

## Research Article

# Exploring the Regional Structure of the Worldwide Air Traffic and Route Networks

Issa Moussa Diop <sup>1</sup>, Cherif Diallo <sup>1</sup>, Chantal Cherifi <sup>2</sup> and Hocine Cherifi <sup>3</sup>

<sup>1</sup>LACCA Lab, Gaston Berger University, Saint-Louis, Senegal

<sup>2</sup>DISP Lab, University of Lyon 2, Lyon, France

<sup>3</sup>LIB EA 7534, University of Burgundy Franche-Comté, Dijon, France

Correspondence should be addressed to Issa Moussa Diop; diop.issa-moussa@ugb.edu.sn

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The topological structure of the world air transportation network has been the subject of much research. However, to better understand the reality of air networks, one can consider the traffic, the number of passengers, or the distance between flights. This paper studies the weighted world air transportation network through the component structure, recently introduced in the network literature, by using the number of flights. The component structure is based on the community or multiple core-periphery structures and splits the network into local and global components. The local components capture the regional flights of these two mesoscopic structures (dense parts). The global components capture the inter-regional flights (links between the dense parts). We perform a comparative analysis of the world air transportation network and its components with their weighted counterparts. Moreover, we explore the strength and the s-core of these networks. Results display fewer local components well delimited and more global components covering the world than the unweighted world air transportation network. Centrality analysis reveals the difference between the top airports with high traffic and the top airports with high degrees. This difference is more pronounced in the global air network and the largest global component. Core analysis shows similitude between the s-core and the k-core for the local and global components, even though the latter includes more airports. For the world air network, the North and Central America-Caribbean airports dominate the s-core, whereas the European airports dominate the k-core.

## 1. Introduction

The concept of a complex network paradigm provides a more profound comprehension of diverse, interconnected systems, including social networks, economics, epidemics, and infrastructure [1]. Infrastructure networks such as power grid networks [2], road networks [3–5], and airport networks [6] receive a lot of attention. The air transport system connects all countries in the world. This infrastructure has a direct impact on society and the global economy. Indeed, millions of people and goods transit through the air every day. The COVID-19 pandemic has significantly impacted the global air transportation network. The virus spread rapidly through air travel, and research has shown the consequences of political shutdowns on national

and international economies. Reference [7] provides further insights into these repercussions. Thus, understanding the air transportation system can help policymakers make decisions that can improve or affect this system. Network science provides a simple way to represent and understand this infrastructure. Thus, several studies are devoted to the air transportation network, including structure, dynamics, and robustness. In a previous study, we analyzed the world air transportation network through a new mesoscopic structure called the component structure [8]. A network contains two types of components. The dense parts of the network form the local components. The interactions between the local components are called the global components. Therefore, one must extract the dense areas to build the component structure. To do so, the community structure

and the core-periphery structure are good candidates. Indeed, the communities constitute cohesive groups of nodes sparsely connected [9–11]. The core-periphery [12–14] structure contains two groups of nodes (core and periphery) with three types of connections. The core nodes are tightly connected. The periphery nodes are almost not connected. The links between the core and periphery nodes are relatively dense. Networks can exhibit a multi-core-periphery structure [15]. One can extract the dense parts using any algorithm to uncover the communities or the various cores to form the local components. In the world air transportation network, the local components capture the regional destinations, while the global components represent the inter-regional flights. In the previous study, we analyzed the unweighted world transportation network. Therefore, the results of our investigations are related to the network infrastructure. Indeed, it uses no information on the dynamics of the infrastructure. This simplification can lead to centrality anomalies [15, 16] and hide critical information about the flow of flights and passengers in the infrastructure. Indeed, the traffic in the various routes can be pretty different, and failing to integrate these differences can lead to misleading conclusions.

This paper presents a comparative analysis of both weighted and unweighted world transportation networks. The unweighted network focuses on the infrastructure, representing various routes connecting airports, while the weighted network considers the passenger traffic on these routes. The analysis is carried out at three levels: macroscopic, mesoscopic, and microscopic.

- (1) *Macroscopic Level.* This level allows us to identify and highlight the global characteristics that differentiate the two types of networks.
- (2) *Mesoscopic Level.* At this level, we adopt a component structure representation to explore and distinguish the influences of geography and economy. Five geographical areas (local components) of the weighted world air transportation network are investigated. Furthermore, we also study the network formed by inter-regional flights. To facilitate our analysis, we examine and compare the  $k$ -core and  $s$ -core of the network.
- (3) *Microscopic Level.* In this level of analysis, we evaluate and contrast the degree and strength of airports, aiming to compare highly connected airports to those experiencing heavy traffic.

By conducting a comprehensive analysis across these three levels, this paper aims to understand better the differences and interactions between the weighted and unweighted world transportation networks.

The rest of the paper is organized as follows. Section 2 reports a review of related studies of the air transportation network. Section 3 describes the data, and Section 4 examines the network community structure. Section 5 reports the analysis of the component structure. Section 6 explores the topological properties of the local and global components. Section 7 presents the topological properties of the

world air transportation network. Section 8 presents the results of a comparative analysis of the strength centrality of the components and the world air transportation network. Section 9 discusses the results of the core structure analysis. Finally, we conclude in Section 10.

## 2. Literature Review

The literature reports numerous studies of unweighted air transportation networks [17–20]. They cover national, regional, and worldwide networks and include investigations at the macroscopic, mesoscopic, and microscopic scales. All these networks share some common characteristics. They are generally small-world and scale-free. Moreover, some exhibit a community structure, and one can observe that the most connected cities do not have the largest betweenness [17].

In contrast, the weighted air transportation network is not much studied. It is particularly true for the world air transportation network. In the following, we briefly report related studies of the national, regional, and worldwide weighted air transportation networks.

In [20], the author studied the weighted airport network of India. The weight indicates the number of weekly flights between two airports. The distribution of strength reveals the heterogeneity of this network. Moreover, the author showed that the strength of a node correlates well with its degree. Comparing nodes' unweighted and weighted clustering coefficients indicates that the hubs tend to form interconnected groups. However, the airport network in India is disassortative. The weighted assortativity shows that the hubs with many flights are more connected.

In [21], the authors investigated the air transportation network of the United States from 2002 to 2005, by quarter. They explored 16 networks in this period using topological and weighted metrics. Cities are the nodes, and routes between two cities are the edges. They considered three types of weight: the nonstop distance between cities, the average passengers, and the average one-way fare. The degree and the strength distribution of these three attributes exhibit a scale-free behavior. Additionally, they are correlated. Then, high-degree nodes tend to have high-strength links. The analysis of the degree-degree correlation shows a rich-club phenomenon. Indeed, there is large traffic among the hubs. Moreover, the interconnected hubs make long distances at an accessible price due to the multiplication of point-to-point flights. Finally, the authors proposed a weighted network model.

In [22], the authors studied the Australian airport network's structure and dynamic flow. The weights correspond to the number of flights between two airports. The authors found that the strengths of the nodes evolve in the same direction as the degree. Sydney airport, the most prominent hub, handles the highest fraction of traffic. The weighted clustering coefficient is lower than the unweighted. Consequently, the edges with low weights form the topological clustering. The weighted and unweighted degree-degree correlation reveals a disassortative behavior.

In [19], the authors analyzed the characteristics of the Asian international passenger aviation market in 2014 and 2018. The number of passengers weights the links between two airports. To conduct their analysis, they considered 28 Asian top airlines, with 7 low-cost carriers and 21 full-service carriers. The airports of the low-cost carriers increased, and their degree distribution shows the evolution of the low-cost carriers into a hub and spoke system. Developing countries like China and Vietnam influence the air transport network of this region. To characterize the airports' influence, they explored eight centrality measures: the degree, mean association (normalized strength), betweenness, weighted betweenness, page rank, weighted page rank, reverse page rank, and weighted reverse page rank. For most centrality measures, Changi Airport, Incheon Airport, Narita Airport, and Hong Kong Airport are in the top position in 2014 and 2018 [19].

In [23], the authors proposed to extend the definitions of the clustering coefficient and the assortativity to a weighted network. They used the world air transportation network as a typical example. The link weight is the number of seats available on the flights between two airports. They showed that the degree and strength distributions are heavy-tailed. Moreover, there is a linear relationship between a node's degree and strength. The network exhibits the rich-club phenomenon. Indeed, the interconnected neighbors of the hubs handle the most significant proportion of the traffic. The weighted degree-degree correlation is assortative.

In summary, previous studies on weighted networks have focused on analyzing national, regional, and global air transportation networks. Notably, when studying the weighted world air network, the analysis has primarily concentrated on macroscopic properties [23]. This paper investigates the weighted world air transportation network in light of its component structure. We perform an extensive comparative analysis of the route (unweighted) and traffic (weighted) networks at the macroscopic, mesoscopic, and microscopic levels.

### 3. Data and Tools

This section describes the data used to conduct our experiment. Then, two weighted community detection algorithms are applied to the data. The purpose is to compare their community structure. Finally, we also compare the communities from the unweighted and weighted world air transportation network.

**3.1. Data.** This network of 2734 nodes and 16665 links originates from FlightAware [24]. It collects the flights between May 17, 2018, and May 22, 2018 [25]. Nodes represent airports, and links are direct flights between airports. The link weight is the number of flights between two airports. Table 1 reports basic properties (unweighted and weighted). One needs at least 12 flights to reach the most distant airports. On average, four flights are required to reach any destination. The network is not very dense, and

globally, there are few triplets by airports with many flights ( $C^w \geq C$ ). The unweighted assortativity reveals that the hubs tend to connect to the airport with few connections. In addition, the global weighted assortativity shows numerous flights between these hubs and these types of airports ( $K^w \geq K$ ).

**3.2. Tools.** Here, we briefly recall the algorithm to extract the component structure. As it relies on the communities to form the local components, we describe two community detection algorithms that uncover these dense parts of the network. Finally, we present the evaluation measures.

**3.2.1. Extracting the Component Structure.** The process of extracting the component structure described in [8] consists of three steps:

- (1) Extracting the dense areas of the network by using a community or multiple core-periphery detection algorithms.
- (2) Extracting the local components by removing all the links between the dense areas.
- (3) Extracting the global components by removing all the links within the dense areas.

Note that a node can belong to the local and global components. In this paper, weighted community detection algorithms are used [26]. Indeed, they take into account the dynamic flow in the network.

**3.2.2. Extracting the Communities.** We use two popular community detection algorithms: Louvain [27] and Combo [28]. They are weighted nonoverlapping community detection algorithms based on modularity. Our purpose is to evaluate the impact of the community structure variations induced by the community detection algorithms on the component structure.

(1) *Louvain.* Louvain constructs the communities in two phases, repeated iteratively. At first, each node is a community. Then, one evaluates the modularity after grouping neighboring nodes. The second step considers the group of nodes (community), maximizing the modularity as new nodes. The algorithm stops when there are no more modifications, and the modularity can no longer be maximized. Note that the definition of modularity depends on the nature of the network (unweighted or weighted).

(2) *Combo.* Combo combines three strategies to uncover the community structure. Indeed, at the initial stage, all nodes form a community. Then, for several iterations, one can merge communities, split communities, or recombine nodes between communities according to the optimization of the objective functions until any gain is possible. One can use two objective functions (modularity and code length). In this paper, we use the algorithm based on modularity.

TABLE 1: Basic topological properties of the world air transportation network.

	$N$	$ E $	$d$	$L$	$\mu$	$\zeta$	$\zeta^w$	$\lambda$	$\lambda^w$	$\eta$
Network	2734	16665	12	3.86	0.004	0.046	0.007	-0.046	0.048	0.09

$N$  is the network size.  $|E|$  is the number of edges.  $d$  is the diameter.  $L$  is the average shortest path length.  $\mu$  is the density.  $\zeta$  and  $\zeta^w$  are, respectively, the unweighted and weighted average clustering coefficients.  $\lambda$  and  $\lambda^w$  represent, respectively, the unweighted and weighted assortativity, also called the degree-degree correlation coefficient.  $\eta$  is the hub dominance.

**3.2.3. Quality Metrics.** We use the classical metrics such as the modularity and the mixing parameter [29–30] to measure the quality of partitions. In addition, we use the NMI to quantify the similarity of the partitions from the Louvain and Combo weighted community structures.

**3.2.4. Modularity.** Modularity is the most popular one. It compares the actual community structure with a null model without community structure. Its values range between  $-1$  and  $1$ . Several community detection algorithms aim to optimize modularity. The best partition is the one that is nearer to  $1$ . The modularity of weighted networks [27] is defined as follows:

$$Q = \frac{1}{2m} \sum_{i,j} \left[ A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j). \quad (1)$$

$A_{ij}$  is the weight of link between  $i$  and  $j$ ;  $k_i$  is the strength of node  $i$ ;  $c_i$  is the community that  $i$  belongs to; the Kronecker delta  $\delta(a, b)$  is  $1$  if  $a = b$  and  $0$  otherwise.  $m = 1/2 \sum_{i,j} A_{ij}$ .

**3.2.5. Mixing Parameter.** For a weighted network, the mixing parameter  $\mu^w$  of a node  $i$  is the proportion of weight pointing outside its community. The mixing parameter of the network is the average of the nodes' mixing parameters. The communities are well separated when the average mixing parameter is near  $0$ . The smaller the mixing parameter, the stronger the community structure. The mixing parameter of a network is defined as follows:

$$\mu^w = \frac{1}{n} \sum \mu_i^w, \quad (2)$$

where  $\mu_i^w = w^{\text{ext}}/w_i$ ,  $w^{\text{ext}}$  is the sum of weight from node  $i$  to the communities that do not contain  $i$ , and  $w_i$  is the strength of node  $i$ .

**3.2.6. Normalized Mutual Information.** The normalized mutual information measures the similarity of two partitions. The partitions are similar when the NMI is near one and almost independent when close to  $0$ . The NMI of two partitions  $P_1$  and  $P_2$  is defined as follows:

$$\text{NMI}(P_1, P_2) = 2 * \frac{I(P_1, P_2)}{H(P_1) + H(P_2)}, \quad (3)$$

where  $I(P_1, P_2)$  is the mutual information between  $P_1$  and  $P_2$ .  $H(P)$  is the entropy of  $P$ . Here, the sum of entropy normalizes the mutual information.

**3.2.7. Jaccard Index.** The Jaccard index is used to compare the similarity of two sets. It is defined as follows:

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|}. \quad (4)$$

When the two sets are identical, the Jaccard index equals  $1$ . It is equal to zero if the two sets have no element in common.

## 4. Community Structure Analysis

This section first compares the weighted community structures uncovered by Louvain and Combo. Then, we compare the communities of the weighted and unweighted world air transportation network uncovered by Louvain. We use the modularity and the mixing parameter as quality measures of the community structures. We also perform a comparative qualitative evaluation highlighting the similarities and differences between weighted and unweighted networks.

**4.1. Comparing the Weighted Community Structure Uncovered by Louvain and Combo.** The number of communities extracted from the networks by the algorithms is quite different. Louvain uncovers  $17$  communities. The largest contains  $725$  airports, whereas the smallest includes two airports. Combo identifies seven communities. The largest includes  $703$  airports, and the smallest consists of  $70$  airports. Table 2 displays the quality metrics of the two community structures. Their modularity is identical. Its value of  $0.47$  indicates that the communities are dense, with a medium proportion of intercommunity links. The community structure of Louvain contains few connections between the communities with strong weights. In contrast, Combo reveals more intercommunity links with smaller weights. In both cases, the mixing parameter values demonstrate that the communities are well separated.

Although the Louvain algorithm uncovers more than two times more communities than the Combo algorithm, their community structures have numerous similarities. Indeed, Combo groups some communities of Louvain. The high value of the NMI ( $0.87$ ) confirms their similarity. Five communities are very similar at first glance. They cover the same geographical areas. In Figure 1, these areas correspond to the communities with the same color. They are in North and Central America-Caribbean, Europe-Russia-Central Asia, East and Southeast Asia-Oceania, Africa-Middle East-Southern Asia, and South America. In addition, Table 3 shows their Jaccard Index. Its value for all similar communities is greater than  $0.85$ . For North and Central

TABLE 2: Modularity, mixing parameter, and NMI of the community structures discovered by the community detection algorithms of Louvain and Combo.

