\left(1-g_j^2\right)^{1/m}\right)^{1/2} e^{i2\pi\left(1-\otimes_{j=1}^{m}\left(1-\left(\alpha_j/2\pi\right)^2\right)^{1/m}\right)^{1/2}},\\
\otimes_{j=1}^{m} h_j^{1/m} e^{i2\pi\left(\otimes_{j=1}^{m}\left(\beta_j/2\pi\right)^{1/m}\right)}
\end{array} \right].
\end{aligned} \tag{29}
$$

4. The Proposed MAGDM Method

In this section, we design an algorithm based on the proposed CPFMSM operator to address classical MAGDM problems.

This part shall develop a MAGDM method based on a new proposed CPFMSM operator under the CPF environment. For conventional MAGDAM problems, assume that $\mathcal{A}=\{\mathcal{A}_1,\mathcal{A}_2, \ldots, \mathcal{A}_m\}$ be a collection of attributes, $\mathcal{Y}=\{\mathcal{Y}_1,\mathcal{Y}_2, \ldots, \mathcal{Y}_m\}$ be a group of alternatives, and $Đ=\{Đ_1,Đ_2,\ldots,Đ_q\}$ be the family of evaluators. The evaluator $Đ_s\,(s=1,2,\ldots,q)$ provides his information for every alternative $\mathcal{Y}$, i.e., $\mathcal{Y}_i\,(i=1,2,\ldots,n)$ with regard to the attribute $\mathcal{A}_p\,(p=1,2,\ldots,m)$ in terms of the CPFNs, which are represented by $Q_{ip}^s=(g_{ip}^s e^{i2\pi\alpha_{g_{ip}}^s}, h_{ip}^s e^{i2\pi\beta_{h_{ip}}^s})$. Let $D^s=(Q_{ip}^s)_{n\times m}$ be the decision matrices. Therefore, to achieve the best optimal alternative, we propose an innovative MAGDM approach to sort the alternatives employed in the proposed CPFMSM operator. The main steps for the algorithm are stated as follows:

Step 1. The decision matrices can be standardized, by the following transformation manner:

$$Q_{ip}^s=\begin{cases}\left(g_{ip}^s e^{i2\pi\alpha_{g_{ip}}^s}, h_{ip}^s e^{i2\pi\beta_{h_{ip}}^s}\right), \mathcal{A}_p \text{forbenefitattribute},\\ \left(h_{ip}^s e^{i2\pi\beta_{h_{ip}}^s}, g_{ip}^s e^{i2\pi\alpha_{g_{ip}}^s}\right), \mathcal{A}_p \text{forcosattribute}.\end{cases} \tag{30}$$

Step 2. Employ the CPFMSM operator or CPFDMSM operator to fuse all the individual DM matrices $D^s=(Q_{ip}^s)_{n\times m}\,(s=1,2,\ldots,q)$ into another single DM matrix $D=(Q_{ip})_{n\times m}$:

$$Q_{ip}=CPFMSM\left(Q_{ip}^1,Q_{ip}^2,\ldots,Q_{ip}^q\right), \tag{31}$$

or

$$Q_{ip}=CPF\ DM\ SM\left(Q_{ip}^1,Q_{ip}^2,\ldots,Q_{ip}^q\right). \tag{32}$$

Step 3. Employ the CPFMSM operator or CPFDMSM operator to integrate linguistic assessment information $Q_{ip}\,(p=1,2,\ldots,m)$ into the inclusive evaluation value of the alternatives $\mathcal{Y}_i\,(i=1,2,\ldots,n)$:

$$Q_i=CPFMSM\left(Q_{ip},Q_{ip},\ldots,Q_{ip}\right), \tag{33}$$

or

$$Q_i=CPF\ DM\ SM\left(Q_{ip},Q_{ip},\ldots,Q_{ip}\right). \tag{34}$$

Step 4. Calculate the score index $S(Q_i)$ for all evaluation values of $\mathcal{Y}_i\,(i=1,2,\ldots,n)$ based on Definition 6, because if score values fail to distinguish the sorting of different alternatives, it becomes the same. So, we further utilized $E(Q_i)$ for the accuracy value using Definition 7.

Step 5. Employ Definition 8, and sort all of the alternatives $\mathcal{Y}_i\,(i=1,2,\ldots,n)$ to select the best optimum alternative.

Step 6. The end.

4.1. Illustrative Example. Now, in this part, a daily life example is illustrated for the verification and viability of the introduced method. After that, the sensitivity of parameter k is also studied. Furthermore, the comparative analysis is also carried out in brief to address the validity of the approach.

