

Research Article

Super-Twisting Nonsingular Terminal Sliding Mode-Based Robust Impedance Control of Robots

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This paper proposes a super-twisting terminal sliding mode-based robust impedance controller to improve the compliance and robustness in robot-environment interaction. Based on the desired impedance dynamics, an impedance reference trajectory is constructed. Then, based on a super-twisting terminal sliding-mode, the robust impedance controller is designed to guarantee the achievement of the desired impedance dynamics in finite-time through the finite-time convergence of an impedance error. The main contribution of this paper is that the proposed control improves impedance control robustness by using the super-twisting nonsingular terminal sliding-mode without causing the chattering problem. The finite-time stability of the closed-loop control system is validated by theoretical analysis based on the Lyapunov theory, and the control effectiveness is illustrated by simulations on a two-link robot manipulator.

1. Introduction

Compliant behavior of a robot is required in its interaction with its environment to ensure safe interaction [1] but cannot be provided by traditional position control of rigid-link robots. Impedance control proposed by Hogan in 1980s is one of the most popular used active compliance control approaches [2]. In this approach, the desired spring-damping dynamics between robot positions and interaction forces is constructed to improve interaction compliance. This active compliance control has been applied in service robots and industrial robots. In applications, one difficulty in impedance control design comes from robot modeling uncertainties, which hinders the convergence of impedance errors to zero or its small neighborhood and affects the control stability and control robustness. How to improve impedance control robustness has always been a significant research topic.

In the last decades, varieties of robust impedance control approaches including iterative learning impedance control [3–5], adaptive impedance control [6–9], neural network impedance control [10–15], and fuzzy impedance control [16] were proposed to improve impedance control

robustness. However, these control strategies have the following deficiencies. (i) The desired impedance dynamics requires to be factorized in the real space, which restricts the application ranges. (ii) Iterative learning impedance control is mainly applied in repetitive motions, adaptive impedance control requires linear factorization of robot dynamics, while neural/fuzzy impedance control only obtains finiteness of impedance errors and cannot get the convergence of impedance errors to zero.

Sliding-mode control is well known for its strong robust. In [17], a sliding-mode impedance control strategy was proposed based on robust passivity. However, the inherent chattering problem in sliding-mode control severely affects the impedance control performances. To alleviate the chattering problem, a dead-zone strategy was used in designing sliding-mode impedance control [18]. However, this strategy may not effectively decrease chattering. What is worse, the used dead-zone strategy may hinder arriving at the desired sliding surface, which further affects the impedance control performances.

By traditional sliding-mode control, tracking errors asymptotically converge to zero after related variables arriving at the desired sliding surfaces. Terminal sliding mode

control with a nonlinear sliding-mode can guarantee the finite-time convergence of tracking errors, which is better than the asymptotic convergence to some extent [19]. Super-twisting algorithm considered as second-order sliding-mode control has strong control robustness to modeling uncertainties without causing chattering problems [20]. This paper proposes a robust impedance control approach for robots by exploiting the advantages of super-twisting terminal sliding mode. Based on the desired impedance dynamics, an impedance trajectory is constructed. Then, based on a super-twisting sliding mode, a robust impedance controller is designed to guarantee the achievement of the desired impedance through the finite-time convergence of an impedance error. This proposed control approach improves the impedance control robustness using the super-twisting sliding-mode without causing the chattering. The finite-time control stability and the control effectiveness are validated by theoretical analysis and simulation results.

2. Robot Dynamics

Consider the robot arm with the following dynamics:

$$\ddot{q} = f(q, \dot{q}) + \Delta(q, \dot{q}) + M^{-1}(q)(\tau + \tau_e), \quad (1)$$

where q and \dot{q} denote the robot angular vector and velocity vector, respectively; $M(q)$ denotes the inertial matrix; $f(q, \dot{q})$ denotes a known robot function; $\Delta(q, \dot{q})$ denotes robot modeling uncertainties; τ is the control input; and $\tau_e = J^T f_e$ is the interaction force in joint space with J being the Jacobian matrix and f_e the interaction force in work space.

Assumption 1. The desired trajectory q_d and its first- and second-order time derivative are bounded.

Assumption 2. On a compact set Ω , the uncertainty term $\Delta(q, \dot{q})$ satisfies $d_\Delta = [d_{\Delta 1}, \dots, d_{\Delta n}]^T = \hat{\Delta}$, where $|d_{\Delta i}| \leq d_{\max}$, $i = 1, 2, \dots, n$ and d_{\max} is a positive constant.

