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Research Article

Adaptive Control for Complex Systems with Dynamics and Time-Varying Powers

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This work focuses on presenting a control algorithm to investigate nonlinear systems, which contain time-varying powers, inverse dynamics, and uncertainties. First, some appropriate transformations are introduced to obtain a new system. Then, a Lyapunov function, which covers quadratic and high-order components, is recursively constructed for control design. Subsequently, by introducing the neural networks, the uncertain functions encountered during the design are approximated. Based on the inequality techniques, the nonlinear terms are skillfully estimated. By defining the bounds of some unknown parameters and using the adaptive technique, some virtual controllers are selected in each step to dominate the nonlinear functions and guarantee that the derivative of the Lyapunov function satisfies the required form. Finally, a new adaptive controller is constructed and semiglobal practical finite time stability (SGPFS) is guaranteed. The proposed approach is verified with a numerical example.

1. Introduction

Control of complex nonlinear systems is viewed as a challenging issue in practical engineering. Since different systems are modeled with differential equations that have disparate nonlinear characters, the related control schemes constantly vary from each other. Recently, plenty of outstanding approaches have been raised, as shown in the sliding mode control [1], intelligent control [2, 3], event-based control [4], and so on. It should be mentioned that complex systems often have diversified uncertainties, including unknown parameters, uncertain dynamics, and unpredictable disturbances. The uncertainties actually add more obstacles for the system analysis and control synthesis. Besides, practical systems often suffer from time-varying powers [5, 6], which lead to the inapplicability of conventional methods of systems with constant powers. So far, when systems have time-varying powers, lots of control challenges have not been solved. It is interesting to study such systems and raise a feasible control strategy.

For uncertain systems, the adaptive control strategy has been viewed as one of the effective tools for control design, see [7–9]. During the control design steps of nonlinear systems described by differential equations, this strategy utilizes the idea of certain equivalence and always results in a dynamic controller. Particularly, by presenting an adaptive backstepping method [10], the global asymptotic stability was guaranteed for systems that suffered from unknown parameters. By proposing a dynamic event-triggered control-based output-feedback control method, Cao et al. [11] studied the adaptive issue of systems with immeasurable states and input delay. By proposing an adaptive NN fixed-time control method, Cao et al. [12] further designed the controller for systems that included dynamic uncertainties and communication resources. By raising an adaptive fuzzy command filtered method, Li et al. [13] considered systems with uncertain dynamics and ensured the system to be semiglobally stable. Recently, high-order nonlinear system has been a hot topic and many excellent results have been reported, see [14–16]. Specially, by introducing a Lyapunov–Krasovskii functional and employing the adaptive neural network control technique, Duan et al. [14] skillfully designed the adaptive controller for delayed systems. Utilizing one power integrator strategy, Niu et al. [15] studied the adaptive stabilization issue of stochastic systems with arbitrary switching. Based on the idea of homogeneous domination and dynamic control approach, Shen and Zhai [16] designed the controller for uncertain high-order systems. In fact, the system powers in the above works are assumed to be constants. When the system powers are time-varying functions, there are some results discussing the control issues. For instance, Chen et al. [17] presented a feedback control approach for systems that involved time-varying powers. Yoo [18] discussed uncertain systems with time-varying functions and investigated the fault accommodation control issue. However, finite time control issues are still challenging, especially when the system powers vary with a large range.

Finite time control has received lots of concerns in last years. Such kind of control has better system responses such as faster convergent rate, higher tracking precision, and better robustness. There are many splendid results for this control issue. Specially, for low-order systems, Yu et al. [19] studied the tracking control problem by presenting a finite time command filtered backstepping method. Polyakov et al. [20] considered the stability problems and provided a robust control method. As for high-order systems, Gao et al. [21] discussed the output feedback control approach, Chen et al. [22] further considered the output constraint and raised the output feedback method to solve finite stabilization problem. For system with uncertainties, Li et al. [23] investigated the adaptive finite time regulation for lower-order systems. Sun et al. [24] raised one fast finite time control approach and applied it to systems with constant powers. However, the adaptive finite time control issue is still open for systems that have dynamics and time-varying powers. Naturally, we raise the following problem:

> Can we regulate the uncertain system with dynamics and time-varying powers via a finite time control approach?

We will consider the above problem. The study has two advantages as follows:

(i) The considered system has a more general form. In practical engineering, some dynamic models have time-varying powers, see the model of a boiler-turbine unit in [17] and the underactuated mechanical system in reference [25]. However, few results considered control issue of systems with time-varying powers. Also, due to the inaccuracy of measurement or the modeling errors, some inverse dynamics cannot be directly neglected. Besides, the practical systems are often in nontriangular structure and suffer from multiple uncertainties. These factors motivate us to study the more general system, which contains time-varying powers, inverse dynamics, nontriangular structure, and uncertainties. The system is hence more general.

(ii) A new adaptive control approach is presented. Noting that the existing methods of systems are mainly for constant powers, they cannot be applied to the system of this work. Besides, since the finite time control of the system has inverse dynamics, nontriangular structure, and uncertainties are still challenging, we are inspired to raise a new adaptive control approach. In this work, a Lyapunov function, which includes quadratic and high-order components, is constructed. By estimating the complex nonlinear terms with the neural network and employing the adaptive control design, the adaptive controller is skillfully constructed for the considered system. The merits of the method lie in two aspects. One is that, it provides an effective solution for adaptive finite control of complicated nonlinear system. As can be seen, for the complicated system (1), the finite time control problem is still challenging. The presented method overcomes a series of obstacles and finally provides a feasible solution to solve the above problem. The second is that the proposed method adopts few parameter estimations and does not introduce complicated basis functions of neural network in the control input. Hence, the designed adaptive controller has a simple form.

2. Problem Formulation

We study the system as follows:

$$\begin{cases} \dot{\xi} = g(\varrho, \xi, x_1), \\ \dot{x}_1 = a_1(t,x)\lceil x_2 \rceil^{s_1(t)} + f_1(\varrho, \xi, x), \\ \dot{x}_2 = a_2(t,x)\lceil x_3 \rceil^{s_2(t)} + f_2(\varrho, \xi, x), \\ \vdots \\ \dot{x}_n = a_n(t,x)\lceil u \rceil^{s_n(t)} + f_n(\varrho, \xi, x), \end{cases} \tag{1}$$

where $\xi \in \mathscr{R}^m$ and $x = [x_1, \ldots, x_n]^\top \in \mathscr{R}^n$ are state vectors of the system, $\varrho \in \mathscr{R}^l$ denotes a parameter vector, $g \in \mathscr{R}^m$ represents a vector of continuous functions, and $u \in \mathscr{R}$ denotes the input. For $1 \le i \le n$, $a_i > 0$ denotes the control coefficients, $f_i(\cdot)$ are unknown continuous functions, $s_i(t) > 0$ are unknown system powers, and $\lceil \cdot \rceil^{s_i(t)} = \mathrm{sign}(\cdot)|\cdot|^{s_i(t)}$.

System (1) describes a class of complex nonlinear systems. It covers the inverse dynamics, which generates because of the inaccuracy of mathematical modeling or measuring errors and always leads to poor system responses. Also, it contains time-varying powers, which are involved in many models in practical engineering. Besides, the system has unknown coefficients and involves unknown functions that are in nontriangular form. In practical life, many dynamic models can be transformed into system (1). Thus, the research of such kind of systems is of practical significance. However, the corresponding control issue is still challenging. In this work, we will provide a new adaptive control strategy for it.

Assumption 1. There exist unknown constants $\lambda_1 > 0$ and $\lambda_2 > 0$ such that $\lambda_1 \le a_i(t,x) \le \lambda_2$, $1 \le i \le n$.

Assumption 2. There exists $s_m \le s_i(t) \le s_h$, where $0 < s_m \le 1$ and $s_h \ge s_m$ are constants that belong to $\mathscr{R}_{\text{odd}} = \{q_1/q_2 \mid q_1 > 0 \text{ and } q_2 > 0 \text{ are odd integers}\}$.

