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Research Article

Theoretical Model Development for Energy Motion of Dusty Turbulent Flow of Fibre Suspensions in a Rotational Frame

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Fibre suspension has garnered considerable attention in turbulent flows that are used in many industries. Solid particles, such as dust particles, notably affect the turbulent flow field in a rotational frame. In assessing their impacts, the dusty turbulent flow for fibre suspensions needs to be studied in a frame of rotation that can be substantially applied in many industries. This study, therefore, aims to build a theoretical model for the energy motion of dusty turbulent flow of fibre suspensions in a rotational frame. The turbulence momentum equation was considered to formulate the model in presence of dusty fluid rotating flow of fibre suspensions. The newly derived equation was derived in second-order correlation tensors $F_{i,j}$, $W_{i,j}$, $G_{i,j}$, $S_{i,j}$, $X_{i,j}$, $Y_{i,j}$, $Q_{i,j}$, and $R_{i,j}$ at any two points in the flow domain, where the tensors were expressed as space, time, and distance functions. The developed model is a considerable improvement because it takes into account all of the potential influential parameters that could affect the motion of turbulent energy, such as dust particles, suspending particles (fibres), and rotating frame. However, the impact of these parameters on turbulence energy motion must be evaluated in order to assess the performance of turbulence systems utilized in a variety of industries, such as paper manufacturing. The present theoretical development will contribute to open up experimental and numerical research opportunities for the advancement of the industry, science, and technology.

1. Introduction

The flows of turbulent are often found in moving objects, for example, various moving vehicles, which affect flow resistance to the bodies of the objects. Turbulent flow motions define the fluid flow behaviours based on mass conservation, energy, and momentum laws [1]. In the second-order pressure-velocity correlation, Hinze and Uberoi [2] derived the turbulent motion at any two points in the flow domain. The correlations between velocity and pressure fluctuations were taken into account in his study. However, the effect of any potential turbulent motion parameters was not considered. The turbulent flows are greatly affected by the introduction of fibres into the flow. In several parts of the industry, fibre injections into the turbulent flow are used, for example, in environmental engineering, paper manufacturing, chemical engineering, the manufacture of composite materials, and the textile industry. The property of the fibre suspension has a notable effect on product quality. The complexity in turbulent fibre motion arises mainly from the translational and revolutionary dynamics of the particles, which are driven by forces and torques, depending on the form and the orientation of the particles [3]. Over the last thirty years, numerous studies have been undertaken on turbulent flows of fibre suspensions. A preliminary study on turbulent motion of fibre (ice crystals) suspension was conducted by Cho et al. [4]. Atmospheric turbulence associated with their effect on fibre orientation was investigated in their study. The mean fibre orientation was not significantly affected by the atmospheric turbulence. Fluid motion equations were solved with a spectral approach by Lin et al. [5], and the trajectories of fibre were determined on slim body theory. Although a key parameter to define the fibre orientations on a broad scale was found to be the Stokes number, its orientation effect on the fibre is not significant and the ratio of fibre direction has a minimum influence on the orientation distribution of fibre.

Particle motions in the flows of turbulent are a fundamental issue for applications [6], for example, atmospheric clouds, and spray combustion engines. In an accurate understanding between dust particle collisions and growth [7–9], humid terrestrial clouds, cool stars [10], brown dwarves, and cloud production of planets, the interactions between turbulent and the particles have been considered. The evolution of the particles' size is dependent on the particle collision rate which can be significantly increased by turbulent motions [11]. In the presence of small-sized particles, Bhatti et al. [12] studied the electro-osmotic flow of non-Newtonian fluid moving in a sinusoidal fashion in a Darcy–Brinkman–Forchheimer medium. The Jeffrey fluid model was used to investigate non-Newtonian effects. The Debye length approximation, Poisson–Boltzmann equation, and ionic Nernst–Planck equation were employed to construct the mathematical model. The effect of parameters on both fluid and particle phases was also discussed. The present effects have been proven to be advantageous in the design and manufacture of microfluidic devices for determining the transport mechanism.

A precise and exact calculation of the collision rate includes an accurate understanding of the collision rate impacts caused by turbulence and preferential or clustering particle concentrations. Several studies [13–21] have been carried out on the motion of the dusty turbulent flow. The characteristics of the dust particles [22–25] mainly rely on the particle size of the turbulence scale. The particles compete over fluid impacts and collisions on turbulent flow into the neighbourhoods of the particle. In a study conducted by [26], entropy generation and irreversibility processes that occur as a result of partial slip on magnetic dusty fluid caused by a peristaltic wave via a porous conduit were studied. The results demonstrated that the presence of dust particles in the fluid causes the flow to decelerate. In the presence of a strong magnetic field, Mahanthesh [27] investigated the effect of Hall current on the two-phase boundary layer flow of an electrically conducting dusty fluid over a permeable stretched sheet. Using appropriate similarity transformations, the governing equations were reduced from a set of partial differential to ordinary differential equations. The author reported that, in controlling the friction factor on the sheet, the dust particles' mass concentration can be employed as a control parameter. It was also revealed that suction and injection have opposing effects on the formation of the momentum boundary layer. In the presence of dust particles, the effect of Hall current on the time-dependent flow of a nanofluid was also investigated by Gireesha et al. [28]. The boundary layer approximation notion was used for modeling the governing equations for nanofluid as well as dust phases. Increases in heat transfer rate were found to be proportional to Hall current and unsteadiness.

