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Research Article

Adaptive Fuzzy-Sliding Consensus Control for Euler–Lagrange Systems with Time-Varying Delays

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This paper presents an adaptive fuzzy sliding-mode controller for multiple Euler–Lagrange systems communicated with directed topology. Based on the graph theory and Lyapunov–Krasovskii functions, a delay-dependent sufficient condition for the existence of sliding surfaces is given in terms of linear matrix inequalities. The asymptotic stability is analyzed by using the Lyapunov method in the presence of unknown parametric dynamics, actuator faults, and time-varying delays. The usage of adaptive techniques is to adapt the unknown parameters so that the objective of globally asymptotic stability is achieved. Finally, simulation results are provided to illustrate the effectiveness of the proposed control scheme.

1. Introduction

Cooperative control of multiagent systems has attracted much attention in recent years, such as consensus [1–4], formation [5–8], and flocking problems [9–12]. The solving of the consensus problem is an essential and interesting topic of cooperative multiagent research. Basically, the idea of consensus implies that a group of agents can reach an agreement on certain quantities of interest. In graph-based approaches, a directed or indirected graph is popularly applied to describe the communication topology of networked multiagent systems. In the last few years, several methods have been proposed to deal with the consensus problems of the multiagent systems [13–17]. In [13], adaptive synchronization protocols for a heterogeneous multiagent network were investigated, where the interaction between agents is represented by a direct graph. In [14], the leaderless consensus problem over strict-feedback nonlinear multiagent systems with unknown model parameters and control directions was investigated. Also, a robust continuous-time optimization algorithm was presented for multiagent systems with guaranteed fixed-time convergence. Zhang al. presented a robust consensus tracking strategy for multiple unmanned underwater vehicles with switching topology [16]. In [17], the leader-following average consensus problem was addressed for linear multiagent systems.

Besides the first-order or second-order linear models, one important class of multiagent control systems is the so-called Euler–Lagrange systems, which generally describes the dynamic properties of robot manipulators, rigid body systems, and so on. In [18], the leader-following consensus problem was studied of multiple Euler–Lagrange (EL) systems subject to an uncertain leader. In [19], an adaptive sliding mode control technique was proposed for the EL systems with actuator faults and system uncertainties. In addition, a distributed optimal consensus strategy based on an event-triggered scheme for EL multiagent systems was investigated [20]. The model-free optimal consensus problem was addressed for networked Euler–Lagrange systems without velocity measurements [21]. Chen et al. proposed a robust adaptive finite-time tracking control scheme for Euler-Lagrange systems subject to nonparametric uncertainties, unknown disturbances, and input saturation [22]. In [23], a robust adaptive finite-time tracking control scheme was proposed for Euler–Lagrange systems subject to nonparametric uncertainties, unknown disturbances, and input saturation.

The sliding-mode method has been studied for nonlinear systems because of some attractive features, such as robustness to parameter variations and good transient performance. Recently, a sliding-mode controller has been developed to deal with nonlinearities and uncertainties for multiagent systems [24–28]. In [24], the finite-time consensus tracking of multirobotic systems with disturbances was investigated via utilizing integral sliding mode control. In [25], an optimal sliding mode control approach was presented for the consensus of nonlinear discrete-time high-order multiagent systems. Jina et al. investigated the consensus control problem of Takagi–Sugeno fuzzy multiagent systems by using an observer-based distributed adaptive sliding mode control [26]. In addition, the event-triggered tracking control problem of second-order uncertain multiagent systems was addressed by utilizing the distributed sliding-mode control approach [27]. In [28], the consensus tracking problem of networked control systems with disturbances was discussed, where an integral sliding mode protocol was developed to achieve the consensus in a setting time. Recently, fuzzy sliding-mode control has attracted much attention, where the fuzzy mechanism is useful to decrease the chattering behaviors. In [29], an adaptive backstepping fuzzy neural network controller using a fuzzy sliding mode controller was designed to suppress the harmonics of a shunt active power filter. In addition, a fuzzy sliding mode control method was proposed to improve the ability of magnetic levitation feed platform subject to external disturbance [30]. In [31], a fuzzy sliding-mode control was developed to deal with unmodeled dynamics and external disturbances in a human-exoskeleton system.

On the other hand, in multiagent systems, actuator failures generally result in poor system performance or even cause the instability. In [32], an adaptive fixed-time controller was designed for a class of uncertain nonstrict feedback multiagent systems subject to actuator faults and external disturbances. In [33], a robust consensus control strategy was addressed for nonlinear second-order multiagent systems against actuator faults and uncertainties. Dong et al. presented an augmented control system for a quadrotor unmanned aerial vehicle with parameter uncertainties, external disturbance, and the partial loss of actuator effectiveness [34]. In [35], a cooperative fault tolerant control was presented for linear leader-follower networks subject to actuator faults. Moreover, the fault-tolerant leader-following consensus problem was discussed for multiagent systems with input saturation and actuator faults [36]. In [37], the consensus problem was investigated for a class of nonaffine nonlinear multiagent systems with actuator faults of partial loss of effectiveness.

Because of the interactive communication in multiagent systems, the coupling delays between agents become more crucial due to practical considerations. In [38], the consensus problem of discrete-time linear multiagent systems was addressed with unbounded time-varying delays. In [39], the containment control problem of the double-integrator multiagent systems was investigated with time-varying communication delays. Tan et al. discussed the output feedback control problem for a class of nonlinear multiagent systems governed by the high-order strict-feedback model with time delays [40]. Also, the finite-time consensus of leader-following multiagent systems was addressed with multiple time delays over time-varying topology [41]. The leader-following consensus problem was discussed for discrete-time multiagent systems with time-varying delays [42]. In [43], the second-order multiagent networks with time-varying delays were investigated, where a sufficient condition was presented to make all agents asymptotically reach consensus using the linear matrix inequality theory.

In this paper, an adaptive fuzzy sliding-mode fault-tolerant controller (AFSFC) is presented for multiple Euler–Lagrange systems. Also, the parametric uncertainties, actuator faults, and time-varying communication delays are considered. The proposed control scheme is based on adaptive sliding-mode techniques combining with the fuzzy logic strategy. An adaptive algorithm is provided to estimate the unknown parametric vector. Moreover, by employing the Lyapunov–Krasovskii function and linear matrix inequalities (LMIs), a sufficient condition is established such that the resulting sliding-mode dynamics is stable. The main contributions of this paper are stated as follows: (1) an adaptive fuzzy sliding-mode controller is proposed for networked Euler–Lagrange systems with communication time-varying delays. The fuzzy sliding mode and adaptive controller are combined to deal with the parametric uncertainties. (2) For the multiagent systems, provided with controller parameters and communication topology, the maximum tolerated delay of all agents can be determined by using the Lyapunov–Krasovskii analysis and LMIs. (3) The proposed control scheme can be applied to a group of agents with a directed communication topology. The tracking errors are shown to be asymptotically convergent with a directed spanning tree of communication topology. (4) The unknown parametric dynamics and actuator faults can be estimated online using adaptive strategies. (5) The overall closed-loop stability can be preserved using Lyapunov stability analysis in both fault-free and faulty situations. Moreover, the allowable communication delays can be obtained and formulated as some LMIs.

This paper is organized as follows: in Section 2, the dynamic model of Euler–Lagrange systems is presented with the consideration of partial loss of effectiveness faults. The stability of the sliding motion of multiagent systems is investigated in Section 3. In Section 4, an adaptive fuzzy sliding-mode controller for multiagent systems with time-varying delays and actuator faults is discussed. In Section 5, the simulation results are provided for performance comparisons. Finally, the concluding remarks are given in Section 6.

2. Preliminaries

2.1. Fundamentals of Graph Theory. A directed graph $G = (V, E)$ consists of a vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and an edge set $E \subseteq V \times V$, where $(v_j, v_i) \in E$ means that the ith node can receive information from the jth node. $N_i = \{v_j \in V\colon (v_j, v_i) \in E\}$ denotes the neighboring set of v_i. The adjacent matrix $A = [a_{ij}] \in \mathscr{R}^{n\times n}$, where $a_{ij} = 1$, if $(v_j, v_i) \in E$, or $a_{ij} = 0$, otherwise. The degree matrix $D = \mathrm{diag}\{d_1, d_2, \ldots, d_n\} \in \mathscr{R}^{n\times n}$, $d_i = \sum_{j\in N_i} a_{ij}$ of a digraph G is a diagonal matrix.

Lemma 1 (see [44]). *The graph G has a directed spanning tree if and only if there is at least one node with a directed path to all other nodes. If a graph G has a spanning tree, then a right eigenvector L associated with the zero eigenvalue is $1_n = [1, 1, \ldots, 1]^T$, i.e. $\mathscr{L} 1_n = 0$.*

Assumption 1. The graph G has a directed spanning tree.

