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Research Article

Extended Framework for Preventive Maintenance Planning: Risk and Behaviour Analysis of a Proposed Optimization Model

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The considerable increase in the complexity associated with the formulation of maintenance plans has enabled the development of new techniques to bring maintenance scheduling optimization models to more realistic environments. In this sense, a previous optimization model was proposed considering the use of time windows for the formation of grouping schemes under an opportunistic strategy for maintenance activities considering non-negligible execution times, thus offering the possibility of analysing scenarios with limited resources. This article proposes a risk analysis based on the failure probability of each component involved in the maintenance scheduling optimization model, which has the particularity of enabling a greater number of combinations of grouped PM activities. Moreover, it seeks to identify the general behaviour of the optimization model against different scenarios of periodicities and execution times of each maintenance activity. The proposed optimization model is formulated under a mixed integer linear programming (MILP) paradigm and its objective function seeks to minimize the unavailability of the system associated with the execution times of the activities developed, generating different experimental cases, and varying the start time scheduling under a tolerance factor from 0% up to a maximum of 25% for advance or delay. Results show in contrast with the base optimization model, an 8% less unavailability when the tolerance factor is 10%. Finally, it was possible to quantify the risk present in each maintenance schedule, at the same time a behaviour towards advancing PM activities is evidenced by the optimization model proposed over the delay.

1. Introduction

During the last decades, maintenance management has been a fundamental pillar for certain organizations with production processes in their operation, since it enables the correct use and optimization of resources, generating significant savings in terms of performing and programming maintenance activities, where statistics show that a company can save up to 18% of their total costs destined to this work. So much so that its role in modern production systems has become a much more important task in companies that adopt maintenance as an element of the business that generates profit [1]. Given this, it could be safely stated that the objective of maintenance is to contribute to the benefit of the organization in which it is incorporated, developing a necessity for strategy oriented maintenance operations aligned with the objectives of all incumbent organizational levels. This is, recognizing that current production systems operate more efficiently, effectively, and economically to sustain themselves in the long term [2].

The latter has led a large group of researchers to focus their efforts on finding increasingly precise ways to generate maintenance plans that adapt to the requirements of different industries and their different operational contexts. However, given the large number of factors that can be considered within the spectrum of the problem, it has become a task with immense complexity. To this, it must be added that, not too long ago, many mathematical or computational tools had not been developed, therefore the current potential of the hand with technological advancement is tremendous. Given this, as technology evolves, the complexity of modern engineering systems and maintenance systems also do [3]. Therefore, the continuous developments of technical systems and the growing dependence on equipment have led the importance of the effectiveness of maintenance plans to grow [4], which has enabled the finding of different and new techniques to face the complexity of each scenario. In addition, the real contribution of forming efficient maintenance plans is in the minimization of the impact of the costs associated with the execution of these activities within industries that have large-scale production processes, which can reach from 15% to 70% of total production costs [5].

The types of systems used within the framework of this document correspond to “multiunit systems.” This term can refer to a single asset consisting of multiple components (multicomponent system) or a system with multiple assets (multiasset system). Given this, the fact that multiunit systems operate collectively to produce or deliver a service, creating a maintenance policy for each unit separately might not prove efficient [6]. Therefore, maintenance planning should be performed considering all requirements and constraints of system components at the same time. However, in the presence of multicomponent systems, it is necessary to account for the interactions between them, which can be classified into 3 types: economic, structural, and stochastic. Economic dependence is the most common within these [7] and implies that the combination or grouping of maintenance activities will be cheaper than carrying them out separately (positive dependence), while negative economic dependence occurs when the combination of maintenance activities is more expensive than maintaining the components individually [8]. Structural dependence refers to the fact that to carry out a maintenance activity in a certain component, it is necessary to intervene in others. Finally, stochastic dependence implies that the condition of one component influences the condition of others, or when these are subject to failures for common causes. The latter can often be observed in redundant mechanical systems, where the degradation of one component leads to a distribution of the internal force of the system, and therefore overloads other components [9].

As it was mentioned, maintenance planning aims to reduce the negative impact rising from the execution of maintenance activities. Thus, a method to address these impacts is to minimize failure risk through planning focusing mostly on minimizing the cost associated with failure [10]. The latter is known as risk-based maintenance planning and to be performed it may be supported by different policies such as condition-based maintenance (CBM) or preventive maintenance (PM) policies. Risk-based planning is especially attractive for contexts in which failure brings costly consequences and a cost-effective tool is required to reduce the probability of failure [11] or measure the preventive maintenance effectiveness [12]. Most risk-based maintenance planning does not focus on the effects of economic dependence or the system’s unavailability due to scheduled detentions [13]. Moreover, in cases with high uncertainty, it becomes necessary to disregard some costs when availability and reliability are critical [14]. These have pushed the focus away from the context in which high economic dependence is present.

This fosters the development of maintenance plans through the opportunistic maintenance strategy, which is an effective method to reduce interference between maintenance and production operations in a multi-component system [15]. The advantage of this strategy is to be able to carry out several PM activities by stopping the system only once, forming packages or groups of activities. These activity packages and their impact within a certain scheme will be determined by their “time windows.” This term refers to a certain time in which the executions of each PM activity may be advanced or delayed with respect to their tentative moment (or equivalently, default moment) of execution previously defined by the periodicity of each of these activities, enabling other PM activities to be partially or totally executed within the same period of time, thus reducing “set up” costs and increasing the availability of a single machine or a multicomponent system [16]. The formation of these groups of PM activities, in which certain activities can be executed together with others minimizing the downtime of the system, will consequently generate, within an established time horizon, and increased generation of groups. Hence, inefficiency and set up costs will be considerably reduced over the time horizon.

The way in which these schemes are formed is through the development and formulation of an optimization model, which aims to obtain the most appropriate maintenance schedule in terms of the tolerance (time window) assigned to each scenario. Each scenario will consider one fixed tolerance for each activity of the studied scenario. It should be also noted that each model will depend exclusively on the variables and assumptions they incorporate, thus giving them certain characteristics that bring them closer or far from what really happens in certain production processes.

On the other hand, one of the challenges posed by the reduction or minimization of the maintenance costs of multicomponent systems is that maintenance plans that have been developed are not based on the maintenance history or condition of each component, which would imply a considerable increase in the risk of failure [17]. Specifically, there is an inherent cost when wasting the useful life of the components when maintenance activities are advanced; on the other hand, an increase in the probability of failure of the components is generated if the activities are delayed [18], increasing the failure risk and shutdown probability. Given this, it is important to consider and quantify the risk associated with maintenance plans, to prefer those that best suit the requirements of each industry or production process.

Therefore, many studies have been developed that cover maintenance planning from different edges, where each of them incorporates different assumptions and characteristics that are typical of the systems studied. In addition, different techniques are revealed to address both the formation of maintenance plans and the implications they have in terms of risk. In the first place, with respect to maintenance planning, each job differs from the other with respect to the assumptions they incorporate, that is, type of system, types of interactions between components, characteristics of maintenance activities, feasibility of grouping through time windows, and tolerance level thus each of these responds to what is sought in certain problems, situations, or industries. Regarding the interactions between components of a system, commonly, the dependence incorporated into the assessment is the economic one, which refers to the possible decrease in the costs of grouping maintenance activities [5, 16–19]. For its part, stochastic dependence from the point of view of component degradation and its effect on the distribution of the useful life of others has been addressed in conjunction with economic dependence in [2, 8, 20–23]. In fact, several studies have been conducted addressing more accurate and robust modelling, accounting for the interactions, criticality of components [24], and the downstream effect of components that after failure allow normal performance for a limited time [25]. On the other hand, from the point of view of the configuration of the systems, due to the high complexity represented by parallel or hybrid systems such as the one proposed in [26], serial systems are used.

One of these grouping models was early developed in [27] were they sought to maximize the number of groups to, consequently, lower inefficiencies related to availability. They further develop the model in [16] were the objective is to minimize the number of detentions instead of maximizing the groups achieving higher levels of availability and precision in comparison with the original proposed model. More recently, they presented a model available for implementation, also addressing the opportunistic grouping strategy for maintenance scheduling presented in [5] aiming to minimize detentions, but considering the use of resources in the modelling. Now, regarding the characteristics of maintenance activities, it can be considered that their execution times are usually considered negligible, as in [16]; however, in [5], their model is adapted to more realistic contexts, where the incorporation of non-negligible executions stands out, which is an important window of opportunities for the formation of grouping packages.

Furthermore, there are several ways to address the risk associated with scheduling maintenance activities, such as penalty functions, condition-based maintenance, and Bayesian networks. One of the models that raises the penalty functions considering that there is a risk when advancing and delaying maintenance activities corresponds to that carried out in [18], where an optimization model is analysed with a framework similar to the current one, with the particularity of having a dynamic component; that is, maintenance planning can be updated over time considering the existence of events in the short term. Different works emerge that incorporate risk in dynamic contexts [28, 29]; however, from a point of view in which maintenance scheduling is carried out for the entire planning horizon without being subject to possible corrective activities, there is no history of using the probability of failure and corrective cost of each asset using "event space method" for the different groups of activities to be formed within a scheme, as is the method to be developed in the present research.

Considering the above-given theory, this article proposes an extension to the model developed in [5]; that is, maintenance planning is performed by developing a model for optimizing opportunistic maintenance activities in a multicomponent system, incorporating time windows for the formation of grouping packages that allow reducing the downtime of the system by modifying certain key constraints of the original model to facilitate a greater number of groupings. This is achieved by relaxing a certain restriction of the base model that imposes that all maintenance activities that are part of the same working group must begin at the same moment. Although the extension of the original model means adding extra complexity a multiobjective approach such as [30] would improve the solver efficiency, it would at the same time, push to greater changes to the modelling, losing the focus of this article.

The activities can be grouped under two scenarios: their time windows generate an opportunity for grouping when two or more overlap between them; or there is no overlap between the time windows, but since the model contemplates non-negligible execution times, the grouping can be carried out by intercepting the execution times of each activity. In this way, an activity could make use of all the allowed tolerance to delay or advance its execution, without overlapping with the tolerance of another activity, and still be able to generate a grouping package with another activity, as long as its execution times allow it. This will generate two effects: recognizing as joint groupings of activities those that the base model does not consider and provides a new spectrum of possibilities to form groupings, favouring the minimization of the downtime of the system.

Therefore, the main objective of this research is to generate a postoptimal risk analysis on an optimization model that allows a greater number of groupings (and hence a greater minimization in downtime) to recognize the behaviour of the model and deliver valuable information to a decision maker about maintenance scheduling in different scenarios. As mentioned, the quantification of risk is based on the probability of failure of each component; however, it is necessary to emphasize that for each of these components or equipment, parameters are added that define their behaviour by assuming a certain function of the probability distribution of failure, in this case as a Weibull function of scale parameters (α) and shape (β). The quantification of the risk is postoptimal, that is, first a maintenance schedule is obtained disregarding the risk and then each of these schemes proposed by the optimisation model (which differ from the level of tolerance allowed to advance or delay PM activities) are calculated the associated risk (failure probability when delaying and wasted useful life when advancing). We seek to obtain the optimal tolerance levels according to the risk that a decision maker intends to assume in each industry or production line, delivering a series of indicators and numerical results that accompany this decision.

2. Framework Definition

The framework presented in [5] is an extension to the original model developed in [16], which seeks to expand its scope and technical characteristics, incorporating tolerance of time windows, non-negligible duration of preventive maintenance activities, changes in the configuration of the system and the application of limitations to the technical feasibility in the performance of certain activities, thus generating a better adherence to real maintenance scenarios in the industry. Therefore, some of the most important and considerable assumptions to be incorporated in the current model from the original model, except for (iv) and (vi) are discussed as follows:

(i) The system contemplates a multicomponent series configuration, where each of the equipment has only one failure mode: previously defining that the components involved will have exclusively one failure mode, reduces the degree of analytical and computational complexity.

(ii) The execution of grouped PM activities always involves a simultaneous/parallel execution: this point is important to be able to formulate the optimization model, if there were serial executions, the stopping times of the system would be considerably longer, which would not allow to find the minimum unavailability. The latter is approachable when the activities are performed in parallel and the total time of unavailability for the execution of a package of PM activities corresponds to the longest individual execution time of the activities.

