Hindawi
Complexity
Volume 2023, Article ID 1335184, 10 pages
https://doi.org/10.1155/2023/1335184

Research Article

The pth Moment Exponential Synchronization of Drive-Response Memristor Neural Networks Subject to Stochastic Perturbations

Xiaobo Wang,[1,2] **Xuefei Wu**,[3] **Zhe Nie**,[4] **and Zengxian Yan**[5]

[1]College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu 211106, China
[2]Business School, Nanjing Xiaozhuang University, Nanjing 211171, China
[3]Artificial Intelligence School, Shenzhen Polytechnic, Shenzhen 518055, China
[4]Digital Creativity and Animation School, Shenzhen Polytechnic, Shenzhen 518055, China
[5]Guangxi Modern Polytechnic College, Hechi 547000, Guangxi, China

Correspondence should be addressed to Xuefei Wu; wuxuefei@szpt.edu.cn

Received 4 November 2022; Revised 11 February 2023; Accepted 13 May 2023; Published 18 July 2023

Academic Editor: Xinzhi Liu

Copyright © 2023 Xiaobo Wang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this paper, the pth moment exponential synchronization problems of drive-response stochastic memristor neural networks are studied via a state feedback controller. The dynamics of the memristor neural network are nonidentical, consisting of both asymmetrically nondelayed and delayed coupled, state-dependent, and subject to exogenous stochastic perturbations. The pth moment exponential synchronization of these drive-response stochastic memristor neural networks is guaranteed under some testable and computable sufficient conditions utilizing differential inclusion theory and Filippov regularization. Finally, the correctness and effectiveness of our theoretical results are demonstrated through a numerical example.

1. Introduction

Neural networks, which simulate the structure of neurons and synapses in the human brain with mathematical models and combines multilevel conduction to simulate the interconnection structure of neurons, have now been widely used in artificial intelligence. On one hand, the development of the neural network is based on the understanding of biological brain to simulate its working mechanism more closely, such as the proposal and development of the third generation artificial neural network-spiking neural network; the mathematical modeling to simulate the nerve conduction system in the brain and the goal is to understand the way of brain signal transmission in order to help understand the way the brain works from the perspective of computational simulation.

Memristor is one of the best ways for hardware to realize synapses in artificial neural networks. The first nanoscale memristor device is made and set off a boom in memristor research in 2008. Memristors have been utilized as electronic synaptic devices and exhibit a migration of ions that bear striking resemblance to the diffusion procedure of neurotransmitters across neural synapses. This has led to the common tendency of utilizing memristors to character synapses in neural networks, which is broadly employed to store synaptic weights. Extensive experiments have shown that synapses in neural networks simulated by memristors will have great promising advantages. To be specific, the memristor, as a fundamental passive device, has nanoscale size and nonvolatility, achieving the consecutive changes in synaptic weights during the simulation of neural synapses, and also facilitating the integration of computation and storage. Moreover, a neural network structure that is highly integrated can be built by virtue of the memristors, which endow artificial neural networks with the ability not only to learn and memorize but also to perform more various functions.

And then, the modeling of the memristor neural network for the study of its rich dynamic behavior has become a hot research direction because of its broad application prospect. Over the past decade or so, the research studies put more energy into such a few aspects, such as signal processing [1], intelligent vehicle [2], and chaotic circuits [3].

During the past decade, there has been an increasing interest in diverse synchronization problems in complex dynamical networks considering their extensive applications in practice [4–16]. The process of synchronization in a network of dynamic nodes refers to the convergence of all nodes towards a shared behavior, which is driven by specific coupling and/or control protocols. So far, a number of results associated with the synchronization of memristor neural networks have been shown [17–24]. Furthermore, based on the consideration of synchronization efficiency, the selection and improvement of the control strategy is still the focus of our attention. What is worth celebrating is that, many peers have provided abundant case references and conclusions for synchronization problems, including state feedback control [25], impulsive control [26], and quantized control [27]. And the practical application about drive-response stochastic memristor neural networks is more widespread and useful than the memristor neural networks due to the inevitability of unknown stochastic factors, for example, as the authors expound in their studies about the stochastic memristor neural networks and their synchronization [26, 28, 29], the MNN's dynamical behavior is exceptionally sensitive to unexpected stochastic disturbance due to frequent changes, which are caused by signal transmission anomalies or external environmental factors.

More relevant research results are as follows: Guo et al. [30] discussed the synchronization issue of multiple memristor neural networks with time delays by establishing two new integrate-differential inequalities. Zhu et al. [31] studied the synchronization problem of the master and slave memristor neural networks via an event-based impulsive scheme and considered certain state-dependent triggering conditions of nonlinear and linear continuous-time dynamic systems. We have noticed that the pth moment synchronization is more general than the mean square synchronization, where the power p does not need to be greater than 1 since it still gives a metric value for the random variables. For $p = 1$, this is also referred to as convergence in the mean sense and for $p = 2$, this is referred to as convergence in the mean square sense.

As another unexpected factor, time delay is a must to be dealt within the synchronization control process of most complex dynamic networks, including drive-response stochastic memristor neural networks, as it can negatively affect the signal transmission and dynamic behavior. The time delay neural network is an architecture of multilayer artificial neural networks that is specifically designed for classifying patterns with shift-invariance, and for modeling the context at each layer of the network. Moreover, stochastic disturbance and uncertainty occur in some random environments and influence the performance and synchronization efficiency of the system [32].

Based on the discussion mentioned above, the main contributions are outlined as follows: this paper studies the pth moment exponential synchronization of the drive-response stochastic memristor neural networks with time-varying delay. In addition, our model is extended to incorporate the stochastic perturbations, thereby increasing its generality and comprehensiveness compared to previous studies. Different from the mean-square synchronization and almost sure synchronization, we consider the pth moment exponential synchronization of the drive-response memristor neural networks with stochastic perturbations. By using the differential inclusion theory and the Filippov regularization, certain more general synchronization criteria are obtained. The proposed feedback control, the second term of which is capable of eliminating the influences of both mismatched and state-dependent arguments, is simpler and more flexible than the existing results.

The organization of this paper is as follows: in Section 2, we present the model of the drive-response memristor neural network subject to stochastic perturbations, along with some definitions and assumptions. The sufficient condition for the moment synchronization is derived in Section 3. Section 4 includes numerical simulations.