2.2. Dynamic Models of Multiagent Systems. This paper considers a group of n-agent Euler–Lagrange systems, which can be represented as

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{q}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{q}_i)\dot{q}_i + \mathbf{g}_i(\mathbf{q}_i) = \tau_i, \tag{1}$$

where $\dot{\mathbf{q}}_i \in \mathscr{R}^p$ is the vector of joint positions, $\mathbf{M}_i(\mathbf{q}_i) \in \mathscr{R}^{p\times p}$ is the inertia matrix, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathscr{R}^{p\times p}$ is the Coriolis matrix, $\mathbf{g}_i(\mathbf{q}_i) \in \mathscr{R}^p$ is the gravitational vector, $\tau_i \in \mathscr{R}^p$ is the vector of input torques, $i = 1, 2, \ldots, n$. The actuator fault model considered can be described as

$$\boldsymbol{\tau}_i^f = \boldsymbol{\sigma}_i \boldsymbol{\tau}_i + \Delta\boldsymbol{\tau}_i, \tag{2}$$

where σ_i is the effectiveness factor, $\Delta\tau_{\mathbf{i}}$ is an additive fault, $0 < \underline{\sigma}_i \le 1$, and $i = 1, 2, \ldots, n$. With fault model (2), the Euler–Lagrangian dynamics (1) of ith agent can be rewritten as follows:

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \boldsymbol{\sigma}_i \boldsymbol{\tau}_i + \Delta\boldsymbol{\tau}_i. \tag{3}$$

Let σ_i be the lower bound of σ_i, $0 < \underline{\sigma}_i \le \sigma_i$, in which $\underline{\sigma}_i$ is an unknown positive constant. The actuators are fault-free when $\underline{\sigma}_i = 1$ and $\Delta\tau_i = 0$, and $\underline{\sigma}_i \in (0, 1)$ corresponds to the cases with partial loss of effectiveness (PLOE) faults.

Property 1. The Euler–Lagrangian dynamics (1) is linearly parameterizable as

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \mathbf{Y}_{\mathbf{i}}(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i)\Theta_i, \tag{4}$$

where $\mathbf{Y}_i(\cdot) \in \mathscr{R}^{p\times p_\theta}$ is the regression matrix and $\Theta_i \in \mathscr{R}^{p_\theta}$ is a vector of unknown constant parameters.

Property 2. The matrix $\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew symmetric.

Lemma 2 (see [45]). *Given a positive definite matrix $\mathbf{Z}$, the following inequality holds:*

$$2\mathbf{A}^{\mathbf{T}}\mathbf{B} \le \mathbf{A}^{\mathbf{T}}\mathbf{Z}\mathbf{A} + \mathbf{B}^{\mathbf{T}}\mathbf{Z}^{-1}\mathbf{B}, \tag{5}$$

where $\mathbf{A}$ and $\mathbf{B}$ are two matrices with proper dimensions.

Assumption 2. There exists a positive constant $\overline{\tau}_i$ such that $\|\Delta\tau_i\| \le \overline{\tau}_i$.

Lemma 3 (see [46]). *Let $\mathbf{O} = \begin{bmatrix} \mathbf{O}_{11} & \mathbf{O}_{12} \\ \mathbf{O}_{12}^T & -\mathbf{O}_{22} \end{bmatrix}$ be a matrix with proper dimensions, $\mathbf{O}_{22} \rangle 0$. Then, the following equation holds,*

$$\mathbf{O} \langle 0 \ \mathbf{O}_{11} + \mathbf{O}_{12}\mathbf{O}_{22}^{-1}\mathbf{O}_{12}^T \langle 0. \tag{6}$$

3. Stability of Sliding Motion

For the time-delayed multiagent systems (1), a sliding-mode controller will be designed so that the corresponding sliding motion is asymptotically stable. The sliding surface for the ith agent is defined as

$$\mathbf{s}_i = \dot{\mathbf{q}}_i + \mathbf{K}_i \epsilon_i, \tag{7}$$

where $\mathbf{K}_i \in \mathscr{R}^{p\times p}$ is a constant positive diagonal matrix, $\epsilon_i = \sum_{j\in N_i} a_{ij}(\mathbf{q}_i(t - d(t)) - \mathbf{q}_j(t - d(t)))$, $i = 1, 2, \ldots, n$.

Assumption 3. The communication time-varying delay $d(t)$ satisfies that

$$0 \langle d(t) \le \overline{d} \langle \infty, \dot{d}(t) \le \mu \langle 1, \tag{8}$$

where $\overline{d}$ and μ are positive constants.

In the following, the notations $\mathbf{q}_{i,d}$ and $\mathbf{q}_{j,d}$ stand for $\mathbf{q}_i(t - d(t))$ and $\mathbf{q}_j(t - d(t))$, respectively, for simplicity. We denote $\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \ldots, \mathbf{s}_n^T]^T$. The sliding surface of multiagent systems is summarized as

$$\mathbf{s} = \dot{\mathbf{q}} + \mathbf{K}(\mathscr{L} \otimes \mathbf{I}_p)\mathbf{q}_d, \tag{9}$$

where $\dot{\mathbf{q}} = [\dot{\mathbf{q}}_1^T, \dot{\mathbf{q}}_2^T, \ldots, \dot{\mathbf{q}}_n^T]^T$, $\mathbf{q}_d = [\mathbf{q}_{1,d}^T, \mathbf{q}_{2,d}^T, \ldots, \mathbf{q}_{n,d}^T]^T \in \mathscr{R}^{pn}$, and $\mathbf{K} = \mathrm{diag}\{\mathbf{K}_1, \mathbf{K}_2, \ldots, \mathbf{K}_n\} \in \mathscr{R}^{pn\times pn}$. When the sliding mode is achieved, (9) can be equivalently described as

$$\dot{\mathbf{q}} = -\mathbf{K}(\mathscr{L} \otimes \mathbf{I}_p)\mathbf{q}_d. \tag{10}$$

Let the error function between the 1st agent and other agents as

$$\mathbf{e}_{1r} = \mathbf{q}_1 - \mathbf{q}_r, r = 2, 3, \ldots, n. \tag{11}$$

Moreover, (11) can be rewritten as the following augmented form:

$$\mathbf{e} = \mathbf{q}_1 \otimes 1_{n-1} + \mathbf{E}\mathbf{q}, \tag{12}$$

where $\mathbf{e} = [\mathbf{e}_{12}^T, \mathbf{e}_{13}^T, \ldots, \mathbf{e}_{1n}^T]^T$, $\mathbf{q} = [\mathbf{q}_1^T, \mathbf{q}_2^T, \ldots, \mathbf{q}_n^T]^T$, $1_{n-1} = [1, 1, \ldots, 1]^T$, and

$$\mathbf{E} = \begin{bmatrix} 1_p & -1_p & 0_p & \cdots & 0_p \\ 1_p & 0_p & -1_p & \cdots & 0_p \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1_p & 0_p & 0_p & \cdots & -1_p \end{bmatrix} \in \mathscr{R}^{p(n-1)\times pn}, \tag{13}$$

in which 1_p is the p-dimension identity matrix, and 0_p is the p-dimensional matrix with all zeros. Thus, (11) can be rewritten as

$$\mathbf{q} = \mathbf{q}_1 \otimes 1_n + \mathbf{F}\mathbf{e}, \tag{14}$$

where

$$\mathbf{F}=\begin{bmatrix} 0_p & 0_p & \cdots & 0_p \\ -1_p & 0_p & \cdots & 0_p \\ 0_p & -1_p & \cdots & 0_p \\ \vdots & \vdots & \ddots & \vdots \\ 0_p & 0_p & \cdots & -1_p \end{bmatrix} \in \mathscr{R}^{pn\times p(n-1)}. \quad (15)$$

The derivative of (12) along (10) and (13) is derived as

$$\mathbf{e}=-\mathbf{EK}\left(\mathscr{L}\otimes\mathbf{I}_p\right)\left(\mathbf{q}_{1,d}\otimes 1_n\right)-\mathbf{EK}\left(\mathscr{L}\otimes\mathbf{I}_p\right)\mathbf{Fe}_d, \quad (16)$$

where the notation $\mathbf{e}_d$ stands for $\mathbf{e}(t-d(t))$. According to Lemma 1, it is obtained as

$$\dot{\mathbf{e}}=-\mathbf{\Psi}\mathbf{e}_d, \quad (17)$$

where $\Psi=\mathbf{EK}(\mathscr{L}\otimes\mathbf{I}_p)\mathbf{F}$.