Assumption 3. There exists a constant $s \ge s_h - s_m + 1$ ($s \in \mathscr{R}_{\text{odd}}$) and a Lyapunov function $W(\xi)$ such that

$$\dot{W}(\xi) = \frac{\partial W(\xi)}{\partial \xi^{\top}} g(\varrho, \xi, x_1) \le -\left(\|\xi\|^{s_h+1} + \|\xi\|^{s+s_m}\right) + |x_1|\phi_0(x_1) + d_0, \tag{2}$$

where d_0 is a positive constant and $\phi_0(\cdot) \ge 0$ is a continuous function.

Lemma 4 (see [2]). *For a vector $\Xi \in \Lambda_r \subseteq \mathscr{R}^r$, if $h_i(\theta, \Xi)$ is a continuous function, then, a neural network $\Phi_i^{\top} S_i(\Xi)$ exists such that $h_i(\theta, \Xi) = \Phi_i^{\top} S_i(\Xi) + \eta_i(\Xi)$, where Φ_i represents ideal constant weight vector, $\eta_i(\Xi)$ satisfies $|\eta_i(\Xi)| \le \delta_i$ for a constant $\delta_i > 0$, and $S_i(\Xi)$ denotes the Gaussian function vector and satisfies $\|S_i(\Xi)\| \le \bar{s}$, where $\bar{s} > 0$ is a constant.*

Lemma 5 (see [24]). *There exists the inequality as follows:*

$$\left|\pi(\tau,\nu)\tau^{l_1}\nu^{l_2}\right| \le \phi(\tau,\nu)|\tau|^{l_1+l_2} + \frac{l_2}{l_1+l_2}|\pi(\tau,\nu)|^{l_1+l_2/l_2}\left(\frac{l_1}{(l_1+l_2)\phi(\tau,\nu)}\right)^{l_1/l_2}|\nu|^{l_1+l_2}, \tag{3}$$

where $l_1 > 0, l_2 > 0$ are constants and $\pi(\tau,\nu) > 0, \phi(\tau,\nu) > 0$ are functions.

Lemma 6 (see [6]). *Let $0 \le l_1 \le \cdots \le l_n$, $m_1 > 0, \ldots,$ and $m_n > 0$. For $y \in \mathscr{R}$, there exists the following:*

$$m_1|y|^{l_1} + m_n|y|^{l_n} \le \sum_{i=1}^{n} m_i|y|^{l_i} \le \left(\sum_{i=1}^{n} m_i\right)\left(|y|^{l_1} + |y|^{l_n}\right). \tag{4}$$

Remark 7. We emphasize two points as follows: (i) In this paper, the system powers refer to $s_i(t)$, $1 \le i \le n$ in (1) and are not the order n. They are time-varying functions, which are more general than the constants in [14, 15, 22, 24]. From Assumption 1, we see that they can belong to a larger interval $[s_m, s_h]$, which cover the powers smaller than one or bigger than one. (ii) In engineering, there are practical models, which have the form of system (1), see the system [17, 26]. System (1) actually provides a general form to describe similar dynamic models.

Remark 8. Assumptions 1–3 of system (1) are reasonable and much weaker than those of the existing research. Assumption 1 implies that the bounds of control coefficients can be unknown. It is more general than the known cases of [2, 21]. Assumption 2 shows that the powers can be changing in a larger scope. Assumption 3 implies that the inverse dynamics of system (1) satisfies a weaker stability condition. It should be emphasized that the control issue under Assumptions 1–3 is still difficult. Next, we will study the finite control issue and raise an adaptive control strategy.

3. Adaptive Control Design

The adaptive control method of this paper adapts to the control law according to the parameter variation. As can be seen, there are many uncertainties in the nonlinear functions. To design the controller, we adopt the neural networks to approximate these unknown functions and subsequently introduce some unknown ideal constant weight vectors. During the control design steps, the obtained nonlinear bounds contain unknown parameters, see (7), (9), (24), and (26). By defining the maximum value θ_i of those unknown parameters in each step, we can design adaptive laws (18) and (28) to approximate parameters θ_i, $1 \le i \le n$. Then, we construct the virtual controllers with a recursive method by using $\widehat{\theta}_i$ and a new Lyapunov function. In the last step, the adaptive controller is successfully designed. For the details, see Figure 1.

Step 1: Introduce the transformation $z_1 = x_1$, and define $\widetilde{\theta}_1(t) = \widehat{\theta}_1(t) - \theta_1$, where θ_1 will be defined later and $\widehat{\theta}_1(t)$ is an estimation of θ_1. Choosing the function $V_1(\xi, z_1, \widetilde{\theta}_1) = \rho W(\xi) + 1/\lambda_1(z_1^2 + z_1^{s+1}) + 1/(2\epsilon_1\widetilde{\theta}_1^2)$, where $\rho > 0$ and $\epsilon_1 > 0$ are constants, we get the following:

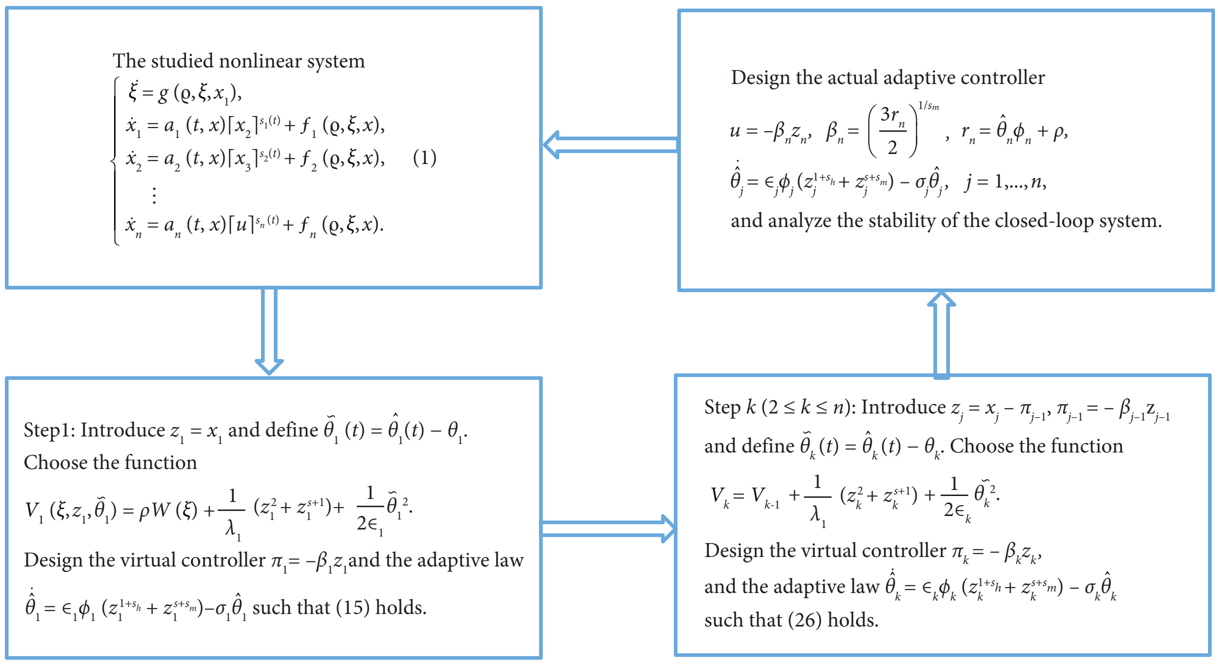

Figure 1: The flowchart of the control method.

