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Research Article

Performance Analysis in Production Systems with Uncertain Data: A Stochastic Data Envelopment Analysis Approach

Seyedeh Fatemeh Bagheri,[1] **Alireza Amirteimoori** [iD],[1] **Sohrab Kordrostami** [iD],[2] **and Mansour Soufi** [iD][3]

[1]*Department of Applied Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran*
[2]*Department of Applied Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran*
[3]*Department of Industrial Management, Rasht branch, Islamic Azad University, Rasht, Iran*

Correspondence should be addressed to Alireza Amirteimoori; aamirteimoori@gmail.com

The problem of determining an optimal benchmark to inefficient decision-making units (DMUs) is an important issue in the field of performance analysis. Previous methods for determining the projection points of inefficient DMUs have only focused on one objective and other features have been ignored. This paper attempts to determine the best projection point for each DMU when the inputs and outputs data are in stochastic form and presents an alternative definition for the best projection by considering three main aspects: technical efficient, minimal cost, and maximal revenue as much as possible. Considering the important role of the electricity industry in the economic growth of each country, a practical example has been implemented on 16 regional electricity companies in Iran in 9 consecutive periods. The efficiency score along with the projection points of the three technical models (BCC model of Banker et al. (1984)), cost, and stochastic revenue are compared with the projection point obtained from the model presented in this article, which simultaneously meets these three objectives, showing the improvement of companies' performance.

1. Introduction

Data envelopment analysis (DEA) was first introduced by Charnes et al. [1] (CCR model) to calculate the relative efficiency of a set of homogeneous decision-making units (DMU) and extended by Banker et al. [2] (BCC model). Traditional DEA models (BCC and CCR models) rely on past information and they are developed in deterministic case, and hence the uncertainty and stochasticity of the data are ignored. In the real world, we are faced with many decisions, most of which are associated with uncertainty. Stochastic data envelopment analysis models to address these shortcomings were proposed by Charnes and Cooper [3]. In the last two decades, performance analysis in an uncertain environment has been studied frequently. See for instances Hossain et al. [4], Mo et al. [5], Rong et al. [6], Amirteimoori et al. [7], and Ghasemi et al. [8].

Some of the differences between DEA and stochastic DEA (SDEA) models have been presented in Table 1. SDEA method is based on stochastic programming methods, which is one of the branches of mathematical programming. Stochastic programming models are divided into E-model and P-model in terms of the type of objective function. E-model based stochastic programming models are used to obtain the expected value or mathematical expectation that the objective function of such models does not include a random variable. To obtain the highest probability of an event occurring, stochastic programming models based on P-model are used so that in the objective function of such models, a random variable is used possibly. Charnes and Cooper [9] proposed a stochastic programming model with potential constraints. They were then introduced by Land et al. [10] E-model-based stochastic programming model. Programming models based on E-model are used to obtain

TABLE 1: Differences between models.

DEA model	SDEA model
The values of inputs and outputs are definite.	The values of inputs and outputs are random.
No accidental error is entered in the model	Random error enters the model in the form of a random component
The model offers only past performance	Allows you to predict performance in the future
The efficiency frontier is very sensitive to small changes in inputs and outputs.	The efficiency frontier is less sensitive to changes in inputs and outputs.
The error level in such models is considered zero.	Efficiency is defined according to the level of error in the model.
In the final model, the correlation between inputs and outputs is not considered.	The correlation between inputs and outputs is considered in such models.

the expected value. The objective function of these models does not include random variables.

The efficiency measure in which, in addition to input and output data, the price of the input of the DMUs is also required and its goal is cost minimization, called cost efficiency, was introduced by Farrell [11]. Färe et al. [12] developed cost efficiency. By studying data envelopment analysis and stochastic cost functions, Cooper and Ton [13] identified some specific problems in stochastic frontier analysis (SFA). Stochastic frontier analysis method is a parametric method. Shiraz et al. [14] formulated the stochastic cost model in a nonlinear form.

Färe et al. [12] have played a key role in the development of revenue efficiency. In addition to input and output data, they have also paid attention to the output of the decision-making unit aiming at maximizing revenue. Lin [15] proposed a method to set revenue targets at the efficient frontier. Shiraz et al. [14] also presented a random income model that is nonlinear.

In data envelopment analysis, DMUs are divided into two categories: efficient and inefficient units. Inefficient units can be efficient by decreasing their inputs level and increasing their outputs level.

Achieving optimal scale size (OSS) has always been of interest to researchers. Cesaroni and Giovannola [16] introduced a new definition of optimal scale size based on minimizing the cost of each unit so that average cost productivity combines scale productivity and allocation and generalizes economic scale size in productivity analysis. In the following, Haghighatpisheh et al. [17] proposed a new definition of optimal scale size that uses both of the cost of inputs and the revenue of outputs. This measure of average cost-revenue efficiency (ACRE) shows the ratio of profitability efficiency to average productivity.

The process of industrialization of developed countries shows proper planning, optimal use of resources, and determining the appropriate pattern as the main goal in the development process of these countries. The electricity industry has an infrastructural and influential role on the economic growth of any country. In recent years, per capita electricity consumption in Iran has increased significantly in parallel with its production rate. Accordingly, efficiency and determining the appropriate model for companies has always been the concern of managers in the electricity industry. In this article, a real and useful example has been

done on Iran's regional electricity distribution companies and it has used random data that can play an important role as a vision in the electricity industry and solving future challenges.

Most studies to find the projection point of each inefficient unit have focused on only one goal. For example, the projection point obtained from the cost model finds an efficient point on efficient frontier with the cost minimization approach. The projection point obtained from the revenue model moves towards the frontier with the revenue maximization approach. Bagheri et al. [18] presented a method that simultaneously tried to meet the projection point of each unit from three perspectives, cost minimization, revenue maximization, and the shortest distance to the efficient frontier as much as possible and it was proved that the efficiency of the model obtained for inefficient units is higher than the unit under evaluation. The results of solving this method for managers to make decisions were based on past information, so we tried to use stochastic programming. In this paper, for each inefficient unit, a projection point is obtained, which is very important in two stages. First, the resulting projection for each unit meets as many as possible the three technical, cost, and revenue concepts. Secondly, random data and stochastic programming in linear form are used in it. Therefore, it will greatly help managers to make better decisions in the future and improve the performance of units.

This paper has been organized as follows: technical, cost, and revenue efficiency with deterministic and random data are examined in Sections 2 and 3, respectively. The random models of the linearly shaped models have been presented in Section 3. In Section 4, the method proposed by Bagheri et al. [18] to determine the ideal projection point has been implemented this time in stochastic programming. In Section 5, we will provide a real and practical example, and finally in Section 6, the conclusion of the article is presented.

2. Deterministic DEA Models

In this section, we briefly review some basic DEA models in deterministic environment. Technical efficiency measure was first introduced by Debreu [19] and Farrell [11]. The traditional DEA-based models CCR model of Charnes et al. [1] and its subsequent extension BCC model of Banker et al. [2] was developed on classical efficiency analysis model of Farrell [11]. The BCC model for evaluating a specific DMU_p

is formulated in the input-oriented deterministic environment as follows:

$$\text{Min}\,\theta$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{ip}, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}, \quad r = 1, \ldots, s, \qquad (1)$$

$$\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0,\, j = 1, \ldots, n,$$

$$\theta_p \text{free variable.}$$

In the above model, θ is the input abatement factor. DMU_p is said to be technically efficient if and only $\theta = 1$ and all slack variables are equal to zero. Corresponding to each inefficient point, a frontier point is determined by reducing inputs. Removing the third convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$ leads to CCR model.

