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Research Article

Generalized Class of Finite Population Variance in the Presence of Random Nonresponse Using Simulation Approach

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In this article, we estimate the finite population variance in random nonresponse using simple random sampling, which may be helpful for data analysis in applied and environmental sciences. For the three distinct random nonresponse techniques by Singh and Joarder [25], we have proposed a generalized class of exponential-type estimators that uses an auxiliary variable. Up to the first order of approximation, expressions of the bias and mean square error of the proposed estimators are obtained. The suggested estimators illustrate their superior performances to the current estimators in the comparable strategies in a comparative analysis using the real and simulated datasets.

1. Introduction

In the survey sampling theory, an estimator of the population parameter(s) can be more precise by effectively using auxiliary information. Examples of this context include the ratio estimator, product estimator, and regression estimation method. When there is a strong correlation between the study variable and the auxiliary variable, ratio estimators are frequently employed to estimate the population parameter. The first estimation of the finite population variance with the known population variance and coefficient of variation of the auxiliary variable was carried out by Das and Tripathi [1]. Kadilar and Cingi [2] proposed some ratio-type variance estimators using ratio estimators in a simple and stratified random sampling. Singh and Malik [3] introduced a new family of estimators using auxiliary attributes under population variance in simple random sampling. Panda and Sahoo [4] suggested a family of exponential estimators for estimating the finite population variance using auxiliary information in simple random sampling. Haq et al. [5] developed estimators of the finite population variance using the information on the study variables under stratified random sampling. Many other researchers have also significantly contributed to estimating the finite population variance. Sunday et al. [6] adopted a variable step hybrid block method for the approximation of Kepler Problem by integrating the Lagrange polynomial with limits of integration selected at special points. Juraev et al. [7, 8] used regularization formula and matrix factorization for explicit form of the approximate solutions of the Cauchy Problem.

The issue of nonresponse in human-related surveys affects practically all questionnaire designs. Nonresponse occurs when some survey participants choose only to complete part of the questionnaire or when the interviewers fail to approach survey nonrespondents. In sample surveys, nonresponse is a possible source of errors. The estimation of population parameters exhibits significant variance and nonresponse bias due to the missing data. It is only possible to obtain data from some units in the chosen sample due to nonresponse. Nonresponse diminishes the specified sample on one hand and the estimator's effectiveness on the other. The existence of nonresponse can occasionally be random. Hansen and Hurwitz [9] were the first to study the nonresponse problem in a postal survey. In addition, Argyros [10], Akbarov et al. [11], and Rubin [12] suggested three different conditions of missingness of response in the sensitive survey: observed at random (OAR), missing at random (MAR), and parameter distinctness (PD).

Furthermore, missing at random (MAR) and missing completely at random (MCAR) is prominent, according to Heitjan and Basu [13].

Some researchers such as Singh and Joarder [14] and Ahmed et al. [15] suggested estimating finite population variance under random nonresponse using auxiliary information. Singh et al. [16], Pankov et al. [17], Musaev [18], Noor and Noor [19], and Singh and Khalid [20, 21] proposed a strategy to estimate the population mean and variance in the presence of random nonresponse using two-phase successive sampling. Bhushan and Pratap Pandey [22] presented some ratio and product-type estimators of finite population variance in the presence of random nonresponse using auxiliary information. Khalid and Singh [23] suggested some imputation methods for missing data problems due to random nonresponse in two-occasion successive sampling. Many others have dealt with the issue of random nonresponse using auxiliary information to estimate the finite population variance. Motivated by the above-mentioned work and looking at the importance of handling the problems of random nonresponse in survey sampling, we suggest a generalized class of exponential-type estimators of finite population variance in the presence of random nonresponse under three different strategies.

The rest of the article is presented as follows: Section 2 discusses the methodology, and some existing estimators are briefly reviewed in Section 3. Section 4 introduces our proposed generalized class of estimators along with the expressions of biases and mean square errors (MSEs). Section 5 elaborates the efficiency comparison of existing and proposed estimators. In Sections 6 and 7, the results of our empirical study based on real and simulated data are presented, and finally, Section 8 concludes our study.

2. Methodology and Notations

Let $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_N)$ denote a population of N units from which a simple random sample of size n is drawn without replacement. If $r\,(r = 0, 1, 2, \ldots, (n-2))$ denotes the number of sampling units on which information could not be obtained due to a random nonresponse, then the remaining $(n-r)$ units can be treated as a simple random sample from Ω. We are interested in estimating the finite population variance under random nonresponse and assume $r < (n-1)$, *i.e.*, $0 \le r \le (n-2)$. Singh and Joarder [14] assumed the discrete distribution as

$$P(r) = \frac{n-r}{(nq+2p)} \binom{n-2}{r} p^r q^{n-2-r}, \tag{1}$$

where p is the probability of nonresponse, $q = 1 - p$, and $\binom{n-2}{r}$ represents the total number of ways to obtain r nonresponses out of a possible $(n-2)$.

We consider a finite population of N distinct objects and a sample of size n is drawn by simple random sampling without replacement (SRSWOR) from a population of N distinct objects. Let y_i and x_i be the i^{th} observations of the study variable y and auxiliary variable x, respectively, which are correlated with a proper amount of correlation ρ_{yx} in Ω.

Let $\overline{Y} = N^{-1}\sum_{i=1}^{N} y_i$ and $\overline{X} = N^{-1}\sum_{i=1}^{N} x_i$ be the population means of the study and auxiliary variables along with their sample means $\overline{y}^* = (n-r)^{-1}\sum_{i=1}^{n-r} y_i$ and $\overline{x}^* = (n-r)^{-1}\sum_{i=1}^{n-r} x_i$, respectively. Let $s_y^{*2} = (n-r-1)^{-1}\sum_{i=1}^{n-r}(y_i - \overline{y}^*)^2$ and $s_x^{*2} = (n-r-1)^{-1}\sum_{i=1}^{n-r}(x_i - \overline{x}^*)^2$ be the response sample variances and are conditionally unbiased estimators of the population variances of $S_y^{*2} = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \overline{Y})^2$ and $S_x^{*2} = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \overline{X})^2$, respectively. Let $\mu_{ls} = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \overline{Y})^l (x_i - \overline{X})^s$ be the population covariance between l and s and $\lambda_{ls} = (\mu_{ls}/(\mu_{20})(\mu_{02}))$ be the lsth moment ratio. Furthermore, we define $f_1 = ((1/nq + 2p) - (1/N)), f_2 = ((1/n) - (1/N))$ and $f_3 = ((1/nq + 2p) - (1/n))$.

The maximum-likelihood estimators of p, λ_{ls} and μ_{ls} defined by Singh and Joarder [14] are specified as

$$\begin{aligned}
\widehat{p} &= \frac{(n-1+r) - \sqrt{(n-1+r)^2 - 4rn(n-3)/(n-2)}}{2(n-3)}, \\
\widehat{\lambda}_{ls}^* &= \frac{\widehat{\mu}_{ls}^*}{(\widehat{\mu}_{20}^*)(\widehat{\mu}_{02}^*)}, \\
\widehat{\mu}_{ls}^* &= \frac{\sum_{i=1}^{N}(y_i - \overline{y}^*)^l (x_i - \overline{x}^*)^s}{(n-1+r)}.
\end{aligned} \tag{2}$$

If $r = 0$, then $\widehat{p} = 0$, and if $r = (n-2)$, then $\widehat{p} = 1$; thus $\widehat{p}$ is an admissible estimator of response probability p.

We investigate the impact of random nonresponse of the study and auxiliary variables of the generalized class of variance estimators under the following three strategies as discussed by Singh and Joarder [14]:

Strategy I: when S_x^2 and corresponding estimate s_x^{*2} is used

Strategy II: when S_x^2 and corresponding estimate s_x^2 is used

Strategy III: when s_x^2 and corresponding estimate s_x^{*2} is used

To obtain the expressions of biases and mean squared error (MSEs) of the existing and proposed estimators, we consider the following relative error terms: let $e_o = (s_y^{*2} - S_y^2/S_y^2)$, $e_1 = (s_x^{*2} - S_x^2/S_x^2)$, and $e_2 = (s_x^2 - S_x^2/S_x^2)$, such that $E(e_i) = 0$, (for $i = 0, 1, 2$). Also, $E(e_o^2) = f_1(\lambda_{40} - 1)$, $E(e_1^2) = f_1(\lambda_{04} - 1)$, $E(e_2^2) = f_2(\lambda_{04} - 1)$, $E(e_o e_1) = f_1(\lambda_{22} - 1), E(e_o e_2) = f_2(\lambda_{22} - 1)$, and $E(e_1 e_2) = f_2(\lambda_{04} - 1)$.