$$\begin{aligned} \dot{V}_1 &= \rho\dot{W}(\xi) + \frac{1}{\lambda_1}(2z_1 + (s+1)z_1^s)\dot{z}_1 + \frac{1}{\epsilon_1}\tilde{\theta}_1\dot{\hat{\theta}}_1, \\ &\le -\rho\left(\|\xi\|^{s_h+1} + \|\xi\|^{s+s_m}\right) + \rho|z_1|\phi_0(\xi, x_1) + \rho d_0 + \frac{1}{\lambda_1}(2z_1 + (s+1)z_1^s), \\ &\quad \cdot\left(a_1\lceil x_2\rceil^{s_1(t)} - a_1\lceil \pi_2\rceil^{s_1(t)} + a_1\lceil \pi_2\rceil^{s_1(t)} + f_1\right) + \frac{1}{\epsilon_1}\tilde{\theta}_1\dot{\hat{\theta}}_1. \end{aligned} \tag{5}$$

By Lemma 4, there exist some neural networks such that

$$\begin{aligned} \phi_0(x_1) &= \Phi_0^{\mathrm{T}}S_0 + \eta_0(t), |\eta_0(t)| \le \delta, \\ f_1(\cdot) &= \Phi_1^{\mathrm{T}}S_1 + \eta_1(t), |\eta_1(t)| \le \delta, \end{aligned} \tag{6}$$

where $\Phi_i, S_i, i = 0, 1$ are defined in Lemma 4, $\eta_0(t), \eta_1(t)$ are the approximation errors, and $\delta > 0$ is a constant. By Lemma 5, it yields from (6) that

$$\rho|z_1|\varphi_0(x_1) \le \rho|z_1|\left(|\Phi_0^{\mathrm{T}}S_0| + |\eta_0|\right) \le b_{10}\left(\|\Phi_0\|^{1+s_h} + 1\right)z_1^{s_h+1} + d_{10}, \tag{7}$$

where $b_{10} > 0$ is a constant and $d_{10} = 1/(s_h+1)1/(b_{10}s_h + b_{10})^{1/s_h}(\|S_0\|^{(1+s_h)/s_h} + \eta_0^{(1+s_h)/s_h})\rho^{(1+s_h)/s_h}$. Utilizing (6), it leads to

$$\frac{1}{\lambda_1}(2z_1 + (s+1)z_1^s)f_1 \le \frac{s+1}{\lambda_1}\left(|z_1| + |z_1|^s\right)\left(|\Phi_1^{\mathrm{T}}S_1| + |\eta_1|\right). \tag{8}$$

By Lemma 5, we obtain the following:

$$\frac{s+1}{\lambda_1}|z_1|\|\Phi_1^{\mathrm{T}}S_1| \le \frac{s+1}{\lambda_1}|z_1|\|\Phi_1\| \cdot \left(\|S_1\|^{(1/sh)}\right)^{s_h} \le b_{11}\left(\frac{\|\Phi_1\|}{\lambda_1}\right)^{1+s_h} z_1^{s_h+1} + d_{11}, \tag{9}$$

where b_{11} is a positive constant and $d_{11} = 1/(s_h+1)1/(b_{11}s_h+b_{11})^{1/s_h}\|S_1\|^{(1+s_h)/s_h}(s+1)^{(1+s_h)/s_h}$. Similarly, by Lemma 5, we get the following:

$$\frac{s+1}{\lambda_1}|z_1\|\eta_1| \le b_{12}\left(\frac{1}{\lambda_1}\right)^{s_h+1} z_1^{s_h+1} + d_{12},$$

$$\frac{s+1}{\lambda_1}|z_1|^s|\Phi_1^\top S_1| \le b_{13}\left(\frac{\|\Phi_1\|}{\lambda_1}\right)^{(s+s_m)/s_m} z_1^{s_m+s} + d_{13}, \tag{10}$$

$$\frac{s+1}{\lambda_1}|z_1|^s|\eta_1| \le b_{14}\left(\frac{1}{\lambda_1}\right)^{(s+s_m)/s} z_1^{s_m+s} + d_{14},$$

where b_{12}, b_{13}, b_{14} are positive constants,

$$d_{12} = \frac{s_h}{1+s_h}\left(\frac{1}{b_{12}+b_{12}s_h}\right)^{1/s_h} \eta_1^{(1+s_h)/s_h}(s+1)^{(1+s_h)/s_h},$$

$$d_{13} = \frac{s}{s+s_m}\left(\frac{s}{b_{13}+b_{13}s_m}\right)^{s/s_m} \cdot \|S_1\|^{(s+s_m)/s_m}(s+1)^{(s+s_m)/s_m},$$

$$d_{14} = \frac{s_m}{s+s_m}\left(\frac{s}{b_{14}+b_{14}s_m}\right)^{s/s_m} \eta_1^{(s+s_m)/s_m}(s+1)^{(s+s_m)/s_m}. \tag{11}$$

Defining $\theta_1 = \max\{\|\Phi_0\|^{1+s_h}+1, (\|\Phi_1\|/\lambda_1)^{1+s_h}, (1/\lambda_1)^{s_h+1}, (\|\Phi_1\|/\lambda_1)^{(s+s_m)/s_m}, (1/\lambda_1)^{(s+s_m)/s_m}\}$, and $\varphi_1 = \max\{b_{10}+b_{11}+b_{12}, b_{13}+b_{14}\}$, it follows from (10)–(14) that

$$\rho|z_1|\phi_0(\xi, x_1) + \frac{1}{\lambda_1}(2z_1+(s+1)z_1^s)f_1 \le \theta_1\phi_1\left(z_1^{s_h+1}+z_1^{s_m+s}\right) + \sum_{j=0}^{4} d_{1j}. \tag{12}$$

Substituting (12) into (5), it follows that

$$\begin{aligned}\dot V_1 \le & -\rho\left(\|\xi^{s_h+1} + \|\xi^{s+s_m}\right) + \frac{1}{\lambda_1}(2z_1+(s+1)z_1^s)\left(a_1\lceil x_2\rceil^{s_1(t)} - a_1\lceil\pi_1\rceil^{s_1(t)}\right)\\ & + \frac{1}{\epsilon_1}\tilde\theta_1\dot{\hat\theta}_1 + \theta_1\phi_1\left(z_1^{s_h+1}+z_1^{s_m+s}\right) + d_1 + \frac{a_1}{\lambda_1}(2z_1+(s+1)z_1^s)\lceil\pi_1\rceil^{s_1(t)},\end{aligned} \tag{13}$$

where $d_1 = \rho d_0 + \sum_{j=0}^4 d_{1j}$. Now, choosing the virtual controller, we get the following:

$$\pi_1\left(\hat\theta_1, \xi, x_1\right) = -\left(\frac{3r_1}{2}\right)^{1/s_m} z_1 =: -\beta_1 z_1, \tag{14}$$

where $r_1 = \hat\theta_1\phi_1 + 2\rho$. It is deduced that

$$\frac{a_1}{\lambda_1}(2z_1+(s+1)z_1^s)\lceil\pi_1\rceil^{s_1(t)} \le -3r_1\left(|z_1|^{1+s_1(t)} + |z_1|^{s+s_1(t)}\right), \tag{15}$$

By (16) and Lemma 6, we get

$$\frac{a_1}{\lambda_1}(2z_1+(s+1)z_1^s)\lceil\pi_1\rceil^{s_1(t)} \le -r_1\left(z_1^{1+s_h}+z_1^{s+s_m}\right). \tag{16}$$

Substituting (16) into (13), it yields that

$$\begin{aligned}\dot V_1 \le & -\rho\left(\|\xi\|^{s_h+1}+\|\xi\|^{s+s_m}\right) - 2\rho\left(z_1^{1+s_h}+z_1^{s+s_m}\right) + \frac{a_1}{\lambda_1}(2z_1+(s+1)z_1^s)\left(\lceil x_2\rceil^{s_1(t)} - \lceil\pi_1\rceil^{s_1(t)}\right)\\ & + \frac{1}{\epsilon_1}\tilde\theta_1\left(\dot{\hat\theta}_1 - \epsilon_1\phi_1\left(z_1^{1+s_h}+z_1^{s+s_m}\right)\right) + d_1,\end{aligned} \tag{17}$$

where ϵ_1 is a constant. Choosing the first adaptive law, we get the following:

$$\dot{\hat\theta}_1 = \epsilon_1\phi_1\left(z_1^{1+s_h}+z_1^{s+s_m}\right) - \sigma_1\hat\theta_1, \tag{18}$$

and substituting it into (17), we get the following:

$$\begin{aligned}\dot{V}_1 \le &-\rho\left(\|\xi\|^{s_h+1} + \|\xi\|^{s+s_m}\right) - 2\rho\left(z_1^{1+s_h} + z_1^{s+s_m}\right) + \frac{a_1}{\lambda_1}\left(2z_1 + (s+1)z_1^s\right)\left(\lceil x_2 \rfloor^{s_1(t)}\right.\\ &\left.-\lceil \pi_1 \rfloor^{s_1(t)}\right) - \frac{\sigma_1}{\epsilon_1}\tilde{\theta}_1\hat{\theta}_1 + d_1,\end{aligned} \tag{19}$$

where σ_1 is a constant.