Emergency management is a suitable term for handling the hazards of disasters and accidents. This management involves the establishment of essential response mechanism and the selection of emergency steps by the government and some other public organizations in the process of response, avoidance, recovery, and clearance of the emergency situation. This emergency management includes a sequence of essential measures, usage of technology, science, management techniques, and plans. This management will help to certify the activities associated with the emergency management, safety of the public life, health, property, and the promotion of healthy and safe society. In the current years, the frequently occurring natural disasters have caused significant loss and damage to the global economy and to the human lives. To minimize the damage caused by great disasters and accidents, the center for emergency management will seek out various solutions depending on the kind of accidents and will invite experts from different fields for the evaluation of alternative emergency plan. This assessment of the emergency alternative is a significant step in the emergency management program. This is basically derived from the classical problem of decision-making, which has gained the consideration of various scholars and researchers. So, here, we implement the presented approach to deal with the assessment problem of the selection of the suitable emergency alternative for the emergency management plan. Four suitable alternatives are opted for further assessment, after a sequence of screening. Four alternatives $\{\mathcal{Y}_1,\mathcal{Y}_2,\mathcal{Y}_3,\mathcal{Y}_4\}$ are selected by three evaluators $\{Đ_1,Đ_2,Đ_3\}$ for effective modeling of the characteristic of alternatives. The following attributes are taken into account by discussion with experts, which are presented as follows: $\mathcal{A}_1$, used for preparation capability; $\mathcal{A}_2$, used for rescue capability; $\mathcal{A}_3$, used for restoring capability; and $\mathcal{A}_4$, used for reaction ability. According to the assessment and experience, the individual three CPFDM matrices are given by evaluators $\{Đ_1,Đ_2, Đ_3\}$ and are presented in Tables 1–3, respectively.

4.1.1. DM Process

Step (1). As all the attributes are of similar type, the standardization process is omitted.

Step (2). The CPFMSM operator or the CPFDMSM is employed to combine all the singular decision-making matrices into another single matrix and is presented in Tables 4-5 (take$k=2$).

Table 1: Complex Pythagorean fuzzy decision matrix given by expert $Đ_1$.

	A_1	A_2	A_3	A_4
$\mathscr{Y}_1$	$\begin{pmatrix} 0.30e^{i2\pi(0.50)} \\ 0.20e^{i2\pi(0.40)} \end{pmatrix}$	$\begin{pmatrix} 0.25e^{i2\pi(0.56)} \\ 0.35e^{i2\pi(0.40)} \end{pmatrix}$	$\begin{pmatrix} 0.20e^{i2\pi(0.70)} \\ 0.45e^{i2\pi(0.28)} \end{pmatrix}$	$\begin{pmatrix} 0.25e^{i2\pi(0.65)} \\ 0.15e^{i2\pi(0.20)} \end{pmatrix}$
$\mathscr{Y}_2$	$\begin{pmatrix} 0.45e^{i2\pi(0.55)} \\ 0.28e^{i2\pi(0.42)} \end{pmatrix}$	$\begin{pmatrix} 0.26e^{i2\pi(0.46)} \\ 0.32e^{i2\pi(0.36)} \end{pmatrix}$	$\begin{pmatrix} 0.31e^{i2\pi(0.31)} \\ 0.43e^{i2\pi(0.48)} \end{pmatrix}$	$\begin{pmatrix} 0.30e^{i2\pi(0.58)} \\ 0.18e^{i2\pi(0.32)} \end{pmatrix}$
$\mathscr{Y}_3$	$\begin{pmatrix} 0.40e^{i2\pi(0.29)} \\ 0.28e^{i2\pi(0.22)} \end{pmatrix}$	$\begin{pmatrix} 0.29e^{i2\pi(0.64)} \\ 0.35e^{i2\pi(0.29)} \end{pmatrix}$	$\begin{pmatrix} 0.35e^{i2\pi(0.41)} \\ 0.51e^{i2\pi(0.28)} \end{pmatrix}$	$\begin{pmatrix} 0.37e^{i2\pi(0.39)} \\ 0.19e^{i2\pi(0.42)} \end{pmatrix}$
$\mathscr{Y}_4$	$\begin{pmatrix} 0.43e^{i2\pi(0.65)} \\ 0.20e^{i2\pi(0.12)} \end{pmatrix}$	$\begin{pmatrix} 0.29e^{i2\pi(0.56)} \\ 0.30e^{i2\pi(0.29)} \end{pmatrix}$	$\begin{pmatrix} 0.38e^{i2\pi(0.61)} \\ 0.44e^{i2\pi(0.33)} \end{pmatrix}$	$\begin{pmatrix} 0.33e^{i2\pi(0.48)} \\ 0.10e^{i2\pi(0.39)} \end{pmatrix}$

Table 2: Complex Pythagorean fuzzy decision matrix given by expert $Đ_2$.

