Racism Dissemination Model and Simulation Analysis Considering Crowd Classification with Intervention Strategies

1. Introduction

In real-life activities, all humankind has its social environment and network [1, 2]. These networks are linked to formulating an extensive and complex multicultural social network [3, 4]. Throughout this linkage, individuals can disseminate manifold information, such as personal opinions or mindsets toward things or behaviors, and influence each other [5]. Meanwhile, in the network formation, race-based segregation insidiously perpetuates racism [6]. Racism and racial bigotry bear multiple shapes and challenge everyone’s daily life [7, 8]. The derivative perpetuates racial crowd mistreatment in public policies, institutional rules, and cultural manifestations [9]. Racism can be manifested at structural and individual levels [10, 11]. The myriad articulations of racism can influence numerous activities of the community’s straightforward and indirect pathways. Structural racism has been and sustains the fundamental causality of persistent multiple services disparities in the community [12]. Recent research works verified that people who experience racism more than rates of sicknesses. In the health system, racism is an absolute cause of health

The blow of racism is unexpected and ultimately harms everyone’s social harmony. Develop empathy about how it impacts people and the wider community, thereby allowing to change mindsets and behaviors. The racism awareness programs are structured to support controlling the issues of disparity and the absence of endorsement of others among individuals [14]. It enhances to provide that social network settings are comprehensive and reward cohesion without racial imbalance or racism. Antiracist education enables pupils to understand the destructive results of racism and nurtures them to resist and stop wherever it emerges [15, 16]. In addition, confining racist networks and activity is also worthwhile to remove racism [17]. Systems science approaches developed to advance essential social aspects suggest a scientific method in backing community decision-making to reduce the effects of racism.

System dynamic models are used to compute and analyze physical, biological, and social phenomena in real-world problems. Numerous dynamic system models of many types have been formulated and studied to provide an understanding of the transmission of infectious diseases [18–21]. Recently, various research studies real-life phenomena using fractional order models [22, 23]. The process of rumor spread is similar to the process of epidemic spreading; many researchers study the propagation of rumors in social networks [24–26]. In opinion dynamics, researchers have constructed some achievements in information spreading in social networks [27, 28] and its control [29]. Based on emotion contagion and opinion dynamic emotion diffusion model was designed and analyzed [30]. Furthermore, some researchers study racial segregation dynamics and interconnected social impacts [31–34]. Chen and Wang studied the rumor-spreading model considering rumor credibility, correlation, and crowd classification based on personality [35]. We comprehend from previous research that rumors, misinformation, and all categories of crud information and opinions are spreading on social networks. It pivots out that racist speech is also one of these categories of crud information and opinions. The contagiousness of racism in multiracial societies acts as the trigger for hate crimes and physical violence. Mamo recently studied the dynamical process of racism propagation on cyberspace [36, 37]. Recently, Teklu and Terefe studied animosity and racism and violence codynamic dynamics using compartment models [38, 39].

Nevertheless, we review that none of the previous research incorporates racism dissemination dynamics with the crowd classification. In each information dissemination process, individuals will convince, reluctant, or reject the information disseminated by hostile individuals [40]. In idea-building modeling, one must assume the personality varieties of an individual within the total population. An individual’s personality represents inherent human attributes that conscript behavior preference. Constantly some people immediately assemble racist ideology. However, some people are calm and desire to follow assurance instantly. The varieties of individual essence are the factor that affects the racism disseminating process and should consider in the study of racism propagation. Furthermore, antiracism education boosts community resilience and removes the susceptible radical personality. Community awareness programs foster personal opinions against racism and nullify radical personalities. Antiracism education is a proactive measure to mitigate the propagation and remove the harmful threat of racism. Yet, some radical racists strive in their toxic attitude and intentionally disseminate racism in a multicultural society. Racist individuals threaten social solidarity and introduce socioeconomic flux and political crises. Regulations against racial discrimination and hate crimes allow for confining racist and racist assertions via social networks. We consider racist confinement within treatment as one of the racism-consolidating intervention strategies. The racist confinement could reduce the racism dissemination rate by restricting racist individuals and their racist ideas.

In the next section, we introduce the model assumption that governs the model formulation process. After that, in Section 3, we present the theoretical analysis, obtaining the racism-free equilibrium and computing the effective reproduction number. We prove the local and global stability of equilibrium points. In Section 4, we evaluate model parameter sensitivity and its interpretation. In Section 5, we carried out a numerical simulation of dynamical systems, which confirmed theoretical analysis and assessed the impact of intervention strategies. Eventually, Section 6 is reserved for the conclusion.

2. Model Description

The propagation of racism is a peril to social, political, and cultural stability and has threatened the harmony of various nations. The spread of racism may diffuse via a social connection process in which racist ideologies bear complex contagiousness. Consequently, to propose compartmental models for the study of racialism is valid. Accordingly, we formulate the governing mathematical model for the study of racism dissemination and assess the impact of intervention strategies by splitting the entire population into seven mutually exclusive compartments, namely, calm susceptible ($S_1$), radical susceptible ($S_2$), awarded susceptible ($S_3$), hesitant ($H$), racist ($R$), confined racist ($C$), and immunized ($I$). The model considers susceptible individuals’ crowd categories based on personality and fosters cohesion via antiracism campaigns and racist custody as intervention strategies.

