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Research Article

Investigation of Behavior on Solutions of Lane–Emden Complex Differential Equations by a Random Differential Transformation Method

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In this study, Lane–Emden complex differential equations have been randomized by selecting random variables for the coefficients, initial conditions, and force functions of complex differential equations. In addition, by finding the solutions of the obtained random complex differential equations, the probability characteristics of the solutions are examined. The random version of the differential transformation method (DTM) is used to obtain an analytical solution close to the solution of the random differential equation. Approximate expected values and variances are calculated using the approximate solution method. The solutions of some random complex differential equations obtained by using random probability characteristics with gamma, normal, and beta distributions were obtained by the DTM method. The probability characteristics obtained are shown graphically with the help of the MAPLE program.

1. Introduction

Lane–Emden equations are used in various fields of science. Many problems in a wide variety of fields can be analyzed using Lane–Emden equations because some differential equations defined by differential equations with initial or boundary values can be converted into Lane–Emden complex differential equations. Differential equations in biology, geophysics, economics, and radiation are just a few of the areas where Lane–Emden [1] complex differential equations are used.

First, Zhou [2] proposed the application of the differential transformation concept (in one dimension) to solve linear and nonlinear initial value problems in electrical circuit analysis. Nonlinear differential equations using the one-dimensional differential transformation method [3–6] are also solved. In addition, complex partial differential equations were considered. Solving complex partial differential equations by two-dimensional differential transformation methods was proposed by Kuang Chen and Huei Ho [7]. In this study, the numerical solutions of some ordinary and complex differential equation systems are analyzed using the differential transformation method (DTM) and compared with the solutions of other numerical methods. The most fundamental works in the theory of complex differential equations are “Pseudo-Analytic Functions Theory” by Bers [8] and “Generalized Analytic Functions” by Vekua [9]. Second-order complex differential equations were solved by using differential transformation methods and Adomian decomposition methods [10, 11].

These methods can be used for calculations of some ordinary and partial differential equations. In the first section, the basic definition and properties of the differential transformation method are given. In the second section, the examples are solved using the differential transformation method and compared with the solutions of other numerical methods. First-order Lane–Emden complex differential equations with variable coefficients are analyzed, and probability characteristics are investigated. Approximate analytical solutions and the obtained probability characteristics of complex ordinary and partial differential equations are graphed using the MAPLE Program [12–14].

In this paper, we follow an approach similar to the literature on systems of random complex differential equations. Components of the Lane–Emden equations are transformed into random variables with several probability distributions. A random differential transformation method (DTM) is applied to find exact or approximate solutions of Lane–Emden complex differential equations given with random components. Probability characteristics of random Lane–Emden equations are given with graphs by using approximate solutions obtained by DTM. A few numerical examples are also given with uniform, beta, and gamma distributions, respectively.

2. Differential Transform Method

For the differential transformation method [2–4] of a function with one variable, the transformation of the $k-$th derivative for a function is given as

$$W(k)=\frac{1}{k!}\left[\frac{d^k w(z)}{dz^k}\right]_{z=z_0}, \tag{1}$$

whereas the inverse transformation is given as

$$w(z)=\sum_{k=0}^{\infty} W(k)\left(z-z_0\right)^k. \tag{2}$$

The following theorems are obtained by using (1) and (2).

Theorem 1 (see [2, 3]). *If* $w(z)=g(z)\pm h(z)$, *then* $W(k)=G(k)\pm H(k)$.

Theorem 2 (see [2, 3]). *If* $w(z)=cg(z)$, *then* $W(k)=cG(k)$, *where* c *is a constant.*

Theorem 3 (see [2, 3]). *If* $(z)=(d^n g(z)/dz^n)$, *then* $W(k)=((k+n)!/k!)G(k+n)$.

Theorem 4 (see [2, 3]). *If* $(z)=z^m(d^n g(z)/dz^n)$, *then* $W(k)=((k-m+n)!/(k-m)!)G(k-m+n)$.

Theorem 5 (see [2, 3]). *If* $w(z)=g(z)h(z)$, *then* $W(k)=\sum_{k_1=0}^{k} G(k_1)H(k-k_1)$.

Theorem 6 (see [2, 3]). *If* $w(z)=z^n$, *then* $W(k)=\delta(k-n)$, *where* $\delta(k-n)=\begin{cases}1, k=n\\ 0, k\neq n\end{cases}$.

Theorem 7 (see [2, 3]). *If* $w(z)=g_1(z)g_2(z)\ldots g_{n-1}(z)\,g_n(z)$, *then* $W(k)=\sum_{k_{n-1=0}}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}\cdots\sum_{k_2=0}^{k_3}\sum_{k_1}=0^{k_2}$ $G_1(k_1)G_2(k_2-k_1)\cdots G_{n-1}(k_{n-1}-k_{n-2})G_n(k-k_{n-1})$.