Lemma 4 (See [47]). *The matrix Ψ is Hurwitz if and only if the communication topology G of the multiagent systems has a directed spanning tree.*

Theorem 1. *Suppose that the communication graph of a multiagent Euler–Lagrange system of (1) has a directed spanning tree. From Assumption 3, the error dynamics of (15) is asymptotically stable, if the following inequalities hold:*

$$\begin{gathered}\begin{bmatrix} -\mathbf{P\Psi}-\mathbf{\Psi}^{\mathbf{T}}\mathbf{P}+\mathbf{Q} & \mathbf{P\Psi} \\ \mathbf{\Psi}^T P & -(\overline{d}\mathbf{R})^{-1}\end{bmatrix}\langle 0, \\ \begin{bmatrix} -\widetilde{\mu}\mathbf{Q} & \mathbf{\Psi} \\ \mathbf{\Psi}^T & (\overline{d}\mathbf{R})^{-1}\end{bmatrix}\langle 0,\end{gathered} \quad (18)$$

where P, Q, and R are symmetric positive definite matrices of $\mathscr{R}^{(pn-1)\times(pn-1)}$ and $\widetilde{\mu}=1-\mu$.

Proof. The Lyapunov–Krasovskii function [48] is chosen as

$$V_s=\mathbf{e}^T\mathbf{Pe}+\int_{t-d(t)}^{t}\mathbf{e}^T(s)\mathbf{Qe}(s)ds+\int_{-\overline{d}}^{0}\int_{t+\theta}^{t}\dot{\mathbf{e}}^T(s)\mathbf{R}\dot{e}(s)dsd\theta. \quad (19)$$

The time derivative of V_s along (15) is derived as

$$\begin{aligned}\dot{V}_s\leq & -2\mathbf{e}^T\mathbf{P\Psi e}_d+\mathbf{e}^T\mathbf{Qe}-\widetilde{\mu}\mathbf{e}_a^T\mathbf{Qe}_d+\overline{d}\mathbf{e}_a^T\mathbf{\Psi}^T\mathbf{R\Psi e}_d \\ & -\int_{t-d}^{t}\epsilon^T(s)\mathbf{R}\dot{e}(s)ds.\end{aligned} \quad (20)$$

It is noted that $\mathbf{e}_d=\mathbf{e}-\int_{t-d(t)}^{t}\mathbf{e}(s)\mathrm{d}s$. Then, from Lemma 2 and Assumption 3, it leads to the following inequality:

$$\begin{aligned}-2\mathbf{e}^T\mathbf{P\Psi}e_d &= -2\mathbf{e}^T P\mathbf{\Psi}\left(\mathbf{e}+\int_{t-d(t)}^{t}\dot{\mathbf{e}}(s)\mathrm{d}s\right) \\ &\leq -2\mathbf{e}^T\mathbf{P\Psi e}+\overline{d}\mathbf{e}^T\mathbf{\Psi}^T\mathbf{PR}^{-1}\mathbf{P\Psi e} \\ &\quad +\int_{t-d(t)}^{t}\dot{e}^T(s)\mathbf{R}\dot{e}(s)\mathrm{d}s.\end{aligned} \quad (21)$$

Substituting (19) into (18), it results in

$$\begin{aligned}\dot{V}_s\leq & -2\mathbf{e}^T\mathbf{P\Psi e}+\mathbf{e}^T\mathbf{Qe}-\widetilde{\mu}\mathbf{e}_d^T\mathbf{Qe}_d+\overline{d}\mathbf{e}^T\mathbf{\Psi}^T\mathbf{PR}^{-1}\mathbf{P\Psi e} \\ & +\overline{d}\mathbf{e}_d^T\mathbf{\Psi}^T\mathbf{R\Psi e}_d \\ = & \begin{bmatrix}\mathbf{e}\\ \mathbf{e}_d\end{bmatrix}^T\begin{bmatrix}\mathbf{\Omega}_1 & 0\\ 0 & \mathbf{\Omega}_2\end{bmatrix}\begin{bmatrix}\mathbf{e}\\ \mathbf{e}_d\end{bmatrix},\end{aligned} \quad (22)$$

where $\Omega_1=-\mathbf{P}\Psi-\Psi^T\mathbf{P}+\mathbf{Q}+\overline{d}\Psi^T\mathbf{PR}^{-1}\mathbf{P}\Psi$ and $\Omega_2=-\widetilde{\mu}\mathbf{Q}+\overline{d}\Psi^T\mathbf{R}\Psi$. From Lemma 3, $\dot{V}_s$ is negative if (16) holds. It implies that the error dynamics of (15) is asymptotically stable. This completes the proof. □

Remark 1. It is noted that Theorem 1 proposed a delay-dependent sufficient condition of stability for the sliding surfaces (9) based on the LMIs (16). The error function (12) with time-varying communication delay will converge to zero as $t\longrightarrow\infty$. Also, the allowable bound of time-varying delays can be obtained. Since it provides only sufficient conditions for stability, the results could be likely conservative.

4. Consensus Controller Design

4.1. Fault-Free Cases. Note that in the fault-free case, i.e., $\sigma_i=1$ and $\Delta\tau_i=0$, the torques acting on the dynamic system (1) are designed as

$$\boldsymbol{\tau}_i=\boldsymbol{\tau}_{eq,i}+\boldsymbol{\tau}_{sw,i}, \quad (23)$$

where $\tau_{eq,i}$ is the equivalent control action and $\tau_{sw,i}$ is the switching controller. To obtain the equivalent control action $\tau_{eq,i}$, the state trajectory is desired to stay in the sliding surface, i.e., $\dot{s}_i=0$. From (7), it gives that $\dot{s}_i=\ddot{\mathbf{q}}_i+\mathbf{K}_i\dot{\epsilon}_i=0$. From (1), with the equivalent control action $\tau_i=\tau_{eq,i}$, the equivalent control action $\tau_{eq,i}$ can be derived as

$$\boldsymbol{\tau}_{eq,i}=-\mathbf{M}_i\mathbf{K}_i\dot{\epsilon}_i-\mathbf{C}_i\mathbf{K}_i\epsilon_i+\mathbf{g}_i, \quad (24)$$

where $\dot{\epsilon}_i=(1-\dot{d}(t))\sum_{j\in N_i}a_{ij}(\dot{q}_{i,d}-\dot{q}_{j,d})$. We consider the unknown parameter vector Θ_i of (4). Let the estimation of Θ_i be defined as

$$\widetilde{\Theta}_i=\mathbf{\Theta}_i-\widehat{\Theta}_i, \quad (25)$$

where $\widehat{\Theta}_i$ is the estimation of Θ_i. The adaptive law of $\widehat{\Theta}_\mathbf{i}$ is designated as follows:

$$\dot{\widehat{\Theta}}_\mathbf{i}=\Gamma\mathbf{Y}_i^T\left(\mathbf{q}_i,\dot{q}_i,\epsilon_i,\dot{\epsilon}_i\right)\mathbf{s}_i, \quad (26)$$

where $\Gamma\in\mathbf{R}^{p\times p\theta}$ is a constant positive definite matrix.

Let $\mathbf{s}_i$ and $\tau_{sw,i}$ be the input and output variables of a switching control system, respectively. Therefore, the switching system is represented by a single input-output fuzzy logic system. The fuzzy system is a collection of the fuzzy IF-THEN rules in the form of

$$\text{Rule k : IF } s_{\mathbf{ri}} \text{ is } M_{ki}, \text{THEN } \tau_{sw,ri} \text{ is } F_{(6-k)i}, \quad k=1,2,\ldots,5, \quad (27)$$

where the M_{ki} and $F_{(6-k)i}$ are the input and output fuzzy sets, $\mathbf{s}_i=[s_{1i},s_{2i},\ldots,s_{pi}]^T$, $\tau_{sw,i}=[\tau_{sw,1i},\tau_{sw,2i},\ldots,\tau_{sw,pi}]^T$, $r=1,2,\ldots,p$, respectively. The triangular input and singleton output membership functions are shown in Figure 1.

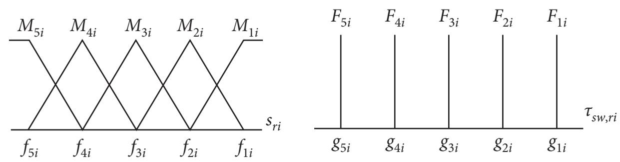

Figure 1: Membership functions of the fuzzy switching controller.