For each inefficient unit, there is one or more efficient units on the efficient frontier that is called the reference set. The reference set of a specific inefficient DMU (denoted by E_o) is defined as follows:

$$E_o = \left\{ j \mid \lambda_j^* > 0 \right\} (j \in \{1, \ldots, n\}). \qquad (2)$$

In technical efficiency models we have less information on inputs and outputs of DMUs. In other words, all we know the quantitative values of inputs and outputs and there is no information on inputs and outputs prices. More information will be available if we also have the costs of the inputs and the prices of the outputs. In this case, we are motivated to calculate the allocative efficiency of the DMUs. We first introduce cost allocative model. Suppose that $C = (c_1, c_2, \ldots, c_m)^t$ is the vector of input prices and $\overline{X} = (\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_m)^t$ is cost minimizing vector of input quantities. Farrell (1957) proposed a measure of cost efficiency. The cost efficiency model in VRS environment is formulated as follows:

$$\text{Min} \sum_{i=1}^{m} c_i \overline{x}_i,$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \overline{x}_i, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}, \quad r = 1, \ldots, s, \qquad (3)$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad \lambda_j \geq 0, \quad j = 1, \ldots, n,$$

The cost efficiency of DMU_p is defined as the ratio of the optimal cost to the actual cost as follows:

$$CE_p = \frac{\sum_{i=1}^{m} c_i \overline{x}_i}{\sum_{i=1}^{m} c_i x_{ip}}. \qquad (4)$$

Clearly, $CE_p \leq 1$ and when this score is equal to one, then DMU_p is called cost efficient, otherwise we say DMU_p is cost inefficient.

Now suppose that we are given information on the price of the outputs. In this case, the revenue efficiency model in VRS environment is formulated as follows:

$$\text{Max} \sum_{r=1}^{s} p_r \overline{y}_r,$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ip}, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq \overline{y}_r, \quad r = 1, \ldots, s, \qquad (5)$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad \lambda_j \geq 0,\, j = 1, \ldots, n.$$

In the above equation, $P = (p_1, p_2, \ldots, p_s)^t$ is vector of output prices and $\overline{Y} = (\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_s)^t$ is revenue maximizing vector of output quantities.

The revenue efficiency of DMU_p is defined as the ratio of optimal revenue to the actual revenue, i.e., $RE_p = \sum_{r=1}^{s} p_r \overline{y}_r / \sum_{r=1}^{s} p_r y_{rp}$.

It is easy to see that $RE_p \geq 1$. If this score is equal to one, then DMU_p is called revenue efficient, otherwise we say DMU_p is revenue inefficient.

3. Basic DEA Models in Stochastic Environment

As we stated before, in many real applications, we often encounter uncertainty, so it is necessary to generalize the models to the uncertainty mode, especially the random mode. SDEA models were first proposed by Charnes and Cooper (1959). Land et al. (1993) introduced the CCR model in the stochastic mode considering the inputs and outputs and estimating the probability distribution prevailing over them, after estimating the efficiency in the future, that by considering the convexity condition, the BCC model is obtained randomly in the nature of the input. In this model, we show the random input and output values by $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$, respectively. These values show the mathematical expectations of random inputs and outputs. Also, a_{ij} and b_{rj} are the standard deviations of random inputs and outputs, respectively. Using a random variable error structure, Cooper et al. [20] proposed the following input-oriented DEA model:

$$\text{Min}\,\theta$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} - \varnothing^{-1}(\propto)\sigma(p_i^+ + p_i^-) \le \theta x_{ip}, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{ip} = p_i^+ - p_i^-, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} + \varnothing^{-1}(\propto)\sigma(q_r^+ + q_r^-) \ge y_{rp}, \quad r = 1, \ldots, s, \qquad (6)$$

$$\sum_{j=1}^{n} \lambda_j b_{rj} - \theta b_{rp} = q_r^+ - q_r^-, \quad r = 1, \ldots, s,$$

$$\sum_{j=1}^{n} \lambda_j = 1\lambda_j, \quad \ge 0, j = 1, \ldots, n,$$

$$p_i^+, p_i^- \ge 0, \quad i = 1, \ldots, m, \; q_r^+, q_r^- \ge 0, r = 1, \ldots, s.$$

The first two constraints in model (4) are related to input variables and the next two constraints are related to output variables. The goal is to minimize the level of inputs by the abatement factor θ. σ has a standard normal distribution and $\varnothing^{-1}$ shows the inverse of the standard normal distribution function. The positive deviation variables $p_i^+, p_i^-, q_r^+, q_r^-$ have been used to transform the model into a linear form.

Corresponding to each inefficient DMU, the projection point was obtained from the above stochastic model through the formula $(\sum_{j=1}^{n} \lambda_j x_{ij}, \sum_{j=1}^{n} \lambda_j y_{rj})$.

Producing a certain level of outputs from the least cost of inputs is called cost efficiency, in which information about the price of inputs is considered. In the following, we present the cost efficiency model in stochastic environment. Inspired by the deterministic form of a stochastic model, we assume that c_i is the i-th input price whose value is constant for all units. Also, $\overline{x_i}$ is considered to be the ideal input whose value is unknown.

Suppose that x_{ij} and y_{rj} are the mathematical expectations of the inputs and outputs of $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$, respectively. Moreover, suppose a_{ij} and b_{rj} are their corresponding standard deviations. The cost efficiency model is formulated as follows:

$$\text{Min}\, \sum_{i=1}^{m} c_i \overline{x}_i,$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} - \varnothing^{-1}(\propto)\sigma(p_i^+ + p_i^-) \le \overline{x}_i, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{ip} = p_i^+ - p_i^-, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} + \varnothing^{-1}(\propto)\sigma(q_r^+ + q_r^-) \ge y_{rp}, \quad r = 1, \ldots, s,$$

$$\sum_{j=1}^{n} \lambda_j b_{rj} - b_{rp} = q_r^+ - q_r^-, \quad r = 1, \ldots, s,$$

$$\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \ge 0, \quad j = 1, \ldots, n,$$

$$p_i^+, p_i^- \ge 0, \quad i = 1, \ldots, m, \; q_r^+, q_r^- \ge 0, r = 1, \ldots, s. \qquad (7)$$

In this model, the objective is to minimize the total cost of inputs. This model is formulated under variable returns to scale and the cost efficiency of DMU_p is defined as follows:

$$CE_p = \frac{\sum_{i=1}^{m} c_i \overline{x}_i}{\sum_{i=1}^{m} c_i x_{ip}}. \qquad (8)$$

Corresponding to each inefficient point, the efficient projection point is obtained from the above stochastic model through the formula $(\sum_{j=1}^{n} \lambda_j x_{ij}, \sum_{j=1}^{n} \lambda_j y_{rj})$. It can be easily shown that the obtained projection point is efficient at the desired confidence level. σ has a standard normal distribution and $\varnothing^{-1}$ shows the inverse of the standard normal distribution function in the above linear model.