2.1. Model Assumptions. To develop the dynamics of racism in multicultural societies, we designate the following scene of explicit assumptions about the phenomenon that discloses clear distinctions with epidemic models:

(A1) The susceptible crowds are separated into three subgroups, namely, calm susceptible ($S_1$), radical susceptible ($S_2$), and aware susceptible ($S_3$). We assume that the portion of the recruited susceptible
Complexity

who entered a calm group and radical group at a rate of $\psi \Lambda$ and $(1 - \psi) \Lambda$, respectively, where $\psi$ is the probability of calmness and $\Lambda$ is the susceptible recruited rate. Calm and radical susceptible individuals carry antiracism education and progress into awarded class $S_i$ at a rate of $\xi$. We consider $\mu$ the natural death rate.

(A2) Racist individuals vigorously proselytize to persuade those conceivable to join their group. The convincing contagious of each racist individual is represented by a parameter $\beta$. We assume that each racist has $\beta$ effective contacts per unit of time. Thus, $\beta RS_i$ individuals will leave the group $S_i$ per unit of time (where $i = 1, 2, 3$).

(A3) The racist essence of the opinion being propagated compromises that the overhead dealings may result in a conviction, active rejection, and hesitation. We will model this convection, hesitation, and rejection founded on the following subsequent assumptions:

(i) The fraction $\beta$ those who left radical and calm susceptible groups by interaction with the racist adhere to the group $R$ and $H$, respectively. Established on their personalities, radical susceptible directly join racist groups without hesitation and rejection. Nevertheless, calm individuals request substantial proof and time to convince or reject the notion, and as late as their decision, those individuals enter the hesitant group.

(ii) An antiracism education creates awareness about the devastating consequence of racism in multicultural societies, enhances the denial strength, and discharges radicalism. The fraction of $\theta$ awarded susceptible interaction with the racists joins the hesitant group, while the rest, by the resistance, transfers to the compartment $I$ without being racist. The parameter $\theta$ will depend on the efficiency of the antiracism education operated or intentional reluctance.

(A4) We consider $1/\sigma$ as the average indecision period of the $H$ group, and $\sigma$ is the transfer rate. Furthermore, behind the termination of hesitancy, some are persuaded at a probability of $\kappa$, but the complement rejects and evolves to immunize group $I$.

(A5) Various motivations, failure of conviction in the ideology desire to immunize racists, can assemble the determination to exit the radical group. Accordingly, we assume racists recover at a rate $\omega$, which depends on the group’s proficiency to retain its members. It will authorize a $\omega R$ discharge of individuals per unit of time that will pass from compartment $R$ to compartment $I$.

(A6) Across-the-board racism diffusion creates socioeconomic flux and political inequality. To bypass such threats, designing antiracism regulations and proclamations, inspecting media content before broadcasting, and confining racists are imperative. We assume that based on regulations and declarations prohibiting racists and their racist activity could stem the spread of racism. The racist individuals confined at a rate of $\eta$ join group $C$.

(A7) We assume that racists confinement performs at a period of $1/\gamma$ (where $\gamma$ is the release rate). Confinement fades with a probability of $\rho$ because of the inefficiency of custody or deliberate temptation. However, at a rate $(1 - \rho)\gamma$, confined racists recovered and join to the immunized group $I$.

Established on the above-given assumptions and conditions of hypothesis and the schematic diagram in Figure 1, a system dynamics of racism dissemination considering the roles of racist confine and community awareness within the crowd classification model can be described by the following equation:

\[
\begin{align*}
\frac{dS_1}{dt} &= \Lambda \psi - \beta S_1 R - (\xi + \mu)S_1, \\
\frac{dS_2}{dt} &= \Lambda (1 - \psi) - \beta S_2 R - (\xi + \mu)S_2, \\
\frac{dS_3}{dt} &= \xi (S_1 + S_2) - \beta S_3 R - \mu S_3, \\
\frac{dH}{dt} &= \beta S_1 R + \beta S_2 R - (\sigma + \mu)H, \\
\frac{dR}{dt} &= \beta S_3 R + \sigma \kappa H + \rho \gamma C - (\eta + \omega + \mu)R, \\
\frac{dC}{dt} &= \eta R - (\gamma + \mu)C, \\
\frac{dI}{dt} &= (1 - \kappa)\alpha H + (1 - \theta)\beta S_3 R + (1 - \rho)\gamma C + \omega R - \mu I.
\end{align*}
\]

with the initial conditions

\[
\begin{align*}
S_1(0) &> 0, S_2(0) > 0, S_3(0) \geq 0, H(0) \geq 0, R(0) \geq 0, C(0) \geq 0, I(0) \geq 0.
\end{align*}
\]

2.2. Basic Properties of the Model. For system (1), describing the dynamics of racism with the behavioral status of individuals, all state variables and parameters of the model are non-negative. It is essential to confirm that all of the solutions with initial conditions (2) remain positive for all $t \geq 0$. Consider the pursuing social-feasible region for the model:

\[
\Omega = \{S_1, S_2, S_3, H, R, C, I\} \in \mathbb{R}_+^7: \text{N} (t) \leq \frac{\Lambda}{\mu}.
\]
**Theorem 1.** The region $\Omega$ is positively-invariant to the model (1).

