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Research Article

Decision-Making Techniques Based on *q*-Rung Orthopair Probabilistic Hesitant Fuzzy Information: Application in Supply Chain Financing

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The influence of COVID-19 on individuals, businesses, and corporations is indisputable. Many markets, particularly financial markets, have been severely shaken and have suffered significant losses. Significant issues have arisen in supply chain networks, particularly in terms of financing. The COVID-19 consequences had a significant effect on supply chain financing (SCF), which is responsible for finance supply chain components and improved supply chain performance. The primary source of supply chain financing is financial providers. Among financial providers, the banking sector is referred to as the primary source of financing. Any hiccup in the banking operational systems can have a massive influence on the financing process. In this study, we attempted to comprehend the key consequences of the COVID-19 epidemic and how to mitigate COVID-19's impact on Pakistan's banking industry. For this, three extended hybrid approaches which consists of TOPSIS, VIKOR, and Grey are established to address the uncertainty in supply chain finance under *q*-rung orthopair probabilistic hesitant fuzzy environment with unknown weight information of decision-making experts as well as the criteria. The study is split into three parts. First, the novel *q*-rung orthopair probabilistic hesitant fuzzy (qROPHF) entropy measure is established using generalized distance measure under qROPHF information to determine the unknown weights information of the attributes. The second part consists of three decision-making techniques (TOPSIS, VIKOR, and GRA) in the form of algorithm to tackle the uncertain information under qROPHF settings. Last part consists of a real-life case study of supply chain finance in Pakistan to analyze the effects of emergency situation of COVID-19 on Pakistani banks. Therefore, to help the government, we chose the best alternative form list of consider five alternatives (investment, government support, propositions and brands, channels, and digital and markets segments) by using proposed algorithm that minimize the effect of COVID-19 on supply chain finance of Pakistani banks. The results indicate that the proposed techniques are applicable and effective to cope with ambiguous data in decision-making challenges.

1. Introduction

The origins of supply chain finance (SCF) may be traced back to the 1970s, when the influence of trade-credit adjustments and inventory regulations on net cash flow was explored by Budin and Eapen [1]. At the turn of the twenty-first century, the first formal definition of SCF appeared. Stemmler [2] identified financial flows into the physical supply chain as a key SCF feature. According to the findings of this study, SCF is an essential component of supply chain management.

In recent years, SCF has played a crucial role in operational and financial transactions and has attracted the interest of academics and business [3–5]. In literature, there are various definitions of SCF. The SCF is defined as the intercompany optimization of financing to maximize the value of all participating firms by consolidating financing activities with consumers, suppliers, and service providers [6]. Many authors contribute many techniques to tackle the supply chain financing problems [7–10].

Buyers and suppliers are looking for ways to improve working capital efficiency and liberate cash stuck in the financial supply chain in the face of an economic slump caused by a global pandemic and the increasing complexity of global supply chains. Integration of financial and physical supply chains is required in a global economy where supply networks are becoming increasingly complex [11]. In integrating financial and physical supply chains, financial service providers serve a significant role in meeting capital requirements throughout the supply chain [12]. Due to the strategic relationship between suppliers, purchasers, and financial service providers, it would be difficult for either side to fail to deliver on mutual contractual promises [13]. Among all financial service providers, banks are the conventional and primary source of financial resources for supply chain finance. According to [14], about 30% of finance supply in the supply chain finance belongs to global/regional banks. Also, 50% of this finance supply belongs to domestic banks. Anything that affects financial networks has unavoidable and irreversible consequences for supply chain financing activity. Due to the emergence of the coronavirus (COVID-19) in the early 2020s, numerous markets were hit by the pandemic's ramifications. Financial markets and banks were no exception. As a consequence, banks' financial supply had significant challenges. A vast number of banks have been exposed to a variety of risks and uncertainties. Banks should develop and coordinate emergency/crisis management implementation plans so that they can be carried out in an emergency with the least amount of disruption to the bank. The unpredictability of loss in the process through which financial institutions such as banks deliver supply chain financial services is known as supply chain finance risk [15]. As a result, in order to minimize risk, a risk assessment is required to investigate the hazards of supply chain financing. Risk recognition, evaluation, measurement, and control are the fundamental aspects of risk management in SCF [16].

As previously stated, SCF and particularly financial suppliers like banks have faced numerous uncertainties, particularly in emergency situations such as pandemics. Fuzzy set [17] is a handy and reliable strategy to cope with such ambiguities in decision-making problems [7–9, 18, 19]. By proposing the notion of positive and negative grades, Atanassov [20] established the fortuitous of intuitionistic fuzzy sets (IFSs) as a refinement of FSs. IFS theory has been applied by a number of academics in a variety of fields [21–23]. Furthermore, Yager [24] developed the Pythagorean fuzzy set (PFS) theory that is more effective and superior to IFS theory in resolving with challenges that IFS theory cannot answer. The PFS has been applied by a number of academics in a variety of fields [25–27]. However, if a decision-maker faces such obstacles that the sum of the squares of both degrees cannot be surpassed from a unit interval, there are several concerns. Furthermore, Yager discovered the theory of q-rung orthopair fuzzy set (QROFS), which employs the rule that the sum of the q-power of both degrees cannot beyond from the unit interval and is more effective and superior to PFS theory in answering issues that PFS theory cannot respond.

The limitation of QROFS would be that the total of the membership degree (MD) power and the nonmembership degree (NMD) power of q may be less than or identical to one. Clearly, the greater the rung q, the more the bounding condition is fulfilled by orthopair and thus the greater the space of fuzzy data that can be expressed by QROFSs [28]. In ability to cope with both the complete absence of clarity and flouted information, QROFSs are better and more efficient than IFS and PFS. Figure 1 depicts the distinction between them.

Many researchers contribute to the QROFS to tackle the uncertain and ambiguity in decision-making problems (DMPs) such as Wei et al. [29] established the novel list of aggregation operators (AGOp) based on QROF information and discussed their application in DMPs. Liu and Wang [30] proposed algebraic norm-based lists of QROF AGOp under QROF environment. Liu and Wang [31] proposed Archimedean Bonferroni mean-based lists of QROF AGOp under QROF environment and discussed their application to tackle uncertainty in DMPs. Peng et al. [32] established the list of exponential operations-based AGOp under QROF environment. Shu et al. [33] presented the integrations rules using QROFs and also discussed applicability in DMPs. Peng and Liu [34] discussed the information measures under q-ROF settings. Ali [35] discussed the novel operational rules and algebraic relations under QROF information. Xing et al. stated that [36], based on the QROF interaction, a new multicriteria group decision-making methodology has been developed. Gao et al. [37] stated the continuities, derivatives, and differentials of QROF functions and discussed in their applicability in real word problems. Wei et al. [38] established the novel list of aggregation operators based on Maclaurin symmetric mean under QROF information and also discussed their application related to emerging technology. Habib et al. [39] proposed the fuzzy competition graphs under QROF settings and also presented the real-life application related to soil ecosystem. Liu et al. [40] established the novel list of AGOp based on power Maclaurin symmetric mean under QROF information. Yang and Pang [41] proposed partitioned Bonferroni mean-based lists of QROF AGOp under QROF environment. Liao et al. [42] established the QROF GLDS approach for BE angel capital investment assessment in China.

Hesitancy is a common occurrence in our universe. In real life, choosing one of the finest options (alternative) under list of attributes is difficult. Experts are having difficulty making decisions due to the data's ambiguity and hesitation. Torra [43] proposed the concept of hesitant fuzzy set to overcome the hesitancy (HFS). Khan et al. [44] proposed an improved version of hesitating fuzzy sets called

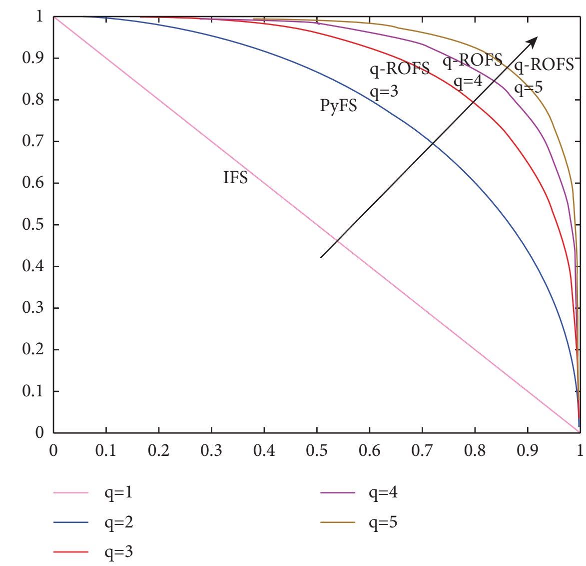

Figure 1: q-ROFSs space.

the Pythagorean hesitant fuzzy set. Overhead ideas can be utilized to efficiently determine randomness. However, the frameworks described above are incapable of dealing with circumstances in which a specialist's rejection is a critical factor in the decision-making process. For example, in the staffing procedure, a board of five professionals is regarded to determine the best applicant, and three of them are rejected from making any judgement. While using existing strategies to examine the information, the number of decision-makers is kept to three rather than five, i.e., the rejected professionals are completely ignored, and the choice is made solely on the basis of the three professionals' preferences. It results in significant data loss and may result in inadequate grades. To deal with such situations, Xu and Zhou [45] proposed a new concept known as probabilistic hesitant fuzzy sets (PHFSs).

As a result of the foregoing motivation, this study provides three novel expanded decision-making techniques, which are "technique for order of preference by similarity to ideal solution" (TOPSIS), "VlseKriterijumska Optimizacija I Kompromisno Resenje" (VIKOR) and "Grey" (GRA) approaches to tackle the uncertain information in real-life decision-making problems under q-rung orthopair probabilistic hesitant fuzzy environment with unknown weight information. To determine the unknown weight information of the decision-making experts as well as the weight information of the consider attributes/criteria, entropy measure based on generalized distance measures is provided under q-rung orthopair probabilistic hesitant fuzzy environment. Meanwhile, this study provides a weight calculation technique that can effectively deal with the bias expert's extreme value and solve the problem of huge differences of opinion among experts. Furthermore, the proposed method will produce a more accurate evaluation result by employing the improved relative closeness formula. As a result, the following are the primary characteristics of this paper's innovations: First, it introduces the q-rung orthopair probabilistic hesitant fuzzy set, which is a novel idea (q-ROPHFS). The motivation for the new concept is that while only positive membership degrees are considered with probabilistic information in probabilistic hesitant fuzzy sets. However, the innovative concept of q-ROPHFS is defined by the presence of both positive and negative hesitant grades, with the restriction that the square sum of both hesitant grades be not more than one.