	Modularity	Mixing parameter	NMI
Louvain	0.47	0.043	0.87
Combo	0.47	0.046	

America-Caribbean and Europe-Russia-Central Asia, it is greater than 0.9, indicating a high similarity between the communities uncovered by the two algorithms. The twelve other communities from Louvain, mainly located in Canada and Alaska, are grouped into Combo. To sum up, although the community structures uncovered by Louvain and Combo differ, the largest communities have a lot in common. Therefore, in the rest of the paper, we adopt the communities of Louvain to build the structure of the components used in subsequent analysis.

*4.2. Comparing Weighted and Unweighted Community Structures Uncovered by Louvain.* Louvain uncovered fewer communities in the weighted network (17 communities) than in the unweighted one (27 communities). The weighted network contains five large communities and 12 small communities. In contrast, there are seven large and 20 small communities in the unweighted network. Three correspond to similar areas with the unweighted communities (North America, Africa-Middle East-Southern Asia, and South America). Moreover, their Jaccard index with their unweighted counterpart reported in Table 4 is very high. The two other large communities in the weighted network regroup into two communities four communities in the unweighted network. The Europe-Russia-Central Asia community breaks into Europe and Russia-Central Asia-Transcaucasia communities. In the same way, the community covering the East-Southeast Asia-Oceania region separates into the East-Southeast Asia community and Oceania. Weighted Louvain tends to group nodes linked by heavyweights in the same community. The Jaccard index reported in Table 4 shows that the regrouped weighted communities are very similar to the unweighted communities. These results reveal several flights between Europe and Russia and East-Southeast Asia and Oceania, even if the number of connections between these regions is limited. Apart from grouping communities of the unweighted network, the communities of the weighted network present some particular singularities. Indeed, some airports belong to communities far from their geographical area.

The airports in North and West Africa are in the Europe-Russia-Central Asia community in the weighted network, while they are in the Africa-Middle East-Southern Asia in the unweighted network. Indeed, these airports have more flights to Europe despite sharing more connections to the other airports in Africa and the Middle East. John F Kennedy Airport in the United States is in the Europe-Russia-Central Asia community. Likewise, Frankfurt Airport in Germany and London Heathrow in the United Kingdom are,

respectively, in the Africa-Middle East-India region and the North and Central America-Caribbean area. These examples confirm that the communities of the weighted network correspond to areas of influence, while in the weighted network, they correspond to geographical regions.

The large communities of the weighted network include several small communities of the unweighted network. Moreover, like large communities, a few small communities of the unweighted network are grouped in the weighted network.

The modularity and the mixing parameter values reported in Table 5 show a higher community structure strength for the unweighted network. Nevertheless, in both cases, the community structure strength is in a medium range indicating a clear community structure. The mixing parameter values corroborate these findings.

## 5. Component Structure

We categorize the uncovered components as large or small. The large components include more than 100 airports and cover large geographical areas. In this section, we describe their features and compare them to the component structure of the unweighted network [8].

*5.1. Local Components.* The local components are the dense parts of the networks. They correspond to the 17 communities uncovered by Louvain in the weighted network. There are five large and twelve small local components. The large local components do not reflect strict geographical divisions. They correspond more to political, cultural, historical, and economic divides. For example, some African airports in Morocco, Tunisia, and West Africa belong to the European component. It is because of the solid economic and historical ties these countries share with Europe. The small local components are in a single country (the United States, Canada, French Polynesia, Greenland, Israel, Australia, and United Arab Emirates) or a few countries (Caribbean).

*5.1.1. Large Local Components.* The large local components cover (1) North and Central America-Caribbean (725 airports), (2) Europe-Russia-Central Asia (683 airports), (3) East-Southeast Asia-Oceania (630 airports), (4) Africa-Middle East-Southern Asia (313 airports), and (5) South America (201 airports). Altogether, they regroup more than 93% of the world's airports. One can distinguish two typical behaviors when comparing the large local components of the weighted and unweighted network illustrated in Figure 2. In the first case, the components are very similar. In the second case, separated components in the unweighted network merge into a single component.

There are three similar components (North and Central America-Caribbean, Africa-Middle East-Southern Asia, and South America). We quantify their similarity using the Jaccard index. The higher the similarity is, the closer the Jaccard index is to 1.

The North and Central America-Caribbean weighted component contains 10% more airports than the unweighted. Their Jaccard index is high (0.81). Note, however,

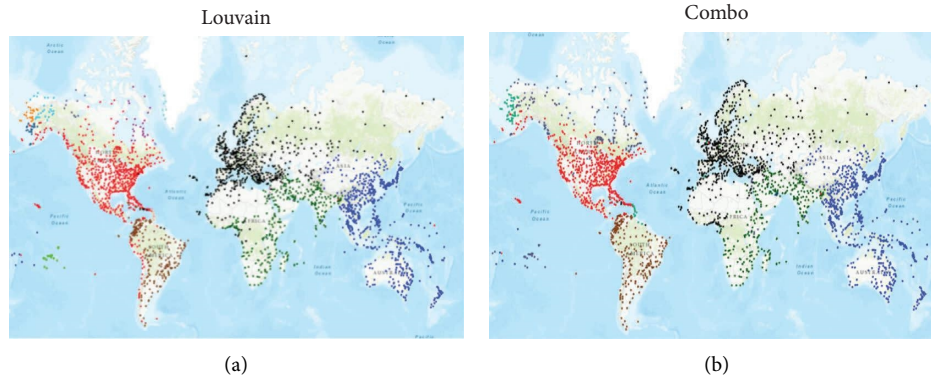


FIGURE 1: (a) The communities the Louvain community detection algorithm identifies. It includes eighteen communities. (b) The communities uncovered by the Combo community detection algorithm. It contains seven communities. For both, each color represents a community. Similar communities have similar colors. One can observe that the Combo algorithm regroupes the communities of the Louvain algorithm located in Canada and Alaska.

TABLE 3: The Jaccard index of the five similar communities uncovered by Louvain and Combo.

Community	Jaccard index
North and Central America-Caribbean	0.94
Europe-Russia-Central Asia	0.98
East and Southeast Asia-Oceania	0.85
Africa-Middle East-Southern Asia	0.88
South America	0.85

TABLE 4: The Jaccard index of the communities uncovered by the weighted and unweighted Louvain algorithm.

Community	Jaccard index
North and Central America-Caribbean	0.81
Europe-Russia-Central Asia	0.87
East and Southeast Asia-Oceania	0.97
Africa-Middle East-Southern Asia	0.87
South America	0.78

Three are similar (North and Central America-Caribbean, Africa-Middle East-Southern Asia, and South America), and two are regrouped by the weighted algorithm. The East and Southeast Asia-Oceania regroupes the East and Southeast Asia component and the Oceania component. Europe-Russia-Central Asia regroupes the Europe component and the Russia-Central Asia component.

that 39 airports of the unweighted component disappear in the weighted component. They are mainly in French Antilles and Venezuela. More surprisingly, John F Kennedy airport is not in the weighted component. Additionally, some new airports emerge in the weighted component (107). Most are in Canada, Alaska, Peru, and Chile. London Heathrow, the most important airport in the United Kingdom, also appears in the weighted component. Indeed, it has several destinations in the United States, and numerous flights from North America land at this airport. The most unexpected airports in this component are the Marshall Airport in the Marshall Islands and the Osmani Airport in Bangladesh. The first collaborates with United Airlines situated in the

TABLE 5: Quality metrics of the community structures uncovered by the unweighted and weighted Louvain community detection algorithms: modularity, mixing parameter, and NMI.

	Modularity	Mixing parameter	NMI
Weighted	0.47	0.14	0.78
Unweighted	0.63	0.13	

United States. The second has flights to and from London Heathrow.

The Jaccard index of the unweighted (336 airports) and weighted (313 airports) Africa-Middle East-Southern Asia component is also high (0.87). The 35 airports of the unweighted component that disappear in the weighted are in the Middle East and West Africa. In contrast, eleven new airports emerged (6 are in Kenya, 2 in West Africa, and 3 in Europe) in the weighted component. The Frankfurt am Main Airport, the largest airport in Germany, is unexpected in this component. Indeed, it has high traffic with Saudi Arabia and India. The Rzeszów-Jasionka airport in Poland and Araxos airport in Greece are also in this component. Both airports have dense traffic with Frankfurt am Main.

The Jaccard index of the unweighted (215 airports) and weighted (201 airports) South America components is 0.78. Thirty-two airports in this unweighted disappeared in the weighted. These airports are in Chile and Peru. As aforementioned, the airports of these two countries are now in the weighted North and Central America-Caribbean component. The airports appearing in the weighted South America component are in Venezuela, Colombia, and Cuba.

There are two merged components. The Europe-Russia-Central Asia component regroupes the European and Russia-Central Asia-Transcaucasia components from the unweighted world air transportation network. Similarly, the “East and Southeast Asia-Oceania” component includes the unweighted network’s East and Southeast Asia and Oceania components. We join the unweighted European (493 airports) and Russia-Central Asia-Transcaucasia (112 airports) components and the unweighted East and Southeast Asia



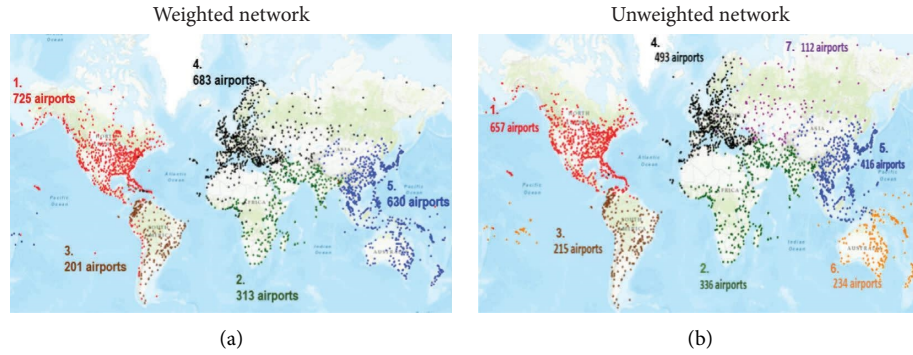


FIGURE 2: (a) The airports in the large components of the weighted network. (b) The airports in the large components of the unweighted network. Each color is associated with a component. The North and Central America-Caribbean component is red (1). The Africa-Middle East-India component is green (2). The South American component is brown (3). The Europe-Russia-Central Asia component is black (4). The Europe-Russia-Central Asia component is black (4). It is divided into Europe in black (4) and Russia in purple (7) in the unweighted network. The East and South-East Asia-Oceania component (5) is divided into East and South-East Asia in blue (5) and Oceania in orange (6) in the unweighted network.

(416 airports) and Oceania (234 airports) components to compare them with their counterparts in the weighted network.

The Jaccard index of the European-Russia component is 0.87. Nine airports present in the unweighted disappear, while 87 new airports appear in the weighted component. They are mainly in Norway, West and North Africa, Iran, the United Arab Emirates, and the French Antilles. Beyond their geographical localization, the countries of this weighted component have political, historical, and economic relations. Indeed, from the airports in Moscow and St. Petersburg, the Russian airports join the other world regions throughout Europe. North Africa, West Africa, and the French Antilles are associated with Europe politically and historically. It translates into high traffic between these regions.

The unweighted and weighted East and Southeast Asia-Oceania components are very similar. Indeed, their Jaccard index equals 0.97. These two regions, through the largest airports, have significant traffic. In the weighted component, twenty-three airports in French Polynesia disappeared, and three airports from Russia and India emerged. Vladivostok and Yuzhno-Sakhalinsk airports are near China and North Korea. Their exchanges are with China, Japan, and South Korea. The Trichy airport in India has heavy traffic with Malaysia, Singapore, and Sri Lanka.

**5.1.2. Small Local Components.** Figure 3 presents the small local components uncovered in the weighted (Figure 3(a)) and unweighted (Figure 3(b)) network for comparative purposes. They are either in a single country (the United States, Canada, French Polynesia, Greenland, Israel, Australia, and United Arab Emirates) or cover a few countries or subregions (Caribbean). The biggest small component in Alaska contains 30 airports. The smallest includes two airports. In the following, we concentrate on small components with a size greater than five airports.

Alaska has three small components with comparable sizes (around 27 airports). These components have a star shape. Indeed, most of the traffic goes through a leading

airport. Fairbanks Airport, in Northeast Alaska, leads the first component. The Nome Airport dominates the Northwest. The third component in the Southwest is centered around the Bethel Airport. These components are also in the unweighted world air transportation network. It reveals the isolation of Alaska in terms of traffic and connections.

Canada possesses two small components. The largest contains 25 airports, mainly on Nunavut and Quebec coasts. It is dominated by Iqaluit and Quujuaq airports. The second includes ten airports in the Northwest Territories. The Inuvik airport serves several flights in this component. Note that the unweighted network merges these two components.

The significant small component in Greenland contains ten airports. The most frequent flights operate through Godthaab/Nuuk, Kangerlussuaq, Ilulissat, and Sisimiut airports. This component is in the unweighted network. Indeed, in Greenland, air transport is a must.

Twenty-two airports in French Polynesia constitute a small local component. This country is a set of islands. Thus, air transportation is very developed. The major airport of this country, Faa'a airport, reaches 18 airports with 547 flights. The Bora Bora airport is the second most important, with 302 flights. This component is not in the unweighted network. So, the flow of flights between the airports of French Polynesia is essential, even though the connections are not dense.

The last significant small component includes French Antilles, Quebec, and Ontario airports. Indeed, there is a strong community from French Antilles based in these two cities of Canada. Therefore, there is high traffic between these airports. The major airports in Trinidad and Tobago (Piarco Airport and Tobago-Crown Point Airport) capture most of the traffic. This component also exists in the unweighted network.

The 13 small unweighted components that disappear in the weighted network are aggregated into the weighted large components. Among them, four are located in North America, five are located in Europe, and four in Africa.

Globally, the small weighted local components cover 6.6% of airports. They are in North and Central

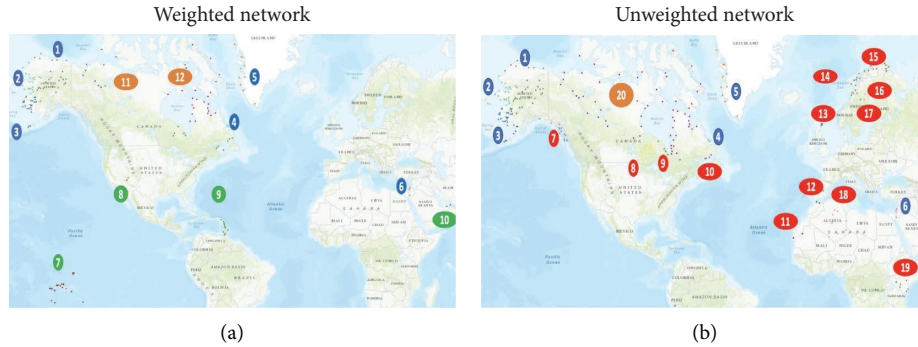


FIGURE 3: Small local components in the weighted (a) and unweighted (b) world air transportation network. The red circles are the components disappearing in the weighted network. The blue circles are similar small components in both networks. The green circles are the components that do not appear in the unweighted network. The orange circles are the components splitting in the weighted network. Geographical areas outside the figure do not contain small components.

America-Caribbean (3 in Alaska and 2 in Canada and 1 in the Caribbean), Europe (1 in Greenland and 1 in Israel), East and Southeast Asia-Oceania (1 in French Polynesia and 1 in Australia), and Africa-Middle East-Southern Asia (1 in the United Arab Emirates). Among them, five components include less than five airports.