(iii) The execution of PM activities returns the component to its initial operational conditions (perfect maintenance): defining that the components return to their original state of life after the PM activity is executed is an extremely accurate assumption in the context in which computational and analytical complexity is intended to be avoided. If not, the analysis should incorporate degradation models, such as those discussed in [31], which propose the incorporation of the concept of "virtual age," which precisely refers to imperfect maintenance actions.

(iv) It is considered that there are technical feasibility constraints: the incorporation of this assumption promotes and supports the main objective in [5], that is, to adapt the original model to real industrial environments, since it limits the simultaneous execution of certain PM activities due to technical nonfeasibility, which may be attributable to lack of personnel, shortage of tools, etc. Since the scope of this investigation does not contemplate this restriction, the analysis of the technical nonfeasibility of some activities is set aside, considering it as an input parameter that any combination of activities can be executed without problems.

(v) Duration of the executions of the PM activities nonnegligible: in the same way as in the previous case, this assumption is also important to adapt and extend the original model to real production environments, since in these the duration of PM activities translates into a productive cessation, directly impacting on the inefficiency costs.

(vi) Simultaneous start of grouped activities: although this is not considered as an assumption within the article, the constraints associated with the base optimization model support it and it translates into a limitation for the system to form grouping packages that further minimize the unavailability of the system.

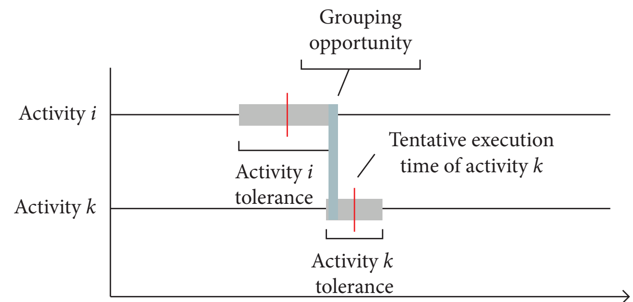


Figure 1: Feasibility of opportunistic grouping between the activities i and k.

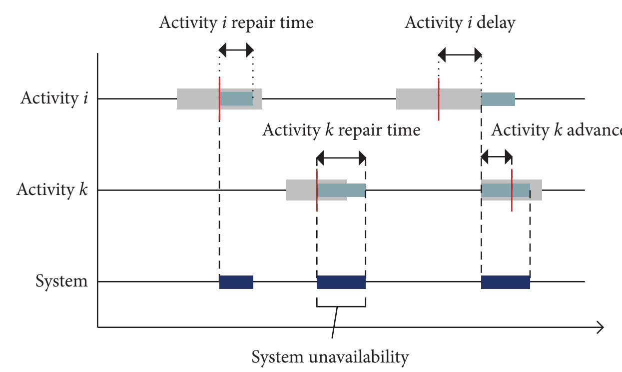


Figure 2: Scheme of grouping two activities of different components configured in series.

On the other hand, from this discussion emerge the differences and key contributions that are implemented in this research for a correct extension of the work carried out in [5], which are divided into two essential points:

2.1. Nonsimultaneous Start of Grouped Activities. For the original model to establish the opportunity to group two or more maintenance activities, it is necessary that the time windows (with their respective tolerances) of these activities are intercepted in some sections (see Figure 1).

That being said, based on what has been mentioned, the following grouping scheme arises, consisting of the same two activities i and k, where the execution of any of the maintenance activities (belonging to different components) always implies a stop of the system. In addition, when there is grouping, both activities may or may not be executed in parallel, hence the total downtime of the system associated with a package of activities is given by the total time in which the maintenance activities are developed. Furthermore, it is assumed that when all PM activities associated with a certain clustering package have ended, the system returns to normal operation immediately (see Figure 2).

As mentioned, restricting the optimization model to grouped activities starting at the same moment limits certain activities, at particular moments, from being able to generate work packages that at first sight seem to be obvious. As an example, Figure 3 shows a grouping scheme subject to tolerances in which the activity i is grouped with the activity k. In this sense, it is necessary to emphasize that although the time windows of each of the grouped activities are not intercepted; in the same way, a grouping package was formed. This grouping is generated by advancing the execution of the activity i and delaying that of the activity k, where it is also appreciated that the time in which the activity i begins is within the execution time of the activity k. That is, the activities make use of all their tolerance (to delay or advance, as the case may be), to allow the minimization of unavailability to be the greatest. Given the above-mentioned theory, in general terms, relaxing this restriction allows the grouping of activities to have different start and end moments; then, the total time out of service of the system in each grouping package will be defined as the period between the earliest start time and the latest completion time of the executions. This enables situations in which could even generate that when there are no time windows available to advance or delay activities, grouping schemes can be formed, which *a priori* could imply times of the unavailability of the system considerably reduced in lower tolerance percentages compared to the extended model.

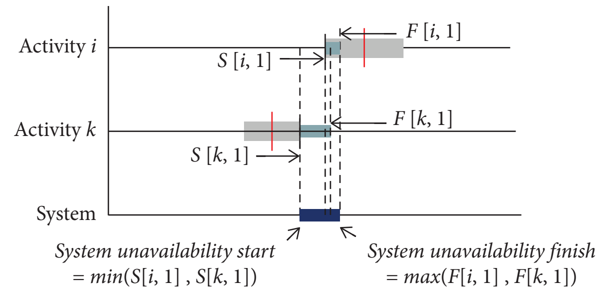


Figure 3: Example of a grouping scheme possible by relaxing the start restriction on grouped activities.

2.2. Risk Analysis. As mentioned, this research main objective is to incorporate a postoptimal analysis of the risk associated with the use of time windows under an extended framework and optimization model, in order to provide a decision maker with the necessary information to opt for the planning of maintenance activities that best suits the defined requirements and know the behaviour of the model itself. In this context, it is proposed that the risk associated with the change in the tentative moments of execution is represented by the increase or decrease in the probability of failure of the components, that is, both the advance and delay will include a certain level of risk, so that each activity, regardless of whether or not it is grouped with others, will have its own risk component determined by its probability of failure at that moment. However, this is not trivial when there are grouping packages, since for activities to begin their executions, it is necessary to assume that none of the components will present a previous failure state, so the reliability of such a scenario is conditioned on the behaviour of the individual probability of failure of each of the equipment or components and the number of them. In practical terms, the fact of forming an opportunistic preventive maintenance scheme will imply that certain activity executions are performed at their tentative moment, or as part of a grouping package in which the risk is analysed together with other activities that are part of it. On the other hand, it is necessary to establish that the fact of advancing a PM activity will always be beneficial in terms of the probability of failure; hence, there must be some form of associated penalty. That is why the quantification of the risk associated with each grouping scheme is composed of 3 parts.

(a) Tentative risk: the first of these refers to the risk that involves executing a certain activity that has not been grouped, and therefore, is executed at its tentative moment. Usually, a random variable is the way in which such a risk is quantified is determined by the cumulative failure probability function of the component or failure state, which is determined by those responsible for having developed the maintenance plans. Given this, the distribution for the random variable will be considered as an input parameter, and considering its widely use to model failure and survival [32–34], it will correspond to a Weibull distribution of scale and shape parameters equally incorporated as exogenous parameters from the previous maintenance planning. The mathematical expression of this quantification is detailed as follows:

$$R_{\text{tentative}} = \left(1 - e^{-\left(rs_{i,j} - rf_{i,j-1}/\alpha\right)^{\beta}}\right) * C_{pf_i}. \qquad (1)$$

Here, $rs_{i,j}$ represents the moment of the beginning of the execution pair (i, j). Furthermore, $rf_{i,j-1}$ represents the moment of completion of the previous execution of (i, j). In addition, C_{pfi} refers to the cost associated (in monetary units) to all the management and activities involved in the preventive intervention of the component i, assumed as an arbitrary parameter.

(b) Grouping risk: as mentioned, grouping implies assuming that none of the components involved will fail before the PM activity execution; since otherwise, it would not be possible to execute these activities within the planned interval. Given this, this assumption involves a risk, which corresponds to all scenarios in which at least one of the grouped components fails. That is why the number of scenarios to be analysed will depend exclusively on the number of activities (n) within the grouping package, generating a total of 2^n cases, however, because one of the combinations corresponds to the scenario where none of the equipment fails; that is, the case that is not intended to be studied since it delivers a scenario of good functioning, the number of cases to be analysed corresponds to $2^n - 1$. As an example, a hypothetical case is developed where

Table 1: Analysis of a two-components case, which shows how to calculate each of the three fault scenarios costs.

Sce	Comp 1	Comp 2	Occurrence prob.	Cost
1	Fault	No fault	$P_1 = F(t)_1 * (1 - F(t)_2)$	$C_1 = P_1 * C_{cf_1} + P_1 * p_1 * C_d$
2	No fault	Fault	$P_2 = (1 - F(t)_1) * F(t)_2$	$C_2 = P_2 * C_{cf_2} + P_2 * p_2 * C_d$
3	Fault	Fault	$P_3 = F(t)_1 * F(t)_2$	$C_3 = P_3 * (C_{cf_1} + C_{cf_2}) + P_3 * \max(p_1, p_2) * C_d$

there is a grouping package of two different component activities to be executed at a certain moment t (see Table 1). Given this, the number of scenarios to analyse will be $2^2 - 1 = 3$.

As can be seen in the table, by generating different scenarios for each grouping scheme will deliver different probabilities of occurrences for them, and therefore, different associated costs. In the first of the scenarios, it is observed that the component i is in a state of failure while the second is not, given this, the probability of occurrence of said scenario corresponds to the probability of failure of the first component by the reliability of the second (probability of nonfailure). Moreover, the cost associated with each scenario has two components. The first one is the cost associated with the corrective repair of the component or components that present a state of failure, while the second term corresponds to the cost of inefficiency that would be incurred by having to stop the system to perform such activity. The first term is calculated by multiplying the probability of occurrence of the scenario under study by the costs of repairing the asset or assets that have failed (C_{cf}). On the other hand, the second component is the one resulting from the multiplication between the probability of occurrence of the scenario by the time it takes to carry out the maintenance activity by the cost of stopping the system (C_d). Finally, the total risk associated to this package corresponds to the sum of every scenario cost, i.e., $C_1 + C_2 + C_3$. In addition, it is necessary to emphasize that when there are joint failure scenarios, such as the third scenario in the table, the cost of inefficiency will be calculated based on the maximum execution time of the components that fail at the same time, since it is assumed that they can be executed in parallel without any problem.