By using the centroid defuzzification technique, the output $\tau_{sw,ri}$ of the fuzzy system is

$$\tau_{sw,ri} = \frac{\sum_{k=1}^{5} g_{ki}\mu_{ki}(s_{ri})}{\sum_{k=1}^{5}\mu_{ki}(s_{ri})} = -\sum_{k=1}^{5}|g_{ki}|\operatorname{sgn}(s_{ri})\mu_{ki}(s_{ri}), \quad (28)$$

where g_{ki} is the value of the corresponding fuzzy output and $\mu_{ki}(s_{ri})$ is the firing strength of the antecedent membership function, and sgn(.) is a standard sign function. We rewrite (25) as the following augmented form:

$$\boldsymbol{\tau}_{sw,i} = -\prod_{\mathbf{i}} \operatorname{sgn}(\mathbf{s}_i), \quad (29)$$

where $\operatorname{sgn}(\mathbf{s}_i) = [\operatorname{sgn}(s_{1i}), \operatorname{sgn}(s_{2i}), \ldots, \operatorname{sgn}(s_{pi})]^T$ and

$$\prod_{\mathbf{i}} = \operatorname{diag}\left\{\sum_{k=1}^{5}|g_{ki}|\mu_{ki}(s_{1i}), \sum_{k=1}^{5}|g_{ki}|\mu_{ki}(s_{2i}), \ldots, \sum_{k=1}^{5}|g_{ki}|\mu_{ki}(s_{pi})\right\}. \quad (30)$$

Remark 2. Triangular and Gaussian membership functions are typical membership functions chosen for the process of fuzzy inference. In this paper, the reason of choosing triangular functions as the input membership functions is to reduce the computation complexity in the calculation of firing strengths in (25).

Theorem 2. *We consider a multiagent Euler–Lagrange system of (1) with a directed spanning-tree communication graph. From (7), the state trajectories of (1) will be driven onto the sliding surface $\mathbf{s}_i = 0$ with the adaptive fuzzy sliding-mode controller (AFSC) (21), (22), (26), and the adaptive law (24).*

Proof. Let the Lyapunov function be chosen as

$$V = \sum_{i=1}^{n}\frac{1}{2}\left(\mathbf{s}_i^T\mathbf{M}_i\mathbf{s}_i + \widetilde{\Theta}_i^T\Gamma^{-1}\widetilde{\Theta}_i\right). \quad (31)$$

The time derivative of V can be expressed as

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n}\left(\mathbf{s}_i^T\mathbf{M}_i\dot{\mathbf{s}}_i + \frac{1}{2}\mathbf{s}_i^T\dot{M}_i\mathbf{s}_i + \widetilde{\Theta}_i^T\Gamma^{-1}\dot{\widetilde{\Theta}}_i\right)\\ &= \sum_{i=1}^{n}\Big(\mathbf{s}_i^T\left(\boldsymbol{\tau}_{sw,i} + \mathbf{Y}_i(\mathbf{q}_i, \dot{q}_i, \epsilon_i, \dot{\epsilon}_i)\widetilde{\Theta}_i - \mathbf{C}_i\mathbf{s}_i\right) \quad (32)\\ &\quad + \frac{1}{2}\mathbf{s}_i^T\dot{M}_i\mathbf{s}_i - \widetilde{\Theta}_i\Gamma^{-1}\dot{\hat{\Theta}}_{\mathbf{i}}\Big).\end{aligned}$$

From Property 1, (29) can be rewritten as

$$\dot{V} = \sum_{i=1}^{n}\left(\mathbf{s}_i^T\left(\boldsymbol{\tau}_{sw,i} + \mathbf{Y}_i(\mathbf{q}_i, \dot{q}_i, \epsilon_i, \dot{\epsilon}_i)\widetilde{\Theta}_i\right) - \widetilde{\Theta}_i^T\Gamma^{-1}\dot{\hat{\Theta}}_i\right). \quad (33)$$

Substituting (24) and (26) into (30), it yields

$$\dot{V} = \sum_{i=1}^{n}\mathbf{s}_i^T\boldsymbol{\tau}_{sw,i} = -\sum_{i=1}^{n}\mathbf{s}_i^T\prod_{\mathbf{i}}\operatorname{sgn}(\mathbf{s}_i) < 0. \quad (34)$$

Therefore, it implies that $\mathbf{s}_i = 0$ and that the system trajectory is enforced on the sliding surfaces. The proof is completed. □

4.2. Fault Cases. In this section, we consider the controller design for multiagent systems with PLOE and unknown effectiveness fault, i.e., $0 < \sigma_i < 1$ and $\Delta\tau_i \neq 0$. From (7), the dynamical system (3) can be rewritten as

$$\mathbf{M}_i\dot{s}_i + \mathbf{C}_i\mathbf{s}_i - \mathbf{M}_i\mathbf{K}_i\dot{\epsilon}_i - \mathbf{C}_i\mathbf{K}_i\epsilon_i + \mathbf{g}_i = \sigma_i\boldsymbol{\tau}_i + \Delta\boldsymbol{\tau}_i. \quad (35)$$

For the multiagent systems of n agents, a faulty actuator of the ith agent can be described as

$$\boldsymbol{\tau}_i = \boldsymbol{\tau}_{eq,i} + \boldsymbol{\tau}_{sw,i} + \boldsymbol{\tau}_{c,i}, \quad (36)$$

where the auxiliary controller $\tau_{c,i}$ is provided to compensate the faulty influences.

The auxiliary controller can be expressed as

$$\boldsymbol{\tau}_{c,i} = -\left[\frac{1}{\hat{\underline{\sigma}}_i}\operatorname{sgn}(\mathbf{s}_i)\|\boldsymbol{\tau}_i\| + \hat{\bar{\boldsymbol{\tau}}}_i\operatorname{sgn}(\mathbf{s}_i)\right]. \quad (37)$$

The adaptive algorithms of fault are given as

$$\dot{\hat{\underline{\sigma}}}_i = \phi_i\left(\frac{1}{\hat{\underline{\sigma}}_i}\operatorname{sgn}(\mathbf{s}_i)\|\boldsymbol{\tau}_i\|\right), \quad (38)$$

$$\dot{\hat{\bar{\boldsymbol{\tau}}}}_i = \varsigma_i\|\mathbf{s}_i\|, \quad (39)$$

where ϕ_i and ς_i are positive constants. To make the design concepts more concise and clearer, the overall adaptive fuzzy sliding-mode fault-tolerant controller (AFSFC) structure is shown in Figure 2.

Theorem 3. *We consider a multiagent Euler–Lagrange system of (3) with a directed spanning-tree communication graph. From (7), the state trajectories of the system (32) will be driven onto the surface $\mathbf{s}_i = 0$ with the AFSFC (33), the auxiliary controller (34), and the adaptive laws (24), (35), (36).*

Proof. Let the Lyapunov function be chosen as

$$V_f = V + \sum_{i=1}^{n}\frac{1}{2}\left(\frac{1}{\phi_i}\tilde{\underline{\sigma}}_i^2 + \frac{1}{\varsigma_i}\tilde{\bar{\tau}}_i^2\right), \quad (40)$$

where $\tilde{\underline{\sigma}}_i = \underline{\sigma}_i - \hat{\underline{\sigma}}_i$, and $\tilde{\bar{\tau}}_i = \bar{\tau}_i - \hat{\bar{\tau}}_i$. The time derivative of V_f can be expressed as

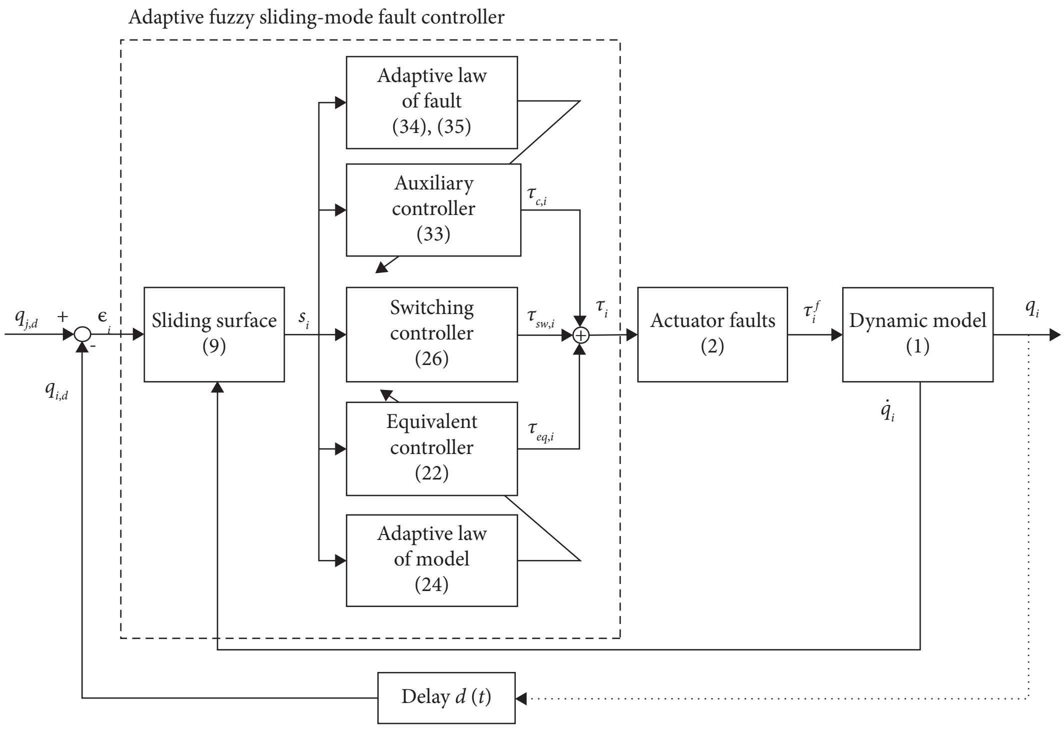

Figure 2: Structure of adaptive fuzzy-sliding fault-tolerant control.