2. Preliminaries and Problem Definition

2.1. Notations. Let R and R^n be the set of real numbers and the $n-$dimensional Euclidean space, respectively. Given a vector or matrix, $\|\cdot\|$ denotes its standard 2-norm and $|\cdot|$ implies the entries' absolute values, that is, $|A| = (|a_{ij}|)_{m\times n}$, let $\lambda_{\min}(A)$ be the smallest eigenvalue of A. The maximum element of a vector x is denoted as $\max\{x\}$ and set $\{1, 2, \ldots, n\}$ is defined as N.

Let $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ be a complete probability space equipped with a filtration $\{F_t\}_{t\geq 0}$, that is, right continuous with F_0 and contains all the $P-$null sets. $C([-\tau, 0]; R^n)$ shall denote the collection of continuous functions ϕ from $[-\tau, 0]$ to R^n with the uniform norm $\|\phi\|^2 = \sup_{-\tau\leq s\leq 0}\phi(s)^T\phi(s)$ and $C^2_{F_0}([-\tau, 0]; R^n)$ the family of all F_0 measurable, $C([-\tau, 0]; R^n)$-valued stochastic variables $\xi = \{\xi(\theta)\colon -\tau \leq \theta\leq 0\}$ such that $\int_{-\tau}^{0} \mathrm{E}|\xi(s)|^2 \mathrm{d}s \leq \infty$, where E means the corresponding expectation operator regarding the given probability measure P.

2.2. Problem Formulation. A memristor neural network subject to stochastic perturbations and time-varying delays is formulated by the stochastic differential functional equations [33].

$$\mathrm{d}x i(t) = \left\{-\mathrm{d}i(xi(t))xi(t) + \sum_{j=1}^{n} aij(xj(t))fj(xj(t)) + \sum_{j=1}^{n} bij(xj(t))fj(xj(t-\tau j(t)))\right\}\mathrm{d}t + \sigma i(xi(t))\mathrm{d}w(t), i, j \in N, \tag{1}$$

where $x_i(t) \in R$ is ith system's voltage, $di(xi(t)) > 0$ stands for the reset rate, $f_j(\cdot)$ is an active function, $\tau j(t)$ is the varying time delay, $\sigma i(xi(t))$ denotes the noise strength function, $\omega(t)$ represents the standard one-dimensional Brownian motion satisfying $\mathrm{E}d\omega = 0$ and $\mathrm{E}(d\omega)^2 = dt$, and the connection weights $aij(xj(t))$ and $b_{ij}(xj(t))$ are defined as follows:

$$
\begin{aligned}
d_i(xi(t)) &= \frac{1}{C_i}\left[\sum_{j=1}^{N}\left(\frac{1}{R_{ij}^f} + \frac{1}{R_{ij}^g}\right) + W_i(x_i(t))\right] \\
&= \begin{cases} \underline{d}_i, & |x_i(t)| \le T_i, \\ \bar{d}_i, & |x_i(t)| > T_i, \end{cases} \\
a_{ij}(x_j(t)) &= \frac{W_{ij}^a(x_j(t))}{C_i}\mathrm{sgn}_{ij} = \begin{cases} \underline{a}_{ij}, & |x_j(t)| \le T_i, \\ \bar{a}_{ij}, & |x_j(t)| > T_i, \end{cases} \\
b_{ij}(x_j(t)) &= \frac{W_{ij}^b(x_j(t))}{C_i}\mathrm{sgn}_{ij} = \begin{cases} \underline{b}_{ij}, & |x_j(t)| \le T_i, \\ \bar{b}_{ij}, & |x_j(t)| > T_i, \end{cases} \\
\mathrm{sgn}_{ij} &= \begin{cases} 1, & i \ne j, \\ -1, & i = j, \end{cases}
\end{aligned}
\tag{2}
$$

where $W_i(x_i(t))$ is the memductance of memristor parallel with capacitor C_i, W_{ij}^a and W_{ij}^b are the memductances of the memristor R_{ij}^f and R_{ij}^g, $T_i > 0$ is the switching jump bound, and $\underline{d}_i$, $\bar{d}_i$, $\underline{a}_{ij}$, $\bar{a}_{ij}$, $\underline{b}_{ij}$, and $\bar{b}_{ij}$ are constants with $\underline{d}_i \ne \bar{d}_i$, $\underline{a}_{ij} \ne \bar{a}_{ij}$, and $\underline{b}_{ij} \ne \bar{b}_{ij}$. We assume that $0 < \tau_1 \le \tau_i(t) \le \tau_2 < \infty$ for all $t \ge 0, i \in \mathrm{N}$ and the identical condition of equation (1) satisfies $x_i(t) = \Phi_i(t) \in C([t_0 - \tau_2, t_0], R)$.

Let $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$, $D(x(t)) = \mathrm{diag}_n\{d_i(x_i(t))\}$, $A(x(t)) = (a_{ij}(x_j(t)))_{n\times n}$, $B(x(t)) = (b_{ij}(x_j(t)))_{n\times n}$, $f(x(t)) = \mathrm{diag}_{n\times 1}\{f_i(x_i(t))\}$, $f(x(t-\tau(t))) = \mathrm{diag}_{n\times 1}\{f_i(x_i(t-\tau_i(t)))\}$, and $\sigma(x(t)) = [\sigma(x_1(t)), \sigma(x_2(t)), \ldots, \sigma(x_n(t))]^T$. Then, system (1) can be written in the vector form as follows:

$$
\begin{aligned}
dx(t) = {} & \{-D(x(t))x(t) + A(x(t))f(x(t)) + B(x(t-\tau(t)))f(x(t-\tau(t)))\}dt \\
& + \sigma(x(t))d\omega(t),
\end{aligned}
\tag{3}
$$

and the response stochastic memristor neural networks with controllers are as follows:

$$
\begin{aligned}
dy(t) = {} & \{-D(y(t))y(t) + A(y(t))f(y(t)) + B(y(t-\tau(t)))f(y(t-\tau(t))) + U(t)\}dt \\
& + \sigma(y(t))d\omega(t),
\end{aligned}
\tag{4}
$$

where $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^T$, $U(t) = \mathrm{diag}_{n\times 1}\{u_i(t)\}d_i(y_i(t)), a_{ij}(y_i(t))$,and $b_{ij}(y_j(t))$ are defined similarly, and $u_i(t)$ represents the controller to be developed later. $y_i(t) = \psi_i(t) \in C([t_0 - \tau_2, t_0], R)$ is the initial conditions for system (4). Usually, $\psi_i(t) \ne \phi_i(t)$ for $i \in \mathrm{N}$.