3. Existing Estimators

Singh and Joarder [14] presented the following three usual estimator strategies for estimating the finite population variance:

Strategy I

$$\widehat{v}_1 = s_y^{*2}\left(\frac{S_x^2}{s_x^{*2}}\right). \quad (3)$$

Strategy II

$$\widehat{v}_2 = s_y^{*2}\left(\frac{S_x^2}{s_x^2}\right). \quad (4)$$

Strategy III

$$\widehat{v}_3 = s_y^{*2}\left(\frac{s_x^2}{s_x^{*2}}\right). \quad (5)$$

Theorem 1. *The mean squared error* (MSE) *of* $\widehat{v}_1$ *is given by*

$$\text{MSE}\left(\widehat{v}_1\right) = f_1 S_y^4\left(\lambda_{40} + \lambda_{04} - 2\lambda_{22}\right). \quad (6)$$

Theorem 2. *The mean squared error* (MSE) *of* $\widehat{v}_2$ *is given by*

$$\text{MSE}\left(\widehat{v}_2\right) = S_y^4\left[f_1\left(\lambda_{40} - 1\right) + f_2\left(\lambda_{04} - 2\lambda_{22} + 1\right)\right]. \quad (7)$$

Theorem 3. *The mean squared error* (MSE) *of* $\widehat{v}_3$ *is given by*

$$\text{MSE}\left(\widehat{v}_3\right) = S_y^4\left[f_1\left(\lambda_{40} - 1\right) + f_3\left(\lambda_{04} + 2\lambda_{22} - 3\right)\right]. \quad (8)$$

Ahmed et al. [15] suggested the following three strategies for estimating the population variance. Each strategy is based on three different types of estimators.

Strategy I

$$\begin{aligned} t_{d_1} &= s_y^{*2}\left(\frac{S_x^2}{s_x^{*2}}\right)^{d_1}, \\ t_{g_1} &= s_y^{*2} + g_1\left(S_x^2 - s_x^{*2}\right), \\ t_{k_1} &= \frac{s_y^{*2}S_x^2}{k_1 s_x^{*2} + \left(1 - k_1\right)S_x^2}. \end{aligned} \quad (9)$$

Strategy II

$$\begin{aligned} t_{d_2} &= s_y^{*2}\left(\frac{S_x^2}{s_x^2}\right)^{d_2}, \\ t_{g_2} &= s_y^{*2} + g_2\left(S_x^2 - s_x^2\right), \\ t_{k_2} &= \frac{s_y^{*2}S_x^2}{k_2 s_x^2 + \left(1 - k_2\right)S_x^2}. \end{aligned} \quad (10)$$

Strategy III

$$\begin{aligned} t_{d_3} &= s_y^{*2}\left(\frac{s_x^2}{s_x^{*2}}\right)^{d_3}, \\ t_{g_3} &= s_y^{*2} + g_3\left(s_x^2 - s_x^{*2}\right), \\ t_{k_3} &= \frac{s_y^{*2}s_x^2}{k_3 s_x^{*2} + \left(1 - k_3\right)s_x^2}, \end{aligned} \quad (11)$$

where d_i, g_i, and k_i $(i = 1, 2, 3)$ are suitably chosen constants.

The optimum values of the appropriate constants d_i, g_i, and k_i $(i = 1, 2, 3)$ are $d_{i\,(\text{opt})} = k_{i\,(\text{opt})} = \left(\left(\lambda_{22} - 1\right)/\left(\lambda_{04} - 1\right)\right)$ and $g_{i\,(\text{opt})} = \left(S_y^2\left(\lambda_{22} - 1\right)/S_x^2\left(\lambda_{04} - 1\right)\right)$.

Theorem 4. *Following are the expressions of the minimum mean square error of* t_{d_i}, t_{g_i} *and* t_{k_i} $(i = 1, 2, 3)$, *respectively:*

$$\begin{aligned} \text{MSE}\left(t_{d_i}\right)_{\min} &= \text{MSE}\left(t_{g_i}\right)_{\min} = \text{MSE}\left(t_{k_i}\right)_{\min} \\ &= S_y^4\left[f_1\left(\lambda_{40} - 1\right) - f_i\frac{\left(\lambda_{22} - 1\right)^2}{\left(\lambda_{04} - 1\right)}\right]. \end{aligned} \quad (12)$$

Bhushan and Pratap Pandey [22] proposed three different strategies from the outlined two methodologies mentioned above. Each strategy is based on three different types of estimators.

Strategy I

$$\begin{aligned} T_{\pi_1} &= \sigma_1 s_y^{*2}\left(\frac{S_x^2}{s_x^{*2}}\right)^{\pi_1}, \\ T_{b_1} &= \sigma_1^* s_y^{*2} + b_1\left(S_x^2 - s_x^{*2}\right), \\ T_{\mu_1} &= \frac{\sigma_1' s_y^{*2}S_x^2}{\mu_1 s_x^{*2} + \left(1 - \mu_1\right)S_x^2}. \end{aligned} \quad (13)$$

Strategy II

$$\begin{aligned} T_{\pi_2} &= \sigma_2 s_y^{*2}\left(\frac{S_x^2}{s_x^2}\right)^{\pi_2}, \\ T_{b_2} &= \sigma_2^* s_y^{*2} + b_2\left(S_x^2 - s_x^2\right), \\ T_{\mu_2} &= \frac{\sigma_2' s_y^{*2}S_x^2}{\mu_2 s_x^2 + \left(1 - \mu_2\right)S_x^2}. \end{aligned} \quad (14)$$

Strategy III

$$T_{\pi_3}=\sigma_3 s_y^{*2}\left(\frac{s_x^2}{s_x^{*2}}\right)^{\pi_3},$$

$$T_{b_3}=\sigma_3^{*} s_y^{*2}+b_3\left(s_x^2-s_x^{*2}\right), \quad (15)$$

$$T_{\mu_3}=\frac{\sigma_3' s_y^{*2} s_x^2}{\mu_3 s_x^{*2}+(1-\mu_3)s_x^2}.$$

Theorem 5. *The following are the expressions for minimum mean square error of the estimators* T_{π_i}, T_{b_i} *and* T_{μ_i} $(i=1,2,3)$, *respectively:*

$$\mathrm{MSE}\left(T_{\pi_i}\right)_{\min}=S_y^4\left[1-\frac{H_i^2}{G_i}\right], \quad (16)$$

$$\mathrm{MSE}\left(T_{b_i}\right)_{\min}=\frac{S_y^4\mathrm{MSE}\left(t_{g_i}\right)}{S_y^4+\mathrm{MSE}\left(t_{g_i}\right)}, \quad (17)$$

$$\mathrm{MSE}\left(T_{\mu_i}\right)_{\min}=S_y^4\left[1-\frac{H_i'^2}{G_i'}\right], \quad (18)$$

where $G_i=1+f_1(\lambda_{40}-1)+f_i\{2\pi_i^2(\lambda_{04}-1)+\pi_i(\lambda_{04}-4\lambda_{22}+3)\}$, *and*

$$H_i=1+f_i\left\{\frac{\pi_i^2}{2}(\lambda_{04}-1)+\frac{\pi_i}{2}(\lambda_{04}-2\lambda_{22}+1)\right\},$$

$$G_i'=1+f_1(\lambda_{40}-1)+f_i\left\{3\mu_i^2(\lambda_{04}-1)-4\pi_i(\lambda_{22}-1)\right\},$$

$$H_i'=1+f_i\left\{\mu_i^2(\lambda_{04}-1)+\mu_i(\lambda_{22}-1)\right\},$$

$$\sigma_{i(\mathrm{opt})}=\frac{H_i}{G_i},$$

$$\sigma_{i(\mathrm{opt})}'=\frac{H_i'}{G_i'}, \quad (19)$$

$$\pi_i=\mu_i=\frac{(\lambda_{22}-1)}{(\lambda_{04}-1)},$$

$$b_{i(\mathrm{opt})}=\sigma_{i(opt)}^{*}\frac{S_y^2(\lambda_{22}-1)}{S_x^2(\lambda_{04}-1)},$$

$$\sigma_{i(\mathrm{opt})}^{*}=\frac{1}{\left[1+f_1(\lambda_{40}-1)-f_i\left\{\left((\lambda_{22}-1)^2/(\lambda_{04}-1)\right)\right\}\right]}.$$

4. The Proposed Generalized Class of Variance Estimators

Motivated by Yadav et al. [24] and Muneer et al. [25], we propose a generalized class of exponential-type estimators of the finite population variance using auxiliary variable in the presence of random nonresponse. These estimators are used in three different random nonresponse strategies and are defined as follows:

Strategy I

$$T_1=s_y^{*2}\left[t_1\left(\frac{S_x^2}{s_x^{*2}}\right)+t_2\left(\frac{s_x^{*2}}{S_x^2}\right)\right]\left[\alpha\exp\left(\frac{a\left(S_x^2-s_x^{*2}\right)}{a\left(S_x^2+s_x^{*2}\right)+2b}\right)+(1-\alpha)\exp\left(\frac{a\left(s_x^{*2}-S_x^2\right)}{a\left(s_x^{*2}+S_x^2\right)+2b}\right)\right]. \quad (20)$$

Strategy II

$$T_2 = s_y^{*2}\left[t_1\left(\frac{S_x^2}{s_x^2}\right) + t_2\left(\frac{s_x^2}{S_x^2}\right)\right]\left[\alpha \exp\left(\frac{a\left(S_x^2 - s_x^2\right)}{a\left(S_x^2 + s_x^2\right) + 2b}\right) + (1-\alpha)\exp\left(\frac{a\left(s_x^2 - S_x^2\right)}{a\left(s_x^2 + S_x^2\right) + 2b}\right)\right]. \quad (21)$$

Strategy III

$$T_3 = s_y^{*2}\left[t_1\left(\frac{s_x^2}{s_x^{*2}}\right) + t_2\left(\frac{s_x^{*2}}{s_x^2}\right)\right]\left[\alpha \exp\left(\frac{a\left(s_x^2 - s_x^{*2}\right)}{a\left(s_x^2 + s_x^{*2}\right) + 2b}\right) + (1-\alpha)\exp\left(\frac{a\left(s_x^{*2} - s_x^2\right)}{a\left(s_x^{*2} + s_x^2\right) + 2b}\right)\right], \quad (22)$$

where a and b are known population parameters of an auxiliary variable and also t_1 and t_2 are the arbitrary constants and $\alpha = 0, 1$.