**5.2. Global Components.** Figure 4 shows the global components extracted from the weighted world air transportation network. There are one large and 11 small global components. The large global component contains 557 airports (20.44% of the world's airports). It covers the world. The small global components, which include 36 airports, are principally in the North and Central America-Caribbean region.

The Jaccard index between the weighted and unweighted large global components is not high (0.63). The large global component of the weighted world air network contains 14 more airports than the unweighted. However, their content is very different. Indeed, 100 airports disappear from the weighted largest global component, and 144 new airports integrate it. Airports disappear in the largest weighted global component because Oceania and Russia merged with their neighboring regions reducing the intercomponent links. New airports appear because of the singularities in the North and Central America-Caribbean, Europe-Russia-Central Asia, and Africa-Middle East-India components. Indeed, highly connected airports not localized in their natural geographical components, such as London Heathrow and John F Kennedy, increase the links between the local components attracting new airports in the global component. Moving an airport from one local component to another modifies the large connected component drastically.

The small global components include 36 North and Central America-Caribbean airports and East and Southeast Asia-Oceania. Their size ranges from 4 airports to 2 airports. Canada contains most of them (11). Only two components are shared with the unweighted world air transportation network. In the following, we neglect these components.

## 6. Topological Properties of the Large Components

This section investigates the clustering coefficient and degree-degree correlation. We recall the definitions of these topological properties in weighted networks.

**6.1. Clustering Coefficient.** The clustering coefficient  $C_i$  of a node  $i$  reflects the cohesiveness of its neighbors. The closer it is to 1, the more interconnected its neighbors are. Whether  $C(k) \approx k^{-1}$ , the network has a hierarchical organization [32]. The weighted clustering coefficient  $C^w$  of a node  $i$  is defined as follows [23]:

$$C_i^w = \frac{1}{s_i(k_i - 1)} \sum_{j,h} \frac{(w_{ij} + w_{ih})}{2} a_{ij} a_{ih} a_{ij} j h. \quad (5)$$

One can define  $C(k)$  and  $C^w(k)$  as the average clustering coefficient of nodes with degree  $k$  for, respectively, unweighted and weighted networks. The relation  $C^w > C$  indicates that the interconnected triplets tend to be formed by links with high weights. The opposite  $C^w < C$  shows that lower-weight edges produce interconnected triplets.

**6.1.1. Large Local Components.** Figure 5 represents the average (weighted and unweighted) clustering coefficient versus degree for the large local components.

The average unweighted clustering coefficient decreases monotonically with the degree. Low-degree nodes have a large clustering coefficient. In contrast, hubs are less cohesive. This characteristic is shared with numerous networks such as the India air transport network [20], the actor-network, and the World Wide Web network. Fitting  $C(k)$  by the law  $k^{-\gamma}$ , we observe that  $\gamma \approx 0.3$  for the Africa-Middle East-Southern Asia and the South America components. For the other components,  $\gamma \approx 0.2$ .

In contrast, the average weighted clustering coefficient is independent of the degree. This result differs from previous analysis with real-world networks [20, 23]. Moreover, weighted average clustering coefficients are always lower





FIGURE 4: (a) The airports in the large global component. They are distributed all over the world. (b) The 11 small global components are circled. Their size ranges between 2 and 3 airports. Most of them are located in North America.

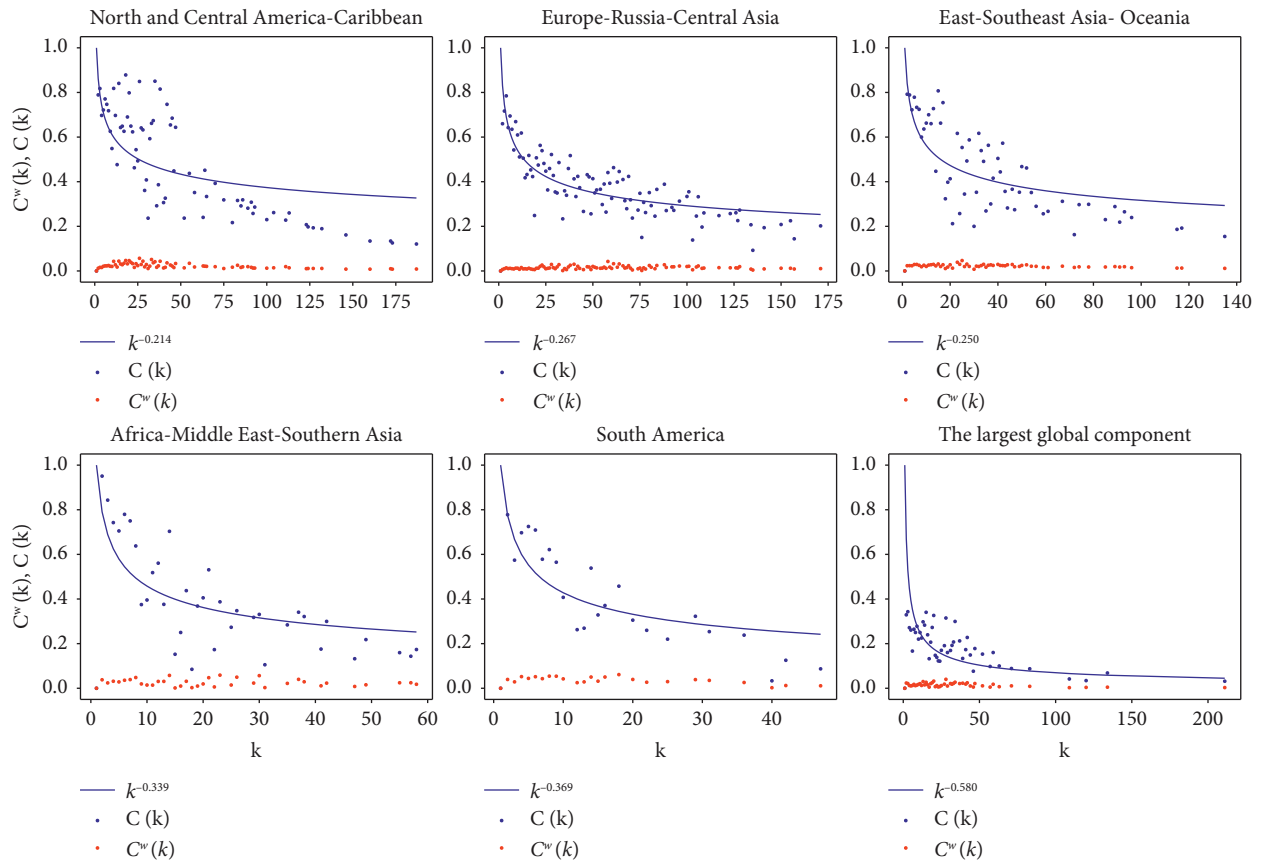


FIGURE 5: Average clustering coefficient versus degree for the large components. The blue dots represent the average clustering coefficient for the unweighted network. The red dots represent the average clustering coefficient for the weighted network. The blue curve is the estimated  $k^{-\beta}$  function. Weighted average clustering coefficients are lower than their unweighted equivalent. The interconnected triplets tend to be formed by low-weight edges in the large local components.

than their unweighted equivalent. It shows that interconnected triplets tend to be formed by edges with low weight in the large local components.

**6.1.2. Large Global Component.** Figure 5 reports the evolution of the average clustering coefficient versus the degree of the large global components. Overall, its behavior is similar to the local components. For the unweighted network, the coefficient of the fitted power law function is

$\gamma \approx 0.58$ . Consequently, there are fewer interconnected triplets in the global component compared to the local ones. Indeed, the global component includes long-distance traffic airports. Among these airports, many are hubs in their country in a hub and spoke configuration with poor rerouting capacity. Like large local components, the unweighted average clustering coefficient of a degree node  $k$  is greater than the weighted average clustering coefficient. Thus, the connected triplets are mainly between nodes with low-weight edges.

**6.2. Degree-Degree Correlation.** The degree-degree correlation  $k_{nn}(k)$  assess the relation between neighbor nodes. It measures the probability that a node of degree  $k$  connects with a node of degree  $k'$ . If high-degree nodes tend to connect to high-degree nodes, the network is said assortative. It is disassortative if high-degree nodes tend to connect to low-degree nodes. The weighted degree-degree correlation is defined as follows [23]:

$$k_{nn,i}^w = \frac{1}{s_i} \sum_{j=1}^N a_{ij} w_{ij} k_j. \quad (6)$$

One can compare weighted ( $k^w$ ) and unweighted ( $k_{nn,i}$ ) degree-degree correlation. If  $k^w > k_{nn,i}$ , edges with high weights tend to connect high-degree nodes. If  $k^w < k_{nn,i}$  the edge weights connect the degree nodes with opposite magnitudes (low or high).

**6.2.1. Large Local Components.** Figure 6 presents the average degree-degree correlation evolution versus the degree for the large local components. One can observe that the weighted degree-degree correlation is greater than the unweighted degree-degree correlation. It indicates a high fraction of traffic transit between hubs in the local components.

However, the curve trend of the unweighted and weighted degree-degree correlation as a function of  $k$  is different for the large local components. Indeed, above a threshold  $k$ , the weighted and unweighted degree-degree correlation of the North-Central America-Caribbean ( $k \approx 70$ ), and South America ( $k \approx 25$ ), decreases. The high-degree nodes tend to connect to low-degree nodes. Thus, the disassortative aspect of these unweighted and weighted components appears clearly. The Europe-Russia-Central Asia component's degree-degree correlation decreases slowly compared to the South America and North-Central America-Caribbean components. Moreover, the degree-degree correlation of the high-degree nodes is higher. It is also less disassortative. In the East and Southeast Asia-Oceania and the Africa-Middle East-Southern Asia components, the unweighted degree-degree correlation shows a characteristic that is slightly disassortative. The average weighted degree-degree correlation of these components is relatively constant.

**6.2.2. Large Global Component.** Figure 6 reports the average degree-degree correlation as a function of degree for the global component. The unweighted network curve decreases. Therefore, the global component is disassortative. Indeed, the largest hubs ( $k > 100$ ) connect with low-degree nodes ( $k < 20$ ). The weighted network curve exhibits a similar evolution. However,  $K^w(k) > K(k)$  for most of degree node  $k$ . Therefore, even though the hubs connect with small-degree nodes, they accumulate more flights.

### 6.3. Strength Distribution

**6.3.1. Large Local Components.** Figure 7 represents the strength distributions of the large local components. We

perform a goodness-of-fit evaluation with the Kolmogorov-Smirnov test (KS) using the power law, truncated power law, log-normal, and stretched exponential distributions. Results reported in Table 6 reveal that the log-normal distribution better fits the large local components.

Moreover, they are heavy-tailed, like the degree distribution of the unweighted large local components of the world air transportation network. One can expect this result. Indeed, the higher the node degree, the higher its weight. Figure 8 shows this behavior. One can see the relation between the strength and the degree for each large local component. Indeed, the average strength and the degree are linked by the relation ( $s(k) = k^\beta$ ). Even though for all the large local components,  $\beta$  ranges between 2.1 and 2.3, this organization is similar to results reported in [20] where  $\beta \approx 1.43$ .

One can differentiate three types of large local components if we focus on the exponent of the strength versus degree curve ( $\beta$ ). The Europe-Russia-Central Asia component forms the first category. It includes several hubs with almost the same traffic. Indeed, the traffic increases slowly as a function of degree. The second category consists of the North-Central America-Caribbean and the East-Southeast Asia-Oceania components. These components also contain several hubs. However, the traffic concentrates in a few hubs. The Africa-Middle East-Southern Asia and the South America components are in the last category. There are fewer hubs and less traffic compared to the other types. The strength as a function of degree increases faster. Consequently, few hubs accumulate most of the traffic.

**6.3.2. Large Global Component.** Figures 7 and 8 show the strength distribution and the strength as a function of  $k$  of the large global component. Like the large local components, the log-normal law better approximates its strength distribution according to the KS test. One can also see that the strength distribution is heavy-tailed. In addition, the distribution parameters are similar to those of the East-Southeast Asia-Oceania component. As found with the degree-degree correlation, the degree's strength shows that the higher the node degree is, the more traffic it accumulates. Airports with fewer connections have at least more than 100 flights. It is not the case in the large local components. The  $\beta$  exponent is comparable to those of North and Central America-Caribbean and the East-Southeast Asia-Oceania components.

## 7. Topological Properties of the World Air Transportation

This section investigates the topological properties of the world air transportation network. We also perform a comparative analysis with the large components.

**7.1. Clustering Coefficient.** Figure 9(a) presents the average clustering coefficient as a function of degree  $k$  for the weighted and unweighted world air transportation network. Similar to the large components, the unweighted clustering

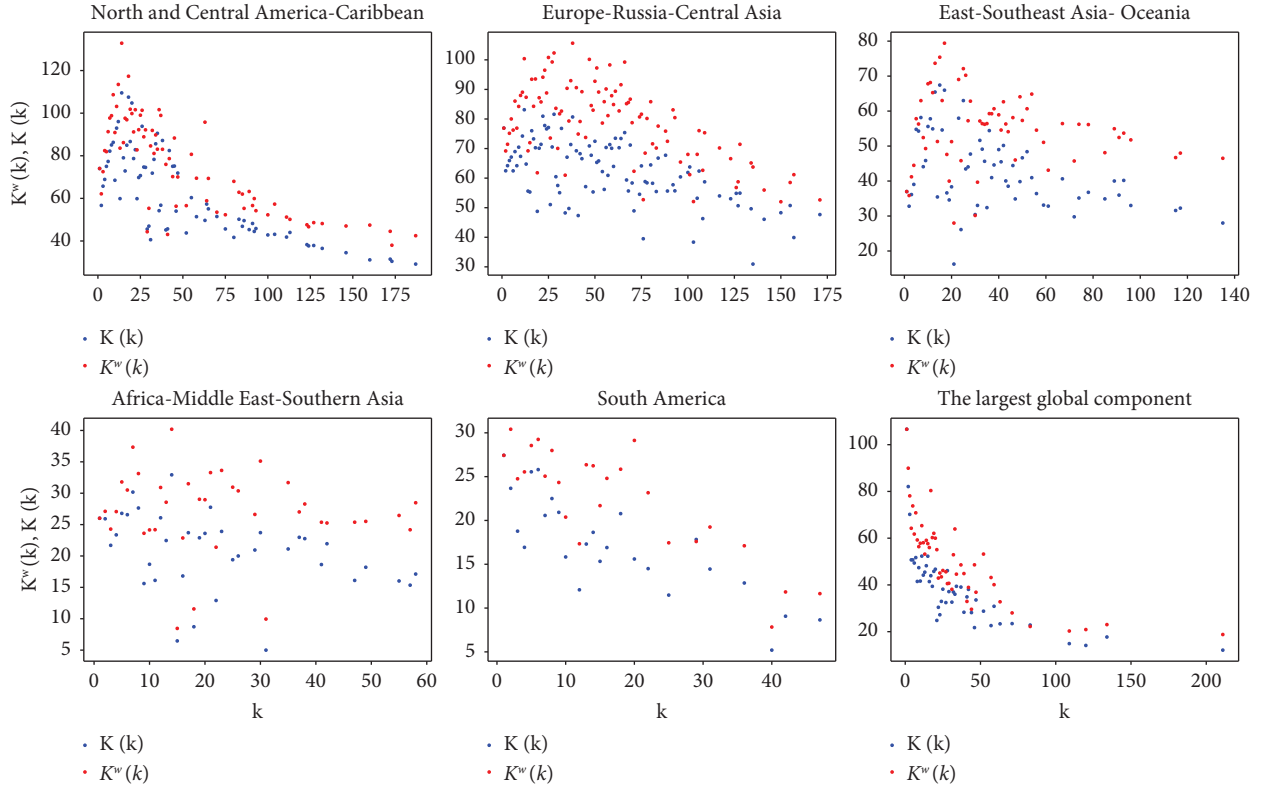


FIGURE 6: Average degree-degree correlation versus degree of the large components. The blue dots represent the average degree-degree correlation for the unweighted network. The red dots represent the average degree-degree correlation for the weighted network. The weighted degree-degree correlation is more significant than the unweighted degree-degree correlation. A high fraction of traffic is concentrated between major hubs in the local components. This concentration of traffic at these hubs is essential for optimizing the overall efficiency and connectivity of the air transportation network.

coefficient decreases monotonically as the degree increases. The function  $k^{-\gamma}$  (with  $\gamma \approx 0.27$ ) gives a good approximation of their relation. The exponent value estimated is identical to the Europe-Russia-Central Asia component. The weighted average clustering is very low and almost independent of the degree. As it is always below the unweighted average clustering coefficient, one can conclude that edges with low degrees tend to constitute the triplets.