3. Differential Conversion Method for Nonlinear Functions

Situation 1. $f(w)=e^{aw}$ (exponential Linearity). From the transformation definition [5–7],

$$\begin{aligned}F(0)&=\left[e^{aw(z)}\right]_{z=0}\\&=e^{aw(0)}\\&=e^{aW(0)}.\end{aligned} \tag{3}$$

Now, taking the derivative $f(w)=e^{aw}$ relative to z,

$$\begin{aligned}\frac{df(w)}{dz}&=ae^{aw}\frac{\mathrm{d}w(z)}{\mathrm{d}z}\\&=af(w)\frac{\mathrm{d}w(z)}{\mathrm{d}z},\end{aligned} \tag{4}$$

can be found (2). If the differential conversion method is applied to both sides of the expression,

$$(k+1)F(k+1)=a\sum_{m=0}^{k}(m+1)W(m+1)F(k-m), \tag{5}$$

and if k is written instead of $(k+1)$,

$$F(k)=a\sum_{m=0}^{k-1}\frac{m+1}{k}W(m+1)F(k-1-m), k\geq 1, \tag{6}$$

and it is found in the form. From equations (5) and (6), $f(w)=e^{aw}$,

$$F(k)=\begin{cases}e^{aw}, & k=0\\ a\sum_{m=0}^{k-1}\frac{m+1}{k}W(m+1)F(k-1-m), & k\geq 1.\end{cases} \tag{7}$$

Situation 2. $f(w)=\ln(a+bw), a+bw>0$ (logarithmic linearity). By the definition of transformation [5–7],

$$\begin{aligned}F(0)&=\left[\ln(a+bw(z))\right]_{z=0}\\&=\ln(a+bw(0))\\&=\ln(a+bW(0)).\end{aligned} \tag{8}$$

Also, if the derivative of $f(w)=\ln(a+bw)$ relative to z' is taken,

$$\frac{\mathrm{df}(w(z))}{\mathrm{dz}}=\frac{b}{a+bw}\frac{\mathrm{dw}(z)}{\mathrm{dz}}, \tag{9}$$

and either as equivalent,

$$a\frac{df(w)}{dz} = b\left(\frac{dw(z)}{dz} - w\frac{df(w)}{dz}\right). \quad (10)$$

If the differential transformation of equation (10) is taken, the following expression is obtained:

$$aF(k+1) = b\left[W(k+1) - \sum_{m=0}^{k}\frac{m+1}{k+1}F(m+1)W(k-m)\right], \quad (11)$$

and if k is written instead of $k+1$,

$$aF(k) = b\left[W(k) - \sum_{m=0}^{k-1}\frac{m+1}{k}F(m+1)W(k-1-m)\right], \quad k \geq 1, \quad (12)$$

is obtained. If $k = 1$ is taken in equation (12),

$$F(1) = \frac{b}{a + bW(0)}W(1), \quad (13)$$

can be found.

If equation (12) for $k \geq 2$ is to be rewritten,

$$F(k) = \frac{b}{a + bW(0)}\left[W(k) - \sum_{m=0}^{k-2}\frac{m+1}{k}F(m+1)W(k-1-m)\right], \quad (14)$$

is obtained. Therefore, from (8), (13), and (14) by combining expressions $f(w) = \ln\ (a + bw)$, the differential transformation of the function can be calculated as follows [1]:

$$F(k) = \begin{cases} \ln\ (a + bW(0)), & k = 0, \\ \dfrac{b}{a + bW(0)}W(1), & k = 1, \\ \dfrac{b}{a + bW(0)}\left[W(k) - \displaystyle\sum_{m=0}^{k-2}\frac{m+1}{k}F(m+1)W(k-1-m)\right], & k \geq 2. \end{cases} \quad (15)$$

4. Lane–Emden Random Complex Differential Equations

Many studies have been conducted on singular initial value problems that can be modelled with the quadratic nonlinear ordinary differential equation. One of the equations in this category is the following random Lane–Emden-type complex differential equations:

$$\begin{gathered} w''(z) + \frac{n}{z}w'(z) + f(z,w) = g(z), \\ w(0) = A, w'(0) = B, \end{gathered} \quad (16)$$

where equations in the form are called complex Lane–Emden type equations for the real number $n \epsilon \mathbb{R}$ in the range $n \geq 0$. For the real number $n \epsilon \mathbb{R}$ and $n \geq 0$, A and B are constant and $f(z,w)$ is a continuous function. The analytic solution of equation (16) is always possible in the singular point neighbourhood of $z = 0$ for the above initial conditions [15]. Astrophysicists Lane [16] Since the first work was done by Lane and Robert Emden [17], the equations were named by a combination of the genealogical names of these researchers. Let us examine the solution behavior by randomizing complex Lane–Emden equations.

5. Numerical Examples

In this section, solutions of random Lane–Emden complex differential equations are obtained with the help of the differential transformation method (DTM) and some examples for variances and expected values of different probability distributions of these solutions are given. Each example contains complex differential equations with random effect terms with different probability distributions added. In recent years, some first-order random differential models and equations have been solved using mean-square computations [18–21].

Example 1. We consider the random complex Lane–Emden differential equation.

$$w''(z) + \frac{3}{z}w'(z) + 3w = 3Bz^3 + 3Az^2 + 15Bz + 8A, \quad (17)$$

where A and B are random variables with a gamma distribution within the interval [22], i.e.,

$$w(0) = 0, w'(0) = 0. \quad (18)$$

$A \sim G(\alpha = 1, \beta = 2)$ with parameters $\alpha = 1$ and $\beta = 2$. Multiplying both sides of equation (17) by z, we obtain

$$zw''(z) + 3w'(z) + 3zw(z) = 3Bz^4 + 3Az^3 + 15Bz^2 + 8Az, \tag{19}$$

and then, if the differential transformation of both sides is taken,

$$W(k+1) = \frac{1}{(k+1)(k+3)}\left[-3\sum_{r=0}^{k}\delta(r-1)W(k-r) + 3B\delta(k-4) + 3A\delta(k-3) + 15B\delta(k-2) + 8A\delta(k-1)\right], \tag{20}$$

is obtained. Initial conditions in (18), $z_0 = 0$, can be converted as

$$W(0) = 0, W(1) = 0. \tag{21}$$

Equation (21) has $W(2) = A$ for $k = 1$ if it is written instead in the expression (20) for $k = 1$. If we follow the same procedure for$k = 2, W(3) = B$.