Our objective is to achieve the pth moment exponential synchronization by designing a suitable controller $u_i(t)$.

Definition 1. The drive system (3) and response system (4) are said to be pth moment exponentially synchronized if

$$\sum_{i=1}^{N} \mathrm{E}\|x(t) - y(t)\|^{p} \leq Ke^{-\kappa t}, \tag{5}$$

where $K > 0$ and $\kappa > 0$, for any initial data $x(t_0), y(t_0) \in C_{F0}^{b}([-\tau, 0]; R^n)$.

Remark 2. Compared with the exponential synchronization [18, 19, 30], the pth moment exponential synchronization is used to measure the systems' subject to the stochastic noise, which is in practice. The pth moment exponential synchronization is more general than the mean square synchronization [34], where $p = 2$.

The control scheme is designed as follows:

$$U(t) = -Ke(t) - \eta \operatorname{sgn}(e(t)), \tag{6}$$

where $K = \operatorname{diag}_n\{k_i\}$ in which k_i and η will be determined later.

Remark 3. The control is basic and useful to deal with the pth moment exponential synchronization problem, which includes two terms the first term adjusting by a proportional gain constant K is proportional control to produce an output value that is proportional to the current error value and the second term adjusting by a proportional gain constant μ is capable of eliminating the influences of mismatched and state-dependent arguments concurrently, which is simpler and more flexible than the existing results.

Utilizing the differential inclusion theory and the Filippov regularization, equation (1) has been rewritten as follows:

$$\begin{aligned} \mathrm{d}x_i(t) \in &\left\{ -\bar{co}\left[\underline{d}_i, \bar{d}_i\right] x_i(t) + \sum_{j=1}^{n} \bar{co}\left[\underline{a}_{ij}, \bar{a}_{ij}\right] f_j\left(x_j(t)\right) + \sum_{j=1}^{n} \bar{co}\left[\underline{b}_{ij}, \bar{b}_{ij}\right] f_j\left(x_j\left(t - \tau_j(t)\right)\right) + s_i \right\} \mathrm{d}t \\ &+ \sigma_i\left(x_i(t)\right) \mathrm{d}\omega(t), i, j \in \mathrm{N}. \end{aligned} \tag{7}$$

Therefore, some measurable functions $\theta_i^1(t) \in \bar{co}[0,1] \theta_{ij}^2(t) \in \bar{co}[0,1]$ and $\theta_{ij}^2(t) \in \bar{co}[0,1]$ exist from the measurable selection theorem [35] such that

$$\begin{aligned} &\left\{ \mathrm{d}x_i(t) = -\left[\theta_i^1(t)\underline{d}_i + \left(1 - \theta_i^1(t)\right)\bar{d}_i\right] x_i(t) + \sum_{j=1}^{n} \left[\theta_{ij}^2(t)\underline{a}_{ij} + \left(1 - \theta_{ij}^2(t)\right)\bar{a}_{ij}\right] f_j\left(x_j(t)\right) + \sum_{j=1}^{n} \left[\theta_{ij}^3(t)\underline{b}_{ij} + \left(1 - \theta_{ij}^3(t)\right)\bar{b}_{ij}\right] f_j\left(x_j\left(t - \tau_j(t)\right)\right) + s_i \right\} \\ &\mathrm{d}t + \sigma_i\left(x_i(t)\right) d\omega(t), i, j \in N. \end{aligned} \tag{8}$$

The response system (4) can be similarly written as follows:

$$\begin{aligned} \mathrm{d}y_i(t) = &\left\{ -\left[\theta_i^4(t)\underline{d}_i + \left(1 - \theta_i^4(t)\right)\bar{d}_i\right] y_i(t) + \sum_{j=1}^{n} \left[\theta_{ij}^5(t)\underline{a}_{ij} + \left(1 - \theta_{ij}^5(t)\right)\bar{a}_{ij}\right] f_j\left(y_j(t)\right) + \sum_{j=1}^{n} \left[\theta_{ij}^6(t)\underline{b}_{ij} + \left(1 - \theta_{ij}^6(t)\right)\bar{b}_{ij}\right] \right. \\ &\left. f_j\left(y_j(t - \tau(t))\right) + s_i \right\} \mathrm{d}t + u_i(t) + \sigma_i\left(y_i(t)\right) \mathrm{d}\omega(t), i, j \in N, \end{aligned} \tag{9}$$

with certain measurable functions $\theta_i^4(t) \in \bar{co}[0,1]$, $\theta_{ij}^5(t) \in \bar{co}[0,1]$, and $\theta_{ij}^6(t) \in \bar{co}[0,1]$.

Denote $e_i(t) = y_i(t) - x_i(t)$ and take the following error stochastic memristor neural network into consideration.

$$
\begin{aligned}
de_i(t) = \Big\{ & -\big[\theta_i^4(t)\underline{d}_i + \big(1-\theta_i^4(t)\big)\bar{d}_i\big] y_i(t) \\
& + \big[\theta_i^1(t)\underline{d}_i + \big(1-\theta_i^1(t)\big)\bar{d}_i\big] x_i(t) \\
& + \sum_{j=1}^{n} \tilde{a}_{ij}(t)\tilde{f}_j\big(e_j(t)\big) \\
& + \sum_{j=1}^{n} \Delta a_{ij}(t) f_j\big(x_j(t)\big) \\
& + \sum_{j=1}^{n} \tilde{b}_{ij}(t)\tilde{f}_j\big(e_j\big(t-\tau_j(t)\big)\big) \\
& + \sum_{j=1}^{n} \Delta b_{ij}(t) f_j\big(x_j\big(t-\tau_j(t)\big)\big) + u_i(t) \Big\} \mathrm{d}t \\
& + u_i(t) + \tilde{\sigma}_i(e_i(t))\mathrm{d}\omega(t), i, j \in N,
\end{aligned} \tag{10}
$$

where $\tilde{f}_j(e_j(t)) = f_j(y_j(t)) - f_j(x_j(t))$, $\tilde{a}_{ij}(t) = \theta_{ij}^5(t)\underline{a}_{ij} + (1-\theta_{ij}^5(t))\bar{a}_{ij}$, $\Delta a_{ij}(t) = (\theta_{ij}^5(t) - \theta_{ij}^2(t))(\underline{a}_{ij} - \bar{a}_{ij})$, $\tilde{f}_j(e_j(t-\tau_j(t))) = f_j(y_j(t-\tau_j(t))) - f_j(x_j(t-\tau_j(t)))$, and $\tilde{b}_{ij}(t) = \theta_{ij}^6(t)\underline{b}_{ij} + (1-\theta_{ij}^6(t))\bar{b}_{ij}$.

