Hindawi
Complexity
Volume 2023, Article ID 8713812, 12 pages
https://doi.org/10.1155/2023/8713812

Research Article

An Alternative Statistical Model to Analysis Pearl Millet (Bajra) Yield in Province Punjab and Pakistan

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Received 11 June 2022; Revised 27 August 2022; Accepted 5 April 2023; Published 28 April 2023

Academic Editor: Teddy Craciunescu

Background. A country's agriculture reflects a backbone and performs a vital part in the betterment of the economy and individuals. Facts and figures of the agriculture sector offer a solid foundation and factual pathway intended for upcoming decisions in favor of a country. Accordingly, the probability models have a more significant influence not only in reliability engineering, hydrology, ecology, and medicine but also in agriculture sciences. *Objective*. The primary objective of this study is to propose a reliable and efficient model for pearl millet yield analysis, thereby empowering decision-makers to make informed decisions about their farming practices. With the successful implementation of this model, farmers can potentially increase their pearl millet yield, leading to higher incomes and improved livelihoods for the rural population of Pakistan. *Model*. This study proposes a novel probability model, namely, the alpha transformed odd exponential power function (ATOE-PF) distribution, for analyzing pearl millet yield in Punjab, Pakistan. *Data*. For data collection, two secondary data sets are explored that are electronically available on the site of the Directorate of Agriculture (Economics and Marketing) Punjab, Lahore, Pakistan. *Results*. The maximum likelihood estimation technique is used for estimating the model parameters. For the selection of a better fit model, we follow some accredited goodness of fit tests. The efficiency and applicability of the ATOE-PF distribution are discussed over the province of Punjab (with RMSE = 4.9176) and Pakistan (with RMSE = 4.5849). Better estimates and closest fit to data among the well-established neighboring models offer robust evidence in support of ATOE-PF distribution as well.

1. Introduction

Being an inhabitant of the agricultural country of Pakistan, our masses' primary source of income relies on agriculture. It has a dynamic role in developing the country's foreign exchange, economic growth, and employment. Over the last 40 years, it has had an outstanding contribution to the development of Pakistan's economy [1]. 65% fluctuating share of Pakistan's population, 18.9% gross domestic production (GDP), and

42.3% of the labor force ultimately dependent on agriculture [2]. The total land area of Pakistan is 196.72 million acres, and 66.97 million acres are harvested, along with 20.51 million acres not harvested [3]. Reference [4] categorized Pakistan's crops into food (wheat and rice) and cash food (cotton, maize, sugarcane), as both the crops have a 6.5% contribution to Pakistan's GDP.

One of the oldest cultivated food a pearl millet, which the locals call Bajra. It is a fifth-ranked crop in Pakistan after sorghum, maize, rice, and wheat. This crop is significant for fodder and grain, along with high nutritional contents for poultry and livestock. From 2010 to 2011, this crop yielded 346 thousand tons with a grown area of 548 thousand hectares. However, it was quite an impressive increase (by 18%) as compared to 2009-2010 production [5]. Worldwide, pearl millet's cultivation area is 31 million hectares [6], though, in Pakistan, 0.50 million hectares area along with 0.33 million tones production [7]. Pearl millet's low yield in Pakistan incorporates many factors, including nonstandard crop, inappropriate time of seeding, fluctuating weather intimidations, competitor cereals, and watering issues [8]. Reference [9] explored it as the feeding of the pet birds. It is expected that if Pakistan imports 61,000 tons of pearl millet by 2030, it will be considered the second leading importer country after China [10].

1.1. Probability Models Used for Different Field Crops. Several statistical techniques to model crop yield have been developed and discussed in the past. For this, one can extend his knowledge by reading from [11–32] and many others.

2. Materials and Methods

2.1. Punjab and Pakistan Area and Production. Crop pearl millet has a very high potential of growing with dry heat and drought tolerance along with the low rainfall area (less than 350 mm) circumstances. Consequently, Sindh (Sanghar, Hyderabad, Nawabshah, Kairpur, and Dadu); Punjab (Gujranwala, Bahawalnagar, Rawalpindi, Gujrat, Chakwal, Mianwali, and Attock); Balochistan (Sibbi, Lorali, and Khuzdar); and NWFP (Bannu, D. I. Khan, and Karak) are considered the most suitable and favorable districts (cities) for appropriate cultivation.