The decision-makers in q-ROPHFS are limited to a single domain and ignore the negative membership degree and its likelihood of occurrence. In comparison to others, every negative reluctant membership degree has some preferences. For example, in DM-problems, decision-makers may express their view in the form of multiple alternative values; for example, if one DM delivers values 0.3, 0.4, and 0.6 for positive membership degree with matching preference values 0.1 and 0.9, the other may reject. The proposed concept considers the possibility of a higher rejection level with reluctance. Despite HFSs and q-ROHFSs, the information about chances will decrease. The probability of occurrence with positive and negative membership degrees provides extra information about the DMs' level of disagreement. The innovative notion of Pythagorean probabilistic hesitant fuzzy set is proposed to deal with uncertainty in decision-making situations. The following is a list of the paper's originality:

(1) q-ROPHFS is a generalization of the PHFS

(2) q-ROPHF extended TOPSIS and VIKOR methods-based algorithms using unidentified weight data of attributes as well as the decision-making experts

(3) The q-ROPHF multicriteria group decision-making problems are tackled by the proposed three generalized decision-making techniques

(4) We apply the distance measures and entropy measure to determine the unknown information of weights

(5) To demonstrate the efficacy and application of the suggested technique, a case study related to supply chain finance is considered to analyze the effect of emergency situation of COVID-19 on Pakistani banks

(6) A comparative study is proposed based on the GRY method to validate the proposed methodologies

The rest of this manuscript is organized as follows. Section 2 briefly retrospect's some basic concepts of q-ROFSs, HFSs, and probabilistic fuzzy set theory. A novel notion of q-rung orthopair probabilistic hesitant fuzzy sets is presented in Section 3. Section 4 highlights a novel entropy measures based on the generalized distance measures under QROPHFSs to determine the unknown weight information of the attributes. Section 5 is devoted the three decision-making methodologies based on TOPSIS, VIKOR, and GRA to address the uncertainty in decision-making problems and also presents the numerical illustration related to finance supply chain is consider analyzing the effects of emergency situation of COVID-19 on Pakistani banks in Section 6. The comparative study of the proposed technique using extended GRY method is presented in Section 7. Section 8 concludes this study.

2. Preliminaries

In this segment, we briefly recall the rudiments of fuzzy sets and their generalized structures like intuitionistic FSs (IFSs), q-rung orthopair FSs (QROFSs), hesitant FSs (HFSs), and q-rung orthopair hesitant FSs (q-ROHFSs). The following are some related definitions.

Definition 1 (see [28]). Let S be a fixed set. A q-ROFSs in S is defined as follows:

$$s = \{(m, P_s(m), N_s(m)) \mid m \in S\}, \tag{1}$$

for each $m \in S$, $P_s(m)$ and $N_s(m) \in [0,1]$ are known to be positive and negative MGs, respectively. In addition, $0 \le (P_s(m))^q + (N_s(m))^q \le 1$ for all $m \in S$.

We shall symbolize the q-rung orthopair fuzzy number (q-ROFN) by a pair $s = (P_s, N_s)$. Conventionally, $Ɇ_m = \sqrt[q]{1 - P_s^q - N_s^q}$ is the degree of hesitancy of $m \in S$ to s.

Definition 2 (see [28]). Let $s_1, s_2, s \in q-$ROFNs, the operational rules are defined as follows:

(1) $s_1 \le s_2$ iff $P_{s_1}(m) \le P_{s_2}(m)$ and $N_{s_1}(m) \ge N_{s_2}(m)$ $\forall$ $m \in S$

(2) $s_1 = s_2$ iff $s_1 \le s_2$ and $s_2 \le s_1$

(3) $s_1 \cup s_2 = \{\max(P_{s_1}(m), P_{s_2}(m)), \min(N_{s_1}(m), N_{s_2}(m)) \mid m \in S\}$

(4) $s_1 \cap s_2 = \{\min(P_{s_1}(m), P_{s_2}(m)), \max(N_{s_1}(m), N_{s_2}(m)) \mid m \in S\}$

(5) $s^c = \{N_s(m), P_s(m) \mid m \in S\}$

Definition 3 (see [28]). Let $s_1, s_2, s \in q-$ROFNs with $h > 0$, the basic operational rules are defined as follows:

(1) $s_1 \otimes s_2 = (P_{s_1} P_{s_2}, \sqrt[q]{(N_{s_1})^q + (N_{s_2})^q - (N_{s_1})^q (N_{s_2})^q});$

(2) $s_1 \oplus s_2 = (\sqrt[q]{(P_{s_1})^q + (P_{s_2})^q - (P_{s_1})^q (P_{s_2})^q}, N_{s_1} N_{s_2});$

(3) $h \cdot s = (\sqrt[q]{1 - (1 - (P_s)^q)^h}, (N_s)^h);$

(4) $(s)^h = ((P_s)^h, \sqrt[q]{1 - (1 - (N_s)^q)^h}).$

Definition 4 (see [43]). Let S be a fixed set. A HFSs in S is defined as follows:

$$s = \{(m, P_s(m)) \mid m \in S\}, \tag{2}$$

for each $m \in S, P_s(m)$ be a set of some values in $[0,1]$ known to be hesitant membership grade (MG).

Definition 5 (see [43]). Let $s_1, s_2 \in$ HFS, the operational rules are defined as follows:

(1) $s_1^c = \cup_{h \in P_{s_1}(m)} \{1 - h\}$

(2) $s_1 \cup s_2 = P_{s_1}(m) \vee P_{s_2}(m) = \cup_{h_1 \in P_{s_1}, h_2 \in P_{s_2}} \max\{h_1, h_2\}$

(3) $s_1 \cap s_2 = P_{s_1}(m) \wedge P_{s_2}(m) = \cup_{h_1 \in P_{s_1}, h_2 \in P_{s_2}} \min\{h_1, h_2\}$

Definition 6. Let S be a fixed set. A PHFS in S is defined as follows:

$$s = \{(m, P_s(m), N_s(m)) \mid m \in S\}, \tag{3}$$

for each $m \in S, P_s(m)$ and $N_s(m) \in [0,1]$ are the set of some values in $[0,1]$ known to be positive and negative hesitant MGs. In addition $(\max(P_s(m)))^2 + (\min(N_s(m)))^2 \le 1$ and $(\min(P_s(m)))^2 + (\max(N_s(m)))^2 \le 1$.

We will use pair $s = (P_s, N_s)$ to represent the Pythagorean hesitant fuzzy number (PHFN).

Definition 7. Let $s_1, s_2 \in$ PHFS, the operational rules are defined as follows:

(1) $s^c = \{N_s(m), P_s(m) \mid m \in S\}$

(2) $s_1 \cup s_2 = \left\{ \begin{matrix} P_{s_1}(m) \vee P_{s_2}(m), \\ N_{s_1}(m) \wedge N_{s_2}(m) \end{matrix} \right\} = \left\{ \begin{matrix} \cup_{h_1 \in P_{s_1}, h_2 \in P_{s_2}} \max(h_1, h_2) \\ \cup_{r_1 \in N} \min(r_1, r_2) \end{matrix} \right\}$

(3) $s_1 \cap s_2 = \left\{ \begin{matrix} P_{s_1}(m) \wedge P_{s_2}(m), \\ N_{s_1}(m) \vee N_{s_2}(m) \end{matrix} \right\} = \left\{ \begin{matrix} \cup_{h_1 \in P_{s_1}, h_2 \in P_{s_2}} \min(h_1, h_2), \\ \cup_{r_1 \in N_{s_1}, r_2 \in N_{s_2}} \max(r_1, r_2) \end{matrix} \right\}$

Definition 8. Let S be a fixed set. A PPHFS s in S is defined as follows:

$$s = \left\{ \left(m, \frac{P_s(m)}{p_s}, \frac{N_s(m)}{n_s} \right) \mid m \in S \right\}, \tag{4}$$

for each $m \in S, P_s(m)$ and $N_s(m) \in [0,1]$ are the set of some values in $[0,1]$ and $p_\circ \epsilon n_\circ$ are probablistics terms. Here, $P_\circ(m)/p_\circ$ and $N_\circ(m)/n_\circ$ are known to be positive and negative probabilistic hesitant MGs. In addition $0 < \alpha_i, \beta_i < 1$ and $0 < p_i, n_i \le 1$ with $\sum_{i=1}^k p_i \le 1, \sum_{i=1}^k n_i \le 1$ (k is a positive integer to describe the number of elements contained in PPHFS, where $\alpha_i \in P_s(m), \beta_i \in N_s(m), p_i \in p_s, n_i \in n_s$) $(\max(P_s(m)))^2 + (\min(N_s(m)))^2 \le 1$ and $(\min(P_s(m)))^2 + (\max(N_s(m)))^2 \le 1$.

We will use pair $s = (P_s/p_s, N_s/n_s)$ to represent the Pythagorean probabistics hesitant fuzzy number (PPHFN).