$$\dot V_f = \dot V + \sum_{i=1}^{n}\left(-\frac{1}{\phi_i}\widetilde{\underline{\sigma}}_i\dot{\widehat{\underline{\sigma}}}_i - \frac{1}{\varsigma_i}\widetilde{\overline{\tau}}_i\dot{\widehat{\overline{\tau}}}_i\right)$$
$$= \sum_{i=1}^{n}\left(\mathbf{s}_i^T\mathbf{M}_i\dot s_i + \frac{1}{2}\mathbf{s}_i^T\dot M_i\mathbf{s}_i - \widetilde{\Theta}_i^T\Gamma^{-1}\dot{\widehat{\Theta}}_i - \frac{1}{\phi_i}\widetilde{\underline{\sigma}}_i\dot{\widehat{\underline{\sigma}}}_i - \frac{1}{\varsigma_i}\widetilde{\overline{\tau}}_i\dot{\widehat{\overline{\tau}}}_i\right). \tag{41}$$

Substituting (32) and (33) into (38), it yields

$$\dot V_f = \sum_{i=1}^{n}\left(\mathbf{s}_i^T\left(\mathbf{M}_i\mathbf{K}_i\epsilon_i + \mathbf{C}_i\mathbf{K}_i\epsilon_i + \mathbf{g}_i + \sigma_i\boldsymbol{\tau}_i + \sigma_i\boldsymbol{\tau}_{c,i} + \Delta\boldsymbol{\tau}_i\right) - \widetilde{\Theta}_i^T\Gamma^{-1}\dot{\widehat{\Theta}}_i - \frac{1}{\phi_i}\widetilde{\underline{\sigma}}_i\dot{\widehat{\underline{\sigma}}}_i - \frac{1}{\varsigma_i}\widetilde{\overline{\tau}}_i\dot{\widehat{\overline{\tau}}}_i\right). \tag{42}$$

From Assumption 2 and Theorem 2, the time derivative of V can be described as

$$\dot V_f \le \sum_{i=1}^{n}\left(\mathbf{s}_i^T\mathbf{Y}_i\left(\mathbf{q}_i, \dot q_i, \epsilon_i, \dot\epsilon_i\right)\widetilde{\Theta}_i + \mathbf{s}_i^T\boldsymbol{\tau}_{sw,i} + \|\mathbf{s}_i\|\|\boldsymbol{\tau}_i\| + \sigma_i\mathbf{s}_i^T\boldsymbol{\tau}_{c,i} + \overline{\tau}_i\|\mathbf{s}_i\| - \widetilde{\Theta}_i^T\Gamma^{-1}\dot{\widehat{\Theta}}_i - \frac{1}{\phi_i}\widetilde{\underline{\sigma}}_i\dot{\widehat{\underline{\sigma}}}_i - \frac{1}{\varsigma_i}\widetilde{\overline{\tau}}_i\dot{\widehat{\overline{\tau}}}_i\right). \tag{43}$$

From (43), one has

$$\sigma_i\mathbf{s}_i^T\boldsymbol{\tau}_{c,i} = -\frac{\sigma_i}{\widehat{\underline{\sigma}}_i}\|\mathbf{s}_i\|\|\boldsymbol{\tau}_i\| - \sigma_i\widehat{\overline{\tau}}_i\|\mathbf{s}_i\| \le -\frac{\underline{\sigma}_i}{\widehat{\underline{\sigma}}_i}\|\mathbf{s}_i\|\|\boldsymbol{\tau}_i\| - \widehat{\overline{\tau}}_i\|\mathbf{s}_i\|$$
$$= -\left(1 + \frac{\widetilde{\underline{\sigma}}_i}{\widehat{\underline{\sigma}}_i}\right)\|\mathbf{s}_i\|\|\boldsymbol{\tau}_i\| - \widehat{\overline{\tau}}_i\|\mathbf{s}_i\|, \tag{44}$$

where $\forall -1 \le -\sigma_i \le -\underline{\sigma}_i < 0$.

Substituting (41) into (40), it leads to

$$\dot V_f \le \sum_{i=1}^{n}\left(\mathbf{s}_i^T\mathbf{Y}_i\left(\mathbf{q}_i, \dot q_i, \epsilon_i, \dot\epsilon_i\right)\widetilde{\Theta}_i + \mathbf{s}_i^T\boldsymbol{\tau}_{sw,i} + \widetilde{\overline{\tau}}_i\|\mathbf{s}_i\| - \frac{\underline{\sigma}_i}{\widehat{\underline{\sigma}}_i}\|\mathbf{s}_i\|\|\boldsymbol{\tau}_i\| - \widetilde{\Theta}_i^T\Gamma^{-1}\dot{\widehat{\Theta}}_i - \frac{1}{\phi_i}\widetilde{\underline{\sigma}}_i\dot{\widehat{\underline{\sigma}}}_i - \frac{1}{\varsigma_i}\widetilde{\overline{\tau}}_i\dot{\widehat{\overline{\tau}}}_i\right). \tag{45}$$

From (24), (35) and (36), one has $\widetilde{\Theta}_i^T\Gamma^{-1}\dot{\widehat{\Theta}}_i = -\mathbf{s}_i^T\mathbf{Y}_i(\mathbf{q}_i, \dot{q}_i, \epsilon_i, \dot{\epsilon}_i)\widetilde{\Theta}_i$, $1/\varsigma_i\widetilde{\overline{\tau}}_i\dot{\widetilde{\overline{\tau}}}_i = \widetilde{\overline{\tau}}_i\|\mathbf{s}_i\|$, and $1/\phi_i\widetilde{\underline{\sigma}}_i\dot{\widehat{\underline{\sigma}}}_i = -\widetilde{\underline{\sigma}}_i(\widehat{\underline{\sigma}}_i)^{-1}\|\mathbf{s}_i\|\|\tau_i\|$. Therefore, (42) can be rewritten as

$$\dot{V}_f \leq \sum_{i=1}^{n}\mathbf{s}_i^T\boldsymbol{\tau}_{sw,i} = -\sum_{i=1}^{n}\mathbf{s}_i^T\prod_{\mathbf{i}}\operatorname{sgn}(\mathbf{s}_i) < 0. \quad (46)$$

Therefore, it implies that $\mathbf{s}_i = 0$ so that the system trajectory is enforced on the sliding surfaces. The proof is completed. □

Remark 3. In summary, with the sliding-mode control, the switching control action $\tau_{sw,i}$ (26) will drive the ith agent to the sliding surface $\mathbf{s}_i$. As the agents converge to the sliding surface, the equivalent controller $\tau_{eq,i}$ (22) will ensure the agents stay on the sliding surface. Moreover, the existence of unknown parameters and actuator faults can be resolved by using the adaptive law (24) and auxiliary controller (34).

5. Simulation Results

The simulations are conducted for 2-DOF (degree of freedom) with six robot agents. The Euler–Lagrange model (1) is obtained as [49].

$$\begin{aligned}
\mathbf{M}_i(\mathbf{q}_i) &= \begin{bmatrix} M_i^{11} & M_i^{12} \\ M_i^{21} & M_i^{22}\end{bmatrix}, \\
\mathbf{C}_i(\mathbf{q}_i, \dot{q}_i) &= \begin{bmatrix} C_i^{11} & C_i^{12} \\ C_i^{21} & C_i^{22}\end{bmatrix}, \\
\mathbf{G}_i &= 0, \\
\mathbf{q}_i &= \begin{bmatrix} q_{1i} \\ q_{2i}\end{bmatrix},
\end{aligned} \quad (47)$$

where $M_i^{11} = \alpha_1 + 2\alpha_2\cos(q_{2i}) + 2\alpha_3\sin(q_{2i})$, $M_i^{12} = M_i^{21} = \alpha_2 + \alpha_3\cos(q_{2i}) + \alpha_4\sin(q_{2i})$, $M_i^{22} = \alpha_2$, $C_i^{11} = -(\alpha_3\sin(q_{2i}) + \alpha_4\cos(q_{2i}))\dot{q}_{2i}$, $C_i^{12} = -(\alpha_3\sin(q_{2i}) + \alpha_4\cos(q_{2i}))(\dot{q}_{1i} + \dot{q}_{2i})$, $C_i^{21} = (\alpha_3\sin(q_{2i}) + \alpha_4\cos(q_{2i}))\dot{q}_{1i}$, $C_i^{22} = 0$, in which $\alpha_1 = I_1 + m_1 l_{c1}^2 + I_e + m_e l_e^2$, $\alpha_2 = I_1 + m_e l_{c1}^2$, $\alpha_3 = m_e l_c l_{ce}\cos(\sigma_e)$, $\alpha_4 = m_e l_c l_{ce}\sin(\sigma_e)$. The parameters of the multiagent system are set as $m_1 = 1$, $m_e = 2$, $l_c = 1$, $l_{c1} = 0.5$, $l_{ce} = 0.8$, $\sigma_e = \pi/6$, $I_e = 0.25$, and $I_1 = 0.15$.