The objective of this paper is to design a robust impedance controller for (1) based on a super-twisting non-singular terminal sliding-mode control approach to achieve the following desired impedance dynamics:

$$-\tau_e = M_d(\ddot{q}_d - \ddot{q}) + D_d(\dot{q}_d - \dot{q}) + K_d(q_d - q), \quad (2)$$

where M_d, D_d , and K_d denote the desired inertial matrix, the desired damping matrix, and the desired stiffness matrix, respectively.

3. Super-Twisting Terminal Sliding-Mode Impedance Control

Passing τ_e through the following filter:

$$M_d \ddot{q}_e + D_d \dot{q}_e + K_d q_e = \tau_e, \quad (3)$$

then the desired impedance dynamics in (2) can be expressed as

$$M_d(\ddot{q}_r - \ddot{q}) + D_d(\dot{q}_r - \dot{q}) + K_d(q_r - q) = 0, \quad (4)$$

where $q_r = q_d + q_e$ is the constructed impedance trajectory.

Remark 1. In (2), M_d, D_d , and K_d are usually chosen as positive definite diagonal matrices that guarantees the following: (i) $q_d - q$ in (2) converges to zero when τ_e equals to zero; (ii) given $\tau_e \in L_\infty$, q, \dot{q} , and \ddot{q} in (2) are bounded and q_e, \dot{q}_e , and \ddot{q}_e in (3) are bounded. Combining $q_e, \dot{q}_e, \ddot{q}_e \in L_\infty$ and Assumption 1, one can obtain $q_r, \dot{q}_r, \ddot{q}_r \in L_\infty$.

Define $e_1 = q_r - q$, $e_2 = \dot{q}_r - \dot{q}$. The impedance error has the following form:

$$\begin{aligned} e_{im} &= M_d(\ddot{q}_d - \ddot{q}) + D_d(\dot{q}_d - \dot{q}) + K_d(q_d - q) + \tau_e, \\ &= M_d(\ddot{q}_r - \ddot{q}) + D_d(\dot{q}_r - \dot{q}) + K_d(q_r - q). \end{aligned} \quad (5)$$

From (5), the desired impedance dynamics in (2) can be realized through the convergence of e_{im} , if e_1, e_2, \dot{e}_2 converge to zero. In the following, super-twisting-based robust control is designed to make e_1, e_2, \dot{e}_2 converge to zero.

Define the following terminal mode:

$$s = \gamma e_1^{\alpha_1/\alpha_2} + e_2, \quad (6)$$

where $s = [s_1, \dots, s_n]^T$ and α_1, α_2 are odd and satisfy $1 < \alpha_1/\alpha_2 < 2$. Based on the dynamics in (1), the dynamics of s satisfies

$$\dot{s} = \gamma \frac{\alpha_1}{\alpha_2} e_1^{\alpha_1/\alpha_2 - 1} e_2 - f(q, \dot{q}) - \Delta - M^{-1}(q)(\tau + \tau_e) + \ddot{q}_r. \quad (7)$$

Design the following super-twisting mode control:

$$\begin{aligned} \tau &= -\tau_e + M(q) \left[\gamma \frac{\alpha_1}{\alpha_2} e_1^{\alpha_1/\alpha_2 - 1} e_2 + \ddot{q}_r + k_1 |s|^{1/2} \text{sgn}(s) \right. \\ &\quad \left. + k_2 \int_0^t \text{sgn}(s(\tau)) d\tau - f(q, \dot{q}) \right]. \end{aligned} \quad (8)$$

where k_1 and k_2 are positive constants and satisfy $k_1^2 > 4k_2$, $k_2 > d_{\max}$. Substituting (8) into (7) yields

$$\dot{s} = -k_1 |s|^{1/2} \text{sgn}(s) - k_2 \int_0^t \text{sgn}(s(\tau)) d\tau - \Delta. \quad (9)$$

Theorem 1. *Design the super-twisting sliding-mode controller in (8) for the robot with dynamics (1). Then, the errors e_1, e_2, \dot{e}_2 converge to zero in finite-time, which further guarantees the finite-time convergence of e_{im} and the realization of the impedance dynamics in (2).*

Proof. Define

$$\begin{aligned} z_{1i} &= s_i, \\ z_{2i} &= k_2 \int_0^t \text{sgn}(s_i(\tau)) d\tau + \Delta_i, \\ x &= [x_1, x_2]^T = [z_{1i}, z_{2i}]^T. \end{aligned} \quad (10)$$