Definition 9. Let $s_1 = (P_{s_1}/p_{s_1}, N_{s_1}/n_{s_1})$ and $s_2 = (P_{s_2}/p_{s_2}, N_{s_2}/n_{s_2})$ are two PyPHFNs with $\alpha > 0$, the operational rules are defined as follows:

(1) $s_1 \oplus s_2 = \left\{ \begin{matrix} \cup_{h_1 \in P_{s_1}(m), h_2 \in P_{s_2}(m), p_1 \in p_{s_1}, p_2 \in p_{s_2}} (\sqrt{h_1^2 + h_2^2 - h_1^2 h_2^2 / p_1 p_2}) \\ \cup_{r_1 \in N_{s_1}(m), r_2 \in N_{s_2}(m), n_1 \in n_{s_1}, n_2 \in n_{s_2}} (r_1 r_2 / n_1 n_2) \end{matrix} \right\}$

(2) $s_1 \otimes s_2 = \left\{ \begin{matrix} \cup_{h_1 \in P_{s_1}(m), h_2 \in P_{s_2}(m), p_1 \in p_{s_1}, p_2 \in p_{s_2}} (h_1 h_2 / p_1 p_2) \\ \cup_{r_1 \in N_{s_1}(m), r_2 \in N_{s_2}(m), n_1 \in n_{s_1}, n_2 \in n_{s_2}} (\sqrt{r_1^2 + r_2^2 - r_1^2 r_2^2 / n_1 n_2}) \end{matrix} \right\}$

(3) $\alpha s_1 = \left\{ \cup_{h_1 \in P_{s_1}(m), p_1 \in p_{s_1}} (\sqrt{1 - (1 - h_1^2)^\alpha} / p_1), \cup_{r_1 \in N_{s_1}(m), n_1 \in n_{s_2}} (r_1^\alpha / n_1) \right\}$

(4) $s_1^\alpha = \left\{ \cup_{h_1 \in P_{s_1}(m), p_1 \in p_{s_1}} (h_1^\alpha / p_1), \cup_{r_1 \in N_{s_1}(m), n_1 \in n_{s_2}} (\sqrt{1 - (1 - r_1^2)^\alpha} / n_1) \right\}$

3. q-Rung Orthopair Probabilistic Hesitant Fuzzy Sets

Definition 10. Let S be a fixed set. APPHFS in S is defined as follows:

$$s = \left\{ \left(m, \frac{P_s(m)}{p_s}, \frac{N_s(m)}{n_s} \right) \middle| m \in S \right\}, \quad (5)$$

for each $m \in S$, $P_s(m)$ and $N_s(m) \in [0,1]$ are the set of some values in $[0,1]$ and $p_s \varepsilon n_s$ are probablistics terms, where $P_s(m)/p_s$ and $N_s(m)/n_s$ are known to be positive and negative probabilistic hesitant *MGs*. In addition $0 \le \alpha_i, \beta_i \le 1$ and $0 \le p_i, n_i \le 1$ with $\sum_{i=1}^k p_i \le 1, \sum_{i=1}^k n_i \le 1$ (k is a positive integer to describe the number of elements contained in PPHFS, where $\alpha_i \in P_s(m), \beta_i \in N_s(m), p_i \in p_s, n_i \in n_s$) $(\max(P_s(m)))^q + (\min(N_s(m)))^q \le 1$ and $(\min(P_s(m)))^q + (\max(N_s(m)))^q \le 1$.

We shall symbolize the q-rung orthopair probabilistics hesitant fuzzy number ($q-$ROPHFN) by a pair $s = (P_s/p_s, N_s/n_s)$.

Definition 11. Let $s_1 = (P_{s_1}/p_{s_1}, N_{s_1}/n_{s_1})$ and $s_2 = (P_{s_2}/p_{s_2}, N_{s_2}/n_{s_2})$ are two $q-$ROPHFNs. Then, the basic operational laws are defined as follows:

(1) $s_1 \cup s_2 = \left\{ \begin{array}{c} \cup_{h_1 \in P_{s_1}(m), p_1 \in p_{s_1}} (\max(h_1/p_1, h_2/p_2)) \\ \cup_{r_1 \in N_{s_1}(m), n_1 \in n_{s_2}} (\min(r_1/n_1, r_2/n_2)) \end{array} \right\}$

(2) $s_1 \cap s_2 = \left\{ \begin{array}{c} \cup_{h_1 \in P_{s_1}(m), p_1 \in p_{s_1}} (\min(h_1/p_1, h_2/p_2)) \\ \cup_{r_1 \in N_{s_1}(m), n_1 \in n_{s_0}} (\max(r_1/n_1, r_2/n_2)) \end{array} \right\}$

(3) $s^c = \{N_s/n_s, P_s/p_s\}$

Definition 12. Let $s_1 = (P_{s_1}/p_{s_1}, N_{s_1}/n_{s_1})$ and $s_2 = (P_{s_2}/p_{s_2}, N_{s_2}/n_{s_2})$ are two $q-$ROPHFNs with $\alpha > 0$, then the operation are defined as follows:

(1) $s_1 \oplus s_2 = \left\{ \begin{array}{c} \cup_{h_1 \in P_{s_1}(m), h_2 \in P_{s_2}(m), p_1 \in p_{s_1}, p_2 \in p_{s_2}} (\sqrt[q]{h_1^q + h_2^q - h_1^q h_2^q} / p_1 p_2) \\ \cup_{r_1 \in N_{s_1}(m), r_2 \in N_{s_2}(m), n_1 \in n_{s_1}, n_2 \in n_{s_2}} (r_1 r_2 / n_1 n_2) \end{array} \right\}$

(2) $s_1 \otimes s_2 = \left\{ \begin{array}{c} \cup_{h_1 \in P_{s_1}(m), h_2 \in P_{s_2}(m), p_1 \in p_{s_1}, p_2 \in p_{s_2}} (h_1 h_2 / p_1 p_2), \\ \cup_{r_1 \in N_{s_1}(m), r_2 \in N_{s_2}(m), n_1 \in n_{s_1}, n_2 \in n_{s_2}} (\sqrt[q]{r_1^q + r_2^q - r_1^q r_2^q} / n_1 n_2) \end{array} \right\}$;

(3) $\alpha s_1 = \left\{ \cup_{h_1 \in P_{s_1}(m), p_1 \in p_{s_1}} (\sqrt[q]{1 - (1 - h_1^q)^\alpha} / p_1), \cup_{r_1 \in N_{s_1}(m), n_1 \in n_{s_2}} (r_1^\alpha / n_1) \right\}$

(4) $s_1^\alpha = \left\{ \cup_{h_1 \in P_{s_1}(m), p_1 \in p_{s_1}} (h_1^\alpha / p_1), \cup_{r_1 \in N_{s_1}(m), n_1 \in n_{s_2}} (\sqrt[q]{1 - (1 - r_1^q)^\alpha} / n_1) \right\}$

Definition 13. For any $q-$ROPHFNs $= (P_s(m)/p_s, N_s(m)/n_{s_1})$. The score value of $q-$ROPHFNs is defined as follows:

$$\alpha(s) = \left(\frac{1}{A_s} \sum_{h_i \in P_{s_i}, p_i \in p_s} h_i \cdot p_i \right)^q - \left(\frac{1}{B_s} \sum_{r_i \in N_s, n_i \in n_s} r_i \cdot n_i \right)^q, \quad (6)$$

where A_s stands for the number of entries in $P_S(m)$ and B_s stands for the number of entries in $N_s(m)$.

Definition 14. For any q-ROPHFN $s = (P_s(m)/p_s, N_s(m)/n_{s_1})$. The accuracy value of $q-$ROPHFNs is defined as follows:

$$\beta(s) = \left(\frac{1}{A_s} \sum_{h_i \in P_{s_i}, p_i \in p_s} h_i \cdot p_i \right)^q + \left(\frac{1}{B_s} \sum_{r_i \in N_s, n_i \in n_s} r_i \cdot n_i \right)^q, \quad (7)$$

where A_s stands for the number of entries in $P_S(m)$ and B_s stands for the number of entries in $N_s(m)$.

Definition 15. Let $s_1 = (P_{s_1}/p_{s_1}, N_{s_1}/n_{s_1})$ and $s_2 = (P_{s_2}/p_{s_2}, N_{s_2}/n_{s_2})$ are two $q-$ROPHFNs. Then, using the Definitions 16 *E* 17, comparison of $q-$ROPHFNs are defined as follows:

(1) If $\alpha(s_1) > \alpha(s_2)$, then $s_1 > s_2$

(2) If $\alpha(s_1) = \alpha(s_2)$ and $\beta(s_1) > \beta(s_2)$, then $s_1 > s_2$

4. Development of an Approach under the q-ROPHFSS

The proposed extended distance measures and the weighted extended distance measurements for the q-ROPHFSs are defined in this section. Furthermore, entropy measures for q-ROPHFS are presented for quantitative evaluation of randomness of a q-ROPHFSs.

4.1. Distance Measure for q-ROPHFSs

Definition 16. For any two $q-$ROPHFSss $= \{s_1, s_2, \ldots, s_n\}$ and $r = \{r_1, r_2, \ldots, r_n\}$, where $s_j = (P_s/p_s(x_j), N_s/n_s(x_j))$ and $r_j = (P_r/p_r(x_j), N_r/n_r(x_j))$, $j = 1, 2, \ldots, n$. For any number $\alpha > 0$, the generalized distance measure between s and r as follows:

$$d(s,r) = \left(\frac{1}{2n} \sum_{j=1}^{n} \left(\begin{array}{c} \left| \left(\frac{1}{A_s} \sum_{h_{s_j} \in P_s, p_{s_j} \in p_s}^{A_s} \left(h_{s_j} p_{s_j} \right) \right)^2 - \left(\frac{1}{A_r} \sum_{h_{r_j} \in P_r, p_{r_j} \in p_r}^{A_r} \left(h_{r_j} p_{r_j} \right) \right)^2 \right|^\alpha \\ \left| \left(\frac{1}{B_s} \sum_{v_{s_j} \in N_s, n_{s_j} \in n_s}^{B_s} \left(v_{s_j} n_{s_j} \right) \right)^2 - \left(\frac{1}{B_r} \sum_{v_{r_j} \in N_r, n_{r_j} \in n_r}^{B_r} \left(v_{r_j} n_{r_j} \right) \right)^2 \right|^\alpha \end{array} \right) \right)^{1/\alpha}, \quad (8)$$

where A_s, B_s, A_r, and B_r are the possible numbers of elements in $P_s/p_s, N_s/n_s$, P_r/p_r, and N_r/n_r.

Definition 17. For any two $q-$ ROPHFSss $= \{s_1, s_2, \ldots, s_n\}$ and $r = \{r_1, r_2, \ldots, r_n\}$, where $s_j = (P_s/p_s(x_j), N_s/n_s(x_j))$ and $r_j = (P_r/p_r(x_j), N_r/n_r(x_j)), j = 1, 2, \ldots, n.$ For any number $\alpha > 0$, the generalized distance measure between s and r as follows:

$$d(s,r) = \left(\frac{1}{2n}\sum_{j=1}^{n} w_j \left(\begin{array}{c} \left|\left(\frac{1}{A_s}\sum_{h_{s_j}\in P_s, p_{s_j}\in p_s}^{A_s}\left(h_{s_j}p_{s_j}\right)\right)^2 - \left(\frac{1}{A_r}\sum_{h_{r_j}\in P_r, p_{r_j}\in p_r}^{A_r}\left(h_{r_j}p_{r_j}\right)\right)^2\right|^{\alpha} \\ \left|\left(\frac{1}{B_s}\sum_{v_{s_j}\in N_s, n_{s_j}\in n_s}^{B_s}\left(v_{s_j}n_{s_j}\right)\right)^2 - \left(\frac{1}{B_r}\sum_{v_{r_j}\in N_r, n_{r_j}\in n_r}^{B_r}\left(v_{r_j}n_{r_j}\right)\right)^2\right|^{\alpha} \end{array}\right)\right)^{1/\alpha}, \tag{9}$$

where $w_j\,(j = 1, 2, \ldots, n)$ be the weights vector assinged for each (x_j) and A_s, B_s A_r ES B_r are the possible numbers of elements in $P_s/p_s, N_s/n_s, P_r/p_r \varepsilon N_r/n_r$.