(c) Useful life cost: considering the previous risks, it is important to note that when a maintenance activity is advanced, its risk will decrease, since its probability of failure decreases. On the other hand, delaying PM activities translates into an increase of the associated risk. This could imply that advancing is always more convenient than delaying; however, this is not the case, since when advancing a maintenance activity and therefore not taking advantage of the total useful life of the component, it has an associated extra risk component which can be quantified as cost. The way to calculate this cost is done by multiplying the total number of weeks that the activities are advanced by a unit cost associated with the useful life of the components (C_{ul}), assumed in this case as 10 (μm) for each week of useful life used.

$$\left(\text{tent}_{\text{exc}_{i,j}} - \left(\text{tent}_{\text{exc}_{i,j-1}} + T_i\right)\right) \cdot C_{ul}, \quad \forall (i, j) \in IJ^+. \quad (2)$$

3. Problem Formulation

The formulation of the optimization model gives shape to a mixed integer linear programming (MILP) problem; that is, the model is made up of linear constraints and objective function, together with real and binary variables. On this occasion, the solver "Gurobi" will be used, which is a specialized and free use tool to solve problems of this nature. The notation and definition of parameters, sets, and decision variables of the problem presented in the previous section is shown as follows:

Sets:

I set of PM activities to perform on the component, indexed by i or n
J_i set of executions of the PM activity $i \in I$ within the horizon T, indexed by j or o
N set of possible grouping pair combinations (i, j, n, o) in the planning horizon T, where $(i, j), (n, o) \in I \times J_i : i \neq n$
K set of possible grouping pair permutations (i, j, n, o) in the planning horizon T, where $(i, j), (n, o) \in I \times J_i : i \neq n$

Parameters:

T planning horizon, in $[u.t]$.
T_i periodicity of the activity i, $i \in \{1, 2, \ldots |I|\}$
M "Big-M" parameter, where $M \gg T$
e tolerance for the execution of the activity i, $i \in \{1, 2, \ldots |I|\}$
p_i activity execution time i, $i \in \{1, 2, \ldots |I|\}$
ε parameter of negligible value, where $\varepsilon \longrightarrow 0$

Decision variables:

S_{ij} start time of the j-th execution of the activity i, $j \in J_i$, $i \in I$
F_{ij} completion time of the j-th execution of the activity i, $j \in J_i$, $i \in I$
$w_{i,j,n,o}$ clustering activation binary variable, where $w_{i,j,n,o} = 1$ if the j-th execution of the activity i is grouped with the o-th execution of the activity n, $j, o \in J_i$, $i, n \in I$
$z_{i,j,n,o}$ instant of start of the system stop associated with the grouping of the activity (i, j) with (i', j') yes and only if $w_{i,j,n,o} = 1$. Otherwise, $z_{i,j,n,o} = 0$
$y_{i,j,n,o}$ instant of completion of the system stop associated with the grouping of the activity (i, j) with (n, o) yes and only if $w_{i,j,n,o} = 1$. Otherwise, $y_{i,j,n,o} = 0$

$rs_{i,j}$ instant of start of the system stop associated with the j-th execution of the activity i, $j \epsilon J_i$, $i \epsilon I$
$rf_{i,j}$ instant of completion of the system shutdown associated with the j-th execution of the activity i, $j \epsilon J_i$, $i \epsilon I$
$r_{i,j}$ total system downtime associated with the j-th execution of the activity i, $j \epsilon J_i$, $i \epsilon I$
$d_{i,j}$ time out of service associated with the j-th execution of the activity i, $j \epsilon J_i$, $i \epsilon I$

Objective function

$$\min \sum_{i\in I, j\in Ji} d_{ij}, \quad \forall (i, j) \in J. \tag{3}$$

Restrictions

$$F_{i,j} \geq S_{n,o} - M \cdot \left(1 - o_{i,j,n,o}\right), \quad \forall (i, j, n, o) \in K, \tag{4}$$

$$S_{\mathrm{n,o}} \geq F_{i,j} - M \cdot \left(1 - o_{n,o,i,j}\right), \quad \forall (i, j, n, o) \in K, \tag{5}$$

$$o_{i,j,n,o} + o_{n,o,i,j} \leq 1 + w1_{i,j,n,o}, \quad \forall (i, j, n, o) \in N, \tag{6}$$

$$o_{i,j,n,o} + o_{n,o,i,j} \geq 1 + w1_{i,j,n,o}, \quad \forall (i, j, n, o) \in N, \tag{7}$$

$$w1_{i,j,n,o} + w2_{i,j,n,o} \leq w_{i,j,n,o}, \quad \forall (i, j, n, o) \in N, \tag{8}$$

$$w1_{i,j,n,o} + w2_{i,j,n,o} \geq w_{i,j,n,o}, \quad \forall (i, j, n, o) \in N, \tag{9}$$

$$rs_{i,j} \leq rs_{n,o} + M \cdot w1_{i,j,n,o} + M \cdot \left(1 - w2_{i,j,n,o}\right), \quad \forall (i, j, n, o) \in N, \tag{10}$$

$$rs_{i,j} \geq rs_{n,o} - M \cdot w1_{i,j,n,o} - M \cdot \left(1 - w2_{i,j,n,o}\right), \quad \forall (i, j, n, o) \in N, \tag{11}$$

$$rs_{i,j} \leq rs_{n,o} - ep + M \cdot w1_{i,j,n,o} + M \cdot w2_{i,j,n,o} + A1_{i,j,n,o} \cdot M, \quad \forall (i, j, n, o) \in N, \tag{12}$$

$$rs_{i,j} \geq rs_{n,o} + ep - M \cdot w1_{i,j,n,o} - M \cdot w2_{i,j,n,o} - \left(1 - A1_{i,j,n,o}\right) \cdot M, \quad \forall (i, j, n, o) \in N. \tag{13}$$

$$z_{i,j,n,o} \leq S_{i,j} + \left(1 - w_{i,j,n,o}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{14}$$

$$z_{i,j,n,o} \leq S_{n,o} + \left(1 - w_{i,j,n,o}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{15}$$

$$z_{i,j,n,o} \leq w_{i,j,n,o} \cdot M, \quad \forall (i, j, n, o) \in N, \tag{16}$$

$$z_{i,j,n,o} \geq S_{i,j} - \left(1 - w_{i,j,n,o}\right) \cdot M - M \cdot VZ1_{ijno}, \quad \forall (i, j, n, o) \in N, \tag{17}$$

$$z_{i,j,n,o} \geq S_{n,o} - \left(1 - w_{i,j,n,o}\right) \cdot M - M \cdot VZ1_{ijno}, \quad \forall (i, j, n, o) \in N, \tag{18}$$

$$VZ1_{ijno} + VZ2_{ijno} = 1, \quad \forall (i, j, n, o) \in N, \tag{19}$$

$$y_{ijno} \geq F_{ij} - \left(1 - w_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{20}$$

$$y_{ijno} \geq F_{no} - \left(1 - w_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{21}$$

$$y_{ijno} \geq w_{ijno} \cdot M, \quad \forall (i, j, n, o) \in N, \tag{22}$$

$$y_{ijno} \leq F_{ij} + \left(1 - w_{ijno}\right) \cdot M + M \cdot VY1_{ijno}, \quad \forall (i, j, n, o) \in N, \tag{23}$$

$$y_{ijno} \leq F_{no} + \left(1 - w_{ijno}\right) \cdot M + M \cdot VY2_{ijno}, \quad \forall (i, j, n, o) \in N, \tag{24}$$

$$VY1_{ijno} + VY2_{ijno} = 1, \quad \forall (i, j, n, o) \in N, \tag{25}$$

$$rs_{ij} \leq z_{ijno} + \left(1 - w_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{26}$$

$$rs_{no} \leq z_{ijno} + \left(1 - w_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{27}$$

$$rs_{ij} \leq S_{ij} + w_{ijno} \cdot M, \quad \forall (i, j, n, o) \in N, \tag{28}$$

$$rs_{no} \leq S_{ij} + w_{ijno} \cdot M, \quad \forall (i, j, n, o) \in N, \tag{29}$$

$$rs_{ij} \geq z_{ijno} - \left(1 - V1_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{30}$$

$$rs_{no} \geq z_{ijno} - \left(1 - V1_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{31}$$

$$rs_{ij} \geq S_{ij} - \left(1 - Ul_{ij}\right) \cdot M, \quad \forall j \in J_i, \forall i \in I, \tag{32}$$

$$Ul_{ij} + \sum_{(a,b,n,o)\in N} Vl_{ijno} \geq 1, \quad \forall j \in J_i, \forall i \in I, \tag{33}$$

$$rf_{ij} \geq y_{ijno} - \left(1 - w_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{34}$$

$$rf_{no} \geq y_{ijno} - \left(1 - w_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{35}$$

$$rf_{ij} \geq F_{ij} - w_{ijno} \cdot M, \quad \forall (i, j, n, o) \in N, \tag{36}$$

$$rf_{no} \geq F_{ij} - w_{ijno} \cdot M, \quad \forall (i, j, n, o) \in N, \tag{37}$$

$$rf_{ij} \leq y_{ijno} + \left(1 - V_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{38}$$

$$rf_{no} \le y_{ijno} + \left(1 - V_{ijno}\right) \cdot M, \quad \forall (i, j, n, o) \in N, \tag{39}$$

$$rf_{ij} \le F_{ij} + \left(1 - U_{ij}\right) \cdot M, \quad \forall j \in J_i, \forall i \in I, \tag{40}$$

$$U_{ij} + \sum_{(i,j,n,o)\in N} V_{ijno} \ge 1, \quad \forall j \in J_i, \forall i \in I, \tag{41}$$

$$V_{ijno} \le w_{ijno}, \quad \forall (i, j, n, o) \in N, \tag{42}$$

$$r_{ij} = rf_{ij} - rs_{ij}, \quad \forall j \in J_i, \forall i \in I, \tag{43}$$

$$r_{ij} \le M \cdot \left(1 - w_{ijno}\right) + r_{no}, \quad \forall (i, j, n, o) \in N, \tag{44}$$

$$r_{ij} \ge - M \cdot \left(1 - w_{ijno}\right) + r_{no}, \quad \forall (i, j, n, o) \in N, \tag{45}$$

$$d_{ij} \ge r_{ij} - M \cdot \sum_{(i,j,n,o)\in N} w_{ijno}, \quad \forall (i, j, n, o) \in N, \tag{46}$$

$$d_{ij} \le r_{ij} + M \cdot \sum_{(i,j,n,o)\in N} w_{ijno}, \quad \forall j \in J_i, \forall i \in I, \tag{47}$$

$$F_{ij} = S_{ij} + p_i, \quad \forall j \in J_i, \forall i \in I, \tag{48}$$

$$S_{ij} \le rf_{i,j-1} + T_i + T_i \cdot e, \quad \forall j \in J_i, \forall i \in I, \tag{49}$$

$$S_{ij} \ge rf_{i,j-1} + T_i - T_i \cdot e, \quad \forall j \in J_i, \forall i \in I, \tag{50}$$

$$S_{ij} \le rf_{i,j-1} + T_i + M\left(\sum_{(a,b,n,o)\in N} w_{abno} + \sum_{(c,d,n,o)\in N} w_{cdno}\right), \quad \forall j \in J_i, \forall i \in I,\ (a,b) = (c,d) = (i,j), \tag{51}$$

$$S_{ij} \ge rf_{i,j-1} + T_i - M\left(\sum_{(a,b,n,o)\in N} w_{abno} + \sum_{(c,d,n,o)\in N} w_{cdno}\right), \quad \forall j \in J_i, \forall i \in I,\ (a,b) = (c,d) = (i,j), \tag{52}$$

$$F_{ij} \ge rf_{ij} - M \cdot \sum_{(i,j,n,o)\in N} w_{ijno}, \quad \forall j \in J_i, \forall i \in I, \tag{53}$$

$$F_{ij} \le rf_{ij} + M \cdot \sum_{(i,j,n,o)\in N} w_{ijno}, \quad \forall j \in J_i, \forall i \in I, \tag{54}$$

$$S_{ij} \ge rs_{ij} - M \cdot \sum_{(i,j,n,o)\in N} w_{ijno}, \quad \forall j \in J_i, \forall i \in I, \tag{55}$$

$$S_{ij} \le rs_{ij} + M \cdot \sum_{(i,j,n,o)\in N} w_{ijno}, \quad \forall j \in J_i, \forall i \in I, \tag{56}$$

$$rs_{ij} \le M \cdot \left(1 - w_{ijno}\right) + rs_{\mathrm{no}}, \quad \forall j \in J_i, \forall i \in I, \tag{57}$$

$$rs_{ij} \ge rs_{\mathrm{no}} - M \cdot \left(1 - w_{ijno}\right), \quad \forall j \in J_i, \forall i \in I, \tag{58}$$

$$rf_{ij} \le M \cdot \left(1 - w_{ijno}\right) + rf_{\mathrm{no}}, \quad \forall j \in J_i, \forall i \in I, \tag{59}$$

$$rf_{ij} \ge rf_{\mathrm{no}} - M \cdot \left(1 - w_{ijno}\right), \quad \forall j \in J_i, \forall i \in I. \tag{60}$$

Now, regarding the development of the optimization model: the set of constraints (3)–(12) are responsible for satisfying the condition that the grouped activities can begin or end at different moments. Constraints (13)–(24) pretend to formulate the variables z_{ij} and y_{ij}, which take the start and end values, respectively, of each of the activities that are grouped, leaving with a value = 0 for those that are not part of any grouping package, which would be used in the group of constraints. On the other hand, restrictions (25)–(41) are responsible for establishing the start and end moments of each of the maintenance activities, whether they are grouped or not. Specifically (25)–(32), determine the moments of beginning (see Figure 4), while (33)–(41) are responsible for establishing the moments of completion of each activity (see Figure 5).

