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Research Article

The Study of a Predator-Prey Model with Fear Effect Based on State-Dependent Harvesting Strategy

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In presence of predator population, the prey population may significantly change their behavior. Fear for predator population enhances the survival probability of prey population, and it can greatly reduce the reproduction of prey population. In this study, we propose a predator-prey fishery model introducing the cost of fear into prey reproduction with Holling type-II functional response and prey-dependent harvesting and investigate the global dynamics of the proposed model. For the system without harvest, it is shown that the level of fear may alter the stability of the positive equilibrium, and an expression of fear critical level is characterized. For the harvest system, the existence of the semitrivial order-1 periodic solution and positive order-q ($q \geq 1$) periodic solution is discussed by the construction of a Poincaré map on the phase set, and the threshold conditions are given, which can not only transform state-dependent harvesting into a cycle one but also provide a possibility to determine the harvest frequency. In addition, to ensure a certain robustness of the adopted harvest policy, the threshold condition for the stability of the order-q periodic solution is given. Meanwhile, to achieve a good economic profit, an optimization problem is formulated and the optimum harvest level is obtained. Mathematical findings have been validated in numerical simulation by MATLAB. Different effects of different harvest levels and different fear levels have been demonstrated by depicting figures in numerical simulation using MATLAB.

1. Introduction

Prey-predator interaction is a crucial topic in theoretical ecology and evolutionary biology. The history of the study about the prey-predator interactions dates back long. The pioneering work to describe the prey-predator interactions in mathematics belongs to the Lotka-Volterra model [1, 2]. Subsequently the model was improved by adding logistic growth term for the prey and variety of population-dependent response functions [3–15]. A prototype model that captures the prey-predator interaction takes the form

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = bx - \mathrm{d}x - cx^2 - yp(x, y), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = e\phi(p(x, y))y - my, \end{cases} \tag{1}$$

where $x(t)$ and $y(t)$ represent the densities of prey and predator population, respectively, b, d, $(d < b)$ and c represent the birth rate, natural death rate, and density-dependent decay rate due to the intraspecies competition, respectively, $p(x, y)$ represents the functional response, e is the efficiency of conversion, m is natural mortality of predator, and ϕ is a monotonically increasing function.

Due to prey-predator interactions, predators always have an impact, direct, indirect, or both, on prey population. In model (1), the term $yp(x, y)$ models the direct impact of predator on prey by catching and killing behavior. Meanwhile fear of predation risk can be regarded as the indirect impact of predator on prey, and some theoretical ecologists and biologists have realised that a prey-predator model should involve not only direct killing but also the fear [16, 17]. The fieldwork of Zanette et al. [18] on song sparrows observed the impact of fear and found a reduction in reproduction by 40% in the number of offspring due to the fear of predation. Based on this phenomenon, Wang et al. [19] incorporated a predator-dependent fear factor into the birth rate of prey in model (1) (i.e., replace b by $b(y) = b/(1 + ky)$) with linear and Holling type-II functional response to explore the effect of fear on population dynamics. The results show that high level of fear could stabilize the system. Das and Samanta [20] investigated the impact of fear in exponential form on a stochastic prey-predator system when the predator is provided additional food. Sahoo and Samanta [21] investigated a two prey-one predator model by including the cost of fear into prey reproduction and switching mechanism in predation. Das et al. [22] developed and explored a predator-prey model incorporating the cost of perceived fear into the birth and death rates of prey species with Holling type-II functional response. Sarkar and Khajanchi [23] and Kumar and Kumari [24] incorporated a form of fear factor into the birth rate of prey by assuming a nonzero minimum cost of fear. The impact of fear has also been investigated on prey-predator systems with prey refuge [25–27], Allee effect [26], hunting cooperation [28], and additional food resource for predator [20, 29].

The study of resource management including fisheries, forestry, and wildlife management has great importance. It is necessary to harvest the population but harvesting should be regulated in such a way that ecological sustainability as well as conservation of the species can be implemented in a long run. Besides, it is always hoped that the sustained ability can be achieved at a high level of productivity and good economic profit. In the past decade, scholars considered different kinds of harvest on the dynamics of the predator-prey system such as continuous harvesting [30–33] and intermittent harvesting [34–37]. Compared to fixed time harvest strategy, the state-dependent harvesting strategy takes the existing resources of species into full consideration and can maintain the sustainability of species in certain level. State-dependent harvested system can be described by the impulsive semidynamical system [38–45]. Recently, Lai et al. [46] proposed and studied a Lotka-Volterra predator-prey system incorporating both continuous harvesting and fear effect; that is,

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{bx}{1+ky} - \mathrm{d}x - cx^2 - pxy - \dfrac{qEx}{a_1 E + a_2 x}, \\ \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = epxy - my, \end{cases} \tag{2}$$

where k is the level of fear, E is the fishing effort used to harvest, q is the catchability coefficient, and a_1 and a_2 are constants. The harvest in (2) is continuous and in Michaelis-Menten type. However, in reality, the harvest of species should consider the aspect of ecological sustainability as well as conservation. Thus, in most cases, species are caught intermittently, not continuously.

To the best of our knowledge, to this day, still no scholars investigated the dynamic behavior of the predator-prey system incorporating both fear effect and intermittent harvesting, which motivated us to study a predator-prey model incorporating fear effect based on state-dependent harvesting strategy. The aim of this study is to check the influence of fear level on the stability of the positive steady state of the system without harvest. Meanwhile, for the harvest system, it mainly discusses the existence of the order-q ($q > 0$) periodic solution, since it provides a possibility to transform the state-dependent harvesting into a cycle one. Meanwhile, in order to make a maximum economic profit in the harvest process, the optimal control problem is discussed. The organization of this study is as follows. In the next section, we introduce the mathematical model for predator-prey system with fear effect based on state-dependent harvesting strategy. In the same section, we present some preliminaries used in the discussion of the system dynamics. Section 3 is dedicated to the existence and stability of semitrivial order-1 and positive order-1 periodic solution. We also study the existence of order-2 and order-3 periodic solution. In Section 4, we demonstrate different effects of different harvest levels and different fear levels by depicting figures in numerical simulation using MATLAB. The paper concludes in Section 5, in which we briefly summarize the biological indications of our analytical findings.

2. Model Formulation and Preliminaries

2.1. Model Formulation. In presence of predator population, the prey population may significantly change their behavior. Fear for predator population enhances the survival probability of prey population, and it can greatly reduce the reproduction of prey population [23]. In this study, we consider a predator-prey model introducing the cost of fear into prey reproduction with Holling type-II functional response and a saturation function ϕ in equation (1); that is,

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{bx}{1+ky} - \mathrm{d}x - cx^2 - \dfrac{pxy}{1+h_1 x}, \\ \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = \dfrac{epxy}{1+(h_1+h_2 p)x} - my. \end{cases} \tag{3}$$

Where the variables, model parameters, and their units/dimensions are given in Table 1. To achieve the commercial purpose of the fishery, it is necessary to harvest the population in such a way that ecological sustainability as well as conservation of the species can be implemented in a long run. The harvest can be continuous or intermittent. In this work, a state-dependent harvest strategy is considered. Let l be the harvest level of prey population; that is, when the density of prey population reaches level l, the harvest is implemented, resulting in a portion of prey and predator being caught. Let E denote the harvest effort, which is dependent on the harvest level l, and let q_1 and q_2 be the catchability coefficients of prey and predator populations. In addition, to avoid the extinction of predator, it is necessary to release a quantity of predator pups, denoted by τ, which is also dependent on level l. Based on this consideration, the model with state-dependent harvesting takes the following form:

Table 1: The description of the model parameters and variables and their units/dimensions.

Symbol	Description	Units/dimensions
x	Density of prey population	Mass
y	Density of predator population	Mass
b	Birth rate of prey population not affected by predators	1/time
d	Natural death rate of prey population	1/time
c	Decay rate due to intraspecies competition	1/mass.1/time
p	Rate of predation	1/mass.1/time
h_1	Handling time	1/mass
e	Conversion rate of prey biomass to predator biomass	Dimensionless
h_2	Conversion time	1/mass
m	Death rate of predator	1/time
k	Level of fear	1/mass
l	Harvesting level of prey population	Mass
E	Harvesting effort	Dimensionless
q_1	Catchability coefficient of prey	Dimensionless
q_2	Catchability coefficient of predator	Dimensionless
τ	Quantity of predator pups released	Mass

$$\begin{cases} \left.\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \frac{bx}{1+ky} - \mathrm{d}x - cx^2 - \frac{pxy}{1+h_1 x} \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= \frac{epxy}{1+(h_1+ph_2)x} - my \end{aligned}\right\}, & x \neq l, \\ \left.\begin{aligned} \Delta x &= -q_1 E x \\ \Delta y &= -q_2 E y + \tau \end{aligned}\right\}, & x = l. \end{cases} \tag{4}$$

Denote $K \triangleq (b-d)/c$. Then K is the carrying capacity of prey population in absence of predator. System (4) is considered in the domain $S = \{(x,y) | 0 \le x \le K, y \ge 0\}$ for ecological practices. The purpose of this paper is to analyze the dynamics of system (4). Besides, it is always hoped that the harvest can be achieved at a good economic profit, and this requires determining an optimal harvest level l. Next, some preliminaries are listed for the analysis of the harvest model (4).

2.2. Preliminaries. Let us consider a general planar system:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = P(x,y), \quad \dfrac{\mathrm{d}y}{\mathrm{d}t} = Q(x,y), & \text{if } \chi(x,y) \neq 0, \\ \Delta x = \alpha(x,y), \quad \Delta y = \beta(x,y), & \text{if } \chi(x,y) = 0, \end{cases} \tag{5}$$

where $(x,y) \in \Omega \subset \mathbb{R}^2$, and $\chi(x,y) = 0$ describes the states at which the harvest is implemented; α and β describe the effects of the harvest strategy. $P(x,y)$ and $Q(x,y)$ are arbitrarily derivative with respect to $(x,y) \in \Omega$; χ, α, and β are linearly dependent on x and y; that is, $\chi_x, \chi_y, \alpha_x, \alpha_y, \beta_x$, and β_y are constant.

The dynamic system constituted by the solution mapping defined by system (5) is called an impulsive semi-continuous dynamic system, denoted as $(\Omega, \pi; I, M_{\mathrm{IMP}})$, where $\pi = (\pi_1, \pi_2)$: $\Omega \times R \longrightarrow \Omega$, $M_{\mathrm{IMP}} \triangleq \{(x,y) | \chi(x,y) = 0\}$, and

$$\begin{aligned} I: M_{\mathrm{IMP}} \longrightarrow N_{\mathrm{PHA}} &= I(M_{\mathrm{IMP}}) \\ &\triangleq \{(x', y') | x' = x + \alpha(x,y), y' = y + \beta(x,y), (x,y) \in M_{\mathrm{IMP}}\}. \end{aligned} \tag{6}$$

Let $\mathbf{z}(t) = (x(t), y(t))$ be the solution of system (5) with initial value $\mathbf{z}(0) = \mathbf{z}_0$. Denote $\gamma(\mathbf{z}, \mathbf{z}_0) = \{\mathbf{z}(t) | t \ge t_0 \text{ with } \mathbf{z}(t_0) = \mathbf{z}_0\}$, also denoted as $\gamma(\mathbf{z})$ in short. Denote $\mathbf{z}_k = \mathbf{z}(t_k^+) \in \gamma(\mathbf{z})$, where $t_k \in \prod \triangleq \{t_k | k = 1, 2, \ldots\}$ with $\mathbf{z}(t_k) \in M_{\mathrm{IMP}}$.

Definition 1 (priodic solution [47–49]). *The solution* $\mathbf{z} = \widehat{\mathbf{z}}(t)$ *of system* (5) *is said to be periodic if there exists* positive *integer* $m \ge 1$ *such that* $\widehat{z}_m = \widehat{\mathbf{z}}_0$. *Denote* $k \triangleq \min\{m \in \mathbb{N}, \widehat{\mathbf{z}}_m = \widehat{\mathbf{z}}_0\}$; *then orbit* $\gamma(\widehat{\mathbf{z}})$ *is said to be an order-* k *periodic orbit of system* (5).

Definition 2 (orbitally stable [47–49]). *An orbit* $\gamma(\widehat{\mathbf{z}})$ *is said to be orbitally stable if, for any* $\varepsilon > 0$, *there is a neighborhood* V *of* $\widehat{\mathbf{z}}$ *so that, for all* $\mathbf{z}$ *in* V, *there is a reparameterization of time (a smooth, monotonic function)* $\widehat{t}(t)$ *such that* $|\mathbf{z}(t) - \widehat{\mathbf{z}}(\widehat{t}(t))| < \varepsilon$ *for all* $t \ge t_0$.