$$
\Delta b_{ij}(t) = \big(\theta_{ij}^6(t) - \theta_{ij}^3(t)\big)\big(\underline{b}_{ij} - \bar{b}_{ij}\big), \tilde{\sigma}_{\mathrm{i}}(e_i(t)) = \sigma_i(y_i(t)) - \sigma_i(x_i(t)). \tag{11}
$$

Set $e(t) = [e_1(t), e_2(t), \ldots, e_n(t)]^T$. Then, the error system (10) is converted into the following form:

$$
\begin{aligned}
\mathrm{d}e(t) = \Big\{ & -D_1(t)y(t) + D_2(t)x(t) + A(t)\tilde{f}(e(t)) + \Delta A(t) f(x(t)) + B(t)\tilde{f}(e(t-\tau(t))) + \Delta B(t) f(x(t-\tau(t))) + U(t) \Big\}\mathrm{d}t \\
& + \tilde{\sigma}(e(t))\mathrm{d}\omega(t),
\end{aligned} \tag{12}
$$

where $D_1(t) = \mathrm{diag}_n\{\theta_i^4(t)\underline{d}_i + (1-\theta_i^4(t))\bar{d}_i\}$, $D_2(t) = \mathrm{diag}_n\{\theta_i^1(t)\underline{d}_i + (1-\theta_i^1(t))\bar{d}_i\}$, $\Delta A(t) = (\Delta a_{ij}(t))_{n\times n}$, $A(t) = (\tilde{a}_{ij}(t))_{n\times n}$, $B(t) = (\tilde{b}_{ij}(t))_{n\times n}$, $\Delta B(t) = (\Delta b_{ij}(t))_{n\times n}$, $\tilde{f}(e(t)) = \mathrm{diag}_{n\times 1}\{\tilde{f}_i(e_i(t))\}$, $\tilde{f}(e(t-\tau(t))) = \mathrm{diag}_{n\times 1}\{\tilde{f}_i(e_i(t-\tau(t)))\}$ and $\tilde{\sigma}(e(t)) = \mathrm{diag}_{n\times 1}\{\tilde{\sigma}_i(e_i(t))\}$, and $U(t) = \mathrm{diag}_{n\times 1}\{u_i(t)\}$.

Assumption 4. $f_i(\cdot)$, $\sigma_i(\cdot)$, and $i \in \{1, 2, \ldots, n\}$ are measurable, satisfying the Lipschitz condition, *i.e.*, there exist positive constants l_i and h_i such that

$$
\begin{aligned}
\big|f_i(x) - f_i(y)\big| &\le l_i|x-y|, \\
\big|\sigma_i(x) - \sigma_i(y)\big| &\le h_i|x-y|,
\end{aligned} \tag{13}
$$

for all $x, y \in R$. And there exist positive constants $m_i, i \in \mathrm{N}$ such that

$$
\big|f_i(x)\big| \le m_i, \tag{14}
$$

for all $x \in R$.

Lemma 5. *For any $x, y \in R^n$ and scalar $\varepsilon > 0$, we have*

$$
x^T y + y^T x \le \varepsilon x^T x + \varepsilon^{-1} y^T y. \tag{15}
$$

Lemma 6 (see [36]). *Let $v(t) \ge 0$ over the interval $t \in (-\infty, +\infty)$ and*

$$
D^+ v(t) \le \alpha(t)v(t) + \beta(t) \sup_{t-\tau(t)\le s\le t} v(s) \text{ for } t > t_0, \tag{16}
$$

where $D^+ v(t) = \overline{\lim}_{h\longrightarrow 0^+}(v(t+h) - v(t))/h$, $\tau(t) > 0$, $\alpha(t) \le 0$, and $\beta(t) \ge 0$ are continuous functions. If there exists a positive constant δ such that

$$
\alpha(t) + \beta(t) \le -\delta < 0 \text{ for } t \ge t_0, \tag{17}
$$

then

$$v(t) \leq \sup_{-\infty \leq s \leq t_0} v(s)e^{-\mu*}(t-t_0), \tag{18}$$

where $\mu^* = \inf_{t \geq t_0}\{\mu: \mu + \alpha(t) + \beta(t)e^{\mu\tau(t)} = 0\}$.

Lemma 7 (see [36]). *Considering an* $n-$ dimensional *stochastic differential equation, we have*

$$dx(t) = f(t, x(t), x(t-\tau))\mathrm{d}t + \sigma(t, x(t), x(t-\tau))\mathrm{d}\omega(t). \tag{19}$$

Let $C^{2,1}(R_+ \times R^n; R_+)$ denote the family of all nonnegative functions $V(t,x)$ on $R_+ \times R^n$, which are twice continuously differentiable in x and once differentiable in t. If $V \in C^{2,1}(R_+ \times R^n; R_+)$, define an operator LV from $R_+ \times R^n$ to R by

$$\begin{aligned} LV(t, x(t)) &= V(t, x(t)) + V_x(t, x(t)) f(t, x(t), x(t-\tau)) \\ &\quad + \frac{1}{2}T_r\left[\sigma(t, x(t), x(t-\tau))^T V_{xx}\right. \\ &\quad (t, x(t))\sigma(t, x(t), x(t-\tau))], \end{aligned} \tag{20}$$

where $V_t(t,x) = \partial V(t,x)/\partial t$, $V_{xx}(t,x) = (\partial^2 V(t,x)/\partial x_i x_j)_{n\times n}$, and $V_x(t,x) = (\partial V(t,x)/\partial x_1, \ldots, \partial V(t,x)/\partial x_n)$. If $V \in C^{2,1}(R_+ \times R^n; R_+)$, then for any $\infty > t > t_0 \geq 0$,

$$EV(t, x(t)) = EV(t_0, x(t_0)) + E\int_{t_0}^{t} LV(s, x(s))\mathrm{d}s, \tag{21}$$

as long as the expectations of the integrals exist.