The six-agent connected network is shown in Figure 3. Form Figures 3(a)–3(f), the directed graph exists the spanning tree. According to Theorem 1, the agents of the proposed topology in Figure 3(a) are asymptotically stable using the AFSFC scheme with the delay bound $\overline{d} = 0.16$ (sec). The parameters of the AFSFC are set as $\widetilde{\mu} = 0.5$, $\mathbf{K}_i = \text{diag}\{1, 1\}$, $f_{1i} = -f_{5i} = 1$, $f_{2i} = -f_{4i} = 0.5$, $f_{3i} = 0$, $g_{1i} = -g_{5i} = 144$, $g_{2i} = -g_{4i} = 12$, $g_{3i} = 1.5$, and $\Gamma_i = \text{diag}\{2, 2\}$. The adaptive laws are set as $\widehat{\Theta}_i = [\widehat{\alpha}_1, \widehat{\alpha}_2, \widehat{\alpha}_3, \widehat{\alpha}_4]^T$, $\phi_i = 1 \times 10^{-4}$, $\varsigma_i = 1$, and

$$\mathbf{Y}_i(\mathbf{q}_i, \dot{q}_i, \epsilon_i, \dot{\epsilon}_i) = \begin{bmatrix} Y_i^{11} & Y_i^{12} & Y_i^{13} & Y_i^{14} \\ Y_i^{21} & Y_i^{22} & Y_i^{23} & Y_i^{24}\end{bmatrix}^T, \quad (48)$$

where $Y_i^{11} = k_{1i}\epsilon_{1i}$, $Y_i^{12} = k_{2i}\dot{\epsilon}_{2i}$, $Y_i^{21} = 0$, $Y_i^{12} = k_{1i}\dot{\epsilon}_{1i} + k_{2i}\dot{\epsilon}_{2i}$, $Y_i^{23} = k_{1i}\cos(q_2)\dot{\epsilon}_{1i} + k_{1i}\dot{q}_{1i}\sin(q_{2i})\epsilon_{1i}$, $Y_i^{24} = k_{1i}\sin(q_2)\dot{\epsilon}_{1i} + k_{1i}\dot{q}_{1i}\cos(q_{2i})\epsilon_{1i}$, and

$$\begin{aligned}
Y_i^{13} &= 2k_{1i}\cos(q_{2i})\dot{\epsilon}_{1i} + k_{2i}\sin(q_{2i})\dot{\epsilon}_{2i} - k_{1i}\dot{q}_{2i}\sin(q_{2i})\epsilon_{1i} \\
&\quad - k_{2i}\dot{q}_{1i}\sin(\dot{q}_{2i})\epsilon_{2i} - k_{2i}\dot{q}_{2i}\sin(\dot{q}_{2i})\epsilon_{2i}, \\
Y_i^{14} &= 2k_{1i}\sin(q_{2i})\dot{\epsilon}_{1i} + k_{2i}\sin(q_{2i})\dot{\epsilon}_{2i} - k_{1i}\dot{q}_{2i}\cos(q_{2i})\epsilon_{1i} \\
&\quad - k_{2i}\dot{q}_{1i}\sin(\dot{q}_{2i})\epsilon_{2i} - k_{2i}\dot{q}_{2i}\cos(\dot{q}_{2i})\epsilon_{2i}.
\end{aligned} \quad (49)$$

In this paper, the time delay is chosen as $d(t) = 0.08 + 0.08\sin(t)$ (sec) for simulations. The initial states of the six agents are set as $\mathbf{q}_1 = [-1, 1]^T$, $\mathbf{q}_2 = [1, 0.5]^T$, $\mathbf{q}_3 = [-2.5, -1]^T$, $\mathbf{q}_4 = [2, -0.5]^T$, $\mathbf{q}_5 = [2.5, 1.5]^T$, $\mathbf{q}_6 = [-2, 1.5]^T$, and the initial velocities and acceleration are zeros. The following indices are considered for the combined formation errors, the integral absolute error (IAE), the integral time absolute error (ITAE), the integral square error (ISE), and the integral time square error (ITSE) [50].

Remark 4. The indices IAE, ITAE, ISE, and ITSE are considered for the performance comparisons. The IAE and ISE indicate the accumulated consensus error of agents, where the weights in the transient and steady-state stages are equal. On the other hand, ITAE and ITSE indicate the time-weighted consensus errors, and these two indices can be better used to highlight the performance superiority in the steady state.

5.1. Fault-Free Cases.

The comparisons between the adaptive fuzzy sliding-mode controller (AFSC) and the proposed adaptive fuzzy sliding-mode fault-tolerant controller (AFSFC) are also given in Figs. 4–9. Figures 4 and 7 show the responses of multiagent systems with the ASC and proposed AFSC, respectively. Lines A1-A6 are the trajectories of agents 1–6, the symbol "o" is initial state positions, and the symbol "⋆" is final state positions. Figures 5 and 8 show the states q_{1i} and q_{2i} of multiagent systems with the ASC and proposed AFSC, respectively. Figures 6 and 9 show the control inputs τ_{1i} and τ_{2i} of multiagent systems with the AFSC and AFSFC, respectively. The position errors are summarized in Table 1. In this case, the AFSC and AFSFC method can support a certain degree of consistence in position responses.

5.2. Fault Cases.

In this case, the effectiveness fault σ_i is present randomly in [0.01 0.5], and the additive faults are set to be $\Delta\tau = [2\text{rand}(.)2\text{rand}(.)]^T$, where rand(.) is a random number uniformly distributed in $[\,1\ \ -1\,]$. The comparisons between the AFSC and the proposed AFSFC are also given in Figs. 10–15. Figures 10 and 13 show the response of multiagent systems with the ASC and proposed AFSC. Figures 11 and 14 show the states q_{1i} and q_{2i} of multiagent systems with the AFSC and AFSFC, respectively. All the agents can achieve consensus. The states q_{1i} and q_{2i} are more stable by the proposed AFSFC compared with the AFSC.

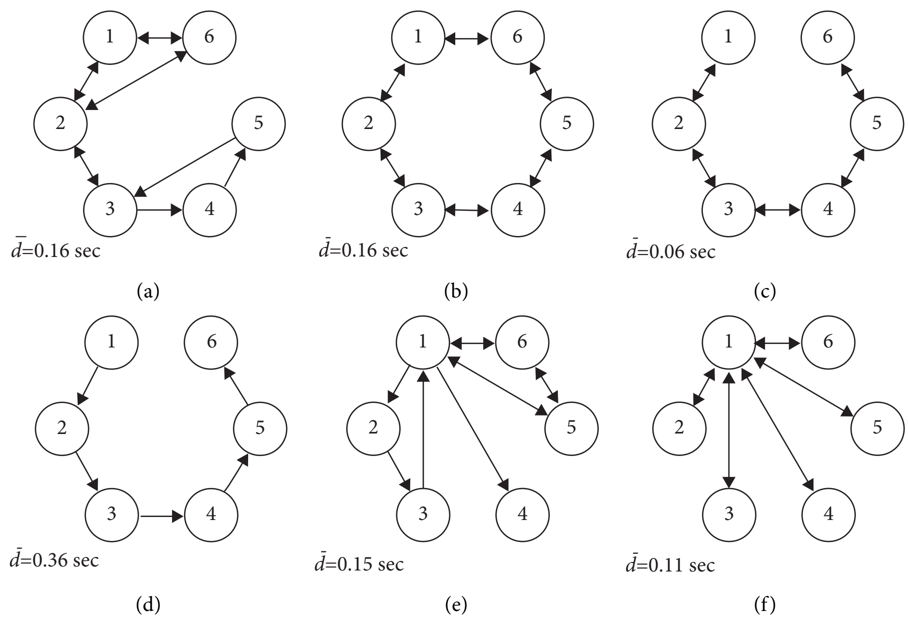

FIGURE 3: Communication topology of a multiagent system. (a) $\overline{d}$ = 0.16 sec. (b) $\overline{d}$ = 0.16 sec. (c) $\overline{d}$ = 0.06 sec. (d) $\overline{d}$ = 0.36 sec. (e) $\overline{d}$ = 0.15 sec. (f) $\overline{d}$ = 0.11 sec.