Now, we propose the revenue efficiency model. As it is known in DEA literature, producing the highest level of outputs from a certain level of inputs is called revenue efficiency. Assume that w_r is the r-th output price whose value is constant for all units. Also, $\overline{y}_r$ is considered to be the r-th optimal output value whose value is unknown. Suppose that x_{ij} and y_{rj} are the mathematical expectations of the inputs and outputs of $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$, respectively. Also, a_{ij} and b_{rj} are the standard deviations of the inputs and outputs $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$, respectively. The revenue efficiency model is formulated in a linear form in a random environment as follows:

$$\text{Max}\, \sum_{r=1}^{s} w_r \overline{y}_r,$$

$$s.t: \sum_{j=1}^{n} \lambda_j x_{ij} - \varnothing^{-1}(\propto)\sigma(p_i^+ + p_i^-) \le x_{ip}, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{ip} = p_i^+ - p_i^-, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} + \varnothing^{-1}(\propto)\sigma(q_r^+ + q_r^-) \ge \overline{y}_r, \quad r = 1, \ldots, s,$$

$$\sum_{j=1}^{n} \lambda_j b_{rj} - b_{rp} = q_r^+ - q_r^-, \quad r = 1, \ldots, s,$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad \lambda_j, \ge 0, \quad j = 1, \ldots, n,$$

$$p_i^+, p_i^- \ge 0, \quad i = 1, \ldots, m \; q_r^+, q_r^- \ge 0, r = 1, \ldots, s. \qquad (9)$$

The first two constraints in the above model are related to input variables and the next two constraints are related to output variables. In this model, the goal is to maximize the revenue of outputs. This model is formulated under variable returns to scale and its efficiency score is obtained from the following formula.

$$RE_p = \frac{\sum_{r=1}^{s} w_r \overline{y}_r}{\sum_{r=1}^{s} w_r y_{rp}}. \tag{10}$$

The projection point is obtained from the above linearized random revenue model through the formula $(\sum_{j=1}^{n} \lambda_j x_{ij}, \sum_{j=1}^{n} \lambda_j y_{rj})$. It can be easily proved that the obtained projection point is efficient at the desired confidence level. σ has a standard normal distribution and $\varnothing^{-1}$ shows the inverse of the standard normal distribution function in the above linear model.

4. The Best Projection Point

In this section, the best projection point in a random environment is determined. As you know, there is an ideal pattern for any inefficient unit. The ideal pattern means the projection point obtained as each decision unit. Bagheri et al. [18] presented a method in which an ideal model was obtained for each unit, which first dominates the unit under evaluation. Therefore, it will be much more efficient. Secondly, in the convex combination of projection points obtained from cost, revenue, and technical models, this projection point is simultaneously examined from three technical, minimum cost, and maximum revenue perspectives. One of the defects of such models is that they rely on past information, so they cannot provide the desired result.

One way to solve this problem is to use random data envelopment analysis. In such models, a random error is added to the model in the form of a random component, in which the efficiency is defined according to the error level ($\propto$.) and the correlation between the input and output variables is taken into account in the model. Toward this end, we proceed as follows:

Step 1: First, we obtain the projection points of the technical, cost, and random revenue models in linear form under the variable returns to scale (models (4)–(6)), which were described in the previous section, naming (x_{ip}^T, y_{rp}^T), (x_{ip}^C, y_{rp}^C), and (x_{ip}^R, y_{rp}^R), such that random cost and revenue models in linear form are an innovation of this article.

Step 2: Then, using the proposed model, we introduce the point distance $(\overline{x}_i, \overline{y}_r)$ from each of the projection points of the BCC, cost, and revenue linearized random models (models (4)–(6)), which are represented by d_1, d_2, d_3, respectively, and minimized under norm 2 in the objective function of the model (8) so that this point of the new projection is located in the convex shell of these points and at the same time overcome the unit under its evaluation. Therefore, the obtained model will be more efficient for each inefficient unit and also more efficient than the unit under evaluation.

$$\text{Min} \, d_1 + d_2 + d_3$$

$$\sqrt{\sum_{i=1}^{m} \left(\overline{x}_i - x_{ip}^T\right)^2 + \sum_{r=1}^{s} \left(\overline{y}_r - y_{rp}^T\right)^2} = d_1, \quad j = 1, \ldots, n,$$

$$\sqrt{\sum_{i=1}^{m} \left(\overline{x}_i - x_{ip}^C\right)^2 + \sum_{r=1}^{s} \left(\overline{y}_r - y_{rp}^C\right)^2} = d_2, \quad j = 1, \ldots, n,$$

$$\sqrt{\sum_{i=1}^{m} \left(\overline{x}_i - x_{ip}^R\right)^2 + \sum_{r=1}^{s} \left(\overline{y}_r - y_{rp}^R\right)^2} = d_3, \quad j = 1, \ldots, n,$$

$$\lambda_1 x_{ip}^T + \lambda_2 x_{ip}^C + \lambda_3 x_{ip}^R = \overline{x}_i, \quad i = 1, \ldots, m \, j = 1, \ldots, n,$$

$$\lambda_1 y_{rp}^T + \lambda_2 y_{rp}^C + \lambda_3 y_{rp}^R = \overline{y}_r, \quad r = 1, \ldots, s \, j = 1, \ldots, n,$$

$$\sum_{k=1}^{3} \lambda_k = 1,$$

$$d_1 \geq 0, d_2 \geq 0, d_3 \geq 0$$

$$\overline{x}_i \geq 0, \overline{y}_r \geq 0, \lambda_k \geq 0$$

$$\tag{11}$$

TABLE 2: Input and output variables.

Input variables	Output variables
(1) Fuel consumption in terms of million liters	
(2) Nominal power	(1) Sending energy to neighboring and overseas power companies at the transmission voltage level
(3) Length of transmission lines	
(4) Capacity of transfer posts	
(5) The length of the above distribution lines	(2) Selling energy to electricity distribution companies and industrial companies and the above distribution voltage level
(6) Capacity of the above distribution posts	

TABLE 3: The mean value inputs and outputs.

DMUs	Electricity distribution companies	The mean value of the first input	The mean value of the second input	The mean value of the third input	The mean value of the fourth input	The mean value of the fifth input	The mean value of the sixth input	The mean value of the first output	The mean value of the second output
1	Azerbaijan	2921	2776	3723	5808	4234	4574	7420	12211
2	Esfahan	3526	2204	3200	8406	5545	6700	6708	20752
3	Kermanshah	2898	2360	3975	7767	5562	5446	7849	13818
4	Tehran	7994	7682	4214	21400	4679	13864	9751	390100
5	Khorasan	3570	3538	1883	4166	8400	6913	2591	16434
6	Khuzestan	2906	2286	3691	12983	3874	8716	7274	28195
7	Zanjan	177	324	1170	3231	2382	2989	2270	6695
8	Semnan	109	269	1002	2401	859	1066	1527	2629
9	Sistan and Baluchestan	1127	1002	3716	2046	2935	2077	411	4468
10	The west	1609	1481	2958	4682	4369	3330	5012	6182
11	Fars	3144	3496	5321	10982	7933	8349	7729	16456
12	Kerman	1756	1858	4371	4967	4302	3490	3399	9577
13	Guilan	1628	1433	1115	3267	1348	2101	6110	4368
14	Mazandaran	2992	2212	2460	6449	3236	4523	5894	9501
15	Hormozgan	3126	2444	2110	6555	3522	4010	3678	9983
16	Yazd	818	930	1561	2514	1659	1868	1851	5294

TABLE 4: Standard deviation of inputs.