Theorem 18. *Let s and r are two* $q-$ ROPHFSs, *the distance measure has the following constraints:*

$(A1)\; 0 \le d(s,r) \le 1$

$(A2)\; d(s,r) = 1 \Longleftrightarrow s = r$

$(A3)\; d(s,r) = d(r,s)$

Proof. Straightforward. □

*4.2. Entropy Measure for q-*ROPHFSs. In decision-making, the weights information of attributes/criteria is crucial. Many researchers study decision-making challenges in fuzzy settings with incomplete or undetermined weight data of attributes. Entropy measure is a conventional concept to determine attribute weight effectively.

Definition 19. The Shannon entropy information $E(E_1, E_2, \ldots, E_n)$ for $q-$ ROPHFS is defined as follows:

$$E = \frac{-1}{(2\log\ (n))}\sum\nolimits_{i=1}^{n}\left(P_s(\log\ P_s) \times p_s + N_s(\log\ N_s) \times n_s\right). \tag{10}$$

List of some properties of entropy measure as follows:

(1) $E(s) = 0 \Longleftrightarrow s$ is a crisp set

(2) $E(s) = 1 \Longleftrightarrow h_{s_i}(m) = v_{s_i}(m) \forall m \in E$

(3) $E(s) \le E(r)$ if $s \le r$

(4) $E(s) = E(s^c)$

5. Multiattribute Decision-Making Techniques for *q*-ROPHFSs

Let $\mathcal{J} = \{\mathcal{J}_1, \mathcal{J}_2, .., \mathcal{J}_m\}$ be a set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of criteria and r be the number of $DMs, DM_f\,(f = 1, 2, 3, \ldots r)$ to demonstrate their perception about "m" alternatives with respect to the "n" criteria by taking q-ROPHFSs $S_{i_i}^{(f)} = (P_{i_j}^{(f)}, N_{i_j}^{(f)})$. The decision matrix of experts information is presented in Table 1.

$$S^{(f)} = \left[S_{ij}^{(f)}\right]_{m\times n} = \left[\left(\frac{P_{S_{ij}}^{(f)}}{p_{S_{ij}}}, \frac{N_{S_{ij}}^{(f)}}{n_{S_{ij}}}\right)\right]_{m\times n}. \tag{11}$$

5.1. q-ROPHF Extended TOPSIS Method. There are two primary parts to this method. First, a method for calculating the weights of criteria/attributes is presented using the proposed entropy measure for q-ROPHFNs. A ranking procedure based on the degree of similarity to the ideal solution is considered in the last part. Steps are presented as follows for the q-ROPHF extended TOPSIS method:

Step 1: first, we collect the data provided by the decision-makers in the form of q-ROPHFNs.

Step 2: we normalized the information defined by DMs in this step, as the decision matrix may have some benefit and cost parameters altogether, as follows:

$$e_{i_j}^{(f)} = \begin{cases} S_{ij}^{(f)}, & \text{for benefit criteria,} \\ \left(S_{ij}^{(f)}\right)^c, & \text{for cost criteria,} \end{cases}$$
$$f = 1, 2, 3, \ldots \check{r}, i = 1, 2, 3, \ldots, m, j = 1, 2, 3, \ldots, n \text{ where } \left(S_{ij}^{(f)}\right)^c \text{ is complement of } S_{ij}^{(f)}, \text{that is } \left(S_{ij}^{(f)}\right)^c = \left(N_{S_{ij}}^{(f)}, P_{S_{ij}}^{(f)}\right). \tag{12}$$

Table 1: Expert information $S^{(f)}$.

	C_1	C_2	$\cdots$	C_n
J_1	$(P_{S_{11}}^{(f)}/p_{S_{11}}, N_{S_{11}}^{(f)}/n_{S_{11}})$	$(P_{S_{12}}^{(f)}/p_{S_{12}}, N_{S_{12}}^{(f)}/n_{S_{12}})$	$\cdots$	$(P_{S_{1n}}^{(f)}/p_{S_{1n}}, N_{S_{1n}}^{(f)}/n_{S_{1n}})$
J_2	$(P_{S_{21}}^{(f)}/p_{S_{21}}, N_{S_{21}}^{(f)}/n_{S_{21}})$	$(P_{S_{22}}^{(f)}/p_{S_{22}}, N_{S_{22}}^{(f)}/n_{S_{22}})$	$\cdots$	$(P_{S_{2n}}^{(f)}/p_{S_{2n}}, N_{S_{2n}}^{(f)}/n_{S_{2n}})$
J_3	$(P_{S_{31}}^{(f)}/p_{S_{31}}, N_{S_{31}}^{(f)}/n_{S_{31}})$	$(P_{S_{32}}^{(f)}/p_{S_{32}}, N_{S_{32}}^{(f)}/n_{S_{32}})$	$\cdots$	$(P_{S_{3n}}^{(f)}/p_{S_{3n}}, N_{S_{3n}}^{(f)}/n_{S_{3n}})$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
J_m	$(P_{S_{m1}}^{(f)}/p_{S_{m1}}, N_{S_{m1}}^{(f)}/n_{S_{m1}})$	$(P_{S_{m2}}^{(f)}/p_{S_{m2}}, N_{S_{m2}}^{(f)}/n_{S_{m2}})$	$\cdots$	$(P_{S_{mn}}^{(f)}/p_{S_{mn}}, N_{S_{mn}}^{(f)}/n_{S_{mn}})$

Step 3: the weights of the parameters are determined in the following manner by the proposed entropy measure. The calculation of entropy corresponding to each criterion is as follows:

$$\begin{aligned} E(S_j) &= E\left(S_{1j}, S_{2_j}, S_{3j}, .., S_{mj}\right), j = 1, 2, 3, \ldots, n \\ &= \frac{-1}{(2\log\ (n))}\sum_{i=1}^{n} (P_s(\log\ P_s) \times p_s \\ &\quad + N_s(\log\ N_s) \times n_s). \end{aligned} \tag{13}$$

then

$$w(S_j) = \frac{(1 - E(S_j))}{\sum_{j=1}^{n}(1 - E(S_j))}. \tag{14}$$

Thus, weights of criteria are found as follows:

$$w(S_j) = (w(S_1), w(S_2), w(S_3), \ldots, w(S_n))^T. \tag{15}$$

Step 4: in this step, the best alternative according to given list of criteria/attribute are determined.

Step 4(a): determine the weighted $\mathrm{NDM}^{(f)}$ using the weights of the assessed parameters in the following manner:

$$\mathrm{NDM}_{i_j}^{(f)} = w(S_j) \cdot S_{i_j}^{(f)} = \left(\begin{array}{c} \cup_{h_i \in P_{S_{ij}}, p_i \in p_{S_{ij}}} \dfrac{\sqrt[q]{1 - \prod_{i=1}^{m}\left(1 - \left(h_{S_{ij}}^{(f)}\right)^q\right)^{w(S_j)}}}{\prod_{i=1}^{m} p_{S_{ij}}^{(f)}} \\ \\ \cup_{v_i \in N_{S_{ij}}, n_i \in n_{S_{ij}}} \dfrac{\prod_{i=1}^{m}\prod_{i=1}^{m}\left(v_{S_{ij}}^{(f)}\right)^{w(S_j)}}{\prod_{i=1}^{m} n_{S_{ij}}^{(f)}} \end{array} \right). \tag{16}$$

for all $f = 1.2.3 \ldots ř$.

Step 4(b): derive $\mathrm{PIS}^{(f)}$ and $\mathrm{NIS}^{(f)}$ for all weighted $\mathrm{NDM}^{(f)}$, as follows for all $DM^{(f)}$:

$$\begin{aligned} \mathrm{PIS}^{(f)} &= \left\{\mathrm{PIS}_j^{(f)}\right\}_{\widetilde{r\times n}} \\ &= \left\{\left(\mathrm{NDM}_{ij}^{(f)}\right): \mathrm{m}\left[s\left(\mathrm{NDM}_{ij}^{(f)}\right)\right]\right\}, \\ \mathrm{NIS}^{(f)} &= \left\{\mathrm{NIS}_j^{(f)}\right\}_{\widetilde{r\times n}} \\ &= \left\{\left(\mathrm{NDM}_{ij}^{(f)}\right): \mathrm{m}\left[s\left(\mathrm{NDM}_{ij}^{(f)}\right)\right]\right\}. \end{aligned} \tag{17}$$

for $j = 1, 2, 3, \ldots, n$.

Step 4(c): consider the weight of the measured requirements. EIS_i^+ and EIS_i^- are denoted using the weighted distance calculation of weighted $\mathrm{NDM}^{(f)}$ from $\mathrm{PIS}^{(f)}$ and $\mathrm{NIS}^{(f)}$ and are measured according to each DM_f in the following manner:

$$\mathrm{EIS}_i^{+(f)} = \left(\frac{1}{2n}\sum_{j=1}^{n} w_j \left(\begin{array}{c} \left|\begin{array}{c}\left(\dfrac{1}{A_s}\displaystyle\sum_{h_{s_j}\in P_s, p_{s_j}\in p_s}^{A_s} h_{\mathrm{NDM}_{ij}}^{(f)} p_{\mathrm{NDM}_{ij}}^{(f)}\right)^2 \\ -\left(\dfrac{1}{A_r}\displaystyle\sum_{h_{r_j}\in P_r, p_{r_j}\in p_r}^{A_r} h_{\mathrm{PIS}_j}^{(f)} p_{\mathrm{PIS}_j}^{(f)}\right)^2\end{array}\right|^{\alpha} + \\ \left|\begin{array}{c}\left(\dfrac{1}{B_s}\displaystyle\sum_{v_{s_j}\in N_s, n_{s_j}\in n_s}^{B_s} v_{\mathrm{NDM}_{ij}}^{(f)} n_{\mathrm{NDM}_{ij}}^{(f)}\right)^2 \\ -\left(\dfrac{1}{B_r}\displaystyle\sum_{v_{r_j}\in N_r, n_{r_j}\in n_r}^{B_r} v_{\mathrm{PIS}_j}^{(f)} n_{\mathrm{PIS}_j}^{(f)}\right)^2\end{array}\right|^{\alpha}\end{array}\right)\right)^{1/\alpha},$$