	A_1	A_2	A_3	A_4
$\mathscr{Y}_1$	$\begin{pmatrix} 0.20e^{i2\pi(0.80)} \\ 0.26e^{i2\pi(0.17)} \end{pmatrix}$	$\begin{pmatrix} 0.15e^{i2\pi(0.55)} \\ 0.42e^{i2\pi(0.35)} \end{pmatrix}$	$\begin{pmatrix} 0.20e^{i2\pi(0.37)} \\ 0.69e^{i2\pi(0.48)} \end{pmatrix}$	$\begin{pmatrix} 0.22^{e} \\ 0.24e^{i2\pi(0.39)} \end{pmatrix}$
$\mathscr{Y}_2$	$\begin{pmatrix} 0.37e^{i2\pi(0.34)} \\ 0.39e^{i2\pi(0.28)} \end{pmatrix}$	$\begin{pmatrix} 0.26e^{i2\pi(0.38)} \\ 0.24e^{i2\pi(0.19)} \end{pmatrix}$	$\begin{pmatrix} 0.43e^{i2\pi(0.19)} \\ 0.32e^{i2\pi(0.47)} \end{pmatrix}$	$\begin{pmatrix} 0.20e^{i2\pi(0.47)} \\ 0.17e^{i2\pi(0.28)} \end{pmatrix}$
$\mathscr{Y}_3$	$\begin{pmatrix} 0.10e^{i2\pi(0.27)} \\ 0.37e^{i2\pi(0.38)} \end{pmatrix}$	$\begin{pmatrix} 0.36e^{i2\pi(0.28)} \\ 0.46e^{i2\pi(0.55)} \end{pmatrix}$	$\begin{pmatrix} 0.41e^{i2\pi(0.46)} \\ 0.37e^{i2\pi(0.29)} \end{pmatrix}$	$\begin{pmatrix} 0.28e^{i2\pi(0.59)} \\ 0.36e^{i2\pi(0.37)} \end{pmatrix}$
$\mathscr{Y}_4$	$\begin{pmatrix} 0.39e^{i2\pi(0.64)} \\ 0.18e^{i2\pi(0.25)} \end{pmatrix}$	$\begin{pmatrix} 0.17e^{i2\pi(0.60)} \\ 0.27e^{i2\pi(0.34)} \end{pmatrix}$	$\begin{pmatrix} 0.18e^{i2\pi(0.49)} \\ 0.55e^{i2\pi(0.38)} \end{pmatrix}$	$\begin{pmatrix} 0.24e^{i2\pi(0.45)} \\ 0.57e^{i2\pi(0.41)} \end{pmatrix}$

Table 3: Complex Pythagorean fuzzy decision matrix given by expert $Đ_3$.

	A_1	A_2	A_3	A_4
$\mathscr{Y}_1$	$\begin{pmatrix} 0.61e^{i2\pi(0.40)} \\ 0.33e^{i2\pi(0.38)} \end{pmatrix}$	$\begin{pmatrix} 0.37e^{i2\pi(0.65)} \\ 0.24e^{i2\pi(0.22)} \end{pmatrix}$	$\begin{pmatrix} 0.29e^{i2\pi(0.57)} \\ 0.59e^{i2\pi(0.32)} \end{pmatrix}$	$\begin{pmatrix} 0.22e^{i2\pi(0.42)} \\ 0.21e^{i2\pi(0.39)} \end{pmatrix}$
$\mathscr{Y}_2$	$\begin{pmatrix} 0.47e^{i2\pi(0.36)} \\ 0.36e^{i2\pi(0.27)} \end{pmatrix}$	$\begin{pmatrix} 0.32e^{i2\pi(0.38)} \\ 0.23e^{i2\pi(0.40)} \end{pmatrix}$	$\begin{pmatrix} 0.17e^{i2\pi(0.58)} \\ 0.30e^{i2\pi(0.44)} \end{pmatrix}$	$\begin{pmatrix} 0.29e^{i2\pi(0.49)} \\ 0.15e^{i2\pi(0.46)} \end{pmatrix}$
$\mathscr{Y}_3$	$\begin{pmatrix} 0.37e^{i2\pi(0.36)} \\ 0.30e^{i2\pi(0.39)} \end{pmatrix}$	$\begin{pmatrix} 0.28e^{i2\pi(0.29)} \\ 0.45e^{i2\pi(0.45)} \end{pmatrix}$	$\begin{pmatrix} 0.21e^{i2\pi(0.57)} \\ 0.43e^{i2\pi(0.31)} \end{pmatrix}$	$\begin{pmatrix} 0.19e^{i2\pi(0.11)} \\ 0.27e^{i2\pi(0.74)} \end{pmatrix}$
$\mathscr{Y}_4$	$\begin{pmatrix} 0.22e^{i2\pi(0.64)} \\ 0.10e^{i2\pi(0.25)} \end{pmatrix}$	$\begin{pmatrix} 0.36e^{i2\pi(0.19)} \\ 0.29e^{i2\pi(0.56)} \end{pmatrix}$	$\begin{pmatrix} 0.29e^{i2\pi(0.53)} \\ 0.56e^{i2\pi(0.83)} \end{pmatrix}$	$\begin{pmatrix} 0.38e^{i2\pi(0.56)} \\ 0.57e^{i2\pi(0.26)} \end{pmatrix}$

Table 4: Collective DM matrix using the CPFMSM operator.