**7.2. Degree-Degree Correlation.** Figure 9(b) illustrates the evolution of the degree-degree correlation as a function of degree  $k$  for the weighted world transportation network. For both weighted and unweighted networks, one can see that the distribution of the points is not monotone. Thus, one cannot conclude about the degree-degree correlation of nodes. This result differs from previous results showing an assortative behavior for the weighted world air transportation network [23]. The Africa-Middle East-Southern Asia component is the only component with similar behavior. Indeed, all the others exhibit disassortative behavior.

**7.3. Strength Distribution.** Like the components, the log-normal distribution best fits the strength distribution represented in Figure 10. In addition, the strength as a function of degree exponent ( $s(k) = k^\beta$  with  $\beta \approx 2.177$ ) is in the range of the local components.

## 8. Centrality Analysis

Centrality analysis investigates the most influential nodes in a network. There are multiple definitions of centrality that exploit either local or global characteristics of the networks [33, 34]. Here, we perform a comparative analysis of the strength (number of flights in an airport) and degree (number of routes in an airport) centralities of the various components. This analysis is in line with recent works considering the community structure to define new centrality measures [35].

### 8.1. Top Five Strength Analysis

**8.1.1. Regional Analysis.** Table 7 reports the top five airports in descending order of the number of flights with airports in their local component (internal strength centrality).

The top five airports in the *North and Central America-Caribbean component* are in the United States. They are in densely populated states such as Georgia, Illinois, Texas, California, and Washington D.C. Hartsfield J Atlanta Airport captures the higher number of flights. It is also the most connected. It is one of the densest in terms of passengers. Located in the second most populous city in the United States, the Los Angeles Airport is the second busiest in terms of flights, although it has a low ranking in terms of the number of routes. Chicago O'Hare Airport ranks third in

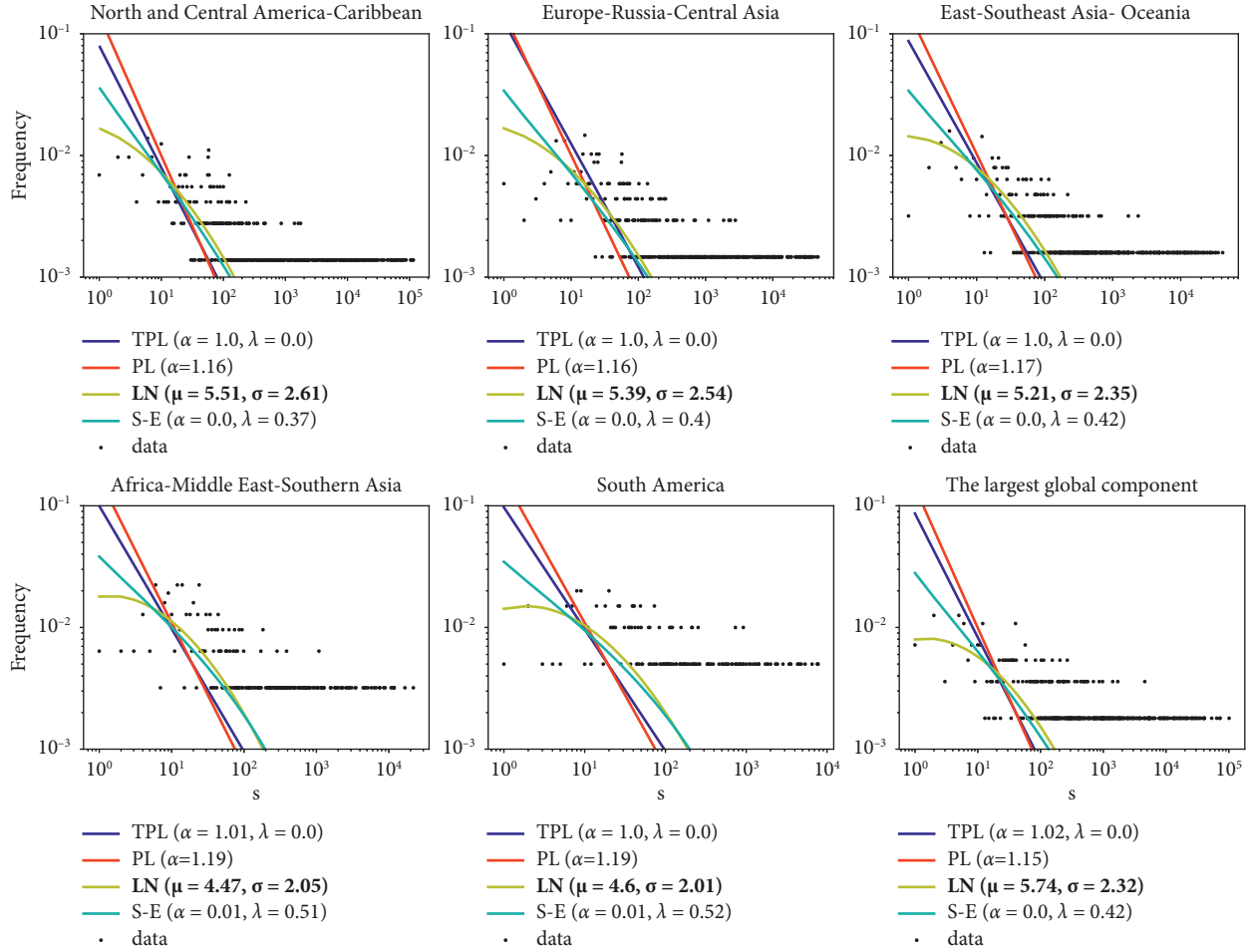


FIGURE 7: Strength distributions of the large local components and the largest global component. Dots denote empirical distribution, and lines denote estimates. The distributions under test include power law (PL), truncated power law (TPL), log-normal (LN), and stretched exponential (S-EXP). The values in bold are the best-fit parameters according to the Kolmogorov–Smirnov test. The large local components have heavy-tailed characteristics.

TABLE 6: KS test for the strength distribution.

	Power law	Truncated power law	Log-normal	Stretched exponential
North and Central America-Caribbean	0.31	0.136	<b>0.044</b>	0.06
Europe-Russia-Central Asia	0.31	0.27	<b>0.041</b>	0.06
East and Southeast Asia	0.32	0.15	<b>0.032</b>	0.05
Africa-Middle East-Southern Asia	0.32	0.15	<b>0.045</b>	0.065
South America	0.33	0.15	<b>0.043</b>	0.18

Distributions under test are power law, truncated power law, log-normal, and stretched exponential. The smallest value (bold) corresponds to the best fit.

the number of flights and destinations. Dallas Fort Worth Airport, the second most connected airport, ranks fourth according to the number of flights. All the airports mentioned above have more than 100 connections and 100000 flights within this component.

This is not the case for the fifth important airport, the Ronald Reagan Washington Airport. It is a national airport in the capital city of the United States. It is a hub for American Airlines and receives several million passengers. Altogether, these airports handle almost 15% of the regional flights. London Heathrow ranks ninth. It operates around

74000 flights. Denver and Houston airports which are in the top five hubs in terms of connections rank, respectively, eleventh and sixteenth [8] when considering the number of flights.

In the *Europe-Russia-Central Asia component*, four out of five top airports are in Europe and one in the United States. Dublin Airport in Ireland is the first with more than 4500 flights. It is the central hub of the important low-cost carrier, Aer Lingus Airlines. Very connected, the Barcelona Airport is the second with numerous flights. Indeed, it is a tourist city receiving several million passengers

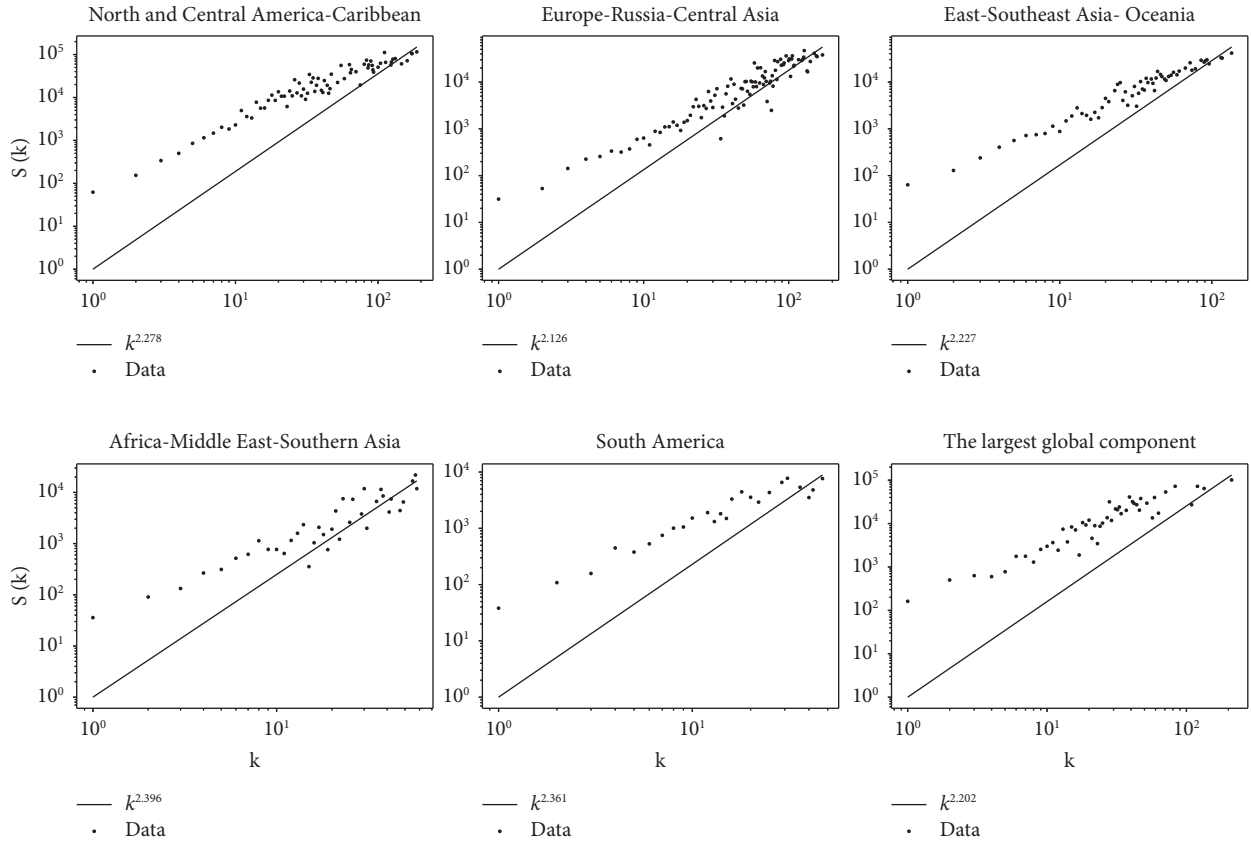


FIGURE 8: Strength as a function of degree of the large local components and the largest global component. Dots denote empirical distribution, and lines denote estimates. The more links an airport has, the more traffic it receives.

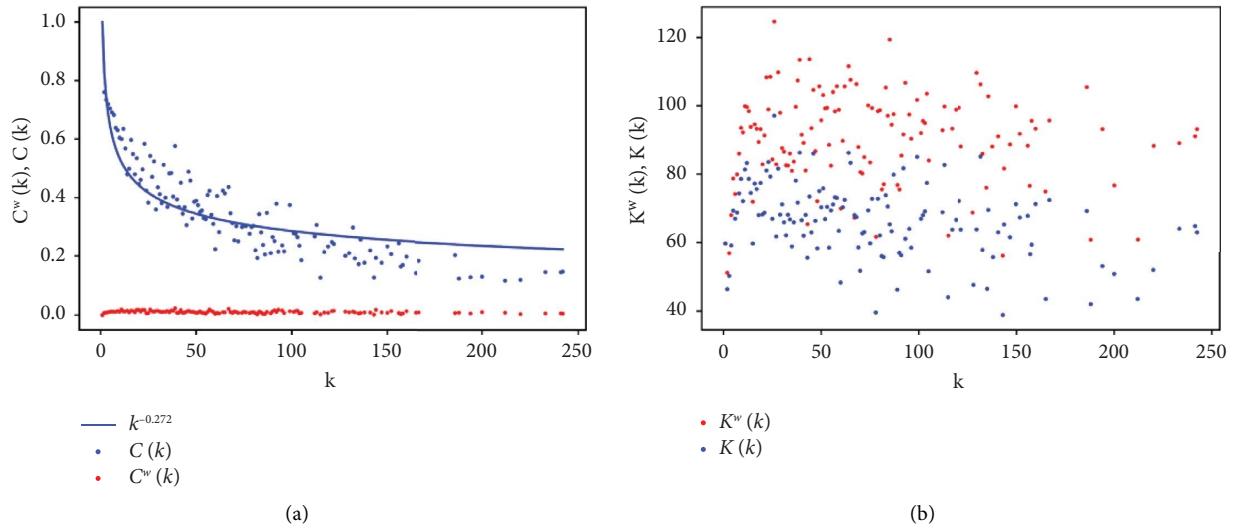


FIGURE 9: (a) The average clustering coefficient as a function of  $k$ . The exponent value estimated is identical to the Europe-Russia-Central Asia component. (b) The average degree correlation of degree  $k$ . For both figures, the red points are the values of the weighted world transportation network, while the blue points are the unweighted world transportation network values. The distribution of the points is not monotone. This behavior is similar to the one of the Africa-Middle East-Southern Asia component.

per year. The largest hub, Charles de Gaulle Airport in the capital city of France, ranks third with considerable traffic. Unexpectedly, John F Kennedy Airport in the United States

ranks fourth with 40 links within this component. This airport has more flights to Europe than several local airports. Indeed, it is the main gate between the United States and



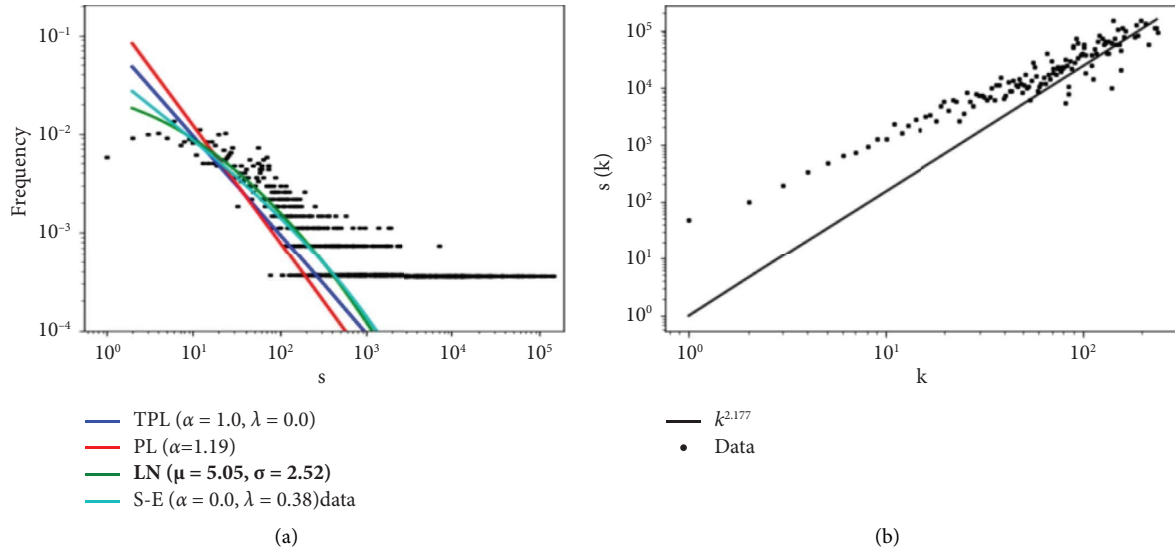


FIGURE 10: The world air transportation network's strength distribution (a). Dots denote empirical distribution, and lines denote estimates. The distributions under test include power law (PL), truncated power law (TPL), log-normal (LN), and stretched exponential (S-EXP). The parameters that best fit the data, as determined by the Kolmogorov–Smirnov test, are shown in bold. The strength as a function of the degree (b) of the large global component. The log-normal distribution is the best fit for the strength distribution.