$$\mathrm{EIS}_i^{-(f)} = \left(\frac{1}{2n}\sum_{j=1}^{n} w_j \left(\begin{array}{c} \left|\begin{array}{c}\left(\dfrac{1}{A_s}\displaystyle\sum_{h_{s_j}\in P_s, p_{s_j}\in p_s}^{A_s} h_{\mathrm{NDM}_{ij}}^{(f)} p_{\mathrm{NDM}_{ij}}^{(f)}\right)^2 \\ -\left(\dfrac{1}{A_r}\displaystyle\sum_{h_{r_j}\in P_r, p_{r_j}\in p_r}^{A_r} h_{\mathrm{NIS}_j}^{(f)} p_{\mathrm{NIS}_j}^{(f)}\right)^2\end{array}\right|^{\alpha} + \\ \left|\begin{array}{c}\left(\dfrac{1}{B_s}\displaystyle\sum_{v_{s_j}\in N_s, n_{s_j}\in n_s}^{B_s} v_{\mathrm{NDM}_{ij}}^{(f)} n_{\mathrm{NDM}_{ij}}^{(f)}\right)^2 \\ -\left(\dfrac{1}{B_r}\displaystyle\sum_{v_{r_j}\in N_r, n_{r_j}\in n_r}^{B_r} v_{\mathrm{NIS}_j}^{(f)} n_{\mathrm{NIS}_j}^{(f)}\right)^2\end{array}\right|^{\alpha}\end{array}\right)\right)^{1/\alpha}. \tag{18}$$

Step 4(d): the revised indices of closeness are calculated in the following manner for all DM alternatives:

$$\mathrm{RCI}_i^{(f)} = \frac{\left(\mathrm{EIS}_i^-(f)\right)}{\left(\mathrm{EIS}_i^{-(f)} + \mathrm{EIS}_i^{+(f)}\right)}, \tag{19}$$

Step 5: alternative selection and pick the most suitable alternative with a minimum distance.

5.2. q-ROPHF Extended VIKOR Method. There are two primary parts to this method. Firstly, a method for calculating the weights of criteria/attributes is presented using the proposed entropy measure for q-ROPHFNs. A ranking procedure based on the degree of similarity to the ideal solution is considered in the last part. Steps are presented as follows for the q-ROPHF extended VIKOR method.

Step 1: first, we collect the data provided by the decision-makers in the form of a-ROPHFNs.

Step 2: we normalized the information defined by DMs in this step, as the decision matrix may have some benefit and cost parameters altogether, as follows:

$$e_{ij}^{(f)} = \begin{cases} S_{ij}^{(f)} & \text{for benefit criteria,} \\ \left(S_{ij}^{(f)}\right)^c & \text{for cost criteria,} \end{cases} \quad (20)$$

$f = 1, 2, 3, \ldots r, \quad i = 1, 2, 3, \ldots, m, \quad j = 1, 2, 3, \ldots, n$ where $(S_{ij}^{(f)})^c$ is complement of $S_{ij}^{(f)}$, that is $(S_{ij}^{(f)})^c = (N_{S_{ij}}^{(f)}, P_{S_{ij}}^{(f)})$.

Step 3: the weights of the parameters are determined in the following manner by the proposed entropy measure. The calculation of entropy corresponding to each criterion is as follows:

$$\begin{aligned} E\left(S_j\right) &= E\left(S_{1j}, S_{2_j}, S_{3j}, .., S_{mj}\right), j = 1, 2, 3, \ldots, n \\ &= \frac{(-1)}{(2\log\ (n))} \sum_{i=1}^{n} \left(P_s \left(\log\ P_s\right) \times p_s \right. \\ &\quad \left. + N_s \left(\log\ N_s\right) \times n_s\right). \end{aligned} \quad (21)$$

then

$$w\left(S_j\right) = \frac{1 - E\left(S_j\right)}{\sum_{j=1}^{n}\left(1 - E\left(S_j\right)\right)}. \quad (22)$$

Thus, weights of criteria are found as follows:

$$w\left(S_j\right) = \left(w\left(S_1\right), w\left(S_2\right), w\left(S_3\right), \ldots, w\left(S_n\right)\right)^T. \quad (23)$$

Step 4: in this step, the best alternative according to given list of criteria/attribute are determined.

Step 4(a): determine the weighted NDM(f) using the weights of the assessed parameters in the following manner:

$$\mathrm{NDM}_{i_j}^{(f)} = w\left(S_j\right) \cdot S_{i_j}^{(f)} = \left(\begin{array}{c} \bigcup_{h_i \in P_{S_{ij}}, p_i \in p_{S_{ij}}} \dfrac{\sqrt[q]{1 - \prod_{i=1}^{m}\left(1 - \left(h_{S_{ij}}^{(f)}\right)^q\right)^{w\left(S_j\right)}}}{\prod_{i=1}^{m} p_{S_{ij}}^{(f)}} \\ \\ \bigcup_{v_i \in N_{S_{ij}}, n_i \in n_{S_{ij}}} \dfrac{\prod_{i=1}^{m}\left(v_{S_{ij}}^{(f)}\right)^{w\left(S_j\right)}}{\prod_{i=1}^{m} n_{S_{ij}}^{(f)}} \end{array} \right) \text{for all } f = 1, 2, 3, \ldots r. \quad (24)$$

Step 4(b): derive PIS$^{(f)}$ and NIS$^{(f)}$ for all weighted NDM$^{(f)}$, as follows for all $DM^{(f)}$:

$$\begin{aligned} \mathrm{PIS}^{(f)} &= \left\{\mathrm{PIS}_j^{(f)}\right\}_{r\times n} \\ &= \left\{\left(\mathrm{NDM}_{ij}^{(f)}\right): m\left[s\left(\mathrm{NDM}_{ij}^{(f)}\right)\right]\right\}, \\ \mathrm{NIS}^{(f)} &= \left\{\mathrm{NIS}_j^{(f)}\right\}_{r\times n} \\ &= \left\{\left(\mathrm{NDM}_{ij}^{(f)}\right): m\left[s\left(\mathrm{NDM}_{ij}^{(f)}\right)\right]\right\}. \end{aligned} \quad (25)$$

for $j = 1, 2, 3, \ldots, n$

Step 4(c): the useful calculation of the q-ROPHF category over the attributes of profit types is obtained from the following formula:

$$M_i = \frac{\sum_{j=1}^{n}\left(w_j d\left(\mathrm{PIS}_j^{+}, e_{ij}\right)\right)}{d\left(\mathrm{PIS}_j^{+}, \mathrm{NIS}_j\right)}, \quad i = 1, 2, 3, \ldots, m. \quad (26)$$

From the following formula, the measure of q-ROPHF regret over the benefit type attribute is obtained.

$$R_i = \max_j \frac{w_j d\left(\mathrm{PIS}_j^{+}, e_{ij}\right)}{d\left(\mathrm{PIS}_j^{+}, \mathrm{NIS}_j^{-}\right)}, \quad i = 1, 2, 3, \ldots, m, \quad (27)$$

where the distance of any two q-ROPHFNs is as follows:

$$d(s,r)=\left(\frac{1}{2n}\sum_{j=1}^{n}\left(\begin{array}{c}\left|\left(\frac{1}{A_s}\sum_{h_{s_j}\in P_s,p_{s_j}\in p_s}^{A_s}\left(h_{s_j}p_{s_j}\right)\right)^2-\left(\frac{1}{A_r}\sum_{h_{r_j}\in P_r,p_{r_j}\in p_r}^{A_r}\left(h_{r_j}p_{r_j}\right)\right)^2\right|^{\alpha}\\ \left|\left(\frac{1}{B_s}\sum_{v_{s_j}\in N_s,n_{s_j}\in n_s}^{B_s}\left(v_{s_j}n_{s_j}\right)\right)^2-\left(\frac{1}{B_r}\sum_{v_{r_j}\in N_r,n_{r_j}\in n_r}^{B_r}\left(v_{r_j}n_{r_j}\right)\right)^2\right|^{\alpha}\end{array}\right)\right)^{1/\alpha}, \quad (28)$$

where A_s, B_s, A_r, and B_r are the possible numbers of elements in P_s/p_s $N_s/n_s, P_r/p_r$ and N_r/n_r.

Step 5: from the following formula, the q-ROPHF compromise measure value of $G_i\,(i=1,2,3,\ldots,m)$ is obtained:

$$G_i = V\left(\frac{(M_i - M^+)}{(M^- - M^+)}\right) + (1-V)\left(\frac{(R_i - R^+)}{(R^- - R^+)}\right), \quad (29)$$

where
$M^+ = \min_i M_i, M^- = \max_i M_i, R^+ = \min_i R_i, R- = \max_i R_i$ and V is the weight of the majority attribute strategy or the overall total utility. The greater the value of V, the greater average of the decision-preferences maker's over various attributes. The value 0.55 also arrives without loss of overview.

We can understand from equation (29) that the q-ROPHF compromise measure combines two components: the distance in terms of group utility is the preceding one. In terms of individual remorse, the finale is the gap. The smaller the value of the compromise measure of q-ROPHF, the superior the alternative would be. So between $G_i\,(i=1,2,3,\ldots,m)$, we need to choose the smallest one.

Step 6(a): rank the $X_i\,(i=1,2,3,\ldots,m)$ alternatives according to M_i, R_i and $G_i\,(i=1,2,3,\ldots,m)$ values. The solution needed must fulfill the two requirements set out as follows:

Condition 1. $G(X_2) - G(X_1) \geq 1/m - 1$ where X_1 and X_2 are the first and second place alternatives in the ranking list, respectively.
Condition 2. The highest graded by M and R should be X_1. It is said that the Condition 1 is an acceptable gain, while the Condition 2 is said to be an acceptable stability. If both circumstances are not met, go to the next stage.

Step 6(b): gain the alternative to compromise: explore the maximum value of M according to the following equation:

$$G\left(X^N\right) - G(X_1) \leq DG. \quad (30)$$

If Condition 1 does not hold, the compromise options are all the alternative $X_1, X_2, X_3, \ldots, X^N$. If the Condition 2 does not hold, then the compromise solutions are the alternatives X_1 and X_2.