From (9), the dynamics of x satisfies

$$\dot{x} = \frac{1}{|x_1|} (Ax + B\bar{d}_\Delta), \quad (11)$$

where $\bar{d}_\Delta = |x_1| d_\Delta$ and satisfies $|\bar{d}_\Delta| \leq d_{\max} |x_1|$, and matrices A and B are defined by

$$A = \begin{bmatrix} \frac{-k_1}{2} & \frac{1}{2} \\ -k_2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (12)$$

Define $C = [1, 0]$. Then, $d_{\max}^2 x^T C^T C x - \bar{d}_{\Delta}^T \bar{d}_{\Delta} \geq 0$, and there exist a positive matrix P and a positive constant ϵ , such that

$$\begin{bmatrix} A^T P + PA + d_{\max}^2 C^T C + \epsilon P & PB \\ B^T P & -1 \end{bmatrix} \langle 0. \quad (13)$$

Consider the following Lyapunov function:

$$V = x^T P x. \quad (14)$$

Taking the time derivative of V with respect to the time t and substituting (13) and (15) into it yields

$$\begin{aligned} \dot{V} &= \frac{1}{|x_1|} \begin{bmatrix} x^T & \bar{d}_{\Delta}^T \end{bmatrix} \begin{bmatrix} A^T P + PA & PB \\ B^T P & 0 \end{bmatrix} \begin{bmatrix} x^T & \bar{d}_{\Delta}^T \end{bmatrix}^T, \\ &\leq \frac{1}{|x_1|} \begin{bmatrix} x^T & \bar{d}_{\Delta}^T \end{bmatrix} \begin{bmatrix} A^T P + PA & PB \\ B^T P & 0 \end{bmatrix} \begin{bmatrix} x^T & \bar{d}_{\Delta}^T \end{bmatrix}^T \\ &\quad + d_{\max}^2 x^T C^T C x - \bar{d}_{\Delta}^T \bar{d}_{\Delta}, \\ &= \frac{1}{|x_1|} \begin{bmatrix} x^T & \bar{d}_{\Delta}^T \end{bmatrix} \begin{bmatrix} A^T P + PA + d_{\max}^2 C^T C & PB \\ B^T P & -1 \end{bmatrix} \begin{bmatrix} x^T & \bar{d}_{\Delta}^T \end{bmatrix}^T, \\ &\leq \frac{-\epsilon}{|x_1|} x^T P x = -\frac{\epsilon}{|x_1|} V. \end{aligned} \quad (15)$$

Since $|x| \leq V^{1/2} / \lambda_{\min}^{1/2}(P)$, then

$$\frac{-1}{|x|} \leq -\frac{\lambda_{\min}^{1/2}(P)}{V^{1/2}}. \quad (16)$$

From (15) and (16), one can obtain

$$\dot{V} \leq -\epsilon \lambda_{\min}^2(P) V^2. \quad (17)$$

Based on (17), V converges to zero in finite-time T_0 which is defined by

$$T_0 = \frac{2V^{1/2}(x(0))}{\epsilon \lambda_{\min}^{1/2}(P)}. \quad (18)$$

From the finite-time convergence of $V(x)$, $x, s_i, z_{2i}, i = 1, 2, \dots, n$ converge to zero in finite-time. From the definitions of s_i and s , we can further obtain the finite-time convergence of e_1, e_2 in the finite-time T_0 . The dynamics of e_2 can be expressed as

$$\dot{e}_2 = \gamma \frac{\alpha_1}{\alpha_2} e_1^{\alpha_2 - 1} e_2 - k_1 |s|^2 \text{sgn}(s) - k_2 \int_0^t \text{sgn}(s(\tau)) d\tau - \Delta. \quad (19)$$

As e_1, e_2, s, z_{2i} converge to zero in the finite-time T_0 , from (19), \dot{e}_2 converges to zero in the finite-time T_0 . The finite-time convergence of e_1, e_2 , and \dot{e}_2 implies the finite-time convergence of the impedance error which guarantees the achievement of the dynamics in (2). \square

Remark 2. If $x_{1i} = 0$, then $z_{1i} = s_i = 0$ and $\dot{s}_i = 0$ which imply $e_1 = 0, e_2 = 0$, and $\dot{e}_2 = 0$. Then, the desired impedance dynamics is achieved.

Remark 3. In comparison with nonfinite-time control, finite-time control can make the robot converge to the desired signal in finite-time. It has been illustrated that finite-time control has faster convergence, better robust, and anti-disturbance performances.

Remark 4. Neural networks (NNs), fuzzy logic (FL), and sliding-mode control can be applied to improve impedance control robustness. However, neural networks- and fuzzy logic-based control typically only achieves infinite-time uniformly ultimately bounded stability owing to the inherent approximation errors of NNs and FL. Compared with NNs- and FL-based impedance control, the proposed super-twisting impedance control can obtain finite-time control stability and has better control robustness.

