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Research Article

Manufacturer's Green Financing and CER Decisions under the Carbon Tax Policy and Capital Constraints

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This study constructs a two-echelon low-carbon supply chain consisting of a single manufacturer and a retailer. The manufacturer's initial capital is zero. The study analyzes three financing modes of the manufacturer by using the Stackelberg game model: external financing (the EF mode), internal financing (the IF mode), and trade credit financing (the TC mode). We found that the IF mode is superior to the EF mode while inferior to the TC mode regarding carbon emission reduction (CER), market demand, and the manufacturer's profit. Additionally, the IF mode is superior to the EF mode regarding the retailer and supply chain's profits and special to the TC mode only when the loan and deposit interest rate meet the specific conditions. The best financing mode for the manufacturer is the TC mode, but he needs to give some compensation to the retailer in order to make the retailer cooperate with him. The impact of carbon tax rate changes on the equilibrium solutions in the three financing modes is not only related to the change range of the carbon tax rate itself but also closely related to the initial carbon emissions. A high carbon tax rate can help stimulate the cleaner manufacturer to reduce carbon emissions, but a moderate carbon tax rate is more applicable to the polluting manufacturer. A numerical example is given to demonstrate some of the conclusions. Finally, the study provides managerial insights based on the analytical results.

1. Introduction

Dealing with climate change is one of the most urgent issues globally. With the massive emission of greenhouse gases represented by carbon dioxide, global warming, rising sea levels, soil erosion, and other environmental pollution, problems are becoming increasingly severe [1]. In response to the climate change disasters and the objective needs of national economic development, many countries actively explore effective strategies to balance environmental preservation and economic development [2, 3]. For example, in 2019, the European Union released the European Green Deal, proposing that Europe will achieve net-zero greenhouse gas emissions by 2050 and become the world's first carbon-neutral region. As a major stakeholder, China has always actively responded to and promoted CER. President Xi Jinping announced that China will strive to achieve peak carbon dioxide emissions by 2030 and carbon neutrality by 2060, demonstrating China's firm commitments and important contributions to the global fight against climate change. Simultaneously, governments have introduced carbon subsidies, carbon limits, carbon trading, carbon taxes, carbon labels, and other related institutional policies. The carbon tax policy is considered a typical global climate governance policy and an effective tool for reducing CO_2 emissions [4]. The carbon tax policy uses taxation to achieve emission reduction in a short period [5], which requires lower management costs and can supplement the control scope that carbon cap-and-trade regulations cannot cover.

Under the influence of the strong intervention of the carbon tax policy, enterprises need to improve their green production capacity to reduce carbon emissions. Producing low-carbon products can not only reduce the carbon tax cost for manufacturers but also increase market demand for the products and facilitate the development of the company's scale. However, the research and development of green technology and the purchase of green equipment incur additional costs. The double pressure of production and reducing carbon emissions makes enterprises face the problem of tight funds. To help manufacturers overcome the green production dilemma, supply chain company offers some solution. Li & Fung (https://www.lifung.com), for instance, offers credit and other lending services to help finance the capital needs of upstream enterprises [6]. Within the supply chain, in March 2022, Guangdong Midea landed the first green supply chain finance business in the home appliance industry, helping their suppliers achieve lower-cost financing. When manufacturers face financial constraints, they can secure loans from the bank [7], trade credit from the retailer [8, 9], or they can apply for advances from a well-funded retailer [10]. In the context of "carbon peaking" and "carbon neutrality," we can expect that low-carbon finance will be applied to more and broader areas for the benefit of society in the future.

Studying firms' green financing decision-making and supply chain operation strategies under the carbon tax policy and capital constraints is relevant. However, few studies have considered the impact of the retailer's surplus funds on supply chain financing and operation decisions after the capital-strapped manufacturer applies for internal financing from the capital-strong retailer. Additionally, the benefits brought by these excess funds to the supply chain have been neglected. Therefore, this study aims to address the green financing and CER decisions of the capital-constrained manufacturer under the carbon tax policy. We constructed three financing models for the manufacturer. When the manufacturer cannot afford the costs of production and investment in green technology, they can apply for bank loans (the EF mode). They can also apply for loans directly from a well-funded retailer, considering the retailer's remaining assets (the IF mode). The retailer can even prepay any shortfall in advance, provided the manufacturer promises a price discount (the TC mode). In our article, we first obtain the equilibrium solutions of the three financing modes and then analyze how the carbon tax rate changes affect these equilibrium solutions and compare several key parameters that the manufacturer focuses on when making decisions. We try to answer the following questions:

(1) From the manufacturer's standpoint, which financing mode can maximize the economic benefits to the manufacturer?
(2) When the manufacturer chooses the financing mode that brings the most economic benefits, can his environmental and social benefits be optimized simultaneously?
(3) Are the manufacturer's and supply chain's economic benefits consistent? If not, how do we reconcile the contradictions between them?

The remainder of this paper is organized as follows. Section 2 presents a related literature review. Section 3 presents the problem description and basic assumptions. Furthermore, we set up three different financing modes, obtain the equilibrium solutions, and analyze the impact of the carbon tax rate changes on the equilibrium solutions in Section 4. Section 5 compares CER, market demand, and profits of manufacturers, retailers, and supply chains under the three financing modes. A numerical simulation is presented in Section 6. Finally, we summarize the conclusions and propose management implications in Section 7.

2. Literature Review

Three streams of literature are related to this work: carbon tax policy, supply chain finance, and low-carbon supply chain with capital constraints. This section reviews related literature and highlights how our study differs from theirs.

2.1. Carbon Tax Policy. Scholars have a strong research interest in business operation management under the carbon tax policy. Krass et al. [11] combined the government's carbon tax regulation and the study of firms' technology choices and found that the effect of a carbon tax on firms' technology choices in the context of mono-oligopoly varied nonmonotonically. Giri et al. [12] considered the government's tax advocacy strategy in a two-echelon system consisting of two competing green manufacturers and one distributor and proposed a nonlinear two-level interaction model to evaluate the equilibrium strategy of the government and the supply chain. Ruidas et al. [13] analyzed the impact of different carbon tax policies on the carbon emissions of product inventory. In contrast to the above studies, some scholars have explored the effectiveness of carbon taxes for emission reduction strategies. Chen and Hao [14] compared the effects of a carbon tax on two competing firms with different operational efficiencies, showing that the carbon tax had a more pronounced impact on CER and the profits of inefficient firms. Wang et al. [15] compared the optimal strategies of supply chain members under different power structures in decentralized and centralized supply chains under carbon tax policies and gauged the impact of a carbon tax on supply chain decisions. Shu et al. [16] constructed a trade-old-for-remanufactured model in the policy context of a carbon tax and carbon subsidy and analyzed the manufacturer's optimal pricing and production strategies in the model, affirming the carbon tax policy's effectiveness in reducing emissions. With different emission tax policies, Yu et al. [17] compared the effects of varying government environmental policies on pricing and total emissions in the supply chain and proposed that the implementation of environmental taxes can both promote CER and incentivize firms to operate green.

In addition, several studies have analyzed the impact of CER on pollution abatement [18]. Cheng et al. [19] believed that when the carbon tax policy and consumer environmental awareness appear simultaneously, it can effectively stimulate manufacturers to increase the CER investment to improve supply chain operational efficiency. Zhang et al. [20] confirmed that when consumers have a level of environmental awareness, the government can achieve the goal of profit maximization in the whole supply chain by setting a reasonable carbon tax rate, which is consistent with the government's low-carbon emission goal. Luo et al. [21] discussed the impact of carbon tax policy formulation on CER decisions in the supply chain. It is believed that carbon tax policy should be tailored to different industries and promote supply chain members to reduce carbon emissions by cultivating consumers' environmental awareness. Zhu et al. [22] examined how the carbon tax policy and consumer environmental awareness affect production and CER decisions.

2.2. Supply Chain Finance. The aforementioned scholars have discussed the operational decisions of low-carbon supply chains, which are primarily based on the premise of sufficient production capital. Many enterprises are inherently prone to financial difficulties due to inventory redundancy problems. In recent years, supply chain financing has gradually solved the problem of financial difficulties for some enterprises, and supply chain financing has gradually differentiated into two main ways: internal funding and external financing in practical application. Xiao et al. [23] discussed the effectiveness of revenue-sharing, buyback contracts, and all-unit quantity discount contracts for coordinating the supply chain in the presence of default cost and proposed a generalized revenue-sharing agreement for coordinating the corresponding centralized financially constrained supply chain. Jing and Seidmann [24] suggested that bank loan financing can mitigate double marginalization of the supply chain when production cost is high. Compared with external funding relying on banks, the external financing of supply chain members does not require additional fixed assets as collateral, and supply chain members are more familiar with each other's business situation, so financing risks are better controlled. For instance, Bellantuono et al. [25] proposed an early payment discount plan and showed that joint adoption of an early payment discount plan with a revenue-sharing plan has better returns. Chen et al. [26] identified that trade credit from suppliers can increase the profits of supply chain members and reduce the risk of default. Lee and Rhee [27] demonstrated trade credit as a tool for achieving supply chain coordination from the supplier's perspective and derived optimal price reduction subsidies and risk premiums in trade credit using buyback/markdown allowance contracts. Li et al. [28] derived the optimal selling price and replenishment cycle for retailers under an advance-cash-credit payment scheme and demonstrated a positive correlation between the percentage of payments made in advance and the selling price. Shen et al. [29] compared three financing models in the supply chain of capital-constrained manufacturers: bank credit financing, dual trade financing from competing retailers, and mixed bank financing and trade credit financing. The effects of retailer competition and interest rates on the choice of financing options are analyzed. Furthermore, some scholars have compared bank/trade finance to advanced payment finance strategies [30]. Zhou et al. [30] discussed supply chain members and overall supply chain benefits in two financing channels: trade credit from the manufacturer to the distributor or advance payment from the platform to the distributor, from the perspective of risk allocation in a three-party supply chain. Different from the above studies, Jiang et al. [31] studied the impact of retailers' credit ratings on supply chain operation management decisions.

2.3. Low-Carbon Supply Chain with Capital Constraints. The carbon tax policy has increased the incentive of enterprises to reduce carbon emissions, but it has also led manufacturers to bear more carbon reduction costs. Therefore, scholars have conducted in-depth studies on financing a low-carbon supply chain under capital constraints. For example, Wua et al. [32], Cao et al. [33], Zhang et al. [34], and Mahato et al. [35] explored the influence of bank financing and trade credit financing on the optimal decision-making of low-carbon supply chain members with capital constraints. Qin et al. [36] analyzed both green financing and cost-sharing contracts using the Stackelberg game model, considering manufacturers' capital constraints and the carbon cap-and-trade mechanism, and found that green finance rates do not always adversely affect CER, and retailers' cost-sharing does not always have a positive impact on CER. Jin et al. [37] discussed the influence of different government intervention policies on manufacturing enterprises' green financing decisions. Lu et al. [38] analyzed the optimal strategy choice between autonomous financing and supply chain cooperative financing when the manufacturer cannot afford the cost of CER under carbon quota control. An et al. [39] constructed a two-echelon supply chain consisting of a capital-constrained manufacturer and a supplier, analyzed the supply chain operational decisions under strict carbon emission regulation when the manufacturer adopted green credit financing, and compared it with social welfare under the traditional trade finance mode. Wang et al. [40] combined the order financing model in green finance and found that a carbon tax and green finance can promote the green transformation of corporate clean technologies. Li et al. [41] constructed a two-echelon low-carbon supply chain model wherein the manufacturer is capital-constrained to use carbon pledge financing and analyze the manufacturer's CER decisions and supply chain members' profits under different power structures. Furthermore, some scholars have designed contracts to coordinate cash-constrained supply chain members. Depending on the presence or absence of prepayment financing, Qin et al. [42] discussed supply chain models when capital-constrained manufacturers use bank and hybrid financing models and extend the models with cap-and-trade regulation. It is also demonstrated that prepayment financing with price discounts can improve supply chain members' profits when the manufacturer is capital-constrained. Wu et al. [43] discussed no-financing, trade credit, and bank credit financing modes in a manufacturer's capital-constrained low-carbon supply chain and designed a cost-sharing contract, which was later confirmed to improve the profitability of supply chain members.

These studies demonstrate the numerous works made by scholars to study carbon tax policy, supply chain finance, and low-carbon supply chains with capital constraints. However, most of the existing studies on carbon tax policy are based on the case of a well-funded supply chain, and the pricing and CER decisions in a low-carbon supply chain under financial constraint deserve equal attention. Although the results of a low-carbon supply chain have been fruitful, some problems still need to be explored, for example, developing and comparing financing modes within a supply chain. Additionally, many studies only analyzed the impact of carbon abatement financing on production decision-making but did not consider the role of the manufacturer's initial capital. In different financing modes, the equilibrium outputs of the manufacturers are not the same, which means that they incur extra production costs under each financing mode. It is inappropriate to consider only the CER cost of the manufacturer without considering the production cost. Therefore, both production and CER costs should be considered when financing. In our study, we assume that the initial capital of the manufacturer is zero and that the costs of production and investing in green technology need to be financed. Based on this notion, we investigate the EF, IF, and TC modes, compare the equilibrium solutions of the three financing modes, and analyze the impact of the carbon tax rate changes on the equilibrium solutions. Table 1 shows how our current model differs from the existing model.

3. Problem Description and Assumption

3.1. Problem Description. This study constructs a two-echelon low-carbon supply chain with the manufacturer as the leader of the Stackelberg game and the retailer as the follower. The manufacturer uses raw materials to produce low-carbon products and sell them to the retailer at wholesale price. Meanwhile, the retailer faces a single-product monopolistic market to sell low-carbon products directly to consumers at the selling price. The government imposes a carbon tax on the carbon dioxide emitted by the manufacturer during production. We only consider that the manufacturer needs to pay a carbon tax for his carbon emission behavior and ignores the carbon emitted by the retailer in the sales process. To reduce the paid carbon tax while satisfying consumers' preferences for low-carbon products, the manufacturer decides to invest in green technology to reduce emissions, which means that the manufacturer needs to spend a considerable amount of funds.

When the initial capital possessed by the manufacturer is not enough to disburse the costs of production and invest in green technology, the manufacturer can apply for loans from financial institutions and pay back the principal and interest of the loan at the end of the production-sale cycle. The manufacturer can borrow from the retailer when the retailer has sufficient funds. Notably, the manufacturer also has a third option, which is to ask the retailer to pay all the money upfront in exchange for promising the retailer a one-time price discount on all items ordered.

According to Zhao and Ji [44] and Long and Wang [45], a CER decision is usually a long-term decision that needs to be made early and prioritized over the pricing decision. Therefore, the game process was divided into three stages. The first stage is the CER decision, wherein the manufacturer decides the level of carbon emissions. The second stage is the wholesale price decision, wherein the manufacturer determines the price charged when the product is delivered to the retailer. The third stage is the selling price decision, wherein the retailer sets a selling price for a low-carbon product per unit. The flowchart of the green financing and CER decision-making is shown in Figure 1.