Europe. Even though it has a higher number of links in this component, Amsterdam Schiphol Airport ranks fifth. It is the principal airport of the Netherlands. The Munich Airport and the London Stansted Airport, which are in the five highest hubs in terms of routes [8], rank, respectively, sixth and sixty-fifth in terms of flights. The Frankfurt Airport, in the same group as the two mentioned above, is now in Africa-Middle East-Southern Asia. The most vital airport in Russia, the Pulkovo Airport, is the fourteenth. The Dubai Airport, which belongs to this component, ranks twenty-fifth, although it is the biggest airport in the Middle East.

In the *East and Southeast Asia-Oceania component*, three out of five most busy airports are in China. The others are in Singapore and Hong Kong. The top airport is Beijing Capital Airport, China's capital city. It is the only one to have more than 40000 flights. Singapore Changi Airport ranks second, even if it is not one of the five most connected airports in this component. It is the primary hub of Singapore Airlines. The third airport is the Shanghai Pudong Airport in Shanghai, a populated city in China. Guangzhou Baiyun Airport is another central hub in China, ranking fourth. Indeed, this airport has the second-largest number of connections and carries millions of passengers. The Hong Kong Airport, with less than 30000 flights, ranks fifth. Chengdu Shuangliu Airport and Taiwan Taoyuan Airport, in the top five destinations, rank, respectively, eleventh and ninth [8]. The Sydney K Smith Airport, the most significant Oceania airport, ranks eighth.

The Indian airports dominate the *Africa-Middle East-Southern Asia component*. Indeed, they are in the top three busiest airports. The two others are in Saudi Arabia and Germany. With its numerous links, the Indira Gandhi Airport handles the highest traffic with more than 20000 flights. The second airport with almost 17000 flights is the Chhatrapati Shivaji Airport, located in Bombay, a metropolis of India. One of the primary airports in India, the Kempegowda Airport serving Bangalore, ranks third. It is a hub of Air Asia. The most connected airport, King Abdulaziz, ranks fourth. The Frankfurt am Main Airport is fifth with 30 internal connections and more than 11000 flights in this component. The most connected airport in the region, the Dubai Airport, is in the Europe-Russia-Central Asia component, and the fifth in terms of destinations, the Addis Ababa Bole Airport, ranks nineteenth in terms of flights.

In the *South America component*, among the top five, four airports are in Brazil and one in Colombia. They have less than 8000 flights. Tancredo Neves Airport is the first. Situated in Belo Horizonte metropolitan area, it is a hub of Azul Brazilian Airlines. It does not have a high number of connections, but its traffic is very dense. The second, Guarulhos G A F Montoro Airport, is the most crucial airport in Brazil. It serves the largest economic and tourist city, São Paulo. El Dorado Airport, located in the capital of Colombia, ranks third. Rio G-T Jobim Airport in Rio de Janeiro, one of the most populated

TABLE 7: The internal strength of an airport is determined by the number of flights within its dominant local component.

Large local component	Airport	City	Country	Internal strength	Local rank	Unweighted local rank
North and Central America-Caribbean	Hartsfield J Atlanta	Atlanta	United States	115410	1	1
	Los Angeles	Los Angeles	United States	111415	2	11
	Chicago O'Hare	Chicago	United States	107554	3	3
	Dallas Fort Worth	Dallas-Fort Worth	United States	104321	4	2
	Ronald Reagan Washington	Washington	United States	87953	5	19
Europe	Dublin	Dublin	Ireland	47105	1	8
	Barcelona	Barcelona	Spain	45408	2	4
	Charles de Gaulle	Paris	France	41216	3	7
	John F Kennedy	New York	United States	38697	4	15
	Amsterdam Schiphol	Amsterdam	Netherlands	38065	5	1
East and Southeast Asia	Beijing Capital	Beijing	China	41320	1	1
	Singapore Changi	Singapore	Singapore	35556	2	12
	Shanghai Pudong	Shanghai	China	33043	3	3
	Guangzhou Baiyun	Guangzhou	China	32318	4	2
	Hong Kong	Hong Kong	Hong Kong	29712	5	8
Africa-Middle East-Southern Asia	Indira Gandhi	Delhi	India	21874	1	3
	Chhatrapati Shivaji	Mumbai	India	16704	2	4
	Kempegowda	Bangalore	India	11860	3	17
	King Abdulaziz	Jeddah	Saudi Arabia	11829	4	2
	Frankfurt am Main	Frankfurt	Germany	11778	5	4
South America	Tancredo Neves	Belo Horizonte	Brazil	7745	1	7
	G G A F Montoro	Sao Paulo	Brazil	7706	2	1
	El Dorado	Bogota	Colombia	7588	3	2
	Rio G-T Jobim	Rio de Janeiro	Brazil	6553	4	6
	Presidente J Kubitschek	Brasilia	Brazil	5366	5	5

The local rank of an airport is determined by its internal strength, ranked in descending order within its local component. The cumulative fraction of connected airports indicates the proportion of airports that the top  $x$  local hubs can access within their respective local components.

cities in Brazil, is fourth. It is less connected among the top five. The fifth airport, the Presidente J Kubitschek Airport, is located in the capital of Brazil.

*8.1.2. Inter-Regional Analysis.* Table 8 lists the top five airports participating in the inter-regional traffic in each large local component. These airports also belong to the large global component.

Four of the five most essential airports for inter-regional traffic in the North and Central American-Caribbean region are in the USA. The other one is in Canada. The first two airports in this ranking are the John F Kennedy Airport and the Newark Liberty Airport. With more connections (120), John F Kennedy airport's (nearly 100000) flight flow is much denser than Newark Liberty airport's flight flow (39 flights and almost 70000 flights). These airports are located in New York state. Indeed, this state, located in the Northeastern United States, is one of the most important ones, with the largest number of residents. Moreover, it is near other regions and includes numerous international institutions and multinational corporations. The Chicago O'Hare Airport and the Los Angeles Airport rank third and fourth. They are in other populated cities, Chicago and Los Angeles, in the United States. These two airports have comparable degrees and traffic. In Canada, the busiest airport, Lester B. Pearson Airport, ranks fifth. It is a gateway to this country and the USA. The top three in this region are in the top 10 of the large global component.

The Frankfurt am Main Airport dominates the inter-regional flow of flights in the Europe-Russia-Central Asia region. Being the most important airport in Germany, it has around 100000 flights with 211 connections. The Charles de Gaulle Airport ranks first with almost 70000 flights. Located in the capital of France, it is the busiest airport in this country. Very connected, the London Heathrow Airport is a third of the list. It serves several flights to different regions of the world. The busiest airport in the Netherlands, the Amsterdam Schiphol Airport ranks fourth. The Munich Airport is the fifth airport with the largest inter-regional traffic. It is the second busiest airport in Germany. All these airports mentioned above are in the top 10 of the global component. One can say that the European airport leads the inter-regional flights.

The top five busiest inter-regional airports in the East and Southeast Asia-Oceania region do not include any airport in Oceania. Indeed, the Narita Airport in Japan's capital city is the busiest inter-regional airport with about 40000 flights. The Beijing Airport ranks second. It is the most connected, with traffic comparable to the Narita Airport. The Suvarnabhumi Airport, the most critical airport in Thailand, is the third, with almost 30000 flights. The Incheon Airport,

the most significant hub in South Korea, is the fourth. Its traffic is around 30000. The fifth airport with the largest international traffic is the Hong Kong airport, with almost 26000 flights. Only Narita and Beijing airports figure in the top 10 airports in the large global component.

While the Indian airports dominate the regional traffic, the busiest inter-regional airports of the Africa-Middle East-Southern Asia region are scattered in the Middle East. The Dubai Airport is by far the most dynamic in this region. Its position and big area promote it. Located in the capital of Qatar, the Hamad airport ranks third. The second-largest airport in the United Arab Emirates ranks fourth. Cairo Airport in Egypt is fifth in this ranking. It has a comparable number of flights to the third and fourth airports (between 12000 and 14000). The first inter-regional airport in this region is not in the top 10 of the large global component. It ranks nineteenth.

The inter-regional airports handling the highest traffic in the South America region are in different countries. The Guarulhos Airport in Brazil is the busiest. El Dorado Airport in Colombia follows. These two airports are also regional hubs. The Ministro Pistarini Airport, located in the capital of Argentina, ranks third. The regional hub and the second most important in Brazil, the Rio G-T Jobim Airport, is the fourth inter-regional airport in this area. No airport in this component is in the top 20 of the large global component. Indeed, the Guarulhos Airport ranks twenty-fourth.

*8.2. Comparison with the World Transportation Network.* Table 9 lists the 25 busiest airports in the world regarding the number of flights. It also includes their local and global strength rank and their degree rank. It shows that the North and Central America-Caribbean region controls a big part of the world's traffic. Indeed, 19 are in this area. Five are in the Europe-Russia-Central Asia region, and one is in the East and Southeast Asia-Oceania region. If we compare the airports' strengths with their degree, one detects that European airports usually deserve more destinations worldwide while North American airports have more flights.

Although 19 airports in the North and Central America-Caribbean area are in the top 25 worldwide airports for their traffic, only six are in the top 25 inter-regional airports. In contrast, they are all in the top 25 regional airports. These results demonstrate that the North and Central America-Caribbean region focuses more on regional traffic. The airports of New York (John F Kennedy and Newark airports) rank, respectively, 3 and 6 for inter-regional traffic. Therefore, New York City is the US gateway. Los Angeles is the busiest airport in the world, but the traffic is well distributed between local and global destinations. Chicago O'Hare and Hartsfield J Atlanta are in the top 5 airports in the world, but it is mainly due to their regional position. Note that the rank

TABLE 8: Strength centrality in the large global component.

Region	Airport	City	Country	External strength	Global rank in the region	Global rank in the component
North and Central America-Caribbean	John F Kennedy	New York	United States	72166	1	3
	Newark Liberty	Newark	United States	40924	2	6
	Chicago O'Hare	Chicago	United States	38539	3	10
	Los Angeles	Los Angeles	United States	35148	4	11
	Lester B. Pearson	Toronto	Canada	34818	5	12
Europe	Frankfurt	Frankfurt	Germany	101024	1	1
	Charles de Gaulle	Charles de Gaulle	Paris	72206	2	2
	London Heathrow	London	United Kingdom	64265	3	4
	Amsterdam Schiphol	Amsterdam	Netherlands	53340	4	5
	Munich	Munich	Germany	40785	5	7
East and Southeast Asia	Narita	Tokyo	Japan	40608	1	8
	Beijing Capital	Beijing	China	39915	2	9
	Suvarnabhumi	Bangkok	Thailand	29477	3	16
	Incheon	Seoul	South Korea	29288	4	17
	Hong Kong	Hong Kong	Hong Kong	25989	5	20
Africa-Middle East-Southern Asia	Dubai	Dubai	United Arab Emirates	27074	1	19
	Indira Gandhi	Delhi	India	16882	2	32
	Hamad	Doha	Qatar	1121	3	40
	Abu Dhabi	Abu Dhabi	United Arab Emirates	13512	4	41
	Cairo	Cairo	Egypt	12803	5	43
South America	Guarulhos-G A F M	São Paulo	Brazil	21733	1	24
	El Dorado	Bogotá	Colombia	13616	2	39
	Ministro Pizarini	Buenos Aires	Argentina	9447	3	50
	Rio G-T Jobim	Rio de Janeiro	Brazil	6103	4	64
	Jorge Chávez	Lima	Peru	4575	5	80

We report the top five inter-regional hubs in each region. An airport's external strength is the number of connections it has with airports in the large global component. The global rank in a region is determined by decreasing external strength in that region. Airports' rank in the global component is based on decreasing external strength.

TABLE 9: The 25 largest nodes in the air transportation network are ranked descendingly according to their strength centrality.

Region	Airport	City	Country	Strength	Worldwide strength rank	Local strength rank	Global strength rank	Worldwide degree rank
North and Central America-Caribbean	Los Angeles	Los Angeles	United States	146563	1	11	11	24
	Chicago O'Hare	Chicago	United States	146093	2	3	10	6
	Hartsfield J Atlanta	Atlanta	United States	135109	4	1	28	5
	Dallas Fort Worth	Dallas	United States	122407	5	4	30	8
	<i>John F Kennedy</i>	<i>New York</i>	<i>United States</i>	<i>110863</i>	8	4	3	<i>14</i>
	Newark Liberty	Newark	United States	106850	9	14	6	19
	San Francisco	San Francisco	United States	104325	10	10	15	52
	Seattle Tacoma	Seattle	United States	98539	11	6	35	67
	Charlotte Douglas	Charlotte	United States	89409	13	7	55	26
	Ronald R Washington	Washington	United States	88348	14	5	248	73
	Lester B. Pearson	Toronto	Canada	85631	15	25	12	23
	McCarran	Las Vegas	United States	83054	16	8	76	33
	Philadelphia	Philadelphia	United States	81950	17	15	30	40
	General E L Logan	Boston	United States	78835	19	19	25	57
	George Bush Houston	Houston	United States	77421	20	16	33	13
	Denver	Denver	United States	76329	22	11	77	12
	Montreal/P E Trudeau	Montreal	Canada	72866	23	18	37	65
	Phoenix Sky Harbor	Phoenix	United States	71831	24	12	224	71
	Minneapolis-St Paul	Minneapolis	United States	70461	25	13	87	39
	<b>London Heathrow</b>	<b>London</b>	<b>United Kingdom</b>	<b>138629</b>	<b>3</b>	<b>9</b>	<b>4</b>	<b>10</b>
	<i>Charles de Gaulle</i>	<i>Paris</i>	<i>France</i>	<i>113422</i>	6	2	2	3
	<i>Frankfurt am Main</i>	<i>Frankfurt</i>	<i>Germany</i>	<i>112802</i>	7	5	1	2
	<i>Amsterdam Schiphol</i>	<i>Amsterdam</i>	<i>Netherlands</i>	<i>91405</i>	12	5	5	1
	<b>Munich</b>	<b>Munich</b>	<b>Germany</b>	<b>77154</b>	<b>21</b>	<b>6</b>	<b>7</b>	<b>9</b>
	East and Southeast Asia	<i>Beijing Capital</i>	<i>Beijing</i>	<i>81235</i>	<i>18</i>	<i>1</i>	<i>9</i>	<i>1</i>

An air transportation network's strength centrality is ranked by its worldwide strength rank. It is computed using the total strength (internal and external). Local rank is the strength centrality rank in the large local components. Internal strength is used to calculate it. Global rank is the strength centrality rank in the global component. External strength is used to calculate it. No indication is given when an airport does not belong to the global component. Top inter-regional airports are in bold. Top regional and inter-regional airports are in italics. Others are top regional airports.



of the regional airports in the world air transportation network can be misleading because they are not the most active in inter-regional traffic.