The motions between a particle of the fluid and the suspended fibre rely on fluid dynamics fundamental of turbulence characteristics with the velocity pressure correlations of fluctuating components. A good number of [15, 29–35] studies have been undertaken in rotating systems to analyze the motion of turbulent flows. The mean rotation induces dynamical influences on turbulent flows through the pressure-stream-rate correlation in the transport equations. The rotation produces the auxiliary body force-like Coriolis and centrifugal force in rotating turbulent flows, accompanied by the turbulent structures. Consequently, the momentum transfer mechanism suffers from complexity. However, most research studies did not take into account or address the rotational impacts of the system at different angles, owing to rotational frame complications. Coriolis force plays a critical role in a turbulent flow rotating system [36, 37]. To measure the impact of the Coriolis force, You et al. [30, 35] directed an experimental-based study on the rotating turbulent flow. The velocity for particle images was considered in this study to calculate the flow profile. Because of the Coriolis effect, the flow domains between the leading and trailing sides were found to be different [30]. Coriolis force widened the vortex nearby the front side and intimidated the vortex nearby the trailing end. The force of Coriolis showed major effects in the flow field on the vortices. It was also revealed that not only the secondary flow but also the Coriolis force influences the flow domain which must not be overlooked.

Due to the simplification of laminar flow, it is shown through comprehensive literature surveys that several research studies were undertaken on assessing the behaviour of laminar fibre motion. However, the turbulent fibre motion studies are still inadequate because of the complexities of fibre motion and turbulence. Flow phenomena generated in the flow domain are complex if it includes any type of solid particle such as dust particles into the turbulent fibre flow. The dust particles can range from 1 to 400 μm diameters, with particles more than 100 μm settling down towards the formation source [38]. The overall range of the particle sizes can be classified into three categories: lower than 1 μm, 1–20 μm, and more than 20 μm, which are referred to as ultrafine, fines, and large particles, respectively. However, the smaller particles are comparatively difficult to remove or separate from the airstream because they have a stronger inclination to stay in suspension.

In a frame of rotation, the dusty turbulent flow becomes more complex. However, some mathematical models were developed on the energy motion of turbulent flow in second-order correlation tensors by considering dust particles [13], fibre suspension [39], dust particles with fibre suspensions [40], a rotating frame [32], and dust particles in a frame of rotation [15]. Nevertheless, any of these articles did not consider all of the parameters that can influence turbulence energy motion, such as dust particles, fibre suspension, and rotational frame. The main fact behind this is because of its complexity level in mathematical modeling. As a result, the previous models cannot be considered fully complete and feasible, and they must be improved by taking into account all of these influential parameters in order to make the model feasible. By considering all of these parameters, the present study thus constructs a model for the energy motion of dusty rotating turbulent flow of fibre suspensions. To address the research gaps and improve the previous models, the present study introduced correlation tensors $W_{i,j}$, $X_{i,j}$, $Y_{i,j}$, and $R_{i,j}$, which represent the correlations between fluid velocities, angular velocities, and fluid velocities, angular velocities produced due to the rotation, and the suspending fluid velocities in the fluid flow domain. The resulting partial differential equation model can be generalized to time-space fractional order, making it valuable to the broader scientific community. Based on a numerical investigation, the DNS (direct numerical simulation) method can be applied to the rotation of dusty rigid fibres with different lengths relative to the neighbouring fluid in the channel flow. Due to its high precision, the spectral element method (SEM) may be used to solve the newly developed equation, and SEM applications are also suggested for future research.

2. Methodology

A turbulent flow domain was considered in a rotational frame to formulate and develop the present model. The fluid flow domain comprised fibre suspensions in the existence of dust particles. The mathematical formulation of the model was directed with the equation of momentum for viscid turbulent incompressible flow. The mathematical consequences were sequentially added to the momentum equation. To find the relationships among the velocities of fluid, fibres, and dust particles in the rotational frame, any two points, C and D, were chosen at a distance r in the turbulent flow field. Point C was regarded as the origin of the system coordinate since no difference exists in representing the relationships of turbulence momentum between the points if any of the two points is assumed as the source (origin). The independent variables ψ_k were chosen to differentiate the influences between distance and position. The method of averaging was used to express first-, second-, and third-order pressure-velocity correlations between the two points, C and D. These correlations were demonstrated in a second-order partial differential equation by introducing second-order correlation tensors and following averaging method and tensor properties.

3. Model Formulation and Development

The equation of momentum and the continuity for viscid turbulent incompressible flow are represented by [32, 41]

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + v\frac{\partial^2 u_i}{\partial x_j \partial x_j} - 2\varepsilon_{ijl}\Omega_j u_l, \qquad \frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

with the fluid velocity components of $u_i(x,t)$, fluid density ρ, pressure $p(x,t)$, suspending fluids' kinematical viscosity v, three-dimensional permutation symbol ε_{ijl}, rotation vector Ω_j, dissipation of turbulence ε, position x, and time t.

For the turbulent flow with dust (solid) particles, the following equations are obtained:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + v\frac{\partial^2 u_i}{\partial x_j \partial x_j} - 2\varepsilon_{ijl}\Omega_j u_l + \frac{KN}{\rho}(v_i - u_i), \tag{2}$$

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{3}$$

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial u_i}{\partial x_j} = -\frac{K}{m_s}(u_i - v_i), \tag{4}$$

with dust (solid) particle velocity components $v_i(x,t)$, dust particle mass for single-spherical $m_s = 4/3\pi R_s^3 \rho_s$ with radius R_s, Stoke's formula for the drag $K = 6\pi R_s \rho v$, $KN/\rho = f$ frequency dimension, and dust particle density number N. It is noted that the term $KN/\rho(v_i - u_i)$ appears in equation (2) due to the solid dust particles existent in the field of turbulent flow [14]. The fluid is presumed to be air and treated as multiphase since this present study comprises fibre suspensions and dust particle interactions into the dusty turbulent flow in the frame of rotation.