	A_1	A_2	A_3	A_4
$\mathscr{Y}_1$	$\begin{pmatrix} 0.3632e^{i2\pi(0.5653)} \\ 0.2664e^{i2\pi(0.3302)} \end{pmatrix}$	$\begin{pmatrix} 0.2576e^{i2\pi(0.5868)} \\ 0.3423e^{i2\pi(0.3298)} \end{pmatrix}$	$\begin{pmatrix} 0.2300e^{i2\pi(0.5501)} \\ 0.5859e^{i2\pi(0.3656)} \end{pmatrix}$	$\begin{pmatrix} 0.2300e^{i2\pi(0.5446)} \\ 0.2020e^{i2\pi(0.3363)} \end{pmatrix}$
$\mathscr{Y}_2$	$\begin{pmatrix} 0.4300e^{i2\pi(0.4155)} \\ 0.3347e^{i2\pi(0.3270)} \end{pmatrix}$	$\begin{pmatrix} 0.2795e^{i2\pi(0.4068)} \\ 0.2650e^{i2\pi(0.3266)} \end{pmatrix}$	$\begin{pmatrix} 0.3054e^{i2\pi(0.3562)} \\ 0.3527e^{i2\pi(0.4636)} \end{pmatrix}$	$\begin{pmatrix} 0.2646e^{i2\pi(0.5133)} \\ 0.1670e^{i2\pi(0.3580)} \end{pmatrix}$
$\mathscr{Y}_3$	$\begin{pmatrix} 0.3018e^{i2\pi(0.3063)} \\ 0.3180e^{i2\pi(0.3370)} \end{pmatrix}$	$\begin{pmatrix} 0.3097e^{i2\pi(0.3952)} \\ 0.4225e^{i2\pi(0.4408)} \end{pmatrix}$	$\begin{pmatrix} 0.3253e^{i2\pi(0.4800)} \\ 0.4395e^{i2\pi(0.2936)} \end{pmatrix}$	$\begin{pmatrix} 0.2806e^{i2\pi(0.3756)} \\ 0.2784e^{i2\pi(0.5281)} \end{pmatrix}$
$\mathscr{Y}_4$	$\begin{pmatrix} 0.3494e^{i2\pi(0.6433)} \\ 0.1640e^{i2\pi(0.2131)} \end{pmatrix}$	$\begin{pmatrix} 0.2748e^{i2\pi(0.4654)} \\ 0.2869e^{i2\pi(0.4061)} \end{pmatrix}$	$\begin{pmatrix} 0.2839e^{i2\pi(0.5434)} \\ 0.5195e^{i2\pi(0.5421)} \end{pmatrix}$	$\begin{pmatrix} 0.3170e^{i2\pi(0.4968)} \\ 0.4687e^{i2\pi(0.3582)} \end{pmatrix}$

Table 5: Collective DM matrix using the CPFDMSM operator.

	A_1	A_2	A_3	A_4
$\mathcal{Y}_1$	$\begin{pmatrix} 0.3878e^{i2\pi(0.5889)} \\ 0.2632e^{i2\pi(0.3216)} \end{pmatrix}$	$\begin{pmatrix} 0.2651e^{i2\pi(0.5886)} \\ 0.3377e^{i2\pi(0.3246)} \end{pmatrix}$	$\begin{pmatrix} 0.2317e^{i2\pi(0.5641)} \\ 0.5782e^{i2\pi(0.3589)} \end{pmatrix}$	$\begin{pmatrix} 0.2302e^{i2\pi(0.5518)} \\ 0.200e^{i2\pi(0.3296)} \end{pmatrix}$
$\mathcal{Y}_2$	$\begin{pmatrix} 0.4318e^{i2\pi(0.4231)} \\ 0.3434e^{i2\pi(0.3224)} \end{pmatrix}$	$\begin{pmatrix} 0.2807e^{i2\pi(0.4078)} \\ 0.2632e^{i2\pi(0.3193)} \end{pmatrix}$	$\begin{pmatrix} 0.3150e^{i2\pi(0.3776)} \\ 0.3494e^{i2\pi(0.4634)} \end{pmatrix}$	$\begin{pmatrix} 0.2659e^{i2\pi(0.5154)} \\ 0.1682e^{i2\pi(0.3523)} \end{pmatrix}$
$\mathcal{Y}_3$	$\begin{pmatrix} 0.3153e^{i2\pi(0.3080)} \\ 0.3162e^{i2\pi(0.3323)} \end{pmatrix}$	$\begin{pmatrix} 0.3111e^{i2\pi(0.4184)} \\ 0.4203e^{i2\pi(0.4319)} \end{pmatrix}$	$\begin{pmatrix} 0.3312e^{i2\pi(0.4837)} \\ 0.4368e^{i2\pi(0.2933)} \end{pmatrix}$	$\begin{pmatrix} 0.2856e^{i2\pi(0.4010)} \\ 0.2736e^{i2\pi(0.5065)} \end{pmatrix}$
$\mathcal{Y}_4$	$\begin{pmatrix} 0.3559e^{i2\pi(0.6434)} \\ 0.1626e^{i2\pi(0.2088)} \end{pmatrix}$	$\begin{pmatrix} 0.2807e^{i2\pi(0.4865)} \\ 0.2872e^{i2\pi(0.3944)} \end{pmatrix}$	$\begin{pmatrix} 0.2907e^{i2\pi(0.5455)} \\ 0.5173e^{i2\pi(0.5030)} \end{pmatrix}$	$\begin{pmatrix} 0.3203e^{i2\pi(0.4985)} \\ 0.4432e^{i2\pi(0.3545)} \end{pmatrix}$

Table 6: Collective evaluation data attained using CPFMSM and CPFDMSM operators.