3. Main Result

This section investigates the pth moment exponential synchronization of the proposed systems.

The selection of the gain K and η to guarantee the pth moment exponential synchronization of system (1) with system (4) will be given in the following theorem:

Theorem 8. *We assumed that Assumption 4 is satisfied. If the gain K satisfies*

$$\begin{aligned} &K > -\bar{D} + \rho_A L + \rho_B^2 L^2 + \left(1 + h^2\right) I_n, \\ &\eta = \max\{\underline{D}T + (\mu_A + \mu_B)M\}, \end{aligned} \tag{22}$$

where u_A, u_B, ρ_A, *and* ρ_B *are positive constants satisfying* $\|\underline{A}(t)\| \leq \rho_A$, $\|\underline{B}(t)\| \leq \rho_B$, $\|\Delta A(t)\| \leq \underline{\mu}_A$, *and* $\|\Delta B(t)\| \leq \mu_B$. $\bar{D} = \mathrm{diag}\{\min\{\underline{d}_1 \bar{d}_1\}, \ldots, \min\{\underline{d}_n \bar{d}_n\}\}$, $\underline{D} = \mathrm{diag}\{|\underline{d}_1 - \bar{d}_1|, \ldots, |\underline{d}_n - \bar{d}_n|\}$, $T = (T_1, \ldots, T_n)^T$, $h = \max_{1 \leq i \leq n}\{h_i\}$, *and* $M = \mathrm{diag}_n\{m_i\}$, *then system (1) and system (4) can reach the* pth *moment exponential synchronization with control scheme (6).*

Proof. We denote $V(t) = \|e(t)\|^p$. Considering Lemma 7 with system (12), we get

$$\mathrm{d}V(t) = LV(t)\mathrm{d}t + p\|e(t)\|^{p-2} y^T(t)\tilde{\sigma}(e(t))\mathrm{d}\omega(t). \tag{23}$$

Integrating from t_0 to t and taking the expectations, it holds

$$EV(t) = EV(t_0) + E\int_{t_0}^{t} LV(s)\mathrm{d}s, \tag{24}$$

in which

$$\begin{aligned} LV(t) &= \frac{p}{2}\|e(t)\|^{p-2}\Big(2y^T(t)\big[-D_1(t)y(t) + D_2(t)x(t) + A(t)\tilde{f}(e(t)) + \Delta A(t) f(x(t)) + B(t)\tilde{f}(e(t-\tau(t))) \\ &\quad + \Delta A(t) f(x(t-\tau(t))) + U(t)\big]\Big) + \tilde{\sigma}^T(e(t))\tilde{\sigma}(e(t))) \\ &\leq \frac{p}{2}\|e(t)\|^{p-2}\Big(2y^T(t)\Big[-\bar{D}_e(t) + \mathrm{diag}\left\{\mathrm{sgn}^T(e(t))\right\}\underline{D}T + A(t)\tilde{f}(e(t)) + \Delta A(t) f(x(t)) + B(t)\tilde{f}(e(t-\tau(t))) \\ &\quad + \Delta B(t) f(x(t-\tau(t))) - Ke(t) - \eta\mathrm{n}(e(t))\big] + \tilde{\sigma}^T(e(t))\tilde{\sigma}(e(t))\Big). \end{aligned} \tag{25}$$

Because $\eta = \max\{\underline{D}T + (\mu_A + \mu_B)M\}$ in (6), one has

$$\begin{aligned} &2y^T(t)(\Delta A(t) f(x(t)) + \Delta B(t) f(x(t-\tau(t))) + I_n\mathrm{sgn}(e(t))\underline{D}T - \eta\mathrm{sgn}(e(t))) + \tilde{\sigma}^T(e(t))\tilde{\sigma}(e(t)) \\ &\leq 2\left|y^T(t)\right|(\underline{D}T + (\mu_A + \mu_B)M) - \eta y^T(t)\mathrm{sgn}(e(t)) + h^2\|e(t)\|^2 \\ &\leq 2\left|y^T(t)\right|\left(\underline{D}T + (\mu_A + \mu_B)M - \eta 1_n\right) + h^2\|e(t)\|^2 \leq h^2\|e(t)\|^2. \end{aligned} \tag{26}$$

By means of Lemma 5 and (25), inequality (26) turns to

$$\begin{aligned} LV(t) &\le \frac{p}{2}\|e(t)\|^{p-2}\Big(2y^T(t)\big[-\bar{D}+\rho_A L+\big(1+h^2\big)I_n-K\big]e(t)+\|\rho_B L\|^2\|e(t-\tau(t))\|^2 \\ &\quad+\Big(\varepsilon+\frac{h^2}{2}\Big)\|e(t)\|^2+\frac{1}{\varepsilon}\|\underline{D}T+(\mu_A+\mu_B)M\|^2\Big) \\ &\le \frac{p}{2}\|e(t)\|^{p-2}\left(2y^T(t)\left[-\bar{D}+\rho_A L+\left(1+\varepsilon+\frac{h^2}{2}I_n\right)-K\right]e(t)+\|\rho_B L\|^2\|e(t-\tau(t))\|^2\right) \\ &\le -a\|e(t)\|^2+b\|e(t-\tau(t))\|^2 \\ &\le -aV(t)+b\sup_{t-\tau(t)\le s\le t}V(s), \end{aligned} \tag{27}$$

where $a=p\lambda_{\min}\{\bar{D}-\rho_A L-(1+h^2)In+K\}, b=p\|\rho_B L\|^2/2$. Using Assumption 4, we get

$$p\|e(t)\|^{p-2}y^T(t)\widetilde{\sigma}(e(t))\le phV(t). \tag{28}$$

By virtue of (27) and (28), we can get

$$\mathrm{d}V(t)\le\left(-aV(t)+b\sup_{t-\tau(t)\le s\le t}\mathrm{V}(s)\right)\mathrm{d}t+phV(t)\mathrm{d}\omega(t). \tag{29}$$