In presence of fibre, the equation of turbulent motion (3) gives

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + v\frac{\partial^2 u_i}{\partial x_j \partial x_j} - 2\varepsilon_{\mathrm{ijl}}\Omega_j u_l + \frac{KN}{\rho}(v_i - u_i) + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{\mathrm{ijlm}}\varepsilon_{\mathrm{lm}} - \frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right], \tag{5}$$

with the apparent viscosity of fibre suspension μ_f, the turbulent intensity of fibre suspension I_{ij}, tensor strain rate $\varepsilon_{lm} = 1/2(\partial u_l/\partial x_m + \partial u_m/\partial x_l)$, and fourth- and second-order tensors for fibre orientation a_{ijlm} and a_{lm}, respectively. The term $\mu_f/\rho\partial/\partial x_j[a_{ijlm}\varepsilon_{lm} - 1/3(I_{ij}a_{lm})\varepsilon_{lm}]$ occurs in equation (5) because of the fibres poured into the turbulent flow domain [42].

For a rotating frame, the turbulence momentum equation is given by [32]

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + v\frac{\partial^2 u_i}{\partial x_j \partial x_j} - 2\varepsilon_{ijl}\Omega_j u_l - 2(\Omega_i u_i \eta_i)\sin\theta, \tag{6}$$

where $-2(\Omega_i u_i \eta_i)\sin\theta = -2(\overline{\Omega} \times \overline{u})$ denotes the Coriolis force, where η, Ω_i, and θ specify unit vector normal to $\overline{u}$ and $\overline{\Omega}$, angular velocity, and the angle makes between $\overline{u}$ and $\overline{\Omega}$, respectively.

Therefore, combining equations (5) and (6), a new equation is obtained as follows:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon \frac{\partial^2 u_i}{\partial x_j \partial x_j} - 2\varepsilon_{ijl}\Omega_j u_l + \frac{KN}{\rho}\left(v_i - u_i\right) + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{\text{ijlm}}\varepsilon_{lm} - \frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] - 2\left(\Omega_i u_i \eta_i\right)\sin\theta. \quad (7)$$

Equation (7) represents the new energy equation of motion for fibre suspended dusty rotating turbulent flow.

Suppose C and D are any two points in the turbulent flow field and assume c and d be the directions along with the points C and D, respectively. Therefore, u_c and u_d can be used as the components of the velocity along with C and D directions. Suppose that $\overline{U_i}$ is the mean velocity, time independent, and constant throughout the considered domain.

Thus, $(U_i = \overline{U}_i + u_i)_C$ and $(U_j = \overline{U}_j + u_j)_D$.

All the terms have a value, which can be calculated from the equations of u_i at C point and of u_j at D point.

At point C, the turbulence momentum can be written from equation (7) for u_i:

$$\frac{\partial u_i}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon \frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\varepsilon_{ikl}\Omega_k u_l + f\left(v_i - u_i\right) + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{\text{iklm}}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right] - 2\left(\Omega_i u_i \eta_i\right)\sin\theta. \quad (8)$$

Since, for incompressible fluids, $(u_i \partial u_k / \partial x_k)_C = 0$, equation (8) yields

$$\frac{\partial}{\partial t}(u_i)_B + \left[\overline{U}_k + (u_k)_C\right]\left(\frac{\partial}{\partial x_k}\right)_C (u_i)_C + \left(u_i \frac{\partial u_k}{\partial x_k}\right)_C = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_C p_C + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_C (u_i)_C - 2\left[\left(\varepsilon_{ikl}\Omega_k u_l\right)_C + \left(\Omega_i u_i \eta_i\right)_C \sin\theta\right] + f\left(v_i - u_i\right)_C + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_C \left[a_{\text{iklm}}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right]_C. \quad (9)$$

Multiplying equation (9) by $(u_j)_D$,

$$\begin{aligned}&\left(u_j\right)_D \frac{\partial}{\partial t}(u_i)_C + \left[\overline{U}_k + (u_k)_C\right]\left(\frac{\partial}{\partial x_k}\right)_C (u_i)_C\left(u_j\right)_D + (u_i)_C\left(\frac{\partial}{\partial x_k}\right)_C (u_k)_C\left(u_j\right)_D \\ &= -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_C p_B\left(u_j\right)_D + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_C (u_i)_C\left(u_j\right)_D - 2\left[\left(\varepsilon_{ikl}\Omega_k u_l\right)_C + \left(\Omega_i u_i \eta_i\right)_C \sin\theta\right]\left(u_j\right)_D \\ &+ f\left(v_i - u_i\right)_C\left(u_j\right)_D + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_C\left[a_{\text{iklm}}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right]_C\left(u_j\right)_D,\end{aligned} \quad (10)$$

where $(u_j)_D$ is taken as constants for the differential system at C point.