Alternative	CPFMSM operator	CPFDMSM operator
$\mathcal{Y}_1$	$\begin{pmatrix} 0.2711e^{i2\pi(0.5618)} \\ 0.3565e^{i2\pi(0.3406)} \end{pmatrix}$	$\begin{pmatrix} 0.2807e^{i2\pi(0.5736)} \\ 0.3505e^{i2\pi(0.3337)} \end{pmatrix}$
$\mathcal{Y}_2$	$\begin{pmatrix} 0.3209e^{i2\pi(0.4243)} \\ 0.2846e^{i2\pi(0.3704)} \end{pmatrix}$	$\begin{pmatrix} 0.3254e^{i2\pi(0.4324)} \\ 0.2850e^{i2\pi(0.3658)} \end{pmatrix}$
$\mathcal{Y}_3$	$\begin{pmatrix} 0.3045e^{i2\pi(0.3911)} \\ 0.3677e^{i2\pi(0.4046)} \end{pmatrix}$	$\begin{pmatrix} 0.3110e^{i2\pi(0.4055)} \\ 0.3642e^{i2\pi(0.3940)} \end{pmatrix}$
$\mathcal{Y}_4$	$\begin{pmatrix} 0.3063e^{i2\pi(0.5388)} \\ 0.3730e^{i2\pi(0.3888)} \end{pmatrix}$	$\begin{pmatrix} 0.3124e^{i2\pi(0.5458)} \\ 0.3647e^{i2\pi(0.3718)} \end{pmatrix}$

Table 7: Score index of all the alternatives of $\mathcal{Y}_i\,(i = 1, 2, \ldots, n)$.

Operator	$\mathcal{Y}_1$	$\mathcal{Y}_2$	$\mathcal{Y}_3$	$\mathcal{Y}_4$
CPFMSM operator	$S\,(Q_1) = 0.1460$	$S\,(Q_2) = 0.0648$	$S\,(Q_3) = 0.0532$	$S\,(Q_4) = 0.0938$
CPFDMSM operator	$S\,(Q_1) = 0.1736$	$S\,(Q_2) = 0.0778$	$S\,(Q_3) =$ -0.0267	$S\,(Q_4) = 0.1242$

Table 8: Sorting of each alternative.

Operator	Order relation of $\mathcal{Y}_i\,(i = 1, \ldots, 4)$	Most optimal
CPFMSM operator	$\mathcal{Y}_1 > \mathcal{Y}_4 > \mathcal{Y}_2 > \mathcal{Y}_3$	$\mathcal{Y}_1$
CPFDMSM operator	$\mathcal{Y}_1 > \mathcal{Y}_4 > \mathcal{Y}_2 > \mathcal{Y}_3$	$\mathcal{Y}_1$

Table 9: Score index values and ranking of alternatives on the basis of different values of k.

Parameter value	$S\,(\mathcal{Y}_1)$	$S\,(\mathcal{Y}_2)$	$S\,(\mathcal{Y}_3)$	$S\,(\mathcal{Y}_4)$	Ranking relation	Optimal selection
$k = 1$	−0.4910	−0.4474	−0.6516	−0.6130	$\mathcal{Y}_2 > \mathcal{Y}_1 > \mathcal{Y}_4 > \mathcal{Y}_3$	$\mathcal{Y}_2$
$k = 2$	0.1460	0.0648	0.0532	0.0938	$\mathcal{Y}_1 > \mathcal{Y}_4 > \mathcal{Y}_2 > \mathcal{Y}_3$	$\mathcal{Y}_1$
$k = 3$	1.2738	1.4303	1.1203	1.2998	$\mathcal{Y}_2 > \mathcal{Y}_4 > \mathcal{Y}_1 > \mathcal{Y}_3$	$\mathcal{Y}_2$

Step (3). The CPFMSM operator or the CPFDMSM is employed to combine linguistic assessment information $Q_{ip}\,(p = 1, 2, \ldots, m)$ presented in Tables 4-5 and to get the inclusive assessment value of alternatives $\mathcal{Y}_i\,(i = 1, 2, \ldots, n)$, which is presented in Table 6 (take $k = 2$).

Table 10: DM matrix from Example 2.