Using the assumptions of identical conditions for the drive and response drive-response stochastic memristor neural networks, there exists a positive constant N_0 such that $\|e(t)\|\le N_0$ for $t\in[t_0-\tau_2,t_0]$. The stochastic differential function is given as

$$\mathrm{d}W(t)=\big(-aW(t)+bN_0^2\big)\mathrm{d}t+phW(t)\mathrm{d}\omega(t), \tag{30}$$

under the identical initial condition as $V(t)$. Applying the results with respect to the existent and unique solutions for stochastic differential equations, it follows that equation (30) involves an only solution $W(t)$ for $t\in[t_0,t_0+\tau_1]$ with $\mathrm{E}W(t)<\infty$. Clearly, $\mathrm{E}V(t)\le\mathrm{E}W(t)<\infty$, $t\in[t_0,t_0+\tau_1]$, and from (27), we also have $\mathrm{E}LV<\infty$ for $t\in[t_0,t_0+\tau_1]$. It follows that

$$\mathrm{E}\frac{d}{\mathrm{d}t}\int_{t_0}^{t}LV(s)\mathrm{d}s=\mathrm{E}LV(t)<\infty. \tag{31}$$

And hence, by using the properties between the derivative and expectations of a random variable (see [35, 36]), we get

$$\mathrm{E}\frac{d}{\mathrm{d}t}\int_{t_0}^{t}LV(s)\mathrm{d}s=\frac{d}{\mathrm{d}t}\left(\mathrm{E}\int_{t_0}^{t}LV(s)\mathrm{d}s\right). \tag{32}$$

Note that (24) and (27) imply that

$$\begin{aligned} \frac{\mathrm{dE\,V}(t)}{\mathrm{d}t} &= \frac{d}{\mathrm{d}t}\left(\mathrm{E}\int_{t_0}^{t}\mathrm{LV}(s)\mathrm{d}s\right)=\mathrm{E}\frac{d}{\mathrm{d}t}\int_{t_0}^{t}\mathrm{LV}(s)\mathrm{d}s \\ &\le -a\mathrm{E}V(t)+b\sup_{t-\tau(t)\le s\le t}\mathrm{E}V(s) \\ &\le -a\mathrm{E}V(t)+b\sup_{t_0-\tau_2\le s\le t_0}\mathrm{E}V(s). \end{aligned} \tag{33}$$

As a result, in light of Lemma 6,

$$\mathrm{EV}(\mathrm{t})\le\left(\sup_{t_0-\tau_2\le s\le t_0}\mathrm{EV}(s)\right)\mathrm{e}^{-\lambda(t-t_0)}, \tag{34}$$

and thus,

$$\mathrm{E}\|e(t)\|^p\le\left(\sup_{t_0-\tau_2\le s\le t_0}\mathrm{E}\|\mathrm{y}(\mathrm{s})\|^p\right)\mathrm{e}^{-\lambda(t-t_0)}, \tag{35}$$

for $t\in[t_0,t_0+\tau_1]$, where $\lambda=\inf_{t\ge t_0}\{\lambda\colon\lambda-a+be^{\lambda(t)\tau}=0\}$ by condition (22). Since

$$\begin{aligned} d\|e(t)\|^p &\le \left(-a\|e(t)\|^p+b\left(\sup_{t_0+\tau_1-\tau_2\le s\le t_0+\tau_1}\mathrm{E}\|y(s)\|^p\right)e^{-\lambda\tau 1}\right)\mathrm{d}t \\ &\quad+2h\|e(t)\|^2\mathrm{d}\omega(t) \\ &\le\big(-a\|e(t)\|^2+bN_0^2\big)\mathrm{d}t+2h\|e(t)\|^p\mathrm{d}\omega(t), \end{aligned} \tag{36}$$

for $t\in[t_0+\tau_1,t_0+2\tau_1]$, one derives

$$\begin{aligned} \mathrm{E}\|e(t)\|^p &\le \left(\sup_{t_0+\tau_1-\tau_2\le s\le t_0+\tau_1}\mathrm{E}\|y(s)\|^p\right)e^{-\lambda(t-t_0-\tau_1)} \\ &\le \left(\sup_{t_0-\tau_2\le s\le t_0}\mathrm{E}\|y(s)\|^p\right)e^{-\lambda(t-t_0)}, \end{aligned} \tag{37}$$

where $t\in[t_0+\tau_1,t_0+2\tau_1]$, and so

$$\mathrm{E}\|e(t)\|^p\le\left(\sup_{t_0-\tau_2\le s\le t_0}\mathrm{E}\|y(s)\|^p\right)e^{-\lambda(t-t_0)}, \tag{38}$$

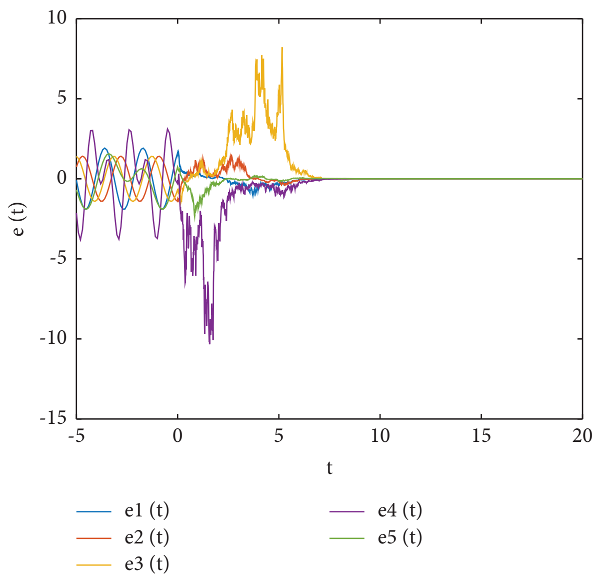

Figure 1: State trajectories of error variables under feedback controller.

where $t \in [t_0, N\tau_1]$ for any natural numbers N following the mathematical induction. By using Hölder′s inequality, (see [36] p.5) one can obtain

$$\mathrm{E}\|e(t)\|^p \leq \left(\sup_{t_0-\tau_2 \leq s \leq t_0} \mathrm{E}\|y(s)\|^p \right)^{p/2} e^{-p\lambda/2\,(t-t_0)}, \tag{39}$$

where $t \in [t_0, T]$ for any constants $T > t_0$. □

Remark 9. We present a rigorous proof of the main result in this paper, where the moment inequalities and martingale inequalities are crucial in demonstrating the above-mentioned theorem. The pth moment exponential synchronization is more general than the mean square case.