3.2. Model Assumptions. To facilitate modeling and analysis, some assumptions are proposed as follows.

Assumption 1. Market demand faced by the retailer is expressed as a function of selling price and the final carbon emissions. Following Lu et al. [38], we have

$$D_i = a - bp_i - \theta e_i. \quad (1)$$

The market demand function implies that market demand is negatively correlated with the selling price and final carbon emissions. In the market demand function, a is the potential market scale, b is the sensitivity coefficient of market demand to the selling price and, θ is the level of consumers' environmental awareness. Moreover, e_i is the final carbon emissions and $i = 1, 2, 3$ represents the EF, IF, and TC modes, respectively.

Assumption 2. To control the amount of carbon dioxide discharged into the atmosphere, the government imposes a carbon tax on the manufacturer. The carbon tax rate is t, which means that for each unit of carbon dioxide emitted by a manufacturer, t units of carbon tax must be paid to the government at the end of the production-sales cycle. If the final carbon emissions per unit product after investing in green technology are e_i, then the total carbon tax paid by the manufacturer to the government is te_iD_i.

Assumption 3. The cost of investing in green technology for the manufacturer is $(1/2)k(e_0 - e_i)^2$, where k is the cost coefficient of CER and e_0 is the initial carbon emissions per unit product. Sun and Yang [46] and Song et al. [47] found that when the investment in CER is increased, the cost of investment in green technology and the amount of CER have a quadratic relationship. This relationship denotes that as the amount of CER increases, the cost of CER increases, and the marginal cost of CER increases.

Assumption 4. To simplify the analysis process, we set the initial capital of the manufacturer to zero without affecting the conclusion. Simultaneously, we assume that the retailer's capital is sufficient and can meet the manufacturer's capital demand for production and investment in green technology.

Table 1: A comparison between this paper and other studies.

References	Carbon tax	Production cost financing	Green technology financing	Retailer surplus capital
Jin et al. [37]		√	√	
Giri et al. [12]	√			
Ruidas et al. [13]	√			
Wang et al. [15]	√			
Yu et al. [17]	√			
Xiao et al. [23]		√	√	
Jing and Seidmann [24]		√		
Lee and Rhee [27]		√		
Chen et al. [26]		√		
Jiang et al. [31]		√		
Qin et al. [36]			√	
Lu et al. [38]			√	
An et al. [39]			√	
Zhang et al. [20]			√	
Li et al. [41]			√	
This paper	√	√	√	√

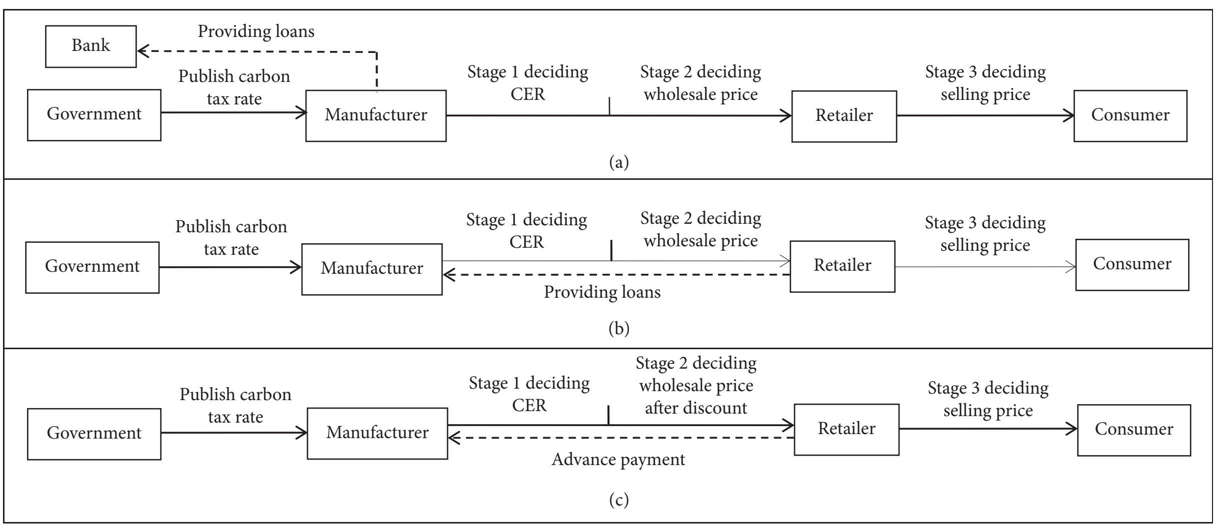


Figure 1: Flowchart of the green financing and CER decision-making.

Table 2 summarizes the main notations and definitions used in this study.

4. Model Establishment and Solution

4.1. External Financing (The EF Mode). In the EF mode, the amount of capital required by the manufacturer is $cD_1 + (1/2)k(e_0 - e_1)^2$, which includes the cost of producing low-carbon products and investing in green technology. The manufacturer obtains all funds by applying for bank financing. The action order of supply chain members is as follows: in the CER decision-making phase, the manufacturer determines how much carbon emissions to reduce per unit of low-carbon product based on the carbon tax rate announced by the government. In the pricing decision-making phase, the manufacturer and retailer play the Stackelberg game. The manufacturer is the leader of the game and, therefore, has the priority to make decisions. After the manufacturer determines the wholesale price, the retailer determines the selling price for the low-carbon product.

The manufacturer's profit function is expressed as follows:

$$\pi_1^M(w_1, e_1) = (w_1 - c - te_1)D_1 - \frac{1}{2}k(e_0 - e_1)^2 - \left(cD_1 + \frac{1}{2}k(e_0 - e_1)^2\right)r_1. \tag{2}$$

The retailer's profit function is expressed as follows:

$$\pi_1^R(p_1) = (p_1 - w_1)D_1 + Br_2, \tag{3}$$

where $D_1 = a - bp_1 - \theta e_1$.

In formula (2), the first item is the gross profit from selling low-carbon products to the retailer minus the carbon tax paid to the government, the second item is the cost of investing in green technology, and the third item is the interest paid to the bank on loans. In formula (3), the first

Table 2: Notations and their definitions.

	Implications
Parameters	
a	Potential market size
b	Sensitivity coefficient of market demand to the selling price
θ	Level of consumers' environmental awareness
e_0	Initial carbon emission per unit of low-carbon product
c	Production cost per unit of low-carbon product
t	Carbon tax rate
k	The cost coefficient of CER
r_1	Loan annual interest rate
r_2	Deposit annual interest rate, $r_1 > r_2$
B	The retailer's initial capital
Decision variables	
e_i	Final carbon emission per unit of low-carbon product
Δe_i	CER per unit of low-carbon product
w_i	Wholesale price per unit of low-carbon product
p_i	Selling price per unit of low-carbon product
Functions	
D_i	Market demand for low-carbon products
π_i^M	The manufacturer's profit
π_i^R	The retailer's profit
π_i^{SC}	The supply chain system's profit
$*$	The superscript "$*$" indicates the optimal solution

item is the profit from selling low-carbon products to consumers, and the second item is the interest earned on the funds held in the bank.

Theorem 5. *In the EF mode, the optimal CER, wholesale price, selling price, and market demand are given, respectively, by*

$$\begin{aligned}\Delta e_1^* &= \frac{(\theta + bt)\xi_1}{\chi_1},\\ w_1^* &= c + \frac{2k(1+r_1)(\xi_1 + 2\mathrm{bte}_0) - \gamma_1}{\chi_1},\\ p_1^* &= w_1^* + \frac{k(1+r_1)\xi_1}{\chi_2},\\ D_1^* &= \frac{kb(1+r_1)\xi_1}{\chi_1},\end{aligned} \tag{4}$$

where $\xi_1 = a - bc(1+r_1) - (\theta + bt)e_0 > 0$, $\chi_1 = 4kb(1+r_1) - (\theta + bt)^2 > 0$, *and* $\gamma_1 = (\theta + bt)((a - bc)t + c\theta r_1)$.

Proof of Theorem 5. Please refer to Appendix A.

Substituting e_1^*, w_1^*, and p_1^* into $\pi_1^M(w_1, e_1)$ and $\pi_1^R(p_1)$, we obtain the manufacturer, retailer, and supply chain system's profits that are expressed as

$$\begin{aligned}\pi_1^{M*} &= \frac{k(1+r_1)\xi_1^2}{2\chi_1},\\ \pi_1^{R*} &= \frac{k^2 b(1+r_1)^2\xi_1^2}{\chi_1^2} + Br_2,\\ \pi_1^{SC*} &= \frac{k\left(6kb(1+r_1)^2 - (\theta + \mathrm{bt})^2(1+r_1)\right)\xi_1^2}{2\chi_1^2} + Br_2.\end{aligned} \tag{5}$$

Social responsibility and sustainable development are important issues in the development of modern enterprises. As a member of society, an enterprise needs to undertake social responsibility and promote sustainable development while pursuing economic benefits. John Elkington first proposed the concept of the triple bottom line in 1997. He believes that no company can only pursue profit; economic, environmental, and social benefits are the triple bottom line that enterprises must adhere to. These three indicators are also regarded as the "troika" to evaluate the value of enterprises. In our paper, we mainly discuss the financing mode selection from the perspective of the manufacturer. The manufacturers are first concerned about the economic benefits of each financing mode, followed by environmental and social benefits. In this article, we represent these three indicators in terms of profit, CER, and market demand, respectively. Therefore, analyzing the impact of the carbon tax rate changes on these three indicators is necessary, as they will affect the manufacturer's financing decision-making. □

Corollary 6. *Regarding CER and the carbon tax rate in the EF mode, we have the following:*

(i) *When* $e_0 > e_0^E$ *and* $0 < t < t_1^E$, *there is* $\partial \Delta e_1^*/\partial t > 0$

(ii) *When* $e_0 > e_0^E$ *and* $t_1^E < t < t_0^E$, *there is* $\partial \Delta e_1^*/\partial t < 0$

(iii) *When* $0 < e_0 < e_0^E$ *and* $0 < t < t_0^E$, *there is* $\partial \Delta e_1^*/\partial t > 0$

where $e_0^E = a - bc(1+r_1)/\sqrt{4kb(1+r_1)}$, $t_1^E = 4kb(1+r_1)e_0 - \sqrt{\Delta_1^E}/2/b(a - bc(1+r_1)) - \theta/b$, $t_0^E = \sqrt{4kb(1+r_1)} - \theta/b$, *and* $\Delta_1^E = 16kb(1+r_1)(4kb(1+r_1)e_0^2 - (a - bc(1+r_1))^2)$.

Proof of Corollary 6. Please refer to Appendix B. □

Corollary 7. *Regarding market demand and the carbon tax rate in the EF mode, we have the following:*

(i) When $0 < e_0 < e_0^E$ and $0 < t < t_2^E$, there is $\partial D_1^*/\partial t < 0$

(ii) When $0 < e_0 < e_0^E$ and $t_2^E < t < t_0^E$, there is $\partial D_1^*/\partial t > 0$

(iii) When $e_0 > e_0^E$ and $0 < t < t_0^E$, there is $\partial D_1^*/\partial t < 0$

where $t_2^E = (a - bc(1 + r_1)) - \sqrt{\Delta_2^E}/2/\mathrm{be}_0 - \theta/b$ and $\Delta_2^E = 4((a - bc(1 + r_1))^2 - 4kb(1 + r_1)e_0^2)$.

Proof of Corollary 7. Please refer to Appendix C. □

Corollary 8. *Regarding the manufacturer's profit and the carbon tax rate in the EF mode, we have the following:*

(i) When $0 < e_0 \le e_0^E$ and $0 < t \le t_3^E$, there is $\partial \pi_1^{M*}/\partial t \le 0$

(ii) When $e_0 > e_0^E$ and $0 < t \le \widehat{t}_3^E$, there is $\partial \pi_1^{M*}/\partial t \le 0$

where $t_3^E = 4kb(1 + r_1)e_0/b(a - bc(1 + r_1)) - \theta/b$ and $\widehat{t}_3^E = a - bc(1 + r_1)/be_0 - \theta/b$.

Proof of Corollary 8. Please refer to Appendix D. □

4.2. Internal Financing (the IF Mode). In the IF mode, the costs of producing low-carbon products and investing in green technology, which should be paid for by the manufacturer, add up $cD_2 + (1/2)k(e_0 - e_2)^2$. As the initial capital of the manufacturer is zero, all funds needed by the manufacturer are borrowed from the retailer. At the end of the production-sale cycle, the manufacturer pays a certain amount of interest in addition to repaying the principal to the retailer. The interest rate acceptable to the manufacturer is not higher than that provided by the bank. Otherwise, the manufacturer chooses a loan from the bank. Notably, the interest rate acceptable to the retailer must not be lower than that obtained by depositing funds in the bank. Otherwise, the retailer will choose to deposit funds in the bank. To facilitate the comparison between the EF and IF modes, we assume that the manufacturer borrows from the retailer at the same interest rate as the bank. The decision-making sequence of the IF mode was consistent with that of the EF mode. First, the manufacturer determines how much carbon is emitted per unit of low-carbon product. Then, the manufacturer and retailer play a Stackelberg game; that is, the manufacturer first decides the wholesale price, and the retailer determines the selling price after observing the manufacturer's decision-making.

The manufacturer's profit function is expressed as follows:

$$\begin{aligned}\pi_2^M &= (w_2 - c - te_2)D_2 - \frac{1}{2}k(e_0 - e_2)^2 \\ &\quad - \left(cD_2 + \frac{1}{2}k(e_0 - e_2)^2\right)r_1.\end{aligned} \tag{6}$$

The retailer's profit function is expressed as follows:

$$\begin{aligned}\pi_2^R &= (p_2 - w_2)D_2 + \left(cD_2 + \frac{1}{2}k(e_0 - e_2)^2\right)r_1 \\ &\quad + \left(B - cD_2 - \frac{1}{2}k(e_0 - e_2)^2\right)r_2,\end{aligned} \tag{7}$$

where $D_2 = a - \mathrm{bp}_2 - \theta e_2$.

In formula (6), the first item is the gross profit from selling low-carbon products to the retailer minus the carbon tax paid to the government, the second item is the cost of investing in green technology, and the third item is the interest paid to the retailer on loans. In formula (7), the first item is the profit from selling low-carbon products to consumers, the second item is the interest on the loan from the manufacturer, and the third item is the interest earned on the remaining funds deposited in the bank after deducting the loan given to the manufacturer.

Theorem 9. *In the IF mode, the optimal CER, wholesale price, selling price, and market demand are presented as follows:*

$$\begin{aligned}\Delta e_2^* &= \frac{(\theta + \mathrm{bt})\xi_2}{\chi_1}, \\ w_2^* &= c(1 + r_1) + \frac{2k(1 + r_1)(\xi_2 + 2\mathrm{bte}_0) - \gamma_2}{\chi_1}, \\ p_2^* &= w_2^* + c(r_2 - r_1) + \frac{k(1 + r_1)\xi_2}{\chi_1}, \\ D_2^* &= \frac{kb(1 + r_1)\xi_2}{\chi_1},\end{aligned} \tag{8}$$

where $\xi_2 = a - bc(1 + r_2) - (\theta + bt)e_0 > 0$ *and* $\gamma_2 = (\theta + bt)t(a - bc(1 + r_2))$.

Proof of Theorem 9. Please refer to Appendix E.