DMUs	Electricity distribution companies	Deviation of the first input criterion	Deviation of the second input criterion	Deviation of the third input criterion	Deviation of the fourth input criterion	Deviation of the fifth input criterion	Deviation of the sixth input criterion	Deviation of the first output criterion	Deviation of the second output criterion
1	Azerbaijan	1089.883	1237.542	215.0558	1070.038	270.2961	775.2903	831.979	1673.598
2	Esfahan	1498.915	705.8272	319.7671	1188.98	428.8438	995.109	933.7435	2313.444
3	Kermanshah	322.6686	0	289.0778	1296.611	561.172	693.5971	678.1084	1340.566
4	Tehran	2489.094	2504.389	83.29466	1720.807	249.0723	1123.076	1623.369	3402.231
5	Khorasan	881.8044	908.9224	431.5762	771.0019	502.0667	926.4356	1356.391	1607.083
6	Khuzestan	297.5013	217.1843	366.632	2131.656	276.4489	1200.906	2192.867	2713.754
7	Zanjan	237.1139	313.7117	335.8794	695.6687	375.4917	513.3965	681.8732	918.4726
8	Semnan	134.3652	264.0417	280.8594	654.7603	106.6865	160.7389	812.4838	315.395
9	Sistan and Baluchestan	196.5782	197.9394	496.7202	603.1526	300.5495	327.6523	185.0081	685.6501
10	The west	371.9556	476.3024	396.2865	1144.02	255.3703	425.3939	1682.792	763.4743
11	Fars	1379.946	1118.915	634.9756	2660.342	739.18	1423.841	4040.326	2041.423
12	Kerman	224.6264	256.3416	285.8916	713.4466	312.9601	314.8651	626.4647	1252.81
13	Guilan	632.6855	562.7966	150.9967	497.6073	109.0138	286.4961	697.3034	606.6424
14	Mazandaran	251.6188	6.63325	53.64699	924.1342	529.2353	893.8775	472.1165	1412.197
15	Hormozgan	118.1397	216	377.599	1084.704	1882.73	824.2087	667.4624	2207.234
16	Yazd	185.1756	195.287	315.0286	648.9068	73.67496	224.0982	903.4788	1074.818

TABLE 5: Technical efficiency numbers, cost and random revenue in three error levels.

DMUs	Electricity distribution companies	The stochastic efficiency of companies		
		$\propto$ = 0.05	$\propto$ = 0.3	$\propto$ = 0.5
1	Azerbaijan	TE = 1 CE = 1 RE = 3.29	TE = 1 CE = 1 RE = 3.49	TE = 1 CE = 1 RE = 3.54
2	Esfahan	TE = 0.91 CE = 0/62 RE = 3.46	TE = 0.88 CE = 0.59 RE = 3.55	TE = 0.86 CE = 0.57 RE = 3.60
3	Kermanshah	TE = 1 CE = 1 RE = 3.79	TE = 1 CE = 1 RE = 4.34	TE = 1 CE = 1 RE = 4.60
4	Tehran	TE = 1 CE = 1 RE = 1	TE = 1 CE = 1 RE = 1	TE = 1 CE = 1 RE = 1
5	Khorasan	TE = 0.78 CE = 0.39 RE = 2.08	TE = 0.77 CE = 0.36 RE = 2.13	TE = 0.76 CE = 0.35 RE = 2.15
6	Khuzestan	TE = 1 CE = 0.81 RE = 2.49	TE = 1 CE = 0.70 RE = 2.79	TE = 1 CE = 0.62 RE = 2.95
7	Zanjan	TE = 1 CE = 0.73 RE = 1	TE = 1 CE = 0.70 RE = 1	TE = 1 CE = 0.69 RE = 1
8	Semnan	TE = 1 CE = 1 RE = 1	TE = 1 CE = 1 RE = 1	TE = 1 CE = 1 RE = 1
9	Sistan and Baluchestan	TE = 1 CE = 0.55 RE = 1	TE = 1 CE = 0.53 RE = 1	TE = 1 CE = 0.52 RE = 1
10	The west	TE = 0.94 CE = 0.59 RE = 3.74	TE = 0.81 CE0.55 RE = 3.97	TE = 0.76 CE = 0.52 RE = 4.7
11	Fars	TE = 1 CE = 1 RE = 5.3	TE = 1 CE = 0.89 RE = 5.58	TE = 0.90 CE = 0.68 RE = 5.70
12	Kerman	TE = 0.63 CE = 0.44 RE = 4.15	TE = 0.61 CE = 0.43 RE = 4.24	TE = 0.60 CE = 0.42 RE = 4.27
13	Guilan	TE = 1 CE = 1 RE = 1	TE = 1 CE = 1 RE = 1.21	TE = 1 CE = 1 RE = 1.40
14	Mazandaran	TE = 0.80 CE = 0.56 RE = 4.84	TE = 0.70 CE = 0.53 RE = 5.01	TE = 0.65 CE = 0.51 RE = 5.03
15	Hormozgan	TE = 0.53 CE = 0.51 RE = 5.72	TE = 0.53 CE = 0.45 RE = 5.93	TE = 0.52 CE = 0.42 RE = 6.1
16	Yazd	TE = 1 CE = 0.77 RE = 1.02	TE = 1 CE = 0.75 RE = 1.07	TE = 1 CE = 0.75 RE = 1.09

TE: technical efficiency, CE: cost efficiency, and RE: revenue efficiency.

TABLE 12: Comparing ideal cost and revenue with observed one.

DMU	Electricity distribution companies	Observed cost	Observed revenue	$\propto = 0.05$		$\propto = 0.3$		$\propto = 0.5$	
				Ideal cost	Ideal revenue	Ideal cost	Ideal revenue	Ideal cost	Ideal revenue
1	Azerbaijan	64869	46682	64869	46682	64869	46682	64869	46682
2	Esfahan	82515	61628	49525.05	63197.13	48718.95	62461.3	47259.5	61873.59
3	Kermanshah	78727	51183	78731	51183	78731	51183	78731	51183
4	Tehran	163677	809453	163689	809453	163689	809453	163689	809453
5	Khorasan	72017	40641	25976.57	42635.91	26570.28	41505.5	26915.9	41074.78
6	Khuzestan	100157	78212	86018.87	81868.52	82519.32	82819.31	64110.27	82608.58
7	Zanjan	30236	20200	30236	20200	30236	20200	30236	20200
8	Semnan	17806	9839	17806	9839	17806	9839	17806	9839
9	Sistan and Baluchestan	35547	10169	35547	10169	35547	10169	35547	10169
10	The west	52068	27400	31034.99	29389.36	28884.55	28146.23	28863.41	27739.03
11	Fars	110172	56099	110172	56099	96631.43	210432.8	75343.34	56320.16
12	Kerman	58237	29351	24871.5	30382.8	25958.79	29965.9	26578.33	29753.85
13	Guilan	29348	27066	29348	27066	29348	27066	29348	27066
14	Mazandaran	60126	36684	35456.75	38660.06	30563.11	37062.47	30816.2	36767.75
15	Hormozgan	59832	31000	25772.51	32923.34	25486.15	31614.52	25352.1	31000
16	Yazd	26018	5553	22159.96	16913.13	22411.55	16627.34	22567.87	16509.86