Definition 3 (asymptotic orbital stability [47–49]). $\gamma(\widehat{\mathbf{z}})$ *is said to be asymptotically orbitally stable if it is orbitally stable and additionally* V *may be chosen so that, for all* $\mathbf{z} \in V$, *there exists a constant* $\tau(\mathbf{z})$ *such that* $|\mathbf{z}(t) - \widehat{\mathbf{z}}(t - \tau(\mathbf{z}))| \longrightarrow 0$ *as* $t \longrightarrow \infty$.

Definition 4 (Poincaré map). *Let* $N_\omega \triangleq \{S \in N_{\mathrm{PHA}} | \exists T_S > 0 \text{ such that } \pi(S, T_S) \in M_{\mathrm{IMP}}\}$. *Define the Poincaré map* ϕ_N*:* $N_\omega \longrightarrow N_{\mathrm{PHA}}$ *as follows:* $\phi_N(S) = S^+ \triangleq I(\pi(S,T_S)) = \pi(S,T_S) + (\alpha(\pi(S,T_S)), \beta(\pi(S,T_S)))$.

Remark 1. If there exists a point $L \in N_\omega$ *and* $q > 0$ *such that* $\phi_N^q(L) = L$ *and* $\phi_N^j(L) \neq L\ (j < q)$*, that is,* L *is a fixed point of* ϕ_N^q*, then system (4) admits an order-q periodic solution.*

Lemma 1 (analogue of Poincaré criterion [47–49]). *The order-qT-periodic solution* $\mathbf{z}(t) = (\xi(t), \eta(t))$ *of system (5) is orbitally asymptotically stable and enjoying the property of the asymptotic phase if the multiplier* μ_2 *satisfies the condition* $|\mu_2^q| < 1$*, where*

$$\mu_2^q = \prod_{j=1}^{q} \Delta_j \exp\left(\int_0^T \left[\frac{\partial P}{\partial x}(\xi(t), \eta(t)) + \frac{\partial Q}{\partial y}(\xi(t), \eta(t)) \right] \mathrm{d}t \right), \tag{7}$$

with

$$\Delta_j = \frac{P_+((\partial\beta/\partial y)(\partial\phi/\partial x) - (\partial\beta/\partial x)(\partial\chi/\partial y) + \partial\chi/\partial x)}{P(\partial\chi/\partial x) + Q(\partial\chi/\partial y)} + \frac{Q_+((\partial\alpha/\partial x)(\partial\chi/\partial y) - (\partial\alpha/\partial y)(\partial\phi/\partial x) + (\partial\chi/\partial y))}{P(\partial\chi/\partial x) + Q(\partial\chi/\partial y)}, \tag{8}$$

and $P_+ = P(\xi(\tau_j^+), \eta(\tau_j^+))$, $Q_+ = Q(\xi(\tau_j^+), \eta(\tau_j^+))$, *and* $P, Q, \partial\alpha/\partial x, \partial\alpha/\partial y, \partial\beta/\partial x, \partial\beta/\partial y, \partial\chi/\partial x, \partial\chi/\partial y$ *are calculated at the point* $(\xi(\tau_j), \eta(\tau_j))$.

3. Main Works

Define

$$e_M \triangleq \frac{m + (h_1 + h_2 p) m K}{pK}. \tag{9}$$

Then e_M is called the critical value of the conversion; that is, when $e \le e_M$, the conversion is not enough to maintain the survival of predator and the predator population will go to extinction. Thus, in this work, it is reasonable to assume that $e > e_M$.

Besides, denote

$$\begin{aligned}
x^* &\triangleq \frac{m}{ep - mh_1 - pmh_2}, \\
y^* &\triangleq \frac{\sqrt{(p + k(d + cx^*)(1 + h_1 x^*))^2 - 4pk(d + cx^* - b)(1 + h_1 x^*)} - p - k(d + cx^*)(1 + h_1 x^*)}{2pk}, \\
k^* &\triangleq -\frac{ph_1[h_1(d + cx^* - b) + c(1 + h_1 x^*)]}{c(1 + h_1 x^*)^2 [c(1 + h_1 x^*) + h_1(d + cx^*)]}, \\
T^* &= x^* \left[\frac{ph_1 y^*}{(1 + h_1 x^*)^2} - c \right], \\
D^* &= \left[\frac{bk}{(1 + ky^*)^2 + p/1 + h_1 x^*} \right] \frac{epx^* y^*}{(1 + h_1 x^* + h_2 p x^*)^2} > 0.
\end{aligned} \tag{10}$$

For system (3), the following result holds.

Theorem 1. *There are three equilibria for system (3) when* $e > e_M$*: two boundary saddles* $O(0,0)$ *and* $E(K,0)$ *and one positive equilibrium* $E^*(x^*, y^*)$*. Moreover, one of the two following cases holds:*

(i) $E^*(x^*, y^*)$ *is a stable focus or node in case of* $k > k^*$

(ii) $E^*(x^*, y^*)$ *is unstable in case of* $k < k^*$*, and a unique stable limit cycle exists, denoted by* Γ_{LC}

Proof. The Jacobian matrix $J(\overline{E})$ of model (3) at the equilibrium $\overline{E}(\overline{x}, t\overline{y})$ is

$$\begin{pmatrix} \dfrac{b}{1 + k\overline{y}} - d - 2c\overline{x} - \dfrac{p\overline{y}}{(1 + h_1\overline{x})^2} & -\dfrac{kb\overline{x}}{(1 + k\overline{y})^2} - \dfrac{p\overline{x}}{1 + h_1\overline{x}} \\ \dfrac{ep\overline{y}}{(1 + h_1\overline{x} + h_2 p\overline{x})^2} & \dfrac{ep\overline{x}}{1 + h_1\overline{x} + h_2 p\overline{x}} - m \end{pmatrix}. \tag{11}$$

It is obvious that $O(0,0)$ and $E(K,0)$ are saddles. At the equilibrium $E^*(x^*,y^*)$, the characteristic equation $\lambda^2 - T^*\lambda + D^* = 0$. If $y^* < y_M \triangleq a(1+h_1x^*)^2/ph_1$ holds, then $T^* < 0$. By equation (6), $y^* < y_M$ if and only if $k > k^*$. Thus, the positive equilibrium $E^*(x^*,y^*)$ is locally asymptotically stable in case of $k > k^*$ and unstable in case of $k < k^*$. In this case, there exists a unique stable limit cycle for system (3). □

3.1. Semitrivial Order-1 Periodic Solution for $\tau = 0$. When $\tau = 0$, there is $y(t) \equiv 0$ for $t > 0$ with $y(0) = 0$. In this case, system (3) is reduced to the following system:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = (b-d)x\left(1-\dfrac{x}{K}\right), & x \neq l, \\ \Delta x = -q_1 E x, & x = l. \end{cases} \tag{12}$$

Setting $x_0 = (1-q_1E)l$, the solution of equation $\mathrm{d}x/\mathrm{d}t = (b-d)x(1-x/K)$ with $x(0) = x_0$ is

$$x(t) = \frac{K(1-q_1E)l\,\exp((b-d)t)}{(K-(1-q_1E)l) + (1-q_1E)l\,\exp((b-d)t)}. \tag{13}$$

Let

$$T \triangleq \frac{1}{r}\ln\left(\frac{K-(1-q_1E)l}{(1-q_1E)(K-l)}\right). \tag{14}$$

Then there is $x(T) = l$ and $x(T^+) = (1-q_1E)l = x_0$ by impulse effect. Thus, the following result holds.

Theorem 2. *System (4) with $\tau = 0$ has a semitrivial order-1 periodic solution for $(n-1)T < t \le nT$:*

$$\begin{cases} \overline{\xi}(t) = \dfrac{K(1-q_1E)l\,\exp((b-d)(t-(n-1)T))}{(K-(1-q_1E)l) + (1-q_1E)l\,\exp((b-d)(t-(n-1)T))}, \\ \overline{\eta}(t) = 0, \end{cases} \tag{15}$$

which is orbitally asymptotically stable when $R_0 < 1$, where

$$R_0 \triangleq (1-q_2E)\left(\frac{(1+(h_1+h_2p)l)(K-(1-q_1E)l)}{(1+(1-q_1E)(h_1+h_2p)l)(K-l)}\right)^{ep/c(1+K(h_1+h_2p))}\left(\frac{(1-q_1E)(K-l)}{K-(1-q_1E)l}\right)^{m/(b-d)}. \tag{16}$$

Proof. To discuss the stability of $(\overline{\xi}(t),\overline{\eta}(t))$, let us consider a small disturbance δ_0. The trajectory starting from $B_1((1-q_1E)l,\delta_0)$ is denoted by $(\widehat{\xi}(t),\widehat{\eta}(t))$. This disturbed trajectory first intersects the harvest set M_{IMP} at point $B_2(l,y_1)$ when $t = T+\delta t$, and then it jumps to point $B_2^+((1-q_1E)l,\delta_1)$. Thus, there is

$$\begin{aligned} \widehat{\xi}(0) &= (1-q_1E)l, \\ \widehat{\eta}(0) &= \delta_0, \\ \widehat{\xi}(T+\delta t) &= l, \\ \widehat{\eta}(T+\delta t) &= y_1. \end{aligned} \tag{17}$$

Let $\delta x = \widehat{\xi}(t) - \overline{\xi}(t)$ and $\delta y = \widehat{\eta}(t) - \overline{\eta}(t)$. Then $\delta x_0 = \widehat{\xi}(0) - \overline{\xi}(0) = 0$, and $\delta y_0 = \widehat{\eta}(0) - \overline{\eta}(0) = \delta_0$. Setting $\delta y_1 \triangleq y_1$, for $0 < t < T$, the variables δx and δy can be expressed by the relation

$$\begin{pmatrix} \delta x(t) \\ \delta y(t) \end{pmatrix} = \Phi(t)\begin{pmatrix} 0 \\ \delta_0 \end{pmatrix} + o\left(\delta_0^2\right), \tag{18}$$

where $\Phi(t) = (\phi_{ij})_{2\times 2}$ is the fundamental solution satisfying the variation equation.

$$\begin{aligned} \Phi'(t) &= \begin{pmatrix} b-d-2c\overline{\xi}(t) & -kb\overline{\xi}(t) - \dfrac{p\overline{\xi}(t)}{1+h_1\overline{\xi}(t)} \\ 0 & \dfrac{ep\overline{\xi}(t)}{1+(h_1+ph_2)\overline{\xi}(t)} - m \end{pmatrix}\Phi(t), \\ \Phi(0) &= I_2. \end{aligned} \tag{19}$$

According to the first-order Taylor expansion on $\widehat{\eta}(t)$, there is $\delta y_1 = \widehat{\eta}(T+\delta t) \approx \delta y(T) = \phi_{22}(T)\delta_0$, where

$$\phi_{22}(T) = \left(\frac{(1+(h_1+h_2p)l)(K-(1-q_1E)l)}{(1+(1-q_1E)(h_1+h_2p)l)(K-l)}\right)^{ep/c(1+K(h_1+h_2p))}\left(\frac{(1-q_1E)(K-l)}{K-(1-q_1E)l}\right)^{m/(b-d)}. \tag{20}$$

By impulse effect, there is $\delta_1 = (1 - q_2 E)\,\delta y_1 = ((1 - q_2 E)\phi_{22}(T)\delta_0$. Thus, if inequality (10) holds, there is $\delta_1 < \delta_0$. By the arbitrary of δ_0, it concludes that the order-1 semitrivial periodic solution is orbitally asymptotically stable. □

Corollary 1. *The semitrivial order-1 periodic solution $(\overline{\xi}(t), \overline{\eta}(t))$ is orbitally asymptotically stable if one of the two following cases is satisfied: (i) $l \le x^*$ and (ii) $l > x^*$ and $E > E^* \triangleq \max\{0, (\phi_{22}(T) - 1)/\phi_{22}(T)q_2\}$.*

3.2. Positive Order-K Periodic Solution for $\tau > 0$.

Since the harvest may cause the extinction of predator when $\tau = 0$, in order to keep the predator species from going extinct, it is necessary to reduce the harvest strength and release a certain quantity of predator pups.