Substituting e_2^*, w_2^*, and p_2^* into $\pi_2^M(w_2, e_2)$ and $\pi_2^R(p_2)$, we obtain the manufacturer, retailer, and supply chain system's profits and are expressed as

$$\begin{aligned}\pi_2^{M*} &= \frac{k(1 + r_1)\xi_2^2}{2\chi_1}, \\ \pi_2^{R*} &= \frac{k\left(2\mathrm{kb}(1 + r_1)^2 + (\theta + \mathrm{bt})^2(r_1 - r_2)\right)\xi_2^2}{2\chi_1^2} + Br_2, \\ \pi_2^{SC*} &= \frac{k\left(6\mathrm{kb}(1 + r_1)^2 - (\theta + \mathrm{bt})^2(1 + r_2)\right)\xi_2^2}{2\chi_1^2} + Br_2.\end{aligned} \tag{9}$$

□

Corollary 10. *Regarding CER and the carbon tax rate in the IF mode, we have the following:*

(i) When $e_0 > e_0^I$ and $0 < t < t_1^I$, there is $\partial \Delta e_2^*/\partial t > 0$

(ii) When $e_0 > e_0^I$ and $t_1^I < t < t_0^I$, there is $\partial \Delta e_2^*/\partial t < 0$

(iii) When $0 < e_0 < e_0^I$ and $0 < t < t_0^I$, there is $\partial \Delta e_2^*/\partial t > 0$

where $e_0^I = a - bc(1 + r_2)/\sqrt{4kb(1 + r_1)}$, $t_1^I = 4kb(1 + r_1) e_0 - \sqrt{\Delta_1^I}/2/b(a - bc(1 + r_2)) - \theta/b$, $t_0^I = t_0^E = \sqrt{4kb(1 + r_1)} - \theta/b$, and $\Delta_1^E = 16kb(1 + r_1)(4kb(1 + r_1)e_0^2 - (a - bc(1 + r_2))^2)$.

Proof of Corollary 10. Please refer to Appendix F. □

Corollary 11. *Regarding market demand and the carbon tax rate in the IF mode, we have the following:*

(i) When $0 < e_0 < e_0^I$ and $0 < t < t_2^I$, there is $\partial D_2^*/\partial t < 0$

(ii) When $0 < e_0 < e_0^I$ and $t_2^I < t < t_0^I$, there is $\partial D_2^*/\partial t > 0$

(iii) When $e_0 > e_0^I$ and $0 < t < t_0^I$, there is $\partial D_2^*/\partial t < 0$

where $t_2^I = (a - bc(1 + r_1)) - \sqrt{\Delta_2^I}/2/be_0 - \theta/b$ and $\Delta_2^I = 4((a - bc(1 + r_2))^2 - 4kb(1 + r_1)e_0^2)$.

Proof of Corollary 11. Please refer to Appendix G. □

Corollary 12. *Regarding the manufacturer's profit and the carbon tax rate in the IF mode, we have the following:*

(i) When $0 < e_0 \le e_0^I$ and $0 < t \le t_3^I$, there is $\partial \pi_2^{M*}/\partial t \le 0$

(ii) When $e_0 > e_0^I$ and $0 < t \le \hat{t}_3^I$, there is $\partial \pi_2^{M*}/\partial t \le 0$

where $t_3^I = 4kb(1 + r_1)e_0/b(a - bc(1 + r_2)) - \theta/b$ and $\hat{t}_3^I = a - bc(1 + r_2)/be_0 - \theta/b$.

Proof of Corollary 12. Please refer to Appendix H. □

4.3. Trade Credit Financing (the TC Mode). In the TC mode, the amount of capital the manufacturer needs to set aside for production and investing in green technology is $cD_3 + (1/2)(e_0 - e_3)^2$. The manufacturer has no initial capital but is financed neither by loans from the bank nor by loans from the retailer. The manufacturer negotiates with the retailer to have the latter prepay the costs of producing low-carbon products and investing in green technology and gives the retailer a wholesale price discount on all products ordered. In this way, although the retailer loses a small amount of interest income, it also saves part of the procurement cost. Therefore, the retailer can agree to the contract proposed by the manufacturer. The decision-making sequence of the TC mode is that the manufacturer decides on CER first and then determines the wholesale price after giving the optimal price discount. Finally, the retailer decides the selling price according to the wholesale price provided by the manufacturer.

The manufacturer's profit function is expressed as follows:

$$\pi_3^M = (w_3 - c - te_3)D_3 - \frac{1}{2}k(e_0 - e_3)^2. \quad (10)$$

The retailer's profit function is expressed as follows:

$$\pi_3^R = (p_3 - w_3)D_3 + \left(B - cD_3 - \frac{1}{2}k(e_0 - e_3)^2\right)r_2, \quad (11)$$

where $D_3 = a - bp_3 - \theta e_3$.

In formula (10), the first item is the gross profit from selling low-carbon products to the retailer minus the carbon tax paid to the government, and the second item is the cost of investing in green technology. In formula (11), the first item is the profit from selling low-carbon products to consumers, and the second item is the interest earned by depositing the remaining funds in the bank after paying the manufacturers' production cost and CER investment. In particular, w_3 is the wholesale price after the discount provided by the manufacturer.

Theorem 13. *In the TC mode, the optimal CER, wholesale price, selling price, and market demand are presented as follows:*

$$\begin{aligned}
\Delta e_3^* &= \frac{(\theta + \mathrm{bt})\xi_2}{\chi_3},\\
w_3^* &= c + \frac{2k(\xi_2 + 2\mathrm{bte}_0) - \gamma_2}{\chi_3},\\
p_3^* &= w_3^* + \mathrm{cr}_2 + \frac{k\xi_2}{\chi_3},\\
D_3^* &= \frac{\mathrm{kb}\xi_2}{\chi_3},
\end{aligned} \quad (12)$$

where $\chi_3 = 4kb - (\theta + bt)^2 > 0$.

Proof of Theorem 13. Please refer to Appendix I.

Substituting e_3^*, w_3^*, and p_3^* into $\pi_3^M(w_3, e_3)$ and $\pi_3^R(p_3)$, we obtain the manufacturer, retailer, and supply chain system's profits and are expressed as

$$\begin{aligned}
\pi_3^{M*} &= \frac{k\xi_2^2}{2\chi_3},\\
\pi_3^{R*} &= \frac{k\left(2\mathrm{kb} - (\theta + \mathrm{bt})^2 r_2\right)\xi_2^2}{2\chi_3^2} + Br_2,\\
\pi_3^{SC*} &= \frac{k\left(6\mathrm{kb} - (\theta + \mathrm{bt})^2(1 + r_2)\right)\xi_2^2}{2\chi_3^2} + Br_2.
\end{aligned} \quad (13)$$

□

Corollary 14. *Regarding CER and the carbon tax rate in the TC mode, we have the following:*

(i) When $e_0 > e_0^T$ and $0 < t < t_1^T$, there is $\partial \Delta e_3^*/\partial t > 0$

(ii) When $e_0 > e_0^T$ and $t_1^T < t < t_0^T$, there is $\partial \Delta e_3^*/\partial t < 0$

(iii) When $0 < e_0 < e_0^T$ and $0 < t < t_0^T$, there is $\partial \Delta e_3^*/\partial t > 0$

where $e_0^T = a - bc(1 + r_2)/\sqrt{4kb}$, $t_1^T = 4kbe_0 - \sqrt{\Delta_1^T}/2/b(a - bc(1 + r_2)) - \theta/b$, $t_0^T = \sqrt{4kb} - \theta/b$, and $\Delta_1^T = 16kb(4kbe_0^2 - (a - bc(1 + r_2))^2)$.

Proof of Corollary 14. Please refer to Appendix J. □

Corollary 15. *Regarding market demand and the carbon tax rate in the TC mode, we have the following:*

(i) When $0 < e_0 < e_0^T$ and $0 < t < t_2^T$, there is $\partial D_3^*/\partial t < 0$

(ii) When $0 < e_0 < e_0^T$ and $t_2^T < t < t_0^T$, there is $\partial D_3^*/\partial t > 0$

(iii) When $e_0 > e_0^T$ and $0 < t < t_0^T$, there is $\partial D_3^*/\partial t < 0$

where $t_2^T = (a - bc(1 + r_2)) - \sqrt{\Delta_2^T}/2/\mathrm{be}_0 - \theta/b$ and $\Delta_2^T = 4((a - bc(1 + r_2))^2 - 4kbe_0^2)$.

Proof of Corollary 15. Please refer to Appendix K. □

Corollary 16. *Regarding the manufacturer's profit and the carbon tax rate in the TC mode, we have the following:*

(i) When $0 < e_0 \leq e_0^T$ and $0 < t \leq t_3^T$, there is $\partial \pi_3^{M*}/\partial t \leq 0$

(ii) When $e_0 > e_0^T$ and $0 < t \leq \hat{t}_3^T$, there is $\partial \pi_3^{M*}/\partial t \leq 0$

where $t_3^T = 4kbe_0/b(a - bc(1 + r_2)) - \theta/b$ and $\hat{t}_3^T = \hat{t}_3^I = a - bc(1 + r_2)/\mathrm{be}_0 - \theta/b$.

Proof of Corollary 16. Please refer to Appendix L.

From Corollaries 6, 10, and 14, we know that the impact of the carbon tax rate changes on CER is not only related to the initial carbon emissions but also to the value of the carbon tax rate itself in the three modes. There is a threshold value for the initial carbon emissions. When the initial carbon emissions are below the threshold, raising the carbon tax rate will incentivize the manufacturer to increase CER. When the initial carbon emissions are greater than the threshold, in the process of the continuous increase of the carbon tax rate, CER shows a trend of increasing first and then decreasing.

From Corollaries 7, 11, and 15, we know that the impact of the carbon tax rate changes on the market demand for low-carbon products is related to the initial carbon emissions and the range of the carbon tax rate. For different financing modes, there is a threshold for the initial carbon emissions. When the initial carbon emissions are greater than the threshold, these equilibrium solutions decrease with an increase in the carbon tax rate. When the initial carbon emissions are less than this threshold, an increase in the carbon tax rate in a lower range will lead to a reduction in these equilibrium solutions, but an increase in the carbon tax rate in a higher degree will promote growth in these equilibrium solutions.

From Corollaries 8, 12, and 16, we know that, regardless of the manufacturer's initial carbon emission level, the manufacturer's profit decreases with an increase in the carbon tax rate. In other words, although increasing the carbon tax rate will stimulate the manufacturer to reduce carbon emissions and thus promote the growth of market demand, the contribution of the evolution of market demand to profit is not sufficient to offset the side effect of the increase in the carbon tax rate on profit. Consequently, the manufacturer's profit tends to decline with an increase in the carbon tax rate. □

5. Models Comparison

Based on Theorems 5, 9, and 13, we can further compare the EF, IF, and TC modes on CER, market demand, the manufacturer's profit, the retailer's profit, and the supply chain's profit.

Corollary 17. *Comparing the EF and IF modes, we obtain the following:*

(i) $\Delta e_1^* < \Delta e_2^*$

(ii) $D_1^* < D_2^*$

(iii) $\pi_1^{M*} < \pi_2^{M*}$

(iv) $\pi_1^{R*} < \pi_2^{R*}$

(v) $\pi_1^{SC*} < \pi_2^{SC*}$

Proof of Corollary 17. Please refer to Appendix M.

As can be seen from Corollary 17, the IF mode is better than the EF mode for the main supply chain equilibrium solutions. From the perspective of the manufacturer, profit is undoubtedly the primary goal for the manufacturer to choose finance modes. The most favored financing mode will be the one which can bring the greatest economic benefits to the manufacturer. Under the corporate social responsibility requirement, the manufacturer must consider not only economic benefits but also environmental benefits (such as CER) and social benefits (such as the quantity of product supplied, i.e., market demand). If the manufacturer gets the most economic benefits, environmental and social benefits can also be the best; it will be a "multi-win" situation. In terms of the EF and IF modes, the former is inferior to the latter in the aspect of CER, market demand, the manufacturer's profit, the retailer's profit, and the supply chain system's profit. Compared to the IF mode, the EF mode is a strictly inferior strategy for the manufacturer and is excluded in the first place. □

Corollary 18. *Comparing the IF and TC modes, we obtain the following:*

(i) $\Delta e_2^* < \Delta e_3^*$

(ii) $D_2^* < D_3^*$

(iii) $\pi_2^{M*} < \pi_3^{M*}$

(iv) For π_2^{R*} and π_3^{R*}, combined with the initial carbon emissions, the discussion is as follows.

Case 1: When $0 < e_0 < e_0^T = a - bc(1 + r_2)/\sqrt{4kb}$,

(a) If $0 < \sqrt{r_1^2 + 12r_1(1 - 2r_2) - 16r_2(1 - r_2) + 4} < 1$, it meets $0 < t < t_1^R$, and there is $\pi_3^{R*} < \pi_2^{R*}$; it meets $t_1^R < t < t_2^R$, and there is $\pi_3^{R*} > \pi_2^{R*}$; it meets $t_2^R < t < t_0^T$, and there is $\pi_3^{R*} < \pi_2^{R*}$

(b) If $1 < \sqrt{r_1^2 + 12r_1(1 - 2r_2) - 16r_2(1 - r_2) + 4} < 3$, it meets $0 < t < t_2^R$, and there is $\pi_3^{R*} > \pi_2^{R*}$; it meets $t_2^R < t < t_0^T$, and there is $\pi_3^{R*} < \pi_2^{R*}$

(c) If $\sqrt{r_1^2 + 12r_1(1 - 2r_2) - 16r_2(1 - r_2) + 4} > 3$, it meets $0 < t < t_0^T$, and there is $\pi_3^{R*} > \pi_2^{R*}$

Case 2: When $e_0 > e_0^T = a - bc(1 + r_2)/\sqrt{4kb}$,

(a) If $0 < \sqrt{r_1^2 + 12r_1(1 - 2r_2) - 16r_2(1 - r_2) + 4} < 1$, it meets $0 < t < t_1^R$, and there is $\pi_3^{R*} < \pi_2^{R*}$; it *meets* $t_1^R < t < t_2^R$*, and there is* $\pi_3^{R*} > \pi_2^{R*}$*; and it meets* $t_2^R < t < \hat{t}_3^T$*, and there is* $\pi_3^{R*} < \pi_2^{R*}$

(b) *If* $1 < \sqrt{r_1^2 + 12r_1(1 - 2r_2) - 16r_2(1 - r_2) + 4} < 3$*, it meets* $0 < t < t_2^R$*, and there is* $\pi_3^{R*} > \pi_2^{R*}$*; it meets* $t_2^R < t < \hat{t}_3^T$*, and there is* $\pi_3^{R*} < \pi_2^{R*}$

(c) *If* $\sqrt{r_1^2 + 12r_1(1 - 2r_2) - 16r_2(1 - r_2) + 4} > 3$*, it meets* $0 < t < \hat{t}_3^T$*, and there is* $\pi_3^{R*} > \pi_2^{R*}$

(v) *For* π_2^{SC*} *and* π_3^{SC*}*, combined with the initial carbon emissions, the discussion is as follows.*