4. Simulation Results

To show the control effectiveness, simulations are conducted on a two-link robot manipulator (see Figure 1) with the dynamics in (1), where $f(q, \dot{q}) = 0$,

$$\begin{aligned} \Delta(q, \dot{q}) &= M^{-1}(q) (C(q, \dot{q})\dot{q} + G(q) + F\dot{q}), \\ M(q) &= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \\ C(q, \dot{q}) &= \begin{bmatrix} -w_3 \sin(q_2) \dot{q}_2 & -w_3 \sin(q_2) (\dot{q}_1 + \dot{q}_2) \\ w_3 \sin(q_2) \dot{q}_1 & 0 \end{bmatrix}, \\ G &= \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}, F = \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix}, \\ J &= \begin{bmatrix} -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix}, \\ M_{11} &= w_1 + 2w_3 \cos(q_2), \\ M_{22} &= w_2, \\ M_{12} &= w_3 \cos(q_2) + w_2, M_{21} = M_{12}, \\ g_1 &= w_4 g \cos(q_1) + w_5 g \cos(q_1 + q_2), g_2 = w_5 g \cos(q_1 + q_2). \end{aligned} \quad (20)$$

The related parameters in the above equations are defined by

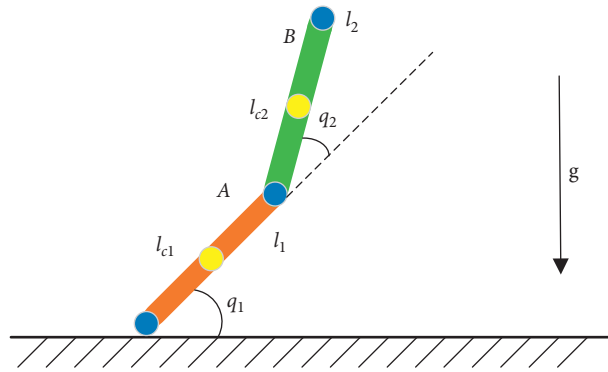


FIGURE 1: Two-link robot arm.

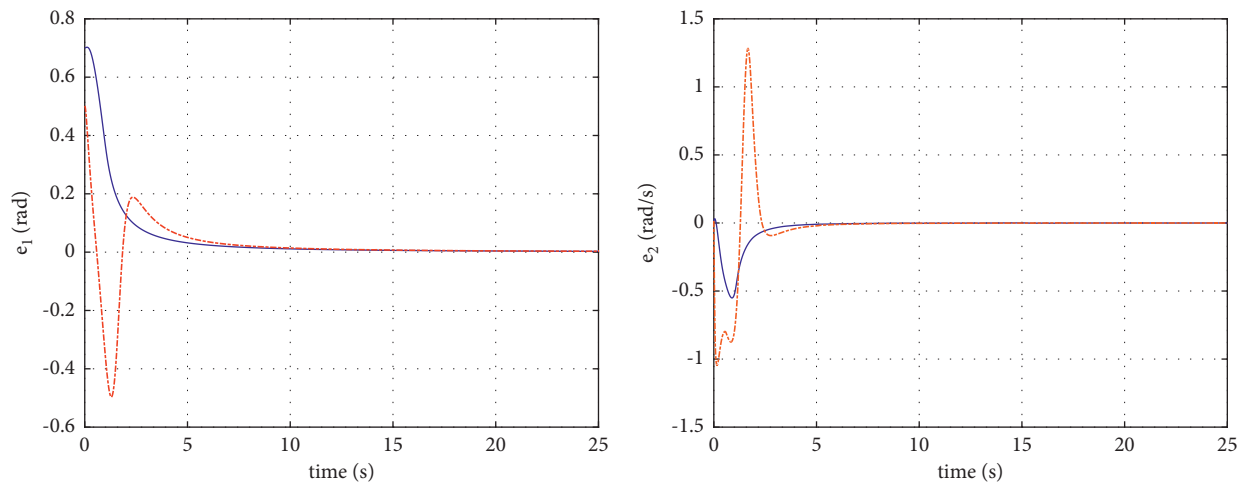
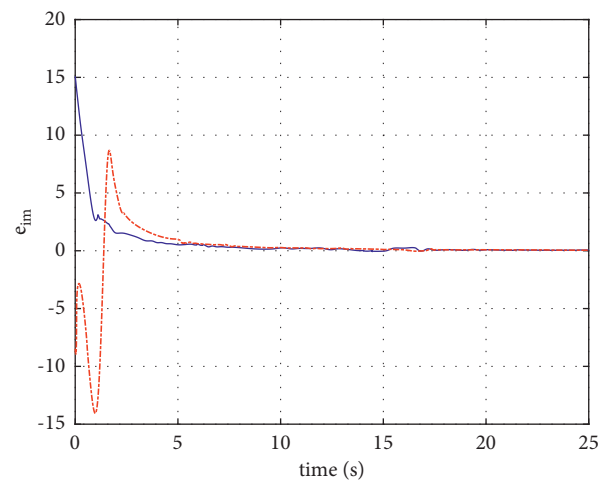
FIGURE 2: Performances of auxiliary errors e_1 and e_2 .