If continued in this way, $W(k) = 0$ becomes $k \geq 6$. Using the inverse transformation rule,

$$\begin{aligned} w(z) &= \sum_{k=0}^{\infty} W(k)z^k = W(0) + W(1)z + W(2)z^2 + W(3)z^3 + W(4)z^4 + \ldots \\ &= Az^2 + Bz^3, \end{aligned} \tag{22}$$

which converges efficiently to the exact solution $Az^2 + Bz^3$.

Using the tools of mean-square calculus and DTM, the expectation and the variance of any random function can be obtained as [23, 24]

$$\begin{aligned} E[u(x)] &= \sum_{k=0}^{n} E[U(k)]x^k \\ \mathrm{Var}[u(x)] &= \sum_{i=0}^{n}\sum_{j=0}^{n} cov(U(i), U(j))x^{i+j}. \end{aligned} \tag{23}$$

The numerical characteristics of the approximate solutions of the random Lane–Emden equation calculated by random DTM are obtained by the following operations:

$$\begin{aligned} &A, B \sim G(\alpha, \beta) \\ M_z(t) &= E\left[e^{tz}\right] \\ &= \frac{1}{(1-\beta t)^{\alpha}}, \end{aligned} \tag{24}$$

and when $z \sim G(\alpha, \beta)$,

$$\begin{aligned} E[z] &= \alpha\beta, E\left[z^2\right] = \left(\alpha + \alpha^2\right)\beta^2, \\ \mathrm{Var}[z] &= \alpha\beta^2, \\ E[w(z)] &= E\left[Az^2 + Bz^3\right] \\ &= E[A]z^2 + E[B]z^3 \\ &= \alpha\beta\left(z^2 + z^3\right), \\ \mathrm{Var}[w(z)] &= Var\left[Az^2 + Bz^3\right] \\ &= \mathrm{Var}[A]z^4 + Var[B]z^6 \\ &= \alpha\beta^2\left(z^4 + z^6\right). \end{aligned} \tag{25}$$

If $z \sim G(\alpha, \beta)$ is specifically chosen as $\alpha = 1, \beta = 2$,

$$\begin{aligned} E[w(z)] &= \alpha\beta\left(z^2 + z^3\right) \\ &= 2\left(z^2 + z^3\right). \end{aligned} \tag{26}$$

The expectations can be given in a single graph for a comparison with the deterministic results of equation (1) as above (Figure 1). Maximum and minimum values of expected values of the random variables are obtained as follows: $w(z)$ takes its maximum value as 57.6888 and its minimum value as 0.

$$\begin{aligned} \mathrm{Var}[w(z)] &= \alpha\beta^2\left(z^4 + z^6\right) \\ &= 4\left(z^4 + z^6\right). \end{aligned} \tag{27}$$

If the variance found for the selected parameter values is plotted with MATLAB (2013a), the graph in Figure 2 is obtained.

The results for the confidence intervals of the expectations are given below (Figure 3). Here, three standard deviations are used to obtain the confidence intervals, and the dashed line shows the upper end of the confidence interval, whereas the dash-dot lines are the lower ends of the interval.

The variances of $w(z)$ are given above (Figure 2). Extremum values of the variances of the random variables are obtained as follows: $\min[\mathrm{Var}(w(z))] = 0$ and $\max[\mathrm{Var}(w(z))] = 2069.3$.

Confidence intervals for expected values of random variables are equal to

$$(E[w(z)] - K.\mathrm{std}(w(z)), E(w(z)) + K.std(w(z)), \tag{28}$$

and these can be obtained through standard deviations. For $K = 3$, this formula gives approximately 99% confidence intervals for the approximate expected value of the normally distributed random variable [22]. If the 99% confidence interval is plotted with MATLAB (2013a), the graph shown in Figure 3 is obtained. Known as the three-sigma rule, this popular rule indicates that about 99.73% of values for a normally distributed variable are within about three standard deviations of the mean. Therefore, using appropriate parameters, we will compare the variations of the results for two continuous distributions with limited and unlimited support, respectively. Appropriate parameters will ensure that almost all possible values for random effects are drawn from the same range for both distributions.

The confidence intervals of $w(z)$ are given in Figure 3. The extremum values of the confidence intervals are as follows: $\min\,(E(w(z)) - 3\mathrm{std}(w(z))) = 0$ and $\max\,(E(w(z)) + 3\mathrm{std}(w(z))) = 193.6102$. Here, $K = 3$ gives an approximate 99% confidence interval.

Example 2. We consider the random complex Lane–Emden equation.

$$w''(z) + \frac{7}{z}w'(z) + z^2 w = Bz^6 + Az^5 + 40Bz^2 + 27Az, A, B \sim N\left(\mu, \sigma^2\right), \tag{29}$$

and initial conditions are

$$w(0) = 0, w'(0) = 0. \tag{30}$$

Multiplying both sides of equation (29) by z,

$$zw''(z) + 7w'(z) + z^3 w(z) = Bz^7 + Az^6 + 40Bz^3 + 8Az^2, \tag{31}$$

is obtained. If the differential transformation of both sides is taken,

$$W(k+1) = \frac{1}{(k+1)(k+7)}\left[-\sum_{r=0}^{k}\delta(r-3)W(k-r) + B\delta(k-7) + A\delta(k-6) + 40B\delta(k-3) + 27A\delta(k-2)\right], \tag{32}$$