Case 1: When $0 < e_0 < e_0^T = a - bc(1 + r_2)/\sqrt{4kb}$*,*

(a) *If* $\sqrt{8kr_1(1 + 2(r_1 - r_2) - r_1r_2)/b(3r_1^2 + 2r_1(1 - 2r_2))} - \theta/b < t_0^T$*, it meets* $0 < t < \sqrt{8kr_1(1 + 2(r_1 - r_2) - r_1r_2)/b(3r_1^2 + 2r_1(1 - 2r_2))} - \theta/b$*, and there is* $\pi_3^{SC*} > \pi_2^{SC*}$*; it meets* $\sqrt{8kr_1(1 + 2(r_1 - r_2) - r_1r_2)/b(3r_1^2 + 2r_1(1 - 2r_2))} - \theta/b < t < t_0^T$*, and there is* $\pi_3^{SC*} < \pi_2^{SC*}$

(b) *If* $\sqrt{8kr_1(1 + 2(r_1 - r_2) - r_1r_2)/b(3r_1^2 + 2r_1(1 - 2r_2))} - \theta/b > t_0^T$*, it meets* $0 < t < t_0^T$*, and there is* $\pi_3^{SC*} > \pi_2^{SC*}$

Case 2: When $e_0 > e_0^T = a - bc(1 + r_2)/\sqrt{4kb}$*,*

(a) *If* $\sqrt{8kr_1(1 + 2(r_1 - r_2) - r_1r_2)/b(3r_1^2 + 2r_1(1 - 2r_2))} - \theta/b < \hat{t}_3^T$*, it meets* $0 < t < \sqrt{8kr_1(1 + 2(r_1 - r_2) - r_1r_2)/b(3r_1^2 + 2r_1(1 - 2r_2))} - \theta/b$*, and there is* $\pi_3^{SC*} > \pi_2^{SC*}$*; it meets* $\sqrt{8kr_1(1 + 2(r_1 - r_2) - r_1r_2)/b(3r_1^2 + 2r_1(1 - 2r_2))} - \theta/b < t < \hat{t}_3^T$*, and there is* $\pi_3^{SC*} < \pi_2^{SC*}$

(b) *If* $\sqrt{8kr_1(1 + 2(r_1 - r_2) - r_1r_2)/b(3r_1^2 + 2r_1(1 - 2r_2))} - \theta/b > \hat{t}_3^T$*, it meets* $0 < t < \hat{t}_3^T$*, and there is* $\pi_3^{SC*} > \pi_2^{SC*}$

where $t_1^R = \sqrt{kb[1 - \sqrt{r_1^2 + 12r_1(1 - 2r_2) - 16r_2(1 - r_2) + 4}]} - \theta/b$*,*

$t_2^R = \sqrt{kb[1 + \sqrt{r_1^2 + 12r_1(1 - 2r_2) - 16r_2(1 - r_2) + 4}]} - \theta/b$*,*

$t_0^T = \sqrt{4kb} - \theta/b$*, and* $\hat{t}_3^T = a - bc(1 + r_2)/be_0 - \theta/b$*.*

Proof of Corollary 18. Please refer to Appendix N.

Corollary 18 shows that the economic, environmental, and social benefits of the TC mode are better than those of the IF mode. Combined with the results of Corollary 17, the optimal financing mode for the manufacturer, as the financing decision maker, is the TC mode. At the same time, we can also see that in the two ways of IF and TC, the retailer and supply chain's profits depend on the initial carbon emissions of the manufacturer and the carbon tax rate imposed by the government. Although the optimal financing mode of the manufacturer is the TC mode, if the retailer's profit is smaller than that of the IF mode, then the retailer has every reason to refuse to prepay the production cost and CER investment to the manufacturer. In this case, if the supply chain system's profit in the TC mode is even higher, the manufacturer can design a reasonable coordination contract to give a certain amount of profit compensation to the retailer to ensure that the retailer's final profit is not lower than the profit in the IF mode, and then, the TC mode will be adopted by both parties. However, suppose the level of the supply chain system's profit in the IF mode is higher. In that case, the coordination contract cannot effectively coordinate the supply chain, and the manufacturer can only choose the IF mode. □

6. Numerical Analysis

6.1. Parameter Settings. In this section, we provide a numerical example to illustrate the impact of the carbon tax rate change on CER, market demand, the manufacturer, retailer, and supply chain system's profits. Based on the above analysis, we compare the equilibrium solutions of different financing modes. Referring to the parameter setting of the demand function by Zhang and Qin [48], we let $a = 100, b = 1, c = 20$, and $\theta = 1$. Other parameters are assigned as follows: $k = 40$, $r_1 = 10\%$, $r_2 = 5\%$, and $V_R = 3000$. According to the given parameters, we obtain $e_0^E = 5.88, e_0^I = 5.96$, and $e_0^T = 6.25$. For the convenience of the following elaboration, we refer to the manufacturers with the initial carbon emissions per unit product $e_0 < \min\{e_0^E, e_0^I, e_0^T\}$ as the clean manufacturer and those with the initial carbon emissions per unit product $e_0 > \min\{e_0^E, e_0^I, e_0^T\}$ as the polluting manufacturer. We used $e_0 = 4$ as a clean manufacturer and $e_0 = 7$ as a polluting manufacturer. When examining the influence of the carbon tax rate change on the equilibrium solutions of the three financing modes, it is necessary to ensure that the value range of the carbon tax rate meets the constraints for the three financing modes. In our study, when the initial carbon emissions are $e_0 = 4$, there is $0 < t < 7.10$; when the initial carbon emissions are $e_0 = 7$, there is $0 < t < 10.14$. Without loss of generality, the carbon tax rate ranges from 0 to 7 for the clean manufacturer, and for the polluting manufacturer, the carbon tax rate ranges from 0 to 10.

6.2. Sensitivity Analysis

6.2.1. The Impact of the Carbon Tax Rate Changes on CER. Figure 2 describes the impact of the carbon tax rate changes on CER for clean and polluting manufacturers. As illustrated in Figure 2(a), for the clean manufacturer, CER increases with an increase in the carbon tax rate. As shown in Figure 2(b), for the polluting manufacturer, CER first increased and then decreased with an increase in the carbon tax rate, and the number of carbon emissions reduced by the polluting manufacturer was less than that facilitated by the clean manufacturer under the same carbon tax rate. Hence, it can be considered that increasing the carbon tax rate has a relatively effective incentive effect on the clean manufacturer while stimulating the polluting manufacturer to reduce carbon emissions has a limited effect. In terms of the three scenarios, the abatement effect is best in the TC mode, and the IF mode is generally better than the EF mode.

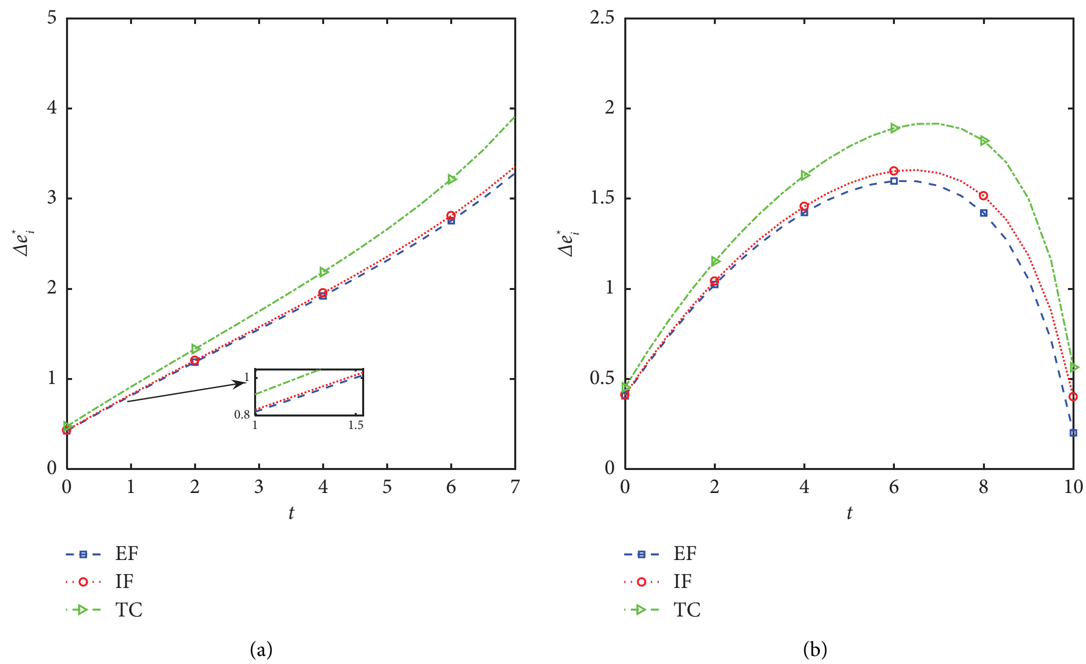


FIGURE 2: The impact of the carbon tax rate changes on CER. (a) $e_0 = 4$. (b) $e_0 = 7$.

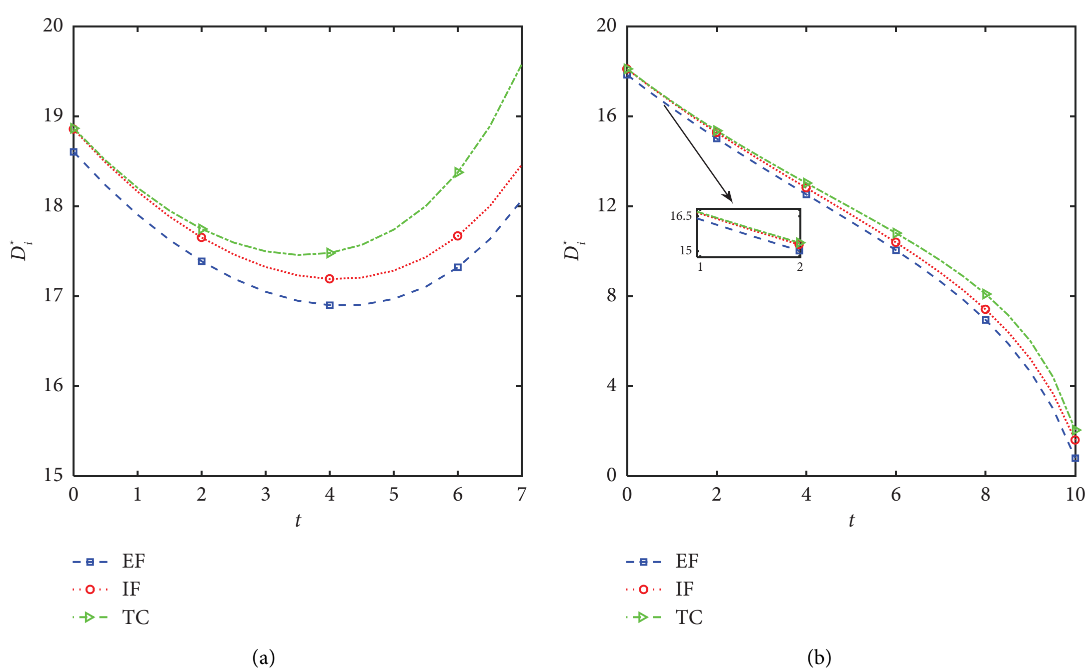
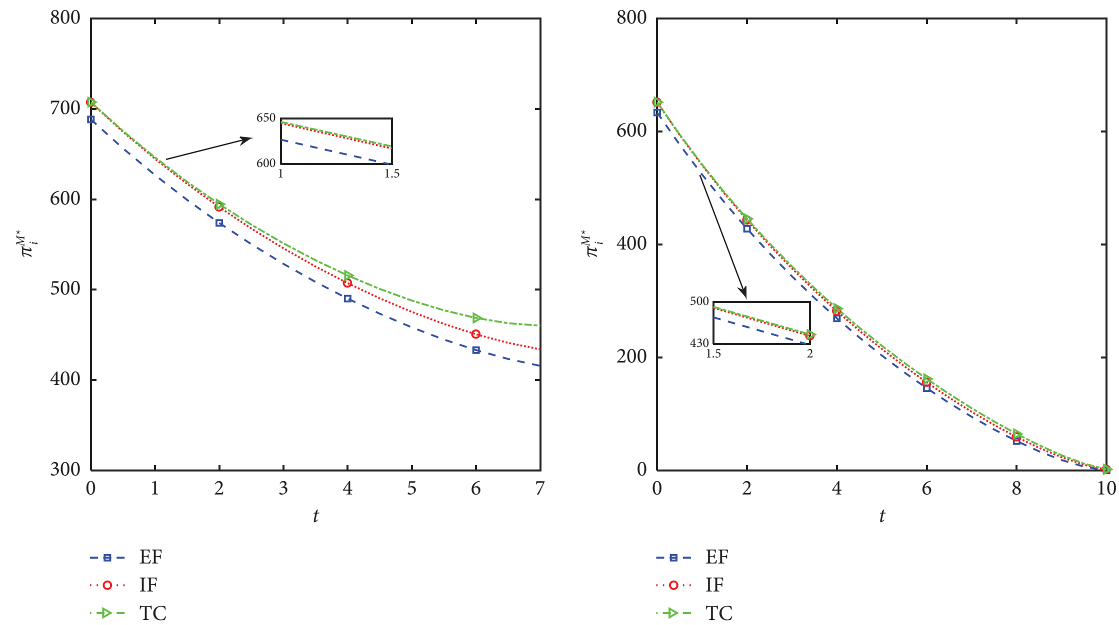


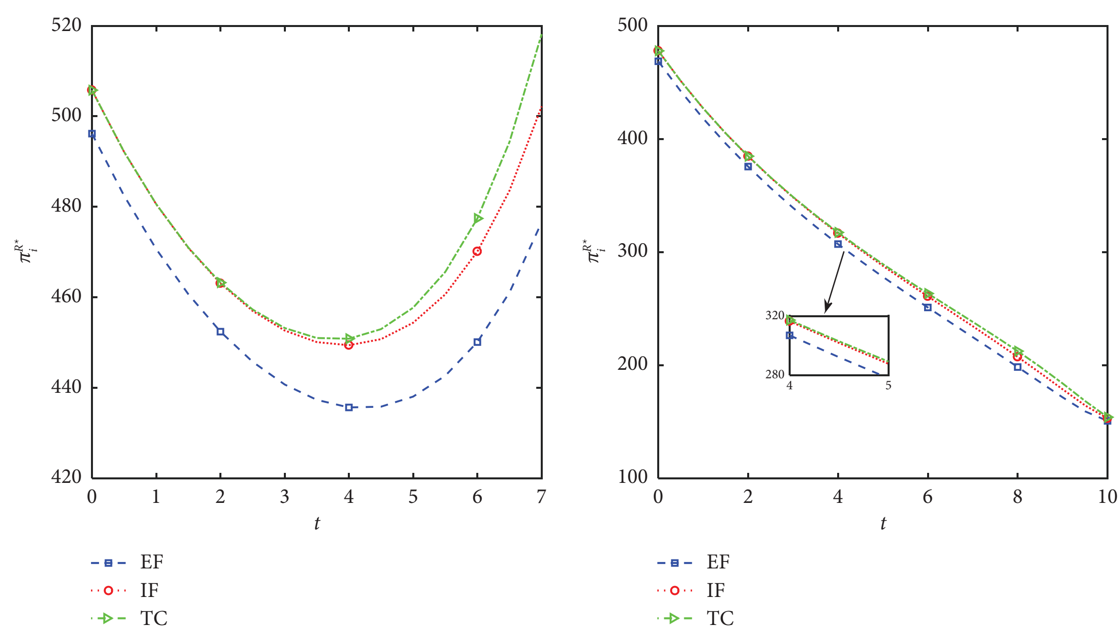
FIGURE 3: The impact of the carbon tax rate changes on market demand. (a) $e_0 = 4$. (b) $e_0 = 7$.