*Proof.* Consider $(S_1, S_2, S_3, H, R, C, I)$ is the solution of model (1), we obtain the following equation:

$$\begin{align*}
\frac{dS_1}{dt} |_{S_{1,0}} &= \Lambda \psi > 0, \\
\frac{dS_2}{dt} |_{S_{2,0}} &= \Lambda (1 - \psi) > 0,
\end{align*}$$

$$\begin{align*}
\frac{dS_3}{dt} |_{S_{3,0}} &= \xi (S_1 + S_2) \geq 0, \\
\frac{dH}{dt} |_{H_{t=0}} &= \beta S_1 R + \theta \beta S_2 R \geq 0,
\end{align*}$$

$$\begin{align*}
\frac{dR}{dt} &= \alpha H + \rho \gamma C \geq 0, \\
\frac{dC}{dt} |_{C_{t=0}} &= \eta R \geq 0,
\end{align*}$$

$$\begin{align*}
\frac{dI}{dt} |_{I_{t=0}} &= (1 - \kappa) \sigma H + (1 - \theta) \beta S_2 R + (1 - \rho) \gamma C + \omega R \geq 0,
\end{align*}$$

with the continuity of the solution of the system (1), $S_1 (t) > 0, S_2 (t) > 0, S_3 (t) > 0, H (t) > 0, R (t) > 0, C (t) > 0$ for all $t \geq 0$. The summation of all of the equations in the model (1) gives the following equation:

$$\begin{align*}
\frac{dN}{dt} &= -\mu (S_1 (t) + S_2 (t) + S_3 (t) + H (t)) \\
+ R (t) + C (t) + I (t))
\end{align*}$$

$$N (t) \leq \int_{0}^{t} \left[ \frac{dN}{dt} + \mu N \right] dt$$

Since all parameter values of mode 1 are positive, from (5), it follows that

$$\begin{align*}
\frac{dN}{dt} &\leq \Lambda - \mu N.
\end{align*}$$

Furthermore, the solution of equation (5) is as follows:

$$\begin{align*}
0 \leq N (t) \leq \left( N (0) e^{-\mu t} - \frac{\Lambda}{\mu} e^{-\mu t} + \frac{\Lambda}{\mu} \right), \forall t \geq 0.
\end{align*}$$

At any time $t > 0$, the total population is less than the upper bound $(\lim_{t \to \infty} N (t) \leq \Lambda/\mu)$ if $N (0) \leq \Lambda/\mu$. Hence, for system (1), all trajectories stay inside the region $\Omega$. Thus, the region $\Omega$ is positively-invariant and attracts all solutions trajectories of the model (1); in other words, the existence and uniqueness of solutions are provided. Therefore, model (1) is mathematically and socially well-posed [41].

\[ \square \]

3. **Theoretical Analysis of the Model**

In this section, we provide the threshold value, $\mathcal{R}_c$, which is the effective reproduction number of the model (1). The number is defined as the average number of susceptible individuals recruited by each racist individual while remaining in the racist group. We bear it as racism strength because it is the size of the recruitment and retention capability of the racist group. This parameter is equivalent to the basic reproduction number in epidemiological models [42–44]. Then, we investigate the stability properties of the racism-free equilibrium of the system (1), which is equal to

$$E_0 = \left( S_1^0, S_2^0, S_3^0, 0, 0, 0, 0 \right)$$

$$\begin{align*}
\begin{pmatrix}
\psi \Lambda \\
(1 - \psi) \Lambda \\
\Lambda \xi
\end{pmatrix}
\begin{pmatrix}
\frac{\xi}{\mu + \xi} \\
\frac{\mu}{\mu + \xi} \\
0
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}.
\end{align*}$$

**Lemma 1.** The effective reproduction number $\mathcal{R}_c$ of the system (1) is given by the following equation:

$$\mathcal{R}_c = \frac{\beta \Lambda (\gamma + \mu) (\mu ((1 - \psi) (\sigma + \mu) + \kappa \psi) + \theta \kappa \sigma \xi)}{\mu (\sigma + \mu) (\xi + \mu) (\gamma ((1 - \rho) \eta + \omega + \mu) + \mu (\eta + \omega + \mu))}.$$
Using the next-generation matrix approach the reproduction number \( \mathcal{R}_0 \) reduces to basic reproduction number \( \mathcal{R}_0 \), given by the following equation:

\[
\mathcal{R}_0 = \frac{\beta \Lambda (\kappa \psi + (1 - \psi) (\mu + \sigma))}{\mu (\sigma + \mu) (\omega + \mu)}.
\]

3.1. Local Stability of Racism-Free Equilibrium. In the absence of racism (by setting racist compartment \( R = 0, H = 0 \)), we get a unique racism-free equilibrium (8). In the absence of intervention strategies, the efective reproduction number (\( \mathcal{R}_e \)) reduces to basic reproduction number (\( \mathcal{R}_0 \)), given by the following equation:

\[
\mathcal{R}_e = \frac{\beta \Lambda (\gamma + \mu) (\mu (\gamma (1 - \psi) (\sigma + \mu) + \kappa \psi) + \theta \kappa \sigma \xi)}{\mu (\sigma + \mu) (\xi (\gamma (1 - \rho) \eta + \omega + \mu) + \mu (\eta + \omega + \omega))}.
\]