	A_1	A_2	A_3	A_4
$\mathscr{Y}_1$	$\begin{pmatrix} 0.70e^{i2\pi(0.80)} \\ 0.36e^{i2\pi(0.95)} \end{pmatrix}$	$\begin{pmatrix} 0.55e^{i2\pi(0.55)} \\ 0.62e^{i2\pi(0.35)} \end{pmatrix}$	$\begin{pmatrix} 0.89e^{i2\pi(0.37)} \\ 0.69e^{i2\pi(0.48)} \end{pmatrix}$	$\begin{pmatrix} 0.78e^{i2\pi(0.56)} \\ 0.84e^{i2\pi(0.39)} \end{pmatrix}$
$\mathscr{Y}_2$	$\begin{pmatrix} 0.67e^{i2\pi(0.34)} \\ 0.69e^{i2\pi(0.28)} \end{pmatrix}$	$\begin{pmatrix} 0.56e^{i2\pi(0.38)} \\ 0.59e^{i2\pi(0.85)} \end{pmatrix}$	$\begin{pmatrix} 0.69e^{i2\pi(0.59)} \\ 0.78e^{i2\pi(0.47)} \end{pmatrix}$	$\begin{pmatrix} 0.87e^{i2\pi(0.47)} \\ 0.97e^{i2\pi(0.73)} \end{pmatrix}$
$\mathscr{Y}_3$	$\begin{pmatrix} 0.90e^{i2\pi(0.27)} \\ 0.96e^{i2\pi(0.67)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.28)} \\ 0.85e^{i2\pi(0.79)} \end{pmatrix}$	$\begin{pmatrix} 0.65e^{i2\pi(0.46)} \\ 0.76e^{i2\pi(0.48)} \end{pmatrix}$	$\begin{pmatrix} 0.81e^{i2\pi(0.59)} \\ 0.77e^{i2\pi(0.37)} \end{pmatrix}$
$\mathscr{Y}_4$	$\begin{pmatrix} 0.67e^{i2\pi(0.64)} \\ 0.78e^{i2\pi(0.25)} \end{pmatrix}$	$\begin{pmatrix} 0.57e^{i2\pi(0.60)} \\ 0.48e^{i2\pi(0.88)} \end{pmatrix}$	$\begin{pmatrix} 0.33e^{i2\pi(0.49)} \\ 0.66e^{i2\pi(0.38)} \end{pmatrix}$	$\begin{pmatrix} 0.24e^{i2\pi(0.45)} \\ 0.87e^{i2\pi(0.81)} \end{pmatrix}$

Table 11: Values of the score function and ranking of alternatives of Example 2.

Approaches	Value of score function	Sorting
CIFWA operator propounded in [46]	Impotent to calculate	Impotent to calculate
CIFBM operator [47] $(p = q = 1)$	Impotent to calculate	Impotent to calculate
CPFS in [29]	Impotent to calculate	Impotent to calculate
Cq-ROF2TLMSM operator propounded in [48] $(k = 2 \text{and} q = 3)$	$S\ (\mathscr{Y}_1) = 0.8924$, $S(\mathscr{Y}_2) = 0.7290$ $S(\mathscr{Y}_3) = 0.7036$, $S(\mathscr{Y}_4) = 0.6228$	$\mathscr{Y}_1 > \mathscr{Y}_2 > \mathscr{Y}_3 > \mathscr{Y}_4$
CPFMSM operator propounded in this manuscript $(k = 2)$	$S\ (\mathscr{Y}_1) = 0.1451$, $S(\mathscr{Y}_2) = -0.2820$ $S(\mathscr{Y}_3) = -0.3099$, $S(\mathscr{Y}_4) = -0.3744$	$\mathscr{Y}_1 > \mathscr{Y}_2 > \mathscr{Y}_3 > \mathscr{Y}_4$

Table 12: Comparative analysis with previous existing operators.

Operator	$S\ (\mathscr{Y}_1)$	$S\ (\mathscr{Y}_2)$	$S\ (\mathscr{Y}_3)$	$S\ (\mathscr{Y}_4)$	Ranking
CIFMSM [7]	0.1467	0.1129	0.0826	0.1149	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$
CIFDMSM [7]	0.1772	0.1066	0.0499	0.1547	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$
CIFWA [46]	0.7027	0.6900	0.6663	0.7023	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$
CIFBM [47]	0.8752	0.8474	0.8268	0.8720	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$
Cq-ROF2TLWMSM [48] for $q = 1$	0.8764	0.8483	0.8275	0.8750	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$
Cq-ROF2TLWMSM [48] for $q = 2$	0.8752	0.8474	0.8268	0.8720	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$
Cq-ROF2TLWMSM [48] for $q = 3$	0.8736	0.8474	0.8268	0.8720	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$
Cq-ROF2TLWDMSM [48] for $q = 2$	0.1802	0.1607	0.1688	0.1706	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_3 > \mathscr{Y}_2$
CPFMSM	0.1460	0.0648	0.0532	0.0938	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$
CPFDMSM	0.1736	0.0778	−0.0267	0.1242	$\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$

Step (4). The score functions $S(Q_i)$ calculated for each alternative $\mathscr{Y}_i (i = 1, 2, \ldots, n)$ by employing Definition 6 are presented in Table 7.

Steps (5). All the alternatives $\mathscr{Y}_i (i = 1, 2, \ldots, n)$ are ranked by utilizing Definition 8, and the best optimum alternative is attained from them, which is shown in Table 8 as follows.