Step k $(2 \le k \le n)$. In step $k-1$, we assume that there exist transformations as follows:

$$\begin{aligned} z_j &= x_j - \pi_{j-1},\\ \pi_{j-1} &= -\beta_{j-1} z_{j-1},\\ &2 \le j \le k-1,\end{aligned} \tag{20}$$

a candidate Lyapunov function V_{k-1}, and the adaptive laws

$$\dot{\hat{\theta}}_j = \phi_j\left(z_j^{1+s_h} + z_j^{s+s_m}\right) - \sigma_j\hat{\theta}_j, 2 \le j \le k-1, \tag{21}$$

such that

$$\begin{aligned}\dot{V}_{k-1} \le &-\rho\left(\|\xi\|^{s_h+1} + \|\xi\|^{s+s_m} + \sum_{j=1}^{k-2}\left(z_j^{1+s_h} + z_j^{s+s_m}\right)\right) - 2\rho\left(z_{k-1}^{1+s_h} + z_{k-1}^{s+s_m}\right) - \sum_{j=1}^{k-1}\frac{\sigma_j}{\epsilon_j}\tilde{\theta}_j\hat{\theta}_j\\ &+\frac{a_{k-1}}{\lambda_1}\left(2z_{k-1} + (s+1)z_{k-1}^s\right)\left(\lceil x_k \rfloor^{s_{k-1}(t)} - \lceil \pi_{k-1} \rfloor^{s_{k-1}(t)}\right) + d_{k-1},\end{aligned} \tag{22}$$

where β_j and ϕ_j are smooth functions and σ_j, ϵ_j and d_{k-1} are positive constants. In this step, we define $\tilde{\theta}_k(t) = \hat{\theta}_k(t) - \theta_k$, where θ_k denotes the unknown constant and $\hat{\theta}_k(t)$ is an estimation of θ_k, and we choose $V_k = V_{k-1} + 1/\lambda_1\,(z_k^2 + z_k^{s+1}) + 1/(2\epsilon_k\tilde{\theta}_k^2)$, where $\epsilon_k > 0$ is a constant. We obtain the following:

$$\begin{aligned}\dot{V}_k &\le \dot{V}_{k-1} + \frac{1}{\lambda_1}\left(2z_k + (s+1)z_k^s\right)\dot{z}_k + \frac{1}{\epsilon_k}\tilde{\theta}_k\dot{\hat{\theta}}_k\\ &\le -\rho\left(\|\xi\|^{s_h+1} + \|\xi\|^{s+s_m} + \sum_{j=1}^{k-2}\left(z_j^{1+s_h} + z_j^{s+s_m}\right)\right) - 2\rho\left(z_{k-1}^{1+s_h} + z_{k-1}^{s+s_m}\right)\\ &\quad+\frac{a_{k-1}}{\lambda_1}\left(2z_{k-1} + (s+1)z_{k-1}^s\right)\left(\lceil x_k \rfloor^{s_{k-1}(t)} - \lceil \pi_{k-1} \rfloor^{s_{k-1}(t)}\right)\\ &\quad-\sum_{j=1}^{k-1}\frac{\sigma_j}{\epsilon_j}\tilde{\theta}_j\hat{\theta}_j + d_{k-1} + \frac{1}{\lambda_1}\left(2z_k + (s+1)z_k^s\right)\dot{z}_k + \frac{1}{\epsilon_k}\tilde{\theta}_k\dot{\hat{\theta}}_k.\end{aligned} \tag{23}$$

Following the proof in Appendix, we have the following:

$$\begin{aligned}&\frac{a_{k-1}}{\lambda_1}\left(2z_{k-1} + (s+1)z_{k-1}^s\right)\left(\lceil x_k \rfloor^{s_{k-1}(t)} - \lceil \pi_{k-1} \rfloor^{s_{k-1}(t)}\right)\\ &\le \rho\left(z_{k-1}^{1+s_h} + z_{k-1}^{s+s_m}\right) + \theta_{k1}\phi_{k1}\left(z_k^{1+s_h} + z_k^{s+s_m}\right) + d_{k0},\end{aligned} \tag{24}$$

where $\theta_{k1} > 0$ is an unknown constant, d_{k0} is a positive constant, and $\varphi_{k1} > 0$ is a smooth function. It is deduced that

$$\dot{z}_k = a_k\lceil x_{k+1} \rfloor^{s_k} + f_k - \sum_{j=1}^{k-1}\frac{\partial \pi_{k-1}}{\partial \hat{\theta}_j}\dot{\hat{\theta}}_j - \sum_{j=1}^{k-1}\frac{\partial \pi_{k-1}}{\partial x_j}\left(a_j\lceil x_{j+1} \rfloor^{s_j} + f_j\right). \tag{25}$$

Utilizing Lemma 4, there exists a neural network $f_k - \sum_{j=1}^{k-1}\partial\pi_{k-1}/\partial\widehat{\theta}_j\dot{\widehat{\theta}}_j - \sum_{j=1}^{k-1}\partial\pi_{k-1}/\partial x_j(a_j\lceil x_{j+1}\rceil^{s_j} + f_j) = \Phi_k^{\top}S_k + \eta_k(t)$, where Φ_k, S_k are defined in Lemma 4, $\eta_k(t)$ is the approximation error, and $|\eta_k(t)| \leq \delta$. By the proof in Appendix, it follows that

$$
\begin{aligned}
&\frac{1}{\lambda_1}\left(2z_k + (s+1)z_k^s\right)\left(f_k - \sum_{j=1}^{k-1}\frac{\partial\pi_{k-1}}{\partial\widehat{\theta}_j}\dot{\widehat{\theta}}_j - \sum_{j=1}^{k-1}\frac{\partial\pi_{k-1}}{\partial x_j}\left(a_j\lceil x_{j+1}\rceil^{s_j} + f_j\right)\right)\\
&= \frac{1}{\lambda_1}\left(2z_k + (s+1)z_k^s\right)\left(\Phi_k^{\top}S_k + \eta_k(t)\right)\\
&\leq \theta_{k2}\phi_{k2}\left(z_k^{1+s_h} + z_k^{s+s_m}\right) + \sum_{j=1}^{4}d_{kj},
\end{aligned}
\tag{26}
$$

where θ_{k2} and $d_{k1},\ldots,d_{k4}$ are positive constants and $\phi_{k2} > 0$ is a smooth function. Defining $\theta_k = \max\{\theta_{k1},\theta_{k2}\}$ and $\varphi_k = \varphi_{k1} + \varphi_{k2}$, substituting (24) and (26) into (23), and considering $\lceil x_{k+1}\rceil^{s_k} = \lceil x_{k+1}\rceil^{s_k} - \lceil\pi_k\rceil^{s_k} + \lceil\pi_k\rceil^{s_k}$, we have the following:

$$
\begin{aligned}
\dot{V}_k \leq & -\rho\left(\|\xi\|^{s_h+1} + \|\xi\|^{s+s_m} + \sum_{j=1}^{k-2}\left(z_j^{1+s_h} + z_j^{s+s_m}\right)\right) - \sum_{j=1}^{k-1}\frac{\sigma_j}{\epsilon_j}\widetilde{\theta}_j\widehat{\theta}_j + d_{k-1} + \sum_{j=0}^{4}d_{kj}\\
& + \frac{1}{\epsilon_k}\widetilde{\theta}_k\dot{\widehat{\theta}}_k + \theta_k\varphi_k\left(z_k^{1+s_h} + z_k^{s+s_m}\right) + \frac{a_k}{\lambda_1}\left(2z_k + (s+1)z_k^s\right)\cdot\left(\lceil x_{k+1}\rceil^{s_k(t)} - \lceil\pi_k\rceil^{s_k(t)} + \lceil\pi_k\rceil^{s_k(t)}\right).
\end{aligned}
\tag{27}
$$