In this context, the Jacobian matrix of the system (1) at racism-free equilibrium point \( E_0 \) is

\[
J(E_0) = \begin{pmatrix}
-(\mu + \xi) & 0 & 0 & 0 & \frac{\beta \Lambda \psi}{\mu + \xi} & 0 & 0 \\
0 & -(\mu + \xi) & 0 & 0 & \frac{(1 - \psi) \beta \Lambda}{\mu + \xi} & 0 & 0 \\
\xi & \xi & -\mu & 0 & \frac{\beta \Lambda \xi}{\mu (\mu + \xi)} & 0 & 0 \\
0 & 0 & 0 & -(\mu + \sigma) & \frac{\beta \theta \Lambda \xi}{\mu (\mu + \xi)} + \frac{\beta \Lambda \psi}{\mu + \xi} & 0 & 0 \\
0 & 0 & 0 & \kappa \sigma & -(\eta + \mu + \omega) + \frac{(1 - \psi) \beta \Lambda}{\mu + \xi} & \gamma \rho & 0 \\
0 & 0 & 0 & 0 & \eta & -(\gamma + \mu) & 0 \\
0 & 0 & 0 & (1 - \kappa) \sigma & \frac{(1 - \theta) \beta \Lambda \xi}{\mu (\mu + \xi)} + \omega & (1 - \rho) \gamma & -\mu
\end{pmatrix}.
\]

where \( a_0 = 1, a_1 = (\mu + \xi) (\gamma + \eta + 3 \mu + \sigma + \omega) - \beta \Lambda (1 - \psi) /\mu + \xi, a_2 = \sigma + \mu / \gamma + \mu (\gamma (1 - \rho) \eta + \omega + \mu) + \mu (\eta + \omega + \omega) + (1 - \mathcal{R}_e) + \beta \Lambda (\gamma + \mu) (1 - \psi) (\xi + \mu), \) and \( a_3 = (\sigma + \mu) / (\gamma (1 - \rho) \eta + \omega + \mu) + \mu (\eta + \omega + \omega) (1 - \mathcal{R}_e) \). Here, we get \( a_0 > 0, a_1 > 0, a_3 > 0, \) and \( a_1 a_2 - a_0 a_3 > 0 \) if the effective reproduction number \( \mathcal{R}_e < 1 \). Hence, using Ruth–Hurwitz
stability criteria all the eigenvalues of \( f(E_0) \) have negative real part if \( R_\varepsilon < 1 \). Therefore, the racism-free equilibrium point \( E_0 \) is locally asymptotically stable.

The social implication of Theorem 2 denotes that the racist group does not continue to conceive a reasonable capacity for tempting susceptible individuals or retaining its contemporary fellows if \( R_\varepsilon < 1 \).

### 3.2. Global Stability of Racism-Free Equilibrium

**Theorem 3.** The racism-free equilibrium \( E_0 \) of system (1) is globally asymptotically stable in \( \Omega \) if \( R_\varepsilon < 1 \).

**Proof.** Suppose \( (S_1, S_2, S_3, H, R, C, I) \) be a non-negative solution of system (1). To complete the proof, it is sufficient to show that this non-negative solution tends to the racism-free equilibrium \( E_0 \) at \( t \) approaches to positive infinity.

According to the first, second, and third equations of system (1), we obtain the following equation:

\[
\begin{align*}
\frac{dS_1}{dt} &= \lambda S_1 - \beta S_1 R - (\xi + \mu S_1), \\
\frac{dS_2}{dt} &= \lambda (1 - \psi) - \beta S_2 R - (\xi + \mu S_2), \\
\frac{dS_3}{dt} &= \xi (S_1 + S_2) - \beta S_3 - \mu S_3.
\end{align*}
\]

For any \( \varepsilon > 0 \), it exists \( T_\varepsilon \), when \( t > T_\varepsilon \), we have \( S_1 \leq S_1^0 + \varepsilon, S_2 \leq S_2^0 + \varepsilon, \) and \( S_3 \leq S_3^0 + \varepsilon \).

Hence, the fourth, fifth, sixth, and seventh equations of the system (1) with \( S_1 \leq S_1^0 + \varepsilon, S_2 \leq S_2^0 + \varepsilon, \) and \( S_3 \leq S_3^0 + \varepsilon \), give the inequality

\[
\begin{align*}
\frac{dH}{dt} &\leq \beta (S_1^0 + \varepsilon) R + \theta \beta (S_2^0 + \varepsilon) R - (\sigma + \mu) H, \\
\frac{dR}{dt} &\leq \beta (S_1^0 + \varepsilon) R + \sigma H + \rho \gamma C - (\eta + \omega + \mu) R, \\
\frac{dC}{dt} &\leq \eta R - (\gamma + \mu) C, \\
\frac{dI}{dt} &\leq (1 - \kappa) \sigma H + (1 - \theta) \beta (S_1^0 + \varepsilon) R + (1 - \rho) \gamma C + \omega R - \mu I.
\end{align*}
\]

(16)

Define an auxiliary linear system by (17), namely

\[
\begin{align*}
\frac{dH}{dt} &= \beta (S_1^0 + \varepsilon) R + \theta \beta (S_2^0 + \varepsilon) R - (\sigma + \mu) H, \\
\frac{dR}{dt} &= \beta (S_1^0 + \varepsilon) R + \sigma H + \rho \gamma C - (\eta + \omega + \mu) R, \\
\frac{dC}{dt} &= \eta R - (\gamma + \mu) C, \\
\frac{dI}{dt} &= (1 - \kappa) \sigma H + (1 - \theta) \beta (S_1^0 + \varepsilon) R + (1 - \rho) \gamma C + \omega R - \mu I.
\end{align*}
\]

(18)

For \( R_\varepsilon < 1 \), we can fix an \( \varepsilon \) small enough such that \( R_\varepsilon (\varepsilon) < 1 \), each eigenvalue of the characteristic equation of the Jacobian matrix at the racism-free equilibrium of system (18) lies in the left half plane. Thus, each non-negative solution of system (18) satisfies \( \lim_{t \to \infty} H(t) = 0, \lim_{t \to \infty} R(t) = 0, \lim_{t \to \infty} C(t) = 0, \) and \( \lim_{t \to \infty} I(t) = 0 \).