4.1. Properties of the Proposed Generalized Class of Variance Estimators. The properties of the proposed estimators are given in the following theorems:

Theorem 6. *The biases of the estimators $T_i\,(i = 1, 2, 3)$ are given by*

$$\text{Bias}\left(T_i\right) = S_y^2\left[A_i t_1 + B_i t_2 - 1\right]. \quad (23)$$

Theorem 7. *The mean square error of the estimators $T_i\,(i = 1, 2, 3)$ is given by*

$$\text{MSE}\left(T_i\right) = S_y^4\left[1 - 2A_i t_1 - 2B_i t_2 + C_i t_1^2 + D_i t_2^2 + 2E_i t_1 t_2\right]. \quad (24)$$

The optimum values for the appropriate constants of t_1 and t_2 are $t_{1\,(\text{opt})} = (A_iD_i - B_iE_i/C_iD_i - E_i^2)\,(i = 1, 2, 3)$ and $t_{2\,(\text{opt})} = (B_iC_i - A_iE_i/C_iD_i - E_i^2)$. The details for strategy I are given in the Appendix.

Theorem 8. *The minimum* MSE *of the estimators $T_i\,(i = 1, 2, 3)$ is given by*

$$\text{MSE}\left(T_i\right)_{\min} = S_y^4\left(1 + \frac{2A_iB_iE_i - A_i^2D_i - B_i^2C_i}{C_iD_i - E_i^2}\right), \quad (25)$$

where $\beta_i = (aS_x^2/2\,(aS_x^2 + b))$ for $(i = 1, 2, 3)$,

$$\begin{aligned}
A_i &= 1 + f_i\left\{\left(\lambda_{04} - 1\right)\left(1 - \beta_i + 2\alpha\beta_i - \frac{\beta_i^2}{2} + 2\alpha\beta_i^2\right) - \left(\lambda_{22} - 1\right)\left(1 - \beta_i + 2\alpha\beta_i\right)\right\},\\
B_i &= 1 + f_i\left\{\left(\lambda_{04} - 1\right)\left(\beta_i - 2\alpha\beta_i - \frac{\beta_i^2}{2} + 2\alpha\beta_i^2\right) + \left(\lambda_{22} - 1\right)\left(1 + \beta_i - 2\alpha\beta_i\right)\right\},\\
C_i &= 1 + f_1\left(\lambda_{40} - 1\right) + f_i\left\{\left(\lambda_{04} - 1\right)\left(3 - 4\beta_i + 8\alpha\beta_i + 4\alpha^2\beta_i^2\right) - 4\left(\lambda_{22} - 1\right)\left(1 - \beta_i + 2\alpha\beta_i\right)\right\},\\
D_i &= 1 + f_1\left(\lambda_{40} - 1\right) + f_i\left\{\left(\lambda_{04} - 1\right)\left(1 + 4\beta_i - 8\alpha\beta_i + 4\alpha^2\beta_i^2\right) + 4\left(\lambda_{22} - 1\right)\left(1 + \beta_i - 2\alpha\beta_i\right)\right\},\\
E_i &= 1 + f_1\left(\lambda_{40} - 1\right) + f_i\left[4\beta_i\left\{\alpha^2\beta_i\left(\lambda_{04} - 1\right) + \left(\lambda_{22} - 1\right)\left(1 - 2\alpha\right)\right\}\right].
\end{aligned} \quad (26)$$

The special cases for the strategies I–III are shown in Table 1.

5. Efficiency Comparison

In this section, the existing and proposed estimators are compared theoretically in terms of minimum mean square errors.

(i) From (5) and (25), we have

$\text{MSE}\left(T_i\right)_{\min} < MSE\left(\hat{v}_1\right)$if,

$$f_1\left(\lambda_{40} + \lambda_{04} - 2\lambda_{22}\right) - \left(1 + \frac{2A_iB_iE_i - A_i^2D_i - B_i^2C_i}{C_iD_i - E_i^2}\right) > 0. \quad (27)$$

Table 1: Some members of the proposed class of estimators.

a	b	T$_1$	T$_2$	T$_3$
1	0	$T_1^{(1)}$	$T_2^{(1)}$	$T_3^{(1)}$
1	$\beta_2(x)$	$T_1^{(2)}$	$T_2^{(2)}$	$T_3^{(2)}$
1	C_x	$T_1^{(3)}$	$T_2^{(3)}$	$T_3^{(3)}$
1	ρ_{yx}^*	$T_1^{(4)}$	$T_2^{(4)}$	$T_3^{(4)}$
$\beta_2(x)$	C_x	$T_1^{(5)}$	$T_2^{(5)}$	$T_3^{(5)}$
C_x	$\beta_2(x)$	$T_1^{(6)}$	$T_2^{(6)}$	$T_3^{(6)}$
C_x	ρ_{yx}^*	$T_1^{(7)}$	$T_2^{(7)}$	$T_3^{(7)}$
ρ_{yx}^*	C_x	$T_1^{(8)}$	$T_2^{(8)}$	$T_3^{(8)}$
$\beta_2(x)$	ρ_{yx}^*	$T_1^{(9)}$	$T_2^{(9)}$	$T_3^{(9)}$
ρ_{yx}^*	$\beta_2(x)$	$T_1^{(10)}$	$T_2^{(10)}$	$T_3^{(10)}$

(ii) From (7) and (25), we have

$$\text{MSE}(T_i)_{\min} < MSE(\widehat{v}_2)\ \text{if},$$
$$f_1(\lambda_{40}-1)+f_2(\lambda_{04}-2\lambda_{22}+1)-\left(1+\frac{2A_iB_iE_i-A_i^2D_i-B_i^2C_i}{C_iD_i-E_i^2}\right)>0. \tag{28}$$

(iii) From (8) and (25), we get

$$\text{MSE}(T_i)_{\min} < MSE(\widehat{v}_3)\ \text{if},$$
$$f_1(\lambda_{40}-1)+f_3(\lambda_{04}+2\lambda_{22}-3)-\left(1+\frac{2A_iB_iE_i-A_i^2D_i-B_i^2C_i}{C_iD_i-E_i^2}\right)>0. \tag{29}$$

(iv) From (12) and (25), we derive

$$\text{MSE}(T_i)_{\min} < MSE\left(t_{d_i}\right)_{\min} = \text{MSE}\left(t_{g_i}\right)_{\min} = \text{MSE}\left(t_{k_i}\right)_{\min}\text{if},$$
$$f_1(\lambda_{40}-1)-f_i\frac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}-\left(1+\frac{2A_iB_iE_i-A_i^2D_i-B_i^2C_i}{C_iD_i-E_i^2}\right)>0. \tag{30}$$

(v) From (16) and (25), we get

$$\text{MSE}(T_i)_{\min} < MSE\left(T_{\pi_i}\right)_{\min}\text{if},$$
$$\left(1-\frac{H_i^2}{G_i}\right)-\left(1+\frac{2A_iB_iE_i-A_i^2D_i-B_i^2C_i}{C_iD_i-E_i^2}\right)>0. \tag{31}$$

(vi) From (17) and (25), we have

$$\text{MSE}(T_i)_{\min} < MSE\left(T_{b_i}\right)_{\min}\text{if},$$
$$\frac{\text{MSE}\left(t_{g_i}\right)}{S_y^4+\text{MSE}\left(t_{g_i}\right)}-\left(1+\frac{2A_iB_iE_i-A_i^2D_i-B_i^2C_i}{C_iD_i-E_i^2}\right)>0. \tag{32}$$

Table 2: Data summary for population-I and -II.