Likewise, the turbulence momentum equations for u_j are obtained at D point:

$$\frac{\partial u_j}{\partial t}+(\overline{U}_k+u_k)\frac{\partial u_j}{\partial x_k}=-\frac{1}{\rho}\frac{\partial p}{\partial x_j}+v\frac{\partial^2 u_j}{\partial x_k\partial x_k}-2\varepsilon_{jkl}\Omega_k u_l$$
$$+f\left(v_j-u_j\right)+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{\mathrm{jklm}}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]-2\left(\Omega_j u_j\eta_j\right)\sin\theta. \tag{11}$$

By using the condition of incompressible fluids, $(u_j\partial u_k/\partial x_k)_D=0$, equation (11) gives

$$\frac{\partial}{\partial t}\left(u_j\right)_D+[\overline{U}_k+(u_k)_D]\left(\frac{\partial}{\partial x_k}\right)_D\left(u_j\right)_D+\left(u_j\frac{\partial u_k}{\partial x_k}\right)_D=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_D p_D+v\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_D\left(u_j\right)_D$$
$$-2\left[\left(\varepsilon_{jkl}\Omega_k u_l\right)_D+\left(\Omega_j u_j\eta_j\right)_D\sin\theta\right]+f\left(v_j-u_j\right)_D+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_D\left[a_{\mathrm{jklm}}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]_D. \tag{12}$$

Multiplying equation (12) by $(u_i)_C$, the following is obtained:

$$(u_i)_C\frac{\partial}{\partial t}\left(u_j\right)_D+[\overline{U}_k+(u_k)_D]\left(\frac{\partial}{\partial x_k}\right)_D\left(u_j\right)_D(u_i)_C+\left(u_j\right)_D\left(\frac{\partial}{\partial x_k}\right)_D(u_k)_D(u_i)_C$$
$$=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_D p_D(u_i)_C+v\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_D\left(u_j\right)_D(u_i)_C-2\left[\left(\varepsilon_{jkl}\Omega_k u_l\right)_D+\left(\Omega_j u_j\eta_j\right)_D\sin\theta\right](u_i)_C$$
$$+f\left(v_j-u_j\right)_D(u_i)_C+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_D\left[a_{\mathrm{jklm}}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]_D(u_i)_C, \tag{13}$$

where $(u_i)_C$ are taken as constants for differential systems at D point.

Summing equations (10) and (13),

$$\frac{\partial}{\partial t}(u_i)_C\left(u_j\right)_D+\left[\left(\frac{\partial}{\partial x_k}\right)_C(u_i)_C(u_k)_C\left(u_j\right)_D+\left(\frac{\partial}{\partial x_k}\right)_D(u_i)_C(u_k)_D\left(u_j\right)_D\right]+\overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_C(u_i)_C\left(u_j\right)_D+\left(\frac{\partial}{\partial x_k}\right)_D(u_i)_C\left(u_j\right)_D\right]$$
$$=-\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_C p_C\left(u_j\right)_D+\left(\frac{\partial}{\partial xj}\right)_D p_C(u_i)_C\right]+v\left[\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_C+\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_D\right](u_i)_C\left(u_j\right)_D$$
$$-2\left[(\varepsilon_{ikl}\Omega_k u_l)_C\left(u_j\right)_D+\left(\varepsilon_{jkl}\Omega_k u_l\right)_D(u_i)_C\right]-2\left[(\Omega_i u_i\eta_i)_C\left(u_j\right)_D+\left(\Omega_j u_j\eta_j\right)_D(u_i)_C\right]\sin\theta$$
$$+f\left[(v_i-u_i)_C\left(u_j\right)_D+\left(v_j-u_j\right)_D(u_i)_C\right]$$
$$+\frac{\mu_f}{\rho}\left[\left(\frac{\partial}{\partial x_k}\right)_C\left(a_{\mathrm{iklm}}\varepsilon_{lm}-\frac{1}{3}I_{ik}a_{lm}\varepsilon_{lm}\right)_C\left(u_j\right)_D+\left(\frac{\partial}{\partial x_k}\right)_D\left(a_{\mathrm{jklm}}\varepsilon_{lm}-\frac{1}{3}I_{jk}a_{lm}\varepsilon_{lm}\right)_D(u_i)_C\right]. \tag{14}$$

In order to represent the relationships of turbulence momentum between D and C point, no difference exists if C or D is assumed as the source (origin) of the system coordinate. Point C was regarded as origin here. Suppose the variables ψ_k are independent and used to differentiate the influences between distance and position, where $\psi_k = (x_k)_D - (x_k)_C$.

Therefore, the following relations are obtained:

$$\left(\frac{\partial}{\partial x_k}\right)_C = -\frac{\partial}{\partial \psi_k},$$
$$\left(\frac{\partial}{\partial x_k}\right)_D = \frac{\partial}{\partial \psi_k}, \qquad (15)$$
$$\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_C = \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_D = \frac{\partial^2}{\partial \psi_k \partial \psi_k}.$$

Using these relations and averaging all the terms, equation (15) yields

$$\begin{aligned}
&\frac{\partial}{\partial t}\overline{(u_i)_C(u_j)_D} - \frac{\partial}{\partial \psi_k}\overline{(u_i)_C(u_k)_C(u_j)_D} + \frac{\partial}{\partial \psi_k}\overline{(u_i)_C(u_k)_D(u_j)_D} = -\frac{1}{\rho}\left[-\frac{\partial}{\partial \psi_i}\overline{p_C(u_j)_D} + \frac{\partial}{\partial \psi_j}\overline{p_D(u_i)_C}\right]\\
&+ 2v\frac{\partial^2}{\partial \psi_k \partial \psi_k}\overline{(u_i)_C(u_j)_D} + f\left[\overline{(v_i)_C(u_j)_D} - 2\overline{(u_i)_C(u_j)_D} + \overline{(u_i)_D(v_j)_C}\right]\\
&- 2\left[\overline{(\varepsilon_{ikl}\Omega_k u_l)_C(u_j)_D} + \overline{(\varepsilon_{jkl}\Omega_k u_l)_D(u_i)_C}\right] - 2\left[\overline{(\Omega_i u_i \eta_i)_C(u_j)_D} + \overline{(\Omega_j u_j \eta_j)_D(u_i)_C}\right]\sin\theta\\
&- \frac{\mu_f}{\rho}\frac{\partial}{\partial \psi_k}\left[\overline{(a_{\mathrm{iklm}}\varepsilon_{lm})_C(u_j)_D} - \frac{1}{3}\overline{(I_{ik}a_{lm}\varepsilon_{lm})_C(u_j)_D} - \overline{(a_{\mathrm{jklm}}\varepsilon_{lm})_D(u_i)_C} + \frac{1}{3}\overline{(I_{jk}a_{lm}\varepsilon_{lm})_D(u_i)_C}\right].
\end{aligned} \qquad (16)$$