Considering that the expression under the radical of the first three constraints is greater than or equal to zero, therefore, the model can be rewritten as follows:

$$
\text{Min} \, d_1 + d_2 + d_3
$$

$$
s.t. \sum_{i=1}^{m} \left(\overline{x}_i - x_{ip}^T \right)^2 + \sum_{r=1}^{s} \left(\overline{y}_r - y_{rp}^T \right)^2 = d_1^2, \quad j = 1, \ldots, n,
$$

$$
\sum_{i=1}^{m} \left(\overline{x}_i - x_{ip}^C \right)^2 + \sum_{r=1}^{s} \left(\overline{y}_r - y_{rp}^C \right)^2 = d_2^2, \quad j = 1, \ldots, n,
$$

$$
\sum_{i=1}^{m} \left(\overline{x}_i - x_{ip}^R \right)^2 + \sum_{r=1}^{s} \left(\overline{y}_r - y_{rp}^R \right)^2 = d_3^2, \quad j = 1, \ldots, n,
$$

$$
\lambda_1 x_{ip}^T + \lambda_2 x_{ip}^C + \lambda_3 x_{ip}^R = \overline{x}_i \quad i = 1, \ldots, m \, j = 1, \ldots, n,
$$

$$
\lambda_1 y_{rp}^T + \lambda_2 y_{rp}^C + \lambda_3 y_{rp}^R = \overline{y}_r, \quad r = 1, \ldots, s \, j = 1, \ldots, n,
$$

$$
\sum_{k=1}^{3} \lambda_k = 1,
$$

$$
d_1 \geq 0, d_2 \geq 0, \, d_3 \geq 0,
$$

$$
\overline{x}_i \geq 0, \, \overline{y}_r \geq 0, \lambda_k \geq 0.
$$

(12)

In the above model, we intend to minimize the total sum of the distance between projection points and ideal points from each of the projection points obtained from the BCC random, cost, and revenue models under norm 2 (L2) in the objective function of the proposed model so that the ideal point are in the convex combination of these points and at the same time dominate the unit under evaluation. This projection point obtained for each unit simultaneously meets the three technical, cost, and revenue objectives as much as possible. Secondly, random data and technical, cost,

and revenue stochastic models are used in a linear form that has not been addressed so far, so obtaining such a pattern with these scores is very important.

5. Practical Example

The electricity industry has an infrastructural and influential role on the economic growth of any country. Iran Regional Electricity Company is one of the most important companies in the field of electricity industry, whose main task is to

TABLE 6: Projection points obtained from different models at the error level of 0.05.

DMUs	Companies	0.05 Projection points of random models
1	Azerbaijan	TE = (2921, 2776, 3723, 5808, 4234, 4574, 7420, 12211) CE = (2921, 2776, 3723, 5808, 4234, 4574, 7420, 12211) RE = (2337.70, 2197.15, 3625.07, 5676.51, 3119.44, 4204.98, 2226.99, 76203.28)
2	Esfahan	TE = (2223.96, 1915.04, 2056.28, 5657.61, 2589.11, 3770.64, 6708, 21300.80) CE = (2315.68, 21547.78, 2201.58, 5054.89, 2640.87, 3564.31, 6708, 21380.93) RE = (2094.71, 2129.11, 1916.77, 7688.32, 2945.51, 5656.92, 4105.28, 100754.13)
3	Kermanshah	TE = (2898, 2360, 3975, 7767, 5562, 5446, 7849, 13818) CE = (2898, 2360, 3975, 7767, 5562, 5446, 7849, 13818) RE = (1903.34, 1948.97, 1842.25, 7243.52, 2889.28, 5390.68, 3922.14, 91367.89)
4	Tehran	TE = (7994, 7682, 4214, 21400, 4679, 3864, 9751, 390100) CE = (7994, 7682, 4214, 21400, 4679, 3864, 9751, 390100) RE = (7994, 7682, 4214, 21400, 4679, 3864, 9751, 390100)
5	Khorasan	TE = (987.19, 1050.11, 1429.09, 3200.08, 1497.03, 2074.81, 2684.91, 16624.70) CE = (740.23, 802.17, 1143.70, 3284.25, 1108.46, 1764.05, 2869.79, 17008.05) RE = (1054.04, 1115.63, 1816.35, 4153.73, 1612.42, 2480.12, 2105.98, 40070.06)
6	Khuzestan	TE = (2906, 2286, 3691, 12983, 3874, 8716, 7274, 28195) CE = (2867.07, 2911.31, 3663.17, 7969.76, 4984.47, 5876.36, 7274, 28770.86) RE = (1938.28, 1981.86, 1855.86, 7324.73, 2899.55, 5439.29, 3955.57, 93081.56)
7	Zanjan	TE = (177, 324, 1170, 3231, 2382, 2989, 2270, 6695) CE = (430.05, 529.39, 1053.42, 2736.14, 975.79, 1362.63, 2332.38, 6965.58) RE = (177, 324, 1170, 3231, 2382, 2989, 2270, 6695)
8	Semnan	TE = (109, 269, 1002, 2401, 859, 1066, 1527, 2629) CE = (109, 269, 1002, 2401, 859, 1066, 1527, 2629) RE = (109, 269, 1002, 2401, 859, 1066, 1527, 2629)
9	Sistan and Baluchestan	TE = (1127, 1002, 3716, 2046, 2935, 2077, 411, 4468) CE = (150.14, 307.68, 1018.76, 2500.12, 878.93, 1132.77, 1569.91, 4650.52) RE = (1127, 1002, 3716, 2046, 2935, 2077, 411, 4468)
10	The west	TE = (1418.26, 1265.45, 1276.62, 3828.85, 1698.70, 2662.49, 5425.06, 6219.54) CE = (1457.53, 1309.99, 1116.90, 3250.39, 1300.24, 2026.10, 5509.21, 6275.16) RE = (1492.03, 1469.44, 2589.53, 4482.77, 2234.03, 3025.13, 1999.49, 49129.09)
11	Fars	TE = (3144, 3496, 5321, 10982, 7933, 8349, 7729, 16456) CE = (3144, 3496, 5321, 10982, 7933, 8349, 7729, 16456) RE = (2969.64, 2952.66, 2257.48, 9721.92, 3202.61, 6874.12, 4942.60, 143667.32)
12	Kerman	TE = (976.84, 952.22, 1314.61, 3054.18, 1295.44, 1800.64, 3569.72, 9715.72) CE = (844.49, 856.15, 1101.82, 3069.52, 1117.79, 1694.73, 3478.60, 9973.50) RE = (1619.31, 1600.36, 2525.83, 4930.55, 2216.15, 3263.94, 2241.68, 57804.96)
13	Guilan	TE = (1628, 1433, 1115, 3267, 1348, 2101, 6110, 4368) CE = (1628, 1433, 1115, 3267, 1348, 2101, 6110, 4368) RE = (1628, 1433, 1115, 3267, 1348, 2101, 6110, 4368)
14	Mazandaran	TE = (1788.77, 1555.52, 1816.97, 4450.99, 2494.05, 3084.56, 6059.26, 9649.62) CE = (1672.51, 1486.96, 1156.34, 3502.43, 1380.89, 2238.87, 6018.83, 9900.02) RE = (1774.99, 1835.11, 1683.03, 6426.53, 1691.42, 3800.82, 3276.03, 84508.40)
15	Hormozgan	TE = (984.32, 963.99, 1113.55, 3156.53, 1163.45, 1793.73, 3892.66, 10323.81) CE = (939.82, 932.02, 1114.31, 3153.67, 1151.12, 1774.94, 3734.24, 10860.31) RE = (1776.79, 1836.96, 1681.38, 6419.56, 1666.99, 3772.96, 3266.50, 84584.74)
16	Yazd	TE = (818., 930, 1561, 2514, 1659, 1868, 1851, 5294) CE = (271.05, 403.50, 1033.82, 2602.57, 920.82, 1232.31, 1898.77, 5663.27) RE = (380.40, 480.75, 1558.01, 2510.97, 1297.13, 1382.27, 1388.16, 6630.61)

produce, distribute, and transmit electricity. In this practical example, fuel consumption and nominal power are used in the power generation process. The nominal power of a propulsion device has been written by the manufacturer on its specifications plate for certain conditions in terms of horsepower or megawatts. In small machines, the nominal power is specified in kilowatts. The power transmission section is done by posts and lines of the transmission network. The substation is part of a network, which has been concentrated in a given location and is used to selectively connect and disconnect electrical circuits within a network. Power distribution is also used to send and sell energy to

TABLE 7: Projection point obtained from different models and revenue at the error level of 0.3.