Parameters	Population-I	Population-II
N	34	32
n	12	10
r	3	2
p	0.3251	0.2173
q	0.6749	0.7827
$\overline{y}^*$	5.33	49.5625
$\overline{x}^*$	13.12	19.375
s_y^{*2}	42.6250	45.8512
s_x^{*2}	172.7870	48.5535
$\widehat{\lambda}_{22}^*$	1.1905	1.1002
$\widehat{\lambda}_{04}$	3.4547	1.8287
$\widehat{\lambda}_{40}^*$	4.2072	1.5037
ρ_{yx}^*	0.6167	0.7077
f_1	0.0849	0.0898
f_2	0.0539	0.0688
f_3	0.031	0.0210
$d_i = k_i = \pi_i = \mu_i\,(i = 1, 2, 3)$	0.0776	0.1209

(vii) From (18) and (25), we get

$$\text{MSE}\left(T_i\right)_{\min} < MSE\left(T_{\mu_i}\right)_{\min}\text{if},$$

$$\left(1 - \frac{H_i^{'2}}{G_i^{'}}\right) - \left(1 + \frac{2A_iB_iE_i - A_i^2D_i - B_i^2C_i}{C_iD_i - E_i^2}\right) > 0. \tag{33}$$

6. Empirical Study

In this section, we have chosen two populations to exhibit the performances of the estimators, given as follows:

Population-I (Cochran [26]; p: 182): let y be the number of paralytic polio cases in the placebo group and x be the number of placebo children

Population-II (Maddala [27]; p: 108): the data present experience and salary structure of the University of Michigan economists from 1983 to 1984. Let y be the salary (thousands of dollars) and x be the years of experience (defined as years since receiving Ph.D.)

The descriptive statistics for populations I and II are shown in Table 2.

The percentage relative efficiency (PRE) is given by

$$\text{PRE} = \frac{\text{MSE}\left(\widehat{v}_i\right)}{\text{MSE}\left(T_i^{(j)}\right)} \times 100\,\text{for}, \quad i = 1, 2, 3\,\text{and}\,j = 1, 2, \ldots, 10. \tag{34}$$

Tables 3 and 4 show the percentage relative efficiency (PRE) of the existing and proposed generalized class of estimators under random nonresponse using population-I and -II, respectively, for the two choices of $\alpha = 0$ and 1.

Interpretation of the results:

From Tables 3 and 1, we report that

(i) The proposed generalized class of estimators possesses more efficiency by using real datasets

(ii) The estimators under strategy I are always more efficient than strategy II and strategy III

(iii) In Tables 3 and 4, the proposed generalized class of estimators provides notable gain over existing estimators for estimating the finite population variance in the presence of random nonresponse while considering the auxiliary information

7. Simulation Study

A simulation study is conducted using R software to obtain the efficiency of the proposed estimators under simple random sampling. The following steps are used to perform the simulation study:

(i) A population size of 1000 is generated using a bivariate normal distribution at different values of covariance matrices by assuming a positive correlation between Y and X variables

(ii) We take a sample size of $n = 400$ and the number of nonrespondents $r = 160$ for the population-III and population-IV at $\rho_{yx}^* = 0.6, 0.7, 0.8$ and 0.9

(iii) Using the random nonresponse technique, the population is repeated 1000 times to obtain the percentage relative efficiency (PRE) of the proposed and existing estimators using the samples obtained

The descriptive statistics of population-III and population-IV are shown in Table 5.

Tables 6–9 show the values of percentage relative efficiency (PRE) of the existing and proposed generalized class of estimators under random nonresponse for population-III at $n = 300$ and $r = 160$. Moreover, Tables 10–13 show the percentage relative efficiency (PRE) of the existing and proposed generalized class of estimators under random nonresponse for population-IV at $n = 300$ and $r = 160$.

The bold values in the tables represent the maximum percentage relative efficiency among the proposed and existing estimators.

Table 3: PRE of the existing and proposed class of estimators for Population-I.

	Strategy I		Strategy II		Strategy III	
	Estimators	PRE	Estimators	PRE	Estimators	PRE
α	$\hat{\nu}_1$	100	$\hat{\nu}_2$	100	$\hat{\nu}_3$	100
	$t_{d_1} = t_{g_1} = t_{k_1}$	165.4202	$t_{d_2} = t_{g_2} = t_{k_2}$	141.4628	$t_{d_3} = t_{g_3} = t_{k_3}$	132.5071
	T_{π_1}	212.7663	T_{π_2}	181.2239	T_{π_3}	169.2532
	T_{b_1}	210.2550	T_{b_2}	179.8692	T_{b_3}	168.5269
	T_{μ_1}	210.8537	T_{μ_2}	179.8692	T_{μ_3}	168.5269
0	$T_1^{(1)}$	241.0492	$T_2^{(1)}$	195.4399	$T_3^{(1)}$	176.4999
	$T_1^{(2)}$	210.2550	$T_2^{(2)}$	**195.9277**	$T_3^{(2)}$	176.7353
	$T_1^{(3)}$	241.3512	$T_2^{(3)}$	195.5829	$T_3^{(3)}$	176.5697
	$T_1^{(4)}$	241.2356	$T_2^{(4)}$	195.5282	$T_3^{(4)}$	176.543
	$T_1^{(5)}$	241.1371	$T_2^{(5)}$	195.4816	$T_3^{(5)}$	176.5202
	$T_1^{(6)}$	242.0688	$T_2^{(6)}$	195.9219	$T_3^{(6)}$	176.7349
	$T_1^{(7)}$	241.2353	$T_2^{(7)}$	195.5281	$T_3^{(7)}$	176.5429
	$T_1^{(8)}$	241.5365	$T_2^{(8)}$	195.6705	$T_3^{(8)}$	176.6124
	$T_1^{(9)}$	241.1034	$T_2^{(9)}$	195.4656	$T_3^{(9)}$	176.5124
	$T_1^{(10)}$	**242.6782**	$T_2^{(10)}$	195.2089	$T_3^{(10)}$	**176.8746**
1	$T_1^{(1)}$	**267.7469**	$T_2^{(1)}$	**206.572**	$T_3^{(1)}$	**181.4741**
	$T_1^{(2)}$	267.1681	$T_2^{(2)}$	206.3553	$T_3^{(2)}$	**181.4741**
	$T_1^{(3)}$	267.5575	$T_2^{(3)}$	206.5074	$T_3^{(3)}$	181.4473
	$T_1^{(4)}$	267.6401	$T_2^{(4)}$	206.5321	$T_3^{(4)}$	181.4576
	$T_1^{(5)}$	267.6965	$T_2^{(5)}$	206.5532	$T_3^{(5)}$	181.4663
	$T_1^{(6)}$	267.1691	$T_2^{(6)}$	206.3557	$T_3^{(6)}$	181.3844
	$T_1^{(7)}$	267.6403	$T_2^{(7)}$	206.5322	$T_3^{(7)}$	181.4567
	$T_1^{(8)}$	267.4689	$T_2^{(8)}$	206.468	$T_3^{(8)}$	181.431
	$T_1^{(9)}$	267.7158	$T_2^{(9)}$	206.5604	$T_3^{(9)}$	181.4693
	$T_1^{(10)}$	266.8310	$T_2^{(10)}$	206.2289	$T_3^{(10)}$	181.3318

Table 4: PRE of the existing and proposed class of estimators for Population-II.

	Strategy I		Strategy II		Strategy III	
	Estimators	PRE	Estimators	PRE	Estimators	PRE
α	$\hat{\nu}_1$	100	$\hat{\nu}_2$	100	$\hat{\nu}_2$	100
	$t_{d_1} = t_{g_1} = t_{k_1}$	230.2757	$t_{d_2} = t_{g_2} = t_{k_2}$	191.6483	$t_{d_3} = t_{g_3} = t_{k_3}$	160.6724
	T_{π_1}	243.33	T_{π_2}	202.0618	T_{π_3}	168.8815
	T_{b_1}	241.2674	T_{b_2}	200.8619	T_{b_3}	168.4757
	T_{μ_1}	236.6938	T_{μ_2}	200.8619	T_{μ_3}	168.4757
0	$T_1^{(1)}$	259.2237	$T_2^{(1)}$	210.6027	$T_3^{(1)}$	171.4473
	$T_1^{(2)}$	260.4927	$T_2^{(2)}$	211.2481	$T_3^{(2)}$	171.6249
	$T_1^{(3)}$	259.483	$T_2^{(3)}$	210.7349	$T_3^{(3)}$	171.5598
	$T_1^{(4)}$	259.7294	$T_2^{(4)}$	210.8604	$T_3^{(4)}$	171.5264
	$T_1^{(5)}$	259.3661	$T_2^{(5)}$	210.6753	$T_3^{(5)}$	171.5099
	$T_1^{(6)}$	**262.4703**	$T_2^{(6)}$	**212.2455**	$T_3^{(6)}$	171.605
	$T_1^{(7)}$	**262.4703**	$T_2^{(7)}$	211.2946	$T_3^{(7)}$	171.5173
	$T_1^{(8)}$	259.5887	$T_2^{(8)}$	210.7888	$T_3^{(8)}$	171.5888
	$T_1^{(9)}$	259.5026	$T_2^{(9)}$	210.7449	$T_3^{(9)}$	171.4911
	$T_1^{(10)}$	260.9819	$T_2^{(10)}$	211.4958	$T_3^{(10)}$	**171.6693**

Table 4: Continued.