Table 1 provides valuable information on the coordinates and region of Pakistan and Punjab. It is a useful resource for researchers and other stakeholders who are interested in understanding the geography and location of the region, and can be used for various analytical and research purposes.

Figure 1: A graphic representation of the area and pearl millet output in Punjab and Pakistan. The figure uses a map of the region along with data on pearl millet production to provide an easy-to-understand overview of the cultivation of pearl millet in this area.

Figure 2: An illustration of the ultimate shape of the pearl millet crop. This figure provides a clear visual reference for the physical appearance of the crop, which can be useful for those who are not familiar with it.

Table 1: Pakistan and Punjab geography condition.

Coordinates	Region
Pakistan 30.3753°N, 72.7097°E	South Asia
Punjab 31.17040 N, 72.70970 E	Pakistan

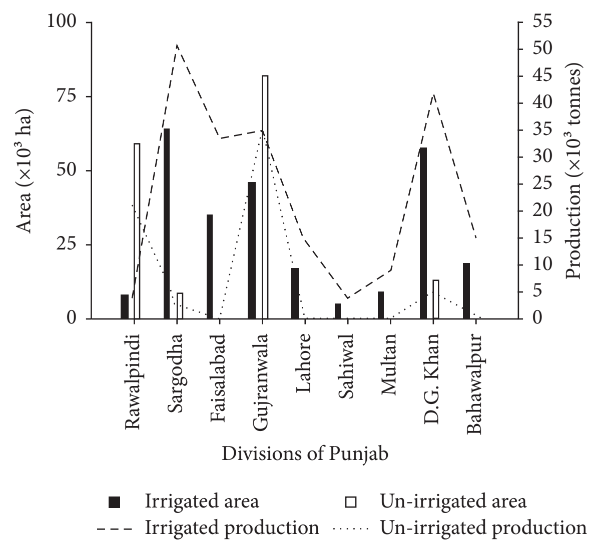
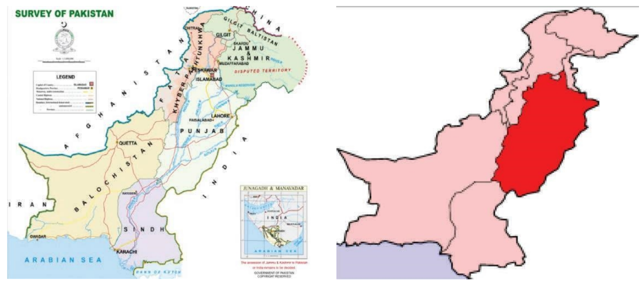


Figure 1: Area and production of pearl millet in Punjab and Pakistan.

Figure 2: Pearl millet picture.

Figure 3 includes two panels; the left panel displays a map of Pakistan, while the right panel displays a map of the Punjab province in Pakistan. The map of Punjab shows the major cities in the province, as well as the locations of pearl millet farms, providing valuable information on the geographic distribution of pearl millet cultivation in the region. The use of a map in this figure helps to provide a clear and visual representation of the information, making it easier for the audience to understand the distribution of pearl millet cultivation in the region.

2.2. Pakistan Climate Conditions. Pakistan experiences a significant amount of climatic variability. Despite the fact the summer months of April to September are fairly nice, the



(a) (b)

Figure 3: Map of Pakistan (a) map of Punjab (b)

winter is brutally chilly in the high mountains in the north and north west. The Indus Valley's plains experience sweltering heat in the summer and freezing conditions in the winter. The southern coastline region experiences a mild climate. Rainfall is generally insufficient. The lower Indus plain's northern regions receive an average annual rainfall of 16 centimeters, whereas the Himalayan area gets an average annual rainfall of 120 centimeters. Rainfall occurs late in the summer and has a monsoonal origin. Humidity is comparatively low because of the heavy rains and wide diurnal temperature fluctuation. High humidity only exists along the coastal strip.