is obtained. Initial conditions in (30), $z_0 = 0$, can be converted as

$$W(0) = 0, W(1) = 0. \tag{33}$$

If (33) is written in place of the equations in the expression (32) for $k = 1, W(2) = 0$ is found for $k = 1$. If we follow the same procedure, $W(3) = A$ for $k = 2$ and $W(4) = B$ for $k = 3$. If continued in this way, $W(k) = 0$ becomes $k \geq 6$. Using the inverse transformation rule,

$$\begin{aligned} w(z) &= \sum_{k=0}^{\infty} W(k)z^k = W(0) + W(1)z + W(2)z^2 + W(3)z^3 + W(4)z^4 + \ldots \\ &= Az^3 + Bz^4, \end{aligned} \tag{34}$$

which converges efficiently to the exact solution $w(z) = Az^3 + Bz^4$.

This is also the complete solution of equation (29). If $A \sim N(\mu, \sigma^2)$ is normally distributed [22], we obtain

$$\begin{aligned} M_z(t) &= E\left[e^{tz}\right] \\ &= e^{\mu t + \frac{1}{2}\sigma^2 t^2}, \end{aligned} \tag{35}$$

$$E[z] = \mu, E\left[z^2\right] = \mu^2 + \sigma^2, \mathrm{Var}[w(z)] = \sigma^2,$$

and when $z \sim N(\mu, \sigma^2)$,

$$\begin{aligned} E[w(z)] &= E\left[Az^3 + Bz^4\right] \\ &= E[A]z^3 + E[B]z^4 \\ &= \mu\left(z^3 + z^4\right), \\ \mathrm{Var}[w(z)] &= \mathrm{Var}\left[Az^3 + Bz^4\right] \\ &= \mathrm{Var}[A]z^3 + \mathrm{Var}[B]z^8 \\ &= \sigma^2\left(z^6 + z^8\right). \end{aligned} \tag{36}$$

Specifically, if $z \sim N(\mu = 1, \sigma^2 = 4)$ is selected, $E[w(z)] = (z^3 + z^4)$.

The expectations can be given in a single graph for a comparison with the deterministic results of equation (1) as above (Figure 4). Maximum and minimum values of expected values of the random variables are obtained as follows: $w(z)$ takes its maximum value as 81.5843 and its minimum value as 0.

$$\mathrm{Var}[w(z)] = 4\left(z^6 + z^8\right). \tag{37}$$

The variances of $w(z)$ are given above (Figure 5). Extremum values of the variances of the random variables are obtained as follows: $\min\,[\mathrm{Var}(w(z))] = 0$ and $\max\,[\mathrm{Var}(w(z))] = 1651.2$.

The confidence intervals of $w(z)$ are given in Figure 6. The extremum values of the confidence intervals are as

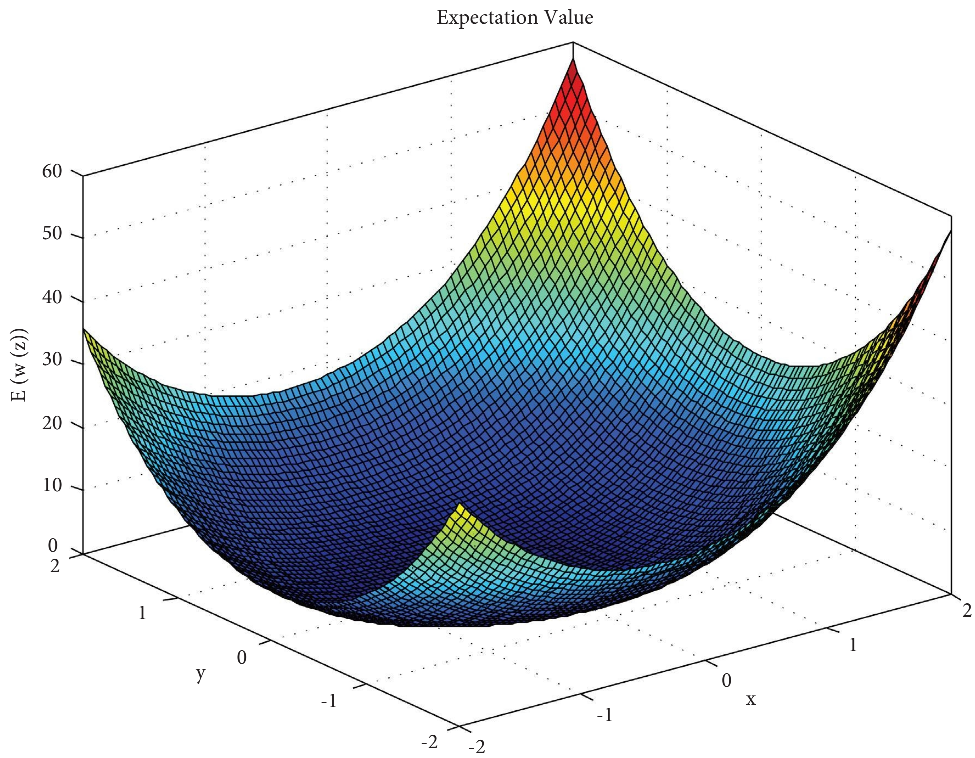

Figure 1: Expected value of equation (17) for $\alpha = 1, \beta = 2$.