4. Numerical Examples

This part presents one numerical example to demonstrate the main result. We consider a drive memristor neural network [33] with 5 nodes and parameters $T = 2, \tau(t) = 0.01e^t/(e^t + 1)$ and

$$\begin{aligned}
\underline{A} &= \begin{bmatrix} 4.1 & 1 & 1.1 & 0.7 & 1.3 \\ 1 & 1.1 & 0.9 & 1.2 & 0 \\ 1.1 & 0.9 & 2 & 1.5 & 0.8 \\ 0.7 & 1.2 & 1.5 & 2.6 & 1.1 \\ 1.3 & 0 & 0.8 & 1.1 & 0.8 \end{bmatrix}, \\
\bar{A} &= \begin{bmatrix} 2.1 & 1 & 1.1 & 0.7 & 1.5 \\ 0.7 & 1.4 & 2.4 & 3.1 & 2.8 \\ 2.2 & 2.9 & 3 & 4.3 & 2.9 \\ 0.9 & 3.2 & 5.5 & 1.4 & 4.3 \\ 2.6 & 2.2 & 2.8 & 1.8 & 2.4 \end{bmatrix}, \\
\underline{B} &= \begin{bmatrix} 4.1 & 1 & 1.5 & 1.7 & 1.3 \\ 1 & 1.1 & 0.9 & 1.2 & 0 \\ 1.1 & 0.9 & 2 & 1.5 & 0.8 \\ 0.7 & 1.2 & 1.5 & 2.6 & 1.1 \\ 1.3 & 0 & 0.8 & 1.1 & 0.8 \end{bmatrix}, \\
\bar{B} &= \begin{bmatrix} 2.1 & 1 & 1.7 & 1.7 & 1.5 \\ 0.7 & 1.4 & 2.4 & 3.1 & 2.8 \\ 2.2 & 2.9 & 3 & 4.3 & 2.9 \\ 0.9 & 3.2 & 5.5 & 1.4 & 4.3 \\ 2.6 & 2.2 & 2.8 & 1.8 & 2.4 \end{bmatrix}.
\end{aligned} \tag{40}$$

At node i, we take

$$d_i(x_i(t)) = \begin{cases} 0.9, & |x_i(t)| \le T, \\ 1.1, & |x_i(t)| > T, \end{cases}$$
$$a_{ij}(x_j(t)) = \begin{cases} \underline{a}_{ij}, & |x_j(t)| \le T, \\ \bar{a}_{ij}, & |x_j(t)| > T, \end{cases} \quad (41)$$
$$b_{ij}(x_j(t)) = \begin{cases} \underline{b}_{ij}, & |x_j(t)| \le T, \\ \bar{b}_{ij}, & |x_j(t)| > T, \end{cases}$$

and $\mathrm{f}_i(x_i(t)) = \sin(x_i(t)/3)$, $f_i(x_i(t-\tau(t))) = \sin(x_i(t-\tau(t))/3)$, $\sigma_i(x_i(t)) = x_i(t)$, and $i \in \{1, 2, \ldots, 5\}$.

$$d_i(y_i(t)) = \begin{cases} 0.9, & |e_i(t)| \le T, \\ 1.1, & |e_i(t)| > T, \end{cases},$$
$$a_{ij}(y_j(t)) = \begin{cases} \underline{a}_{ij}, & |e_j(t)| \le T, \\ \bar{a}_{ij}, & |e_j(t)| > T, \end{cases}, \quad (42)$$
$$b_{ij}(y_j(t)) = \begin{cases} \underline{b}_{ij}, & |e_j(t)| \le T, \\ \bar{b}_{ij}, & |e_j(t)| > T. \end{cases}$$

By some calculations, one can obtain that $u_A = 12.7$, $\rho_A = 6.6$, $u_B = 20.7$, and $\rho_B = 7.4$. We set $K = 10I_5$ and $\eta = 0.1$, which means the conditions in Theorem 8 holds. Moreover, Figure 1 shows that the trajectories reach synchronization.

5. Conclusion

In this paper, we studied the synchronization problem for the drive-response memristor neural networks subject to stochastic perturbations and time-varying delays. The phenomenon of the pth moment exponential synchronization is obtained. Some testable and computable sufficient conditions are derived to ensure the pth moment exponential synchronization of these drive-response stochastic memristor neural networks utilizing the Filippov theory and the Lyapunov stable theory, which is more general than the mean-square synchronization.

In future, we will improve on the control strategy, such as data-sampled control, quantization, intermittent control, and impulsive control. And the results will be extended to the pth moment synchronization of the general networked dynamical systems.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work has been supported by the Shenzhen Science and Technology Program under grant JCYJ20160428092427867, the Department of Education of Guangdong Province Technological Innovation Program under grant 2017GKTSCX060, and fund projects: General Project of Philosophy and Social Sciences Research in Jiangsu Universities "Research on Supply Chain Configuration Mode and Cooperation Strategy of New Energy Vehicle After-Sales Service Responsibility" (2021SJA0486).

References

[1] F. Alibart, L. Gao, B. D. Hoskins, and D. B. Strukov, "High precision tuning of state for memristive devices by adaptable variation-tolerant algorithm," *Nanotechnology*, vol. 23, no. 7, Article ID 075201, 2012.

[2] X.-H. Chang, Y. Liu, and M. Shen, "Resilient control design for lateral motion regulation of intelligent vehicle," *Transactions on Mechatronics*, vol. 24, no. 06, pp. 2488–2497, 2019.

[3] B. Bo-Cheng, X. Jian-Ping, Z. Guo-Hua, M. Zheng-Hua, and Z. Ling, "Chaotic memristive circuit: equivalent circuit realization and dynamical analysis," *Chinese Physics B*, vol. 20, no. 12, Article ID 120502, 2011.

[4] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, and C. Zhou, "The synchronization of chaotic systems," *Physics Reports*, vol. 366, no. 1-2, pp. 1–101, 2002.