Now, choosing the adaptive law, we get the following:

$$
\dot{\widehat{\theta}}_k = \epsilon_k\phi_k\left(z_k^{1+s_h} + z_k^{s+s_m}\right) - \sigma_k\widehat{\theta}_k, \tag{28}
$$

where $\sigma_k \geq 0$ is a constant and selecting the virtual control, we get the following:

$$
\pi_k = -\left(\frac{3r_k}{2}\right)^{1/s_m} z_k =: -\beta_k z_k, \tag{29}
$$

where $r_k = \widehat{\theta}_k\phi_k + 2\rho$ and the derivative of V_k satisfies the following:

$$
\begin{aligned}
\dot{V}_k \leq & -\rho\left(\|\xi\|^{s_h+1} + \|\xi\|^{s+s_m} + \sum_{j=1}^{k-2}\left(z_j^{1+s_h} + z_j^{s+s_m}\right)\right) - 2\rho\left(z_k^{1+s_h} + z_k^{s+s_m}\right) - \sum_{j=1}^{k}\frac{\sigma_j}{\epsilon_j}\widetilde{\theta}_j\widehat{\theta}_j + d_k\\
& + \frac{a_k}{\lambda_1}\left(2z_k + (s+1)z_k^s\right)\left(\lceil x_{k+1}\rceil^{s_k(t)} - \lceil\pi_k\rceil^{s_k(t)}\right),
\end{aligned}
\tag{30}
$$

where $d_k = d_{k-1} + \sum_{j=0}^{4}d_{kj}$. This completes the recursive design.

4. Main Results

Theorem 9. *If Assumptions 1–3 are satisfied, system (1) has an adaptive controller as follows:*

$$
\begin{aligned}
&u = -\beta_n z_n, \beta_n = \left(\frac{3r_n}{2}\right)^{1/s_m}, r_n = \widehat{\theta}_n\phi_n + \rho,\\
&\dot{\widehat{\theta}}_j = \epsilon_j\phi_j\left(z_j^{1+s_h} + z_j^{s+s_m}\right) - \sigma_j\widehat{\theta}_j, j = 1,\ldots,n,
\end{aligned}
\tag{31}
$$

where β_n, ϕ_j are smooth functions and $\rho, \epsilon_j, \sigma_j$ are constants. $z_1 = x_1 - \pi_1(\xi)$ and $z_2,\ldots,z_{n-1}$ are defined in (20). Moreover, for constants $0 < \alpha < 1$ and $\rho_0 > 0$, if $W(\xi)$ satisfies

$W^{\alpha}(\xi) \le \rho_0(\|\xi\|^{s_h+1} + \|\xi\|^{s+s_m})$, then the closed-loop system is SGPFS.

Proof. Choose $V_n = \rho W(\xi) + 1/\lambda_1 \sum_{j=1}^{n}(z_j^2 + z_j^{s+1}) + \sum_{j=1}^{n} 1/2\epsilon_j \widetilde{\theta}_j^2$. Following the steps of Section 3, we construct the adaptive controller (31) such that we get the following:

$$\dot{V}_n \le -\rho\left(\|\xi\|^{s_h+1} + \|\xi\|^{s+s_m}\right) + \sum_{j=1}^{n}\left(z_j^{1+s_h} + z_j^{s+s_m}\right)\Bigg) - \sum_{j=1}^{n}\frac{\sigma_j}{\epsilon_j}\widetilde{\theta}_j\widehat{\theta}_j + d_n. \tag{32}$$

Since $0 < s_m < s_j(t) < s_h$, $s_m \le 1$, and $s \ge 1$, it follows that $(z_j^2 + z_j^{s+1})^{(s+s_m)/(s+1)} \le (z_j^2)^{(s+s_m)/(s+1)} + z_j^{s+s_m} \le (z_j^2)^{(s_h+1)/2} + 1 + z_j^{s+s_m} = z_j^{s_h+1} + 1 + z_j^{s+s_m}$, and

$$\left(\frac{1}{\lambda_1}\sum_{j=1}^{n}\left(z_j^2 + z_j^{s+1}\right)\right)^{(s+s_m)/(s+1)} \le \left(\frac{1}{\lambda_1}\right)^{(s+s_m)/(s+1)}\sum_{j=1}^{n}\left(z_j^{1+s_h} + z_j^{s+s_m}\right) + n\left(\frac{1}{\lambda_1}\right)^{(s+s_m)/(s+1)}, \tag{33}$$

which further indicates that

$$-\rho\sum_{j=1}^{n}\left(z_j^{1+s_h} + z_j^{s+s_m}\right) \le -\rho\lambda_1^{(s+s_m)/(s+1)}\left(\frac{1}{\lambda_1}\sum_{j=1}^{n}\left(z_j^2 + z_j^{s+1}\right)\right)^{(s+s_m)/(s+1)} + n\rho. \tag{34}$$

By Lemma 5, it leads to the following:

$$\left(\sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2\epsilon_j}\right)^{(s+s_m)/(s+1)} \le \sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2\epsilon_j} + 1. \tag{35}$$

Also, we get the following:

$$-\rho\left(\|\xi^{s_h+1} + \|\xi^{s+s_m}\right) \le -\frac{\rho}{\rho_0}W^{\alpha}(\xi). \tag{36}$$

Considering $-\sigma_j/\epsilon_j\widetilde{\theta}_j\widehat{\theta}_j \le -\sigma_j/2\epsilon_j\widetilde{\theta}_j^2 + \sigma_j/2\epsilon_j\theta_j^2$, $\sum_{j=1}^{n}\sigma_j/2\epsilon_j\widetilde{\theta}_j^2 \ge \min_{1\le j\le n}\{\sigma_j\}\ \sum_{j=1}^{n}\ \sigma_j/2\epsilon_j\widetilde{\theta}_j^2 \ge \min_{1\le j\le n}\{\sigma_j\}$. $(\sum_{j=1}^{n}\sigma_j/2\epsilon_j\widetilde{\theta}_j^2)^{(s+s_m)/(s+1)} - \min_{1\le j\le n}\{\sigma_j\}(1 - s_m/(s+1)((s+s_m)/(s+1))^{(s+s_m)/(s+1)}$ and (32)–(36), we have the following:

$$\dot{V}_n \le -c(\rho W(\xi))^{\alpha} - c\left(\frac{1}{\lambda_1}\sum_{j=1}^{n}\left(z_j^2 + z_j^{s+1}\right) + \sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2\epsilon_j}\right)^{(s+s_m)/(s+1)} + d_{n+1}, \tag{37}$$