6.2.2. The Impact of the Carbon Tax Rate Changes on Market Demand. Figure 3 describes the impact of the carbon tax rate changes on market demand for clean and polluting manufacturers. As shown in Figure 3(a), for the clean manufacturer, the market demand first decreases and then increases with an increase in the carbon tax rate. As shown in Figure 3(b), the polluting manufacturer's market demand continues to decrease with an increase in the carbon tax rate. This phenomenon shows that a more stringent carbon tax policy is beneficial for increasing the market demand for a clean manufacturer with small initial carbon emissions, but for a polluting manufacturer with large initial carbon emissions, imposing a high carbon tax will only cause a continuous decline in market demand. It is also clear from Figure 3 that when the cleaning manufacturer lacks capital, the IF mode yields a larger market demand than the EF mode, and market demand is always higher in the TC mode. There is little difference in market demand for the polluting manufacturer under different financing modes.



(a) (b)

Figure 4: The impact of the carbon tax rate changes on the manufacturer's profits. (a) $e_0 = 4$. (b) $e_0 = 7$.



(a) (b)

Figure 5: The impact of the carbon tax rate changes on the retailer's profits. (a) $e_0 = 4$. (b) $e_0 = 7$.

6.2.3. The Impact of the Carbon Tax Rate Changes on the Manufacturer's Profits. As presented in Figure 4, regardless of the manufacturer's initial carbon emissions, the manufacturer's profits continue to decrease with an increase in the carbon tax rate. Increasing the carbon tax rate will stimulate the manufacturer to reduce carbon emissions, thus increasing market demand to a certain extent and contributing to an increase in the manufacturer's profits. On the other hand, increasing the carbon tax rate directly leads to a substantial decline in the marginal profits of the manufacturer selling low-carbon products, and the side effects on the total profits of the manufacturer are so great that they exceed the positive side and lead to a decline in profits. As shown in Figure 4(a), for a clean manufacturer, the profit of the EF mode is significantly lower than that of the IF and TC mode, while the profit advantage of the TC mode over the IF mode gradually emerges with an increase in the carbon tax rate. As Figure 4(b) shows, for the polluting manufacturer,

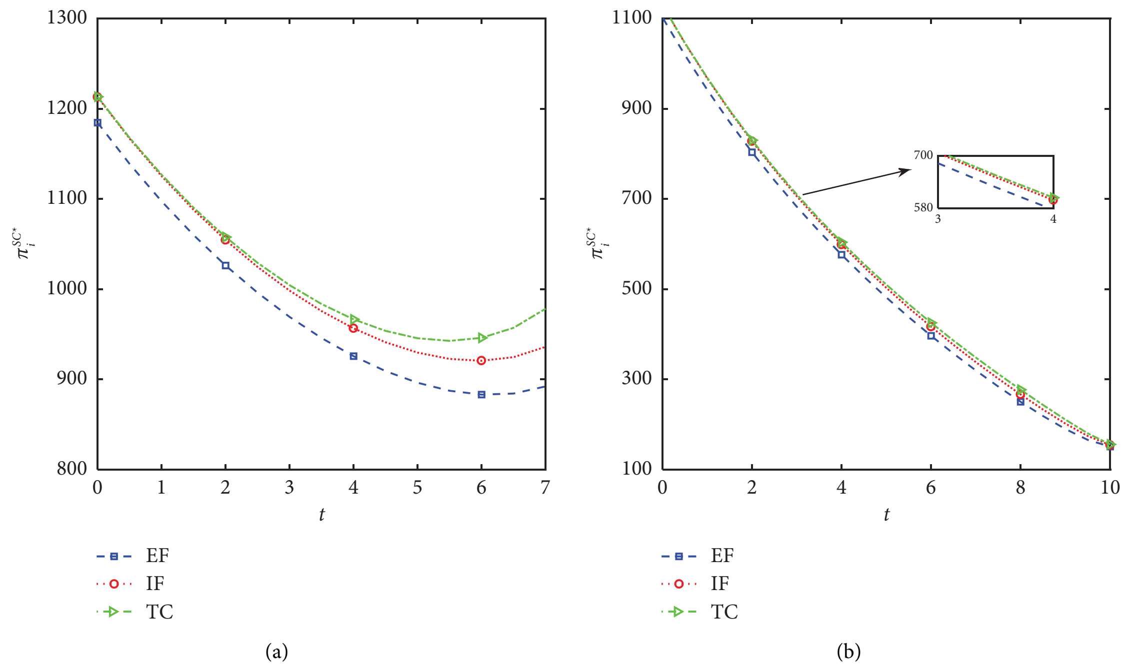


Figure 6: The impact of the carbon tax rate changes on the supply chain system's profits. (a) $e_0 = 4$. (b) $e_0 = 7$.

the profits of the three financing modes are almost the same, and the profit difference does not change significantly with an increase in the carbon tax rate.

6.2.4. The Impact of the Carbon Tax Rate Changes on the Retailer's Profits. From Figure 5, it can be seen that when the initial carbon emissions are low, the retailer's profits tend to decrease and then increase with an increase in the carbon tax rate, and when the initial carbon emissions are high, the retailer's profits continue to decrease with an increase in the carbon tax rate. On the other hand, if the initial carbon emissions of the manufacturer are low, the retailer can always make more profits by lending excess funds to the manufacturer or by advancing the costs of production and CER investment required by the manufacturer rather than by depositing extra funds in the bank. This result provides a theoretical basis for the retailer to accept the financing terms proposed by the manufacturer. However, when the initial carbon emissions are high, the profit advantage in the IF and TC modes is fragile.

6.2.5. The Impact of the Carbon Tax Rate Changes on the Supply Chain System's Profits. As shown in Figure 6, for a clean manufacturer, the supply chain system's profits first decrease and then increase with an increase in the carbon tax rate. By comparing Figures 5(a) and 6(a), it can be seen that although the profits of the supply chain system and the retailer both decline first and then increase with an increase in the carbon tax rate, the rebound time of the supply chain system's profit lags behind the latter, and the rebound amplitude is smaller because the manufacturer's profits shrink when the carbon tax rate is raised. The EF mode is clearly inferior, and when the carbon tax rate is high, the advantage of the TC mode is more prominent. For a polluting manufacturer, the supply chain system's profits decline with an increase in the carbon tax rate, and there is no difference in the earnings of the three financing modes.

7. Main Conclusions and Managerial Implications

From the research in this article, we draw the following conclusions.

(1) By comparing the EF, IF, and TC modes, it is found that in the TC mode, CER, market demand, and the manufacturer's profit are the best, and under certain conditions, the retailer and supply chain system also have the highest profit level. In contrast, the EF mode is the worst. Therefore, the EF mode is the first to be eliminated and will not be one of the options for the manufacturer. Because the manufacturer is the dominant player in the supply chain, that is, the manufacturer has the initiative to choose the financing mode. Whether considering economic, environmental, or social benefits, the TC mode is the dominant strategy for the manufacturer.

(2) The impact of carbon tax policy on the supply chain is complicated. Raising the carbon tax rate does not guarantee lower carbon emissions. Our research shows that the effects of the carbon tax rate on key parameters of the supply chain are not only related to the carbon tax rate itself but also to the manufacturer's initial carbon emissions. If the manufacturer's initial carbon emissions are low, then the government increases the carbon tax rate, which can reduce carbon emissions. Otherwise, the conclusion is the

opposite. Moreover, the impact of the carbon tax rate on market demand is not monotonous and related to the manufacturer's initial carbon emissions. However, regardless of the financing mode, increasing the carbon tax rate will directly reduce the profit level of the manufacturer.

This study can provide the following management inspiration.

(1) For the government, imposing a carbon tax would force the manufacturer to reduce carbon emissions. For reducing emissions, carbon tax policies are effective for clean and polluting manufacturers. The government could set a higher carbon tax rate for a clean manufacturer, whereas, for a polluting manufacturer, a moderate carbon tax rate might be more effective in reducing carbon emissions. The marginal cost of CER increases rapidly. Therefore, when the cost of reducing one unit of carbon emissions exceeds the benefits of tax savings and increased demand, the manufacturer is reluctant to continue reducing carbon emissions. Appropriate CER subsidies may help further stimulate the manufacturer to continue cutting emissions.

(2) The manufacturer is the supply chain leader and has the initiative to choose the financing mode, but the TC mode is the most appropriate for him. In the TC mode, the main indicators are optimal, such as economic benefits (profit level), social benefits (market demand), and environmental benefits (CER). Although the advantages of market demand and profit for the polluting manufacturer are not obvious, an appropriate carbon tax rate still establishes the advantage of CER, thus providing more low-carbon products and increasing competitiveness.

(3) For the retailer, because the loan interest rate is higher than the deposit interest rate, it is worse for the retailer to earn interest from the bank on excess capital than to lend it to the manufacturer. Thus, the EF mode is an outcome that the retailer is unwilling to accept. Fortunately, the EF mode is not an option for the manufacturer. Although the optimal financing mode of the manufacturer is the TC mode, Corollary 18 shows that the retailer's profit may be less than it is in the IF mode. As a result, the retailer will refuse the TC mode. The manufacturer can induce the retailer to accept "prepayment + discount" by designing properly coordinated contracts. Of course, if the profit of the supply chain system in the TC mode is less than that in the IF mode, then the manufacturer's efforts are in vain.

There are some limitations in this research. First, our paper does not consider the retailer's participation in CER action during the selling process. Future research could consider increasing the retailer's green sales efforts on low-carbon products. In addition, we discussed the comparison of three financing modes under the fixed interest rate and did not analyze the impact of the interest rate changes on the supply chain parameters. If financial markets are hit hard, and the interest rates are unstable, will the previously dominant financing mode still be the first choice for the manufacturer? This issue also deserves our attention.

Appendix

A. Proof of Theorem 5

Proof of Theorem A.5. Taking the second-order derivative of π_1^R with respect to p_1, we obtain $\partial^2\pi_1^R/\partial p_1^2 = -2b < 0$. It indicates that π_1^R is a strictly concave function in regard to p_1. Let us set $\partial\pi_1^R/\partial p_1 = 0$, we get $p_1(w_1) = a - \theta e_1 + bw_1/2b$. Substituting $p_1(w_1)$ into the profit function of the manufacturer, then π_1^M is expressed by $\pi_1^M = -(1+r_1)(k(e_0-e_1)^2 + (c+te_1-w_1)(a-\theta e_1 - bw_1))/2$. Taking the second-order derivative of π_1^M with respect to w_1, we obtain $\partial^2\pi_1^M/\partial w_1^2 = -b < 0$. Therefore, π_1^M it is a strictly concave function in regard to w_1. Making $\partial\pi_1^M/\partial w_1 = 0$, so there is $w_1 = a + \mathrm{bc}(1+r_1) + (\mathrm{bt}-\theta)e_1/2b$. Substituting w_1 and $p_1(w_1)$ into π_1^M, then π_1^M is expressed by $\pi_1^M = (a - \mathrm{bc}(1+r_1) + (\theta+\mathrm{bt})e_1)^2 - 4\mathrm{kb}(1+r_1)(e_0-e_1)^2/8b$. When $d^2\pi_1^M/\mathrm{d}e_1^2 = (\theta+\mathrm{bt})^2/4b - k(1+r_1) < 0$, that is $(\theta+\mathrm{bt})^2 - 4\mathrm{kb}(1+r_1) < 0$, the manufacturer has a maximum profit. Taking $d\pi_1^M/de_1 = 0$, then we solve for $e_1^* = 4\mathrm{kb}\ (1+r_1)e_0 - (\theta+\mathrm{bt})(a-\mathrm{bc}(1+r_1))/4\mathrm{kb}(1+r_1) - (\theta+\mathrm{bt})^2$. Simultaneously, we can get expressions for Δe_1^*, w_1^*, p_1^*, and D_1^*. □

B. Proof of Corollary 6

Proof of Corollary B.6. Taking the first-order derivative of Δe_1^* with respect to t, we obtain

$$\frac{\partial\Delta e_1^*}{\partial t} = \frac{b\big((a-\mathrm{bc}(1+r_1))(\theta+\mathrm{bt})^2 - 8\mathrm{kb}(1+r_1)e_0(\theta+\mathrm{bt}) + 4\mathrm{kb}(1+r_1)(a-\mathrm{bc}(1+r_1))\big)}{\big(4\mathrm{kb}(1+r_1)-(\theta+\mathrm{bt})^2\big)^2}. \tag{B1}$$

Let $f_1^E(\theta+\mathrm{bt}) = (a-\mathrm{bc}(1+r_1))(\theta+\mathrm{bt})^2 - 8\mathrm{kb}(1+r_1)e_0(\theta+\mathrm{bt}) + 4\mathrm{kb}(1+r_1)(a-\mathrm{bc}(1+r_1))$ show if that $f_1^E(\theta+\mathrm{bt})$ has the same monotonicity as $\partial\Delta e_1^*/\partial t$. Obviously, $f_1^E(\theta+bt)$ is a quadratic function with a parabola going upward, and its discriminant is given by

$$\Delta_1^E = 16\mathrm{kb}(1+r_1)\big(4\mathrm{kb}(1+r_1)e_0^2 - (a-\mathrm{bc}(1+r_1))^2\big). \tag{B2}$$

It is not difficult to find that when the manufacturer's initial carbon emissions $e_0 > e_0^E = a - \mathrm{bc}(1+r_1)/\sqrt{4\mathrm{kb}(1+r_1)}$, there is $\Delta_1^E > 0$. At this point, $f_1^E(\theta+\mathrm{bt})$

has two intersections with the horizontal axis, represented by

$$\theta + bt_1^E = \frac{4kb(1+r_1)e_0 - \sqrt{\Delta_1^E}/2}{a - bc(1+r_1)},$$
$$\theta + b\widetilde{t}_1^E = \frac{4kb(1+r_1)e_0 + \sqrt{\Delta_1^E}/2}{a - bc(1+r_1)}. \tag{B3}$$

Due to $e_0 > e_0^E = a - bc(1+r_1)/\sqrt{4kb(1+r_1)}$, there is $\theta + b\widetilde{t}_1^E > 4kb(1+r_1)e_0/a - bc(1+r_1) > \sqrt{4kb(1+r_1)}$. Considering the second-order condition of the manufacturer's profit maximization, it satisfies $0 < \theta + bt < \sqrt{4kb(1+r_1)}$. Therefore, the intersection $\theta + b\widetilde{t}_1^E$ does not meet the condition, and it will not be discussed. Thus, when $0 < \theta + bt < \theta + bt_1^E$, we have $f_1^E(\theta + bt) > 0$, that is, $\partial\Delta e_1^*/\partial t > 0$, and when $\theta + bt_1^E < \theta + bt < \sqrt{4kb(1+r_1)}$, we have $f_1^E(\theta + bt) < 0$, that is, $\partial\Delta e_1^*/\partial t < 0$. Additionally, when the initial carbon emissions meet $0 < e_0 < e_0^E$, there is $\Delta_1^E < 0$, which indicates that $f_1^E(\theta + bt)$ has no intersection with the horizontal axis. At the moment, there is $f_1^E(\theta + bt) > 0$, that is, $\partial\Delta e_1^*/\partial t > 0$. □