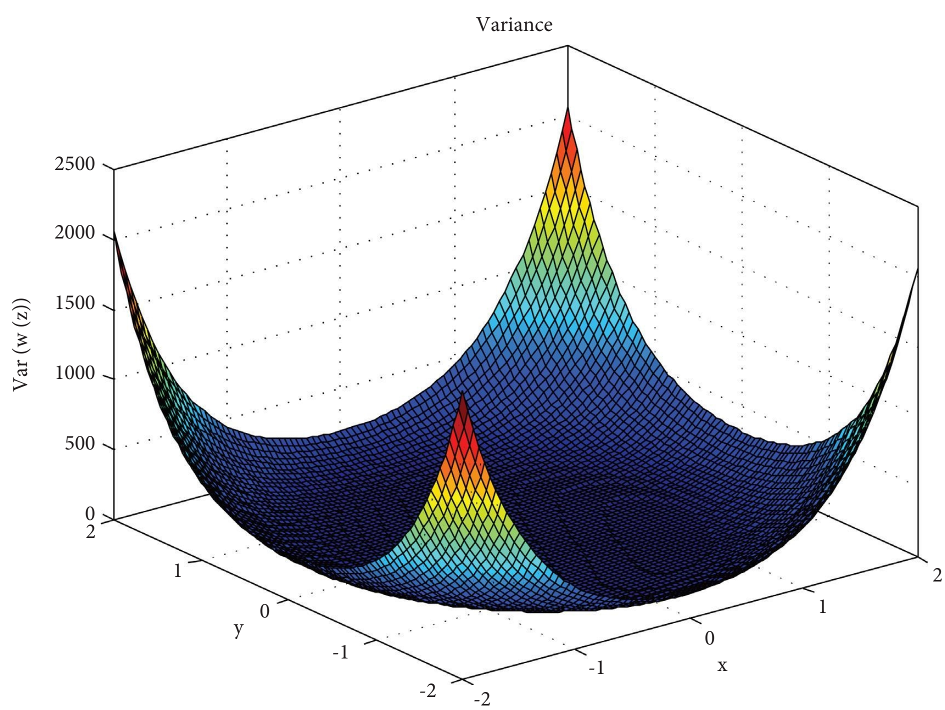

Figure 2: Variance of equation (17) for $\alpha = 1, \beta = 2$.

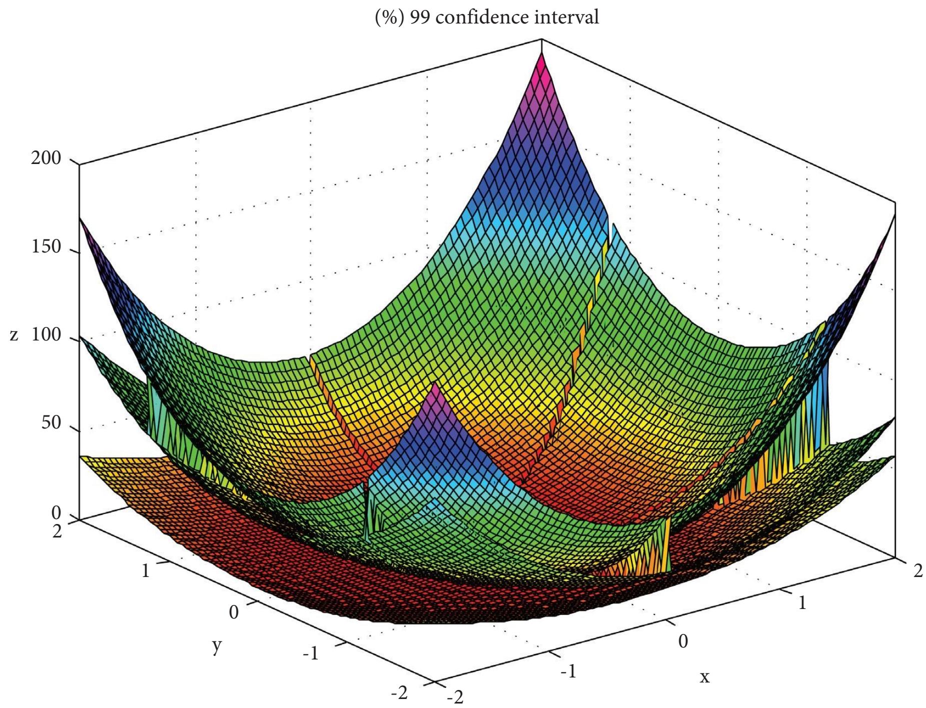

Figure 3: The 99% confidence interval of equation (17) for $\alpha = 1, \beta = 2$.

follows: min $(E(w(z)) - 3\mathrm{std}(w(z))) = 0$ and max $(E(w(z)) + 3\mathrm{std}(w(z))) = 466.4606$. Here, $K = 3$ gives an approximate 99% confidence interval.

Example 3. We consider the random complex Lane–Emden equation.

$$w''(z) + \frac{4}{z}w'(z) + zw(z) = Az^7 + 54Az^4, A \sim \mathrm{Beta}(\alpha, \beta), \tag{38}$$

and initial conditions are

$$w(0) = 0, w'(0) = 0. \tag{39}$$

Multiplying both sides of equation (38) by z,

$$zw''(z) + 4w'(z) + z^2 w(z) = Az^8 + 54Az^5, \tag{40}$$

is obtained.