FIGURE 3: Performances of the impedance error.

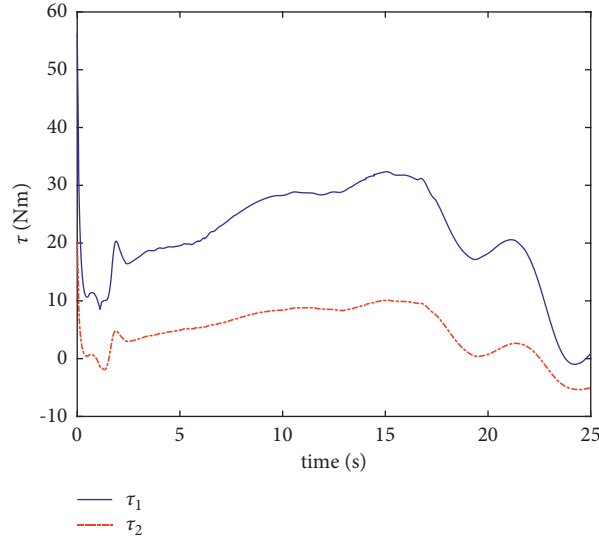


FIGURE 4: The super-twisting terminal sliding-mode impedance control.

$$\begin{aligned}
 w_1 &= I_1 + I_2 + m_1 l_{c1}^2 + m_2 l_1^2 + m_2 l_{c2}^2, w_2 = m_2 l_{c2}^2 + I_2, w_3 = m_2 l_1 l_{c2}, w_4 = m_1 l_{c1} + m_2 l_1, w_5 = m_2 l_{c2}, w_6 = v_1, w_7 = v_2, \\
 m_1 &= m_2 = 1.5 \text{ kg}, l_1 = l_2 = 0.8 \text{ m}, l_{c1} = l_{c2} = 0.4 \text{ m}, I_1 = I_2 = 0.3 \text{ kg} \cdot \text{m}^2, \\
 v_1 &= v_2 = 0.3 \text{ N} \cdot \text{m} \cdot \text{s}, g = 9.8 \frac{\text{m}}{\text{s}^2}.
 \end{aligned} \tag{21}$$

Choose the interaction force $f_e = [5 \sin(0.2t), 5 \cos(0.2t)]^T$, the robot initial position $q(0) = \dot{q}(0) = [0, 0]^T$, the desired trajectory $q_d = [0.5 + 0.2 \cos(\pi t/3)]^T$, and the desired inertial matrix, the desired damping, the desired stiffness as I , $10I$, and $20I$, respectively. Design the control parameters as $\gamma = 3$, $\alpha_1 = 5$, $\alpha_2 = 3$, $k_1 = 8$, and $k_2 = 10$. Figures 2 and 3 depict the performances of tracking errors e_1, e_2 and the impedance error e_{im} by the proposed super-twisting sliding-mode impedance control in Figure 4. Under the proposed robust impedance controller, the auxiliary tracking errors e_1 and e_2 converge to zero after 10 second, which guarantees the convergence of the impedance error e_{im} . From the simulation results, the super-twisting terminal sliding-mode impedance control effectively improves the impedance control robustness without causing the chattering problem in sliding-mode control.

5. Conclusions

This paper proposes a super-twisting terminal sliding-mode impedance controller for robots to improve the compliance and robustness of robot-environment interaction.

By finite-time control theory, we validate the finite-time control stability and the impedance control robustness by theoretical analysis. The control effectiveness is illustrated by simulations on a two-link robot arm. The main contribution and innovation of this paper lie in the super-twisting terminal sliding-mode impedance controller which improves the impedance control robustness without causing chattering.

The desired impedance dynamics in (2) has infinite-time stability. In the near coming future, we will construct finite-time impedance dynamics and design robust impedance control to achieve the desired finite-time impedance dynamics using sliding-mode control or sliding-mode observer.

Data Availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also form part of an ongoing study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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