C. Proof of Corollary 7

Proof of Corollary C.7. Taking the first-order derivative of D_1^* with respect to t, we obtain

$$\frac{\partial D_1^*}{\partial t} = \frac{kb^2(1+r_1)\left(-e_0(\theta+bt)^2 + 2(a - bc(1+r_1))(\theta+bt) - 4kb(1+r_1)e_0\right)}{\left(4kb(1+r_1) - (\theta+bt)^2\right)^2}. \tag{C1}$$

Let $f_2^E(\theta + bt) = -e_0(\theta+bt)^2 + 2(a - bc(1+r_1))(\theta+bt) - 4kb(1+r_1)e_0$ show if that $f_2^E(\theta + bt)$ has the same monotonicity as $\partial D_1^*/\partial t$. Obviously, $f_2^E(\theta + bt)$ is a quadratic function with a parabola going downward, and its discriminant is given by

$$\Delta_2^E = 4\left((a - bc(1+r_1))^2 - 4kb(1+r_1)e_0^2\right). \tag{C2}$$

It is not difficult to find that when the manufacturer's initial carbon emissions $0 < e_0 < e_0^E$, there is $\Delta_2^E > 0$. At this point, $f_2^E(\theta + bt)$ has two intersections with the horizontal axis, represented by

$$\theta + bt_2^E = \frac{(a - bc(1+r_1)) - \sqrt{\Delta_2^E}/2}{e_0},$$
$$\theta + b\widetilde{t}_2^E = \frac{(a - bc(1+r_1)) + \sqrt{\Delta_2^E}/2}{e_0}. \tag{C3}$$

Due to $0 < e_0 < e_0^E$, then there is $\theta + b\widetilde{t}_2^E > a - bc(1+r_1)/e_0 > \sqrt{4kb(1+r_1)}$. Considering the second-order condition of the manufacturer's profit maximization, it satisfies $0 < \theta + bt < \sqrt{4kb(1+r_1)}$. Therefore, the intersection $\theta + b\widetilde{t}_2^E$ does not meet the condition, and it will not be discussed. Thus, when $0 < \theta + bt < \theta + bt_2^E$, we have $f_2^E(\theta + bt) < 0$, that is, $\partial D_1^*/\partial t < 0$, and when $\theta + bt_2^E < \theta + bt < \sqrt{4kb(1+r_1)}$, we have $f_2^E(\theta + bt) > 0$, that is, $\partial D_1^*/\partial t > 0$. Additionally, when the initial carbon emissions meet $e_0 > e_0^E$, there is $\Delta_2^E < 0$, which indicates that $f_2^E(\theta + bt)$ has no intersection with the horizontal axis. At the moment, there is $f_2^E(\theta + bt) < 0$, that is, $\partial D_1^*/\partial t < 0$. □

D. Proof of Corollary 8

Proof of Corollary D.8. Taking the first-order derivative of π_1^{M*} with respect to t, we obtain

$$\frac{\partial \pi_1^{M*}}{\partial t} = \frac{kb(1+r_1)(a - bc(1+r_1) - (\theta+bt)e_0)((a - bc(1+r_1))(\theta+bt) - 4kb(1+r_1)e_0)}{\left(4kb(1+r_1) - (\theta+bt)^2\right)^2}. \tag{D1}$$

From Theorem A.5, we know that there is $4kb(1+r_1) - (\theta+bt)^2 > 0$, $a - bc(1+r_1) - (\theta+bt)e_0 \geq 0$, and $\Delta e_1^* \leq e_0$, which indicates that $\theta + bt$ needs to satisfy both $\theta + bt < \sqrt{4kb(1+r_1)}$, $\theta + bt \leq a - bc(1+r_1)/e_0$, and $\theta + bt \leq 4kb(1+r_1)e_0/a - bc(1+r_1)$, that is, $\theta + bt < \min\{\sqrt{4kb(1+r_1)}, a - bc(1+r_1)/e_0, 4kb(1+r_1)e_0/a - bc(1+r_1)\}$. Next, we discuss two cases.

Case 1: $0 < e_0 \leq e_0^E = a - bc(1+r_1)/\sqrt{4kb(1+r_1)}$

When $0 < e_0 \leq e_0^E$, there is $\min\{\sqrt{4kb(1+r_1)}, a - bc(1+r_1)/e_0, 4kb(1+r_1)e_0/a - bc(1+r_1)\} = 4kb(1+r_1)e_0/a - bc(1+r_1)$, that is, $0 < \theta + bt \leq 4kb(1+r_1)e_0/a - bc(1+r_1)$. Therefore, we have $(a - bc(1+r_1))(\theta+bt) - 4kb(1+r_1)e_0 \leq (a - bc(1+r_1))4kb(1+r_1)e_0/a - bc(1+r_1) - 4kb(1+r_1)e_0 = 0$, that is, $\partial\pi_1^{M*}/\partial t \leq 0$.

Case 2: $e_0 > e_0^E = a - bc(1+r_1)/\sqrt{4kb(1+r_1)}$

When $e_0 > e_0^E$, there is $\min\{\sqrt{4\text{kb}(1+r_1)}, a-\text{bc}(1+r_1)/e_0, 4\text{kb}(1+r_1)e_0/a-\text{bc}(1+r_1)\} = a-\text{bc}(1+r_1)/e_0$, that is $0<\theta+\text{bt}\le a-\text{bc}(1+r_1)/e_0$. Therefore, we have $(a-\text{bc}(1+r_1))(\theta+\text{bt})-4\text{kb}(1+r_1)\ e_0 \le (a-\text{bc}(1+r_1))^2/e_0 - 4\text{kb}(1+r_1)e_0 \le 0$, that is, $\partial\pi_1^{M*}/\partial t \le 0$. □

E. Proof of Theorem 9

Proof of Theorem E.9. Taking the second-order derivative of π_2^R with respect to p_2, we obtain $\partial\pi_2^R/\partial p_2^2 = -2b < 0$. It indicates that π_2^R is a strictly concave function in regard to p_2. Let us set $\partial\pi_2^R/\partial p_2 = 0$, we get $p_2(w_2) = a-\theta e_2 + b(c(r_2-r_1)+w_2)/2b$. Substituting $p_2(w_2)$ into the manufacturer's profit function, then π_2^M is expressed by $\pi_2^M = -(1+r_1)\ (k(e_0-e_2)^2 + \ (c+te_2-w_2)(a-\theta e_2+bcr_1-b(cr_2+w_2)))/2$. Taking the second-order derivative of π_2^M with respect to w_2, we can obtain $\partial^2\pi_2^M/\partial w_2^2 = -b < 0$. Therefore, π_2^M is a strictly concave function in regard to w_2. Making $\partial\pi_2^M/\partial w = 0$, so there is $w_2 = a+\text{bc}(1+r_1)+bc(r_1-r_2)+(bt-\theta)e_2/2b$. Substituting w_2 and $p_2(w_2)$ into π_2^M, then π_2^M is expressed by $\pi_2^M = (a-\text{bc}(1+r_2)-(\theta+\text{bt})e_2)^2 - 4kb(1+r_1)\ (e_0-e_2)^2/8b$. When $d^2\pi_2^M/de_2^2 = (\theta+bt)^2/4b - k(1+r_1) < 0$, that is, $(\theta+\text{bt})^2 - 4\text{kb}\ (1+r_1) < 0$, the manufacturer has a maximum profit. Taking $d\pi_2^M/de_2 = 0$, then we solve for $e_2^* = 4\text{kb}(1+r_1)e_0 - (\theta+\text{bt})(a-bc(1+r_2))/4\text{kb}(1+r_1) - (\theta+bt)^2$. Simultaneously, we can get expressions for Δe_2^*, w_2^*, p_2^*, and D_2^*. □

F. Proof of Corollary 10

Proof of Corollary F.10. Taking the first-order derivative of Δe_2^* with respect to t, we obtain

$$\frac{\partial\Delta e_2^*}{\partial t} = \frac{b\big((a-\text{bc}(1+r_2))(\theta+\text{bt})^2 - 8\text{kb}(1+r_1)e_0(\theta+\text{bt}) + 4\text{kb}(1+r_1)(a-\text{bc}(1+r_2))\big)}{\big(4\text{kb}(1+r_1)-(\theta+\text{bt})^2\big)^2}. \tag{F1}$$

Let $f_1^I(\theta+\text{bt}) = (a-\text{bc}(1+r_2))(\theta+\text{bt})^2 - 8\text{kb}(1+r_1)e_0(\theta+\text{bt}) + 4\text{kb}(1+r_1)(a-\text{bc}(1+r_2))$ show if that $f_1^I(\theta+\text{bt})$ has the same monotonicity as $\partial e_2^*/\partial t$. Obviously, $f_1^I(\theta+bt)$ is a quadratic function with a parabola going upward, and its discriminant is given by

$$\Delta_1^I = 16\text{kb}(1+r_1)\big(4\text{kb}(1+r_1)e_0^2 - (a-\text{bc}(1+r_2))^2\big). \tag{F2}$$

It is not difficult to find that when the manufacturer's initial carbon emissions $e_0 > e_0^I = a-\text{bc}(1+r_2)/\sqrt{4\text{kb}(1+r_1)}$, there is $\Delta_1^I > 0$. At this point, $f_1^I(\theta+bt)$ has two intersections with the horizontal axis, represented by

$$\theta+\text{bt}_1^I = \frac{4\text{kb}(1+r_1)e_0 - \sqrt{\Delta_1^I}/2}{a-\text{bc}(1+r_2)},$$
$$\theta+b\widetilde{t}_1^I = \frac{4\text{kb}(1+r_1)e_0 + \sqrt{\Delta_1^I}/2}{a-\text{bc}(1+r_2)}. \tag{F3}$$

Due to $e_0 > e_0^I = a-\text{bc}(1+r_2)/\sqrt{4\text{kb}(1+r_1)}$, then there is $\theta+b\widetilde{t}_1^I > 4\text{kb}(1+r_1)e_0/a-\text{bc}(1+r_2) > \sqrt{4\text{kb}(1+r_1)}$. Considering the second-order condition of the manufacturer's profit maximization, it satisfies $0<\theta+bt<\sqrt{4\text{kb}(1+r_1)}$. Therefore, the intersection $\theta+b\widetilde{t}_1^I$ does not meet the condition, and it will not be discussed. Thus, when $0<\theta+\text{bt}<\theta+\text{bt}_1^I$, we have $f_1^I(\theta+bt)>0$, that is, $\partial\Delta e_2^*/\partial t > 0$, and when $\theta+bt_1^I<\theta+\text{bt}<\sqrt{4\text{kb}(1+r_1)}$, we have $f_1^I(\theta+\text{bt})<0$, that is, $\partial\Delta e_2^*/\partial t<0$. Additionally, when the initial carbon emissions meet $0<e_0<e_0^I$, there is $\Delta_1^I<0$, which indicates that $f_1^I(\theta+\text{bt})$ has no intersection with the horizontal axis. At the moment, there is $f_1^I(\theta+\text{bt})>0$, that is, $\partial\Delta e_1^*/\partial t>0$. □

G. Proof of Corollary 11

Proof of Corollary G.11. Taking the first-order derivative of D_2^* with respect to t, we obtain

$$\frac{\partial D_2^*}{\partial t} = \frac{\text{kb}^2(1+r_1)\big(-e_0(\theta+\text{bt})^2 + 2(a-\text{bc}(1+r_2))(\theta+\text{bt}) - 4\text{kb}(1+r_1)e_0\big)}{\big(4\text{kb}(1+r_1)-(\theta+\text{bt})^2\big)^2}. \tag{G1}$$

Let $f_2^I(\theta+\text{bt}) = -e_0(\theta+\text{bt})^2 + 2(a-\text{bc}(1+r_2))(\theta+\text{bt}) - 4\text{kb}(1+r_1)e_0$ show if that $f_2^I(\theta+\text{bt})$ has the same monotonicity as $\partial D_2^*/\partial t$. Obviously, $f_2^I(\theta+\text{bt})$ is a quadratic function with a parabola going downward, and its discriminant is given by

$$\Delta_2^I = 4\big((a-\text{bc}(1+r_2))^2 - 4\text{kb}(1+r_1)e_0^2\big). \tag{G2}$$

It is not difficult to find that when the manufacturer's initial carbon emissions $0<e_0<e_0^I$, there is $\Delta_2^I>0$. At this point, $f_2^I(\theta+\text{bt})$ has two intersections with the horizontal axis, represented by

$$\theta + bt_2^I = \frac{(a - bc(1 + r_2)) - \sqrt{\Delta_2^I}/2}{e_0},$$
$$\theta + b\widetilde{t}_2^I = \frac{(a - bc(1 + r_2)) + \sqrt{\Delta_2^I}/2}{e_0}. \tag{G3}$$

Due to $0 < e_0 < e_0^I$, then there is $\theta + b\widetilde{t}_2^I > a - bc(1 + r_2)/e_0 > \sqrt{4kb(1 + r_1)}$. Considering the second-order condition of the manufacturer's profit maximization, it apparently satisfies $0 < \theta + bt < \sqrt{4kb(1 + r_1)}$. Therefore, the intersection $\theta + b\widetilde{t}_2^I$ does not meet the condition, and it will not be discussed. Thus, when $0 < \theta + bt < \theta + bt_2^I$, we have $f_2^I(\theta + bt) < 0$, that is, $\partial D_2^*/\partial t < 0$, and when $\theta + bt_2^I < \theta + bt < \sqrt{4kb(1 + r_1)}$, we have $f_2^I(\theta + bt) > 0$, that is, $\partial D_2^*/\partial t > 0$. Additionally, when the initial carbon emissions meet $e_0 > e_0^I$, there is $\Delta_2^I < 0$, which indicates that $f_2^I(\theta + bt)$ has no intersection with the horizontal axis. At the moment, there is $f_2^I(\theta + bt) < 0$, that is, $\partial D_2^*/\partial t < 0$. □

H. Proof of Corollary 12

Proof of Corollary H.12. Taking the first-order derivative of π_2^{M*} with respect to t, we obtain

$$\frac{\partial \pi_2^{M*}}{\partial t} = \frac{kb(1 + r_1)(a - bc(1 + r_2) - (\theta + bt)e_0)((a - bc(1 + r_2))(\theta + bt) - 4kb(1 + r_1)e_0)}{\left(4kb(1 + r_1) - (\theta + bt)^2\right)^2}. \tag{H1}$$

From Theorem E.9, we know that there is $4kb(1 + r_1) - (\theta + bt)^2 > 0$, $a - bc(1 + r_2) - (\theta + bt)e_0 \geq 0$, and $\Delta e_2^* \leq e_0$, which indicates that $\theta + bt$ needs to satisfy both $\theta + bt < \sqrt{4kb(1 + r_1)}$, $\theta + bt \leq a - bc(1 + r_2)/e_0$, and $\theta + bt \leq 4kb(1 + r_1)e_0/a - bc(1 + r_2)$, that is, $\theta + bt < \min\{\sqrt{4kb(1 + r_1)}, a - bc(1 + r_2)/e_0, 4kb(1 + r_1)e_0/a - bc(1 + r_2)\}$. Next, we discuss two cases.