Since system (18) is a linear system, the racism-free equilibrium of system (18) is globally asymptotically stable. By the comparison principle, it is easy to see that each non-negative solution of the fourth, fifth, sixth, and seventh equation of system (1) satisfies \( \lim_{t \to \infty} H(t) = 0, \lim_{t \to \infty} R(t) = 0, \lim_{t \to \infty} C(t) = 0, \) and \( \lim_{t \to \infty} I(t) = 0 \). From the first, second, and third equations of system (1), since \( \lim_{t \to \infty} H(t) = 0, \lim_{t \to \infty} R(t) = 0, \lim_{t \to \infty} C(t) = 0, \) and \( \lim_{t \to \infty} I(t) = 0 \), then

\[
\begin{align*}
\frac{dS_1}{dt} &= \lambda \psi - (\xi + \mu) S_1, \\
\frac{dS_2}{dt} &= \lambda (1 - \psi) - (\xi + \mu) S_2, \\
\frac{dS_3}{dt} &= \xi (S_1 + S_2) - (\mu + \xi) S_3.
\end{align*}
\]

(19)

Thus, \( \lim_{t \to \infty} S_1(t) = \lambda \psi \xi / \mu + \mu, \lim_{t \to \infty} S_2(t) = (1 - \psi) \lambda / \xi + \mu, \) and \( \lim_{t \to \infty} S_3(t) = \lambda \xi / \mu \xi + \mu \) then the \( E_0 \) is globally asymptotically stable.

### 3.3. Existence and Local Stability of Racism-Addicted Equilibrium

System (1) has racism-addicted equilibrium, denoted by \( E^* = (S_1^*, S_2^*, S_3^*, H^*, R^*, C^*, I^*) \). To find the racism-addicted equilibrium of system (1), we solve for \( S_1^*, S_2^*, S_3^*, H^*, C^*, I^* \). After some algebraic simplification, we have the following equation:

\[
\begin{align*}
S_1^* &= \frac{\psi \Lambda}{K_1}, S_2^* = \frac{(1 - \psi) \Lambda}{K_1}, \\
S_3^* &= \frac{\xi \Lambda}{K_1 K_2}, H^* = \frac{\beta \Lambda (\psi K_2 + \xi) R^*}{K_1 K_2 (\sigma + \mu)}, \\
C^* &= \frac{\eta R^*/(\gamma + \mu)}{I^* = \frac{(1 - \kappa) \sigma H^* + (1 - \theta) \beta (S_1^0 + \varepsilon) R^* + (1 - \rho) \gamma C^* + \omega R^*}{(1 - \kappa) \sigma H^* + (1 - \theta) \beta (S_1^0 + \varepsilon) R^* + (1 - \rho) \gamma C^* + \omega R^*}
\end{align*}
\]

(20)
where \( K_1 = \beta R^* + \xi + \mu \), and \( K_2 = \beta R^* + \theta + \mu \). We substitute \( S_1^*, H^*\), and \( C^* \) into the fifth equation of system (1) to obtain the following equation:

\[
a R^* + b R^* + c = 0, \quad (21)
\]

where

\[
a = \beta^2 (\sigma + \mu) (\eta \rho - (\gamma + \mu) (\eta + \mu + \omega)),
\]

\[
b = \beta ((\gamma + (\mu)(\sigma + \mu)(\omega + \mu)(\mathcal{R}_0 - 1) + \eta (\sigma + \mu) (2 \gamma \rho - (\gamma + \mu))),
\]

\[
c = \mu (\sigma + \mu) (\xi + \mu) (\gamma ((1 - \rho)\eta + \omega + \mu) + \mu (\eta + \mu + \omega))(\mathcal{R}_c - 1).
\]

Let \( f(R^*) = a R^* + b R^* + c \), then \( f(0) < 0 \), and the differentiation of roots is greater than zero. Therefore, \( f(R^*) \) has only one positive root in \((0, \infty)\), if \( \mathcal{R}_c > 1 \), namely

\[
R^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \quad (23)
\]

\[
J(E^*) = \begin{pmatrix}
-K_1 & 0 & 0 & 0 & -\beta S_1^* & 0 & 0 \\
0 & -K_1 & 0 & 0 & -\beta S_2^* & 0 & 0 \\
\xi & \xi & -K_2 & 0 & -\beta S_3^* & 0 & 0 \\
\beta R^* & 0 & \theta \beta R^* & -(\sigma + \mu) & \beta (\beta S_3^* + S_1^*) & 0 & 0 \\
0 & \beta R^* & 0 & \kappa \sigma & \beta S_2^* - (\eta + \omega + \mu) & \rho \gamma & 0 \\
0 & 0 & 0 & \eta & -\gamma + \mu & 0 & 0 \\
0 & 0 & (1 - \theta) \beta R^* & (1 - \kappa) \sigma & (1 - \theta) \beta S_3^* + \omega & (1 - \rho) \gamma & -\mu \\
\end{pmatrix}, \quad (24)
\]

where \( S_1^*, S_2^*, \) and \( S_3^* \) are obtained from (20), and \( R^* \) is obtained from (23). The characteristic polynomial of \( J(E^*) \) is

\[
P(\lambda) = (\lambda + \mu) (\lambda + K_1) (C_0 \lambda^5 + C_1 \lambda^4 + C_2 \lambda^3 + C_3 \lambda^2 + C_4 \lambda + C_5). \quad (25)
\]