6. Numerical Applications of Proposed Methodology

6.1. Case Study: COVID-19 and Pakistan

The Economic Fallout. Pakistan's financial industry is still in its early stages of growth, although it controls the bulk of the market. Following the onset of the coronavirus epidemic, numerous industries have been impacted by the virus's devastation. The halting of the economy's growth came as a huge shock to a wide range of industries and businesses. The longer the COVID-19 pandemic continues, the more severe the outbreak's impact on the world economy will ultimately to recession. Many small and medium enterprises, especially service businesses are experiencing cash flow issues. The need for banks to continue to assist the economy is unavoidable. The principal repercussions of the COVID-19 outbreak on the Pakistani financial sector are anticipated to cause liquidity issues for businesses and families. The impact of a pandemic like COVID-19 on Pakistan's financial industry can be divided into four categories:

6.2. Exchange Rate (C_1). The exchange rate fluctuated due to the uncertainty over when the COVID-19 epidemic would cease. Since the first case was reported in Pakistan on February 26, 2020, the nominal exchange rate versus the US dollar has been steadily increasing. Reduced employment, decreased country exports, increased bankruptcies, increased credit volume, high risk of loan nonrepayment, and a fall in tourism earnings may be cited as factors for the exchange rate fluctuation.

6.3. Interest Rate (C_2). Prior to the outbreak of the pandemic, banks took the initiative to lower lending interest rates. Banks conveyed their thanks by granting their clients a lower interest rate. Following the outbreak of the epidemic, Pakistan's central bank was obliged to hike interest rates due to economic and banking sector pressures. Certain enterprises, such as hotel cooperatives, have asked banks to cut lending rates. The effort was effective, and practically all banks amended their interest rate policies with the goal of assisting the most vulnerable businesses.

6.4. Customer Behavior Changes (C_3). COVID-19 reduced consumption and influenced consumer behavior as their demands changed. Banks should classify their clients according to a variety of criteria and dimensions. In response to changing consumer behavior, banks must change their service sectors and present unique Opportunities to their clients. Clients' consumption habits altered throughout the epidemic. Customers who were keen to invest in gold and foreign currencies before to the outbreak replaced those who were prepared to take out loans because to low interest rates. They are averse to taking out bank loans.

6.5. Credit Risk, Loss of Income, and Liquidity (C_4). As the crisis worsens expect a drop in income due to difficulties recovering loans and lower transaction volumes. These have a detrimental influence on liquidity, the amount of risky assets, profitability, and capital adequacy. Banks should undertake scenario analysis based on numerous assumptions in these conditions in order to manage any potential financial effects. During the COVID-19 pandemic, this study believes these four criteria to be substantial hazards for the banking industry as the concrete nucleus of supply chain financing. We may approach it as an MCDM problem and solve it using a decision-making technique if we consider the needs and characteristics of decision-making problems. The goal of this study was to look at the banking sector's top risk factors as a result of the COVID-19 pandemic, calculate the banking sector's performance index, and recommend some risk management techniques to preserve financial management efficiency.

In this scenario, the government must prepare an action plans that prioritizes the social and economic well-being of Pakistanis; a plan that lays out the multifaceted aspects of the COVID-19 response, as well as a clear policy for effectively and successfully minimizing, mitigating, and managing the pandemic's negative consequences. This also entails mobilizing technical and financial resources through government-owned sources, donor assistance, and collaboration with development partners to define and develop new economic priorities; making precise plans to keep economic activities and jobs; achieve food security; and protecting the health and social needs of the most vulnerable in a coordinated and effective manner. Following are the five alternatives on the base of which Pakistani bank sectors select the best socioeconomic response plan to protect the needs and rights of people living under the duress of the pandemic.

6.6. Investment ($\mathcal{J}_1$). The economy was expected to expand up moderately before the outbreak of COVID-19, due to structural improvements. In the medium to long term, Pakistan will need to double its private investment rate and human capital investment, increase more revenue, simplify the business regulatory regime, integrate with global value chains, and sustainably manage its natural endowments in order to recover from the effects of COVID-19 and continue on its path to becoming an upper middle-income country.

To reduce the negative economic effects of the outbreak, focus on economic recovery strategies that promote small and medium companies owned by women. During this epidemic, strong market connections should be developed for homebased female workers who can sell their products while working from home.

6.7. Government Support ($\mathcal{J}_2$). Cities and counties took the brunt of increased public health costs while losing "own-source" money from parking penalties, user fees, restaurant taxes, conferences, airports, and other sources. While dealing with the loss of small companies, school closures, and other issues, local governments must provide assistance to vulnerable populations.

6.8. Propositions and Brands ($\mathcal{J}_3$). Customers would be expecting availability in the form of discounts, free services, payment deferrals, and other benefit packages to adjust for their loss of income during this time. Brands must also express empathy and support for their users. To take advantage of the scenario, banks must be aware of the potential and develop mechanisms to gather, monitor, and identify all the potential for improvement that arise as a result of greater digital banking usage.

6.9. Channels and Digital ($\mathcal{J}_4$). Customers' participation with digital platforms will grow throughout this period since they are the primary way of performing transactions. Cheque clearance, for example, has been delayed. As a result, digital channels will become increasingly important.

6.10. MarKets Segments ($\mathcal{J}_5$). Corporates and individuals, who are banking customers, are under a lot of pressure. As economic activity slows, household income will suffer, and SMEs' operations would likely slow as well. Banks may be able to help companies in the health and pharmaceuticals sectors remodel and renovate hospitals and expand their manufacturing capacity.

We have looked at the present scenario and identified several particular subjects as options that the banking sector should think about and address while taking the required steps to deal with this "new normal."

6.11. Using Extended TOPSIS Methodology

Step 1: information of decision-makers in the form of q-ROPHFNs is given in Table 2

Step 2: expert normalized information is provided in Table 3 as follows

Step 3: using the proposed entropy measure of *q*-ROPHFNs, compute the weight information for attributes/criteria as follows:

$$w = \left(w_1 = 0.221877, w_2 = 0.231623, w_3 = 0.193205, w_4 = 0.353295\right). \tag{31}$$

Step 4(a): in Tables 4 and 5, the weighted normalized matrix is computed as follows:

Step 4(b): in Tables 6 and 7, positive and negative ideal solutions are determined as follows:

Table 2: Expert information.

$S^{(f)}$	C_1	C_2	C_3	C_4
J_1	(0.2/0.6, 0.3/0.4) (0.2/0.6, 0.5/0.4)	(0.2⁄1) (0.4/0.6, 0.4/0.4)	(0.3/0.9, 0.4/0.1) (0.8/0.7, 0.2/0.3)	(0.2/0.4, 0.5/0.6) (0.2⁄1)
J_2	(0.4/0.3, 0.8/0.7) (0.6⁄1)	(0.4/0.4, 0.5/0.6) (0.8/0.7, 0.4/0.3)	(0.8⁄1) (0.5/0.6, 0.3/0.4)	(0.3/0.2, 0.4/0.8) (0.2/0.5, 0.2/0.5)
J_3	(0.8/0.5, 0.6/0.5 (0.2/0.5, 0.2/0.5)	(0.3⁄1) (0.4/0.4, 0.8/0.6)	(0.4/0.5, 0.6/0.5) (0.7/0.9, 0.2/0.1)	(0.2/0.1, 0.3/0.9) (0.8⁄1)
J_4	(0.2/0.4, 0.8/0.6) (0.5⁄1)	(0.5/0.5, 0.6/0.5) (0.5/0.5, 0.2/0.5)	(0.8⁄1) (0.4/0.2, 0.4/0.8)	(0.3/0.5, 0.4/0.5) (0.2/0.9, 0.5/0.1)
J_5	(0.7/0.3, 0.4/0.7) (0.3/0.6, 0.8/0.4)	(0.4⁄1) (0.2/0.8, 0.4/0.2)	(0.7/0.4, 0.2/0.6) (0.2/0.4, 0.4/0.6)	(0.6/0.3, 0.2/0.7) (0.3⁄1)

Table 3: Expert normalized information.

$S^{(f)}$	C_1	C_2	C_3	C_4
J_1	(0.2/0.6, 0.3/0.4) (0.2/0.6, 0.5/0.4)	(0.2⁄1) (0.4/0.6, 0.4/0.4	(0.3/0.9, 0.4/0.1) (0.8/0.7, 0.2/0.3)	(0.2/0.4, 0.5/0.6) (0.2⁄1)
J_2	(0.4/0.3, 0.8/0.7) (0.6⁄1)	(0.4/0.4, 0.5/0.6) (0.8/0.7, 0.4/0.3	(0.8⁄1) (0.5/0.6, 0.3/0.4)	(0.3/0.2, 0.4/0.8) (0.2/0.5, 0.2/0.5)
J_3	(0.8/0.5, 0.6/0.5) (0.2/0.5, 0.2/0.5)	(0.3⁄1) (0.4/0.4, 0.8/0.6	(0.4/0.5, 0.6/0.5) (0.7/0.9, 0.2/0.1)	(0.2/0.1, 0.3/0.9) (0.8⁄1)
J_4	(0.2/0.4, 0.8/0.6) (0.5⁄1)	(0.5/0.5, 0.6/0.5) (0.5/0.5, 0.2/0.5	(0.8⁄1) (0.4/0.2, 0.4/0.8)	(0.3/0.5, 0.4/0.5) (0.2/0.9, 0.5/0.1)
J_5	(0.7/0.3, 0.4/0.7) (0.3/0.6, 0.8/0.4)	(0.4⁄1) (0.2/0.8, 0.4/0.2	(0.7/0.4, 0.2/0.6) (0.2/0.4, 0.4/0.6)	(0.6/0.3, 0.2/0.7) (0.3⁄1)

Table 4: Weighted normalized information (a).

$S^{(f)}$	C_1	C_2
J_1	(0.042197/0.6, 0.077811/0.4) (0.699705/0.6, 0.857449/0.4)	(0.043113⁄1) (0.808775/0.6, 0.808775/0.4)
J_2	(0.120697/0.3, 0.383614/0.7) (0.892347⁄1)	(0.1233/0.4, 0.174515/0.6) (0.949628/0.7, 0.808775/0.3)
J_3	(0.383614/0.5, 0.229262/0.5) (0.699705/0.5, 0.699705/0.5)	(0.079497⁄1) (0.808775/0.4, 0.949628/0.6)
J_4	(0.042197/0.4, 0.383614/0.6) (0.857449⁄1)	(0.174515/0.5, 0.234106/0.5) (0.851676/0.5, 0.688815/0.5)
J_5	(0.298316/0.3, 0.120697/0.7) (0.765571/0.6, 0.951695/0.4)	(0.1233⁄1) (0.688815/0.8, 0.808775/0.2)

Table 5: Weighted normalized information (b).