If the differential transformation of both sides is taken,

$$W(k+1) = \frac{1}{(k+1)(k+4)}\left[-\sum_{r=0}^{k}\delta(r-2)W(k-r) + A\delta(k-8) + 54A\delta(k-5)\right], \tag{41}$$

is obtained. Initial conditions in (39), $z_0 = 0$, can be converted as

$$W(0) = 0, W(1) = 0. \tag{42}$$

Equation (42) is found in $W(2) = 0$ for $k = 1$ if it is written instead in the expression (41) for $k = 1$. If we follow the same procedure for $k = 2, W(3) = 0$; for $k = 3, W(4) = 0$; for $k = 4, W(5) = 0$; and for $k = 5, W(6) = A$. If continued in this way, $W(k) = 0$ becomes $k \geq 7$. Using the inverse transformation rule,

$$\begin{aligned} w(z) &= \sum_{k=0}^{\infty} W(k)z^k = W(0) + W(1)z + W(2)z^2 + W(3)z^3 + W(4)z^4 + W(5)z^5 + W(6)z\hat{6}\ldots \\ &= Az^6, \end{aligned} \tag{43}$$

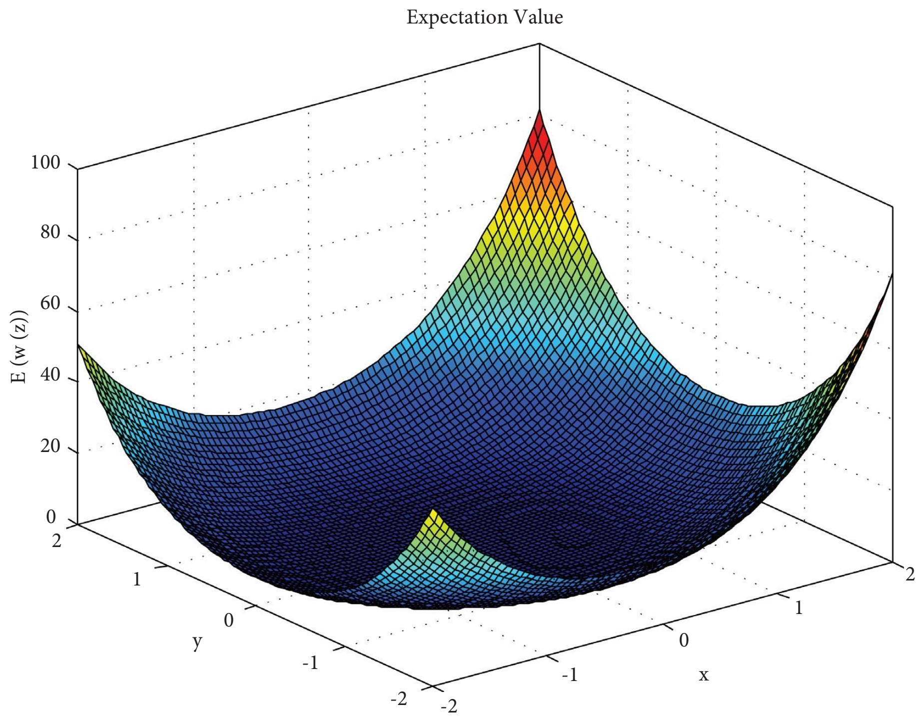

Figure 4: Expected value of equation (29) for $\mu = 1, \sigma = 2$.

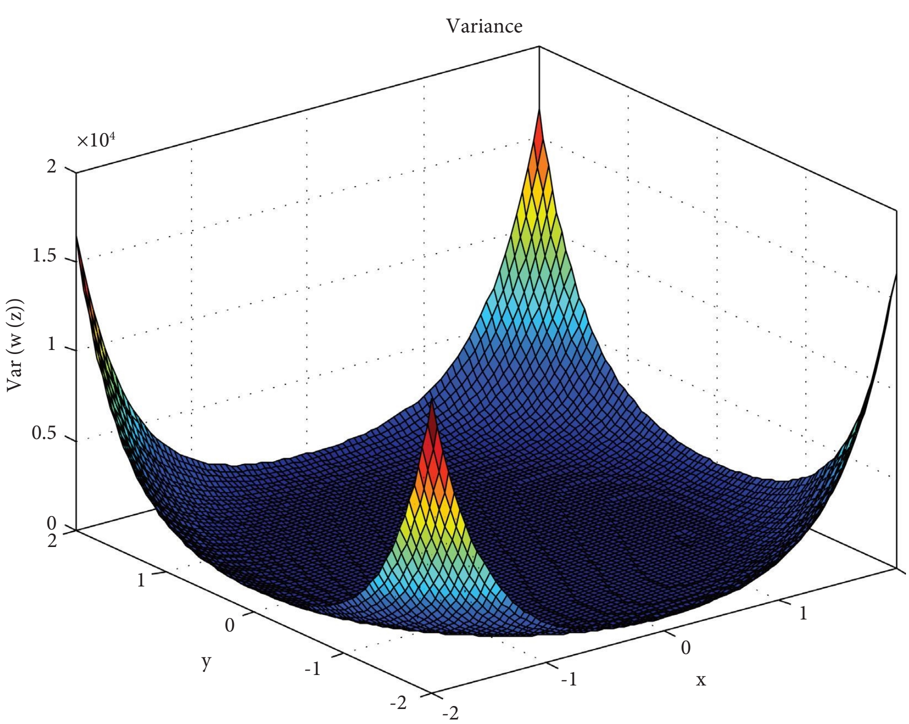

Figure 5: Variance of equation (29) for $\mu = 1, \sigma = 2$.