Case 1: $0 < e_0 \leq e_0^I = a - bc(1 + r_2)/\sqrt{4kb(1 + r_1)}$

When $0 < e_0 \leq e_0^I$, there is $\min\{\sqrt{4kb(1 + r_1)}, a - bc(1 + r_2)/e_0, 4kb(1 + r_1)e_0/a - bc(1 + r_2)\} = 4kb(1 + r_1)e_0/a - bc(1 + r_2)$, that is, $0 < \theta + bt \leq 4kb(1 + r_1)e_0/a - bc(1 + r_2)$. Therefore, we have $(a - bc(1 + r_2))(\theta + bt) - 4kb(1 + r_1)e_0 \leq (a - bc(1 + r_2))4kb(1 + r_1)e_0/a - bc(1 + r_2) - 4kb(1 + r_1)e_0 = 0$, that is, $\partial \pi_2^{M*}/\partial t \leq 0$.

Case 2: $e_0 > e_0^I = a - bc(1 + r_2)/\sqrt{4kb(1 + r_1)}$

When $e_0 > e_0^{IF}$, there is $\min\{\sqrt{4kb(1 + r_1)}, a - bc(1 + r_2)/e_0, 4kb(1 + r_1)e_0/a - bc(1 + r_2)\} = a - bc(1 + r_2)/e_0$, that is, $0 < \theta + bt \leq a - bc(1 + r_2)/e_0$. Therefore, we have $(a - bc(1 + r_2))(\theta + bt) - 4kb(1 + r_1)e_0 \leq (a - bc(1 + r_2))^2/e_0 - 4kb(1 + r_1)e_0 \leq 0$, that is, $\partial \pi_2^{M*}/\partial t \leq 0$. □

I. Proof of Theorem 13

Proof of Theorem I.13. Taking the second-order derivative of π_3^R with respect to p_3, we obtain $\partial \pi_3^R/\partial p_3^2 = -2b < 0$. It indicates that π_3^R is a strictly concave function in regard to p_3. Let us set $\partial \pi_3^R/\partial p_3 = 0$, we get $p_3(w_3) = a - \theta e_3 + bcr_2 + bw_3/2b$. Substituting $p_3(w_3)$ into the profit function of the manufacturer, then π_3^M is expressed by $\pi_3^M = -(k(e_0 - e_3)^2 + (c + te_3 - w_3)(a - \theta e_3 - bcr_2 - bw_3))/2$. Taking the second-order derivative of π_3^M with respect to w_3, we can obtain $\partial^2 \pi_3^M/\partial w_3^2 = -b < 0$. Therefore, π_3^M is a strictly concave function in regard to w_3. Making $\partial \pi_3^M/\partial w_3 = 0$, so there is $w_3 = a + bc(1 - r_2) + (bt - \theta)e_3/2b$. Substituting w_3 into $p_3(w_3)$ into π_3^M, then π_3^M is expressed by $\pi_3^M = (a - bc(1 + r_2) - (\theta + bt)e_3)^2 - 4kb(e_0 - e_3)^2/8b$. When $d^2\pi_3^M/de_3^2 = (\theta + bt)^2/4b - k < 0$, that is, $(\theta + bt)^2 - 4kb < 0$, the manufacturer has a maximum profit. Taking $\partial \pi_3^M/\partial w_3 = 0$, then we solve for $e_3^* = 4kbe_0 - (\theta + bt)(a - bc(1 + r_2))/4kb - (\theta + bt)^2$. Simultaneously, we can get expressions for Δe_3^*, w_3^*, p_3^*, and D_3^*. □

J. Proof of Corollary 14

Proof of Corollary J.14. Taking the first-order derivative of Δe_3^* with respect to t, we obtain

$$\frac{\partial \Delta e_3^*}{\partial t} = \frac{b\left((a - bc(1 + r_2))(\theta + bt)^2 - 8kbe_0(\theta + bt) + 4kb(a - bc(1 + r_2))\right)}{\left(4kb(1 + r_1) - (\theta + bt)^2\right)^2}. \tag{J1}$$

Let $f_1^T(\theta+\mathrm{bt}) = (a-\mathrm{bc}(1+r_2))(\theta+\mathrm{bt})^2 - 8\mathrm{kb}e_0(\theta+\mathrm{bt}) + 4\mathrm{kb}(a-\mathrm{bc}(1+r_2))$ show if that $f_1^T(\theta+\mathrm{bt})$ has the same monotonicity as $\partial e_3^*/\partial t$. Obviously, $f_1^T(\theta+bt)$ is a quadratic function with a parabola going upward, and its discriminant is given by

$$\Delta_1^T = 16kb\left(4\mathrm{kb}e_0^2 - (a-\mathrm{bc}(1+r_2))^2\right). \tag{J2}$$

It is not difficult to find that when the manufacturer's initial carbon emissions $e_0 > e_0^T = a-\mathrm{bc}(1+r_2)/\sqrt{4\mathrm{kb}}$, there is $\Delta_1^T > 0$. At this point, $f_1^T(\theta+\mathrm{bt})$ has two intersections with the horizontal axis, represented by

$$\begin{aligned}\theta+\mathrm{bt}_1^T &= \frac{4\mathrm{kb}e_0 - \sqrt{\Delta_1^T}/2}{a-\mathrm{bc}(1+r_2)},\\ \theta+b\tilde{t}_1^T &= \frac{4\mathrm{kb}e_0 + \sqrt{\Delta_1^T}/2}{a-\mathrm{bc}(1+r_2)}.\end{aligned} \tag{J3}$$

Due to $e_0 > e_0^T = a-\mathrm{bc}(1+r_2)/\sqrt{4\mathrm{kb}}$, then there is $\theta+b\tilde{t}_1^T > 4\mathrm{kb}e_0/a-\mathrm{bc}(1+r_2) > \sqrt{4\mathrm{kb}}$. Considering the second-order condition of the manufacturer's profit maximization, it satisfies $0<\theta+\mathrm{bt}<\sqrt{4\mathrm{kb}}$. Therefore, the intersection $\theta+b\tilde{t}_1^T$ does not meet the condition, and it will not be discussed. Thus, when $0<\theta+\mathrm{bt}<\theta+\mathrm{bt}_1^T$, we have $f_1^T(\theta+\mathrm{bt})>0$, that is, $\partial\Delta e_3^*/\partial t>0$, and when $\theta+\mathrm{bt}_1^T<\theta+\mathrm{bt}<\sqrt{4\mathrm{kb}}$, we have $f_1^T(\theta+\mathrm{bt})<0$, that is, $\partial\Delta e_3^*/\partial t<0$. Additionally, when the initial carbon emissions meet $0<e_0<e_0^T$, there is $\Delta_1^T<0$, which indicates that $f_1^T(\theta+bt)$ has no intersection with the horizontal axis. At the moment, there is $f_1^T(\theta+\mathrm{bt})>0$, that is, $\partial\Delta e_3^*/\partial t>0$. □

K. Proof of Corollary 15

Proof of Corollary K.15. Taking the first-order derivative of D_3^* with respect to t, we obtain

$$\frac{\partial D_3^*}{\partial t} = \frac{\left(\mathrm{kb}^2 - e_0(\theta+\mathrm{bt})^2 + 2(a-\mathrm{bc}(1+r_2))(\theta+\mathrm{bt}) - 4\mathrm{kb}e_0\right)}{\left(4\mathrm{kb}-(\theta+\mathrm{bt})^2\right)^2}. \tag{K1}$$

Let $f_2^T(\theta+\mathrm{bt}) = -e_0(\theta+\mathrm{bt})^2 + 2(a-\mathrm{bc}(1+r_2))(\theta+\mathrm{bt}) - 4\mathrm{kb}e_0$ show if that $f_2^T(\theta+\mathrm{bt})$ has the same monotonicity as $\partial D_3^*/\partial t$. Obviously, $f_2^T(\theta+\mathrm{bt})$ is a quadratic function with a parabola doing downward, and its discriminant is given by

$$\Delta_2^T = 4\left((a-\mathrm{bc}(1+r_2))^2 - 4\mathrm{kb}e_0^2\right). \tag{K2}$$

It is not difficult to find that when the manufacturer's initial carbon emissions $0<e_0<e_0^T$, there is $\Delta_2^T>0$. At this point, $f_2^T(\theta+\mathrm{bt})$ has two intersections with the horizontal axis, represented by

$$\begin{aligned}\theta+\mathrm{bt}_2^T &= \frac{(a-\mathrm{bc}(1+r_2)) - \sqrt{\Delta_2^T}/2}{e_0},\\ \theta+b\tilde{t}_2^T &= \frac{(a-\mathrm{bc}(1+r_2)) + \sqrt{\Delta_2^T}/2}{e_0}.\end{aligned} \tag{K3}$$

Due to $0<e_0<e_0^T$, then there is $\theta+b\tilde{t}_2^T > a-\mathrm{bc}(1+r_2)/e_0 > \sqrt{4kb}$. Considering the second-order condition of the manufacturer's profit maximization, it satisfies $0<\theta+\mathrm{bt}<\sqrt{4\mathrm{kb}}$. Therefore, the intersection $\theta+b\tilde{t}_2^T$ does not meet the condition, and it will not be discussed. Thus, when $0<\theta+\mathrm{bt}<\theta+\mathrm{bt}_2^T$, we have $f_2^T(\theta+\mathrm{bt})<0$, that is, $\partial D_3^*/\partial t<0$, and when $\theta+bt_2^T<\theta+\mathrm{bt}<\sqrt{4kb}$, we have $f_2^T(\theta+\mathrm{bt})>0$, that is, $\partial D_3^*/\partial t>0$. Additionally, when the initial carbon emissions meet $e_0>e_0^T$, there is $\Delta_2^T<0$, which indicates that $f_2^T(\theta+\mathrm{bt})$ has no intersection with the horizontal axis. At the moment, there is $f_2^T(\theta+\mathrm{bt})<0$, that is, $\partial D_3^*/\partial t<0$. □

L. Proof of Corollary 16

Proof of Corollary L.16. Taking the first-order derivative of π_3^{M*} with respect to t, we obtain

$$\frac{\partial\pi_3^{M*}}{\partial t} = \frac{\mathrm{kb}(a-\mathrm{bc}(1+r_2)-(\theta+\mathrm{bt})e_0)((a-\mathrm{bc}(1+r_2))(\theta+\mathrm{bt})-4\mathrm{kb}e_0)}{\left(4\mathrm{kb}-(\theta+\mathrm{bt})^2\right)^2}. \tag{L2}$$

From Theorem I.13, we know that there is $4\mathrm{kb}-(\theta+\mathrm{bt})^2>0$, $a-\mathrm{bc}(1+r_2)-(\theta+bt)e_0\geq 0$ and $\Delta e_3^*\leq e_0$, which indicates that $\theta+\mathrm{bt}$ needs to satisfy both $\theta+\mathrm{bt}<\sqrt{4\mathrm{kb}}$, $\theta+\mathrm{bt}\leq a-\mathrm{bc}(1+r_2)/e_0$, and $\theta+\mathrm{bt}\leq 4\mathrm{kb}e_0/a-\mathrm{bc}(1+r_2)$, that is, $\theta+\mathrm{bt}<\min\{\sqrt{4\mathrm{kb}}, a-\mathrm{bc}(1+r_2)/e_0, 4\mathrm{kb}e_0/a-\mathrm{bc}(1+r_2)\}$. Next, we discuss two cases.

Case 1: $0 < e_0 \le e_0^{TC} = a - \mathrm{bc}(1 + r_2)/\sqrt{4\mathrm{kb}}$

When $0 < e_0 \le e_0^T$, there is $\min\{\sqrt{4\mathrm{kb}}, a - \mathrm{bc}\ (1 + r_2)/e_0, 4\mathrm{kb}\ e_0/a - \mathrm{bc}(1 + r_2)\} = 4\mathrm{kb}e_0/a - \mathrm{bc}(1 + r_2)$, that is $0 < \theta + bt \le 4\mathrm{kb}e_0/a - \mathrm{bc}(1 + r_2)$. Therefore, we have $(a - \mathrm{bc}(1 + r_2))\ (\theta + \mathrm{bt}) - 4\mathrm{kb}e_0 \le (a - \mathrm{bc}(1 + r_2))4\mathrm{kb}\ e_0/a - \mathrm{bc}(1 + r_2)\ - 4\mathrm{kb}e_0 = 0$, that is, $\partial\pi_3^{M*}/\partial t \le 0$.

Case 2: $e_0 > e_0^T = a - \mathrm{bc}(1 + r_2)/\sqrt{4\mathrm{kb}}$

When $e_0 > e_0^T$, there is $\min\{\sqrt{4\mathrm{kb}}, a - \mathrm{bc}\ (1 + r_2)/e_0, 4\mathrm{kb}e_0/a - \mathrm{bc}(1 + r_2)\} = a - \mathrm{bc}(1 + r_2)/e_0$, that is, $0 < \theta + bt \le a - \mathrm{bc}(1 + r_2)/e_0$. Therefore, we have $(a - bc(1 + r_2))(\theta + bt) - 4kbe_0 \le (a - \mathrm{bc}(1 + r_2))^2/e_0 - 4\mathrm{kb}e_0 \le 0$, that is, $\partial\pi_3^{M*}/\partial t \le 0$. □

M. Proof of Corollary 17

Proof of Corollary M.17. Comparing the EF and IF modes.

(i) Regarding Δe_1^* and Δe_2^*, we have the following:

$$\frac{\Delta e_2^*}{\Delta e_1^*} = \frac{a - \mathrm{bc}(1 + r_2) - (\theta + \mathrm{bt})e_0}{a - \mathrm{bc}(1 + r_1) - (\theta + \mathrm{bt})e_0} > 1. \quad \text{(M1)}$$

Therefore, there is $\Delta e_2^* > \Delta e_1^*$.

(ii) Regarding D_1^* and D_2^*, we have the following:

$$\frac{D_2^*}{D_1^*} = \frac{a - \mathrm{bc}(1 + r_2) - (\theta + \mathrm{bt})e_0}{a - \mathrm{bc}(1 + r_1) - (\theta + \mathrm{bt})e_0} > 1. \quad \text{(M2)}$$

Therefore, there is $D_2^* > D_1^*$.

(iii) Regarding π_1^{M*} and π_2^{M*}, we have the following:

$$\frac{\pi_2^{M*}}{\pi_1^{M*}} = \frac{a - \mathrm{bc}(1 + r_2) - (\theta + \mathrm{bt})e_0}{a - \mathrm{bc}(1 + r_1) - (\theta + \mathrm{bt})e_0} > 1. \quad \text{(M3)}$$

Therefore, there is $\pi_2^{M*} > \pi_1^{M*}$.

(iv) Regarding π_1^{R*} and π_2^{R*}, we have the following:

$$\frac{\pi_2^{R*} - Br_2}{\pi_1^{R*} - Br_2} = \frac{\left(2\mathrm{kb}(1 + r_1)^2 + (\theta + \mathrm{bt})^2 (r_1 - r_2)\right)(a - \mathrm{bc}(1 + r_2) - (\theta + \mathrm{bt})e_0)^2}{2\mathrm{kb}(1 + r_1)^2 (a - \mathrm{bc}(1 + r_1) - (\theta + \mathrm{bt})e_0)^2} > 1. \quad \text{(M4)}$$

Therefore, there is $\pi_2^{R*} > \pi_1^{R*}$.