DMUs	Companies	0.3 Projection points of random models
1	Azerbaijan	TE = (2921,2776,3723,5808,4234,4574,7420,12211) CE = (2921,2776,3723,5808,4234,4574,7420,12211) RE = (2396.61,2248.41,3691.32,5766.85,3177.07,4288.80,2248.41,78212.43)
2	Esfahan	TE = (2215.49,1910.07,2044.56,5532.79,2595.41,3689.31,6708, 20920.39) CE = (2340.91,2160.66,2208.97,4866.56,2555.11,3449.95, 6733.10, 20949.90) RE = (2146.40, 2177.76,1936.90,7808.47,2960.70,5728.83,4154.75,103289.41)
3	Kermanshah	TE = (2898,2360,3975,7767,5562,5446,7849,13818) CE = (2898,2360,3975,7767,5562,5446,7849,13818) RE = (2185.89,2217.48,1912.23,7710.10,2547, 5266.38,4000.71,105019.24)
4	Tehran	TE = (7994, 7682, 4214,21400,4679,3864, 9751, 390100) CE = (7994, 7682, 4214,21400,4679,3864, 9751, 390100) RE = (7994, 7682, 4214,21400,4679,3864, 9751, 390100)
5	Khorasan	TE = (971.93,1039.61,1434.32,3181.68,1499.94,2066.24,2618.40,16491.39) CE = (672.80,749.37,1136.52,3233.86,1085.69,1711.99,2679.17,16619.22) RE = (1076.22,1132.88,1862.31,4162.08,1648.58,2505.82,2094.15,40386.29)
6	Khuzestan	TE = (2906, 2286, 3691,12983,3874, 8716, 7274, 28195) CE = (3112.07,2958.78, 3687.80, 6426.26,4190.80,4921.63,7492.31, 28357.32) RE = (2148.19,2179.45,1937.60, 7812.63,2961.23,5731.32,4156.47,103377.29)
7	Zanjan	TE = (177, 324,1170, 3231, 2382, 2989,2270, 6695) CE = (413.74, 516.30,1051.06,2720.52, 969.99,1348.28, 2289.98,6781.83) RE = (177, 324,1170, 3231, 2382, 2989,2270, 6695)
8	Semnan	TE = (109, 269,1002, 2401, 859,1066,1527, 2629) CE = (109, 269,1002, 2401, 859,1066,1527, 2629) RE = (109, 269,1002, 2401, 859,1066,1527, 2629)
9	Sistan and Baluchestan	TE = (1127, 1002, 3716,2046,2935, 2077, 411, 4468) CE = (147.61,305.30,1017.73,2494.03,877.71,1128.67,1567.27,4526.31) RE = (1127, 1002, 3716,2046,2935, 2077, 411, 4468)
10	The west	TE = (1270.64,1164.25,1139.12,3315.03,1627.56,2369.27, 5122.45, 6194.40) CE = (1345.69,1224.53,1109.06,3189.23,1264.47,1951.23,5168.97, 6215.15) RE = (1484.06,1474.44,2458.10,4620.08,2141.94,3047.94, 2112.58,51423.40)
11	Fars	TE = (3144, 3496, 5321,10982, 7933, 8349,7729,16456) CE = (3968.25,3477.71,4025.19,10630.16,5376.55,7213.93,8248.45,92843.70) RE = (3094.35,3070.05,2306.04,10011.78,3239.25,7047.62, 5061.95, 149784.07)
12	Kerman	TE = (1031.86,990.28,1503.94,3011.20,1436.16,1856.41,3448.12, 9613.69) CE = (823.21,838.96,1098.54,3048,1110.11,1675.42, 3424.48,9704.16) RE = (1710.73,1670.73,2723.01,4955.52,2370.54,3367.72, 2185.55,58945.75)
13	Guilan	TE = (1628, 1433,1115, 3267,1348, 2101, 6110,4368) CE = (1628, 1433,1115, 3267,1348, 2101, 6110,4368) RE = (336.35,482.74,1094.61,2948.81,969.14,1435.01,1764.13,13801.11)
14	Mazandaran	TE = (1635.34,1456.78,1158.58,3503.21,1446.07,2302.12, 5956.02, 9555.20) CE = (1641.19,1462.11,1152.38,3475.58,1370.01,2212.92, 5934.02,9629.27) RE = (2094.46, 2076.21,2420.36,6440.61,2240.50,4118.56,3020.48, 87420.82)
15	Hormozgan	TE = (933.79,924.64,1108.58,3121.04,1146.59,1755.89, 3747.37,10091.75) CE = (919.27,914.20,1108.81,3119.98,1142.56,1749.69,3695.82,10263.53) RE = (1908.51,1943.64,1911.08,6521.97,1851.98,3937.66,3237.15, 87368.43)
16	Yazd	TE = (818., 930,1561, 2514,1659,1868,1851, 5294) CE = (257.05,391.92,1031.15,2585.58,915.51,1218.16,1866.19, 5412.24) RE = (381.96,482.07,1560.03,2513.02,1298.84,1384.40,1388.47,6677.91)

companies. Iranian electricity distribution companies are responsible for managing and coordinating subsidiary units and supplying the production, transmission and sale of electricity and are subsidiaries of Tavanir specialized parent company.

Considering the importance of this industry in the economic growth and development of the country, the model is evaluated based on data related to 16 regional electricity companies in Iran, in a 9-year period (from 2005 to 2014) derived from the statistical yearbook of the Ministry

TABLE 8: Projection point obtained from different models at the error level of 0.5.