For $0 \le x \le K$, define

$$y_{\widetilde{L}}(x) \triangleq \frac{\sqrt{[p + k(d + cx)(1 + h_1 x)]^2 + 4pk(b - d - cx)(1 + h_1 x)} - [p + k(d + cx)(1 + h_1 x)]}{2pk}. \tag{21}$$

Let N_0 and M_0 denote the intersection point between $y = y_{\widetilde{L}}(x)$ and the phase set N_{PHA} and the harvest set M_{IMP}, respectively; G_0 denotes the intersection point between $y = \tau$ and the phase set N_{PHA}; in general $\tau \le \tau_{\max} \triangleq y_{N_0} = y_{\widetilde{L}}((1 - q_1 E)l)$. For a point S on N_{PHA} with $0 \le y_s \le y_{N_0}$, if the trajectory of system (4) starting from $S((1 - q_1 E)l, y_s)$ intersects the harvest set M_{IMP}, then it defines a function relationship between y and x for $(1 - q_1 E)l \le x \le l$, denoted by $y = y(x, y_s)$, which satisfies

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{epxy/1 + (h_1 + ph_2)x - my}{bx/1 + ky - dx - cx^2 - pxy/1 + h_1 x} \\ &\doteq \kappa(x, y), \\ y((1 - q_1 E)l, y_s) &= y_s. \end{aligned} \tag{22}$$

By equation (22), the function $y = y(x, y_s)$ can be expressed as follows:

$$y(x, y_s) = y_s + \int_{(1-q_1 E)l}^{x} \kappa(u, y(u, y_s))\mathrm{d}u. \tag{23}$$

Define $\underline{l} \triangleq \max\{l \mid y(l, y_{N_0}) \le y_{\widetilde{L}}(l)\}$. When $l \le \underline{l}$, the trajectory of system (4) starting from N_0 will intersect the harvest set M_{IMP}, and denote the intersection point by N_1; that is, $N_1 = \pi(N_0, T_0)$ for some $T_0 > 0$. Define $\tau_f \triangleq y_{\widetilde{L}}((1 - q_1 E)l) - (1 - q_2 E)\pi_2(N_0, T_0)$.

3.2.1. Existence of Order-1 Periodic Solution.

By Theorem 1, the dynamic behavior of system (3) varies with the model parameter k. Thus, the discussions will be divided according to parameter k and harvest level l.

Case I: $k > k^*$

Case I-1: $l \le x^*$

Since $x \equiv (1 - q_1 E)l$ on N_{PHA}, map ϕ_N in Definition 4 is only a function of y. Next, the Poincaré map ϕ_N will be characterized and its main property will be analyzed.

For $0 \le y_s \le y_{N_0} = y_{\widetilde{L}}((1 - q_1 E)l)$, by Definition 4, there is $\phi_N(y_s) = (1 - q_1 E)y(l, y_s) + \tau$. Meanwhile, for $y_s > y_{N_0}$, there exists a unique $y_s' \in (0, y_{N_0})$ and $T_s > 0$ such that $y_s' = \pi_2(((1 - q_1 E)l, y_s), T_s)$. Then $\phi_N(y_s) = (1 - q_2 E)y(l, \pi_2(((1 - q_1 E)l, y_s), T_s)) + \tau$. To sum up, there is

$$\phi_N(y_s) = \begin{cases} (1 - q_2 E)y(l, y_s) + \tau, & 0 \le y_s \le y_{N_0}, \\ (1 - q_2 E)y(l, \pi_2(((1 - q_1 E)l, y_s), T_s)) + \tau, & y_s > y_{N_0}. \end{cases} \tag{24}$$

Property 1. For system (4), when $l \le x^*$, the Poincaré map ϕ_N defined by equation (24) has the following properties:

(i) *ϕ_N is continuous on $[0, +\infty)$. Moreover, ϕ_N is increasing on $[0, y_{\widetilde{L}}((1 - q_1 E)l)]$ and decreasing on $(y_{\widetilde{L}}((1 - q_1 E)l), +\infty)$*

(ii) *ϕ_N is continuously differentiable on $[0, +\infty)$, and ϕ_N is concave on $[0, y_{\widetilde{L}}((1 - q_1 E)l)]$*

(iii) *There exists a horizontal asymptote $\phi_N = \tau$; that is, $\phi_N(y_s) \longrightarrow \tau$ when $y_s \longrightarrow +\infty$*

Define $f_N(y) \triangleq \phi_N(y) - y$. The following result holds.

Theorem 3. *There exists a unique positive order-1 periodic solution for system (4) when $0 < l \le x^*$ and $0 < \tau \le \tau_{\max}$.*

Proof. By Remark 1, the existence of order-1 periodic solution is equivalent to the existence of a point $L \in N_{\text{PHA}}$ such that y_L is a fixed point of ϕ_N. By Property 1 (i), f_N is continuous on $[0, +\infty)$. Since $f_N(0) = \phi_N(0) = \tau > 0$ and $f_N(y) = \phi_N(y) - y \longrightarrow -\infty$ as $y \longrightarrow +\infty$, by the intermediary property of continuous function, there exists at least one $y_L > 0$ such that $f_N(y_L) = 0$; that is, $\phi_N(y_L) = y_L$. Thus, the trajectory of system (4) starting from $L((1-q_1E)l, y_L)$ forms an order-1 periodic orbit.

Next, the location and uniqueness of the order-1 periodic orbit will be analyzed. By Property 1 (i), ϕ_N achieves its maximum at $y = y_{N_0} = y_{\tilde{L}}((1-q_1E)l)$. It is obvious that $f_N(\tau) > 0$.

If $\tau = \tau_f$, then $\phi_N(y_{N_0}) = y_{N_0}$.

If $\tau < \tau_f$, then $f_N(y_{N_0}) < 0$, which means that $y_L \in (\tau, y_{N_0})$, as shown in Figure 1(a). Since $\phi_N(y)$ is concave on $[0, y_{N_0}]$, y_L is unique.

If $\tau > \tau_f$, there is $f_N(y_{N_0}) > 0$; that is, $\phi_N(y_{N_0}) > y_{N_0}$. Since ϕ_N is decreasing on $[y_{N_0}, +\infty)$, $\phi_N(\phi_N(y_{N_0})) < \phi_N(y_{N_0})$; that is, $f_N(\phi_N(y_{N_0})) < 0$. Besides, define $y_\tau^1 \triangleq \max\{y|\phi_N(y) = y_{N_0}\}$ and $y_\tau^2 \triangleq \min\{y|\phi_N(y) = y_{N_0}\}$. Denote $y_{\min} \triangleq \min\{y_\tau^1, \phi_N(y)\}$. Then there is $y_L \in (y_{\tilde{L}}((1-q_1E)l), y_{\min})$ and y_L is unique, as shown in Figure 1(b)). □

Case I-2: $l > x^*$: in case of $x^* < l \leq \underline{l}$, the trajectory of system (4) starting from N_0 will intersect the harvest set M_{IMP}. When $l > \underline{l}$, the trajectory starting from point N_0 does not intersect the harvest set M_{IMP}. Denote $y_{M_2} = \max\{y|\phi_N(y) = y_{M_0}\}$ and $y_{M_1} = \min\{y|\phi_N(y) = y_{M_0}\}$. Then the domain of ϕ_N is $[0, y_{M_1}] \cup t[y_{M_2}, +\infty)$. Define $\tau_{M_1} \triangleq y_{M_1} - (1-q_2E)y_{\tilde{L}}(l)$ and $\tau_{M_2} \triangleq y_{M_2} - (1-q_2E)y_{\tilde{L}}(l)$.

Theorem 4. *There exists a positive order-1 periodic solution for system (4) when (i) $l \leq \underline{l}$ and $\tau \in (0, \tau_{\max}]$ or (ii) $l > \underline{l}$ and $\tau \in (0, \tau_{M_1}] \cup [\tau_{M_2}, \tau_{\max}]$.*

Proof. When $x^* < l \leq \underline{l}$, similar to the proof of Theorem 3, system (4) admits an order-1 periodic solution. For $l > \underline{l}$, if $y_{M_1} > (1-q_2E)y_{\tilde{L}}(l)$, then, for $\tau \in (0, \tau_{M_1}]$, there is $\phi_N(y_{M_1}) \leq y_{M_1}$. Combining with $\phi_N(y_{G_0}) > y_{G_0}$, it can be concluded that system (4) admits an order-1 periodic solution. For $\tau \in [\tau_{M_2}, \tau_{\max}]$, there is $\phi_N(y_{M_2}) \geq y_{M_2}$ and $\phi_N(y_{\phi_N(y_{M_2})}) < \phi_N(y_{M_2})$; thus, there exists $y_L \in [y_{M_2}, \phi_N(y_{M_2})]$ such that $\phi_N(y_L) = y_L$; that is, system (4) admits an order-1 periodic solution. □ □

Case II: $k < k^*$: in this case, the trajectory of system (4) starting from N_0 will intersect the harvest set M_{IMP}.

Theorem 5. *There exists a positive order-1 periodic solution for system (4) when (i) $l \leq \underline{l}$ and $\tau \in (0, \tau_{\max}]$ or (ii) $l > \underline{l}$ and $\tau \in (0, \tau_{M_1}]$. Moreover, the order-1 periodic solution is unique when $l \leq x^*$.*

3.2.2. Stability of the Order-1 Periodic Solution. Let $(\xi(t), \eta(t))(0 \leq t \leq T)$ be an order-1 periodic solution of system (4). Denote $\xi_0 = \xi(0)$, $\xi_1 = \xi(T)$, $\eta_0 = \eta(0)$, and $\eta_1 = \eta(T)$. Define

$$\Theta(\eta_1) \triangleq \ln\left(\frac{(1-q_2E)\eta_1 + \tau}{(1-q_2E)\eta_1} \frac{(b/1+k\eta_1) - d - cl - (p\eta_1/1+h_1l)}{\left|(b/1+k((1-q_2E)\eta_1+\tau)) - d - c(1-q_1E)l - (p((1-q_2E)\eta_1+\tau)/1+h_1(1-q_1E)l)\right|}\right). \quad (25)$$

Theorem 6. *The order-1T-periodic solution $(\xi(t), \eta(t))$ is orbitally asymptotically stable if*

$$\int_{0^+}^{T}\left[\frac{ph_1\xi(t)\eta(t)}{(1+h_1\xi(t))^2} - c\xi(t)\right]dt < \Theta(\eta_1). \quad (26)$$

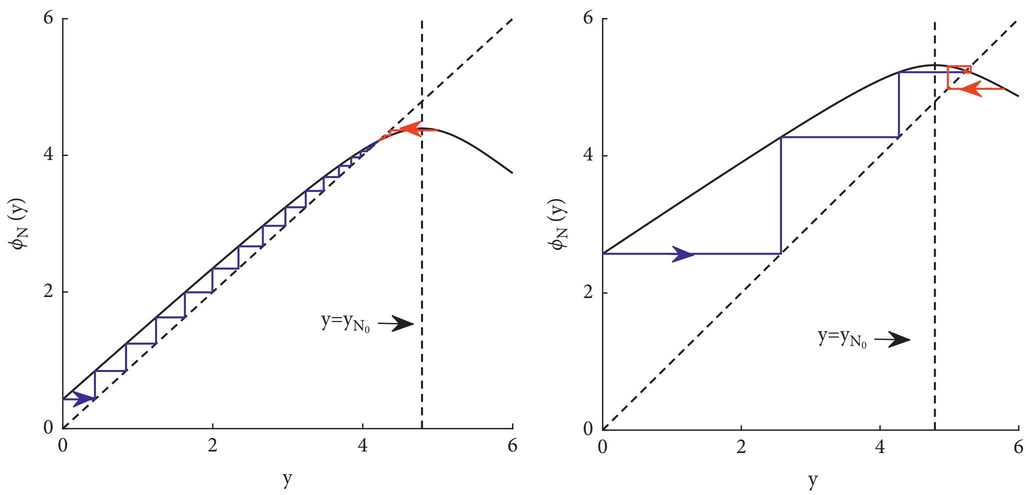
Proof. In system (4), there is

$$\begin{aligned} P(x,y) &= \frac{bx}{1+ky} - dx - cx^2 - \frac{pxy}{1+h_1x}, \\ Q(x,y) &= \frac{epxy}{1+(h_1+ph_2)x} - my, \\ \chi(x,y) &= x - l, \\ \alpha(x,y) &= -q_1Ex, \\ \beta(x,y) &= -q_2Ey + \tau. \end{aligned} \quad (27)$$