(v) Regarding $\pi_1^{\mathrm{SC}*}$ and $\pi_2^{\mathrm{SC}*}$, we have the following:

$$\frac{\pi_2^{\mathrm{SC}*} - Br_2}{\pi_1^{\mathrm{SC}*} - Br_2} = \frac{\left(6\mathrm{kb}(1 + r_1)^2 - (\theta + \mathrm{bt})^2 (1 + r_2)\right)(a - \mathrm{bc}(1 + r_2) - (\theta + \mathrm{bt})e_0)^2}{\left(6\mathrm{kb}(1 + r_1)^2 - (\theta + \mathrm{bt})^2 (1 + r_1)\right)(a - \mathrm{bc}(1 + r_1) - (\theta + \mathrm{bt})e_0)^2} > 1. \quad \text{(M5)}$$

Therefore, there is $\pi_2^{SC*} > \pi_1^{SC*}$. □

N. Proof of Corollary 18

Proof of Corollary N.18. Comparing the IF and TC modes.

(i) Regarding Δe_2^* and Δe_3^*, we have the following:

$$\frac{\Delta e_3^*}{\Delta e_2^*} = \frac{4\mathrm{kb}(1 + r_1) - (\theta + \mathrm{bt})^2}{4\mathrm{kb} - (\theta + \mathrm{bt})^2} > 1. \quad \text{(N1)}$$

Therefore, there is $\Delta e_3^* > \Delta e_2^*$.

(ii) Regarding D_2^* and D_3^*, we have the following:

$$\frac{D_3^*}{D_2^*} = \frac{4\mathrm{kb}(1 + r_1) - (\theta + \mathrm{bt})^2}{4\mathrm{kb} - (\theta + \mathrm{bt})^2} > 1. \quad \text{(N2)}$$

Therefore, there is $D_3^* > D_2^*$.

(iii) Regarding π_2^{M*} and π_3^{M*}, we have the following:

$$\frac{\pi_3^{M*}}{\pi_2^{M*}} = \frac{4\mathrm{kb}(1 + r_1) - (\theta + \mathrm{bt})^2}{4\mathrm{kb} - (\theta + \mathrm{bt})^2} > 1. \quad \text{(N3)}$$

Therefore, there is $\pi_3^{\mathrm{M}*} > \pi_2^{\mathrm{M}*}$.

(iv) Regarding π_2^{R*} and π_3^{R*}, we have the following:

$$\frac{\pi_3^{R*} - Br_2}{\pi_2^{R*} - Br_2} = \frac{\left(2\mathrm{kb} - (\theta + \mathrm{bt})^2 r_2\right)\left(4\mathrm{kb}(1 + r_1) - (\theta + \mathrm{bt})^2\right)^2}{\left(4\mathrm{kb} - (\theta + \mathrm{bt})^2\right)^2\left(2\mathrm{kb}(1 + r_1)^2 + (\theta + \mathrm{bt})^2 (r_1 - r_2)\right)}, \quad \text{(N4)}$$

$(2\mathrm{kb} - (\theta + bt)^2 r_2)\ (4\mathrm{kb}(1 + r_1) - (\theta + bt)^2)^2 - (4\mathrm{kb} - (\theta + bt)^2)^2\ (2\mathrm{kb}(1 + r_1)^2 + (\theta + bt)^2\ (r_1 - r_2)) = r_1(\theta + bt)^2 - (\theta + \mathrm{bt})^4 + (2\mathrm{kb}(2 - r_1 + 4r_2)(\theta + \mathrm{bt})^2 + 16k^2b^2\ (r_1 - 2r_2 - r_1 r_2))$. Let $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta + bt)^2 = -(\theta + bt)^4 + 2\mathrm{kb}\ (2 - r_1 + 4r_2)(\theta + bt)^2 + 16k^2b^2(r_1 - 2r_2 - r_1 r_2)$, we know that $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta + \mathrm{bt})^2$ have the same monotonicity as $\pi_3^{R*} - \pi_2^{R*}$. Obviously, $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta + \mathrm{bt})^2$ is a quadratic function with a parabola doing downward, and its discriminant is given by

$$\Delta_{\pi_3^{R*}-\pi_2^{R*}} = 4k^2b^2\left(r_1^2 + 12r_1(1-2r_2) - 16r_2(1-r_2) + 4\right). \tag{N5}$$

If $r_1^2 + 12r_1(1-2r_2) - 16r_2(1-r_2) + 4 < 0$, there is $\Delta_{\pi_3^{R*}-\pi_2^{R*}} < 0$, it indicates that $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2$ has no intersection with the horizontal axis. At the moment, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 < 0$, that is, $\pi_3^{R*} - \pi_2^{R*} < 0$. When $r_1^2 + 12r_1(1-2r_2) - 16r_2(1-r_2)+4 > 0$, we can obtain $\Delta_{\pi_3^{R*}-\pi_2^{R*}} > 0$. At this point, $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2$ have two intersections with the horizontal axis, represented by

$$\begin{aligned}\left(\theta+bt_1^R\right)^2 &= kb\left[1-\sqrt{r_1^2+12r_1(1-2r_2)-16r_2(1-r_2)+4}\,\right],\\ \left(\theta+bt_2^R\right)^2 &= kb\left[1+\sqrt{r_1^2+12r_1(1-2r_2)-16r_2(1-r_2)+4}\,\right].\end{aligned} \tag{N6}$$

It means $\theta + bt_1^R = \sqrt{kb[1-\sqrt{r_1^2+12r_1(1-2r_1)-16r_2(1-r_2)+4}\,]}$ and $\theta + bt_1^R = \sqrt{kb[1+\sqrt{r_1^2+12r_1(1-2r_1)-16r_2(1-r_2)+4}\,]}$. Due to $0 < \theta + bt < \min\{\sqrt{4kb}, a - bc(1+r_2)/e_0\}$, then we know that if $0 < e_0 < a - bc(1+r_2)/\sqrt{4kb}$, there is $0 < \theta + bt < \sqrt{4kb}$; if $e_0 > a - bc(1+r_2)/\sqrt{4kb}$, there is $0 < \theta + bt < a - bc(1+r_2)/e_0$. Next, we discuss two cases.

Case 1: $0 < e_0 < a - bc(1+r_2)/\sqrt{4kb}$, that is, $0 < \theta + bt < \sqrt{4kb}$

(a) Considering the situation $0 < \sqrt{r_1^2+12r_1(1-2r_2)-16r_2(1-r_2)+4} < 1$
When $0 < \theta+bt < \theta+bt_1^R$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 < 0$, that is, $\pi_3^{R*} - \pi_2^{R*} < 0$; when $\theta+bt_1^R < \theta+bt < \theta+bt_2^R$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 > 0$, that is, $\pi_3^{R*} - \pi_2^{R*} > 0$, and when $\theta+bt_2^R < \theta+bt < \sqrt{4kb}$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 < 0$, that is, $\pi_3^{R*} - \pi_2^{R*} < 0$.

(b) Considering the situation $1 < \sqrt{r_1^2+12r_1(1-2r_2)-16r_2(1-r_2)+4} < 3$
When $0 < \theta+bt < \theta+bt_2^R$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 > 0$, that is, $\pi_3^{R*} - \pi_2^{R*} > 0$; when $\theta+bt_2^R < \theta+bt < \sqrt{4kb}$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 < 0$, that is, $\pi_3^{R*} - \pi_2^{R*} < 0$.

(c) Considering the situation $\sqrt{r_1^2+12r_1(1-2r_2)-16r_2(1-r_2)+4} > 3$
When $0 < \theta+bt < \sqrt{4kb}$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 > 0$, that is, $\pi_3^{R*} - \pi_2^{R*} > 0$.

Case 2: $e_0 > a - bc(1+r_2)/\sqrt{4kb}$, that is, $0 < \theta + bt < a - bc(1+r_2)/e_0$

(a) Considering the situation $0 < \sqrt{r_1^2+12r_1(1-2r_2)-16r_2(1-r_2)+4} < 1$
When $0 < \theta+bt < \theta+bt_1^R$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 < 0$, that is, $\pi_3^{R*} - \pi_2^{R*} < 0$; when $\theta+bt_1^R < \theta+bt < \theta+bt_2^R$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 > 0$, that is, $\pi_3^{R*} - \pi_2^{R*} > 0$, and when $\theta+bt_2^R < \theta+bt < a - bc(1+r_2)/e_0$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 < 0$, that is, $\pi_3^{R*} - \pi_2^{R*} < 0$.

(b) Considering the situation $1 < \sqrt{r_1^2+12r_1(1-2r_2)-16r_2(1-r_2)+4} < 3$
When $0 < \theta+bt < \theta+bt_2^R$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 > 0$, that is, $\pi_3^{R*} - \pi_2^{R*} > 0$; when $\theta+bt_2^R < \theta+bt < a - bc(1+r_2)/e_0$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 < 0$, that is, $\pi_3^{R*} - \pi_2^{R*} < 0$.

(c) Considering the situation $\sqrt{r_1^2+12r_1(1-2r_2)-16r_2(1-r_2)+4} > 3$
When $0 < \theta+bt < a - bc(1+r_2)/e_0$, there is $g_{\pi_3^{R*}-\pi_2^{R*}}(\theta+bt)^2 > 0$, that is, $\pi_3^{R*} - \pi_2^{R*} > 0$.

(v) Regarding π_2^{SC*} and π_3^{SC*}, we have the following:

$$\frac{\pi_3^{SC*} - Br_2}{\pi_2^{SC*} - Br_2} = \frac{\left(6kb-(\theta+bt)^2(1+r_2)\right)\left(4kb(1+r_1)-(\theta+bt)^2\right)^2}{\left(6kb(1+r_1)^2-(\theta+bt)^2(1+r_2)\right)\left(4kb-(\theta+bt)^2\right)^2}, \tag{N7}$$

$g_{\pi_3^{SC*}-\pi_2^{SC*}} = (6kb - (\theta+bt)^2(1+r_2))\,(4kb(1+r_1) - (\theta+bt)^2)^2 - (6kb(1+r_1)^2 - (\theta+bt)^2\ (1+r_2))^2\,(4kb - (\theta+bt)^2) = 2kb\ (\theta+bt)^2(8kbr_1(1+2(r_1-r_2)-r_1r_2)(\theta+bt)^2 - (3r_1^2+2r_1(1-2r_2))3r_1^2+2r_1(1-2r_2))(\theta+bt)^2\ )(3r_1^2+2r_1(1-2r_2))(\theta+bt)^2)\ 3r_1^2+2r_1(1-2r_2)(\theta+bt)^2\ )(8kbr_1(1+2(r_1-r_2)-r_1r_2) - (3r_1^2+2r_1\ (1-2r_2))(\theta+bt)^2)3r_1^2+2r_1\ (1-2r_2))(\theta+bt)^2)8kbr_1\ (1+2(r_1-r_2)-r_1r_2)-$. Due to this $0 < \theta + bt < \min\{\sqrt{4kb}, a - bc(1 + r_2)/e_0\}$, we know that if $0 < e_0 < a - bc(1 + r_2)/\sqrt{4kb}$, there is $0 < \theta + bt < \sqrt{4kb}$; if $e_0 > a - bc(1 + r_2)/\sqrt{4kb}$, there is $0 < \theta + bt < a - bc(1 + r_2)/e_0$. Next, we discuss two cases.

Case 1: $0 < e_0 < a - bc(1 + r_2)/\sqrt{4kb}$

Apparently, the range of $\theta + bt$ is $0 < \theta + bt < \sqrt{4kb}$. If $\sqrt{8kbr_1(1 + 2(r_1 - r_2) - r_1r_2)/3r_1^2 + 2r_1(1 - 2r_2)} < \sqrt{4kb}$, then when $0 < \theta + bt < \sqrt{8kbr_1(1 + 2(r_1 - r_2) - r_1r_2)/3r_1^2 + 2r_1(1 - 2r_2)}$, there is $g_{\pi_3^{SC*} - \pi_2^{SC*}} > 0$, that is, $\pi_3^{SC*} - \pi_2^{SC*} > 0$; when $\sqrt{8kbr_1(1 + 2(r_1 - r_2) - r_1r_2)/3r_1^2 + 2r_1(1 - 2r_2)} < \theta + bt < \sqrt{4kb}$, there is $g_{\pi_3^{SC*} - \pi_2^{SC*}} < 0$, that is, $\pi_3^{SC*} - \pi_2^{SC*} < 0$. If $\sqrt{8kbr_1(1 + 2(r_1 - r_2) - r_1r_2)/3r_1^2 + 2r_1(1 - 2r_2)} > \sqrt{4kb}$, then when $0 < \theta + bt < \sqrt{4kb}$, there is $g_{\pi_3^{SC*} - \pi_2^{SC*}} > 0$, that is, $\pi_3^{SC*} - \pi_2^{SC*} > 0$.

Case 2: $e_0 > a - bc(1 + r_2)/\sqrt{4kb}$

Apparently, the range of $\theta + bt$ is $0 < \theta + bt < a - bc(1 + r_2)/e_0$. If $\sqrt{8kbr_1(1 + 2(r_1 - r_2) - r_1r_2)/3r_1^2 + 2r_1(1 - 2r_2)} < a - bc(1 + r_2)/e_0$, then when $0 < \theta + bt < \sqrt{8kbr_1(1 + 2(r_1 - r_2) - r_1r_2)/3r_1^2 + 2r_1(1 - 2r_2)}$, there is $g_{\pi_3^{SC*} - \pi_2^{SC*}} > 0$, that is, $\pi_3^{SC*} - \pi_2^{SC*} > 0$; when $\sqrt{8kbr_1(1 + 2(r_1 - r_2) - r_1r_2)/3r_1^2 + 2r_1(1 - 2r_2)} < \theta + bt < a - bc(1 + r_2)/e_0$, there is $g_{\pi_3^{SC*} - \pi_2^{SC*}} < 0$, that is, $\pi_3^{SC*} - \pi_2^{SC*} < 0$. If $\sqrt{8kbr_1(1 + 2(r_1 - r_2) - r_1r_2)/3r_1^2 + 2r_1(1 - 2r_2)} > a - bc(1 + r_2)/e_0$, then when $0 < \theta + bt < a - bc(1 + r_2)/e_0$, there is $g_{\pi_3^{SC*} - \pi_2^{SC*}} > 0$, that is, $\pi_3^{SC*} - \pi_2^{SC*} > 0$. □

Data Availability

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Conflicts of Interest

The authors declare that they have no conflicts of interest. There are no professional or other personal interests of any nature or kind in any product, service, and company that could be construed as influencing the position presented in, or the review of, the manuscript entitled.

Authors' Contributions

Yunfeng Zhang conceived and designed the research question. Yunfeng Zhang constructed the models and analyzed the optimal solutions. Yin Qin and Jingting Ma wrote the paper. Yunfeng Zhang, Jingting Ma, and Ying Qin reviewed and edited the manuscript. Tingting Song completed the revision of the manuscript. All authors read and approved the manuscript.

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