	Strategy I		Strategy II		Strategy III	
	Estimators	PRE	Estimators	PRE	Estimators	PRE
	$T_1^{(1)}$	**270.0639**	$T_2^{(1)}$	**215.8255**	$T_3^{(1)}$	**172.7754**
	$T_1^{(2)}$	269.7914	$T_2^{(2)}$	215.702	$T_3^{(2)}$	172.7468
	$T_1^{(3)}$	270.0062	$T_2^{(3)}$	215.7993	$T_3^{(3)}$	172.757
	$T_1^{(4)}$	269.9524	$T_2^{(4)}$	215.7749	$T_3^{(4)}$	172.7624
1	$T_1^{(5)}$	270.0321	$T_2^{(5)}$	215.8111	$T_3^{(5)}$	172.7651
	$T_1^{(6)}$	269.4157	$T_2^{(6)}$	215.5325	$T_3^{(6)}$	172.7499
	$T_1^{(7)}$	269.7726	$T_2^{(7)}$	215.6935	$T_3^{(7)}$	172.7639
	$T_1^{(8)}$	269.983	$T_2^{(8)}$	215.7888	$T_3^{(8)}$	172.7524
	$T_1^{(9)}$	270.0019	$T_2^{(9)}$	215.7973	$T_3^{(9)}$	172.7681
	$T_1^{(10)}$	269.6928	$T_2^{(10)}$	215.6574	$T_3^{(10)}$	172.7399

Table 5: Data summary for Population-III and -IV.

Population-III	Population-IV
$N = 1000$	$N = 1000$
$n = 400$	$n = 400$
$r = 160$	$r = 160$
$\mu = \begin{bmatrix} \overline{Y} & \overline{X} \end{bmatrix} = \begin{bmatrix} 5 & 5 \end{bmatrix}$	$\mu = \begin{bmatrix} \overline{Y} & \overline{X} \end{bmatrix} = \begin{bmatrix} 5 & 5 \end{bmatrix}$
$\Sigma = \begin{bmatrix} 4 & \sigma^*_{xy} \\ \sigma^*_{yx} & 64 \end{bmatrix}$	$\Sigma = \begin{bmatrix} 4 & \sigma^*_{xy} \\ \sigma^*_{yx} & 25 \end{bmatrix}$
$\rho^*_{yx} = 0.6 - 0.9$	$\rho^*_{yx} = 0.6 - 0.9$

Table 6: PRE of the existing and proposed class of estimators of Population-III at $\rho^*_{yx} = 0.6$.

		Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
		$\hat{v}_1$	100	$\hat{v}_2$	100	$\hat{v}_3$	100
		$t_{d_1} = t_{g_1} = t_{k_1}$	1806.071	$t_{d_2} = t_{g_2} = t_{k_2}$	875.0017	$t_{d_3} = t_{g_3} = t_{k_3}$	750.9025
	α	T_{π_1}	1806.208	T_{π_2}	875.2084	T_{π_3}	751.0645
		T_{b_1}	1806.819	T_{b_2}	875.3791	T_{b_3}	751.225
		T_{μ_1}	1581.648	T_{μ_2}	875.3791	T_{μ_3}	751.225
		$T_1^{(1)}$	1838.015	$T_2^{(1)}$	879.6223	$T_3^{(1)}$	755.5621
		$T_1^{(2)}$	1839.658	$T_2^{(2)}$	879.8201	$T_3^{(2)}$	755.7681
ρ^*_{yx}		$T_1^{(3)}$	1838.775	$T_2^{(3)}$	879.7139	$T_3^{(3)}$	755.6574
		$T_1^{(4)}$	1838.359	$T_2^{(4)}$	879.6638	$T_3^{(4)}$	755.6053
	0	$T_1^{(5)}$	1838.276	$T_2^{(5)}$	879.6538	$T_3^{(5)}$	755.5949
		$T_1^{(6)}$	1839.255	$T_2^{(6)}$	879.7717	$T_3^{(6)}$	755.7176
		$T_1^{(7)}$	1838.273	$T_2^{(7)}$	879.6534	$T_3^{(7)}$	755.5945
		$T_1^{(8)}$	1839.269	$T_2^{(8)}$	879.7734	$T_3^{(8)}$	755.7194
		$T_1^{(9)}$	1838133	$T_2^{(9)}$	879.6365	$T_3^{(9)}$	755.5769
		$T_1^{(10)}$	**1840.698**	$T_2^{(10)}$	**879.9448**	$T_3^{(10)}$	**755.898**
		$T_1^{(1)}$	1828.433	$T_2^{(1)}$	878.4372	$T_3^{(1)}$	754.3321
		$T_1^{(2)}$	1828.545	$T_2^{(2)}$	878.4514	$T_3^{(2)}$	754.3468
		$T_1^{(3)}$	1828.482	$T_2^{(3)}$	878.4434	$T_3^{(3)}$	754.3386
		$T_1^{(4)}$	1828.455	$T_2^{(4)}$	878.4399	$T_3^{(4)}$	754.335
0.6	1	$T_1^{(5)}$	1828.449	$T_2^{(5)}$	878.4392	$T_3^{(5)}$	754.3343
		$T_1^{(6)}$	1828.516	$T_2^{(6)}$	878.4476	$T_3^{(6)}$	754.343
		$T_1^{(7)}$	1828.449	$T_2^{(7)}$	878.4392	$T_3^{(7)}$	754.3343
		$T_1^{(8)}$	1828.517	$T_2^{(8)}$	878.4478	$T_3^{(8)}$	754.3431
		$T_1^{(9)}$	1828.44	$T_2^{(9)}$	878.4381	$T_3^{(9)}$	754.3331
		$T_1^{(10)}$	**1828.626**	$T_2^{(10)}$	**878.4616**	$T_3^{(10)}$	**754.3575**

Table 7: PRE of the existing and proposed class of estimators of Population-III at $\rho^*_{yx} = 0.7$.

ρ^*_{yx}	α	Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
0.7	0	$\hat{v}_1$	100	$\hat{v}_2$	100	$\hat{v}_3$	100
		$t_{d_1} = t_{g_1} = t_{k_1}$	1797.573	$t_{d_2} = t_{g_2} = t_{k_2}$	894.6432	$t_{d_3} = t_{g_3} = t_{k_3}$	874.4829
		T_{π_1}	1797.935	T_{π_2}	894.9077	T_{π_3}	874.7322
		T_{b_1}	1798.27	T_{b_2}	894.9933	T_{b_3}	874.8248
		T_{μ_1}	1738.409	T_{μ_2}	894.9933	T_{μ_3}	874.8248
		$T_1^{(1)}$	1831.543	$T_2^{(1)}$	899.6558	$T_3^{(1)}$	880.2593
		$T_1^{(2)}$	1831.367	$T_2^{(2)}$	899.8837	$T_3^{(2)}$	880.5298
		$T_1^{(3)}$	1832.385	$T_2^{(3)}$	899.7612	$T_3^{(3)}$	880.3844
		$T_1^{(4)}$	1831.987	$T_2^{(4)}$	899.7114	$T_3^{(4)}$	880.3253
		$T_1^{(5)}$	1831.832	$T_2^{(5)}$	899.692	$T_3^{(5)}$	880.3023
		$T_1^{(6)}$	1832.919	$T_2^{(6)}$	899.8278	$T_3^{(6)}$	880.4634
		$T_1^{(7)}$	1831.876	$T_2^{(7)}$	899.6975	$T_3^{(7)}$	880.3088
		$T_1^{(8)}$	1832.74	$T_2^{(8)}$	899.8055	$T_3^{(8)}$	880.4369
		$T_1^{(9)}$	1831.695	$T_2^{(9)}$	899.6748	$T_3^{(9)}$	880.2819
		$T_1^{(10)}$	**1834.116**	$T_2^{(10)}$	**899.977**	$T_3^{(10)}$	**880.6405**
	1	$T_1^{(1)}$	1826.578	$T_2^{(1)}$	899.0183	$T_3^{(1)}$	879.5053
		$T_1^{(2)}$	1826.644	$T_2^{(2)}$	899.0269	$T_3^{(2)}$	879.5155
		$T_1^{(3)}$	1826.606	$T_2^{(3)}$	899.022	$T_3^{(3)}$	879.5096
		$T_1^{(4)}$	1826.592	$T_2^{(4)}$	899.0202	$T_3^{(4)}$	879.5075
		$T_1^{(5)}$	1826.587	$T_2^{(5)}$	899.0195	$T_3^{(5)}$	879.5067
		$T_1^{(6)}$	1826.626	$T_2^{(6)}$	899.0246	$T_3^{(6)}$	879.5127
		$T_1^{(7)}$	1826.589	$T_2^{(7)}$	899.0197	$T_3^{(7)}$	879.5069
		$T_1^{(8)}$	1826.619	$T_2^{(8)}$	899.0237	$T_3^{(8)}$	879.5116
		$T_1^{(9)}$	1826.583	$T_2^{(9)}$	899.019	$T_3^{(9)}$	879.506
		$T_1^{(10)}$	1826.678	$T_2^{(10)}$	899.0313	$T_3^{(10)}$	879.5206

Table 8: PRE of the existing and proposed class of estimators of Population-III at $\rho^*_{yx} = 0.8$.