Then

$$\begin{aligned} \frac{\partial P}{\partial x} &= \frac{b}{1+ky} - d - 2cx - \frac{py}{(1+h_1x)^2}, \\ \frac{\partial Q}{\partial y} &= \frac{epx}{1+(h_1+ph_2)x} - m, \\ \frac{\partial \chi}{\partial x} &= 1, \\ \frac{\partial \chi}{\partial y} &= 0, \\ \frac{\partial \alpha}{\partial x} &= -q_1E, \\ \frac{\partial \alpha}{\partial y} &= 0, \\ \frac{\partial \beta}{\partial x} &= 0, \\ \frac{\partial \beta}{\partial y} &= -q_2E. \end{aligned} \quad (28)$$

With a direct calculation, there is



(a)

(b)

Figure 1: Illustration of the Poincaré map ϕ_N for different values of τ. The parameters are taken as $b = 0.7$, $d = 0.2$, $c = 0.005$, $p = 0.1$, $h_1 = 0.036$, $e = 0.44$, $h_2 = 1.44$, $m = 0.2$, $q_1 = 0.8$, $q_2 = 0.6$, and $l = 25\%K$. (a) $\tau = 0.4286$; (b) $\tau = 2.5714$.

$$\frac{P^I_+\left[\left(1+\beta_y'\right)\chi_x'-\beta_x'\chi_y'\right]+Q^I_+\left[\left(1+\alpha_x'\right)\chi_y'-\alpha_y'\chi_x'\right]}{P^I\chi_x'+Q^I\chi_y'} = \left(1-q_2E\right)\frac{P\left(\left(1-q_1E\right)l,\left(1-q_2E\right)y_{L^-}+\tau\right)}{P\left(l,y_{L^-}\right)}, \tag{29}$$

and

$$\begin{aligned}\int_{0^+}^{T}\left(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}\right)_{(\xi(t),\eta(t))}\mathrm{d}t &= \int_{0^+}^{T}\left[\xi\frac{\mathrm{d}t}{\xi}+\frac{\eta\mathrm{d}t}{\eta}\right]+\int_{0^+}^{T}\left[\frac{ph_1\xi\eta}{\left(1+h_1\xi\right)^2}-c\xi\right]\mathrm{d}t \\ &= \ln\left(\frac{\xi(T)}{\xi(0)}\right)+\ln\left(\frac{\eta(T)}{\eta(0)}\right)+\int_{0^+}^{T}\left[\frac{ph_1\xi\eta}{\left(1+h_1\xi\right)^2}-c\xi\right]\mathrm{d}t.\end{aligned} \tag{30}$$

Thus, in case of (26), there is $\mu_2^1 < 1$; then, by Lemma 1, the order-1 T-periodic solution $(\xi(t), \eta(t))$ is orbitally asymptotically stable. □

Theorem 7. *For $l \le x^*$, if $\tau \le \tau_f$, the order-1 periodic solution for system (4) is globally orbitally asymptotically stable.*

Proof. By Theorem 3, system (4) admits a unique order-1 periodic solution when $l \le x^*$. If $\tau \le \tau_f$, there exists a unique $y_L \in (\tau, y_{N_0})$ such that $\phi_N(y_L) = y_L$. Thus, for any y_0, a sequence $\{y_k\}_{k=1,2,\ldots}$ is obtained under ϕ_N; that is, $y_k = \phi_N(y_{k-1})$. If $y_0 < y_L$, then $\{y_k\}$ is a monotonically increasing sequence with $y_k < y_L$, so the limit is y_L. Similarly, if $y_0 \in (y_L, y_{N_0}]$, then $\{y_k\}$ is a monotonically bounded decreasing sequence, and the limit is y_L. If $y_0 > y_{N_0}$, then $y_1 = \phi_N(y_0) \in (0, y_{N_0})$; thus $\{y_k\}_{k=1,2,\ldots}$ is a monotonically bounded sequence with limit y_L. To sum up, by the arbitrariness of y_0, the order-1 periodic solution is globally attractive and so is globally orbitally asymptotically stable. □

3.2.3. Existence of Order-q $(q \ge 2)$ Periodic Solution. For $l \le x^*$, by Theorem 3, if $\tau \le \tau_f$, the order-1 periodic solution is orbitally asymptotically stable and globally attractive, which means that system (4) does not admit order-q $(q \ge 2)$ periodic solution. For $\tau > \tau_f$, there exists unique $y_{L_2} \in (y_{N_0}, \phi_N(y_{N_0}))$ such that $\phi_N(y_{L_2}) = y_{L_2}$. Let $y_{L_1} \in (0, y_{N_0})$ such that $\phi_N(y_{L_1}) = y_{L_2}$. Then $\phi_N^2(y_{L_1}) = \phi_N(y_{L_2}) = y_{L_2}$. Meanwhile, let $y_{N_1} \in (0, y_{L_1})$ and $y_{N_2} \in (y_{L_2}, +\infty)$ such that $\phi_N(y_{N_1}) = \phi_N(y_{N_2}) = y_{N_0}$.

Theorem 8. *For $l \le \underline{l}$ and $\tau > \tau_f$, if (i) $\phi_N^2(y_{N_0}) < y_{N_0}$ or (ii) $\phi_N^2(y_{N_0}) \ge y_{N_0}$ and $\mu_2^1 > 1$ holds, system (4) admits an order-2 periodic solution.*

Proof. Obviously, $\phi_N^2(y_{N_1}) = \phi_N^2(y_{N_2}) = \phi_N(y_{N_0})$. It can be easily checked that $\phi_N^2(y)$ is increasing on $[0, y_{N_1}]$ and $[y_{N_0}, y_{N_2}]$, and $\phi_N^2(y)$ is decreasing on $[y_{N_1}, y_{N_0}]$ and $[y_{N_2}, +\infty)$.

(i) $\phi_N^2(y_{N_0}) < y_{N_0}$. In this case, there is $\phi_N(y_{N_0}) > y_{N_2}$; that is, $\phi_N^2(y_{N_2}) > y_{N_2}$; then $\phi_N^2(\phi_N(y_{N_0})) < \phi_N(y_{N_0})$. Besides, there is $\phi_N^2(y_{L_1}) = y_{L_2} > y_{L_1}$. Thus,

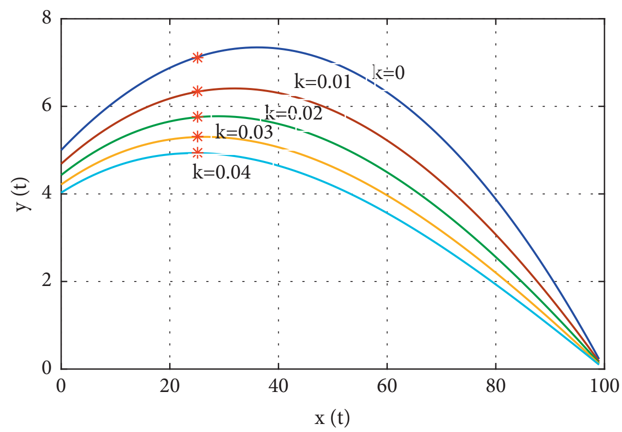


FIGURE 2: The dependence of isoline $\mathrm{d}x/\mathrm{d}t = 0$ on x for different fear level k with parameters given in numerical section. The red asterisk represents the positive equilibrium. For $k = 0, 0.01, 0.02, 0.03$, the positive equilibrium E^* is unstable and, for $k = 0.04$, the positive equilibrium E^* is locally asymptotically stable.

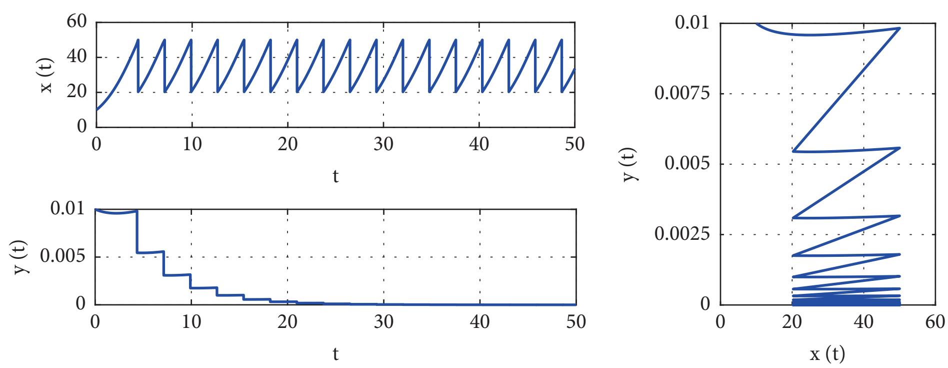


FIGURE 3: The time series evolution of prey population $x(t)$, predator population $y(t)$, and the phase portrait diagram of model (4) for $\tau_2 = 0$ with parameters given in numerical section. The phase portrait diagram shows that the semitrival order-1 periodic solution is orbitally asymptotically stable and predator population tends to extinction.

there exist $y_{P_1} \in (y_{L_1}, y_{N_0})$ and $y_{P_2} \in [y_{N_2}, \phi_N(y_{N_0})]$ such that $\phi_N^2(y_{P_1}) = y_{P_1}$ and $\phi_N^2(y_{P_2}) = y_{P_2}$. Moreover, there are $\phi_N(y_{P_1}) = y_{P_2}$ and $\phi_N(y_{P_2}) = y_{P_1}$.

(ii) $\phi_N^2(y_{N_0}) \geq y_{N_0}$. In this case, there is $\phi_N(y_{N_0}) \leq y_{N_2}$; that is, $\phi_N^2(y_{N_2}) \leq y_{N_2}$. For any $y \in [y_{N_0}, \phi_N(y_{N_0})]$, there is $y_{N_0} \leq \phi_N^2(y_{N_0}) \leq \phi_N(y) \leq \phi_N(y_{N_0})$. Next, it mainly discusses the property of ϕ_N on $[y_{N_0}, \phi_N(y_{N_0})]$. Let $y_0 = y_{N_0}$. Then $y_1 = \phi_N(y_0) = \phi_N(y_{N_0}) > y_{L_2} > y_{N_0}$, $y_2 = \phi_N(y_1) = \phi_N^2(y_0) > y_0$, and $y_3 = \phi_N(y_2) < \phi_N(y_0) = y_1$. Then, under ϕ_N, a sequence $\{y_k\}$ is obtained, where

$$y_0 < y_2 < y_4 < \cdots < y_{L_2} < \cdots < y_5 < y_3 < y_1. \tag{31}$$

Denote $y_{P_1} = \lim_{k\longrightarrow\infty} y_{2k}$ and $y_{P_2} = \lim_{k\longrightarrow\infty} y_{2k+1}$. It is obvious that $y_{P_1} \leq y_{L_2} \leq y_{P_2}$. Since $\mu_2^1 > 1$, then $y_{P_1} < y_{L_2} < y_{P_2}$. Moreover, there is $\phi_N(y_{P_1}) = y_{P_2}$ and $\phi_N(y_{P_2}) = y_{P_1}$. It can be concluded that $\mu_2^2 < 1$, that is, the order-2 periodic solution is orbitally asymptotically stable and globally attractive. □

Theorem 9. *For* $l \leq \underline{l}$ *and* $\tau > \tau_f$*, system (4) admits an order-3 periodic solution if and only if* $\phi_N^2(y_{N_0}) \leq y_{N_1}$*. Moreover, there is at least one order-3 periodic solution when* $\phi_N^2(y_{N_0}) = y_{N_1}$*, and there are at least two order-3 periodic solutions when* $\phi_N^2(y_{N_0}) < y_{N_1}$*.*

Proof. "Necessity." Proof by contradiction. Assume that $\phi_N^2(y_{N_0}) > y_{N_1}$.