$S^{(f)}$	C_3	C_4
J_1	(0.072624/0.9, 0.112682/0.1) (0.957804/0.7, 0.73275/0.3)	(0.053233/0.4, 0.214664/0.6) (0.0566314⁄1)
J_2	(0.359771⁄1) (0.874661/0.6, 0.79246/0.4)	(0.0981/0.2, 0.151974/0.8) (0.566314/0.5, 0.566314/0.5)
J_3	(0.112682/0.5, 0.214307/0.5) (0.9334/0.9, 0.73275/0.1)	(0.053233/0.1, 0.0981/0.9) (0.924192⁄1)
J_4	(0.359771⁄1) (0.837753/0.2, 0.837753/0.8)	(0.0981/0.5, 0.151974/0.5) (0.566314/0.9, 0.782794/0.1)
J_5	(0.279202/0.4, 0.039378/0.6) (0.73275/0.4, 0.837753/0.6)	(0.287021/0.3, 0.053233/0.7) (0.653536⁄1)

Table 6: Positive ideal solution (PIS).

C_1	C_2	C_3	C_3
(0.383614/0.5, 0.229262/0.5)	(0.123/1)	(0.35977/1)	(0.0981/0.2, 0.151974/0.8)
(0.699705/0.5, 0.699705/0.5)	(0.688815/0.8, 0.808775/0.2)	(0.837753/0.2, 0.837753/0.8)	(0.566314/0.5, 0.566314/0.5)

Table 7: Negative ideal solution (NIS).

C_1	C_2	C_3	C_3
(0.120697/0.3, 0.383614/0.7)	(0.079497/1)	(0.112682/0.5, 0.214307/0.5)	(0.053233/0.1, 0.0981/0.9)
(0.892847/1)	(0.808775/0.4, 0.949628/0.6)	(0.93341/0.9, 0.73275/0.1)	(0.924192/1)

Step 4(c): the calculation of the weighted distance of PIS and NIS to the weighted normalized matrix is calculated as follows:

$$\begin{matrix} 0.018493 & 0.054316 & 0.065317 & 0.04416 & 0.024829, \\ 0.084221 & 0.071465 & 0.030669 & 0.080368 & 0.07835. \end{matrix} \tag{32}$$

Step 5: the ranking of the alternative is as follows:

$$0.819957\ 0.568172\ 0.319515 \quad 0.645384 \quad 0.759357. \tag{33}$$

Therefore, $\mathcal{J}_3$ alternative has the least distance, so it is the best alternative in the given list of alternatives.

6.12. Using Extended VIKOR Methodology

Step 1: information of decision-makers in the form of q-ROPHFNs is given in Table 2.

Step 2: expert normalized information is provided in Table 3.

Step 3: using the proposed entropy measure of q-ROPHFNs, compute the weight information for attributes/criteria as follows:

$$w = (w_1 = 0.221877, w_2 = 0.231623, w_3 = 0.193205, w_4 = 0.353295). \tag{34}$$

Step 4(a): in Tables 8 and 9, the weighted normalized matrix is computed as follows.

Step 4(b): in Tables 6 and 7, positive and negative ideal solutions are determined as follows.

Step 4(c): calculations of M_i and R_i is presented in the Table 10.

Step 5: the values of G_i is calculated in the Table 11.

Step 6(a): we can sort the alternatives according to M, R, and G values. Which alternative has lower M, R, and G values, will be the best alternative according to given list of attributes. The values are calculated in Tables 12 and 13.

Step 6(b): according to the value of G, the best alternative is $\mathcal{J}_3$ with $G(\mathcal{J}_3) = 0.024978$ and alternative $\mathcal{J}_1$ is second with $G(\mathcal{J}_1) = 0.035898$ Since $DG = 1/M - 1 = 1/5 - 1 = 0.25, G(\mathcal{J}_1) - G(\mathcal{J}_3) = 0.01092 < 0.25$, which does not fulfill $G(X_1) - G(X_3) \geq DG$. Alternatively, $\mathcal{J}_3$ fulfills condition 2 that it is the best option sorted by M and R. Then, we get $G(\mathcal{J}_1) - G(\mathcal{J}_3) = 0.01092 < 0.25, G(\mathcal{J}_2) - G(\mathcal{J}_3) = 0.796495 > 0.25, \mathcal{J}_1$ and $\mathcal{J}_3$ were therefore a compromise solution. These results indicate that $\mathcal{J}_3$ is the best choice among the five alternatives, while $\mathcal{J}_1$ could be compromise solutions.

7. Comparison Study

In this section, we presented the comparison study of the proposed technique to show the applicability and effectiveness of the proposed methods. In this regards we presented the extended GRY method to validate the proposed techniques.

7.1. q-ROPHF Extended GRA Method. There are two primary parts to this method. First, a method for calculating the weights of criteria/attributes is presented using the proposed entropy measure for q-ROPHFNs. A ranking procedure based on the degree of similarity to the ideal solution is considered in the last part. Steps are presented as follows for the q-ROPHF extended GRA method.

Step 1: first, we collect the data provided by the decision-makers in the form of q-ROPHFNs.

Step 2: we normalized the information defined by DMs in this step, as the decision matrix may have some benefit and cost parameters altogether, as follows:

$$e_{i_j}^{(f)} = \begin{cases} S_{ij}^{(f)} & \text{for benefit criteria,} \\ \left(S_{ij}^{(f)}\right)^c & \text{for cost criteria.} \end{cases} \tag{35}$$

$f = 1, 2, 3, \ldots r, i = 1, 2, 3, \ldots, m, j = 1, 2, 3, \ldots, n$ where $(S_{ij}^{(f)})^c$ is complement of $S_{ij}^{(f)}$, that is $(S_{ij}^{(f)})^c = (N_{S_{ij}}^{(f)}, P_{S_{ij}}^{(f)})$

Step 3: the weights of the parameters are determined in the following manner by the proposed entropy

Table 8: Weighted normalized information (a).

$S^{(f)}$	C_1	C_2
J1	(0.042197/0.6, 0.077811/0.4) (0.699705/0.6, 0.857449/0.4)	(0.043113/1) (0.808775/0.6, 0.808775/0.4)
J2	(0.120697/0.3, 0.383614/0.7) 0.892347/1	(0.1233/0.4, 0.174515/0.6) (0.949628/0.7, 0.808775/0.3)
J3	(0.383614/0.5, 0.229262/0.5) (0.699705/0.5, 0.699705/0.5)	(0.079497/1) (0.808775/0.4, 0.949628/0.6)
J4	(0.042197/0.4, 0.383614/0.6) (0.857449/1)	(0.174515/0.5, 0.234106/0.5) (0.851676/0.5, 0.688815/0.5)
J5	(0.298316/0.3, 0.120697/0.7) (0.765571/0.6, 0.951695/0.4)	(0.1233/1) (0.688815/0.8, 0.808775/0.2)

Table 9: Weighted normalized information (b).

$S^{(f)}$	C_3	C_4
J1	(0.072624/0.9, 0.112682/0.1) (0.957804/0.7, 0.73275/0.3)	(0.053233/0.4, 0.214664/0.6) (0.0566314/1)
J2	(0.359771/1) (0.874661/0.6, 0.79246/0.4)	(0.0981/0.2, 0.151974/0.8) (0.566314/0.5, 0.566314/0.5)
J3	(0.112682/0.5, 0.214307/0.5) (0.9334/0.9, 0.73275/0.1)	(0.053233/0.1, 0.0981/0.9) (0.924192/1)
J4	(0.359771/1) (0.837753/0.2, 0.837753/0.8)	(0.0981/0.5, 0.151974/0.5) (0.566314/0.9, 0.782794/0.1)
J5	(0.279202/0.4, 0.039378/0.6) (0.73275/0.4, 0.837753/0.6)	(0.287021/0.3, 0.053233/0.7) (0.653536/1)

Table 10: The values of M & R.

	J_1	J_2	J_3	J_4	J_5
M	0.731165	1.213087	0.767741	1.463348	1.061134
R	0.299427	0.819749	0.258508	0.828442	0.557226

Table 11: G when $V = 0.5$.

	J_1	J_2	J_3	J_4	J_5
G	0.035898	0.821473	0.024978	1	0.487296

Table 12: Values of M, R and G.

	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$	$\mathcal{J}_4$	$\mathcal{J}_5$
M	0.731165	1.213087	0.767741	1.463348	1.061134
R	0.299427	0.819749	0.258508	0.828442	0.557226
G	0.035898	0.821473	0.024978	1	0.487296

Table 13: Ranking.

Ranking	Best alternative	
M	$\mathcal{J}_1 > \mathcal{J}_3 > \mathcal{J}_5 > \mathcal{J}_2 > \mathcal{J}_4$	$\mathcal{J}_1$
R	$\mathcal{J}_3 > \mathcal{J}_1 > \mathcal{J}_5 > \mathcal{J}_2 > \mathcal{J}_4$	$\mathcal{J}_3$
G	$\mathcal{J}_3 > \mathcal{J}_1 > \mathcal{J}_5 > \mathcal{J}_2 > \mathcal{J}_4$	$\mathcal{J}_3$

Table 14: $K^{(+)}$.

	C_1	C_2	C_3	$\cdots$	C_n
J_1	$(K_{11}^{(+)})$	$(K_{12}^{(+)})$	$(K_{13}^{(+)})$		$(K_{1n}^{(+)})$
J_2	$(K_{21}^{(+)})$	$(K_{22}^{(+)})$	$(K_{23}^{(+)})$		$(K_{2n}^{(+)})$
J_3	$(K_{31}^{(+)})$	$(K_{32}^{(+)})$	$(K_{33}^{(+)})$		$(K_{3n}^{(+)})$
J_m	$(K_{m1}^{(+)})$	$(K_{m2}^{(+)})$	$(K_{m3}^{(+)})$		$(K_{mn}^{(+)})$

Table 15: $K^{(-)}$.