ρ^*_{yx}	α	Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
	0	$\hat{v}_1$	100	$\hat{v}_2$	100	$\hat{v}_3$	100
		$t_{d_1} = t_{g_1} = t_{k_1}$	1820.639	$t_{d_2} = t_{g_2} = t_{k_2}$	912.8881	$t_{d_3} = t_{g_3} = t_{k_3}$	1032.77
		T_{π_1}	1821.361	T_{π_2}	913.2291	T_{π_3}	1033.158
		T_{b_1}	1821.281	T_{b_2}	913.2104	T_{b_3}	1033.135
		T_{μ_1}	1818.351	T_{μ_2}	913.2104	T_{μ_3}	1033.135
		$T_1^{(1)}$	1857.746	$T_2^{(1)}$	918.2996	$T_3^{(1)}$	1040.016
		$T_1^{(2)}$	1859.853	$T_2^{(2)}$	918.5629	$T_3^{(2)}$	1040.378
		$T_1^{(3)}$	1858.718	$T_2^{(3)}$	918.4212	$T_3^{(3)}$	1040.183
		$T_1^{(4)}$	1858.331	$T_2^{(4)}$	918.3728	$T_3^{(4)}$	1040.117
		$T_1^{(5)}$	1858.08	$T_2^{(5)}$	918.3413	$T_3^{(5)}$	1040.073
		$T_1^{(6)}$	1859.334	$T_2^{(6)}$	918.4983	$T_3^{(6)}$	1040.289
		$T_1^{(7)}$	1858.185	$T_2^{(7)}$	918.3545	$T_3^{(7)}$	1040.092
		$T_1^{(8)}$	1858.957	$T_2^{(8)}$	918.4511	$T_3^{(8)}$	1040.224
		$T_1^{(9)}$	1857.946	$T_2^{(9)}$	918.3246	$T_3^{(9)}$	1040.05
		$T_1^{(10)}$	**1860.361**	$T_2^{(10)}$	**918.6263**	$T_3^{(10)}$	**1040.466**

Table 8: Continued.

		Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
0.8	1	$T_1^{(1)}$	**1859.218**	$T_2^{(1)}$	**918.4701**	$T_3^{(1)}$	**1040.252**
		$T_1^{(2)}$	1859.21	$T_2^{(2)}$	918.4694	$T_3^{(2)}$	1040.250
		$T_1^{(3)}$	1859.211	$T_2^{(3)}$	918.4694	$T_3^{(3)}$	1040.251
		$T_1^{(4)}$	1859.213	$T_2^{(4)}$	918.4696	$T_3^{(4)}$	1040.250
		$T_1^{(5)}$	1859.215	$T_2^{(5)}$	918.4697	$T_3^{(5)}$	1040.2507
		$T_1^{(6)}$	1859.21	$T_2^{(6)}$	918.4693	$T_3^{(6)}$	1040.2509
		$T_1^{(7)}$	1859.214	$T_2^{(7)}$	918.4697	$T_3^{(7)}$	1040.2517
		$T_1^{(8)}$	1859.21	$T_2^{(8)}$	918.4693	$T_3^{(8)}$	1040.2506
		$T_1^{(9)}$	1859.216	$T_2^{(9)}$	918.4699	$T_3^{(9)}$	1040.251912
		$T_1^{(10)}$	1859.212	$T_2^{(10)}$	918.4697	$T_3^{(10)}$	1040.2501

Table 9: PRE of the existing and proposed class of estimators of Population-III at $\rho_{yx}^* = 0.9$.

		Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
ρ_{yx}^*	α	$\widehat{v}_1$	100	$\widehat{v}_2$	100	$\widehat{v}_3$	100
		$t_{d_1} = t_{g_1} = t_{k_1}$	1870.658	$t_{d_2} = t_{g_2} = t_{k_2}$	914.7074	$t_{d_3} = t_{g_3} = t_{k_3}$	1219.057
		T_{π_1}	1871.94	T_{π_2}	915.1461	T_{π_3}	1219.665
		T_{b_1}	1871.245	T_{b_2}	915.0021	T_{b_3}	1219.449
		T_{μ_1}	1704.592	T_{μ_2}	915.0021	T_{μ_3}	1219.449
	0	$T_1^{(1)}$	1912.022	$T_2^{(1)}$	920.3575	$T_3^{(1)}$	1228.014
		$T_1^{(2)}$	1914.538	$T_2^{(2)}$	920.6536	$T_3^{(2)}$	1228.496
		$T_1^{(3)}$	1913.179	$T_2^{(3)}$	920.4939	$T_3^{(3)}$	1228.236
		$T_1^{(4)}$	1912.805	$T_2^{(4)}$	920.4498	$T_3^{(4)}$	1228.164
		$T_1^{(5)}$	1912.418	$T_2^{(5)}$	920.4042	$T_3^{(5)}$	1228.09
		$T_1^{(6)}$	1913.918	$T_2^{(6)}$	920.5809	$T_3^{(6)}$	1228.377
		$T_1^{(7)}$	1912.61	$T_2^{(7)}$	920.4268	$T_3^{(7)}$	1228.127
		$T_1^{(8)}$	1913.306	$T_2^{(8)}$	920.5088	$T_3^{(8)}$	1228.26
		$T_1^{(9)}$	1912.289	$T_2^{(9)}$	920.3891	$T_3^{(9)}$	1228.065
		$T_1^{(10)}$	**1914.809**	$T_2^{(10)}$	**920.6854**	$T_3^{(10)}$	**1228.547**
0.9	1	$T_1^{(1)}$	**1923.205**	$T_2^{(1)}$	**921.646**	$T_3^{(1)}$	**1230.116**
		$T_1^{(2)}$	1923.074	$T_2^{(2)}$	921.6313	$T_3^{(2)}$	1230.092
		$T_1^{(3)}$	1923.141	$T_2^{(3)}$	921.6387	$T_3^{(3)}$	1230.104
		$T_1^{(4)}$	1923.161	$T_2^{(4)}$	921.6411	$T_3^{(4)}$	1230.107
		$T_1^{(5)}$	1923.182	$T_2^{(5)}$	921.6435	$T_3^{(5)}$	1230.111
		$T_1^{(6)}$	1923.103	$T_2^{(6)}$	921.6346	$T_3^{(6)}$	1230.097
		$T_1^{(7)}$	1923.171	$T_2^{(7)}$	921.6423	$T_3^{(7)}$	1230.109
		$T_1^{(8)}$	1923.134	$T_2^{(8)}$	921.6381	$T_3^{(8)}$	1230.103
		$T_1^{(9)}$	1923.189	$T_2^{(9)}$	921.6443	$T_3^{(9)}$	1230.113
		$T_1^{(10)}$	1923.062	$T_2^{(10)}$	921.63	$T_3^{(10)}$	1230.089

Table 10: PRE of the existing and proposed class of estimators of Population-IV at $\rho_{yx}^{*} = 0.6$.

		Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
		$\widehat{v}_1$	100	$\widehat{v}_2$	100	$\widehat{v}_3$	100
		$t_{d_1} = t_{g_1} = t_{k_1}$	1701.44	$t_{d_2} = t_{g_2} = t_{k_2}$	844.5068	$t_{d_3} = t_{g_3} = t_{k_3}$	790.8
	α	T_{π_1}	1701.732	T_{π_2}	844.7536	T_{π_3}	844.8611
		T_{b_1}	1702.143	T_{b_2}	844.8611	T_{b_3}	791.0201
		T_{μ_1}	1607.688	T_{μ_2}	844.8611	T_{μ_3}	791.0201
		$T_1^{(1)}$	1731.035	$T_2^{(1)}$	848.9458	$T_3^{(1)}$	795.6826
		$T_1^{(2)}$	1734.098	$T_2^{(2)}$	849.3295	$T_3^{(2)}$	796.1182
ρ_{yx}^{*}		$T_1^{(3)}$	1732.053	$T_2^{(3)}$	849.0737	$T_3^{(3)}$	795.8277
		$T_1^{(4)}$	1732.053	$T_2^{(4)}$	849.0294	$T_3^{(4)}$	795.7775
	0	$T_1^{(5)}$	1731.39	$T_2^{(5)}$	848.9905	$T_3^{(5)}$	795.7333
		$T_1^{(6)}$	1734.331	$T_2^{(6)}$	849.3586	$T_3^{(6)}$	796.1513
		$T_1^{(7)}$	1731.754	$T_2^{(7)}$	849.0362	$T_3^{(7)}$	795.7851
		$T_1^{(8)}$	1732.709	$T_2^{(8)}$	849.156	$T_3^{(8)}$	795.9211
		$T_1^{(9)}$	1731.266	$T_2^{(9)}$	848.9749	$T_3^{(9)}$	795.7156
		$T_1^{(10)}$	**1735.935**	$T_2^{(10)}$	**849.558**	$T_3^{(10)}$	**796.378**
		$T_1^{(1)}$	1725.217	$T_2^{(1)}$	848.1967	$T_3^{(1)}$	794.835
		$T_1^{(2)}$	1725.38	$T_2^{(2)}$	848.218	$T_3^{(2)}$	794.8591
		$T_1^{(3)}$	1725.263	$T_2^{(3)}$	848.2027	$T_3^{(3)}$	794.8418
		$T_1^{(4)}$	1725.246	$T_2^{(4)}$	848.2005	$T_3^{(4)}$	794.8393
0.6	1	$T_1^{(5)}$	1725.232	$T_2^{(5)}$	848.1987	$T_3^{(5)}$	794.8373
		$T_1^{(6)}$	1725.396	$T_2^{(6)}$	848.22	$T_3^{(6)}$	794.8614
		$T_1^{(7)}$	1725.249	$T_2^{(7)}$	848.2008	$T_3^{(7)}$	794.8397
		$T_1^{(8)}$	1725.297	$T_2^{(8)}$	848.2072	$T_3^{(8)}$	794.8468
		$T_1^{(9)}$	1725.227	$T_2^{(9)}$	848.198	$T_3^{(9)}$	794.8365
		$T_1^{(10)}$	**1725.515**	$T_2^{(10)}$	**848.2356**	$T_3^{(10)}$	**794.879**