Constraint (43) is used to generate the variable r_{ij}, which measures the amount of downtime of the system that generates the execution of such execution (44) and (45) determine that, if a grouping between two activities is generated, they must have the same duration in their interval to execute the activities (45) and (46) are used to determine the variable d_{ij}, which delivers the duration of each of the grouping schemes, designating this value only to one of the executions of the grouped activities. In this way, the time out of service of the system can be counted. The restriction (48) is simply responsible for establishing that the time of completion of an activity corresponds to the time of initiation plus the duration of the activity. Restrictions (49) and (50) determine the window of time that each activity must be able to advance or delay its execution in case of forming a grouping package, which depends on the percentage of tolerance and assigned, establishing that tolerance, in terms of time units, is calculated as $e \cdot T_i$. On the other hand, (51) and (52) imply that if a certain execution is not part of some grouping scheme, it must be executed at its tentative moment, which, in turn, is determined by its periodicity. (52)–(55) are responsible for satisfying that, if there is no grouping in a certain execution (i, j), both the individual start and end variables (S_{ij} and F_{ij}) must coincide with the group end start values. In this way, each execution or grouping of them is treated as a group of activities. On the other hand (56)–(59), adjusts the start and end variables of each of the grouping schemes, determining that, if the activity (i, j) is grouped with (n, o) its values rf and rs must be the same.

3.1. Performance Indicators. It is imperative to incorporate the use of performance indicators for the postoptimal analysis and discussion of the results when carrying out the development of this research hand in hand with the computational implementation of the optimization model since it allows to identify the real contribution to the problem raised. Hence, two types of indicators are presented,

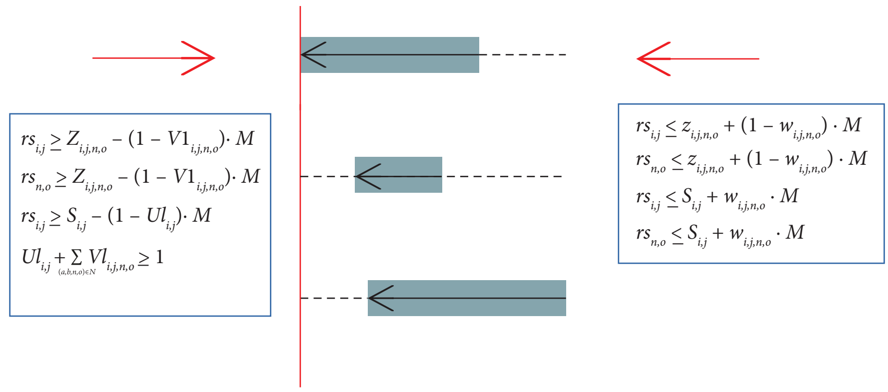


Figure 4: Operation of the constraints associated with the calculation of rs_{ij}.

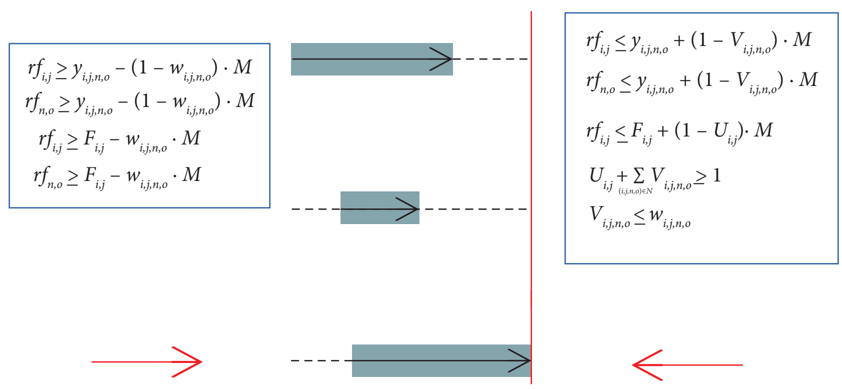


Figure 5: Operation of the constraints associated with the calculation of rf_{ij}.

indicators related to the quality of the grouping scheme (i)–(vi), and the indicators associated with the efficiency of the programming code in terms of the search for feasible solutions (vii), (viii), with a total of 8 indicators. Indicators (ii) to (vi) will allow to know the behaviour of the maintenance activities according to the delay or advance of their moments of execution with respect to the tentative moments. It should be noted that these indicators are extracted from [5].

(i) **Unav**: since this research is an extension to the model proposed by [5], the unavailability of the system in the activity grouping scheme is used as an indicator, which is presented as a direct result of the computational model. This allows to finally make visible the main objective of executing the optimization model, prior to the risk analysis.

(ii) $\mathbf{f_A}$: determines the percentage of activities that are advanced with respect to their tentative moments.

$$f_A = \frac{|IJ^-|}{\sum_{i\in I}|J_i|}. \tag{61}$$

(iii) $\mathbf{f_D}$: determines the percentage of activities that are delayed with respect to their tentative moments.

$$f_D = \frac{|IJ^+|}{\sum_{i\in I}|J_i|}. \tag{62}$$

(iv) $\mathbf{f_J}$: determines the percentage of activities that are executed in their tentative instants.

$$f_J = \frac{|IJ^0|}{\sum_{i\in I}|J_i|}. \tag{63}$$

(v) $\mathbf{A^{AV}}$: indicates the average tolerance usage percentage of those activities that are advanced with respect to their tentative instants.

$$A^{AV} = \frac{\sum_{(i,j)\in IJ^{-0}}\left(\left(t_{\mathrm{exc}_{i,j-1}} + T_i\right) - t_{\mathrm{exc}_{i,j}}\right)/e \cdot T_i}{\sum_{i\in I}|J_i|}. \tag{64}$$

(vi) $\mathbf{D^{AV}}$: indicates the average tolerance usage percentage of those activities that are delayed with respect to their tentative instants.

$$D^{AV} = \frac{\sum_{(i,j)\in IJ^{-0}}\left(t_{\mathrm{exc}_{i,j}} - \left(t_{\mathrm{exc}_{i,j-1}} + T_i\right)\right)/e \cdot T_i}{\sum_{i\in I}|J_i|}. \tag{65}$$

(vii) **Optimality gap:** this indicator refers to the percentage difference between the lower limit and the upper limit of the results obtained when executing the developed optimization model. In general, if the GAP takes values other than 0, it means that the model failed to obtain a point solution, that is, the

Table 2: Input parameters of the maintenance activities involved in the comparative case study.

Act$i$	$p_{i\,[\text{hours}]}$	$T_{i\,[\text{weeks}]}$
1	15	4.2
2	30	6.6
3	40	11.8
4	48	13.2
5	36	5.3

solution will be in a range determined by both limits.

(viii) **Resolution time:** as for the optimality GAP, the resolution times emanate directly from the tool used to solve the optimization model and indicates the time it took to the system to find the solution presented.

Where the sets IJ^-, IJ^+, and IJ^0 are defined as follows:

$$\begin{aligned}
IJ^- &= \left\{(i,j)\colon \text{tent}_{\text{ext}_{i,j}} - \left(\text{tent}_{\text{exc}_{i,j-1}} + T_i\right) < 0,\right.\\
&\forall i \in I, j \in J_i, IJ^+ = \left\{(i,j)\colon \text{tent}_{\text{ext}_{i,j}} - \left(\text{tent}_{\text{exc}_{i,j-1}} + T_i\right) > 0,\right.\\
&\forall i \in I, j \in J_i, IJ^0 = \left\{(i,j)\text{tent}_{\text{ext}_{i,j}} - \left(\text{tent}_{\text{exc}_{i,j-1}} + T_i\right) = 0,\right.\\
&\forall i \in I, j \in J_i.
\end{aligned} \tag{66}$$

4. Results and Discussion

The computational results are obtained from a computer with an Intel Core i5-9300H processor at a speed of 2.40 GHz, with 8 GB of RAM and an operating system built under a 64 bit architecture.

The extension of the optimization model was made using a Python interface, for which the Pyomo optimization model language/tool was used, and the results were obtained through the aforementioned "Gurobi" solver.

The results will be divided into two sections. The first of these will deal with a single case of numerical experimentation with a maximum of 10% tolerance allowed for the formation of grouping schemes, which will be evaluated in both the base optimization model presented in [5] and the one developed in this research, with the aim of comparing the results and experimentally obtaining the benefits of the changes applied to the optimization model. To perform a risk analysis, four different study cases will be presented with a maximum tolerance level of 20%. This will make it possible to know the behaviour in the formation of grouping schemes with greater precision.

4.1. Base Case Comparison. Before executing the model, we give way to the presentation of the numerical experimentation case with which the optimization model is intended to be developed and evaluated. Given this, it is necessary to establish the basic parameters for its operation, which correspond to the number of preventive maintenance activities (PM), their periodicities (T_i), and execution times (p_i). It should be noted that the planning horizon (T) corresponds to 52 weeks, as established in the base case (see Table 2).

First, to determine the impact of the programming algorithm when relaxing the restriction of starting the grouped activities at the same moment, a comparison is made between the results of minimizing the unavailability of the base model and the currently developed one. For this, identical input parameters are assumed for each of the comparative instances, that is, the same execution times, periodicities, and number of activities. The latter is quite relevant, since the number of activities will depend exclusively on the tolerance of each scenario, so it does not affect the results in comparative terms. Given this, the way in which the number of activities is calculated is determined by the following formulation:

$$J_i = \left\{ j \in J_i\colon\ j = 1, 2, \ldots, \left\lfloor \frac{T}{\max\left(p_i,\ i \in I\right) + T_i + T_i \cdot e} \right\rfloor \right\}, \quad \forall i \in I. \tag{67}$$

Equation (67) defines the set J_i, that is, the number of executions of each of the maintenance activities and is the same as that used in [5]. Its operation is given by the division of the planning horizon into as many sections as maintenance activities can be performed for each component or equipment. In this sense, a pessimistic scenario is used, that is, it is assumed that each maintenance cycle is given by the total use of the time windows to delay the activities. In this way, the minimum number of activities that could be executed in the given planning horizon is obtained.

For each model, 10 instances are executed, which differ in the percentage of tolerance that is allowed so that each execution can be advanced or delayed, starting with a value of 0% escalating to 10%. Table 3 shows the results regarding the total unavailability of the system along with their respective variations:

As it can be observed, the results are sensitized with respect to the percentage of tolerance that is allowed for each scheme, which has a resolution of 1%. For each of these instances, 7 different indicators are obtained.

Regarding the Optimality GAP, this time it always acquires values of 0%, which implies that the execution of the optimization model is always yielding a punctual optimum, that is, both the lower and upper limits have the same values. While there are certain differences in the order of 10^{-12} it is considered negligible.

As for the resolution times, up to 8% tolerance relatively low values are obtained, and not greater than 23 seconds; however, when there are tolerances of 9% or 10%, due to the increase in computational complexity and the wider range of possible solutions, times of up to 140 seconds are reached. However, they are considerably low times. It is necessary to establish that the model with which it is compared has notoriously shorter resolution times, reaching in each of the instances values no greater than 2 seconds. This, although it does not imply a major problem when generating results, is explained due to the greater number of restrictions of the proposed model.