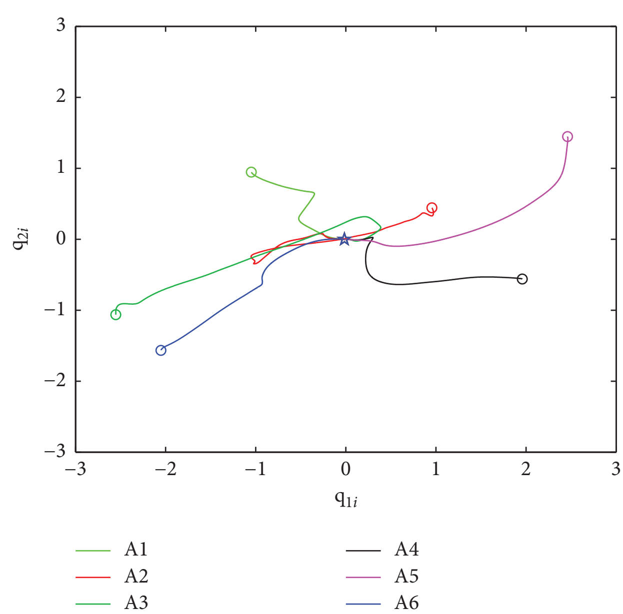

FIGURE 4: Multiagent trajectories and the fault-free case (AFSC).

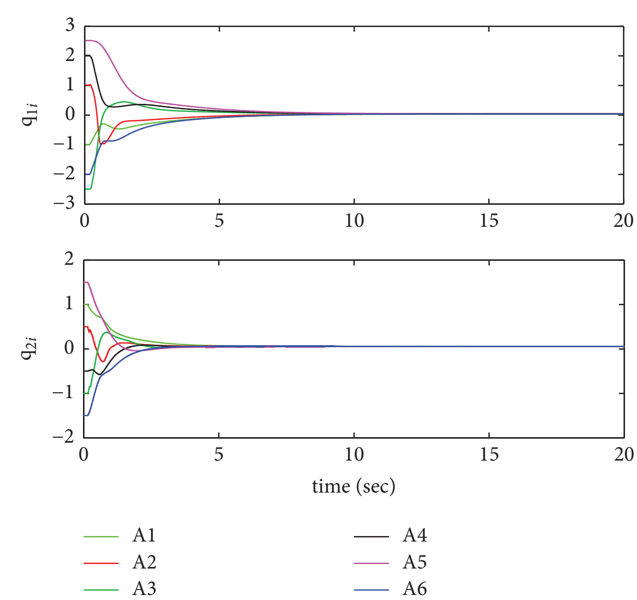

FIGURE 5: Simulation results of states q_{1i} and q_{2i} and the fault-free case (AFSC).

Figures 12 and 15 show the control inputs τ_{1i} and τ_{2i} of multiagent systems with the AFSC and AFSFC, respectively. From Figures 12 and 15, the amplitude of the control inputs is smaller in the first few seconds with the proposed AFSFC. In summary, from Figures 10–15, the performance of the proposed AFSFC is obviously better than the AFSC. Subsequently, the position errors are summarized in Table 2. It can be observed that the proposed AFSFC method extends the performance improvement from 7.07% to 86.84%.

Remark 5. From Figs. 6, 9, 12, and 15, it can be observed that greater inputs are required to deal with the problem of actuator faults. In practical applications, the control inputs should have prescribed limits. The consensus stability issues of multiagent agents with input constraints open a new theoretical problem, which cannot be solved at the current stage and needs to be investigated separately.

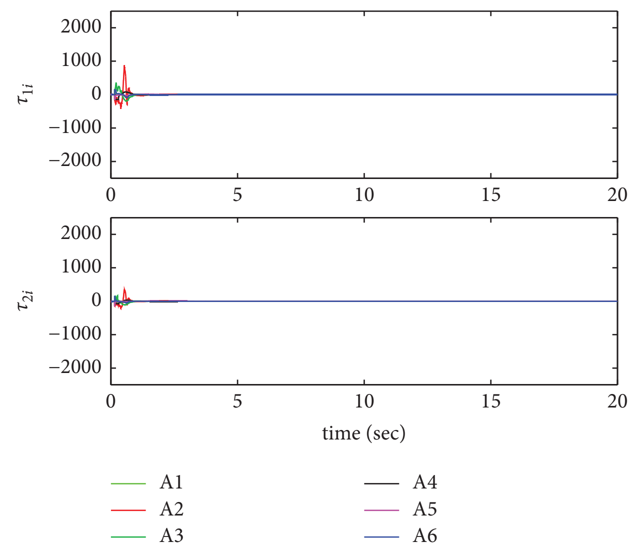

Figure 6: Simulation results of control inputs τ_{1i} and τ_{2i} and the fault-free case (AFSC).

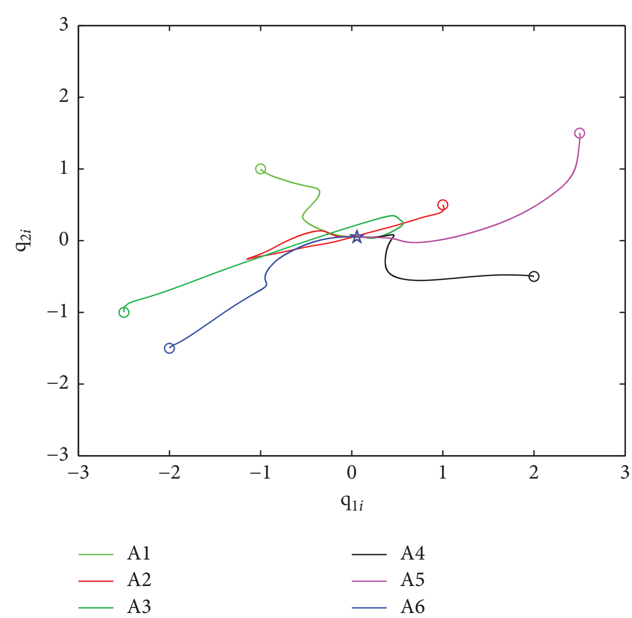

Figure 7: Multiagent trajectories and the fault-free case (AFSFC).

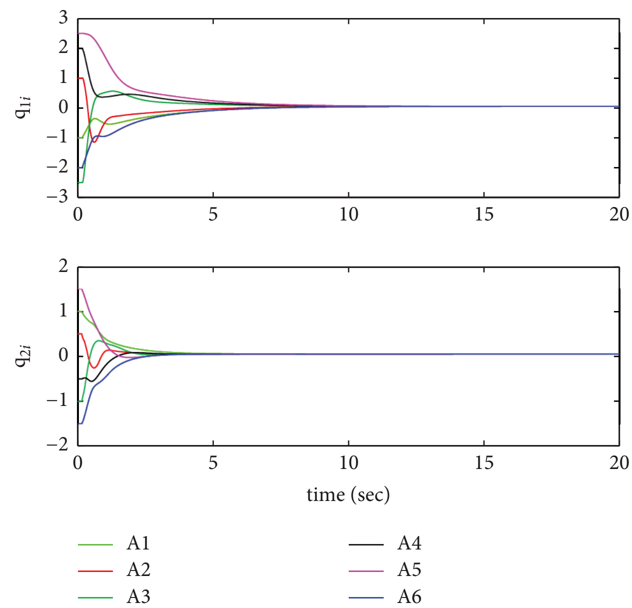

Figure 8: Simulation results of states q_{1i} and q_{2i} and the fault-free case (AFSFC).

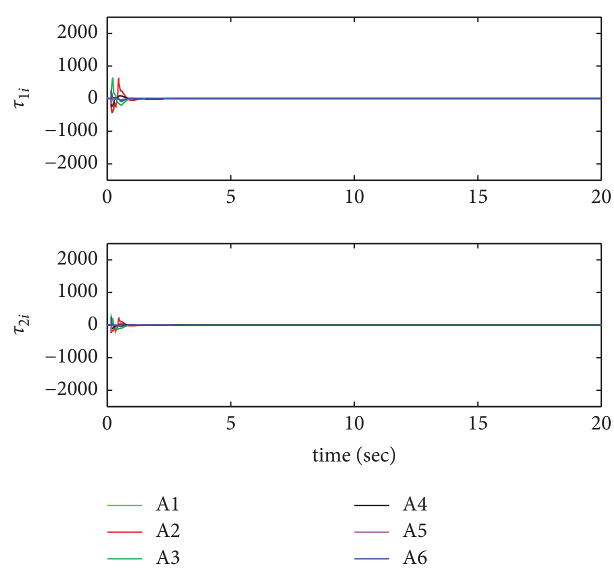

Figure 9: Simulation results of control inputs τ_{1i} and τ_{2i} and the fault-free case (AFSFC).

Table 1: Performance comparisons (fault-free case).