Equation (16) yields the mean energy motion of dusty turbulent flow for fibre suspensions in a rotational frame, where the motion of fibres of turbulent flow is at average speed $\overline{U}_k$ with respect to the system coordinate. Due to the constant derivative, the coefficient term $\overline{U}_k$ was omitted in this equation. The equation consists of the double correlations of velocity $\overline{(u_i)_C(u_j)_D}$, pressure-velocity $\overline{p_C(u_j)_D}$, and triple-velocity such as $\overline{(u_i)_C(u_k)_C(u_j)_D}$, where the terms are located at a certain distance from each other. The correlations of pressure-velocity $\overline{p_D(u_i)_C}$ and $\overline{p_D(u_j)_C}$ form first-order tensor as the pressure is taken as a scalar. Likewise, triple correlations of velocity, $\overline{(u_i)_C(u_k)_D(u_j)_D}$ and $\overline{(u_i)_C(u_k)_C(u_j)_D}$, make third-order tensors. The double correlations are displayed in Figure 1, while the triple correlations are illustrated in Figure 2, respectively, at points C and D, where r is the distance from C to D.

The correlations of the order first, second, and third can be labelled by $(K_{p,j})_{C,D}$, $(W_{i,j})_{C,D}$, and $(S_{ik,j})_{C,D}$, respectively. Thus, the correlations of pressure-velocity and velocity give

$$\begin{aligned}
(K_{i,p})_{C,D} &= \overline{(u_i)_C p_D},\\
(k_{p,j})_{C,D} &= \overline{p_C(u_j)_D},\\
(W_{i,j})_{C,D} &= \overline{(u_i)_C(u_j)_D},\\
(S_{ik,j})_{C,D} &= \overline{(u_i)_C(u_k)_C(u_j)_D},\\
(S_{i,kj})_{C,D} &= \overline{(u_i)_C(u_k)_D(u_j)_D},\\
(F_{i,j})_{C,D} &= \overline{(v_i)_C(u_j)_D},\\
(G_{i,j})_{C,D} &= \overline{(u_i)_C(v_j)_D},
\end{aligned} \qquad (17)$$

where p denotes the pressure which is not an index such as dummy j or i, and therefore, the summation should not be used to p.

The terms of $\overline{(\varepsilon_{\mathrm{jkl}}\Omega_k u_l)_D(u_i)_C}$, $\overline{(\varepsilon_{\mathrm{ikl}}\Omega_k u_l)_C(u_j)_D}$, $\overline{(\Omega_i u_i \eta_i)_C(u_j)_D}$, and $\overline{(\Omega_j u_j \eta_j)_D(u_i)_C}$ form the correlation tensors of second order those can be represented by $M_{i,j}$, $N_{i,j}$, $H_{i,j}$, and $L_{i,j}$, accordingly, while $\overline{(a_{\mathrm{jklm}}\varepsilon_{lm})_D(u_i)_C}$ and $\overline{(I_{jk}a_{lm}\varepsilon_{lm})_D(u_i)_C}$ form the correlation tensor of third order

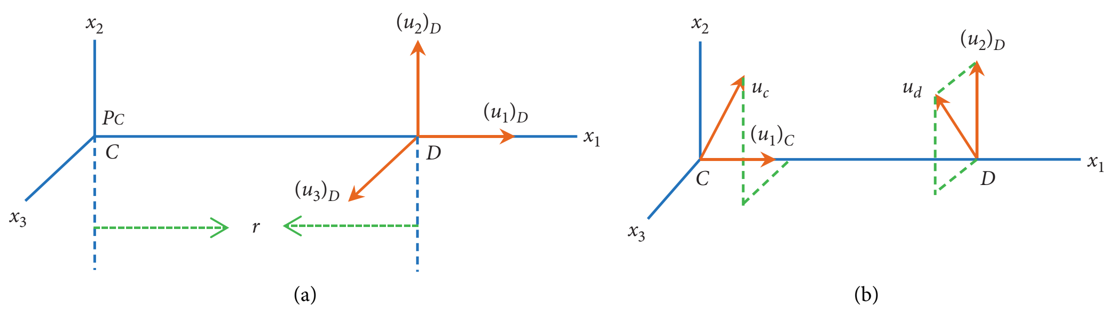

FIGURE 1: Correlations between C and D points. (a) The pressure at C and the velocity at D. (b) Velocities u_c at C and u_d at D.