Obviously, we obtain negative eigenvalues \( \lambda_1 = -\mu \) and \( \lambda_2 = -K_1 \). The eigenvalues of \( J(E^*) \) satisfy the polynomial

\[
C_0 \lambda^5 + C_1 \lambda^4 + C_2 \lambda^3 + C_3 \lambda^2 + C_4 \lambda + C_5, \quad (26)
\]

where

\[
\text{Theorem 4.} \quad \text{If} \quad \mathcal{R}_c > 1, \quad \text{the} \quad \text{racism-addicted equilibrium} \quad E^* \quad \text{of} \quad \text{the} \quad \text{system} \quad (1) \quad \text{is} \quad \text{locally} \quad \text{asymptotically} \quad \text{stable}.
\]

\[
\text{Proof.} \quad \text{The Jacobian matrix of the system} \quad (1) \quad \text{at} \quad E^* \quad \text{is}
\]
3.4. Global Stability of Racism-Addicted Equilibrium

**Theorem 5.** the racism-addicted equilibrium $E^*$ of the system (1) is globally asymptotically stable if $R_e > 1$.

**Proof.** Consider the positive-definite function,

$$
\mathcal{L}(M) = \frac{1}{2}((S_1 - S_1^*) + (S_2 - S_2^*) + (S_3 - S_3^*) + (H - H^*) + (R - R^*) + (C - C^*) + (I - I^*))^2,
$$

where $M = (S_1, S_2, S_3, H, R, C, I)$.

By employing the derivative of the function with respect to $t$ we get the following equation:

$$
\frac{d\mathcal{L}(M)}{dt} = ((S_1 - S_1^*) + (S_2 - S_2^*) + (S_3 - S_3^*) + (H - H^*) + (R - R^*) + (C - C^*) + (I - I^*)) \times \frac{dS_1}{dt} + \frac{dS_2}{dt} + \frac{dS_3}{dt} + \frac{dH}{dt} + \frac{dR}{dt} + \frac{dC}{dt} + \frac{dI}{dt}
$$

$$
= (N - S_1^* - S_2^* - S_3^* - H^* - R^* - C^* - I^*) \times \frac{dN}{dt}
$$

$$
= \left( N - \frac{\Lambda}{\mu} \right) \left( \frac{dN}{dt} \right)
$$

$$
\leq \left( N - \frac{\Lambda}{\mu} \right) (\Lambda - \mu N) \leq - (\Lambda - \mu N)^2 \leq 0.
$$

The derivative of the function $\mathcal{L}(M)$ is greater than zero, and it is equal to zero at the endemic equilibrium point $S_1 = S_1^*, S_2 = S_2^*, S_3 = S_3^*, H = H^*, R = R^*, C = C^*, \text{ and } I = I^*$. The strictly Lyapunov function indicates that the endemic equilibrium ($E^*$) is globally asymptotically stable in the invariant region $\Omega$. Socially, this result implies that racism will continue to exist in the community for a long period of time. Hence, the proof is complete.

The social meaning of Theorem 5, in this case, conforms to racist groups that grew towards more efficient strength in at least one of the aspects of recruitment and retention, which gripped inadequate intervention strategies and vulnerable community cohesion. 

□
In Section 3, the theoretical analysis results investigate whether racism dissemination is suppressed or prevails in society pivoting on the reproduction number $R_e$. Consequently, it is reasonable to analyze the effect of control parameters on the racism dissemination process specified on the reproduction number $R_e$. To estimate how much the parameter sensitizes the system, we illustrate the graph of the parameter versus $R_e$.

We begin by depicting the influence of contact rate $\beta \in (0, 1)$ and awarded hesitancy rate $\theta \in (0, 1)$ on $R_e$ by assigning baseline parameter values $\Lambda = 0.01, \mu = 0.01, \eta = 0.05, \psi = 0.75, \sigma = 0.034, \omega = 0.01, \theta = 0.25, \beta = 0.5, \kappa = 0.6, \rho = 0.01, \gamma = 0.056, \text{ and } \xi = 0.05$.

Figures 2 and 3, we investigate the effect of contact rate $\beta$ and awarded susceptible hesitancy probability $\theta$ on $R_e$. The result illustrates effective reproduction is greater than unity if the value of $\beta > 0.3547$ and $\theta > 0.1024$, respectively. This provides an insight into the racism dissemination that the contact and hesitancy rate below 0.3547 and 0.1024 has not a significant effect. Racist strength advance into a malicious phase if the effective contact rate, $\beta > 0.3547$, and the likelihood of awarded hesitancy is $\theta > 0.1024$. Consequently, declining the contentiousness of racist groups and boosting the rejection strength of susceptible individuals is imperative to construct a social environment beyond racism.

As we noted previously, antiracism education and racist confinement are the proposed intervention strategies. Here, we present the sensitized influence of the rate, $\xi \in (0, 1)$ and $\xi \in (0, 1)$ on $R_e$.