London Heathrow Airport ranks third worldwide. It is mainly due to its position in the inter-regional traffic. In contrast, Charles de Gaulle Airport, Frankfurt am Main Airport, Amsterdam Schiphol Airport, and Munich Airport rank 6, 7, 12, and 21 in the top 25 airports of the world. They are all in the top 25 regional and inter-regional airports. However, they generally exert a stronger influence at the regional level. Note that London Heathrow belongs to the American-Caribbean component. Its traffic is very dense compared with the airport in this component. One can make a similar remark about the Frankfurt am Main Airport, which belongs to the Africa-Middle East-Southern Asia component.

The Beijing Airport is the only one from the East and Southeast Asia region in the top 25 crucial airports. It is the eighteenth worldwide according to the number of flights. In addition, it is the first regional airport and the most connected to the world.

Dubai Airport is the forty-fifth in the world, the first from Africa-Middle East-Southern Asia, even though it deserves many routes. Unexpectedly, it is the twenty-fifth influential regional airport of the Europe-Russia-Central Asia component and the nineteenth inter-regional airport. The most influential airport in South America, the Guarulhos-Governador André F M Airport in São Paulo, Brazil, is the seventy-fifth airport in the world in terms of traffic. In comparison, it is the first in the South America component and the twenty-fourth important inter-regional airport.

To summarize, North America leads the traffic in the world air transportation network. The component structure shows that most of this traffic is regional. Indeed, the large global component, which captures the inter-regional flights, exhibits numerous airports from different world areas essential to the inter-regional traffic. In addition, the large local components display the influential regional airports hidden in the world air network.

**8.3. RBO Analysis.** The ranked-biased overlap (RBO) [36] quantifies the similarity of two ranking lists. A parameter enables the prioritization of higher ranks over lower ones and the extension of the evaluation's depth. Its value ranges between 0 and 1. The higher its value, the more identical the lists. We compare strength with degree centrality of the airports of the large components [8]. We tune the RBO to give equal importance to all ranks. Figure 11 shows the evolution of the RBO of the top-ranked airports for the large components and the world air transportation network. The curves cover the range from top 5 to top 45 sampled with a step of 5.

One can distinguish two types of curves. In the first category, the RBO increases monotonically as the number of airports increases. It includes the

Europe-Russia-Central Asia, Africa-Middle East-Southern Asia, East-Southeast Asia-Oceania, and South America components. Flights in these components do not concentrate on the main hubs. Indeed, the top 5 degree and strength airports are different. In the second category, the RBO decreases to a minimum, and then it increases monotonically. North and Central America-Caribbean, East-Southeast Asia-Oceania, and the global component belong to this category. Their top 5 airports' ranking according to degree or strength is highly similar. There are more differences between the top 10 and 15 airports, so the RBO decreases. Beyond this value, the strength and degree rankings become more homogeneous, so the RBO increases. Looking at the top 45 airports, one can rank the components according to the similarity of strength and degree rankings. East-Southeast Asia-Oceania is the component where traffic (strength) and hub size (degree) are more similar. The Europe-Russia-Central Asia component exhibits the most different ranking. So, in the former, traffic is generated by hubs, while in the latter, airports with few connections can handle a high share of traffic.

The RBO between top strength and degree airports increases monotonically in the world air transportation network. Differences are more pronounced compared to the components. Indeed, the top 5 airports based on degree are concentrated in Europe, while the top 5 based on strength are in the United States. This difference illustrates the different focus in the two regions. Indeed, in the USA, airlines focus on regional traffic, while in Europe, serving many international destinations is a must for companies.

## 9. S-Core Analysis

The s-core [37, 38] analysis is the generalization of the k-core [39] analysis to weighted graphs. The k-core [40] of a graph is the subgraph obtained by recursively removing all the vertices of degree smaller than  $k$  until the degree of all remaining vertices is larger than or equal to  $k$ . By extension, the s-core of a graph is a subnetwork in which a node has at least a strength  $s$ . One can extract the maximum s-core by removing nodes iteratively from the network. Indeed, the  $s_{\min(s_i)}$ -core, where each node has at least a strength 1, is the whole network. One forms the next level, by removing all the nodes with the minimum strength  $s_{\min(s_i+1)}$ . The remaining nodes form  $s_{\min(s_i+1)}$ -core, and so on until one reaches the core number  $\max s_n$ -core for which it is impossible to obtain the  $s_{\min(s_n+1)}$ -core.

This section reports the max s-core analysis of the large weighted components and a comparative investigation with their corresponding max k-core. Additionally, it presents a similar analysis of the world transportation network.

**9.1. Regional Analysis.** Figure 12 reports the maximum s-core values of the five weighted large local components. It also shows the airports it includes.

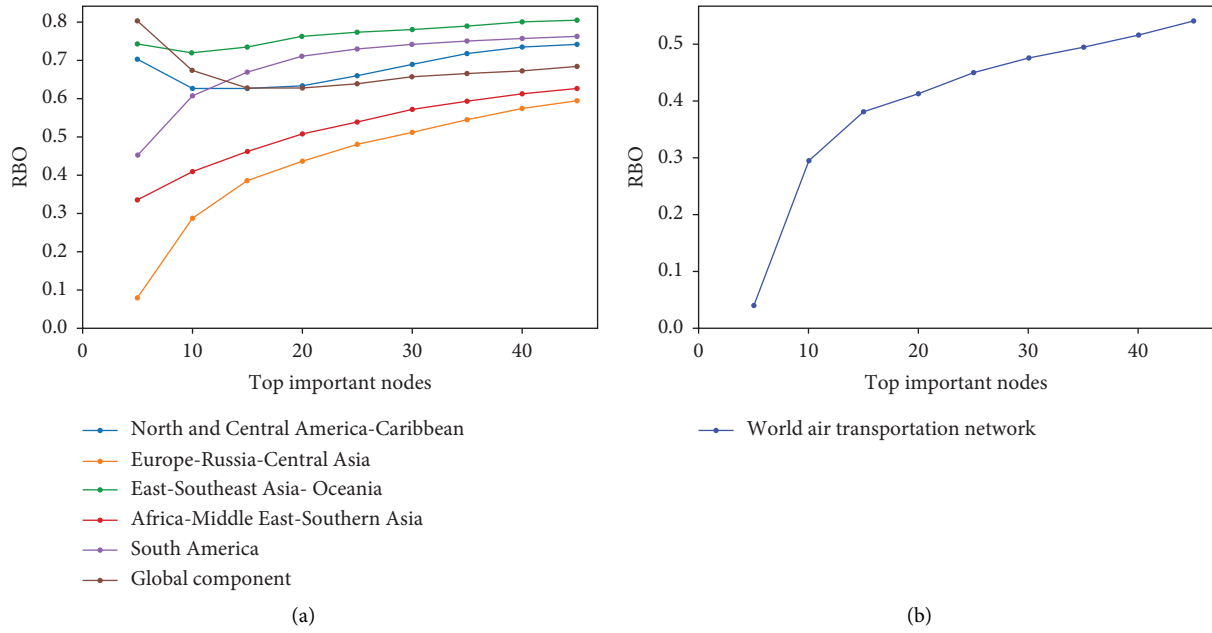


FIGURE 11: The RBO of the large components (a) and the world air transportation network (b). The top 50 hubs of the unweighted (degree) and weighted (strength) are compared by step of 5. Flights in the Europe-Russia Central Asia, Africa-Middle East-Southern Asia, East-Southeast Asia-Oceania, and South America components do not concentrate on the main hubs. In the other large components, the hubs accumulate the traffic.

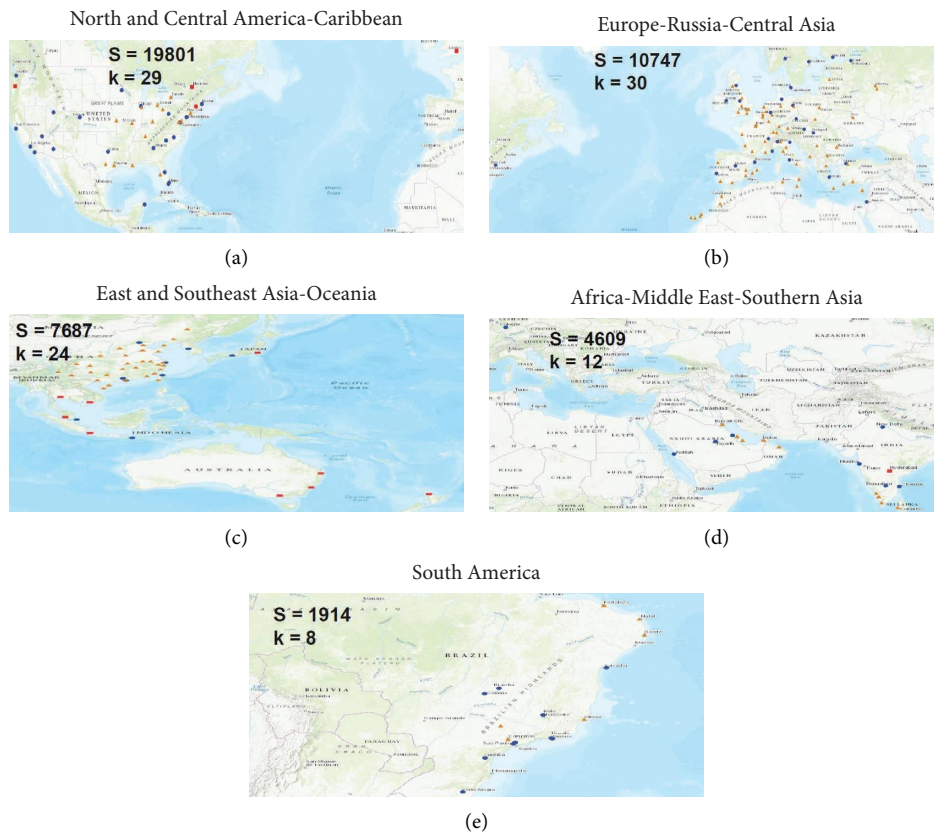


FIGURE 12: The core of the weighted large local components.  $S$  is the maximum  $s$ -core value, and  $K$  is the maximum  $k$ -core value. Blue points represent the airports belonging to the maximum  $s$ -core and the maximum  $k$ -core, the red points are the airports exclusively in the maximum  $s$ -core, and the yellow points are the airports belonging only to the maximum  $k$ -core. The maximum  $s$ -core of the North and Central America-Caribbean component contains 26 airports. The maximum  $s$ -core of Europe-Russia-Central Asia contains 27 airports. The maximum  $s$ -core of East and Southeast Asia-Oceania includes 18 airports. The maximum  $s$ -core of Africa-Middle East-Southern Asia contains 9 airports. The maximum  $s$ -core of South America contains 10 airports.

The max *s*-core of the North and Central America-Caribbean component contains 26 airports. Fort Lauderdale Airport, with 19801 flights, has the lowest traffic. Table 10 lists these airports. Only three are outside the United States: (1) Montreal/P E Trudeau Airport in Montreal, the busiest airport in Canada; (2) Cancun Airport, the second busiest in Mexico; and (3) London Heathrow Airport in the United Kingdom. Their traffic is so high with the US that they can be considered part of the country. Most of the airports are on the coast of the United States. None deserves New York. Indeed, its airports' main traffic is with other world regions. Four airports (Raleigh-Durham, Pittsburgh, Windsor Locks, and Cancun) in the max *s*-core are not among the top 26 airports in terms of traffic. These airports primarily serve other airports in the max *s*-core.

The max *s*-core contains almost half as many airports as the max *k*-core. However, they share 22 airports (84.6% of the max *s*-core airports). The traffic and destinations of these airports are therefore closely related. London Heathrow in the United Kingdom, Portland and Windsor Locks in the United States, and Montreal/Pierre Elliott Trudeau have more traffic than destinations with the other airports of the max *s*-core.

The max *s*-core of the Europe-Russia-Central Asia component has 27 airports, listed in Table 11. Covering several countries in Europe, they are generally in the capital cities. Pulkovo Airport is the only one in the Russian subregion. Ben Gurion Airport has the lowest traffic with 10747 flights. John F Kennedy in the USA has the highest number of flights. Indeed, it is the main connection between Europe and the USA. Charles de Gaulle in France, Munich in Germany, Amsterdam in the Netherlands, and Zurich Airport in Switzerland are not in the max *s*-core. Indeed, the traffic of these airports is more directed towards inter-regional destinations. Five airports in this max *s*-core are not in the 27 top strength airports. These airports are Venice, Marco Polo in Italy, Eleftherios Venizelos in Greece, Budapest Ferenc Liszt in Hungary, Ben Gurion in Israel, and Helsinki Vantaa Airport in Finland.

The max *s*-core contains three times fewer airports than the max *k*-core. All airports in the max *s*-core are in the max *k*-core. John F Kennedy Airport in the USA has a lot of traffic and connections to airports in Europe. Otherwise, no country dominates the max *s*-core, while the max *k*-core airports are mainly in Germany, the United Kingdom, France, Italy, and Spain.

Table 12 contains the 18 airports of the max *s*-core of the East and Southeast Asia-Oceania component. These airports are in different countries, mainly in capitals and megacities. Only four are in Oceania. Kansai Airport in Japan, with its 7687 flights, has the lowest traffic. Note that the airport of Beijing, with the most significant traffic of this component, is the third airport with the weakest traffic in this max *s*-core. Indeed, it has many flights with low-strength airports. There are five airports (Brisbane Airport in Australia, Auckland Airport in New Zealand, Tan Son Nhat Airport in Vietnam, Ngurah Rai Airport in Indonesia, and Kansai Airport in Japan) in the max *s*-core that are absent from the 18 top strength airports. The max *s*-core contains half as many

airports as the *k*-core. Nevertheless, China dominates the max *k*-core, and it is not the case for the max *s*-core, which includes airports from Oceania.

Table 13 lists the nine airports in max *s*-core of the Africa-Middle East-Southern Asia component. They are in three countries (5 in India, 2 in Saudi Arabia, and 1 in Germany). Indeed, there is a lot of traffic between Saudi Arabia and India because these many Indians work in Saudi Arabia. With 4609 flights, Rajiv Gandhi International Airport is the airport with the lowest traffic in the max *s*-core. These airports form a complete graph. They concentrate a large part of the traffic of the component. Indeed, except for Rajiv Gandhi Airport, they are all in the top 9 strength airports. The max *k*-core is twice larger, and it includes the airports of the max *s*-core. Thus, Frankfurt Airport is very connected in terms of traffic and destination with the hubs of this component. Nevertheless, India dominates the max *s*-core and the max *k*-core.

Table 14 gives the ten airports in the max *s*-core of the South America component. They are all in Brazil and mainly on the east coast. Rio Galeão Tom Jobim Airport has the fewest number of flights. Only two airports, Santa Genoveva Airport in Goiania and the Deputado L E Magalhães Airport, are not present in the top 10 strength airports. All the airports in the max *s*-core are in the max *k*-core, which contains almost twice as many airports. Furthermore, Brazil dominates the max *s*-core as it does with the max *k*-core.

To summarize, we observe two typical behaviors for the max *s*-core. In the first one, the airports forming the max *s*-core are mainly in a single country. Indeed, the United States, India, and Brazil dominate the max *s*-core of their component. In the second case, the airports in the max *s*-core are more evenly distributed in the component. It is the case for the East and Southeast Asia-Oceania and Europe-Russia-Central Asia components.

The max *s*-core always contains far fewer airports than the max *k*-core, typically one-half to one-third. In addition, most airports in the max *s*-core are in the max *k*-core. Thus, airports with high traffic tend to have also many connections.

*9.2. Inter-Regional Analysis.* Figure 13(a) shows the max *s*-core of the large global component, and Table 15 lists its 22 airports. These airports are in North America, Europe, and East and Southeast Asia. They are located in capital cities and are the main airports of their country. Suvarnabhumi Airport in Thailand has the lowest traffic with 13196 flights among the max *s*-core airports. Three US airports in the max *s*-core are not in the top 22 strength airports (General Edward Lawrence Logan Airport, Dallas Fort Worth Airport, and Seattle Tacoma Airport). Thus, the airports that concentrate the majority of inter-regional traffic share many flights. The max *s*-core and the max *k*-core have comparable sizes. However, the max *k*-core includes more countries. Three airports (General E L Logan Airport, Dallas Fort Worth Airport, and Seattle Tacoma Airport) are in the max *s*-core and not in the max *k*-core. These airports have many

TABLE 10: 26 max s-core airports of the North and Central America-Caribbean component.