Table 3: Comparative results of total unavailability of the system for different tolerance scenarios between both models.

e (%)	Optimality GAP (%)	Resolution time (s)	Base case unav. (weeks)	Base case opt. (%)	Extended case unav. (sem)	Extended case opt. (%)	Difference (%)
0	0	0.5	5.74	—	5.56	3.1	3.1
1	0	0.6	5.35	6.8	5.26	8.5	1.6
2	0	1.3	5.03	12.5	4.86	15.3	2.8
3	0	1.5	4.86	15.3	4.80	16.4	1.0
4	0	3.8	4.77	16.9	4.60	19.8	3.0
5	0	9.3	4.56	20.6	4.13	28.0	7.4
6	0	8.5	3.83	33.4	3.79	34.0	0.6
7	0	21.6	3.70	35.5	3.70	35.5	0.0
8	0	22.7	3.61	37.1	3.61	37.1	0.0
9	0	140.0	3.61	37.1	3.60	37.3	0.2
10	0	92.4	3.55	38.2	3.06	46.7	8.5
				25		28	2.6

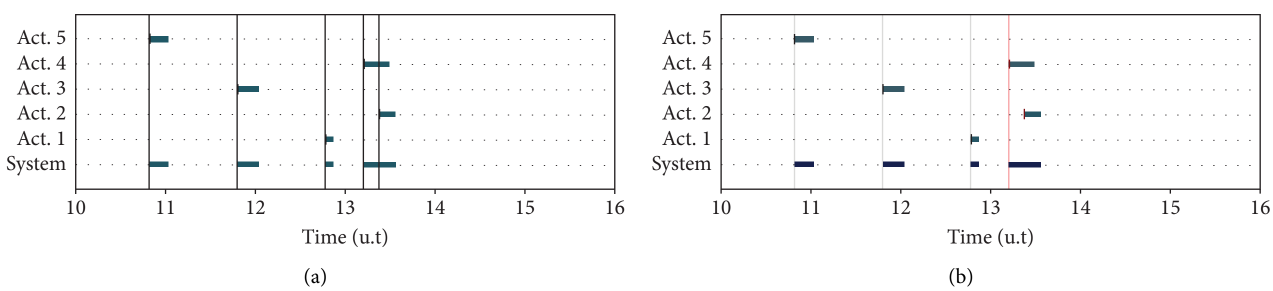


Figure 6: Comparative scheme between both models with $e = 0\%$. (a) Base case. (b) Current case.

Moreover, when analysing the values taken by the objective function in each of the models, that is, the total time (in weeks) of unavailability of the system in a 1year horizon, the optimization model proposed in this research obtains, in the most extreme case, a reduction of 46.7% of the unavailability with respect to the unavailability obtained in the scenario of 0% tolerance with the base model (column 7), in contrast to 38.2% when 10% tolerance is also evaluated in the latter (column 5).

Regarding the graphical results, it is interesting to observe the first scenario for both models, given that there is a variation of 3.1%. In this, since there is no tolerance, the reduction of unavailability of the system is given simply by the fact of having relaxed the restrictions of the optimization model proposed in [5] that limit the start times of every activity that is part of a same group to be equals. This is explained by the fact that the original model cannot count the scenarios where there is overlap in the execution times of the activities as a grouping, thus generating that the periodicities of each of the activities are not respected and a real grouping scenario cannot be chosen. That is, without the need to make use of tolerance, the model can group activities so that the least possible availability is achieved.

In Figure 6, it can be seen that in the executions of activities 2 and 4 there is an overlap; however, the base model does not recognize it. This generates that the next execution of activity 4 has a certain bias when accounting for its periodicity, since, by not recognizing the grouping, the model is assuming that after the activity is executed the associated component immediately returns to operate, which does not turn out to be the case, since activity 2 continues to run. To avoid this bias in the accounting of periodicities, the model currently developed does recognize this as a grouping and both activities schedule the following execution in view of the system resuming at the instant both maintenance activities have been completed.

Furthermore, by correcting the way in which the periodicities of the activities are used would allow for those future ones to be able to form grouping packages with other activities. Moreover, in this same scenario (see Figure 7(b)), the fact of having delayed the subsequent executions of activity 4 generated that the third execution of this activity can generate a grouping package with the sixth execution of activity 2, thus minimizing the times of the unavailability of the system, since now the downtime associated with such executions corresponds to the interval that begins with the execution of activity 4 and ends with the completion of activity 2, in contrast to the base model (see Figure 7(a)) where the time out of service will simply be the sum of both execution times.

This same phenomenon occurs in scenarios where the percentage of tolerance for activities to modify their tentative moments of execution increases, in fact, now not only will the accounting of the periodicities of each activity be corrected (favouring grouping), but the model will be able to advance or delay executions intelligently to obtain the shortest times of unavailability, obtaining even 8.5% less unavailability than in the base model.

On the other hand, analysing only the scenario where the advance or delay of the tentative executions of a

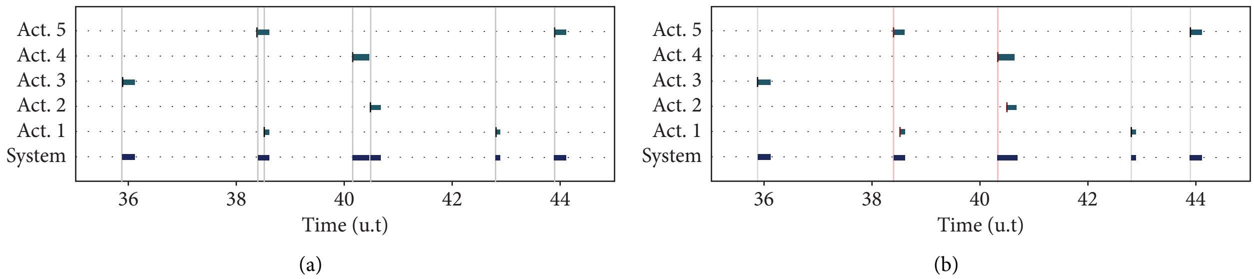


Figure 7: Comparative scheme between both models with $e = 0\%$. (a) Base case. (b) Current case.

maintenance activity is not allowed is not entirely valid, therefore, it is necessary to analyse all the cases reported in Table 2. It can be observed that in all scenarios, from 0% to 6% tolerance there is some degree of difference in the optimization of unavailability. Subsequently, from 7% to 9% there is no difference, to finally, in the case in which the maximum tolerance is defined, generate an 8% optimization above the base case.

These periods in which there is no optimization are related to the periodicity of each activity; since, if the tentative executions of different activities are separated by very large time intervals, it is difficult in all tolerance scenarios to reduce unavailability; hence, the tolerance percentage must be large enough so that their time windows can intersect to form new alternatives to optimize the scheme.

The latter enables the possibility to establish the final results in respect to the comparison between the programming algorithms of both models. For values between 0% and 10% tolerance allowed to modify the tentative moments of execution of maintenance activities, the model developed in the current research generates on average 2.6% extra optimization in each sensitized scenario. However, if only the scenarios where there is indeed a certain degree of improvement were considered, that is, without counting the cases where the tolerance is between the range of 7% to 9%, an optimization of 3.5% is achieved over what is delivered by the base model by tolerance level. Now, in general terms, improvements in the optimization algorithm generate an extra 8.5% in the total unavailability of the system, decreasing it from a value of 5.74 (weeks) to 3.06 (weeks), that is, when activities are allowed to make use of 10% of their tolerances, the unavailability of the system is reduced by 2.68 (weeks), or in percentage terms, 46.7%.

In short, the results presented allow us to recognize the importance of eliminating the bias present in the programming of tasks when there is the possibility of generating grouping due to the overlap of the execution times of the activities. However, it is important to note that in this particular case the results were favourable to be able to generate work packages, such as those shown graphically in Figures 6 and 7, since it is possible, theoretically, to generate a new way of counting the periodicities of the equipment, maintaining or increasing the availability of the system. This is easy to visualize if it were considered that the periodicity of activity number 4 was greater than planned, in this way there would be no possibility of grouping its third execution with the sixth execution of activity 2. Despite this, the fact of determining the precise results will always generate a great added value compared to just delivering an approximation.

Table 4: Parameters associated with the quantification of risk in the case of current experimentation.

Act$i$	C_{cf_i} (μm)	C_{pf_i} (μm)	α	β
1	12	2	7.5	1.8
2	18	3	9.2	1.9
3	30	5	16.4	2.0
4	33	5.5	20.4	1.9
5	15	2.5	8.5	2.2

4.2. Risk Analysis. As mentioned in previous sections, one of the research objectives is to determine which are the optimal tolerance levels based on the risk of all the instances worked by the optimization algorithm. In this sense, the fact of minimizing unavailability and raising awareness regarding the tolerance factor with the developed model is tremendously useful, since it allows to obtain better results and eliminate the existing bias in the base research, however, it is not enough. Given this, it is necessary to obtain the results associated with the risk inherent in the formation of grouping packages, since in this way it would be possible to make coherent decisions based on both variables.

The way in which the risk associated with each individual execution or grouping packages is addressed and quantified is detailed in Section 2, where in general terms it is established that the fact of generating groups of activities implies a risk associated with all scenarios where the system (in series) presents a state of failure. In addition, when a certain execution of some activity is not part of a package of activities to be grouped, it is considered that its cost will be determined exclusively by its probability of failure until its instant of tentative execution.

In order to make visible the risk behaviour associated with each of the grouping schemes, the corresponding analysis will be divided into two parts. The first of these will be an analysis focused on knowing sufficient information to determine the optimal tolerance level in the case of experimentation presented above (see Table 2). Furthermore, with the aim of knowing the behaviour and use of tolerances in general terms, a study will be carried out with 4 different cases of experimentation, which are sensitized from 0% to 25% tolerance, with a resolution of 5%.

Table 5: Number of executions of each of the activities for the different tolerance levels.

e (%)	Act 1	Act 2	Act 3	Act 4	Act 5
0	12	7	4	3	9
1	12	7	4	3	9
2	12	7	4	3	9
3	12	7	4	3	9
4	12	7	4	3	9
5	12	7	4	3	9
6	12	7	4	3	9
7	12	7	4	3	9
8	12	7	4	3	9
9	12	7	4	3	9
10	12	7	4	4	8
11	12	7	4	4	8
12	12	7	4	4	8
13	12	7	4	4	8
14	12	7	4	4	8
15	12	7	4	4	8
16	12	7	4	4	8
17	12	7	4	4	8
18	12	7	4	4	8
19	12	7	4	4	8
20	12	7	4	4	8

Table 6: Number of executions of each of the activities for the different tolerance levels.

e (%)	Cost for attempted executions (μm)	Cost per grouping (μm)	Total cost (μm)	Ind. System (without)
0	230	90	320	5.65
1	214	106	319	5.35
2	177	140	317	4.95
3	173	145	318	4.90
4	91	227	319	4.68
5	83	223	306	4.35
6	83	220	303	4.01
7	78	245	323	3.92
8	69	239	308	3.92
9	82	288	370	3.90
10	54	244	298	3.29
11	54	244	298	2.98
12	39	329	368	2.71
13	39	334	373	2.71
14	39	347	386	2.71
15	39	305	344	2.71
16	39	241	280	2.71
17	39	253	292	2.71
18	39	233	272	2.71
19	39	245	284	2.71
20	39	244	238	2.71

4.2.1. Case Analysis of Base Experimentation. The necessary parameters, as it can be seen in Table 4, are added to the case study to continue with the analysis, incorporating the costs associated with the corrective (C_{cfi}) and preventive intervention (C_{pfi}) and also, since it is considered that the function that best suits the failure behaviour of these is the Weibull distribution, their respective parameters of scale and shape α and β are presented. From the latter, it is possible to obtain the probability of failure associated with each of the programmed executions depending on their actual execution moments (rs_{ij}). In addition, it should be noted that a cost of unavailability of the system (C_d) with a value of 50 (μm), which represents the cost of stopping the system for a unit of time is incorporated.

Now, it is important to note that in the case of the experimentation presented above, a fair comparison is made, where each for tolerance scenario, individually, there is the same amount of activity (J_i) to be executed in the planning horizon. However, in this second part of the analysis, since tolerance cases of different models are not being incorporated, but the different risk values for certain tolerance levels in the same model, the way in which the number of activities per tolerance level is obtained is by testing, that is, find manually and iteratively the number of activities that are achieved to execute within the given planning horizon (see Table 5).

As it can be observed, for all tolerance levels from 0% to 9% there is the same number of executions per activity; however, for scenarios in which the tolerance exceeds this last value, the number of executions is different: one less activity 5 is executed and an extra activity 4 is executed. Now, with the parameters already incorporated in the model, it is possible to calculate the risk associated with each grouping scheme. Given this, the following are the results, in tabular form, of the risk costs sensitized by tolerance level from 0% to 20%, together with the levels of unavailability obtained (see Table 6).