Since $\phi_N^2(y_{N_0}) < y_{L_2}$, if $\phi_N^2(y_{N_0}) \geq y_{N_0}$, system (4) admits a stable order-1 periodic solution or a stable order-2 periodic solution. Moreover, there is $\phi_N(y_{N_0}) \leq y_{N_2}$. If $\tau \leq y_{N_1}$, that is, $\phi_N(\tau) \leq y_{N_0}$, there exist $y_{R_1} \in [0, y_{N_1}]$ and $y_{R_4} \in (y_{N_2}, +\infty)$ such that $\phi_N^2(y_{R_4}) = \phi_N^2(y_{R_4}) = y_{N_0}$. It can be easily checked that ϕ_N^3 is increasing on $[0, y_{R_1}]$, $[y_{N_1}, y_{N_0}]$, and $[y_{N_2}, y_{R_4}]$, and ϕ_N^3 is decreasing on $[y_{R_1}, y_{N_1}]$, $[y_{N_0}, y_{N_2}]$, and $[y_{R_4}, +\infty)$. If $\tau > y_{N_1}$, then R_1 does not exist, ϕ_N^3 is increasing on $[y_{N_1}, y_{N_0}]$ and $[y_{N_2}, y_{R_4}]$, and ϕ_N^3 is decreasing on $[0, y_{N_1}]$, $[y_{N_0}, y_{N_2}]$, and $[y_{R_4}, +\infty)$. In any case, since $\phi_N^3(y_{N_1}) = \phi_N^2(y_{N_0}) \geq y_{N_0} > y_{N_1}$, $\phi_N^3(y_{N_0}) > \phi_N(y_{L_2}) = y_{N_0}$, and $\phi_N^3(y_{R_4}) = \phi_N(y_{N_0}) < y_{R_4}$, it

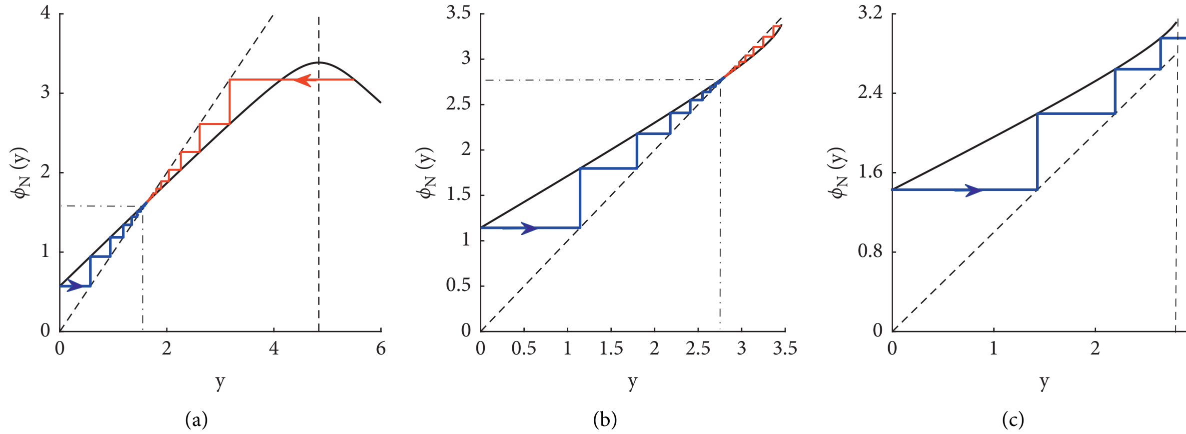


Figure 4: Illustration of the Poincaré map ϕ_N of system (4) for $\tau_2 = 2$ and different harvest level l with parameters given in numerical section: (a) $l = 30\%K$; (b) $l = 50\%K$; (c) $l = 60\%K$. The dotted line represents $v = y$ and the intersection point is the fixed point of the Poincaré map ϕ_N. For $l = 30\%K$ and $l = 50\%K$, the Poincaré map ϕ_N has a unique fixed point; that is, system (4) admits a unique order-1 periodic solution. For $l = 60\%K$, the Poincaré map ϕ_N does not have fixed point; that is, system (4) does not admit order-1 periodic solution.

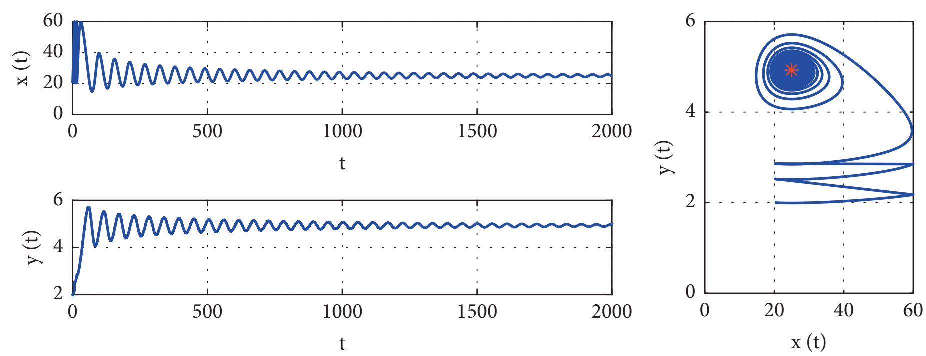


Figure 5: The time series evolution of prey population $x(t)$, predator population $y(t)$, and the phase portrait diagram of model (4) for $l = 60\%K$ and $\tau_2 = 2$ with parameters given in numerical section. The phase portrait diagram shows that the trajectory of system (4) will eventually tend to the positive equilibrium E^*. In this case, the positive equilibrium E^* is globally asymptotically stable.

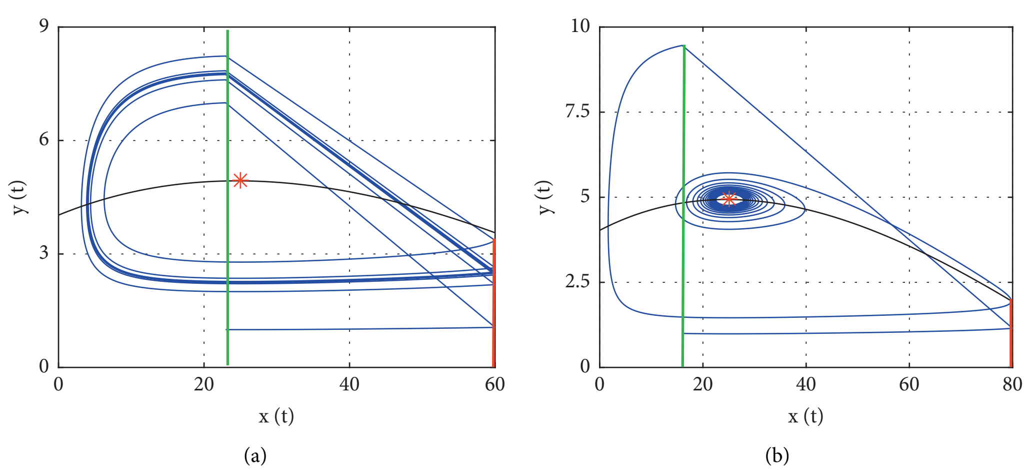


Figure 6: The phase portrait diagram of model (4) for $\tau_2 = 9$ and different harvest level l with parameters given in numerical section: (a) $l = 60\%K$; (b) $l = 80\%K$. For $l = 60\%K$, there is $\tau_2 > \tau_{M_2}$, and system (4) admits an order-1 periodic solution; for $l = 80\%K$, there is $\tau_2 \in (\tau_{M_1}, \tau_{M_2})$, and the trajectory of system (4) will tend to the positive equilibrium E^* after finite impulses.

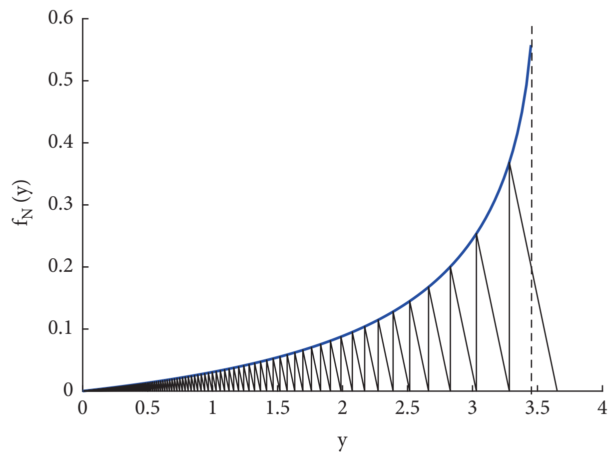


FIGURE 7: Illustration of the successor function $f_N(y) = \phi_N(y) - y$ for $q_2 = 0$ with parameters given in numerical section. The result indicates that the function is always greater than zero, which means that the Poincaré map ϕ_N of system (4) does not have fixed point and the trajectory will tend to the positive equilibrium E^*.

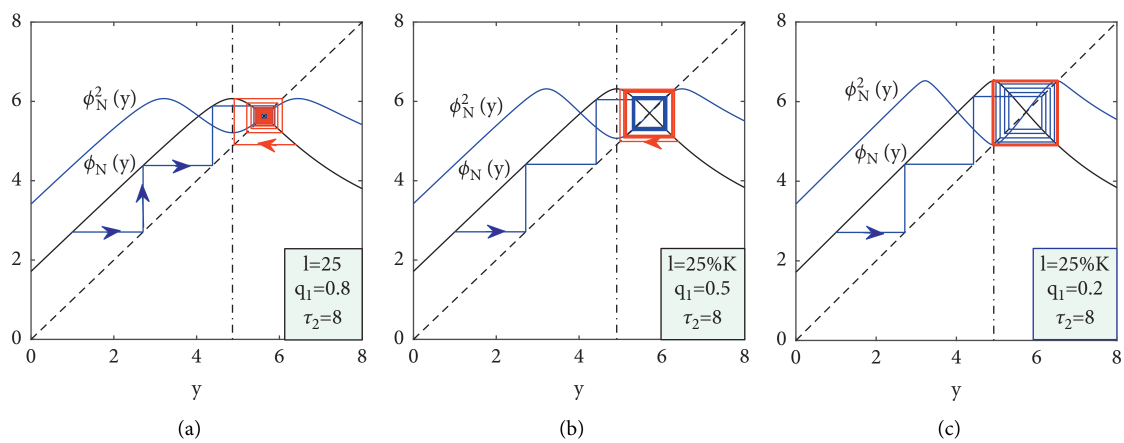


FIGURE 8: Illustration of the Poincaré maps ϕ_N and ϕ_N^2 of system (4) for $l = 25\%K$ and $\tau_2 = 8$ and different values of q_1 with parameters given in numerical section. For $q_1 = 0.8$, system (4) admits a unique orbitally asymptotically stable order-1 periodic solution; for $q_1 = 0.5$, there is $\phi_N^2(4.92) > 4.92$ and $\mu_1 > 1$; in this case, system (4) admits a stable order-2 periodic solution; (c) for $q_1 = 0.2$, there is $\phi_N^2(4.92) < 4.92$, and system (4) admits a stable order-2 periodic solution.

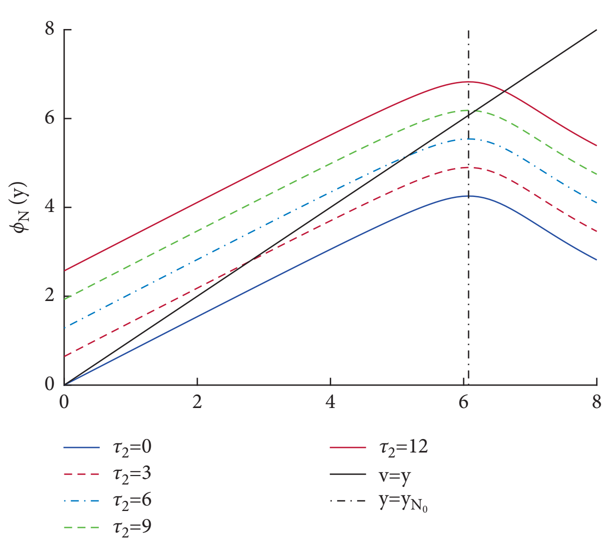


FIGURE 9: Illustration of the Poincaré map ϕ_N of system (4) for $l = 25\%K$ and different values of τ_2 with parameters given in numerical section. The results indicate that ϕ_N has a unique fixed point for any τ_2; that is, system (4) admits a unique order-1 periodic solution.

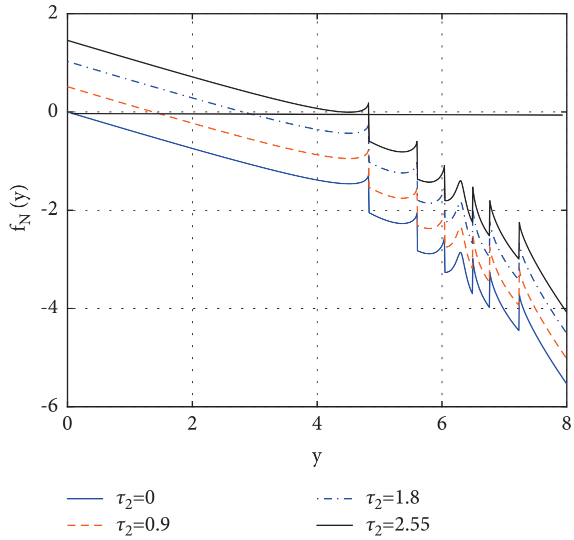


Figure 10: Illustration of the successor function $f_N(y)$ of system (4) for $l = 50\%K$ and different values of τ_2 with parameters given in numerical section. The results indicate that $f_N(y)$ has a zero point in case of $\tau_2 < 2.55$; that is, system (4) admits an order-1 periodic solution when $\tau_2 < 2.55$.