Airport	City	Country	Core strength	Local rank	National rank
Seattle Tacoma	Seattle	United States	50753	6	7
London Heathrow	London	United Kingdom	50191	9	1
Ronald Reagan Washington	Washington	United States	45250	5	10
Los Angeles	Los Angeles	United States	45250	2	1
Montreal/Pierre E Trudeau	Montreal	Canada	41570	18	1
San Francisco	San Francisco	United States	37141	10	10
San Diego	San Diego	United States	37061	17	22
McCarran	Las Vegas	United States	35950	8	11
Phoenix Sky Harbor	Phoenix	United States	35437	12	16
Charlotte Douglas	Charlotte	United States	33968	7	9
Philadelphia	Philadelphia	United States	31443	15	14
Orlando	Orlando	United States	25965	21	21
Hartsfield J Atlanta	Atlanta	United States	25885	1	1
Portland	Portland	United States	25872	24	24
Denver	Denver	United States	25779	11	15
General E L Logan	Boston	United States	24331	19	13
Dallas Fort Worth	Dallas-Fort Worth	United States	23317	4	4
Salt Lake City	Salt Lake City	United States	23296	22	24
Chicago O'Hare	Chicago	United States	22433	3	2
Minneapolis-St P/Wold-Chamberlain	Minneapolis	United States	21173	13	17
Fort Lauderdale Hollywood	Fort Lauderdale	United States	19801	26	26
<b>Raleigh-Durham</b>	<b>Raleigh-Durham</b>	<b>United States</b>	<b>24106</b>	<b>34</b>	<b>32</b>
<b>Miami</b>	<b>Miami</b>	<b>United States</b>	<b>22423</b>	<b>28</b>	<b>19</b>
<b>Bradley</b>	<b>Windsor Locks</b>	<b>United States</b>	<b>20296</b>	<b>40</b>	<b>38</b>
<b>Pittsburgh</b>	<b>Pittsburgh</b>	<b>United States</b>	<b>20125</b>	<b>36</b>	<b>33</b>
<b>Cancun</b>	<b>Cancun</b>	<b>Mexico</b>	<b>20119</b>	<b>27</b>	<b>2</b>

Airports not in the top 26 regional airports with the highest strength airports are in bold. The core degree of a node is its number of flights in the maximum s-core. Local rank is the rank in the large local component. National rank is the airport rank in its country. Airports are ranked from largest to smallest strength.

TABLE 11: 27 max s-core airports of the Europe-Russia-Central Asia component.

Airport	City	Country	Core strength	Local rank	National rank
John F Kennedy	New York	United States	23129	4	21
Dublin	Dublin	Ireland	20467	1	1
Manchester	Manchester	United Kingdom	19058	8	2
Amsterdam Schiphol	Amsterdam	Netherlands	62	5	1
Dubai	Dubai	United Arab Emirates	16916	26	1
Geneva Cointrin	Geneva	Switzerland	16810	16	2
Adolfo S Madrid-Barajas	Madrid	Spain	16789	7	1
Humberto Delgado Airport	Lisbon	Portugal	16308	23	1
Nice-Côte d'Azur	Nice	France	16063	21	3
Oslo Lufthavn	Oslo	Norway	16026	27	2
Barcelona	Barcelona	Spain	15153	2	2
Stockholm-Arlanda	Stockholm	Sweden	15002	17	1
Hamburg	Hamburg	Germany	14642	18	5
Edinburgh	Edinburgh	United Kingdom	14029	22	3
Ataturk	Istanbul	Turkey	13352	10	1
Vienna	Vienna	Austria	12872	20	1
Leonardo da V-F	Rome	Italy	11601	11	1
Malpensa	Milano	Italy	11457	25	2
Copenhagen Kastrup	Copenhagen	Denmark	11350	19	1
Pulkovo	St. Petersburg	Russia	11334	15	1
Berlin-Tegel	Berlin	Germany	11312	9	4
Vaclav Havel Prague	Prague	Czech Republic	11219	24	1
Duesseldorf	Duesseldorf	Germany	10881	12	3
<b>Eleftherios Venizelos</b>	<b>Athens</b>	<b>Greece</b>	<b>13591</b>	<b>28</b>	<b>1</b>
<b>Venice Marco Polo</b>	<b>Venice</b>	<b>Italy</b>	<b>15029</b>	<b>29</b>	<b>3</b>
<b>Helsinki Vantaa</b>	<b>Helsinki</b>	<b>Finland</b>	<b>15523</b>	<b>32</b>	<b>1</b>
<b>Ben Gurion</b>	<b>Tel Aviv</b>	<b>Israel</b>	<b>10747</b>	<b>35</b>	<b>1</b>
<b>Budapest Ferenc Liszt</b>	<b>Budapest</b>	<b>Hungary</b>	<b>10840</b>	<b>37</b>	<b>1</b>

Airports not in the top 27 regional airports with highest strength are in bold. The core strength of a node is its number of flights in the maximum s-core. Local rank is the rank in the large local component. National rank is the airport rank in its country. Airports are ranked from largest to smallest strength.

TABLE 12: 18 max  $s$ -core airports of the East and Southeast Asia-Oceania component.

Airport	City	Country	Core strength	Local rank	National rank
Sydney K Smith	Sydney	Australia	14412	8	1
Shanghai Pudong	Shanghai	China	12981	3	2
Melbourne	Melbourne	Australia	12384	14	2
Hong Kong	Hong Kong	Hong Kong	12202	5	1
Soekarno-Hatta	Jakarta	Indonesia	11662	15	1
Singapore Changi	Singapore	Singapore	10839	2	1
Taiwan Taoyuan	Taipei	Taiwan	9474	9	1
Suvarnabhumi	Bangkok	Thailand	9436	7	1
Incheon	Seoul	South Korea	9410	6	1
Guangzhou Baiyun	Guangzhou	China	8962	4	3
Suvarnabhumi	Kuala Lumpur	Malaysia	8467	10	1
Beijing Capital	Beijing	China	7837	1	1
<b>Brisbane</b>	<b>Brisbane</b>	<b>Australia</b>	<b>9241</b>	<b>20</b>	<b>3</b>
<b>Tan Son Nhat</b>	<b>Ho Chi Minh City</b>	<b>Vietnam</b>	<b>7959</b>	<b>21</b>	<b>1</b>
<b>Narita</b>	<b>Tokyo</b>	<b>Japan</b>	<b>8313</b>	<b>22</b>	<b>1</b>
<b>Auckland</b>	<b>Auckland</b>	<b>Australia</b>	<b>8338</b>	<b>26</b>	<b>1</b>
<b>Ngurah Rai (Bali)</b>	<b>Denpasar</b>	<b>Indonesia</b>	<b>7826</b>	<b>36</b>	<b>2</b>
<b>Kansai</b>	<b>Osaka</b>	<b>Japan</b>	<b>7687</b>	<b>25</b>	<b>3</b>

Airports not in the top 18 regional airports with the highest strength are in bold. Core strength of a node is its number of flights in the maximum  $s$ -core. Local rank is the airport rank in the large local component. National rank is the airport rank in its country. Airports are ranked from largest to smallest strength.

TABLE 13: 9 max  $s$ -core airports of the Africa-Middle East-Southern Asia component.

Airport	City	Country	Core strength	Local rank	National rank
Indira Gandhi	Delhi	India	8699	1	1
Chhatrapati Shivaji	Mumbai	India	7414	2	2
Kempegowda	Bangalore	India	6455	3	3
Frankfurt am Main	Frankfurt	Germany	5907	6	1
King Abdulaziz	Jeddah	Saudi Arabia	5539	5	1
Chennai	Madras	India	4882	8	4
King Fahd	Dammam	Saudi Arabia	4757	4	2
King Khaled	Riyadh	Saudi Arabia	4658	7	1
<b>Rajiv Gandhi</b>	<b>Hyderabad</b>	<b>India</b>	<b>4609</b>	<b>10</b>	<b>5</b>

Airports not in the top 9 regional airports with the highest strength are in bold. The core strength of a node is its number of flights in the maximum  $s$ -core. Local rank is the rank in the large local component. National rank is the airport rank in its country. Airports are ranked from largest to smallest strength.

TABLE 14: 10 max  $s$ -core airports of the South America component.

Airport	City	Country	Core strength	Local rank	National rank
Tancredo Neves	Belo Horizonte	Brazil	4305	2	3
Santos Dumont	Rio de Janeiro	Brazil	3564	6	7
Afonso Pena	Curitiba	Brazil	3500	7	8
Congonhas	Sao Paulo	Brazil	3030	8	9
Guarulhos-Governador A F Montoro	Sao Paulo	Brazil	2887	1	1
Deputado Luis E Magalhães	Salvador	Brazil	2504	10	8
Presidente J Kubitschek	Brasilia	Brazil	1927	4	4
Rio Galeão-Tom Jobim	Rio de Janeiro	Brazil	1914	3	2
<b>Salgado Filho</b>	<b>Porto Alegre</b>	<b>Brazil</b>	<b>2962</b>	<b>11</b>	<b>10</b>
<b>Santa Genoveva</b>	<b>Goiania</b>	<b>Brazil</b>	<b>2095</b>	<b>13</b>	<b>12</b>

Airports not in the top 10 regional airports with the highest strength are in bold. The core strength of a node is its number of flights in the maximum  $s$ -core. Local rank is the rank in the large local component. National rank is the airport rank in its country. Airports are ranked from largest to smallest strength.



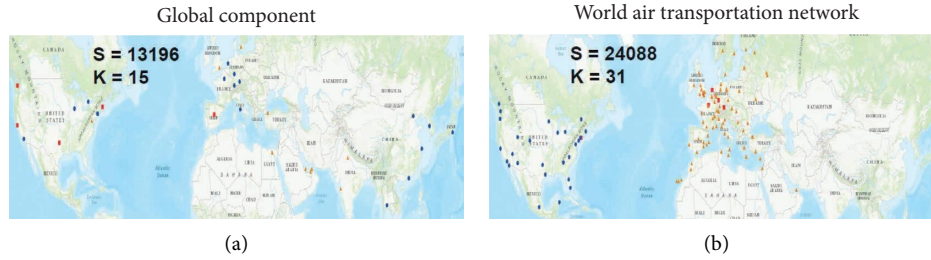


FIGURE 13: (a) The 33 airports in the maximum  $s$ -core of the weighted world air transportation network. (b) The 22 airports in the maximum  $s$ -core of the weighted large global component.  $S$  is the maximum  $s$ -core value, and  $K$  is the maximum  $k$ -core value. Blue points represent the airports belonging to the maximum  $s$ -core and the maximum  $k$ -core, the red points are the airports exclusively in the maximum  $s$ -core, and the yellow points are the airports belonging only to the maximum  $k$ -core.

TABLE 15: 22 max  $s$ -core airports of the global component.

Airport	City	Country	Core strength	Global rank
Frankfurt am Main	Frankfurt	Germany	27927	1
Amsterdam Schiphol	Amsterdam	Netherlands	22890	4
San Francisco	San Francisco	United States	22449	15
Munich	Munich	Germany	22103	7
Chicago O'Hare	Chicago	United States	21893	10
Los Angeles	Los Angeles	United States	21427	11
Charles de Gaulle	Paris	France	20908	2
Newark Liberty	Newark	United States	19757	6
Zurich	Zurich	Switzerland	19578	13
John F Kennedy	New York	United States	18260	3
Narita	Tokyo	Japan	18844	8
Hong Kong	Hong Kong	Hong Kong	17970	20
Beijing Capital	Beijing	China	17445	9
Leonardo da Vinci-Fiumicino	Rome	Italy	15071	14
Shanghai Pudong	Shanghai	China	14494	21
Incheon	Seoul	South Korea	14068	17
Adolfo S Madrid-Barajas	Madrid	Spain	13680	18
Lester B. Pearson	Toronto	Canada	13532	12
Suvarnabhumi	Bangkok	Thailand	13196	16
<b>General E L Logan</b>	<b>Boston</b>	<b>United States</b>	<b>17800</b>	<b>25</b>
<b>Dallas Fort Worth</b>	<b>Dallas-Fort Worth</b>	<b>United States</b>	<b>13588</b>	<b>30</b>
<b>Seattle Tacoma</b>	<b>Seattle</b>	<b>United States</b>	<b>13522</b>	<b>35</b>

Airports not in the top 22 inter-regional airports with the highest strength are in bold. The core strength of a node is its number of links in the maximum  $s$ -core. Global rank is the rank in the largest global component. Airports are ranked from largest to smallest strength.

flights and a moderate number of destinations compared with the other airports in the max  $s$ -core.

*9.3. Comparison with the World Air Transportation Network.* Figure 13(b) represents the max  $s$ -core of the global air transportation network. With 27 airports, North America dominates. There are also five airports in Europe and one in Japan. There are 11 airports listed in Table 16 in the max  $s$ -core of the North and Central America-Caribbean region. Minneapolis-St Paul Airport, with its 24088 flights, has the lowest traffic. Seven airports of the max  $s$ -core of the world air network are not in the top 33 strength airports. The max  $s$ -core global air network has half as many airports as the max  $k$ -core. Moreover, the airports in the max  $s$ -core are mostly absent in the max  $k$ -core. Indeed, the max  $k$ -core is more concentrated in Europe. These results confirm the orientation for high traffic in the USA and high number of

destinations in Europe. Overall, with their high traffic, North American airports lead the weighted world air transportation network, while European airports dominate the unweighted world air transportation network with their high number of destinations. The component structure eliminates this disparity and reveals other important airports worldwide.

## 10. Discussion and Conclusion

This paper investigates the relationship between the weighted and unweighted worldwide air transport network and its impact on its component structure. Table 17 summarizes the main findings. Overall, the weighted network contains fewer components. In both cases, the large local components cover distinct geographical areas. However, their geographical repartition differs slightly. Indeed, the weighted network has five large local components, while the

TABLE 16: 11 airports of the weighted world air transportation network which are not in the max  $s$ -core of the North and Central America-Caribbean component.

Airport	City	Country	Core strength	Worldwide rank	Local rank	Global rank
Frankfurt am Main	Frankfurt	Germany	43819	7	5	1
Charles de Gaulle	Paris	France	38975	6	3	2
Newark Liberty	Newark	United States	33703	9	14	6
George Bush Houston	Houston	United States	29454	20	16	33
Detroit M Wayne County	Houston	United States	28779	26	20	46
Amsterdam Schiphol	Amsterdam	Netherlands	27762	12	5	5
<b>Narita</b>	<b>Tokyo</b>	<b>Japan</b>	<b>25972</b>	<b>38</b>	<b>22</b>	<b>8</b>
<b>Vancouver</b>	<b>Vancouver</b>	<b>Canada</b>	<b>25092</b>	<b>49</b>	<b>33</b>	<b>31</b>
<b>Licenciado Benito Juarez</b>	<b>Mexico City</b>	<b>Mexico</b>	<b>25035</b>	<b>52</b>	<b>31</b>	<b>53</b>
John F Kennedy	New York	United States	24839	8	4	3
Munich	Munich	Germany	24458	21	6	7

Airports not in the top 33 worldwide airports with the highest strength are in bold. The core Strength of a node is its number of flights in the maximum  $s$ -core. Global rank is the rank in the large local component.

unweighted network contains 7. Three components are quite similar (North and Central America-Caribbean, Africa-Middle East-Southern Asia, and South America). The two other components (Europe-Russia-Central Asia and East-Southeast Asia-Oceania) group neighbor components of the unweighted network. Russia-Central Asia is attached to Europe, while Oceania joins East-Southeast Asia. This is due to the substantial traffic between these regions linked to their economic integration. One can observe the same type of phenomenon more locally. Indeed, some major airports integrate regions with which they share a high proportion of their flights. For example, John F Kennedy Airport in the United States belongs to the Europe-Russia-Central Asia component, London Heathrow is in the North and Central America-Caribbean component, and Frankfurt Airport in Europe is in the Africa-Middle East-South Asia component.