To know the behaviour of each of the costs presented, a line graph is generated where these 3 variables are unified:

From the graphical representation in Figure 8 it can be observed how the total risk associated with each grouping scheme behaves together with its two components, the cost of tentative risk and cost of grouping risk. If the first case is analysed, where a 0% tolerance is contemplated to advance or delay maintenance activities, there is a total cost of 320 (μm), in which 230 (μm) correspond to the tentative risk and 90 (μm) represent the cost of grouping. The fact that the costs of tentative risk are greater than those of grouping makes total sense, given that, since there are not so many cases of grouping, most of the activities are executed in their tentative moments. In addition, it is noteworthy that the costs per grouping mean 28.2% of the total costs, since at that level of tolerance there is a fairly small number of grouped activities, so it is evident the importance, in terms of cost, of generating grouping packages.

However, when the percentage of tolerance allowed increases, each of the variables presented has some variation and trend. In particular, grouping costs increase more than tentative costs decrease, causing total costs to increase due to the formation of grouping packages. This is entirely consistent with the implication of increasing tolerance levels, since theoretically and practically increasing time windows so that activities can modify their execution moments favours the creation of grouping packages. It is observed that both cost components have similar increases and reductions. If certain points are analyzed, it is interesting to analyse the scenario in which the results are sensitized with a tolerance 14%, where the total cost reaches the highest value of the entire series and with a total unavailability of 2.71 (sem). In addition, in terms of the trend generated in this cost function, preferably the total cost tends to stabilize, even more, to go down along with the increase in the level of tolerance, except for certain intervals where the local maximums are located. On the other hand, when the tolerance increases from 3% to 4%, a greater increase is generated than the trend with which the grouping costs increase to that level. In the same way, the costs for tentative executions suffer the greatest drop of the entire series. In short, there is no considerable increase or decrease in total costs, however, it is interesting to note that the origin of this behaviour is due to the fact that the generation of grouping packages at that tolerance level is such that their quantity is allowed and increased more than at any other tolerance level. Furthermore, it is evident that the minimum value of unavailability is reached in the scenario of 12%, maintaining this value up to 20%, which is equivalent to a reduction of 52% with respect to unavailability in the scenario in which there is no tolerance. Now, regarding the risk, in this set of tolerance levels there is a downward trend, which is attributable to the preference of advancing maintenance activities by the optimization model, that is, the greater the permissibility to form grouping schemes, the model will choose to advance activities.

To complement the basis for making a correct decision, it is necessary to know the behaviour of the indicators proposed in Section 3.1. In view of this, each of the 5 indicators regarding the grouping schemes determined by the tolerance level is presented (see Table 7).

As for the first three indicators, which measure the percentage of activities advanced, delayed, and executed in their tentative moments (f_A, f_D, and f_J, respectively), it can be observed that as the level of tolerance increases, the number of activities executed in their tentative instant decreases rapidly, going from a 89% when there is no tolerance to only 23% at the highest level. On the other hand, the activities advanced or delayed increase consecutively with the increase in tolerance between 0% and 5%, and then, at the end of the series, show certain downward trends for the activities advanced and upwards for the delayed activities. Now, in order to visualize the behaviour graphically, the following line graph of these three indicators is presented (see Figure 9).

Figure 9 makes it possible to observe the behaviour that each of these indicators has, due to the increase in the level of tolerance allowed. From 0% to 4% tolerance, categorically there is a decrease in the activities carried out in their tentative moments together with an increase in advanced or delayed activities. From 5% to 11% there is a balanced behaviour between the three indicators without implications between them, given that the percentage of activities advanced or delayed do not have a defined behaviour, but in some tolerance levels there is greater advance than delay, while between 12% and 20% there is an upward trend for advanced activities and a decrease for delayed activities. To analyse the two remaining performance indicators, which represent the percentage of use of the tolerance of advanced activities (A^{AV}) or delayed activities (D^{AV}), a line graph is presented to define their behaviour (see Figure 10).

As can be seen, the percentage of use of the tolerance of the advanced activities represented by the green line presents a higher value than the delayed activities in all scenarios, except for two points, corresponding to 3% and 9% tolerance. The latter will eventually make it possible to identify why costs behave in such a way.

In this sense, if one analyses particularly the case in which the tolerance has a 9%, it can be noted that in the cost function a local maximum is reached, with 370 (μm). In addition, the percentage of delayed activities is 8% higher

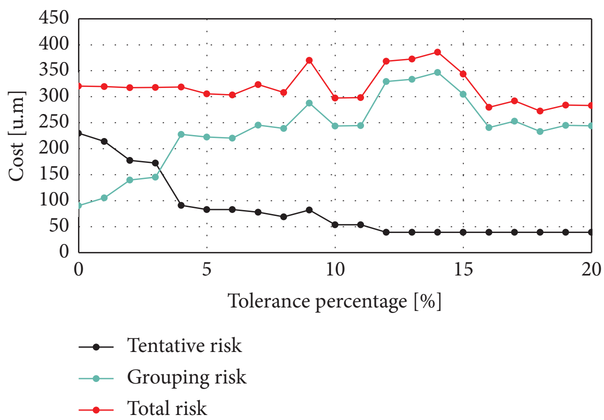


Figure 8: Cost per probability of failure for the different tolerance levels.

Table 7: Variation of the five performance indicators regarding the use of time window tolerances.

e (%)	f_A (%)	f_D (%)	f_J (%)	A^{AV} (%)	D^{AV} (%)
0	11	0	89	—	—
1	20	9	71	42	0
2	17	20	63	18	16
3	14	26	60	12	21
4	23	43	34	47	29
5	40	26	34	46	17
6	37	26	37	32	23
7	37	31	31	30	21
8	31	37	31	32	23
9	29	37	34	18	25
10	34	34	31	36	24
11	31	37	31	31	24
12	43	34	23	41	25
13	43	34	23	39	22
14	43	34	23	38	20
15	43	34	23	36	18
16	51	26	23	39	12
17	51	26	23	41	9
18	51	26	23	42	7
19	57	20	23	42	6
20	71	6	23	43	6

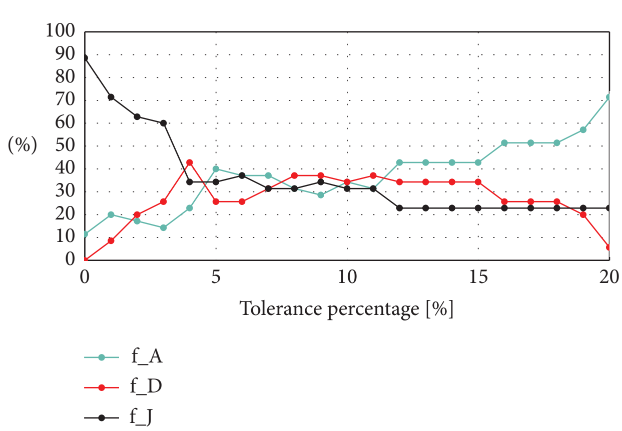


Figure 9: Evolution of the performance indicators of the activities advanced, delayed, or executed in their tentative moments.

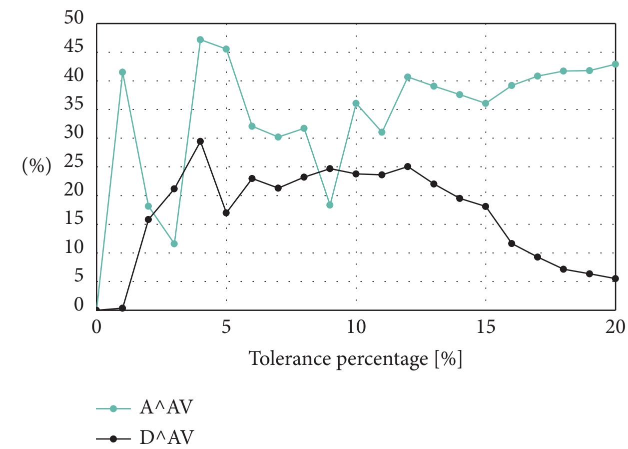


Figure 10: Evolution of the performance indicators of the use of tolerance of delayed or delayed activities.

than those advanced and 3% higher than those executed in their tentative moments. On the other hand, complementing this data with the one exposed in the previous paragraph, it can be explained in greater detail why this increase in costs exists, and it is only due to the fact of decreasing the use of tolerances in advanced activities and, on the other hand, increasing that of delayed activities. This generates that by further delaying the activities, the probability of failure of the components increases, pushing the cost function values to increase.

In short, the risk results are consistent with each of the grouping schemes obtained from the previous optimization of the unavailability of the system. At a higher level of risk, the grouping schemes display a behaviour that favours delaying a greater number of activities, or failing that, using in greater quantity the tolerances of each of these, or, as the cost decreases, a greater number of advanced activities can be observed accompanied by a greater use in the tolerance to advance these activities. However, in order to obtain results with respect to the optimal tolerance level of each grouping scheme, the third risk component is incorporated, which corresponds to the use of the useful life of the components with respect to advance in their execution times. Given this, the results associated with each tolerance level along with their respective graphical representation of this new risk are presented (see Table 8), generated by the sum of costs per grouping, costs for tentative executions, and cost of useful life.

Regarding these results, it can be observed that as the tolerance level increases, the imputed costs per useful life used of each of the components increase, making it clear that the fact of increasing tolerance levels implies an increase in the use of tolerance and therefore, an increase in these useful life costs. Now, with respect to the total costs by risk, that is, if the costs for grouping, tentative executions, and useful life are added, the cost function shown in Figure 11 (green) is obtained, where it is observed that with the increase in tolerance the costs would tend to increase, however, with certain drops, followed by prominent increases.

Table 8: Variation of useful life costs and total costs with respect to tolerance levels.

e (%)	Total cost (μm)	Useful life cost (μm)	Total cost + useful life cost (μm)
0	320	6.6	327
1	319	8.3	328
2	317	7.2	325
3	318	10.1	328
4	319	26.3	345
5	306	56.1	362
6	303	39.5	343
7	323	59.6	383
8	308	65.2	373
9	370	67.0	437
10	298	106.6	404
11	298	104.7	403
12	368	119.2	488
13	373	123.7	496
14	386	127.8	514
15	344	131.2	475
16	280	178.9	459
17	292	196.0	488
18	272	211.7	484
19	284	234.3	518
20	238	222.8	461

Given the above, in order to make a correct decision regarding the optimal tolerance levels, the commented bumps allow to establish which scenario is the most appropriate, however, it will always depend on who makes the decision to choose the tolerance levels. This, given that each decision maker must previously recognize what their objectives and limitations are. For example, if decision makers want to maximize the level of unavailability without sacrificing more than 360 (μm), the most advisable option would be to use a grouping scheme with 6% tolerance. However, if for any reason, it is not technically possible to establish such a tolerance level, it would not be advisable to choose the grouping scheme associated with 5%, since with a level of 4% a relatively similar level of unavailability is obtained without an unjustifiable increase in costs. On the other hand, if the only thing that interests is to minimize the unavailability of the system without cost restrictions, the recommended option would be to use a 16% tolerance. In this way, whenever a decision maker wants to select their appropriate tolerance level, they will have all the necessary information to carry out their choice in the most informed way possible, considering both the costs of failure probability, the costs of useful life, and the unavailability of the system associated with each of the grouping schemes.

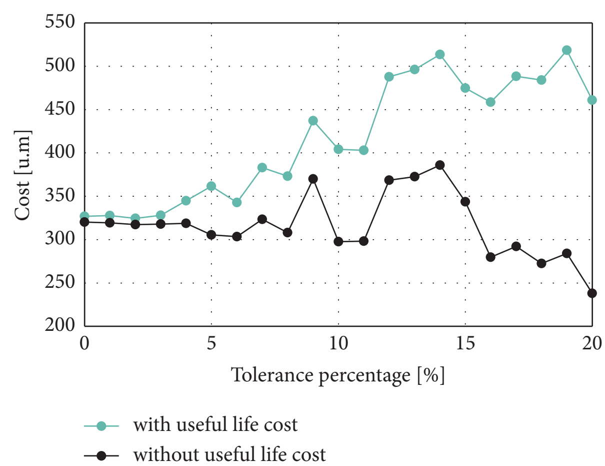


Figure 11: Evolution of total and lifetime costs with respect to each permitted tolerance level.