Table 11: PRE of the existing and proposed class of estimators of Population-IV at $\rho_{yx}^{*} = 0.7$.

		Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
		$\widehat{v}_1$	100	$\widehat{v}_2$	100	$\widehat{v}_3$	100
		$t_{d_1} = t_{g_1} = t_{k_1}$	1709.962	$t_{d_2} = t_{g_2} = t_{k_2}$	860.7556	$t_{d_3} = t_{g_3} = t_{k_3}$	930.6073
	α	T_{π_1}	1710.555	T_{π_2}	861.0686	T_{π_3}	930.9441
		T_{b_1}	1710.61	T_{b_2}	861.0818	T_{b_3}	930.9601
		T_{μ_1}	1709.108	T_{μ_2}	861.0818	T_{μ_3}	930.9601
		$T_1^{(1)}$	1741.892	$T_2^{(1)}$	865.5358	$T_3^{(1)}$	936.7012
		$T_1^{(2)}$	1745.38	$T_2^{(2)}$	865.9785	$T_3^{(2)}$	937.2822
ρ_{yx}^{*}		$T_1^{(3)}$	1743.048	$T_2^{(3)}$	865.683	$T_3^{(3)}$	936.8943
		$T_1^{(4)}$	1743.048	$T_2^{(4)}$	865.6478	$T_3^{(4)}$	936.8481
	0	$T_1^{(5)}$	1742.295	$T_2^{(5)}$	865.5872	$T_3^{(5)}$	936.7686
		$T_1^{(6)}$	1745.647	$T_2^{(6)}$	866.0122	$T_3^{(6)}$	937.3265
		$T_1^{(7)}$	1742.708	$T_2^{(7)}$	865.6568	$T_3^{(7)}$	936.8599
		$T_1^{(8)}$	1743.53	$T_2^{(8)}$	865.7443	$T_3^{(8)}$	936.9748
		$T_1^{(9)}$	1742.197	$T_2^{(9)}$	865.5748	$T_3^{(9)}$	936.7523
		$T_1^{(10)}$	**1746.752**	$T_2^{(10)}$	**866.1515**	$T_3^{(10)}$	**937.5095**

Table 11: Continued.

		Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
0.7	1	$T_1^{(1)}$	1741.286	$T_2^{(1)}$	865.4545	$T_3^{(1)}$	936.5867
		$T_1^{(2)}$	1741.338	$T_2^{(2)}$	865.4545	$T_3^{(2)}$	936.596
		$T_1^{(3)}$	1741.294	$T_2^{(3)}$	865.4485	$T_3^{(3)}$	936.5883
		$T_1^{(4)}$	1741.292	$T_2^{(4)}$	865.4482	$T_3^{(4)}$	936.5878
		$T_1^{(5)}$	1741.288	$T_2^{(5)}$	865.4477	$T_3^{(5)}$	936.5871
		$T_1^{(6)}$	1741.346	$T_2^{(6)}$	865.4555	$T_3^{(6)}$	936.5973
		$T_1^{(7)}$	1741.292	$T_2^{(7)}$	865.4482	$T_3^{(7)}$	936.5879
		$T_1^{(8)}$	1741.3	$T_2^{(8)}$	865.4494	$T_3^{(8)}$	936.5893
		$T_1^{(9)}$	1741.288	$T_2^{(9)}$	865.4476	$T_3^{(9)}$	936.587
		$T_1^{(10)}$	**1741.383**	$T_2^{(10)}$	**865.4606**	$T_3^{(10)}$	**936.6037**

Table 12: PRE of the existing and proposed class of estimators of Population-IV at $\rho^*_{yx} = 0.8$.

		Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
		$\hat{v}_1$	100	$\hat{v}_2$	100	$\hat{v}_3$	100
		$t_{d_1} = t_{g_1} = t_{k_1}$	1765.66	$t_{d_2} = t_{g_2} = t_{k_2}$	876.4591	$t_{d_3} = t_{g_3} = t_{k_3}$	1111.222
	α	T_{π_1}	1766.712	T_{π_2}	876.8572	T_{π_3}	1111.742
		T_{b_1}	1766.253	T_{b_2}	876.7577	T_{b_3}	1111.6
		T_{μ_1}	1682.482	T_{μ_2}	876.7577	T_{μ_3}	1111.6
		$T_1^{(1)}$	1801.633	$T_2^{(1)}$	881.5843	$T_3^{(1)}$	1118.923
		$T_1^{(2)}$	1805.848	$T_2^{(2)}$	882.0978	$T_3^{(2)}$	1119.716
ρ^*_{yx}		$T_1^{(3)}$	1803.024	$T_2^{(3)}$	881.7543	$T_3^{(3)}$	1119.185
		$T_1^{(4)}$	1803.024	$T_2^{(4)}$	881.7318	$T_3^{(4)}$	1119.151
		$T_1^{(5)}$	1802.117	$T_2^{(5)}$	881.6435	$T_3^{(5)}$	1119.014
	0	$T_1^{(6)}$	1806.172	$T_2^{(6)}$	882.1371	$T_3^{(6)}$	1119.777
		$T_1^{(7)}$	1802.937	$T_2^{(7)}$	881.7437	$T_3^{(7)}$	1119.169
		$T_1^{(8)}$	1803.364	$T_2^{(8)}$	881.7958	$T_3^{(8)}$	1119.249
		$T_1^{(9)}$	1802.052	$T_2^{(9)}$	881.6356	$T_3^{(9)}$	1119.002
		$T_1^{(10)}$	**1806.829**	$T_2^{(10)}$	**882.2167**	$T_3^{(10)}$	**1119.9**
0.8	1	$T_1^{(1)}$	**1808.727**	$T_2^{(1)}$	**882.432**	$T_3^{(1)}$	**1120.235**
		$T_1^{(2)}$	1808.596	$T_2^{(2)}$	882.417	$T_3^{(2)}$	1120.211
		$T_1^{(3)}$	1808.672	$T_2^{(3)}$	882.4256	$T_3^{(3)}$	1120.225
		$T_1^{(4)}$	1808.678	$T_2^{(4)}$	882.4264	$T_3^{(4)}$	1120.226
		$T_1^{(5)}$	1808.706	$T_2^{(5)}$	882.4296	$T_3^{(5)}$	1120.231
		$T_1^{(6)}$	1808.591	$T_2^{(6)}$	882.4164	$T_3^{(6)}$	1120.21
		$T_1^{(7)}$	1808.675	$T_2^{(7)}$	882.426	$T_3^{(7)}$	1120.225
		$T_1^{(8)}$	1808.66	$T_2^{(8)}$	882.4243	$T_3^{(8)}$	1120.223
		$T_1^{(9)}$	1808.709	$T_2^{(9)}$	882.4299	$T_3^{(9)}$	1120.231
		$T_1^{(10)}$	1808.581	$T_2^{(10)}$	882.4153	$T_3^{(10)}$	1120.209

Table 13: PRE of the existing and proposed class of estimators of Population-IV at $\rho^*_{yx} = 0.9$.