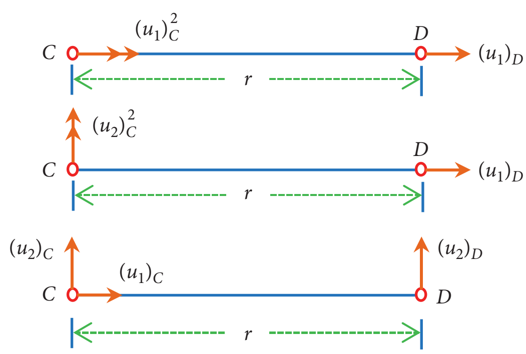

FIGURE 2: Triple correlations for the velocity at C and D.

and expressed as $Q_{i,jk}$ and $R_{i,jk}$, respectively. Consequently, the followings are obtained:

$$\begin{aligned}
\left(M_{i,j}\right)_{C,D} &= \overline{\left(\varepsilon_{ikl}\Omega_k u_l\right)_C \left(u_j\right)_D},\\
\left(N_{i,j}\right)_{C,D} &= \overline{\left(\varepsilon_{jkl}\Omega_k u_l\right)_D \left(u_i\right)_C},\\
\left(D_{i,jk}\right)_{C,D} &= \overline{\left(u_i\right)_C \left(a_{\mathrm{jklm}}\varepsilon_{lm}\right)_D},\\
\left(H_{i,j}\right)_{C,D} &= \overline{\left(\Omega_i u_i \eta_i\right)_C \left(u_j\right)_D},\\
\left(L_{i,j}\right)_{C,D} &= \overline{\left(\Omega_j u_j \eta_j\right)_D \left(u_i\right)_C},\\
\left(Q_{ik,j}\right)_{C,D} &= \overline{\left(a_{\mathrm{iklm}}\varepsilon_{lm}\right)_C \left(u_j\right)_D},\\
\left(R_{i,jk}\right)_{C,D} &= \overline{\left(u_i\right)_C \left(I_{jk} a_{lm}\varepsilon_{lm}\right)_D},\\
\left(R_{ik,j}\right)_{C,D} &= \overline{\left(I_{ik} a_{lm}\varepsilon_{lm}\right)_C \left(u_j\right)_D}.
\end{aligned} \tag{18}$$

Using the above correlation terms, equation (16) yields

$$\begin{aligned}
&\frac{\partial}{\partial t}W_{i,j} - \frac{\partial}{\partial \psi_k}S_{ik,j} + \frac{\partial}{\partial \psi_k}S_{i,kj} = -\frac{1}{\rho}\left(-\frac{\partial}{\partial \psi_i}K_{p,j} + \frac{\partial}{\partial \psi_j}K_{i,p}\right) + 2v\frac{\partial^2}{\partial \psi_k \partial \psi_k}W_{i,j} + f\left(F_{i,j} - 2W_{i,j} + G_{i,j}\right)\\
&\quad - 2\left[\left(M_{i,j} + N_{i,j}\right) + \left(H_{i,j} + L_{i,j}\right)\sin\theta\right] + \frac{\mu_f}{\rho}\frac{\partial}{\partial \psi_k}\left[\left(Q_{i,jk} - Q_{ik,j}\right) + \frac{1}{3}\left(R_{ik,j} - R_{i,jk}\right)\right],
\end{aligned} \tag{19}$$

where the terms of correlations used in equation (19) are to the considering points, C and D.

The double pressure-velocity correlations are zero for incompressible and isotropic turbulence, which means

$$\begin{aligned}
\left(k_{p,j}\right)_{C,D} &= 0,\\
\left(k_{i,p}\right)_{C,D} &= 0.
\end{aligned} \tag{20}$$

In the field of isotropic turbulence, the invariance condition under the reflection to C is followed by

$$\begin{aligned}
&\overline{\left(u_i\right)_C \left(u_k\right)_D \left(u_j\right)_D} = -\overline{\left(u_k\right)_C \left(u_j\right)_C \left(u_i\right)_D},\\
&\text{or}\\
&\left(s_{i,kj}\right)_{C,D} = -\left(s_{kj,i}\right)_{C,D}.
\end{aligned} \tag{21}$$

Thus, the simplification of equation (19) gives

$$\begin{aligned}
&\frac{\partial}{\partial t}W_{i,j} - \frac{\partial}{\partial \psi_k}\left(S_{ik,j} + S_{kj,i}\right) = 2v\frac{\partial^2}{\partial \psi_k \partial \psi_k}W_{i,j} - 2\left[\left(M_{i,j} + N_{i,j}\right) + \left(H_{i,j} + L_{i,j}\right)\sin\theta\right]\\
&\quad + f\left(F_{i,j} - 2W_{i,j} + G_{i,j}\right) + \frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial \psi_k}\left(Q_{jk,i} + Q_{ik,j}\right) + \frac{1}{3}\frac{\partial}{\partial \psi_k}\left(R_{ik,j} + R_{jk,i}\right)\right].
\end{aligned} \tag{22}$$

Table 1: Equation of motions for turbulent flow in second-order tensors of pressure-velocity correlation.