In Figures 4 and 5, the functional interchange of the parameters $\xi$ and $\eta$ with the threshold value $R_e$ depict, respectively. Both figures show that the $R_e$ value decreases if the value of $\xi$ and $\eta$ increases. As we observed in 4, antiracism education is admiringly sensitive at the onset and declines its sensitiveness gradually. This sensitivity analysis suggests that approximately beyond 19.68% of antiracism education coverage with more than 75% racist idea rejection competency (which means less than 25% hesitation probability) will be moderately reduced.
In this section, we conduct an extensive numerical simulation of model (1) to obtain in-depth insight into the system dynamics. For simulation purposes, we set the non-negative initial values of model variables \((S_0, S_1, S_3, H_0, R_0, C_0, I(0)) = (0.889, 0.1, 0.0, 0.01, 0.001, 0.0, 0.0)\) and baseline parameter values are listed in Section 4. Furthermore, we verify the theoretical analysis using numerical simulations and assess the impact of proposed intervention strategies in manifold scenarios.

5.1. Stability Based on Numerical Simulations. In this subsection, we perform the time series evaluation of system 1 to verify the asymptotic stability of the equilibrium points. Given that \(\mathcal{R}_c < 1\), society is in a susceptible state before the arrival of the racist group. According to the result in Section 3, under these conditions, the only equilibrium point of system 1 is \(E^0\). Furthermore, we obtained a unique positive racist-addicted equilibrium point \(E^*\) if \(\mathcal{R}_c > 1\). Now, we explain equilibrium stability numerically by varying the initial conditions.

In Figure 6, we evaluate the dynamical changeover of the system by setting the parameters \(\beta = 0.15\) with baseline parameter values. This time series evaluation shows that all solution trajectories which start from the different initial conditions are stable toward the equilibrium point \(E^0 = (0.35, 0.15, 0.5, 0, 0, 0, 0)\). The result implies that as a time \(t \to \infty\), the densities of individuals in class \(H, R, C,\) and \(I\) decline to zero. Further, we computed the racist strength threshold value operating the formula (13) and obtained the value \(\mathcal{R}_c = 0.5307\). Therefore, the numerical result verifies that the racism-free equilibrium point is locally and globally asymptotically stable and agrees with Theorem 2 and 3. The result gives an insight into a small portion of the racist group that emerges, it will disintegrate, and the population will quickly return to a maximum level of susceptible class. This situation implies that racist groups do not create a suitable capacity for attracting susceptible or retaining their contemporary fellows.

Figure 7 displays the uncertainty of racism dissemination with the parameter setting \(\beta = 0.5, \eta = 0.05, \rho = 0.01, \xi = 0.01, \theta = 0.5\). Based on the formula (9), we obtain \(\mathcal{R}_c = 4.8939\), which is greater than one. This estimation implies that each racist can pollute approximately five susceptible. This time series evaluation shows that the system dynamic solution trajectories are stochastic to their respective extreme values and then locally and globally asymptotically steady towards the racism-addicted equilibrium \(E^* = (0.0599, 0.02, 0.0530, 0.1281, 0.1306, 0.0198, 0.5887)\). Equivalent to Theorem 4 and 5, this numerical simulation displays that racism dissemination persists in the society, if \(\mathcal{R}_c > 1\). Consequently, this claims the status of racist groups that unfolded emphatic convection strength in at least one of the aspects of recruitment and retention, which tempt sickly cohesive community.

5.2. The Impact of Proposed Intervention Strategies. To assess the impact of intervention strategies on the racism dissemination process, we will conduct various numerical simulations to allow a deep insight into the dynamics and its impact. The assessment starts by evaluating the effect of
contact rate on susceptible and racist compartments by setting the values $\beta = 0.01, 0.05, 0.1,$ and 0.5.

Figure 8 shows that as the $\beta$ value increase, racism dissemination rises to a peak level, and the susceptible individual is accordingly contagious with racism. For example, at $\beta = 0.5$, the result indicates approximately 95% of the susceptible individuals switch to the next stage. Besides, the impact of $\beta$ on racist class $R(t)$ simulate in Figure 9, and the result implies that it has positively fostered the racist density. Mass dissemination of racism recorded losses of social cohesion, formulated racial segregation, and boosted structural racism.

Bailey, Zinzi et al. [9] "Structural racism refers to the totality of ways in which societies foster racial discrimination through mutually reinforcing systems of housing, education, employment, earnings, benefits, credit, media, health care, and criminal justice." Thus, to avoid the catastrophic consequences of racism, responsible organizations and officials must introduce bold antiracism education and racism sentiment source restriction. Furthermore, every day, each individual can stand up against racial bigotry and disrespectful attitudes using suitable intervention strategies is compulsory.

Social connections recreated a fundamental role in racism diffusion because the spread of racist ideologies and their eventual structuralism depended not only on personal "infection" via exposure to creation but also on active convection and publicity of that idea through personal
The fight against racism is everyone's struggle, and all are triumphs when we eliminate racial bigotry from our societies. Everyone has played a role in building a world beyond racism. Here, we assess the impact of awareness creation against racism and the likelihood of the intervention to improve the racism rejection mastery of susceptible individuals.

In Figure 10, we simulate the impact of antiracism education using the baseline parameter and varying $\xi = 0.0, 0.05, 0.1, 0.5$ and $\eta = 1$ (which means in the absence of awarded recovery). This numerical result shows the impact of awareness on radical susceptible individuals, and 50% awareness coverage obtains around a 4% reduction of racist density (those individuals upgrade from straightforward convection to hesitation status). As we noted in Figures 4 and 11 also endorsed antiracism education without rejection competency has not retained a remarkable impact on antiracism crusades. Thus, we assess the effects of hesitation or rejection mindset likelihood of awarded susceptible in Figure 11 with 20% antiracism education coverage. In-depth recognized antiracism education improves susceptible competency with high rejection probability. Our numerical result implies that effective antiracism campaigns build the attitude that moves us beyond the narratives of separatism, prejudice, division, anxiety, and hatred of others. Enrich racism rejection probability intensively and lecture to society via all feasible options (such as in school, at work, in public transportation, on social and broadcast media, at home, and on the sports field), eventually eliminating the racist threats. Hence, reasonable antiracism coverage with efficient rejection likelihood is an efficacious intervention measure.