	C_1	C_2	C_3	$\cdots$	C_n
J_1	$(K_{11}^{(-)})$	$(K_{12}^{(-)})$	$(K_{13}^{(-)})$		$(K_{1n}^{(-)})$
J_2	$(K_{21}^{(-)})$	$(K_{22}^{(-)})$	$(K_{23}^{(-)})$		$(K_{2n}^{(-)})$
J_3	$(K_{31}^{(-)})$	$(K_{32}^{(-)})$	$(K_{33}^{(-)})$		$(K_{3n}^{(-)})$
J_m	$(K_{m1}^{(-)})$	$(K_{m2}^{(-)})$	$(K_{m3}^{(-)})$		$(K_{mn}^{(-)})$

measure. The calculation of entropy corresponding to each criterion is as follows:

$$E(S_j) = E\left(S_{1j}, S_{2_j}, S_{3j}, \ldots, S_{mj}\right), j = 1, 2, 3, \ldots, n$$
$$= (-1)/(2\log\ (n)) \sum_{i=1}^{n} (P_s(\log\ P_s) \times p_s$$
$$+N_s(\log\ N_s) \times n_s). \tag{36}$$

then,

$$w(S_j) = \frac{(1 - E(S_j))}{\left(\sum_{j=1}^{n} (1 - E(S_j))\right)}. \tag{37}$$

Thus, weights of criteria are found as follows:

$$w(S_j) = (w(S_1), w(S_2), w(S_3), \ldots, w(S_n))^T. \tag{38}$$

Step 4: in this step, the best alternative according to given list of criteria/attribute are determined.

Step 4(a): determine the weighted NDM(f) using the weights of the assessed parameters in the following manner:

$$\mathrm{NDM}_{i_j}^{(f)} = w(S_j) \cdot S_{i_j}^{(f)} = \begin{pmatrix} \bigcup_{h_i \in P_{S_{ij}}, p_i \in p_{S_{ij}}} \dfrac{\sqrt[q]{1 - \prod_{i=1}^{m}\left(1 - \left(h_{S_{ij}}^{(f)}\right)^q\right)^{w(S_j)}}}{\prod_{i=1}^{m} p_{S_{ij}}^{(f)}} \\ \bigcup_{v_i \in N_{S_{ij}}, n_i \in n_{S_{ij}}} \dfrac{\prod_{i=1}^{m}\left(v_{S_{ij}}^{(f)}\right)^{w(S_j)}}{\prod_{i=1}^{m} n_{S_{ij}}^{(f)}} \end{pmatrix}. \tag{39}$$

for all $f = 1, 2, 3, \ldots r$.

Step 4(b): derive PIS$^{(f)}$ and NIS$^{(f)}$ for all weighted NDM$^{(f)}$, as follows for all $DM^{(f)}$:

$$\begin{aligned} \mathrm{PIS}^{(f)} &= \left\{\mathrm{PIS}_j^{(f)}\right\}_{\widetilde{r\times n}} \\ &= \left\{\left(\mathrm{NDM}_{ij}^{(f)}\right) : \mathrm{m}\left[s\left(\mathrm{NDM}_{ij}^{(f)}\right)\right]\right\}, \\ \mathrm{NIS}^{(f)} &= \left\{\mathrm{NIS}_j^{(f)}\right\}_{\widetilde{r\times n}} \\ &= \left\{\left(\mathrm{NDM}_{ij}^{(f)}\right) : \mathrm{m}\left[s\left(\mathrm{NDM}_{ij}^{(f)}\right)\right]\right\}. \end{aligned} \tag{40}$$

for $j = 1, 2, 3, \ldots, n$.

Step 4(c): measure distance of PIS and NIS with each element of the alternative to determine the positive ideal separation matrix K^+ and negative ideal separation K^- as follow (see Tables 14 and 15):

where

$$d(s, r) = \left(\frac{1}{2n} \sum_{j=1}^{n} \sum_{h_s} \left| \left(\sum_{v_{s_j} \in N_s, n_{s_j} \in n_s}^{B_s} \left(\frac{1}{A_s} h_{s_j} \in p_s\right)\left(h_{s_j} p_{s_j}\right)\right)^2 - \left(\frac{1}{A_r}\right)\left(\sum_{h_{r_j} \in P_r, p_{r_j} \in p_r}^{A_r} \left(h_{r_j} p_{r_j}\right)\right)^2 \right|^{\alpha}\right)^{1/\alpha}. \tag{41}$$

where A_s, B_s, A_r, and B_r are the possible numbers of elements in P_s/p_s $N_s/n_s, P_r/p_r$ and N_r/n_r.

Step 5: determine matrices with Grey coefficients using the following formulas:

Table 16: Grey relational coefficient (φ_{ij}^{+}).

0.833758	0.852003	0.796283	0.876161
0.727805	0.996317	0.431545	0.846628
0.617427	0.975802	0.93921	0.556616
0.993145	0.69808	0.428526	0.872206
0.860693	0.660276	0.678921	1

Table 17: Grey relational coefficient (φ_{ij}^{-}).

0.591037	0.880002	0.989586	0.539976
0.666481	0.979524	0.512448	0.532206
0.505373	0.965965	0.918441	1
0.645767	0.762128	0.509554	0.538953
0.599572	0.731093	0.803636	0.569494

Table 18: Φ_i^{+}.

J_1	0.845724
J_2	0.774739
J_3	0.741121
J_4	0.772987
J_5	0.828369

Table 19: Φ_i^{-}.

J_1	0.716930
J_2	0.661790
J_3	0.866613
J_4	0.608665
J_5	0.658835

$$\begin{aligned}\varphi_{i_j}^{+} &= \frac{\min_{1<i<m}\min_{1<i<m} k_{i_j}^{+} + \rho\left(\min_{1<i<m}\max_{1<i<m} k_{i_j}^{+}\right)}{k_{i_j}^{+} + \rho\left(\min_{1<i<m}\max_{1<i<m} k_{i_j}^{+}\right)},\\ \varphi_{i_j}^{-} &= \frac{\min_{1<i<m}\min_{1<i<m} k_{i_j}^{-} + \rho\left(\min_{1<i<m}\max_{1<i<m} k_{i_j}^{-}\right)}{k_{i_j}^{-} + \rho\left(\min_{1<i<m}\max_{1<i<m} k_{i_j}^{-}\right)},\end{aligned} \tag{42}$$

where $i \in m$, $j \in n$ and $\rho = 0.5$ be fixed coefficient.

Step 6: attributes weight information which is calculated using proposed entropy measure for q-ROPHFSs. Consider that attributes weights $W = \{\rho_1, \rho_2, \rho_3, \ldots, \rho_n\}$ subject to $\rho_j \in [0, 1]$ such that $\sum_{j=1}^{n}\rho_j = 1$. Then, Grey coefficients are obtained as follows:

$$\begin{aligned}\Phi_i^{+} &= \sum\nolimits_{j=1}^{n}\rho_j\varphi_{i_j}^{+}, i \in m,\\ \Phi_i^{-} &= \sum\nolimits_{j=1}^{n}\rho_j\varphi_{ij}^{-}, i \in m.\end{aligned} \tag{43}$$

Step 7: measure the closeness coefficients are obtained as follows:

$$\phi_i = \frac{\Phi_i^{-}}{\Phi_i^{+} + \Phi_i^{-}}, i \in m. \tag{44}$$

Rank the ϕ_i according to descending order. Choose the larger ϕ_i for best alternative.

7.2. Numerical Illustration

Step 1: information of decision-makers in the form of q-ROPHFNs is given in Table 2

Step 2: expert normalized information is provided in Table 3

Step 3: using the proposed entropy measure of q-ROPHFNs, compute the weight information for attributes/criteria as follows:

$$\begin{aligned} w = (w_1 &= 0.221877, w_2 = 0.231623, w_3\\ &= 0.193205, w_4 = 0.353295).\end{aligned} \tag{45}$$

Step 4(a): in Tables 4 and 5, the weighted normalized matrix is computed

Table 20: ϕ_i.

$\mathcal{J}_1$	0.45879
$\mathcal{J}_2$	0.460687
$\mathcal{J}_3$	0.539028
$\mathcal{J}_4$	0.440534
$\mathcal{J}_5$	0.443002

Step 4(b): in Tables 6 and 7, positive and negative ideal solutions are determined

Step 4(c): in order to calculate the positive ideal separation matrix K^+ and negative ideal separation K^-, evaluate the distance of PIS and NIS for each part of the alternative as follows:

$$K^+ = \begin{bmatrix} 0.052936 & 0.049711 & 0.060025 & 0.045647 \\ 0.074864 & 0.028361 & 0.19332 & 0.050647 \\ 0.10571 & 0.031011 & 0.036025 & 0.127931 \\ 0.028764 & 0.082211 & 0.19537 & 0.046297 \\ 0.048223 & 0.092511 & 0.087288 & 0.027897 \end{bmatrix},$$
$$K^- = \begin{bmatrix} 0.190902 & 0.088165 & 0.064896 & 0.220487 \\ 0.155486 & 0.066815 & 0.238883 & 0.225487 \\ 0.243936 & 0.069465 & 0.079371 & 0.06295 \\ 0.164386 & 0.120665 & 0.240933 & 0.221137 \\ 0.186448 & 0.130965 & 0.108133 & 0.202737 \end{bmatrix}. \quad (46)$$

Step 5: calculate the grey relational coefficient of each alternative from K^+ and K^- in the Tables 16 and 17

Step 6: Grey closeness coefficient of each alternative from K^+ and K^- is calculated in Tables 18 and 19 as follows

Step 7: calculated relative closeness coefficient ϕ_i for each alternative is presented in Table 20

Hence, $\mathcal{J}_3$ alternative has the maximum relative closeness coefficient, so it is the best alternative in the given list of alternatives.

8. Conclusion

The impact of the COVID-19 pandemic on the Pakistani financial sector under supply chain finance was investigated using a hybrid decision-making framework. Based on both the literature and banker's judgements, four primary characteristics that have a substantial impact on Pakistan's banking system were identified. By consulting three experts, we were able to identify which of these four factors was the most essential and which was the least important. The advanced TOPSIS, VIKOR, and GRA methods was extended to the q-rung orthopair probabilistic hesitant Fuzzy TOPSIS, VIKOR, and GRA methods with unknown weight information about the decision-making experts as well as the criteria. Three decision-making algorithms are designed to address the uncertain information in supply chain finance for Pakistan's Banks under q-rung orthopair probabilistic hesitant fuzzy environment. To determine the rationality and reliability of the elaborated techniques, a numerical example about supply chain finance in Pakistan to analyze the effects of emergency situation of COVID-19 on Pakistani banks is illustrated. The results indicate that the proposed technique is applicable and effected to tackle the uncertainty and ambiguity in real-life decision-making problems.

In future research, the other techniques like VIKOR, TODAM, and Electric-I, II, and III with real-life problems are investigated under the novel concept of q-rung orthopair probabilistic hesitant fuzzy information.

Data Availability

The data used in this study are hypothetical and can be used by anyone by just citing this article.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

Authors' Contributions

All authors have contributed equally to this article.

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