where $c = \min\{\rho^{1-\alpha}/\rho_0, \rho\lambda_1^{(s+s_m)/(s+1)}, \min_{1\le j\le n}\{\sigma_j\}\}$ and $d_{n+1} = n\rho + \min_{1\le j\le n}\{\sigma_j\}(1-s_m)/(s+1)((s+s_m)/(s+1))^{(s+s_m)/(s+1)} + \sum_{j=1}^{n}\sigma_i/2\epsilon_j\theta_j^2 + d_n$. When $0 < \alpha < s + s_m/s + 1 \le 1$, in view of Lemma 5, one has the following:

$$\left(\frac{1}{\lambda_1}\sum_{j=1}^{n}\left(z_j^2 + z_j^{s+1}\right) + \sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2\epsilon_j}\right)^{\alpha} \le \left(\frac{1}{\lambda_1}\sum_{j=1}^{n}\left(z_j^2 + z_j^{s+1}\right) + \sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2\epsilon_j}\right)^{(s+s_m)/(s+1)} + d_{n+2}, \tag{38}$$

where $d_{n+2} = (s + s_m - \alpha s - \alpha)/(s+s_m)((\alpha s + \alpha)/(s+s_m))^{(\alpha s+\alpha)/(s+s_m-\alpha s-\alpha)}$. Therefore, it follows from (37) that

$$\begin{aligned}\dot{V}_n &\leq -c\left(\rho W(\xi) + \frac{1}{\lambda_1}\sum_{j=1}^{n}\left(z_j^2 + z_j^{s+1}\right) + \sum_{j=1}^{n}\frac{\tilde{\theta}_j^2}{2\epsilon_j}\right)^{\alpha} + d_{n+1} + cd_{n+2} \\ &= -cV_n^{\alpha} + d_{n+1} + cd_{n+2}.\end{aligned} \tag{39}$$

When $0 < (s + s_m)/(s + 1) < \alpha < 1$, similar to (38), we get the following:

$$(\rho W(\xi))^{s+s_m/s+1} \leq (\rho W(\xi))^{\alpha} + d_{n+3}, \tag{40}$$

where $d_{n+3} = (\alpha s + \alpha - s - s_m)/(\alpha s + \alpha)((s + s_m)/(\alpha s + \alpha))^{(s+s_m)/(\alpha s+\alpha-s-s_m)}$. Thus, considering (37) and (40), we obtain the following:

$$\begin{aligned}\dot{V}_n &\leq -c(\rho W(\xi))^{(s+s_m)/(s+1)} - c\left(\frac{1}{\lambda_1}\sum_{j=1}^{n}\left(z_j^2 + z_j^{s+1}\right) + \sum_{j=1}^{n}\frac{\tilde{\theta}_j^2}{2\epsilon_j}\right)^{(s+s_m)/(s+1)} + d_{n+1} + cd_{n+3} \\ &\leq -c\left(\rho W(\xi) + \frac{1}{\lambda_1}\sum_{j=1}^{n}\left(z_j^2 + z_j^{s+1}\right) + \sum_{j=1}^{n}\frac{\tilde{\theta}_j^2}{2\epsilon_j}\right)^{s+s_m/s+1} + d_{n+1} + cd_{n+3} \\ &= -cV_n^{s+s_m/s+1} + d_{n+1} + cd_{n+3}.\end{aligned} \tag{41}$$

Choosing $\alpha_0 = \min\{\alpha, (s + s_m)/(s + 1)\}$ and $d = \max\{d_{n+1} + cd_{n+2}, d_{n+1} + cd_{n+3}\}$, it leads to the following:

$$\dot{V}_n \leq -cV_n^{\alpha_0} + d. \tag{42}$$

By Lemma 4 of reference [23], we see that the solution is SGPFS.

Actually, we can extend the method to the system as follows:

$$\begin{cases}\dot{x}_1 = a_1(t,x)\lceil x_2\rceil^{s_1(t)} + f_1(\varrho, x), \\ \dot{x}_2 = a_2(t,x)\lceil x_3\rceil^{s_2(t)} + f_2(\varrho, x), \\ \vdots \\ \dot{x}_n = a_n(t,x)\lceil u\rceil^{s_n(t)} + f_n(\varrho, x),\end{cases} \tag{43}$$

where the symbols are given in (1). If Assumptions 1 and 2 hold, we can also design a finite-time adaptive controller as (31). For this case, we select $V_n = 1/\lambda_1\sum_{j=1}^{n}(z_j^2 + z_j^{s+1}) + \sum_{j=1}^{n}1/(2\epsilon_j\tilde{\theta}_j^2)$. Utilizing the same design procedures, we can obtain a similar conclusion as Theorem 9. □

Remark 10. During the design of the controller, we adopt some strategies to handle the constraints of the system. To deal with the unknown coefficients a_k, we introduce a term $1/\lambda_1$ in the Lyapunov function. Since $a_k/\lambda_1 \geq 1$, we can obtain $a_k/\lambda_1(2z_k + (s+1)z_k^s)\lceil\pi_k\rceil^{s_k(t)} \leq -r_k(z_k^{1+s_h} + z_k^{s+s_m})$ by using the definition of π_k. Besides, by employing the inequality of Lemma 4, the nonlinear terms that contain a_k can be estimated skillfully. Then, we can define an enough big constant θ_k and design the adaptive law $\widehat{\theta}_k$ to estimate it and utilize $\widehat{\theta}_k$ for the control design. For the constraint of time-varying power $s_i(t)$, we introduce quadratic components and higher-order components in the Lyapunov function, so that nonlinear terms with $s_i(t)$ can be estimated with the appropriate bounds, see (15), (24), and (26) for instance. For the constraint of dynamics and nonlinear functions, we employ the inequality skills and introduce the neural network $\Phi_i^{\top}S_i(\Xi)$ to approximate uncertain terms in each design step, see (6) and (26).

Remark 11. There are some advantages and limitations of this work. The advantages are in three aspects. In detail, this work provides a universal method for system powers in the intervals (0, 1) and $[1, s_h]$. Also, this work considers complicated nonlinear conditions and provides an adaptive controller with a simple form. In addition, the system is rendered semiglobal finite time stable, which is more general than the semiglobal stable. There are also some limitations of this work. For example, despite that the system is complicated, we do not consider the influence of time delay and random disturbances. Also, this work achieves a semiglobal result. A better result is to obtain finite time stabilization globally. Besides, we consider the problem via the state feedback and do not consider the output feedback control.

5. Simulation Example

Example 12. We study the following system:

$$\begin{cases}\dot{\xi} = g(\xi, x_1), \\ \dot{x}_1 = a_1(t,x)\lceil x_2\rceil^{s_1(t)} + \varrho_1\sin x_2, \\ \dot{x}_2 = a_2(t,x)\lceil u\rceil^{s_2(t)} + \varrho_2 x_1 x_2,\end{cases} \tag{44}$$

where ξ, x_1, x_2 are the states, u denotes the control input, $a_1(\cdot), a_2(\cdot)$ are the control coefficients, and ϱ_1, ϱ_2 are unknown parameters. To verify the control method, we assume that $a_1 = 1 + 2/5\sin t, a_2 = 1 + \cos x_1$ and $s_1 = 1 + 1/5\sin t, s_2 = 4/5 + 1/5\cos t$. Then, there are constants λ_1, λ_2 such that $\lambda_1 \leq a_i \leq \lambda_2, i = 1, 2$. Also, there exists $s_m \leq s_i(t) \leq s_h$, where $s_m = 3/5, s_h = 7/5$. Thus, Assumptions 1 and 2 hold. Choosing $W(\xi) = 5/6\xi^{12/5}$ and defining $g = 1/2x_1\sin\xi - \xi^{3/5} - \xi$, $s = 9/5$, we get the following:

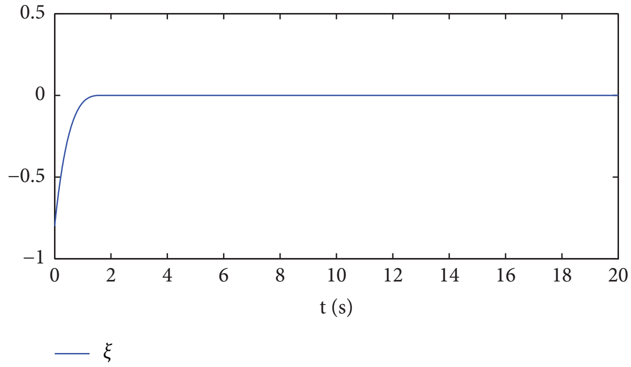

Figure 2: The trajectory of ξ.