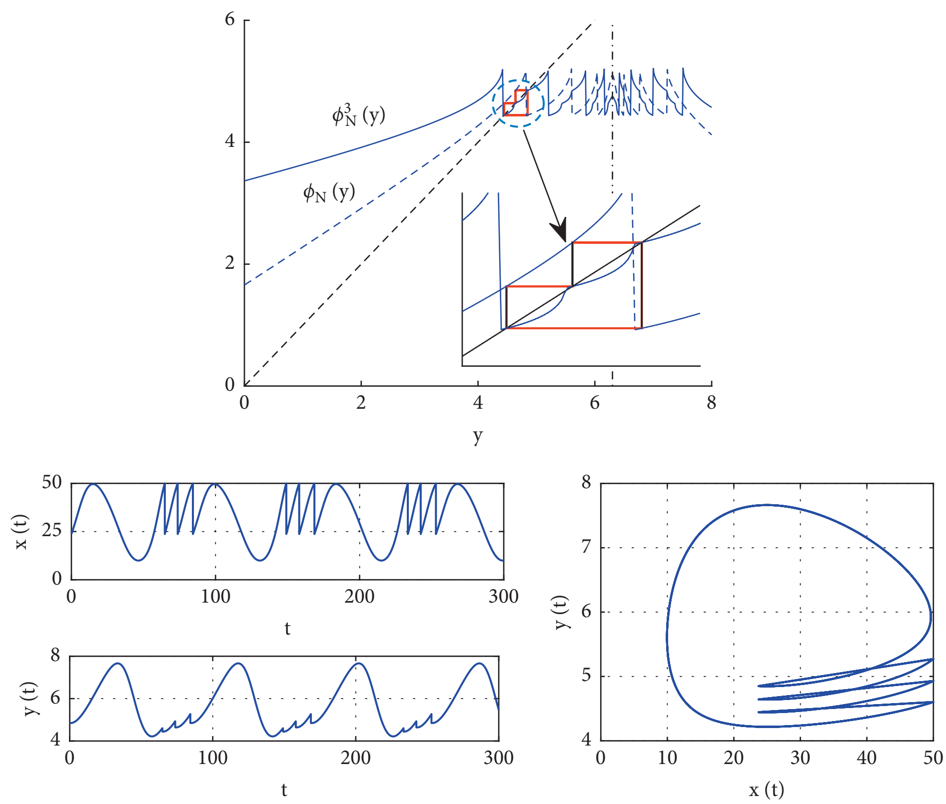


Figure 11: Illustration of functions $\phi_N^3(y)$ and $\phi_N(y)$ of system (4) for $l = 50\%K$ and $\tau_2 = 2.9$ with parameters given in numerical section. The time series evolution of prey population $x(t)$, predator population $y(y)$, and the phase portrait diagram demonstrate the order-3 periodic solution.

can be concluded that $\phi_N^3(y) = y$ if and only if $y = y_{L_2}$; that is, the order-3 periodic solution does not exist.

If $y_{N_1} < \phi_N^2(y_{N_0}) < y_{N_0}$, system (4) simultaneously admits an order-1 periodic solution and order-2 periodic solution. Moreover, there is $\phi_N(y_{N_0}) > y_{N_2}$. If $\tau \le y_{N_1}$, that is, $\phi_N(\tau) \le y_{N_0}$, there exist $y_{R_1} \in [0, y_{N_1}]$, $y_{R_2} \in (y_{L_1}, y_{N_0})$, $y_{R_3} \in (y_{N_0}, y_{L_2})$, and $y_{R_4} \in (y_{N_2}, +\infty)$ such that $\phi_N^2(y_{R_1}) = \phi_N^2(y_{R_2}) = \phi_N^2(y_{R_3}) = \phi_N^2(y_{R_4}) = y_{N_0}$. It can be

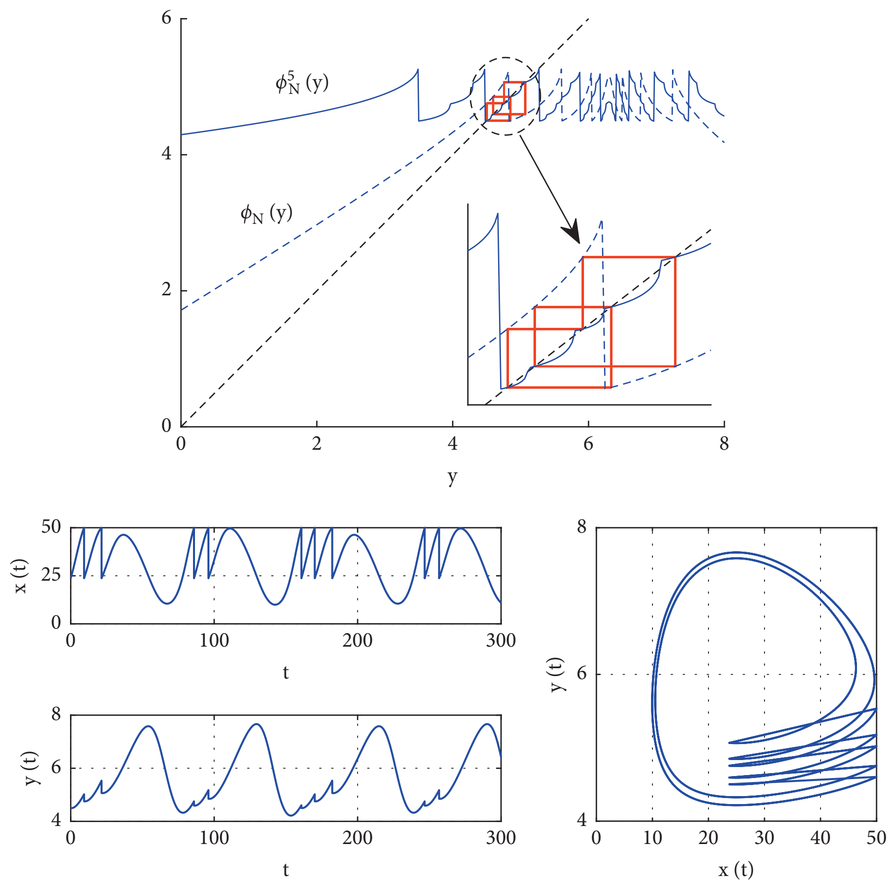


FIGURE 12: Illustration of functions $\phi_N^5(y)$ and $\phi_N(y)$ of system (4) for $l = 50\%K$ and $\tau_2 = 3$ with parameters given in numerical section. The time series evolution of prey population $x(t)$, predator population $y(t)$, and the phase portrait diagram demonstrate the order-5 periodic solution.

easily checked that ϕ_N^3 is increasing on $[0, y_{R_1}]$, $[y_{N_1}, y_{R_2}]$, $[y_{N_0}, y_{R_3}]$, and $[y_{N_2}, y_{R_4}]$, and ϕ_N^3 is decreasing on $[y_{R_1}, y_{N_1}]$, $[y_{R_2}, y_{N_0}]$, $[y_{R_3}, y_{N_2}]$, and $[y_{R_4}, +\infty)$. If $\tau > y_{N_1}$, then R_1 does not exist, ϕ_N^3 is increasing on $[y_{N_1}, y_{R_2}]$, $[y_{N_0}, y_{R_3}]$, and $[y_{N_2}, y_{R_4}]$, and ϕ_N^3 is decreasing on $[0, y_{N_1}]$, $[y_{R_2}, y_{N_0}]$, $[y_{R_3}, y_{N_2}]$, and $[y_{R_4}, +\infty)$. Since $\phi_N^3(y_{N_1}) = \phi_N^2(y_{N_0}) \ge y_{N_0} > y_{N_1}$ and $\phi_N^3(y_{R_4}) = \phi_N(y_{N_0}) < y_{R_4}$, it can be concluded that $\phi_N^3(y) = y$ if and only if $y = y_{L_2}$; that is, the order-3 periodic solution does not exist.

"Sufficiency." If $\phi_N^2(y_{N_0}) \le y_{N_1}$, then there is $\phi_N^3(y_{R_1}) = \phi_N(y_{N_0}) > 0$ and $\phi_N^3(y_{N_1}) = \phi_N^2(y_{N_0}) \le y_{N_1}$. If $\phi_N^2(y_{N_0}) = y_{N_1}$, then there exists an order-3 periodic solution since $\phi_N(y_{N_1}) \ne y_{N_1}$. If $\phi_N^2(y_{N_0}) < y_{N_1}$, then there exists at least one $y_f \in (y_{R_1}, y_{N_1})$ such that $\phi_N^3(y_f) = y_f$ and $\phi_N(y_f) > y_f$; that is, system (4) admits an order-3 periodic solution.

Next, the number of order-3 periodic solutions will be discussed:

(i) When $\phi_N^2(y_{N_0}) = y_{N_1}$, there are $\phi_N^3(y_{N_1}) = y_{N_1}$, $\phi_N^3(y_{N_0}) = y_{N_0}$, and $\phi_N^3(\phi_N(y_{N_0})) = \phi_N(y_{N_0})$. Moreover, there are $y_{N_0} = \phi_N(y_{N_1})$ and $y_{N_1} = \phi_N(\phi_N(y_{N_0}))$; that is, system (4) admits at least one order-3 periodic solution.

(ii) When $\phi_N^2(y_{N_0}) < y_{N_1}$, there are $\phi_N^3(y_{R_1}) = \phi_N^3(y_{R_2}) = \phi_N^3(y_{R_3}) = \phi_N^3(y_{R_4}) = \phi_N(y_{N_0}) > y_{R_4} > y_{R_3} > y_{R_2} > y_{R_1}$, and $\phi_N^3(y_{N_1}) < y_{N_1}$, $\phi_N^3(y_{N_0}) < y_{N_0}$, $\phi_N^3(y_{N_2}) < y_{N_2}$, and $\phi_N^3(\phi_N(y_{N_0})) = \phi_N^2(\tau) < \phi_N(y_{N_0})$; then there exist $y_1 \in (y_{R_1}, y_{N_1})$, $y_2 \in (y_{N_1}, y_{R_2})$, $y_3 \in (y_{R_2}, y_{N_0})$, $y_4 \in (y_{N_0}, y_{R_3})$, $y_5 \in (y_{N_2}, y_{R_4})$, and $y_6 \in (y_{R_4}, \phi_N(y_{N_0}))$ such that $\phi_N^3(y_i) = y_i$ $(i = 1, 2, \ldots, 6)$. Moreover, there are $\phi_N(y_1) = y_3$, $\phi_N(y_3) = y_6$, $\phi_N(y_6) = y_1$; $\phi_N(y_2) = y_4$, $\phi_N(y_4) = y_5$, and $\phi_N(y_5) = y_2$; that is, system (4) admits at least two order-3 periodic solutions. □

3.3. Optimal Harvest Level Determination. To achieve the commercial purpose of the fishery, it is necessary to harvest the population, and it is always hoped that the sustained ability can be achieved at a good economic profit. For the harvest problem, it is necessary to determine the controlled values E and τ and harvest level l, and this in general involves the optimization theory [48, 49].