DMUs	Companies	0.5 Projection points of random models
1	Azerbaijan	TE = (2921,2776,3723,5808,4234,4574,7420,12211) CE = (2921,2776,3723,5808,4234,4574,7420,12211) RE = (2424.07,2272.24,3723,5808,3204.48,4327.67,2257.65,79132.64)
2	Esfahan	TE = (2197.77,1906.40,1994.79,5127.10,2599.98,3422.09, 6708,20752) CE = (2315.77,2134.59,2159.53,4816.42,2500.41,3401.92,6708,20752) RE = (2174.28,2204,1947.75,7873.26,2968.89,5767.61,4181.43,104656.59)
3	Kermanshah	TE = (2898,2360,3975,7767,5562,5446,7849,13818) CE = (2898,2360,3975,7767,5562,5446,7849,13818) RE = (2333.23,2360,1909.46,7767,1951.51,4694.30,3853.60,111935.69)
4	Tehran	TE = (7994, 7682, 4214,21400,4679,3864, 9751, 390100) CE = (7994, 7682, 4214,21400,4679,3864, 9751, 390100) RE = (7994, 7682, 4214,21400,4679,3864, 9751, 390100)
5	Khorasan	TE = (963.76,1033.47,1434.71,3174.20,1498.68,2060.50, 2591,16434) CE = (641.54,724.88,1133.16,3210.31,1075.12,1687.76,2591,16434) RE = (1086.26,1140.70,1883,4166,1664.87,2517.49,2088.89,40531.70)
6	Khuzestan	TE = (2906, 2286, 3691,12983,3874, 8716, 7274, 28195) CE = (2905.31,2742.88,3240.12,6037.62,3695.09,4524.88,7274,28195) RE = (2261.39, 2286,1981.68,8075.74,2994.49,5888.80,4264.80,108929.39)
7	Zanjan	TE = (177, 324,1170, 3231, 2382, 2989,2270, 6695) CE = (406.06,510.13,1049.95,2713.15,967.25,1341.51,2270,6695) RE = (177, 324,1170, 3231, 2382, 2989,2270, 6695)
8	Semnan	TE = (109, 269,1002, 2401, 859,1066,1527, 2629) CE = (109, 269,1002, 2401, 859,1066,1527, 2629) RE = (109, 269,1002, 2401, 859,1066,1527, 2629)
9	Sistan and Baluchestan	TE = (1127, 1002, 3716,2046,2935, 2077, 411, 4468) CE = (146.42,304.18,1017.24,2491.17,877.13,1126.74,1566.03,4468) RE = (1127, 1002, 3716,2046,2935, 2077, 411, 4468)
10	The west	TE = (1227.88,1131.28,1140.06,3310.29,1656.42,2392.23,5012,6182) CE = (1294.02,1185.03,1105.39,3160.77,1247.92,1916.53,5012, 6182) RE = (1486.23,1481,2412.56,4682,2111.03,3064.41,2158.81,52502.82)
11	Fars	TE = (2843.69,2334.76,3751.67,7534.80,5221.84,5257.97,7729,16456) CE = (2947.91,2536.30,3897.17,7260.41,5133.90,5240.15,7729,16456) RE = (3144,3116.78,2325.37,10127.18,3253.84,7116.69,5109.47,152219.20)
12	Kerman	TE = (1058.23,1008.68,1588.66,2993.30,1499.59,1882.68, 3399,9577) CE = (813.19,830.86,1096.99,3037.85,1106.49,1666.32,3399,9577) RE = (1756.,1705.50,2821.36,4967,2447.49,3418.90,2157.07,59495.45)
13	Guilan	TE = (1628, 1433,1115, 3267,1348, 2101, 6110,4368) CE = (1628, 1433,1115, 3267,1348, 2101, 6110,4368) RE = (386.40,529.79,1115,3069.40,993.39,1516.24,1816.33,16260.45)
14	Mazandaran	TE = (1610.73,1437.64,1158.72,3498.34,1461.75,2313.48,5894,9501) CE = (1626.41,1450.38,1150.50,3462.88,1364.88,2200.67, 5894,9501) RE = (2114.01,2091.50,2460,6449,2271.78,4141.44,3010.85,87717.07)
15	Hormozgan	TE = (909.69,905.88,1106.22,3104.19,1138.56,1737.87,3678, 9983) CE = (909.69,905.88,1106.22,3104.19,1138.56,1737.87,3678,9983) RE = (1968.92,1991.45,2027.82,6555,1944.67,4010.,3212.88,88405.76)
16	Yazd	TE = (818., 930,1561, 2514,1659,1868,1851, 5294) CE = (250.51,386.50,1029.89,2577.60,913.03,1211.53,1851, 5294)

of Energy (https://www.moe.gov.ir) according to Jafari et al. [21]. Therefore, for each company, six inputs and two outputs have been considered, which are defined as follows. The selection of these indicators is based on a study of research conducted in the electricity industry.

Input and output variables have been organized in Table 2.

Input and output indices have a normal distribution, the mean value and standard deviation for each of which have been shown in Tables 3 and 4, respectively.

Table 5 shows the efficiency number obtained from technical, stochastic cost, and revenue models at three error levels ($\alpha = 0.05$, 0.3, and 0.5). According to the results obtained from this table, units 1, 3, 4, 6, 7, 8, 9, 10, 13, and 16

TABLE 9: The best projection point at the error level 0.05.

DMUs	Electricity distribution companies	$\propto = 0.05$
1	Azerbaijan	(2921,2776,3723,5808,4234,4574,7420,12211)
2	Esfahan	(2.3E + 3,2053.44,2138.45,5320.41, 2619.40,3658.21, 6701.45,21546.39)
3	Kermanshah	(2.9E + 3,2360, 3975, 7767, 5562, 5446,7849,13818)
4	Tehran	(8.0E + 3,7682,4214,21400,4679,13864,9751,390100)
5	Khorasan	(761.10,823.11,1168.64,3279.76,1141.32,1791.17, 2852.75,17038.83)
6	Khuzestan	(2.9E + 3,2655.77,3628.85,9818.35,4519.93,6921.68, 7191.98,30146.29)
7	Zanjan	(177, 324,1170,3231,2382,2989,2270,6695)
8	Semnan	(109,269,1002,2401,859,1066,1527,2629)
9	Sistan and Baluchestan	(1.1E + 3,1002,3716,2046,2935,2077,411,4468)
10	The west	(1.4E + 3,1289.45,1203.82,3538.60,1499.32,2340.86, 5445.68,6526.16)
11	Fars	(3.1E + 3,3496,5321,10982,7933,8349,7729,16456)
12	Kerman	(844.49,856.15,1101.82,3069.52,1117.79,1694.73, 3478.60,9973.50)
13	Guilan	(1.6E + 3,1433,1115,3267,1348,2101,6110,4368)
14	Mazandaran	(1.7E + 3,1516.75,1424.20,3901.72,1828.32,2587.78, 6016.42,10305.40)
15	Hormozgan	(939.82,932.02,1114.31,3153.67,1151.12,1774.94,3734.24,10860.31)
16	Yazd	(443.18,561.08,1321.06,2553.29,1219.40,1438.99, 1738.57,5848.71)

TABLE 10: The best projection point at the error level 0.3.

DMUs	Electricity distribution companies	$\propto = 0.3$
1	Azerbaijan	(2921,2776,3723,5808,4234,4574,7420,12211)
2	Esfahan	(2.3E + 3,2036.03,2126.43,5206.07,2576.27,3575.27, 6713.56, 21160.31)
3	Kermanshah	(2.9E + 3,2360, 3975, 7767, 5562, 5446,7849,13818)
4	Tehran	(8.0E + 3,7682,4214,21400,4679,13864,9751,390100)
5	Khorasan	(791.66, 864.63,1257.68,3221.82,1250.33,1855.51,2650.64, 16776.79)
6	Khuzestan	(3.0E + 3,2634.24,3638.40,9408.85,4012.92,6647.86, 7297.45,30463.48)
7	Zanjan	(177, 324,1170,3231,2382,2989,2270,6695)
8	Semnan	(109,269,1002,2401,859,1066,1527,2629)
9	Sistan and Baluchestan	(1.1E + 3,1002,3716,2046,2935,2077,411,4468)
10	The west	(1.3E + 3,1195.40,1129.01,3257.20,1448.68,2163.64, 5134.53, 6371.32)
11	Fars	(4.0E + 3,3477.71,4025.19,10630.16,5376.55,7213.93, 8248.45,92843.70)
12	Kerman	(912.28,904.21,1271.26,3039.47,1248.86,1755.89,3429.92,9838.07)
13	Guilan	(1.6E + 3,1433,1115,3267,1348,2101,6110,4368)
14	Mazandaran	(1.6E + 3,1462.07,1152.46,3475.91,1370.78,2213.84, 5934.19,9629.95)
15	Hormozgan	(919.27,914.20,1108.81, 3119.98,1142.56,1749.69, 3695.82,10263.53)
16	Yazd	(467.82,586.17,1340.05,2543.55,1253.07,1468.09, 1733.72,5713.09)

are technical efficiencies. Units 1, 3, 4, 8, and 13 are cost efficient. Revenue efficiency includes units 4, 7, 8, and 9. Therefore, among the companies, only two electricity distribution companies (units 4 and 8) in Tehran and Semnan provinces are efficient at all three levels of error in technical, stochastic cost, and revenue models.