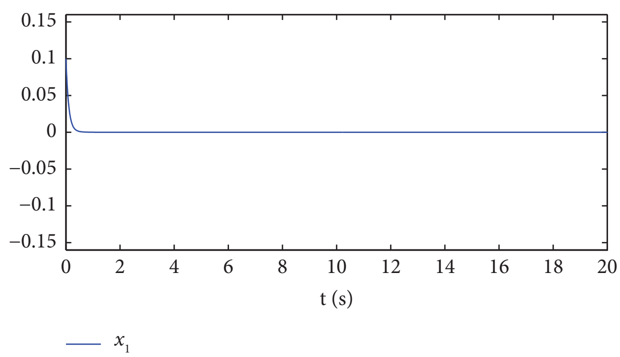

Figure 3: The trajectory of x_1.

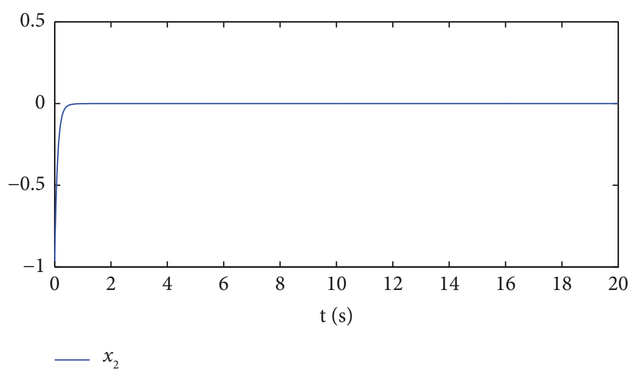

Figure 4: The trajectory of x_2.

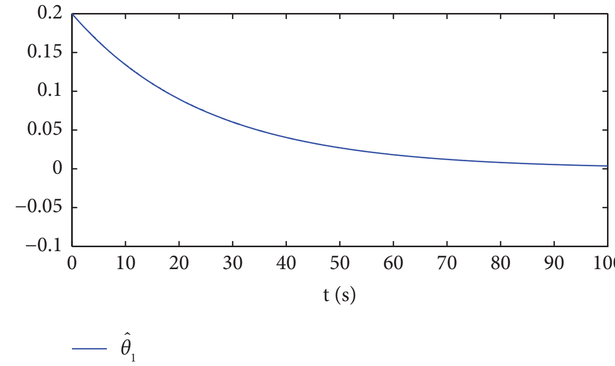

Figure 5: The trajectory of $\widehat{\theta}_1$.

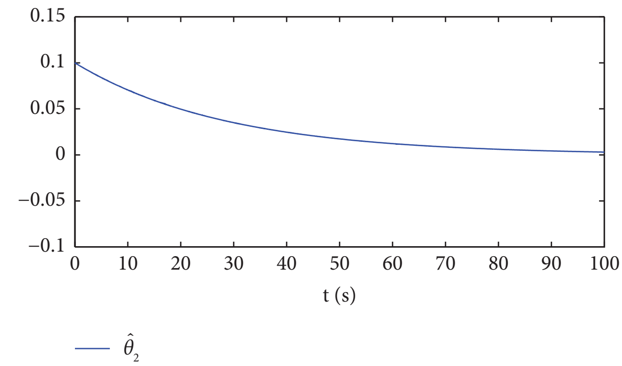

Figure 6: The trajectory of $\widehat{\theta}_2$.

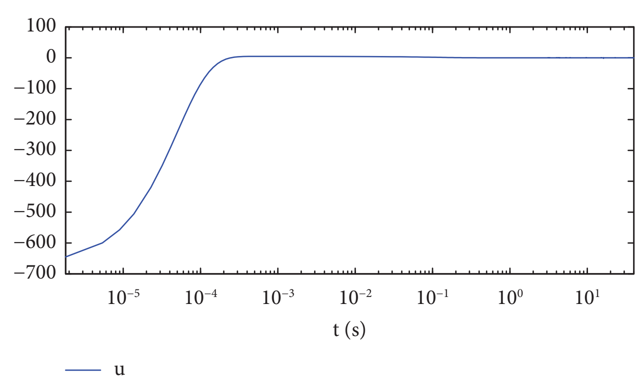

Figure 7: The trajectory of u.

$$\frac{\partial W(\xi)}{\partial \xi} g \leq -\left(\xi^{1+s_h} + \xi^{s+s_m}\right) + |x_1| \phi_0, \tag{45}$$

where $\phi_0 = (x_1^{14/5} + 1)^{1/2}$. Thus, Assumption 3 holds. With Section 3, we obtain the following:

$$\begin{aligned} u &= -\left(\frac{3r_2}{2}\right)^{1/s_m} z_2, \\ \dot{\widehat{\theta}}_1 &= \epsilon_1 \phi_1 \left(z_1^{1+s_h} + z_1^{s+s_m}\right) - \sigma_1 \widehat{\theta}_1, \\ \dot{\widehat{\theta}}_2 &= \epsilon_2 \phi_2 \left(z_2^{1+s_h} + z_2^{s+s_m}\right) - \sigma_2 \widehat{\theta}_2, \end{aligned} \tag{46}$$

where $r_2 = \widehat{\theta}_2 \phi_2 + \rho$, $z_1 = x_1$, $z_2 = x_2 - \pi_2$, $\pi_2 = -(3r_1/2)^{1/s_m} z_1$, $r_1 = \widehat{\theta}_1 \phi_1 + 2\rho$, $\phi_1 = b_{10} + b_{11} + b_{12}$, $\phi_2 = \phi_{21} + \phi_{22}$, $\phi_{21} = (s_h)/(1+s_h)(4/(\rho + \rho s_h))^{1/s_h}(4(s_m + s_h)/s_h)^{(1+s_h)/s_h} + s_m/(s+s_m)(2s/(\rho s + \rho s_m))^{s/s_m}(2(s+1)(1+\phi_{20}))^{(s+s_m)/s_m}$, and $\varphi_{20} = 1 + z_2^2$, $\phi_{22} = \max\{b_{21} + b_{22}, b_{23} + b_{24}\}$.

In the simulation, the parameters are $\epsilon_1 = 0.1$, $\epsilon_2 = 0.01$, $\sigma_1 = 0.04$, $\sigma_2 = 0.035$, $b_{10} = b_{11} = b_{12} = 1$, $\rho = 1$, $b_{21} = b_{22} = b_{23} = b_{24} = 1$, and $\varrho_1 = \varrho_2 = 1/10$. The initial conditions are $\xi(0) = -0.8$, $x_1(0) = 0.1$, $x_2(0) = -0.9$, and $\widehat{\theta}_1(0) = 0.2$, $\widehat{\theta}_2(0) = 0.1$. Figures 2–4 give responses of states ξ, x_1 and x_2. Figures 5 and 6 provide responses of adaptive laws $\widehat{\theta}_1$ and $\widehat{\theta}_2$. Figure 7 supplies the trajectory of input u. Figures 2–7 show that the closed-loop system is SGPFS. Thus, the raised control method is valid.

6. Conclusion

This work has studied the regulation of complex systems. Different from the reported researches, the considered systems involve time-varying powers, inverse dynamics, and uncertainties. Besides, the system powers can be either smaller than one or bigger than one. By constructing a new Lyapunov function and utilizing the adaptive control algorithm and neural network, a semiglobal finite time adaptive controller is successfully designed. Indeed, there are also some interesting but challenging problems. For example, when the system contains time delay, can we design the adaptive controller to guarantee the stability of the system? Can we develop the proposed approach to study stochastic systems? When the system output is the only measured signal, can we construct the state observer and design the output feedback controller?