[5] F. Dorfler and F. Bullo, "Synchronization in complex networks of phase oscillators: a survey," *Automatica*, vol. 50, no. 6, pp. 1539–1564, 2014.

[6] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 863–868, 2007.

[7] X. F. Wang and G. Chen, "Synchronization in scale-free dynamical networks: robustness and fragility," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 49, no. 1, pp. 54–62, 2002.

[8] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[9] J. Wang, C. Xu, J. Feng, M. Z. Q. Chen, X. F. Wang, and Y. Zhao, "Synchronization in moving pulse-coupled oscillator networks," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 62, no. 10, pp. 2544–2554, 2015.

[10] S. Emre Tuna, "Synchronization of harmonic oscillators under restorative coupling with applications in electrical networks," *Automatica*, vol. 75, pp. 236–243, 2017.

[11] Q. Song, F. Liu, G. Wen, J. Cao, and Y. Tang, "Synchronization of coupled harmonic oscillators via sampled position data control," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 63, no. 7, pp. 1079–1088, 2016.

[12] H. Su, M. Z. Q. Chen, J. Lam, and Z. Lin, "Semi-global leader-following consensus of linear multi-agent systems with input saturation via low gain feedback," *IIEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 7, pp. 1881–1889, 2013.

[13] L. Wang, M. Z. Q. Chen, and Q.-G. Wang, "SBounded synchronization of a heterogeneous complex switched network," *Automatica*, vol. 56, pp. 19–24, 2015.

[14] T. Nishikawa and A. E. Motter, "Symmetric states requiring system asymmetry," *Physical Review Letters*, vol. 117, Article ID 114101, 2016.

[15] T. Hou, J. Yu, C. Hu, and Jiang, "Haijun. Finite-time synchronization of fractional-order complex-variable dynamic networks," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 7, pp. 4297–4307, 2021.

[16] C. Hu, H. He, and H. Jiang, "Fixed/preassigned-time synchronization of complex networks via improving fixed-time stability," *IEEE Transactions on Cybernetics*, vol. 51, no. 6, pp. 2882–2892, 2021.

[17] J. Cheng, W. Huang, J. H. Park, and J. Cao, "A hierarchical structure approach to finite-time filter design for fuzzy markov switching systems with deception attacks," *IEEE Transactions on Cybernetics*, vol. 52, no. 8, pp. 7254–7264, 2022.

[18] A. Wu and Z. Zeng, "Exponential stabilization of memristive neural networks with time delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, no. 12, pp. 1919–1929, 2012.

[19] S. Yang, C. Li, and T. Huang, "Exponential stabilization and synchronization for fuzzy model of memristive neural networks by periodically intermittent control," *Neural Networks*, vol. 75, pp. 162–172, 2016.

[20] H. Shen, S. Huo, H. Yan, J. H. Park, and V. Sreeram, "Distributed dissipative state estimation for Markov jump genetic regulatory networks subject to round-robin scheduling," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 3, pp. 762–771, 2019.

[21] X. Yang, J. Cao, and J. Liang, "Exponential synchronization of memristive neural networks with delays: interval matrix method," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 8, pp. 1878–1888, 2016.

[22] J. Feng, S. Chen, J. Wang, and Y. Zhao, "Quasi-synchronization of coupled nonlinear memristive neural networks with time delays by pinning control," *IEEE Access*, vol. 6, pp. 26271–26282, 2018.

[23] J. Cheng, L. Liang, J. H. Park, H. Yan, and K. Li, "A dynamic event-triggered approach to state estimation for switched memristive neural networks with nonhomogeneous sojourn probabilities," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 68, no. 12, pp. 4924–4934, 2021.

[24] S. Chen, J. Feng, J. Wang, and Y. Zhao, "Lmost sure exponential synchronization of drive-response stochastic memristive neural networks," *Applied Mathematics and Computation*, vol. 383, Article ID 125360, 2020.

[25] X. Mao, "Almost sure exponential stabilization by discrete-time stochastic feedback control," *IEEE Transactions on Automatic Control*, vol. 61, no. 6, pp. 1619–1624, 2015.

[26] X. Yang, J. Cao, and J. Qiu, "Pth moment exponential stochastic synchronization of coupled memristor-based neural networks with mixed delays via delayed impulsive control," *Neural Networks*, vol. 65, pp. 80–91, 2015.

[27] J. S. Song and X. H. Chang, "h∞Controller design of networked control systems with a new quantization structure," *Applied Mathematics and Computation*, vol. 376, Article ID 125070, 2020.

[28] J. Cheng, L. Xie, J. H. Park, and H. Yan, "Protocol-based output-feedback control for semi-markov jump systems," *IEEE Transactions on Automatic Control*, vol. 67, no. 8, pp. 4346–4353, 2022.

[29] H. Liu, Z. Wang, B. Shen, and X. Liu, "Event triggered h∞state estimation for delayed stochastic memristive neural networks with missing measurements: the discrete time case," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 8, pp. 3726–3737, 2017.

[30] Z. Guo, S. Yang, and J. Wang, "Global exponential synchronization of multiple memristive neural networks with time delay via nonlinear coupling," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 26, no. 6, pp. 1300–1311, 2014.

[31] W. Zhu, D. Wang, L. Liu, and G. Feng, "Event-based impulsive control of continuous-time dynamic systems and its application to synchronization of memristive neural networks," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 8, pp. 3599–3609, 2017.

[32] X. Mao, *Stochastic Differential Equations and Applications*, Woodhead Publishing, Cambridge, UK, 2nd edition, 2008.

[33] S. Chen, J. Feng, J. Wang, and Y. Zhao, "Almost sure exponential synchronization of drive-response stochastic memristive neural networks," *Applied Mathematics and Computation*, vol. 383, Article ID 125360, 2020.

[34] J.-P. Aubin and A. Cellina, *Differential inclusions: set-valued maps and viability theory*, vol. 264,Springer Science and Business Media, Berlin, Germany.

[35] X. Mao, *Stochastic Differential Equations and Applications*, Elsevier, Amsterdam, Netherlands, 2007.

[36] R. Jeffffrey, *A Fifirst Look at Rigorous Probability Theory*, World Scientifific Publishing Co Pte. Ltd, Singapore, 2006.