4.1.2. Sensitivity Analysis for the Parameter k. As shown in the aforementioned calculations, the parameter has played an important role in process of decision-making. It also affects the final decision. So, in this subsection, we will perform a parameter analysis for the parameter k. For ease, we used the CPFMSM operator to analyze and solve the presented example. We have used the CPFMSM operator with different parameter values to prove the sensitivity of the parameter k in the decision results. Various parameter values are used to incorporate the real example mentioned above and to achieve the sorting results. These results are presented in Table 9 (if $k = 2$).

Table 9 depicts that the alternative result alters when the value of the parameter alters. These alternations show the interrelationship between the attributes in the process of decision-making. For example, if we set $k = 1$ or 3 in the CPFMSM operator, the ranking of the involved alternatives becomes $\mathscr{Y}_2 > \mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_3$ and $\mathscr{Y}_2 > \mathscr{Y}_4 > \mathscr{Y}_1 > \mathscr{Y}_3$, respectively, which is different according to the other situations because, in such situation, the CPFMSM operator will be transformed into CPFAA and CPFGBM operators, respectively. In that situation, the relationships between the discussed attributes are incapable to address the DM problems. We have observed from Table 9 that as the value of parameter k increases, the score value of CPFMSM also increases. This shows that the proposed operator is more flexible and DM may select the value of k according to their real situation and preferences. For solving the decision-making problem and to aggregate the assessment, data

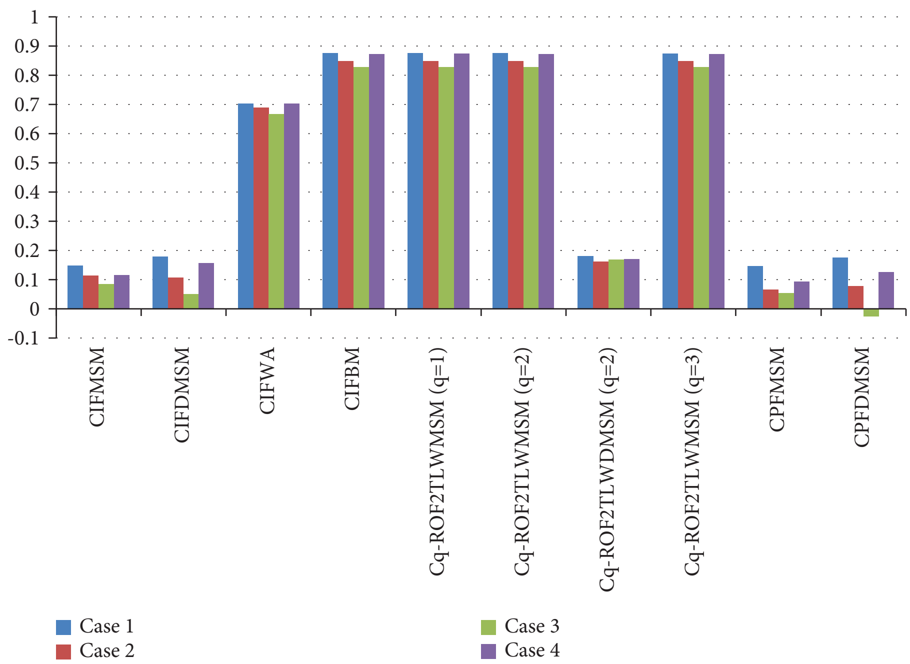

Figure 1: Comparison among different approaches.

decision-making experts have to set $k=2$ in the CPFMSM operator. In diverse values of parameter k, DMs may select an optimal parameter of their own preference and can attain a satisfactory and reasonable decision result. For general values, we can take the value of $k=[n/2]$, for calculation in DM issues, where n denotes the number of attributes and symbol [.] represents the round function [44, 45]. In this situation, the interrelationship between the individual arguments is taken into consideration, while the DM's choice is taken as neutral.

Moreover, by comparing with the existing operators in the literature such as Bonferroni and generalized Bonferroni operators, these operators capture the relationship between two attributes, while the proposed method is superior to the Bonferroni method, as it can capture the relationship among the multiple attributes. Similarly, the BM and generalized BM operators require two and three parameters, respectively, from an infinite rational set, while our suggested operator requires one parameter for processing the CPF data. Therefore, the complexity in the calculation of our suggested approach is more less than the BM and generalized BM operators. Furthermore, the proposed operator can be reduced to some previously suggested operators such as arithmetic averaging, BM, generalized BM, and geometric averaging, respectively, by assigning different values to k such as $k=1$, $k=2$, $k=3$, and $k=m$. Therefore, the suggested operator is more generalized, flexible, and easy to implement for evaluating the DM problem with CPF data.

4.1.3. Generalization Analysis. Now, in this subsection a second numerical example is used to deliberate the generality of the introduced approach. The assessment data presented by the DM experts for the complex linguistic number are presented in Table 10, and then, the case is solved by our propounded approach and the decision results are displayed in Table 11.