Let l be the harvest level, which is a decision variable. Theorems 3 and 4 show that system (4) admits an order-1 periodic solution when $l \le \underline{l}$ or $l > \underline{l}$ with $\tau \le y_{M_1} - (1 - q_2 E) y_{\tilde{L}}(l)$. Since the harvest effort and yield of released

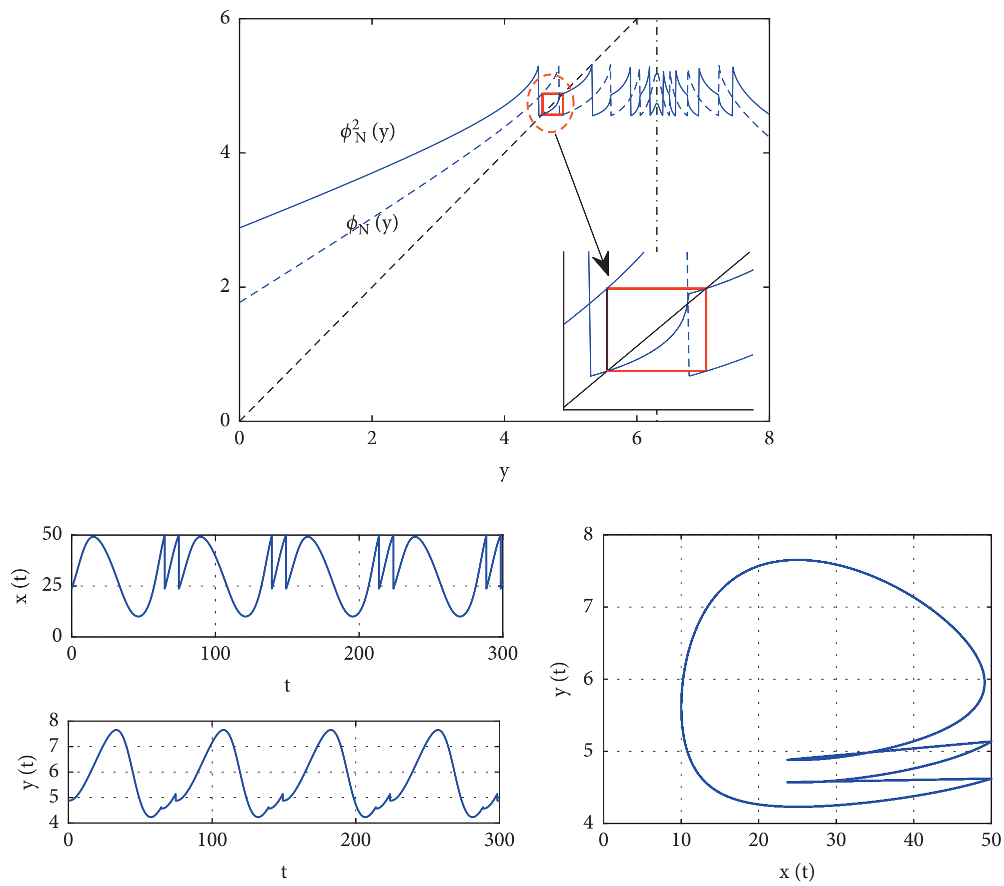


FIGURE 13: Illustration of functions $\phi_N^2(y)$ and $\phi_N(y)$ of system (4) for $l = 50\%K$ and $\tau_2 = 3.1$ with parameters given in numerical section. The time series evolution of prey population $x(t)$, predator population $y(t)$, and the phase portrait diagram demonstrate the order-2 periodic solution.

predator are dependent on the harvest level, then it is assumed that $E(l)$ and $\tau(l)$ take the following forms:

$$E(l) = E_1 + (E_2 - E_1)\frac{l - l_1}{l_2 - l_1},$$
$$\tau(l) = \tau_1 + (\tau_2 - \tau_1)\frac{l - l_1}{l_2 - l_1}, \tag{32}$$

where $l_1 \le l \le l_2$, l_1 and l_2 are minimum and maximum of the harvest level, and E_1 (E_2) and τ_1 (τ_2) are the harvest effort and yield of released predator at the harvest level l_1 (l_2).

Let c_1 and c_2 be the unit selling prices of prey and predator, let c_3 be the unit cost of harvest, and let c_4 be the unit cost in breeding predator. Then the benefits from harvest can be described as $F_{\text{benefit}}(l) = c_1 q_1 E(l) l + c_2 q_2 E(l)\eta(T(l)) - c_3 E^2(l) - c_4\tau(l)$. The objective is to maximize the unit benefits; that is,

$$\max \frac{F_{\text{benefit}}(l)}{T(l)} \text{ such that } l_1 \le l \le \underline{l},$$
$$\underline{l} < l \le \bar{l}, \tau(l) \le y_{M_1} - (1 - q_2 E) y_{\tilde{L}}(l). \tag{33}$$

4. Numerical Simulations and Optimization

In this section, we compute some numerical simulations regarding the existence and stability of the periodic solution for the predator-prey model (4). It is quite difficult to verify the mathematical model simulations with realistic parameter values. We take a hypothetical set of parameter values to illustrate our analytical findings. The model parameters are as follows: $b = 0.7$, $d = 0.2$, $c = 0.005$, $p = 0.1$, $h_1 = 0.036$, $h_2 = 1.44$, $e = 0.44$, and $m = 0.2$. The control parameters are as follows: $E_1 = 0.2$, $E_2 = 1$, and $\tau_1 = 0$.

4.1. Numerical Simulations. Since $e > e_M$, the boundary equilibrium $E(100, 0)$ is unstable, and system (3) has a positive equilibrium E^*. From equation (6), it can be observed that the fear effect factor k only affects the value y^*. Figure 2 illustrates the dependence of isoline $dx/dt = 0$ on x for different fear level k. As illustrated, the positive equilibrium becomes stable from unstable with increasing of k. By equation (6), there is $k^* = 0.0376$. To verify the theoretical results obtained in the above section, the simulations are implemented by considering different combinations of k, q_1, q_2, τ_2, and l.

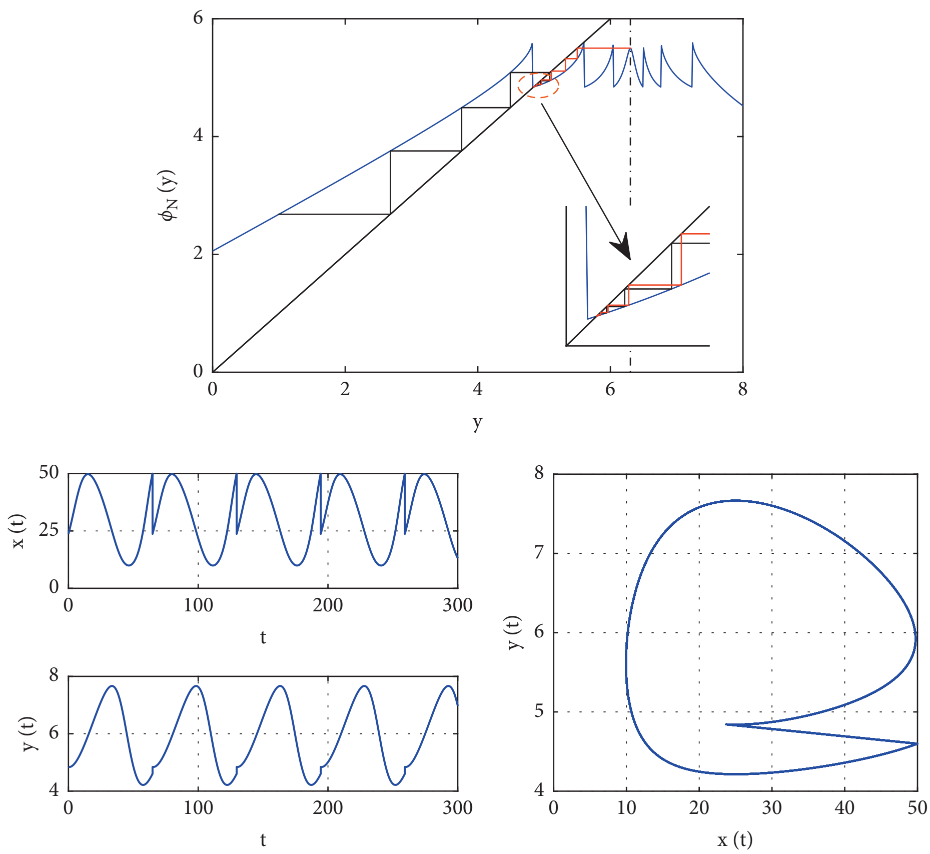


Figure 14: Illustration of function $\phi_N(y)$ of system (4) for $l = 50\%K$ and $\tau_2 = 3.6$ with parameters given in numerical section. The time series evolution of prey population $x(t)$, predator population $y(t)$, and the phase portrait diagram demonstrate the order-1 periodic solution.

Case I: $k = 0.04$, $q_1 = 0.8$, and $q_2 = 0.6$.

I-(1): $\tau_2 = 0$. By Theorem 2, system (4) admits an order-1 semitrivial periodic solution for any $l \le K = 100$, which is expressed by equation (10). Moreover, for $l \le \underline{l} = 34.45$, by Corollary 1, the semitrivial order-1 periodic solution is orbitally asymptotically stable. Meanwhile, for $l = 50\%K$, by Theorem 2, there is $R_0 = 0.5673 < 1$; that is, the order-1 semitrivial periodic solution is orbitally asymptotically stable, as presented in Figure 3.

I-(2): $\tau_2 > 0$. Firstly, for $\tau_2 = 2$, there is $\underline{l} = 34.45\%K$. For $l = 25\%K \le x^*$, Theorem 4 and Theorem 7 indicate that system (4) admits a globally asymptotically stable positive order-1 periodic solution, as illustrated in Figure 1. Meanwhile, for $l > x^*$, function ϕ_N of system (4) for $l = 30\%K, l = 50\%K$, and $l = 60\%K$ is presented in Figure 4. It can be observed that system (4) admits an order-1 periodic solution for $l = 30\%K$ and $l = 50\%K$ since the inequality $\tau < y_{M_1} - (1 - q_2 E) y_{\tilde{L}}(l)$ in Theorem 4 holds. When $l = 60\%K$, the direction of the inequality has changed, and the trajectory of system (4) will tend to the positive equilibrium $E^*(25, 4.93)$ after finite impulses, as shown in Figure 5. But this is not always true; it is dependent on the value of τ_2. As τ_2 goes up to 9, the inequality $\tau > y_{M_2} - (1 - q_2 E) y_{\tilde{L}}(l)$ in Theorem 4 holds; then system (4) admits a positive order-1 periodic solution, as shown in Figure 6(a). However, for $l = 80\%K$, the direction of the inequality is changed again, and the trajectory of system (4) will tend to the positive equilibrium $E^*(25, 4.93)$ after finite impulses, as shown in Figure 6(b).

Case II. $k = 0.04$ and $q_2 = 0$.

II-(1): $\tau_2 = 0$ and $q_1 = 0.8$. Notice from Figure 1 that a higher catching rate for predators (e.g., $q_2 = 0.6$) will cause the predator species extinction for $l = 50\%K$. When the catch for the predator is very small or ignored, that is, $q_2 = 0$, the order-1 semitrivial periodic solution is unstable by Corollary 1 (i.e., $R_0 = 1.02 > 1$). The function $f_N(y) = \phi_N(y) - y$ is presented in Figure 7 and it can be observed that $f_N(y) > 0$, and, for any initial condition, the trajectory of system (4) will eventually tend to the positive equilibrium $E^*(25, 4.93)$.

II-(2): $\tau_2 > 0$. For $\tau_2 = 8$ and $q_1 = 0.8$, by Theorem 3, system (4) admits a unique positive order-1 periodic solution for $l = 25\%K$. To show the existence of order-2 periodic solution, the catching rate for prey q_1 is selected as a key parameter to verify how does the dynamic behavior of the system change. Function ϕ_N for different catching rate for prey is presented in Figure 8. It can be observed that system (4) admits a unique globally asymptotically stable order-1 periodic solution for a higher catching rate for prey, for example, $q_1 = 0.8$, as shown in Figure 8(a). As the catching rate for prey goes down, for example $q_1 = 0.5$, condition (ii) $\phi_N^2(y_{N_0}) \ge y_{N_0}$ and $\mu_2^1 =$