4.2.2. Analysis for Different Instances. Although the corresponding risk analysis has been performed, the question arises of knowing the behaviour of the results provided by the optimization model in general terms, that is, establishing whether the behaviour of using tolerances and distribution of activities is similar for any case of experimentation or will depend exclusively on the input parameters of the model. Given this, 4 new cases are presented below, which differ from the one presented in the previous section in the periodicities and execution times of each of the activities, in addition to reducing the number of components to 4 with the aim of reducing the computational complexity and resolution times of the optimization model. In this sense, the reason why the number of activities is reduced is due to the fact that in these 4 new cases of experimentation, from 0% to 25% tolerance will be sensitized with a resolution of 5%, in order to know the behaviour of the activities with a higher level of freedom to configure the moments of execution of the PM activities. That is, the number of equipment or components, cost parameters, and failure probability distribution parameters are maintained (see Table 9).

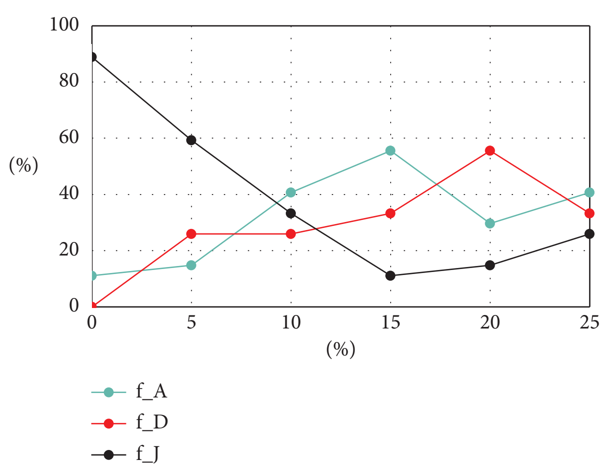
First, the results of unavailability and resolution times for each of the instances analysed are presented (see Table 10). Regarding the resolution times, it can be observed that these increase exponentially as the tolerance level increases, even reaching more than 11 hours in the third case

Table 9: Executions times and periodicities of each PM activity.

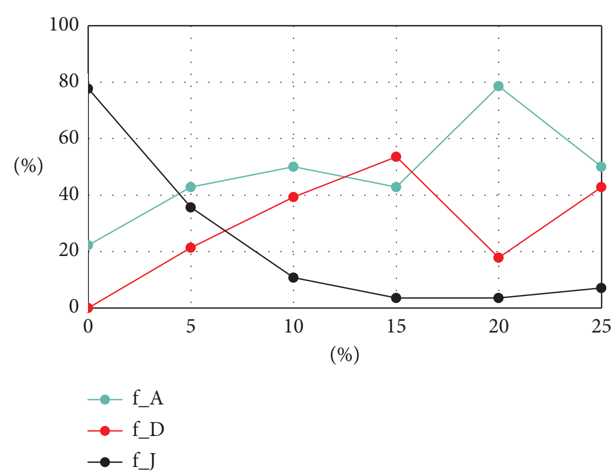
Act	Case 1		Case 2		Case 3		Case 4	
	p_i (hrs)	T_i (wks)	p_i (hrs)	T_i (wks)	p_i (hrs)	T_i (wks)	p_i (hrs)	T_i (wks)
1	15	4	20	5	15	3	15	4
2	30	7	15	6	25	8	20	5
3	40	10	35	9	36	9	30	9
4	48	13	50	10	48	13	48	11

Table 10: Resolution times and unavailability results in order to tolerance factor e.

e (%)	Case 1		Case 2		Case 3		Case 4	
	Res. time (secs)	Unav. (weeks)	Res. time (secs)	Unav. (weeks)	Res. time (secs)	Unav. (weeks)	Res. time (secs)	Unav. (weeks)
0	0.8	4.1	1.0	3.7	1.2	4.1	1.8	4.0
5	3.3	3.4	10.9	3.4	14.3	3.1	5.4	3.4
10	16.8	3.0	49.3	2.4	64.6	2.7	328.6	2.7
15	46.3	2.5	133.0	2.1	1499.5	2.6	808.0	2.2
20	445.7	2.4	1677.2	2.1	13837.9	2.5	2304.5	2.1
25	22850	2.3	22323.2	2.1	40808.8	2.3	18917.7	2.1



(a)



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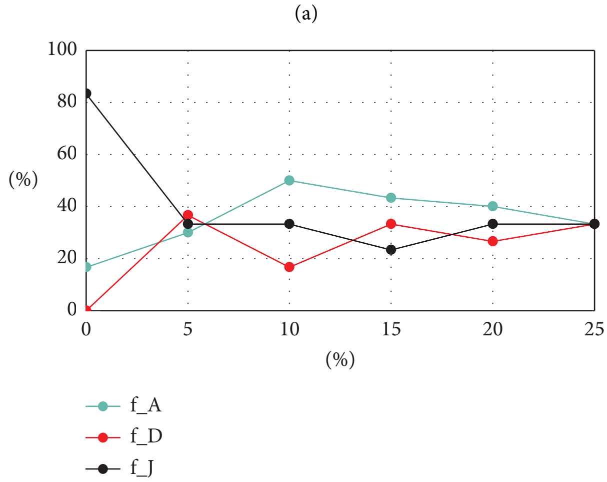

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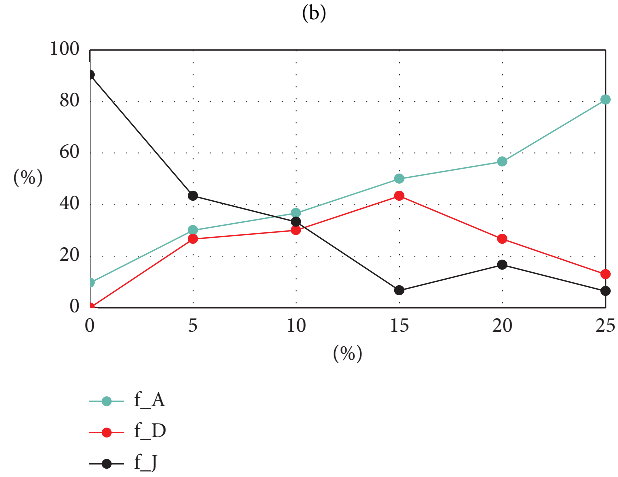

(d)

Figure 12: f_A, f_D and f_J evolution, from 0% to 25% of tolerance factor e. (a) Case study 1. (b) Case study 2. (c) Case study 3. (d) Case study 4.

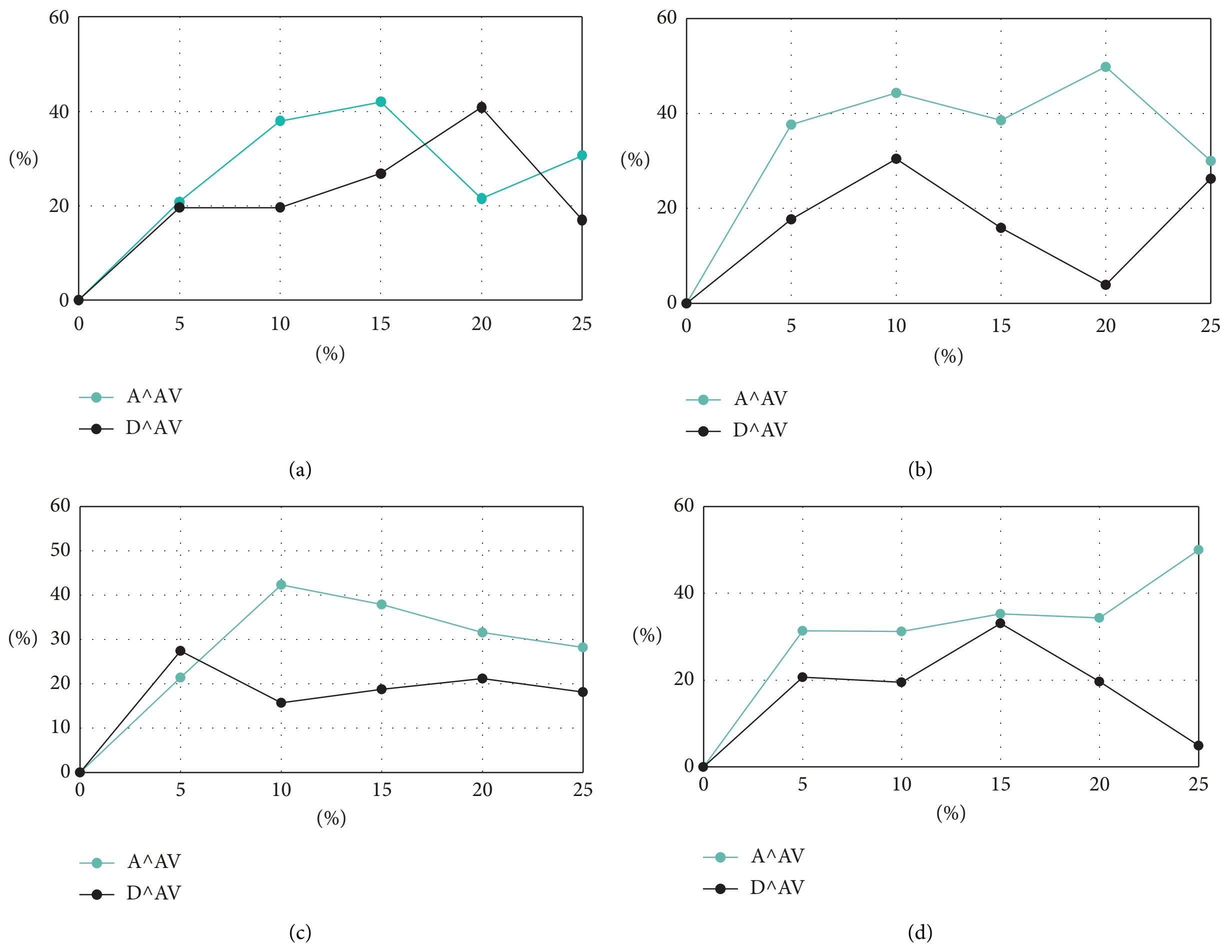

Figure 13: A^{AV} and D^{AV} evolution, from 0% to 25% of tolerance factor e. (a) Case study 1. (b) Case study 2. (c) Case study 3. (d) Case study 4.

of experimentation at 25%. Moreover, regarding the total unavailability of the system in the planning horizon of 52 weeks, it is obtained that at 10% tolerance a reduction of between 28% and 35% is achieved with respect to the scenario in which there is no permissibility for the activities to change their moments of tentative execution, while when the tolerance is 20% the reduction of unavailability varies between 39% and 48%, to finally, in the 25% tolerance, vary between 43% and 48%.

Furthermore, in order to know the behaviour of the number of executions that are advanced, delayed, or executed in their tentative moments, the results associated with the indicators are presented f_A, f_D, and f_J through the following line graphs (see Figure 12). In the first place, it can be observed that as in the case of original experimentation, all new cases show a similar behaviour in terms of the decrease in the activities carried out in their tentative moments together with the increase of those that are delayed or delayed. In the first tolerance range, between 0% and 5%, cases 2, 3, and 4 tend to level the number of executions of each type of activity, with percentages within 20% to 40% each, while case 1 is the only scenario where levelling is reached at 10%. On the other hand, from 5% to 15% tolerance, cases 1 and 3 maintain a certain stabilization, while in cases 2 and 4 the number of activities carried out at their tentative moment tends to decrease considerably along with the increase in delayed or advanced activities. Subsequently, between 15% and 25% tolerance, cases 2 and 4 show a behaviour similar to the base case, where the activities carried out in their tentative moments maintain a downward trend, while the advanced ones tend to increase, which obviously implies a decrease in the delayed ones. It should also be noted that, in general, the activities carried out in advance have a higher proportional percentage than those delayed, implying that the optimization model would tend to privilege the advancement of PM activities for the minimization of total unavailability of the system.