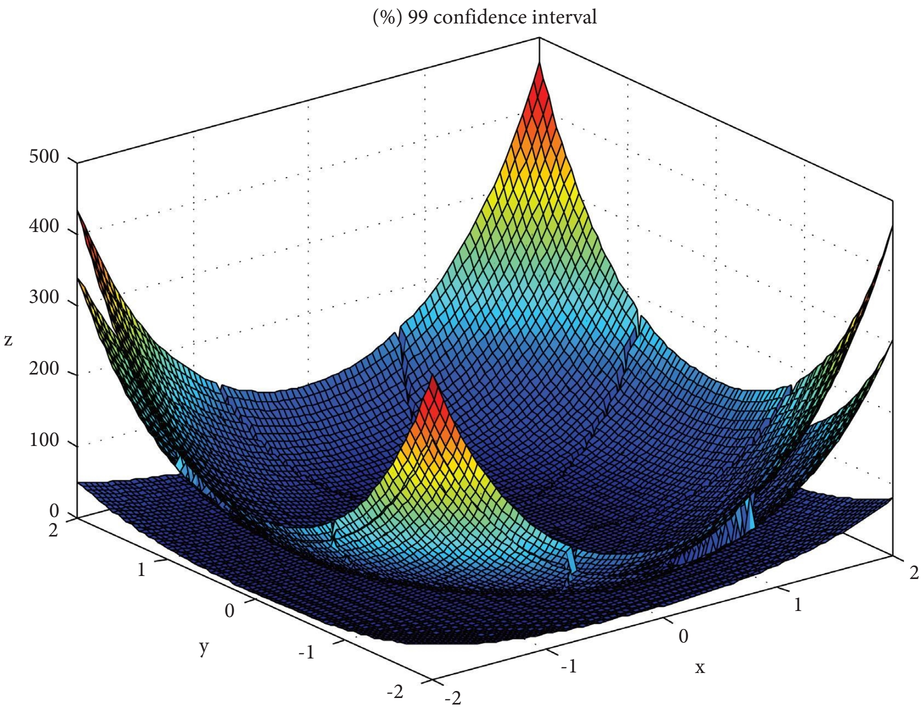

FIGURE 6: The 99% confidence interval of equation (29) for $\alpha = 1, \beta = 2$.

which converges efficiently to the exact solution $w(z) = Az^6$.

This is also the complete solution of equation (38). If $z \sim \text{Beta}(\alpha,)$ is a beta distribution [22],

$$M_z(\alpha, \beta, t) = E\left[e^{tz}\right] = 1 + \sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1}\frac{\alpha+\beta}{\alpha+\beta+r}\right)\frac{t^k}{k!}E[z] = \frac{\alpha}{\alpha+\beta}, \text{Var}[z] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \tag{44}$$

The expected value is

$$\begin{aligned} E[w(z)] &= E\left[Az^6\right] \\ &= E[A]z^6 \\ &= \frac{\alpha}{\alpha+\beta}z^6. \end{aligned} \tag{45}$$

Variance is

$$\begin{aligned} \text{Var}[w(z)] &= \text{Var}\left[Az^6\right] \\ &= \text{Var}[A]z^{12} \\ &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}z^{12}, \end{aligned} \tag{46}$$

Specifically, if $z \sim \text{Beta}(\alpha = 1, \beta = 2)$ is selected, the expected value is

$$\begin{aligned} E[w(z)] &= \frac{\alpha}{\alpha+\beta}z^6 \\ &= \frac{z^6}{3}. \end{aligned} \tag{47}$$

The expectations can be given in a single graph for a comparison with the deterministic results of equation (1) as above (Figure 7). Maximum and minimum values of expected values of the random variables are obtained as follows: $w(z)$ takes its maximum value as 170.6667 and its minimum value as 0.

$$\begin{aligned} \text{Var}[w(z)] &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}z^{12} \\ &= \frac{z^{12}}{18}. \end{aligned} \tag{48}$$

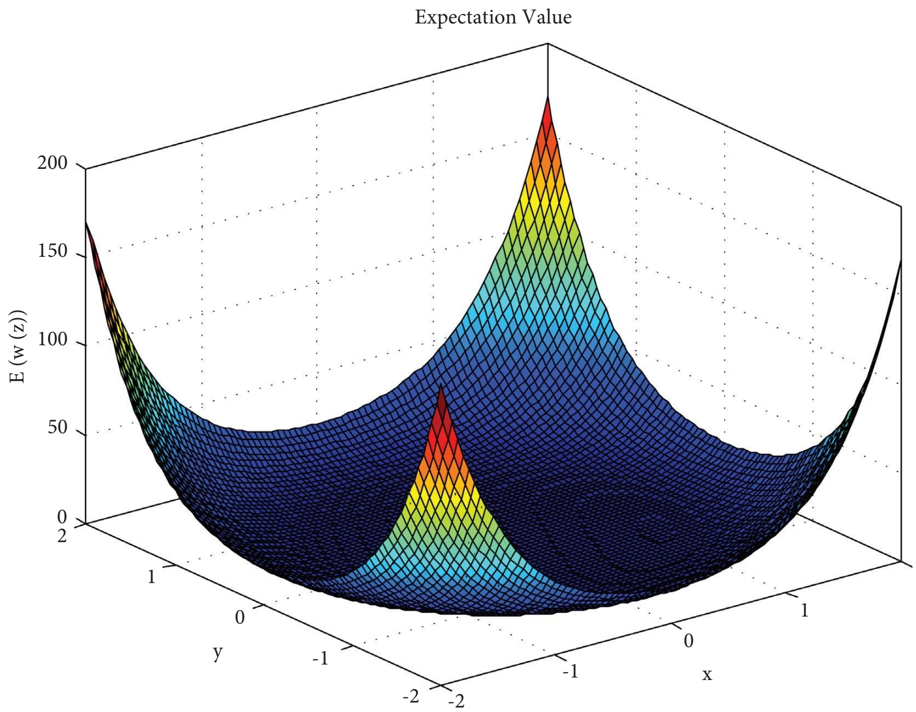

Figure 7: Expected value of equation (38) for $\alpha = 1$, $\beta = 2$.