Racist individuals are the root cause of racism propagation, and they disseminate their racial attitudes through various communication channels (such as social media, broadcasting media, and impersonal connections). An international Convention of the United Nations declared to criminalize hate speech and criminalize membership in racist organizations [47]. Hence, confining or restricting racist perspectives spreading within proper therapy is one of the intervention strategies.

For simulation purposes, we set $\eta = 0.0, 0.01, 0.05, 0.5$ and obtained the time series evaluations of susceptible and racist compartments of system 15. In the absence of racist restriction policy and action, racism disseminates in the
community and aggravates mistrust, founding skepticism on all aspects and pulling apart the social fabric. In addition, numerical simulation Figure 12 in Figure 13 depicts exceeding 95% of susceptible individuals who are contagious with racism. However, beyond 26%, racist restriction using racist confinement intervention virtually reduces the dissemination of racism. Besides, the upgrade of the racist confinement intervention strategy accordingly reduces the racist density (see Figure 13). The time series evaluations imply that being reluctant to fight racism harms everyone’s life directly and indirectly. Meanwhile, beyond marginal values fighting against racism through communication channels and law order has eradicated racism.

In the above-given various scenarios, we assess the sole impact of the control parameter of the model. Here, we will now be evaluating the integrated influence of intervention strategies. The simulations are carried out using baseline parameter values, with various values of the awareness rate $\xi$ and racist confinement rate $\eta$.

In Figure 14, the impacts of integrated interventions are illustrated with various combinations. The scenarios are expressed in the following cases:

(i) Case I: if both $\eta = \xi = 0.0$, the racism dissemination rises to a peak level, and eventually, the racism persists in the community. In this case, no intervention strategies were employed.

(ii) Case II: if $\xi = 0.05$ and $\eta = 0.0$, the racist density declines from 27.89% to 17.89% and reaches its equilibrium slowly. The intervention, moderates the racist density, but racist strength still performed well and spread racism in the community.

(iii) Case III: if $\xi = 0.0$ and $\eta = 0.05$, the racist density decrease from 27.89% to 6.59% and is proximate to the second case and uncertainly steady to its equilibrium. Nevertheless, when we compare antiracism education at the same rate of intervention, racist confinement is more influential in reducing the racist density and affiliated catastrophes.

(iv) Case IV: if both $\eta = \xi = 0.05$, the result shows that the racist density converges to zero and racism-based catastrophes are under control. This result implies that the integrated measures are more effective and reasonable to build the world beyond racism.

6. Conclusion

We established on noted key assumptions a new nonlinear system dynamics compartmental model was formulated. We proved the well-posedness of the developed model via system positivity and boundedness. Using the next-generation matrix approach, we computed racism strength threshold value $R_e$ (effective reproduction number). Throughout the rigorous theoretical analysis, we obtained unique racism-free and racism-addicted equilibrium points $E^0$ and $E^*$, respectively. The racism-free equilibrium point is locally and globally asymptotically stable if $R_e < 1$. In this situation, the racists have weak strength to attract susceptible or retain their members. Besides, the racism-addicted equilibrium $E^*$ exists and is asymptotically stable whenever one racist individual convinced more than one susceptible individual (where $R_e > 1$). Our sensitivity analysis suggested that the racist contract rate and hesitancy probability have a key role to advance the dissemination process at the devastation level. Conversely, antiracism education coverage with a high reasonable level of racism rejection and racist confinement rate reduces and eventually builds weak strength of racism.

Numerous research outcomes are such that in the absence of interventions, racism propagation revs racial segregation, creates a socioeconomic imbalance, and facilitates ethnic cleansing of minority groups. Cogent antiracism education with everyone’s participation reduces the racism dissemination. Confinement of racist activities via law order and individuals’ fight against racism has a mimic role in the reduction of racism burden and dissemination scope. Various numerical simulation studies were carried out to assess the impact of intervention strategies. In the absence of any intervention measures, approximately 95% of the total population will be affected by racists. Our simulation depicted antiracism education with effective rejection ability to avoid racism from the societies. In addition, racist confinement also conquered racist density and build a racism-free social environment. Further, racism dissemination is stemmed by a minimum of 50 50% antiracism education coverage and 95 95% rejection likelihood of awarded individuals. Also, approximately more than 28.5% of racist confinement is rational to mitigate the racism propagation. As compared to a sole intervention impacts racist confinement is more effective than antiracism education. Meanwhile, 5% of combined intervention strategies is adequate to reduce and eliminate the sovereignty of racism in the community. We conclude that every individual joining the antiracism campaign which avails rejection level awareness via a feasible approach has a significant role in the building process of a suitable social environment free from racism.

In general, founded on our rigorous theoretical and numerical, we recommended more intensity on the rejection ability of the awareness process rather than coverage. Furthermore, confined racists with treatment and integrated intervention are more relevant to build harmonized and resilient societies. In this research work, we can not be considered individual heterogeneity, the mean-field theory of social networks, algorithm-based racist message filtration, and the influence of the structured routine of racism. In future work, we will incorporate the list of limitations of our
References