It is shown in Table 11 that the previous methods such as distance measures [29], complex intuitionistic fuzzy weighted averaging (CIFWA) [46], and complex intuitionistic fuzzy BM (CIFBM) [47] are unable to resolve Example 2, while the CPFMSM can successfully solve it. This shows the dominancy of the CPFMSM method, as the CPF and CIF are the particular cases of our presented approach.

4.1.4. Further Contrastive Analyzing. Now, we will perform a comprehensive analysis between the method propounded in this manuscript and the previous approaches. The supremacy of propounded approach is as follows

(1) Comparison with the CIFWA Operator Presented by Garg and Rani [46]. As the CIFWA operator is a basic aggregation operator for the aggregation of data, it supposes that attributes considered in the daily life issues are unrelated; i.e., it deems the significance of attributes, which makes the decision result ill-defined and conspicuous. CPFMSM can satisfy the abovementioned defect and also consider the interrelationship of attributes. Furthermore, CPFMSM shows the individual preference of the decision-makers and the great tendency of order relation of the alternatives using variable parameters. So, the CPFMSM is more useful in addressing the decision-making issues

(2) Comparison with CIFBM Operator Proposed by Garg and Rani. [47]. Although the CIFBM operator is an aggregation operator, used for the aggregation of complex intuitionistic fuzzy data, it considers the interrelationship between the two attributes. CPFMSM operator utilized in this manuscript

considers the relationship between the attributes, and it also minimizes the computational complication in the process of aggregation of information. Moreover, the CPFMSM can resolve those problems, which are not solved by the CIFBM. So, CPFMSM is more universal and gives rational calculation results in the DM process.

(3) Comparison with the Cq-ROFLHM Operator Presented by Liu et al. [49]. The relationship between two attributes is taken into consideration by the HM operator, whereas the CPFMSM operator considers the relationship between the multi-input data arguments. Moreover, the HM has two parameter values, which makes the process of calculation much complicated and complex. This also creates a difficulty for the decision-makers in the selection of a satisfied parameter value, whereas CPFMSM has a single parameter value, which is convenient for the decision-maker to allocate the suitable depending on the need. So, CPFMSM is generally more efficient and effective in handling DM issues.

Furthermore, if we use the proposed operators such as CPFMSM and CPFDMSM, we get the ranking of alternatives as $\mathscr{Y}_1 > \mathscr{Y}_4 > \mathscr{Y}_2 > \mathscr{Y}_3$, for $k = 2$, in which the optimal alternative is the same and coincides with existing operators in the literature [7, 48], whereas the slight difference in the ranking is due to considering different environment and operators. Hence, the best alternative is $\mathscr{Y}_1$. This validates that the propounded operators are effective, rational, and reasonable in the DM process. The score values and ranking of different operators are summarized in Table 12.

A comparison between the existing approaches and the proposed one is presented graphically in Figure 1.

5. Conclusion

In this manuscript, the MSM operator is presented under the CPF environment to solve DM problems. In the past, many aggregation operators were developed under the IFS and PFS environments, in which the range of the consistent membership and nonmembership degree was the subset of real numbers, but now CPFS has fulfilled this deficiency, because in CPFS environment, the range of the MD and NMD is expanded to complex numbers from real numbers within the unit disc. So, the environmental model is the best method for the representation of time variable issues. Moreover, CPFSs are also very efficient in handling the two-dimensional data in one set. In this CPF method, the preferences of the DM experts are expressed in linguistic form. Some aggregation operators such as CPFMSM and CPFDMSM are propounded by the combination of CPF with MSM and DMSM for aggregation of data. Ascending order is used in the paper to rank the alternatives. To express the ambiguous and ill-defined evaluation information, CPFSs are integrated with the MSM operator, which not only solves the problem of the complex uncertain information but also reduces the loss of information in the MAGDM issues. Furthermore, a new MAGDM approach is established here, based on the newly propounded CPFMSM and CPFDMSM operators. A numerical case is carried out to certify the efficiency of the designed approach. The validity of the proposed approach is also shown by a practical problem. Finally, a contrastive and comparative analysis is carried out between the previously proposed methods and our presented approach.

The weight of parameter plays an important role in the ranking process, but we have to ignore it at this stage in order to avoid lengthy calculation while the best alternative remains the same [48]. In near future, we will consider the weighted form of parameter for better optimal results. Furthermore, in future, we will implement the propounded approach to handle many practical problems, for instance, the selection of the best student for doctoral supervision [50] and evaluation of the innovative capability of the universities [51]. Moreover, the proposed operators were limited to two dimensions, but someone can extend the proposed concept for q dimensions. The presented methods can be modified to other generalized fuzzy sets, which will generate different useful structures that will be useful for implementation in science, economics, and technology. Moreover, we will focus on complex fuzzy sets [40, 41], which are the generalization of the picture fuzzy to address the problem of ambiguity and fuzziness. Therefore, the study of spherical fuzzy will be the main concern in this scenario. Furthermore, we will consider approaches of complex fuzzy sets and other theories such as consensus reaching [52, 53], which have a significant role in GDM problems.

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding this study.

Authors' Contributions

All authors contributed equally to this work. All authors read and approved the final manuscript.

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