The projections obtained from the technical, stochastic cost, and revenue models can be seen at three levels of error in Tables 6–8. For each company, the projection points of the technical, cost, and revenue models have been shown in three rows with TE, CE, and RE, respectively. The results have been set in separate tables for three error levels. These tables have been presented in the appendix.

Also, the projection point of the proposed model for each of the electricity distribution companies with three levels of error has been shown in Tables 9–11, which is attached to the article. According to the results obtained for most units, the projection point of the proposed dominant model of the unit under evaluation has been obtained. Dominance means having fewer inputs and more outputs than the unit under evaluation. For example, consider the

electricity distribution company of Khorasan province (unit 5) at the error level of 0.05. The model point of the model presented in this paper shows that all inputs have decreased and all outputs have increased compared to the unit under evaluation of this unit. These results can also be generalized to units 10, 12, 14, and 15. The projection point obtained from the model presented in this paper for units 1, 3, 4, 7, 8, 9, 11, and 13 equals to the unit under evaluation is obtained at an error level of 0.05. These results can be analyzed for other error levels.

In Table 12, the ideal cost and revenue are compared with the observed cost and revenue at three levels of error for all companies. The results in this table show that for all companies in all three levels of error, the ideal cost is not more than the observed cost and the ideal revenue is not less than the observed revenue as per unit, which indicates an improvement in performance of all companies. For example, consider the Electricity Distribution Company of Khuzestan Province (DMU 6) at the error level of 0.05. Although in the proposed model, the second and fifth inputs have been increased, but the ideal cost of this unit is less than the

TABLE 11: The best projection point at error level 0.5.

DMUs	Electricity distribution companies	$\propto = 0.5$
1	Azerbaijan	(2921, 2776, 3723, 5808, 4234, 4574, 7420, 12211)
2	Esfahan	(2.3E + 3, 2015.65, 2073.26, 4983.18, 2553.07, 3416.07, 6704.13, 20880.60)
3	Kermanshah	(2.9E + 3, 2360, 3975, 7767, 5562, 5446, 7849, 13818)
4	Tehran	(8.0E + 3, 7682, 4214, 21400, 4679, 13864, 9751, 390100)
5	Khorasan	(815.53, 891.41, 1299.09, 3200.15, 1303.88, 1891.96, 2586.34, 16657.88)
6	Khuzestan	(2.7E + 3, 2285.41, 2873.81, 6748.02, 3872.99, 4596.19, 7274, 30393.29)
7	Zanjan	(177, 324, 1170, 3231, 2382, 2989, 2270, 6695)
8	Semnan	(109, 269, 1002, 2401, 859, 1066,1527,2629)
9	Sistan and Baluchestan	(1.1E + 3, 1002, 3716, 2046, 2935, 2077, 411, 4468)
10	The west	(1.3E + 3, 1158.05, 1128.84, 3245.29, 1465.54, 2170.53, 5000.49, 6368.78)
11	Fars	(2.9E + 3, 2433.94, 3821.60, 7402.84, 5177.20, 5250.82, 7726.8, 16569.88)
12	Kerman	(949.33, 930.45, 1369.45, 3021.90, 1324.17, 1790.38, 3393.79, 9786.24)
13	Guilan	(1.6E + 3, 1433, 1115, 3267, 1348, 2101, 6110, 4368)
14	Mazandaran	(1.6E + 3, 1444.02, 1155.58, 3483.29, 1416.54, 2261.33, 5892.37, 9545.32)
15	Hormozgan	(909.69, 905.88, 1106.22, 3104.19, 1138.56, 1737.87, 3678, 9983)
16	Yazd	(482.43, 600.76, 1352.32, 2538.99, 1272.95, 1485.24, 1731.40, 5657.83)

observed cost. Also, despite the decrease in the first output of the proposed model for this province, its ideal income has been obtained more than its observed income. This analysis can be generalized to other units.

The input price vector and the output price vector for all units are given a fixed value as follows:

$$(c_1 = 2, c_2 = 1, c_3 = 3, c_4 = 4, c_5 = 3, c_6 = 2, p_1 = 3, p_2 = 2).$$

(13)

For example, the ideal cost and the observed cost along with the ideal revenue and the observed revenue for DMU11 at the error level of 0.03 are obtained from the following formula, which can be generalized to all units.

$$
\begin{aligned}
\text{Observed Cost} &= c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4 + c_5 x_5 + c_6 x_6 \\
&= (2 * 3144) + (3496 * 1) + (5321 * 3) + (10982 * 4) + (7933 * 3) + (8349 * 2) = 110172, \\
\text{Ideal Cost} &= c_1 x_1^* + c_2 x_2^* + c_3 x_3^* + c_4 x_4^* + c_5 x_5^* + c_6 x_6^* \\
&= (2 * 4.00E + 03 * 2) + (3477.71 * 1) + (4025.19 * 3) + (10630.16 * 4) + (5376.55 * 3) + (7213.93 * 2) \\
&= 966314, \\
\text{Observed Cost} &= p_1 y_1 + p_2 y_2 = (7729 * 3) + (16456 * 2) = 56099, \\
\text{Ideal Cost} &= p_1 y_1^* + p_2 y_2^* = (8248.45 * 3) + (92843.7 * 2) = 210433.
\end{aligned}
$$

(14)

6. Conclusions

Decision-making units are divided into two categories: efficient and inefficient units. For each inefficient DMU, there is a projection point on the efficient frontier so that the inefficient unit moves towards the efficient frontier with different approaches and becomes efficient.

In this paper, in the first step, by technical efficiency models, linear stochastic cost, and revenue under variable returns to scale, we identify projection points such as each DMU. Then, through the presented model, a projection point is obtained that the distance from each of the projection points obtained in the first step under norm 2 is the least. The projection point obtained for each inefficient DMU is very important in two ways. First, the projection obtained for each DMU simultaneously meets the three

main aspects of technical efficient, minimal cost, and maximal revenue as much as possible. Secondly, random data and stochastic programming in linear form are used in it. Therefore, it will help managers to make better decisions in the future and will enjoy the benefits of random planning. Considering the importance of the electricity industry in the economic growth and development of each country, a practical example has been implemented on 16 regional electricity companies in Iran in 9 periods. Comparison between cost efficiency and ideal income with observed cost and revenue efficiency showed that the ideal cost of each unit is less than or equal to its observed cost and the ideal revenue of each unit is greater than or equal to the observed revenue of that unit at three levels of error. According to the obtained results, the managers of Iranian electricity distribution companies can plan by identifying inefficient companies,

identifying the projection point of the units and setting goals in order to increase their efficiency before the actual performance of the companies. For further studies, the method presented in this paper can be applied to other types of uncertain data such as fuzzy or interval data and can also be focused on other aspects.

Appendix

Projection points obtained from different models along with the best projection points in three levels of error are given in this section.

Data Availability

All the data used in the application of the model were collected by the authors and they are available per request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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