Now, with respect to the indicators A^{AV} and D^{AV}, their evolution within the same tolerance range is shown below (see Figure 13). On this occasion, the behaviour of the indicators is not so similar with respect to the base case in terms of the trend generated when graphing the results; however, it can be seen that in only 2 of the 24 resolved instances the percentage of use of the time windows to delay activities is greater than that of advancing them, which if it is consistent with what is stated in the development of the case of base experimentation; that is, independent of the number of activities that are delayed or advanced, the use of tolerance is greater for advanced activities and increases with the tolerance level.

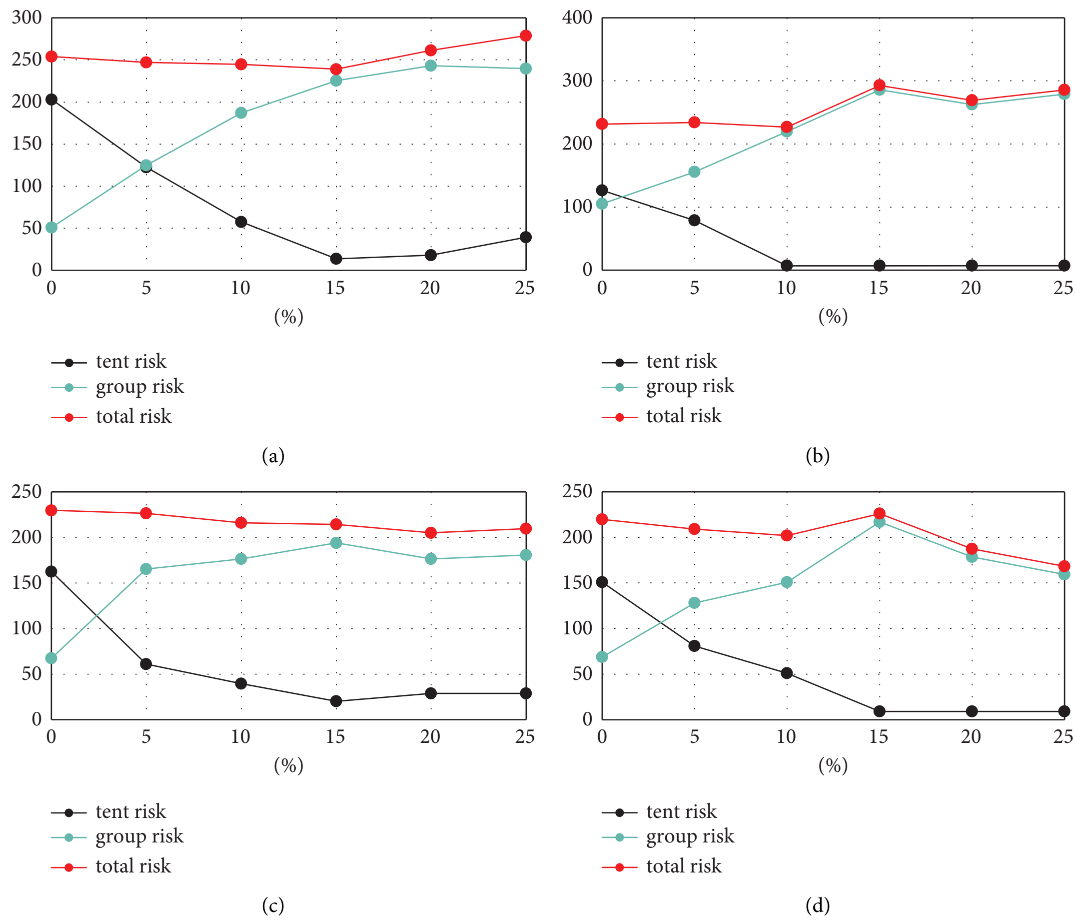

Figure 14: Tentative, grouping and total risk evolution, from 0% to 25% of tolerance factor e. (a) Case study 1. (b) Case study 2. (c) Case study 3. (d) Case study 4.

Finally, in terms of the risk analysis of each of the cases presented, the graphic results of tentative risk, pooling risk, and total risk per instance are shown as follows (see Figure 14). It can be observed that in each of the cases the same behaviour is obtained as in the base case, that is, a prominent increase in the risk by grouping and an abrupt fall in the risk by activities carried out in their tentative moments. On the other hand, in cases 1 and 2, there is an upward trend in risk, while in cases 3 and 4, a decrease, as is obtained in the case of base experimentation. In this sense, the behaviour and trend of total risk, consisting of tentative risk and groping risk, is fully consistent with the results of the performance indicators, however, as in the analysis of the base case, to know the real behaviour of the risk, it is necessary to add the component of risk for useful life, which penalizes the advancement of maintenance activities.

As observed, in each of the cases the trend of the total cost considering useful life is to increase, which establishes and confirms the correct relationship between the fact of penalizing the activities for units of useful life not used of a component. However, it is important to clarify that these results depend exclusively on the cost parameters added to the analysis.

Therefore, it can be observed in Figure 15 that the level of optimal tolerance that emanates from the sensitization of these 4 cases of experimentation does not have a specific value, since it will depend on: the risk that the decision maker intends to assume before the execution of a maintenance plan; the percentage of tolerance in which it is allowed to delay or delay the tentative activities; and what is the value to the that you are satisfied with the optimization of the unavailability of the system. In this sense and given that we are in presence of a risk function that delivers results with local minimums and maximums, the recommendation of the optimal tolerance percentages will vary by sections. On the other hand, since the risk analysis will vary according to the cost parameters of each case of experimentation, the real contribution of this is to generate a practical tool that has the utility of knowing the implications of forming schemes of the grouping of PM activities in terms of the probability of failure of the components, which adapts to different scenarios. However, from the point of view of the optimization model, the results are interesting, since it is found that it tends to advance maintenance activities and to make greater use of tolerance to advance, which is evident in each of the case studies.

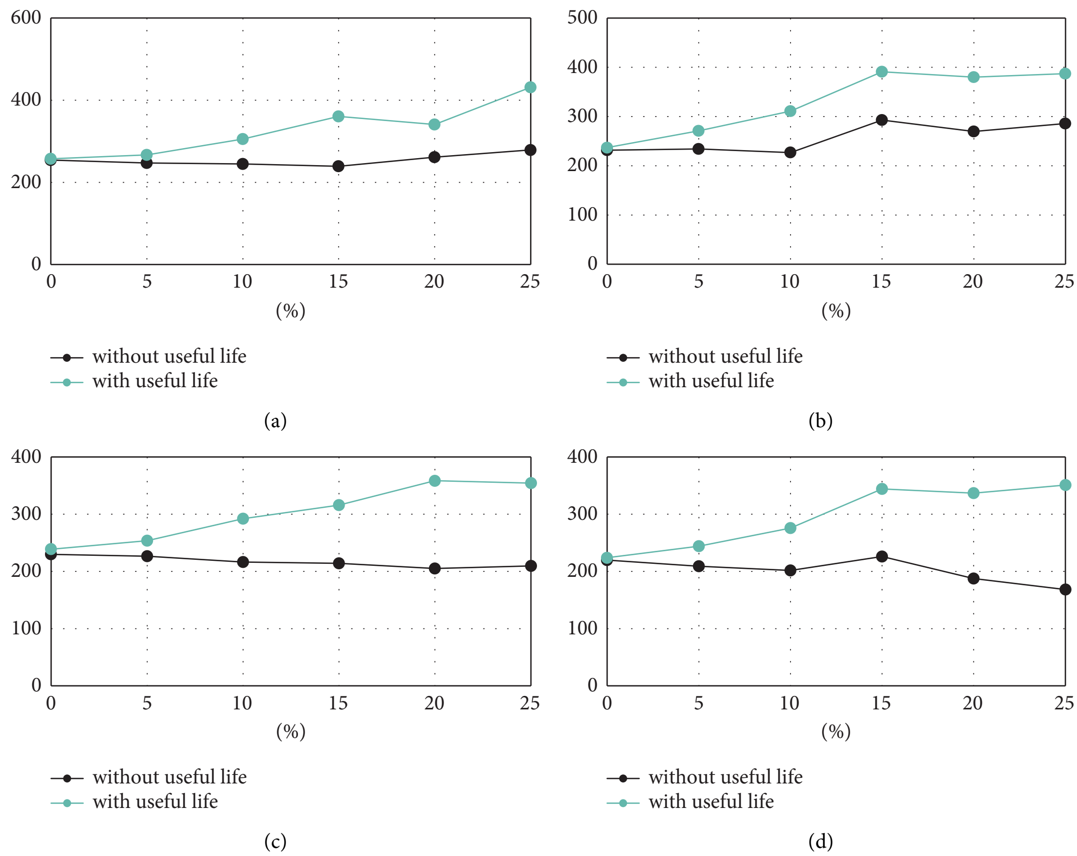

Figure 15: Useful lifetime risk evolution, from 0% to 25% of tolerance factor e. (a) Case study 1. (b) Case study 2. (c) Case study 3. (d) Case study 4.

5. Conclusions

The main objectives of this research are: to be able to provide a decision maker with sufficient tools to choose the best alternative, in terms of risk, within a spectrum of schemes of preventive maintenance activities; and to know the general behaviour of the optimization model for different scenarios. That is why with everything exposed through this article, and in coherence with the objectives set at the beginning, it can be confirmed that these have been fully fulfilled. In specific terms, it has been necessary to meet certain essential milestones, such as the identification of assumptions, criteria, and limitations for the formulation of an optimization model, and the subsequent generation of a tool capable of quantifying risk and behaviour in the formation of grouping schemes.

In terms of results, quite conclusive and interesting behaviours are evidenced with respect to the optimization model developed, particularly in the trend towards the advance in maintenance activities and the formation of new grouping packages that allow to further minimize the times of total unavailability of the system. In contrast with the original framework, the proposed extended framework has achieved:

(1) More flexibility to advance or delay overlapping activities: the original framework forced the start moment of grouped activities and disregarded the importance of overlapping activities.

(2) Risk analysis: the postoptimal analysis along with the proposed performance indicators enables the decision maker to better inform their decisions and have more options available.

Furthermore, although the objectives of the research have been achieved, it is essential to establish that there were certain limitations within the computational formulation of the optimization model. While it is true that the main objective of the research is to establish an optimal tolerance level with respect to the risk represented by a grouping scheme, this analysis is carried out after minimizing the unavailability of the system. Given this, each time the risk level is obtained, it is limited to the fact that the grouping scheme obtained after executing the optimization model disregards the risk associated with the formation of PM activity grouping packages. Therefore, to eventually eliminate the bias present in such a situation, it is advisable to formulate an objective function within the optimization algorithm that incorporates the risk component. For this, one of the best known and most proven methods is that of a multi-objective model, for which it is necessary to establish the weights of each of the variables that are intended to be incorporated, which in this case would correspond to unavailability and the risk of advancing or delaying activities. Another alternative corresponds to that of including within the objective function of the current model, a cost function that involves the risks and costs due to the unavailability of the system, in this way, within the same function both variables would be incorporated. This last recommendation arises after that, in the first instance, it was the way in which it was intended to address the problem, however, the fact of including the risk function within the objective function transformed the nature of the problem, going from being a MILP model to an MINLP model, which corresponds to a mixed integers nonlinear programming. These types of problems have the characteristic of having a high mathematical and computational complexity, so to solve them it is necessary to solve some "solver" capable of handling them since the "Gurobi" that is the one that was used in the research only manages to solve the MILP problems. Given this, using the "Mindtpy" tool, which is able to incorporate a solver of the NP type and another of the MILP type through a degradation algorithm, the problem was addressed, however, the computational complexity of incorporating the Weibull accumulated probability function turned out to be so high that the model proved infeasible. Under this scenario, other tools were sought and unfortunately, those that potentially seemed useful, such is the case of the "Baron" solver. In this context, it is recommended to reduce the computational complexity of the model to incorporate the probability function.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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