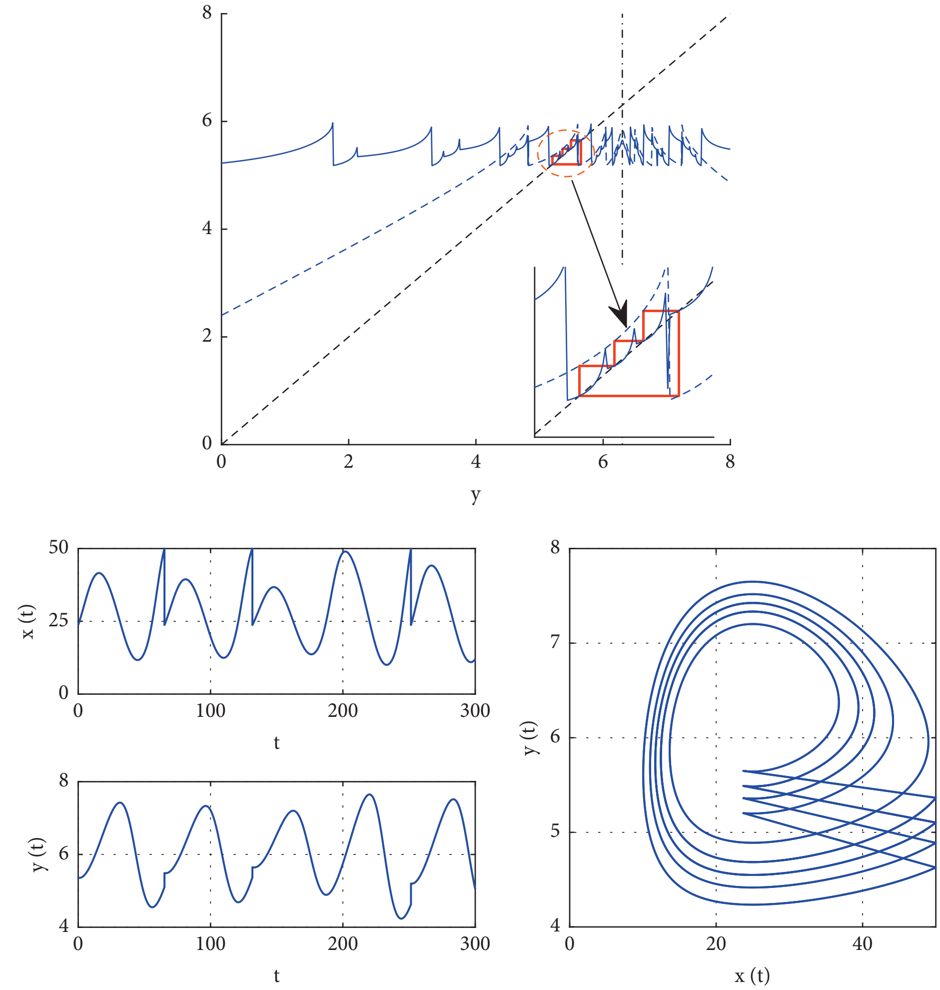


FIGURE 15: Illustration of functions $\phi_N^4(y)$ and $\phi_N(y)$ of system (4) for $l = 50\%K$ and $\tau_2 = 4.2$ with parameters given in numerical section. The time series evolution of prey population $x(t)$, predator population $y(t)$, and the phase portrait diagram demonstrate the order-4 periodic solution.

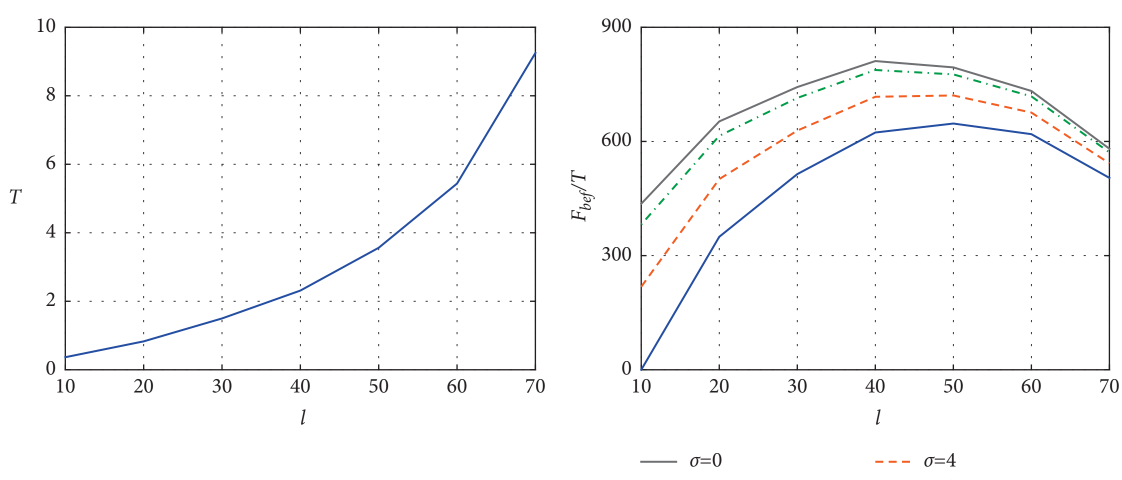


FIGURE 16: The dependence of period T and benefit F_{benefit} on the harvest level l for $k = 0.04$ with parameters given in numerical section.

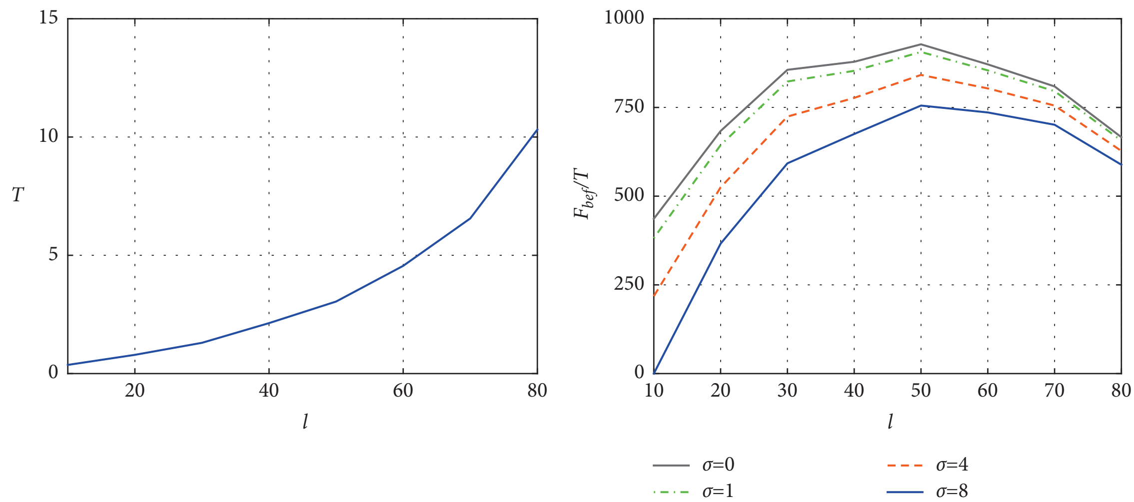


FIGURE 17: The dependence of period T and benefit F_{benefit} on the harvest level l for $k = 0.01$ with parameters given in numerical section.

$1.2612 > 1$ in Theorem 8 holds; then system (4) admits a stable order-2 periodic solution, as shown in Figure 8(b). Meanwhile, for $q_1 = 0.2$, condition (i) $\phi_N^2(y_{N_0}) < y_{N_0}$ in Theorem 8 holds, and then system (4) admits an order-2 periodic solution, as shown in Figure 8(c).

Case III: $k = 0.01$ and $q_1 = q_2 = 0.6$.

The positive equilibrium E^* becomes unstable when $k = 0.01$, and system (3) admits a limit cycle Γ_{LC}. Since the existence and stability of semitrivial order-1 periodic solution for $\tau = 0$ do not depend on the fear factor k, the results are the same as those in Case I-(1) and are omitted hereby. So it mainly discusses the dynamic behavior of system (4) for $\tau_2 > 0$. It is easily checked that $\underline{l} = 32.7\%K$.

Firstly, for $l = 25\%K \leq x^*$, by Theorem 3, system (4) admits a positive order-1 periodic solution for any τ, as shown in Figure 9.

Next, let us consider $l = 50\%K > \underline{l}$. It is easily checked that $\tau_{M_1} = 1.2495$ (i.e., $\tau_2 = 2.1867$); then, by Theorem 4, there exists an order-1 periodic solution for system (4) when $\tau_2 \leq 2.1867$. Here it should be pointed out that the condition given in Theorem 4 is only a sufficient one; in fact, as long as $\tau_2 \leq 2.55$, system (4) admits an order-1 periodic solution, as illustrated in Figure 10.

With an increase of τ_2, the existence of order-1 periodic solution cannot be guaranteed. For example, for any $\tau_2 \in [2.56, 3.58]$, there does not exist order-1 periodic solution for system (4). System (4) admits an order-3 periodic solution for $\tau_2 = 2.9$ (Figure 11), an order-5 periodic solution for $\tau_2 = 3$ (Figure 12), an order-2 periodic solution for $\tau_2 = 3.1$ (Figure 13), an order-1 periodic solution for $\tau_2 = 3.6$ (Figure 14), and an order-4 periodic solution for $\tau_2 = 4.2$ (Figure 15).

4.2. Optimization. To achieve a good economic profit, it is necessary to find a level l^* at which the benefits from harvest are maximal. Assume that $c_1 = 100$, $c_2 = 5c_1 = 500$, and $c_3 = 20\%c_2 = 100$. Denote $\sigma \triangleq c_3/c_1$. In order to sustain the harvest, the releasing yield of predator should not be too large, so, in this part, it is assumed that $\tau_2 = 1$.

For $k = 0.04$, system (4) admits an order-1 periodic solution when $l \leq 70\%K$; the dependencies of period T and benefit F_{benefit} on the harvest level l are presented in Figure 16. It can be seen that period T increases as l increases. Meanwhile the benefit function F_{benefit} climbs up and then declines as l increases. For different σ, F_{benefit} achieves its maximum at different l_σ^*. When the cost of harvest is ignored, that is, $\sigma = 0$, there is $l_0^* = 40\%K$. As σ goes up, F_{benefit} goes down. When $\sigma = 8$, F_{benefit} achieves its maximum at $l_8^* = 50\%K$.

For the case of $k = 0.01$, system (4) admits an order-1 periodic solution when $l \leq l_2$, and the dependencies of period T and benefit F_{benefit} on the harvest level l are presented in Figure 17. The benefit funtion F_{benefit} climbs up and then declines as l increases. For different σ, F_{benefit} almost achieves its maximum at $l_\sigma^* = 50\%K$.

5. Conclusion and Discussions

In this paper, we have discussed the dynamics of a harvested prey-predator model, where the prey is provided with fear effect. For the system without harvest (3), there exists a unique positive equilibrium. To verify the stability of the equilibrium, a critical level of fear factor k^* is characterized (i.e., equation (6)). When the impact of fear on prey is small, that is, $0 \leq k < k^*$, the positive equilibrium is unstable and a limit cycle exists. As the impact of fear grows and exceeds k^*, the positive equilibrium becomes stable and the limit cycle disappears (Figure 2). In any case, predators coexist with prey and the system is persistent.

For the system with harvest (4), if we do not consider the release of predator (i.e., $\tau = 0$), system (4) admits a semitrivial order-1 periodic solution for any harvest level (Figure 3). Moreover, the semitrivial order-1 periodic solution is orbitally stability when the harvest level is not higher than the first equilibrium component (i.e., $l \leq x^*$). Meanwhile, for the case of $l > x^*$, the semitrivial order-1 periodic solution is orbitally stable when a strong harvest intensity is implemented. This means that the system can be disrupted, and predators will go extinct if the harvest is not properly planned. To maintain the ecological health and avoid the extinction of predator populations, it is necessary to reduce the catch rate of predators (Figure 7) or release a certain quantity of predator pups. In the second case, that is, $\tau > 0$, system (4) admits a positive order-1 periodic solution when $l \leq \underline{l}$ (Figures 1, 4, and 9). Moreover, the order-1 periodic solution is orbitally asymptotically stable and globally attractive when $l \leq x^*$ and $\tau \leq \tau_f$. Meanwhile, for $\tau > \tau_f$, system (4) admits an order-2 periodic solution in case of $\phi_N^2(y_{N_0}) \geq y_{N_0}$ and $\mu_2^1 > 1$ (Figure 8). In case of $l > \underline{l}$, system (4) also admits a positive order-1 periodic solution for $\tau \leq \tau_{M_1}$ (Figures 4 and 10) or $\tau \geq \tau_{M_2}$ (Figure 6). But, for $\tau \in (\tau_{M_1}, \tau_{M_2})$, when $k > k^*$, the trajectory of system (4) will tend to the positive equilibrium $E^*(25, 4.93)$ after finite impulses (Figure 5). Meanwhile, in case of $k < k^*$, the dynamic behavior of system (4) depends heavily on parameter τ. For different value of τ, system (4) may admit an order-$k(k = 1, 2, 3, 4, 5)$ periodic solution (Figures 11–15).

To achieve a good economic profit, optimization with $\tau_2 = 1$ is carried out and the results show that the benefits from harvest depend on the unit selling prices of prey and predator, as well as the unit cost of harvest. For given $c_1 = 100$, $c_2 = 500$, and $c_4 = 100$, the benefit function first climbs up and then declines as l increases. For $k = 0.04$, the economic profit F_{benefit} achieves its maximum at $l^* = 40\%K$ when $\sigma = 2$. As σ goes up, F_{benefit} goes down, and F_{benefit} achieves its maximum at $l^* = 50\%K$ when $\sigma = 8$. Meanwhile, in case of $k = 0.01$, F_{benefit} almost achieves its maximum at $l^* = 50\%K$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

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