Appendix

Proof of (24): When $0 < s_{k-1} < 1$, there holds the following:

$$\left|\lceil x_k\rceil^{s_{k-1}} - \lceil \pi_{k-1}\rceil^{s_{k-1}}\right| \le 2\left(|z_k|^{s_m} + |z_k|^{s_h}\right), \tag{A.1}$$

which indicates

$$\begin{aligned}&\frac{a_{k-1}}{\lambda_1}\left(2z_{k-1} + (s+1)z_{k-1}^{s}\right)\left(\lceil x_k\rceil^{s_{k-1}} - \lceil \pi_{k-1}\rceil^{s_{k-1}}\right)\\ &\le \frac{2a_{k-1}}{\lambda_1}\left(2|z_{k-1}| + (s+1)|z_{k-1}|^{s}\right)\left(|z_k|^{s_m} + |z_k|^{s_h}\right).\end{aligned} \tag{A.2}$$

By Lemma 5, we have the following:

$$\frac{4a_{k-1}}{\lambda_1}|z_{k-1}|\left(|z_k|^{s_m} + |z_k|^{s_h}\right) \le \rho z_{k-1}^{1+s_h} + \left(\frac{\lambda_2}{\lambda_1}\right)^{(1+s_h)/s_h}\overline{\phi}_{k1} z_k^{1+s_h} + d_{k0}, \tag{A.3}$$

where $\overline{\phi}_{k1}$ and d_{k0} are constants. Similarly, we have the following:

$$\frac{2a_{k-1}}{\lambda_1}(s+1)|z_{k-1}|^{s}\left(|z_k|^{s_m} + |z_k|^{s_h}\right) \le \rho z_{k-1}^{s+s_m} + \left(\frac{\lambda_2}{\lambda_1}\right)^{s+s_m/s_m}\widetilde{\phi}_{k1} z_k^{1+s_h}, \tag{A.4}$$

where $\varphi_{k0} \ge z_k^{s_h - s_m}$ is a smooth function and $\widetilde{\phi}_{k1} = s_m/(s+s_m)(s/(\rho s + \rho s_m))^{s/s_m}\ (2(s+1)(1+\phi_{k0}))^{(s+s_m)/s_m}$. Considering (A.2)–(A.4), we obtain the following:

$$\begin{aligned}&\frac{a_{k-1}}{\lambda_1}\left(2z_{k-1} + (s+1)z_{k-1}^{s}\right)\left(\lceil x_k\rceil^{s_{k-1}} - \lceil \pi_{k-1}\rceil^{s_{k-1}}\right)\\ &\le \rho\left(z_{k-1}^{1+s_h} + z_{k-1}^{s+s_m}\right) + \theta_{k1}\phi_{k1}\left(z_k^{1+s_h} + z_k^{s+s_m}\right) + d_{k0},\end{aligned} \tag{A.5}$$

where $\theta_{k1} \ge \max\left\{(\lambda_2/\lambda_1)^{(1+s_h)/s_h}, (\lambda_2/\lambda_1)^{(s+s_m)/s_m}\right\}$ and d_{k0} are constants and $\phi_{k1} = \max\left\{\overline{\phi}_{k1}, \widetilde{\phi}_{k1}\right\}$.

When $s_{k-1} \ge 1$, we get the following:

$$\begin{aligned}&\frac{a_{k-1}}{\lambda_1}\left(2z_{k-1} + (s+1)z_{k-1}^{s}\right)\left(\lceil x_k\rceil^{s_{k-1}} - \lceil \pi_{k-1}\rceil^{s_{k-1}}\right)\\ &\le \alpha_k|z_{k-1}|\|z_k|^{s_{k-1}} + \alpha_k|z_{k-1}|^{s}|z_k|^{s_{k-1}} + \alpha_k|z_{k-1}|\|\beta_{k-1}z_{k-1}|^{s_{k-1}-1}|z_k|\\ &\quad + \alpha_k|z_{k-1}|^{s}|\beta_{k-1}z_{k-1}|^{s_{k-1}-1}|z_k|,\end{aligned} \tag{A.6}$$

where $\alpha_k = 2\lambda_2/\lambda_1 s_h(1 + 2^{s_h - 2})(s+1)$. With the help of $|z_k|^{s_{k-1}(t)} \le (|z_k|^{s_m} + |z_k|^{s_h})$, $|z_{k-1}|^{s_{k-1}(t)} \le (|z_{k-1}|^{s_m} + |z_{k-1}|^{s_h})$, similar to (A.5), there is a smooth function ϕ_{k1} and a constant θ_k such that (24) holds.

Proof of (26): By Lemma 5, it yields that

$$\begin{aligned}\frac{2}{\lambda_1}|z_k\|\Phi_k^{\top}S_k| &\le \frac{2}{\lambda_1}|z_1\|\|\Phi_k\| \cdot \left(\|S_k\|^{1/s_h}\right)^{s_h}\\ &\le b_{k1}\left(\frac{\|\Phi_k\|}{\lambda_1}\right)^{1+s_h} z_1^{s_h+1} + d_{k1},\end{aligned} \tag{A.7}$$

where b_{k1} is a constant and $d_{k1} = 1/(s_h + 1)((1/b_{k1} s_h + b_{k1}))^{1/s_h} \|S_k\|^{(1+s_h)/s_h} 2^{(1+s_h)/s_h}$. Similarly, we get the following:

$$\frac{2}{\lambda_1}|z_k\|\eta_k| \le b_{k2}\left(\frac{1}{\lambda_1}\right)^{s_h+1} z_k^{s_h+1} + d_{k2},$$

$$\frac{s+1}{\lambda_1}|z_k|^s|\Phi_k^\top S_k| \le b_{k3}\left(\frac{\|\Phi_k\|}{\lambda_1}\right)^{(s+s_m)/s_m} z_k^{s_m+s} + d_{k3}, \tag{A.8}$$

$$\frac{s+1}{\lambda_1}|z_k|^s|\eta_k| \le b_{k4}\left(\frac{1}{\lambda_1}\right)^{(s+s_m)} z_k^{s_m+s} + d_{k4},$$

where

$$d_{k2} = \frac{s_h}{1+s_h}\left(\frac{1}{b_{k2}+b_{k2}s_h}\right)^{1/s_h} \eta_k^{(1+s_h)/s_h}(s+1)^{(1+s_h)/s_h},$$

$$d_{k3} = \frac{s}{s+s_m}\left(\frac{s}{b_{k3}+b_{k3}s_m}\right)^{s/s_m} \left(\|S_k\|\right)^{(s+s_m)/s_m}(s+1)^{(s+s_m)/s_m},$$

$$d_{k4} = \frac{s_m}{s+s_m}\left(\frac{s}{b_{k4}+b_{k4}s_m}\right)^{s/s_m} \eta_k^{(s+s_m)/s_m}(s+1)^{(s+s_m)/s_m}. \tag{A.9}$$

Let $\theta_{k2} \ge \max\{(\|\Phi_k\|/\lambda_1)^{1+s_h}, (1/\lambda_1)^{s_h+1}, (\|\Phi_k\|/\lambda_1)^{s+s_m/s_m}, (1/\lambda_1)^{s+s_m/s_m}\}$, $\phi_{k2} = \max\{b_{k1} + b_{k2}, b_{k3} + b_{k4}\}$, and b_{k2}, b_{k3}, b_{k4} are constants. It follows that

$$\frac{1}{\lambda_1}\left(2z_k + (s+1)z_k^s\right)\left(\Phi_k^\top S_k + \eta_k\right) \le \theta_k\phi_{k2}\left(z_k^{1+s_h} + z_k^{s+s_m}\right) + \sum_{j=1}^{4} d_{kj}. \tag{A.10}$$

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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