		Strategy I		Strategy II		Strategy III	
		Estimators	PRE	Estimators	PRE	Estimators	PRE
		$\hat{v}_1$	100	$\hat{v}_2$	100	$\hat{v}_3$	100
		$t_{d_1} = t_{g_1} = t_{k_1}$	1888.579	$t_{d_2} = t_{g_2} = t_{k_2}$	882.6787	$t_{d_3} = t_{g_3} = t_{k_3}$	1334.312
	α	T_{π_1}	1890.373	T_{π_2}	883.1826	T_{π_1}	1335.119
		T_{b_1}	1889.118	T_{b_2}	882.9509	T_{b_3}	1334.72
		T_{μ_1}	1467.447	T_{μ_2}	882.9509	T_{μ_3}	1334.72
		$T_1^{(1)}$	1931.552	$T_2^{(1)}$	888.0508	$T_3^{(1)}$	1344.007
		$T_1^{(2)}$	1937.051	$T_2^{(2)}$	888.6385	$T_3^{(2)}$	1345.097
ρ^*_{yx}		$T_1^{(3)}$	1933.354	$T_2^{(3)}$	888.2441	$T_3^{(3)}$	1344.365
		$T_1^{(4)}$	1933.354	$T_2^{(4)}$	888.2392	$T_3^{(4)}$	1344.356
		$T_1^{(5)}$	1932.176	$T_2^{(5)}$	888.1178	$T_3^{(5)}$	1344.131
	0	$T_1^{(6)}$	1937.479	$T_2^{(6)}$	888.6841	$T_3^{(6)}$	1345.181
		$T_1^{(7)}$	1933.45	$T_2^{(7)}$	888.2545	$T_3^{(7)}$	1344.384
		$T_1^{(8)}$	1933.55	$T_2^{(8)}$	888.2652	$T_3^{(8)}$	1344.404
		$T_1^{(9)}$	1932.16	$T_2^{(9)}$	888.1161	$T_3^{(9)}$	1344.128
		$T_1^{(10)}$	**1937.627**	$T_2^{(10)}$	**888.6998**	$T_3^{(10)}$	**1345.21**
		$T_1^{(1)}$	**1951.719**	$T_2^{(1)}$	**890.1608**	$T_3^{(1)}$	**1347.929**
		$T_1^{(2)}$	1951.249	$T_2^{(2)}$	890.1131	$T_3^{(2)}$	1347.841
		$T_1^{(3)}$	1951.548	$T_2^{(3)}$	890.1434	$T_3^{(3)}$	1347.897
		$T_1^{(4)}$	1951.552	$T_2^{(4)}$	890.1439	$T_3^{(4)}$	1347.898
0.9	1	$T_1^{(5)}$	1951.658	$T_2^{(5)}$	890.1546	$T_3^{(5)}$	1347.918
		$T_1^{(6)}$	1951.219	$T_2^{(6)}$	890.11	$T_3^{(6)}$	1347.835
		$T_1^{(7)}$	1951.539	$T_2^{(7)}$	890.1426	$T_3^{(7)}$	1347.895
		$T_1^{(8)}$	1951.531	$T_2^{(8)}$	890.1417	$T_3^{(8)}$	1347.894
		$T_1^{(9)}$	1951.659	$T_2^{(9)}$	890.1547	$T_3^{(9)}$	1347.918
		$T_1^{(10)}$	1951.209	$T_2^{(10)}$	890.109	$T_3^{(10)}$	1347.833

Interpretation of the results:
From Tables 6–13, we report that

(i) The generalized class of exponential-type estimators $T_i\,(i = 1, 2, 3)$ perform better than the usual estimators in the respective strategies

(ii) The proposed optimal estimators achieve a higher gain in terms of percentage relative efficiencies over the existing optimal estimators

(iii) Under strategy I, the estimators provide more efficient results as compared to strategy II and strategy III

8. Conclusion

In our study, we have proposed a generalized class of exponential-type estimators for estimating a finite population variance under random nonresponse using an auxiliary variable. The properties of the proposed classes of estimators have been derived up to the first order of approximation. Based on empirical and simulation studies, the proposed generalized class of estimators is more precise than their existing counterparts. Our study reveals the decrease in mean square error with increased sample size. Thus, the proposed generalized class of estimators is recommended for survey practitioners as it might expand the odds of acquiring progressively efficient results of population variance under random nonresponse conditions. This study can also be extended to incorporate the dual use of auxiliary variables to reduce the survey nonrespondents drastically by using fuzzy statistics.

Appendix

A. Outline of the Derivation of Proposed Strategy I in Section 4

It is possible to write the proposed estimator T_1 under the error transformation as follows:

$$T_1 = S_y^2(1+e_o)\left[t_1(1+e_1)^{-1} + t_2(1+e_1)\right]\left[\alpha\exp\left(\frac{-aS_x^2e_1}{2\left(aS_x^2+b\right)+aS_x^2e_1}\right) + (1-\alpha)\exp\left(\frac{aS_x^2e_1}{2\left(aS_x^2+b\right)+aS_x^2e_1}\right)\right]. \quad (A.1)$$

Equation (A.1) error term is binomially expanded up to order two and can be written as follows:

$$T_1 - S_y^2 = S_y^2\left[t_1\left\{1 - e_1(1-\beta_1+2\alpha\beta_1) + e_1^2\left(1-\beta_1+2\alpha\beta_1 - \frac{\beta_1^2}{2} + 2\alpha\beta_1^2\right) + e_o - e_oe_1(1-\beta_1+2\alpha\beta_1)\right\}\right.$$
$$\left. + + t_2\left\{1 + e_1(1+\beta_1-2\alpha\beta_1) + e_1^2\left(\beta_1 - 2\alpha\beta_1 - \frac{\beta_1^2}{2} + 2\alpha\beta_1^2\right) + e_o + e_oe_1(1+\beta_1-2\alpha\beta_1)\right\} - 1\right]. \quad (A.2)$$

Now, applying the expectation on both sides of equation (A.2) we get the expression of bias of T_1.

Squaring both sides of equation (A.2) by considering the error term up to order two, we get

$$\left(T_1 - S_y^2\right)^2 = S_y^4\left[1 - 2t_1\left\{1 + e_1^2\left(1-\beta_1+2\alpha\beta_1 - 0.5\beta_1^2 + 2\alpha\beta_1^2\right) - e_oe_1(1-\beta_1+2\alpha\beta_1)\right\} + t_1^2\left\{1 + e_o^2 + e_1^2\right.\right.$$
$$\cdot\left(3 - 4\beta_1 + 8\alpha\beta_1 + 4\alpha^2\beta_1^2\right)\big) - 4e_oe_1(1-\beta_1+2\alpha\beta_1)\big\} - 2t_2\left\{1 + e_1^2\left(\beta_1 - 2\alpha\beta_1 - 0.5\beta_1^2 + 2\alpha\beta_1^2\right)\right.$$
$$\left. + e_oe_1(1+\beta_1-2\alpha\beta_1)\right\} + t_2^2\left\{1 + e_o^2 + e_1^2\left(1 + 4\beta_1 - 8\alpha\beta_1 + 4\alpha^2\beta_1^2\right) + 4e_oe_1(1+\beta_1-2\alpha\beta_1)\right\}$$
$$\left. + 2t_1t_2\left\{1 + e_o^2 + 4\alpha^2\beta_1^2e_1^2 + 4\beta_1e_oe_1(1-2\alpha)\right\}\right]. \quad (A.3)$$

Now, taking expectation on both sides of the equation (A.3), we get the MSE of the estimator as

$$\mathrm{MSE}(T_1) = S_y^4\left[1 - 2t_1\left\{1 + f_1\left((\lambda_{04}-1)\left(1-\beta_1+2\alpha\beta_1-0.5\beta_1^2+2\alpha\beta_1^2\right) - (\lambda_{22}-1)(1-\beta_1+2\alpha\beta_1)\right)\right\}\right.$$
$$+ t_1^2\left\{1 + f_1\left((\lambda_{40}-1) + (\lambda_{04}-1)\left(3 - 4\beta_1 + 8\alpha\beta_1 + 4\alpha^2\beta_1^2\right) - 4(\lambda_{22}-1)(1-\beta_1+2\alpha\beta_1)\right)\right.$$
$$- 2t_2\left\{1 + f_1\left((\lambda_{04}-1)\left(\beta_1 - 2\alpha\beta_1 - 0.5\beta_1^2 + 2\alpha\beta_1^2\right) + (\lambda_{22}-1)\left((1+\beta_1-2\alpha\beta_1)\right)\right)\right\}$$
$$+ t_2^2\left\{1 + f_1\left((\lambda_{40}-1)\left(1 + 4\beta_1 - 8\alpha\beta_1 + 4\alpha^2\beta_1^2\right) + 4(\lambda_{22}-1)(1+\beta_1-2\alpha\beta_1)\right)\right\}$$
$$\left. + 2t_1t_2 1 + f_1\left((\lambda_{40}-1) + 4\alpha^2\beta_1^2(\lambda_{04}-1) + 4\beta_1(\lambda_{22}-1)(1-2\alpha)\right)\right]. \quad (A.4)$$

The optimum value of t_1 and t_2 can be obtained by differentiating partially equation (A.4) w.r.t. t_1 and t_2 and then equating it to zero to obtain the minimum MSE of T_1.

Data Availability

All relevant data and its supporting information files are included within the paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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