Turbulent motion	Considered parameters	References
$\frac{\partial}{\partial t}W_{i,j} - S_{i,j} = 2[\upsilon\partial^2/\partial\psi_k\partial\psi_k W_{i,j} - X_{i,j} - Y_{i,j}\sin\theta] + f(F_{i,j} - 2W_{i,j} + G_{i,j}) - \mu_f/\rho(Q_{i,j} - 1/3R_{i,j})$	(i) Momentum for turbulent flow (ii) Fibre suspensions (iii) Dust particles (iv) Rotating frame	This study
$\partial/\partial t W_{i,j} - S_{i,j} = 2[\upsilon\partial^2/\partial\psi_k\partial\psi_k W_{i,j} - X_{i,j}] + f(F_{i,j} - 2W_{i,j} + G_{i,j}) - \frac{\mu_f}{\rho}(Q_{i,j} - 1/3R_{i,j})$	(i) Momentum for turbulent flow (ii) Fibre suspensions (iii) Dust particles	Ahmed et al. [40]
$\partial/\partial t W_{i,j} - S_{i,j} = 2[\upsilon\partial^2/\partial\psi_k\partial\psi_k W_{i,j} - X_{i,j} - Y_{i,j}\sin\theta] + f(F_{i,j} - 2W_{i,j} + G_{i,j})$	(i) Momentum for turbulent flow (ii) Dust particles (iii) Rotating frame	Ahmed [15]
$\partial/\partial t W_{i,j} - S_{i,j} = 2[\upsilon\partial^2/\partial\psi_k\partial\psi_k W_{i,j} - X_{i,j} - Y_{i,j}\sin\theta]$	(i) Momentum for turbulent flow (ii) Rotating frame	Ahmed [32]
$\partial/\partial t W_{i,j} - S_{i,j} = 2\upsilon\partial^2/\partial\psi_k\partial\psi_k W_{i,j}$	(iii) Motion for turbulent flow	Hinze and Uberoi [2]

The terms $\partial/\partial\psi_k(Q_{jk,i} + Q_{ik,j})$, $\partial/\partial\psi_k(S_{ik,j} + S_{kj,i})$, $\partial/\partial\psi_k(R_{ik,j} + R_{jk,i})$, $(M_{i,j} + N_{i,j})$, and $(H_{i,j} + L_{i,j})$ form tensors of second order that can be represented by $Q_{i,j}$, $S_{i,j}$, $R_{i,j}$, $X_{i,j}$, and $Y_{i,j}$, respectively, and expressed as

$$\begin{aligned} Q_{i,j} &= \frac{\partial}{\partial\psi_k}\left(Q_{jk,i} + Q_{ik,j}\right), \\ S_{i,j} &= \frac{\partial}{\partial\psi_k}\left(S_{ik,j} + S_{kj,i}\right), \\ R_{i,j} &= \frac{\partial}{\partial\psi_k}\left(R_{ik,j} + R_{jk,i}\right), \\ X_{i,j} &= \left(M_{i,j} + N_{i,j}\right), \\ Y_{i,j} &= \left(H_{i,j} + L_{i,j}\right). \end{aligned} \tag{23}$$

Therefore, equation (23) yields

$$\begin{aligned} \frac{\partial}{\partial t}W_{i,j} - S_{i,j} &= 2\left[\upsilon\frac{\partial^2}{\partial\psi_k\partial\psi_k}W_{i,j} - X_{i,j} - Y_{i,j}\sin\theta\right] \\ &\quad + f\left(F_{i,j} - 2W_{i,j} + G_{i,j}\right) - \frac{\mu_f}{\rho}\left(Q_{i,j} - \frac{1}{3}R_{i,j}\right). \end{aligned} \tag{24}$$

This newly derived equation expresses the energy motion of dusty rotating turbulent flow of fibre suspensions in 2nd order tensors.

4. Results and Discussion

The energy motion equation of the dusty turbulent flow of fibre suspension (19) was derived in a rotational frame. This new equation was developed in the 2nd order tensors of pressure-velocity and velocity-velocity correlation at any two points C and D of the fluid flow field. All of these tensors were expressed as coordinates of time, space, and distance. The present model was compared to the relevant existing models to assess its feasibility which is summarized in Table 1.

Dynamic behaviours of the particles in turbulent flow fields are a major concern related to the applications. Fibre suspensions in a fluid flow or gas have been known for some time to reduce the shear stress generated by the motion of the fluid through a solid surface under the conditions of turbulent flow. In absence of dust particles into the flow domain, $f = 0$, and consequently, the newly developed equation (24) becomes

$$\begin{aligned} \frac{\partial}{\partial t}W_{i,j} - S_{i,j} &= 2\left[\upsilon\frac{\partial^2}{\partial\psi_k\partial\psi_k}W_{i,j} - X_{i,j} - Y_{i,j}\sin\theta\right] \\ &\quad - \frac{\mu_f}{\rho}\left(Q_{i,j} - \frac{1}{3}R_{i,j}\right). \end{aligned} \tag{25}$$

Equation (25) signifies the energy motion for turbulent flow of fibre suspensions in a rotational frame in the tensors of 2nd order. Here, the correlation tensors, $S_{i,j}, W_{i,j}, X_{i,j}, Y_{i,j}, Q_{i,j}, F_{i,j}, G_{i,j}$, and $R_{i,j}$ express the velocity relationships at any two points C and D (Figure 2) in the flow domain. The tensors, $S_{i,j}$ and $W_{i,j}$, represent the fluid velocity correlations, $Q_{i,j}$ and $R_{i,j}$, indicate the correlations between the suspending fibres and fluid velocities, $F_{i,j}$ and $G_{i,j}$, describe the velocity correlations between fluid and dust particles while $X_{i,j}$ and $Y_{i,j}$ defines the correlations between fluid velocities, and angular velocities between fluid and fibre formed due to the rotation.

Fibre suspensions in the fluid have been known sometimes to reduce the shear stress generated by the motion of the fluid through a solid surface under the conditions of turbulent flow. If fibres are not available in the fluid flow domain, the apparent viscosity $\mu_f = 0$ for suspending fluid; equation (25) thus yields the form

$$\frac{\partial}{\partial t}W_{i,j} - S_{i,j} = 2\left[\upsilon\frac{\partial^2}{\partial\psi_k\partial\psi_k}W_{i,j} - X_{i,j} - Y_{i,j}\sin\theta\right]. \quad (26)$$

Equation (26) expresses the energy motion of turbulent flow in the correlation tensors in a rotating frame.