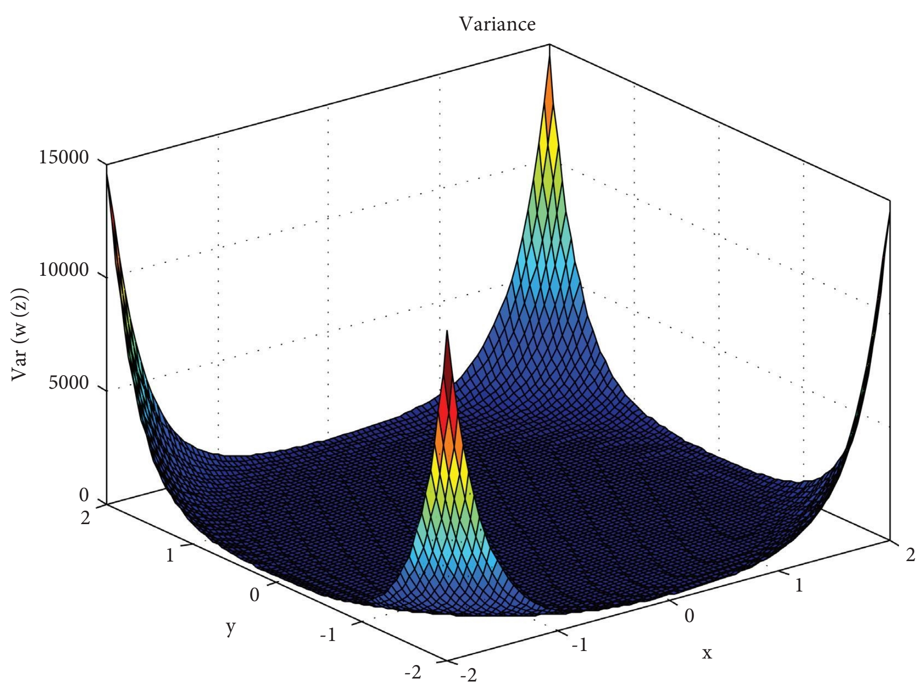

Figure 8: Variance of equation (38) for $\alpha = 1$, $\beta = 2$.

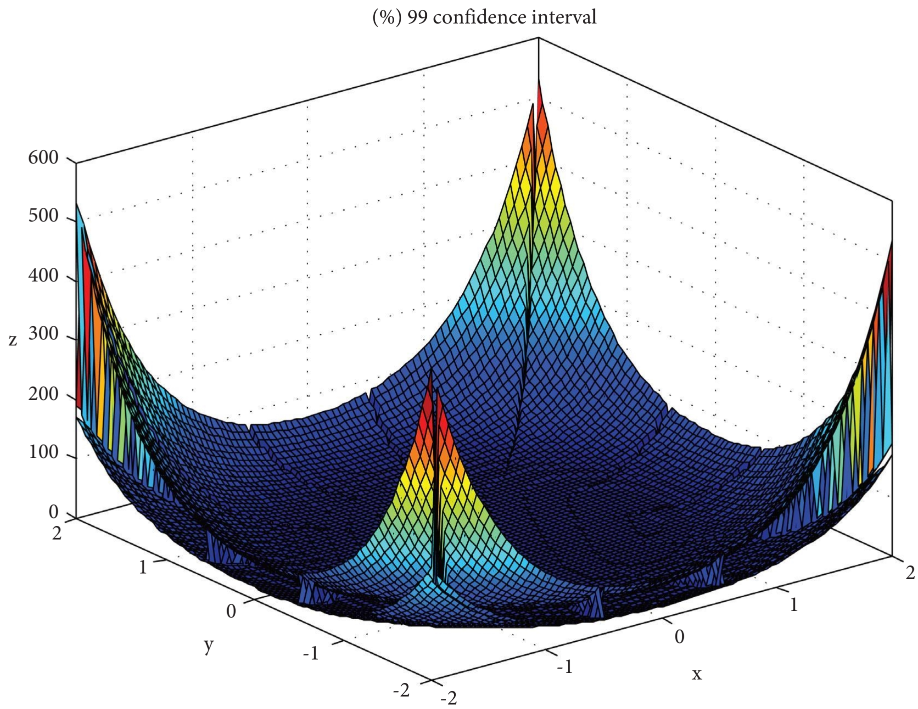

Figure 9: The 99% confidence interval of equation (38) for $\alpha = 1, \beta = 2$.

The variances of $w(z)$ are given above (Figure 8). Extremum values of the variances of the random variables are obtained as follows: $\min [\mathrm{Var}(w(z))] = 0$ and $\max [\mathrm{Var}(w(z))] = 14564$.

The confidence intervals of $w(z)$ are given in Figure 9. The extremum values of the confidence intervals are as follows: $\min (E(w(z)) - 3\mathrm{std}(w(z))) = 0$ and $\max (E(w(z)) + 3\mathrm{std}(w(z))) = 532.7053$. Here, $K = 3$ gives an approximate 99% confidence interval.

6. Conclusion

In this study, a random differential transformation method is applied to solutions of random Lane–Emden complex differential equations. An example of this method is the Lane–Emden complex, which contains gamma, beta, and normally distributed random components. The expected values, variances, and confidence intervals of the complex differential equations solved with the help of the MAPLE program are obtained, and their probability properties are shown in graphics.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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