	IAE	ITAE	ISE	ITSE
AFSC	19454	38888	55358	50697
AFSFC	20697	42587	61819	63843

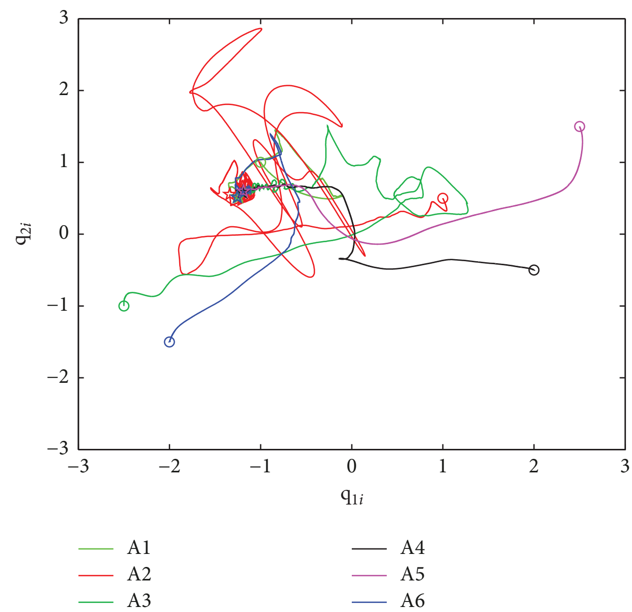

FIGURE 10: Multiagent trajectories and the fault case (AFSC).

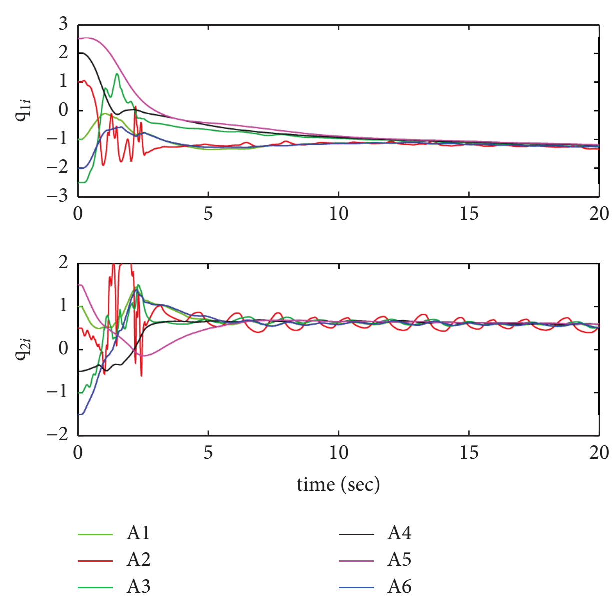

FIGURE 11: The states q_{1i} and q_{2i} and the fault case (AFSC).

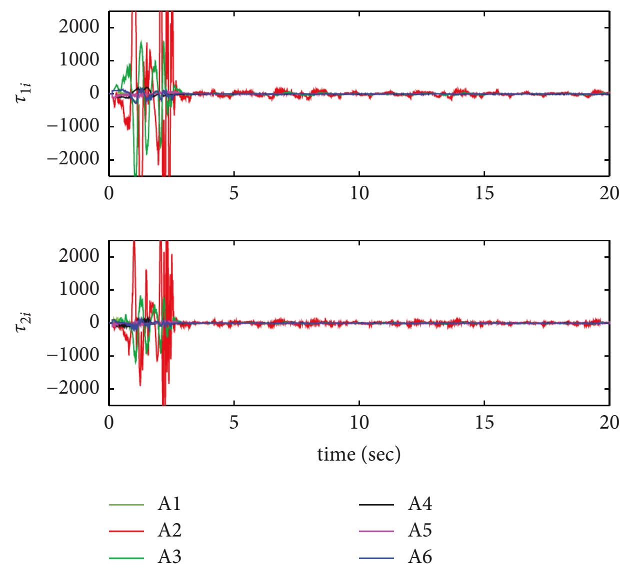

Figure 12: The control inputs τ_{1i} and τ_{2i} and the fault case (AFSC).

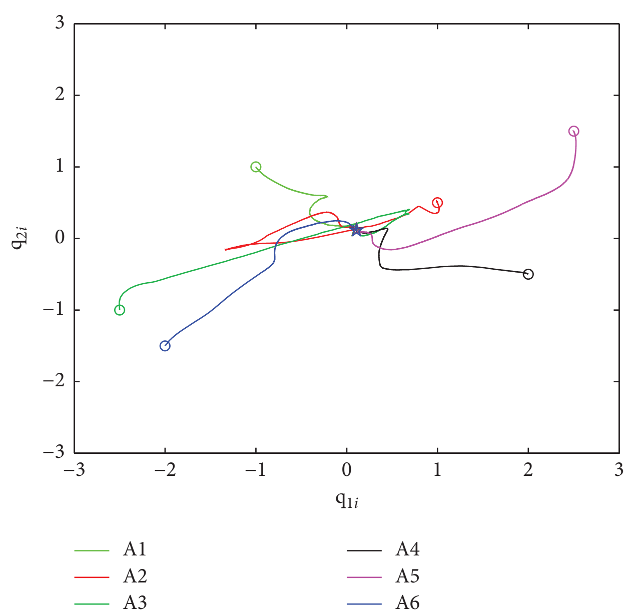

Figure 13: State trajectories and the fault case (AFSFC).

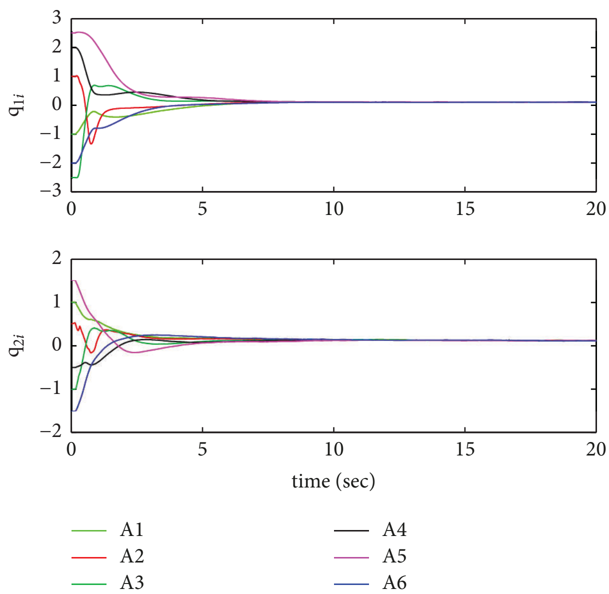

FIGURE 14: The states q_{1i} and q_{2i} and the fault case (AFSFC).

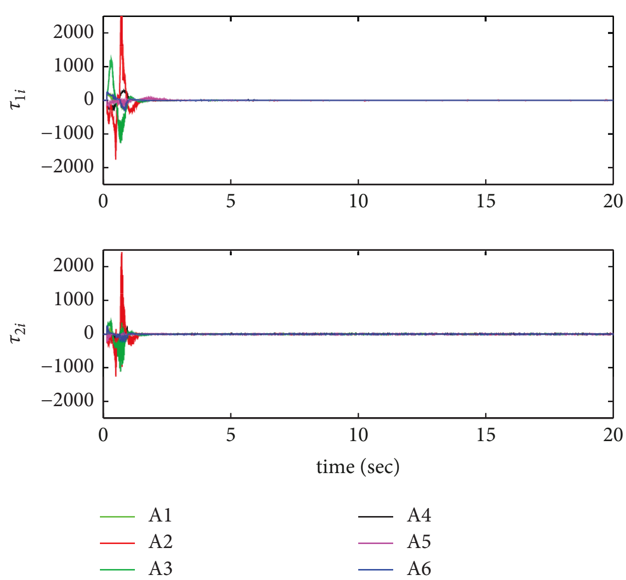

FIGURE 15: The control inputs τ_{1i} and τ_{2i} and the fault case (AFSFC).

TABLE 2: Performance comparisons (fault case).

	IAE	ITAE	ISE	ITSE
AFSC	36241	163310	90090	222250
AFSFC	22046	48783	64378	69119

6. Conclusions

This paper has proposed an adaptive fuzzy sliding-mode fault-tolerant controller for Euler–Lagrange systems in the presence of unknown parametrics, actuator faults, and communication time-varying delays. In the design of the fuzzy sliding-mode fault-tolerant controller, a delay-dependent sufficient condition is derived, and the allowable bound of time delays can be obtained in the form of linear matrix inequalities. Based on the Lyapunov stability theory, the overall stability of the multiagent system is guaranteed such that the desired consensus of agents can be asymptotically attained. Simulation results indicate that the proposed control scheme has superior responses, compared to the AFSC method. Especially, the proposed AFSFC method provides significant improvement in the case of actuator faults. In practical applications, the control inputs should have prescribed limits. Thus, the systematic analysis and synthesis of input-constrained multiagent systems are an interesting topic in the future.

Data Availability

The data that support the findings of this study are available on request from the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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