Owing to the broad applications to both geophysical and engineering fluid mechanics, turbulent flow is commonly researched in the rotational frame, for example, in combustion systems, rotary shafts, rotational heat exchangers, turbine blade passages, centres of the nuclear reactor, rotated heat exchangers, combustion systems, and rotor shafts. In almost all of these complicated systems, a coordinate with nonrotation is usually used to investigate the problems. For these types of rotating systems, the Coriolis effect performs a critical role in the fluid flow field as mentioned earlier. In the nonrotational frame, the Coriolis effects do not produce in the fluid flow field, which means the term due to the Coriolis effect $Y_{i,j}$ vanishes, and therefore, equation (26) gives

$$\frac{\partial}{\partial t}W_{i,j} - S_{i,j} = 2\left[\upsilon\frac{\partial^2}{\partial\psi_k\partial\psi_k}W_{i,j} - X_{i,j}\right]. \quad (27)$$

Equation (27) signifies the energy motion for turbulent flow of 2nd-order correlation tensors.

If no energy dissipation generated by turbulence, $X_{i,j} = 0$, therefore, equation (27) yields

$$\frac{\partial}{\partial t}W_{i,j} - S_{i,j} = 2\upsilon\frac{\partial^2}{\partial\eta_k\partial\eta_k}W_{i,j}. \quad (28)$$

Equation (28) describes the turbulence motion in the 2nd-order tensors which was obtained by Hinze and Uberoi [2].

5. Conclusion and Future Directions

A new energy motion equation was derived for dusty rotating turbulent flows of fibre suspensions using the method of averaging. Fibre suspension in rotating turbulent flow was taken into account in presence of solid (dust) particles to develop the present theoretical model in second-order correlation tensors. The turbulent fibre suspension flow was subjected to both average fluid motion and random motion caused by fluctuating fluid velocity components. Comparing the present theoretical model to previous relevant models revealed a good agreement, as seen in Table 1, in comparison to previous models, the present model represents a significant improvement as it considers all of the potential influential parameters that could affect the motion of turbulent energy, for instance, dust particles, suspending particles (fibres), and rotating frame. However, the impact of these parameters on turbulence energy motion needs to be assessed to measure the performance of the turbulence system used in several industries.

The present model should be optimized in future studies to identify the most influential parameters affecting the turbulence systems. However, taking into account the nonlinearity, discrete nature, constrained parameters, and boundary conditions of turbulence systems, it is extremely difficult to design the most suitable model that accurately represents reality. Considering all of these difficulties, the future study can develop a physical model for a turbulent channel flow that depicts the modeled system in its physical form through both experimental and numerical studies. The difference between angular velocities of fluid and fibre at the fibre centre of mass should be calculated by estimating the slip spin, which enables a direct comparison of fibre rotation as well as fibre orientation with the carrier flow's local parameters. The findings of the future study could provide physical mechanisms underlying fibre rotation in the closed region of the channel walls, emphasizing how particularly fibres spin in relation to the fluid when swept or ejected. Future research in this field can significantly contribute to natural sciences, technologies, and industries.

Nomenclature

$u_i(x,t)$:	Fluid velocity components ($\mathrm{m\ s^{-1}}$)
ρ:	Fluid density ($\mathrm{kg\ m^3}$)
$p(x,t)$:	Pressure (Pa)
υ:	Kinematical viscosity of suspending fluids ($\mathrm{m^2\ s^{-1}}$)
ε_{ijl}:	Three-dimensional permutation symbol
Ω_j:	Rotation vector
ε:	Dissipation of turbulence
x:	Position (m)
$v_i(x,t)$:	Dust (solid) particle velocity components ($\mathrm{m\ s^{-1}}$)
$m_s = 4/3\pi R_s^3\rho_s$:	Dust particle mass (kg)
R_s:	Dust particle radius
$K = 6\pi R_s\rho v$:	Stoke's formula for the drag
$KN/\rho = f$:	Frequency dimension
N:	Dust particle density number
μ_f:	Apparent viscosity of fibre suspension ($\mathrm{kg\ m^{-1}s^{-1}}$)
I_{ij}:	Turbulent intensity of fibre suspension
$\varepsilon_{lm} = 1/2(\partial u_l/\partial x_m + \partial u_m/\partial x_l)$:	Tensor strain rate
a_{lm}:	Second-order tensors for fibre orientation
a_{ijlm}:	Fourth-order tensors for fibre orientation
$-2(\Omega_i u_i \eta_i)\sin\theta = -2(\overline{\Omega}\times\overline{u})$:	Coriolis force
η:	Unit vector normal to $\overline{u}$ and $\overline{\Omega}$
Ω_i:	Angular velocity
θ:	Angle makes between $\overline{u}$ and $\overline{\Omega}$
C, D:	Any two points in the turbulent flow field
c, d:	Directions along with the points C and D, respectively
u_c, u_d:	Components of the velocity along with C and D directions

$\psi_k = (x_k)_D - (x_k)_C$: Considered independent variables
r: Distance between any two points C and D in the flow filed
t: Time (s)
$S_{i,j}$, $W_{i,j}$: Fluid velocity correlations
$Q_{i,j}$, $R_{i,j}$: Correlations between the suspending fibres and fluid velocities
$F_{i,j}$, $G_{i,j}$: Velocity correlations between fluid and dust particles
$X_{i,j}$: Correlations between the velocities of fluid and fibre
$Y_{i,j}$: Angular velocities between fluid and fibre formed due to the rotation.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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