Although the weighted network has more global components than the unweighted, there is also a single large global component in both cases. Although they are of comparable size (around 20% of the world's airports), their content is quite different. Indeed, when components merge, many airports that were linking them disappear from the global component. Additionally, new airports emerge when an airport does not belong to its natural geographical area. For example, most of the airports linked to John F Kennedy located outside the Europe-Russia-Central Asia component integrate the global component.

Analysis of the average weighted clustering coefficient reveals that it is independent of the degree for the large local components. Furthermore, their values are always lower than their unweighted equivalent. Low-traffic airports form triplets in the components and the global air network. In contrast, the average unweighted clustering coefficient decreases monotonically with the degree. Indeed, low-degree nodes tend to have a higher clustering coefficient than hubs. It reflects the lack of adequacy of the hub and spoke configuration for rerouting passengers when necessary. The Africa-Middle East-Southern Asia and the South America components have fewer interconnected hubs than the other local components. Consequently, their configuration is less hub and spoke. The global component exhibits similar behavior. However, the hub and spoke effect is more

pronounced. Indeed, as it contains more hubs, it also has fewer triplets accentuating the rerouting inefficiency of the component.

According to the evolution of the average weighted degree-degree correlation as a function of degree, one can classify the components into two categories. The first category includes the Africa-Middle East-Southern Asia Component. In that case, the degree of the nodes does not exert a significant influence. The second category contains the four other local components and the global component. The weighted degree-degree correlation tends to decrease in these components as the degree increases. The high-degree nodes link more and more with small-strength nodes. In other words, as the degree increases, these components' hub and spoke configuration is more pronounced. Africa-Middle East-Southern Asia component departs from this behavior because air transportation is less mature in this region. The world air transportation network exhibits similar behavior to the Africa-Middle East-Southern Asia component. The same observations are also valid for the unweighted network suggesting that weights are not essential for this property. Additionally, for all the local components and the global air network, it appears that  $K_w(i) \geq K(i)$ . Consequently, airports with high degrees manage a higher number of flights.

The strength of the components follows a log-normal distribution with heavy-tailed characteristics. The strength as a function of degree shows that numerous hubs in the Europe-Russia-Central Asia component manage a comparable number of flights. Some hubs have a lot of traffic in the North-Central America-Caribbean and East-Southeast Asia-Oceania components. Compared to the other components, Africa-Middle East-Southern Asia and South America have fewer hubs and less traffic. However, these hubs handle a high fraction of flights. The strength distribution of the global component and the world air transportation network is also log-normal. Moreover, their hubs also manage most of the traffic.

The centrality analysis shows that the top five high-strength airports are usually in the leading countries of their region (the USA in North and Central America-Caribbean, China in East and Southeast Asia-Oceania, India in Africa-Middle East-Southern Asia, and Brazil in South

TABLE 17: Summary of the main results.

	Weighted	Unweighted
Community structure	Algorithms Communities	Louvain 27 communities
Component structure	Local components	7 large components North and Central America-Caribbean Africa-Middle East-Southern Asia South America
	Global components	Europe Russia East and Southeast Asia Oceania 20 small components 1 large 9 small components
Average clustering coefficient versus degree	Local component	Decrease as a function of degree
	Global component	Decrease as a function of degree
	Worldwide	Decrease as a function of degree
Average degree-degree correlation versus degree	Local components	North and Central America-Caribbean, South America, Europe-Russia-Central Asia, and the East and Southeast Asia-Oceania are more and more disassortative as the degree increases
	Global component	Independent of the degree for the Africa-Middle East-Southern Asia component
	Worldwide	Disassortativity increases with the degree
Degree strength distribution	Local components	Independent of the degree
	Global component	The Africa-Middle East-Southern Asia, Oceania, and the Russia-Central Asia-Transcaucasia components follow a log-normal law
	Worldwide	The North America-Caribbean and the East and Southeast Asia components follow stretched exponential power law
Centrality	Local components	Truncated power law
	Global component	Truncated power law
	Worldwide	Top strength airports and top degree airports of the North and Central America-Caribbean and the East and Southeast Asia-Oceania components are identical
		Top strength airports and top degree airports of the Europe-Russia-Central Asia, Africa-Middle East-Southern Asia, and the South America components are different
		Top strength airports and top degree airports of the components are identical
		Top strength airports and top degree airports of the components are different

TABLE 17: Continued.

	Weighted	Unweighted
	Some airports in the USA and Brazil cover the s-core and the k-core of the North and Central Asia-America and the South America	
Local components	The s-core and the k-core of Europe-Russia-Central Asia and Africa-Middle East-Southern Asia are distributed between several countries	Mainly China airports cover the k-core of East and Southeast Asia-Oceania
Global component	The s-core of the East and Southeast Asia-Oceania is distributed between the two subregions 3 regions (North and Central Asia-America, Europe-Russia-Central Asia, and East and Southeast Asia) cover the s-core	Covered by all the components
Worldwide	North and Central Asia-America component covers mainly the s-core	Europe-Russia-Central Asia component covers mainly the k-core

America). It is more homogeneous in the Europe-Russia-Central Asia component. Furthermore, airports included in components far from their geographical areas are in the top five high-strength airports. The top high-degree airports serve higher number of flights in the North and Central America-Caribbean and the East and Southeast Asia components. It is not the case for the other local components.

The top high-strength inter-regional airports are in several countries except for the USA, which dominates the inter-regional traffic in the North and Central America-Caribbean area. The top five strength inter-regional airports differ from the leading regional airports. It shows that airports are more or less specialized in regional or inter-regional destinations. Furthermore, with a high fraction of the top inter-regional airports, Europe leads the inter-regional traffic. In addition, the top high-degree airports and high-strength airports are similar. Centrality analysis of the world air transportation network shows that North American airports dominate the traffic while Europe-Russia-Central Asia dominates the destinations.

Two categories appear in the max  $s$ -core analysis of the local components. The first includes the Europe-Russia-Central Asia and the East and Southeast Asia components, with the maximum  $s$ -core covering several countries. In the second category, a few countries (one to three) concentrate on the high traffic of the component. The max  $s$ -core of the local components contains far fewer airports (half or one-third) than the max  $k$ -core. Thus, few airports share numerous flights among the airports in the local components with several routes. The maximum  $s$ -core of the global component covers several countries from the North and Central America-Caribbean, Europe-Russia-Central Asia, and the East and Southeast Asia-Oceania regions. These three regions serve the most inter-regional flights. In contrast to the local components, the maximum  $s$ -core and  $k$ -core of the global component have comparable sizes. Consequently, the inter-regional hubs of these components concentrate several numbers of flights.

The max  $s$ -core of the global air network is mainly located in the USA, while the max  $k$ -core airports are in Europe. Then, the most important airports in the  $s$ -core and the  $k$ -core are blurred when considering the global air network instead of the component structure.

This comparative analysis illustrates the essential contribution of the component structure representation for uncovering the regional and inter-regional similarities and differences of the world air transportation network. Indeed, typical network properties have been designed for networks with a homogeneous density. Because of their local density variations, some characteristics can be blurred, hence the importance of decoupling local from global analysis. This representation opens multiple research directions. In future work, we plan to exploit the component structure to gain a better understanding of the robustness of the air transportation network against targeted attacks. Indeed, one can design tailored attacks on the components and inspect their global, regional, or inter-regional impact.

## Data Availability

These data were given to us by other researchers who bought them and did not put them online.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## References

- [1] M. Jusup, P. Holme, K. Kanazawa et al., "Social physics," *Physics Reports*, vol. 948, pp. 1–148, 2022.
- [2] M. Rosas-Casals, "Power grids as complex networks: topology and fragility," in *Proceedings of the 2010 Complexity in Engineering*, pp. 21–26, IEEE, Rome, Italy, February, 2010.
- [3] W. L. Shang, Y. Chen, C. Song, and W. Y. Ochieng, "Robustness analysis of urban road networks from topological and operational perspectives," *Mathematical Problems in Engineering*, vol. 2020, Article ID 5875803, 12 pages, 2020.
- [4] W.-L. Shang, Y. Chen, H. Bi, H. Zhang, C. Ma, and W. Y. Ochieng, "Statistical characteristics and community analysis of urban road networks," *Complexity*, vol. 2020, Article ID 6025821, 21 pages, 2020.
- [5] W.-L. Shang, Z. Gao, N. Daina et al., "Benchmark analysis for robustness of multi-scale urban road networks under global disruptions," *IEEE Transactions on Intelligent Transportation Systems*, pp. 1–11, 2022.
- [6] B. Qian and N. Zhang, "Topology and robustness of weighted air transport networks in multi-airport region," *Sustainability*, vol. 14, no. 11, p. 6832, 2022.
- [7] X. Bao, P. Ji, W. Lin, M. Perc, and J. Kurths, "The impact of covid-19 on the worldwide air transportation network," *Royal Society Open Science*, vol. 8, no. 11, Article ID 210682, 2021.
- [8] I. M. Diop, C. Cherifi, C. Diallo, and H. Cherifi, "Revealing the component structure of the world air transportation network," *Applied Network Science*, vol. 6, no. 1, pp. 92–50, 2021.
- [9] S. Fortunato and D. Hric, "Community detection in networks: a user guide," *Physics Reports*, vol. 659, pp. 1–44, 2016.
- [10] H. Cherifi, P. Gergely, B. K. Szymanski, and X. Lu, "On community structure in complex networks: challenges and opportunities," *Applied Network Science*, vol. 4, no. 1, pp. 1–35, 2019.
- [11] M. E. J. Newman, "Detecting community structure in networks," *The European Physical Journal B (EPJ B) Condensed Matter*, vol. 38, no. 2, pp. 321–330, 2004.
- [12] S. P. Borgatti and M. G. Everett, "Models of core/periphery structures," *Social Networks*, vol. 21, no. 4, pp. 375–395, 2000.
- [13] S. H. Lee, M. Cucuringu, and M. A. Porter, "Density-based and transport-based core-periphery structures in networks," *Physical Review A*, vol. 89, no. 3, Article ID 032810, 2014.
- [14] X. Zhang, T. Martin, and M. E. J. Newman, "Identification of core-periphery structure in networks," *Physical Review A*, vol. 91, no. 3, Article ID 032803, 2015.
- [15] S. Kojaku and N. Masuda, "Finding multiple core-periphery pairs in networks," *Physical Review A*, vol. 96, no. 5, Article ID 052313, 2017.
- [16] S. Rajeh, M. Savonnet, E. Leclercq, and H. Cherifi, "Characterizing the interactions between classical and community-aware centrality measures in complex networks," *Scientific Reports*, vol. 11, no. 1, pp. 10088–10115, 2021.
- [17] R. Guimera, S. Mossa, A. Turttschi, and L. A. Amaral, "The worldwide air transportation network: anomalous centrality, community structure, and cities' global roles," *Proceedings of*



- the National Academy of Sciences*, vol. 102, no. 22, pp. 7794–7799, 2005.
- [18] M. Zanin and F. Lillo, “Modelling the air transport with complex networks: a short review,” *The European Physical Journal Special Topics*, vol. 215, no. 1, pp. 5–21, 2013.
- [19] T. K. Y. Cheung, C. W. H. Wong, and A. Zhang, “The evolution of aviation network: global airport connectivity index 2006–2016,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 133, Article ID 101826, 2020.
- [20] G. Bagler, “Analysis of the airport network of India as a complex weighted network,” *Physica A: Statistical Mechanics and Its Applications*, vol. 387, no. 12, pp. 2972–2980, 2008.
- [21] Z. Xu and R. Harriss, “Exploring the structure of the U.S. intercity passenger air transportation network: a weighted complex network approach,” *Geojournal*, vol. 73, no. 2, pp. 87–102, 2008.
- [22] M. M. Hossain and S. Alam, “A complex network approach towards modeling and analysis of the Australian airport network,” *Journal of Air Transport Management*, vol. 60, pp. 1–9, 2017.
- [23] A. Barrat, M. Barthelemy, R. PastorSatorras, and A. Vespignani, “The architecture of complex weighted networks,” *Proceedings of the National Academy of Sciences*, vol. 101, no. 11, pp. 3747–3752, 2004.
- [24] Flightaware, “Real-time worldwide flight traffic,” 2020, <https://flightaware.com/>.
- [25] L. G. A. Alves, A. Aleta, F. A. Rodrigues, Y. Moreno, and L. A. Nunes Amaral, “Centrality anomalies in complex networks as a result of model over-simplification,” *New Journal of Physics*, vol. 22, no. 1, Article ID 013043, 2020.
- [26] G. K. Orman, L. Vincent, and H. Cherifi, “Qualitative comparison of community detection algorithms,” in *Proceedings of the International conference on digital information and communication technology and its applications*, pp. 265–279, Springer, Dijon, France, June, 2011.
- [27] V. D. Blondel, J.-L. Guillaume, R. Lambiotte, D. Lambiotte, and E. Lefebvre, “Fast unfolding of communities in large networks,” *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2008, no. 10, Article ID P10008, 2008.
- [28] S. Sobolevsky, R. Campari, A. Belyi, and C. O. Ratti, “General optimization technique for high-quality community detection in complex networks,” *Physical Review A*, vol. 90, no. 1, Article ID 012811, 2014.
- [29] M. Jebabli, H. Cherifi, C. Cherifi, and A. Hamouda, “Community detection algorithm evaluation with ground-truth data,” *Physica A: Statistical Mechanics and its Applications*, vol. 492, pp. 651–706, 2018.
- [30] M. Chen, T. Nguyen, and B. K. Szymanski, “On measuring the quality of a network community structure,” in *Proceedings of the 2013 International Conference on Social Computing*, pp. 122–127, IEEE, Alexandria, VA, USA, September, 2013.
- [31] M. E. J. Newman, “Modularity and community structure in networks,” *Proceedings of the National Academy of Sciences*, vol. 103, no. 23, pp. 8577–8582, 2006.
- [32] E. Ravasz and A.-L. Barabási, “Hierarchical organization in complex networks,” *Physical Review*, vol. 67, no. 2, Article ID 026112, 2003.
- [33] L. Lü, D. Chen, X. L. Ren et al., “Vital nodes identification in complex networks,” *Physics Reports*, vol. 650, pp. 1–63, 2016.
- [34] K. Das, S. Samanta, M. Pal, and M. Pal, “Study on centrality measures in social networks: a survey,” *Social Network Analysis and Mining*, vol. 8, no. 1, p. 13, 2018.
- [35] Z. Ghalmane, M. ElHassouni, C. Cherifi, and H. Cherifi, “Centrality in modular networks,” *EPJ Data Science*, vol. 8, no. 1, p. 15, 2019.
- [36] W. Webber, A. Moffat, and J. Zobel, “A similarity measure for indefinite rankings,” *ACM Transactions on Information Systems*, vol. 28, no. 4, pp. 1–38, 2010.
- [37] M. Eidsaa and E. Almaas, “Investigating the relationship between k-core and s-core network decompositions,” *Physica A: Statistical Mechanics and Its Applications*, vol. 449, pp. 111–125, 2016.
- [38] Z. Zhou, H. Huang, Q.-S. Hua, D. Yu, H. Jin, and X. Fu, “Core decomposition and maintenance in weighted graph,” *World Wide Web*, vol. 24, no. 2, pp. 541–561, 2021.
- [39] Y.-X. Kong, G.-Y. Shi, R.-J. Wu, and Y.-C. Zhang, “k-core: theories and applications,” *Physics Reports*, vol. 832, pp. 1–32, 2019.
- [40] Y. X. Kong, G.-Y. Shi, R.-J. Wu, and Y.-C. Zhang, “k-core: theories and applications,” *Physics Reports*, vol. 832, pp. 1–32, 2019.