2.3. Punjab Climate Conditions. In the majority of Punjab's regions, the winters are gloomy and frequently rainy. The weather turns springlike by mid-February and stays that way until mid-April, whenever the summer heat arrives. Punjab is expected to experience the start of the monsoon season around May, although the weather has been unpredictable since the early 1970s. Either as the spring monsoon missed the region or it rained so heavily that flooding occurred. It is very hot in June and July. Media sources indicate that the temperature exceeds 51°C and frequently publish stories about persons who have passed away from the heat, despite the fact that official measurements of the temperature seldom go over 46°C. When the temperature reportedly reached 54°C in Multan in June 1993, temperature records were smashed. The "bars" (monsoon season), which give comfort once it passes, interrupt the intense heat in August. Even though the hottest portion of the summer is passed, colder temperatures will not arrive until late October. One of the most frigid winters in the province's recent history dates back more than 70 years. Temperatures in the Punjab area average from −2° to 45°C; However they may get as high as 50°C (122°F) in the summer and as low as −10°C in the winter. Punjab experiences the following three distinct seasons:

(1) Hot weather (April to June), with temperatures reaching 123 degrees Fahrenheit (51 degrees Celsius).

(2) July to September is the rainy season. Average rainfall per year ranges between 96 cm in the submountain region and 46 cm in the plains.

(3) From October to March, the weather can be cold, foggy, or mild. The temperature drops to 35.6 degrees Fahrenheit (2.0 degrees Celsius).

It should be noted that September through October is the ideal time to harvest the crop known as Bajra.

2.4. Climate Prerequisite. It may be sown at low soil temperatures before reaching 23°C. It germinates best in ideal conditions (25–30°C). The vapor pressure deficit (VPD) caused by the daily maximum temperature of 42°C during blooming directly reduces the pearl millet's ability to set seeds [33]. At 40–45°C (base temperature of 10°C), tillering starts with the main tillers regions of the world depend on precipitation, which typically ranges from 150 to 750 mm (350 mm). Because of its resilience to very hot and dry weather conditions is becoming increasingly important in developing climate-resilient agricultural systems under changing climatic scenarios [34]. The pearl millet requires between 300 and 350 mm of rainfall to thrive. It is important to note that the water requirement of a crop can vary depending on various factors such as soil type, climate, and cultivation practices. The Figure 4 presented in the chart should therefore be considered as general guidelines rather than exact values.

2.5. Data Collection. For this study, we consider secondary data sets. For this, the first data presents the average yield of Bajra in Punjab (1947-48 to 2017-18) (Per Acre/000 Tonnes), and the second data relates to the average yield (Per Acre/000 Tonnes) of Bajra in Pakistan (1947-48 to 2017-18). The datasets are obtained from the agricultural statistics of Pakistan and are available at the electronic address provided in Appendix.

2.6. Model Description. In this paper, we develop a novel two-parameter probability model that performs so well not only in reliability engineering, hydrology, ecology, and medical sciences but has a vital role in agriculture sciences as well. We refer to it as the alpha transformed odd exponential power function (ATOE-PF) distribution. The associated cumulative distribution function (CDF) corresponding to the probability density function (PDF) along with the

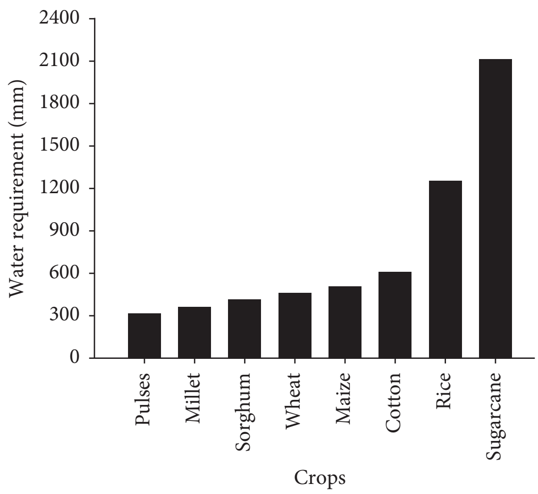


Figure 4: Water requirement of pearl millet in comparison with other crops.

quantile function is, respectively, given by the following equation:

$$
\begin{aligned}
F(x) &= \frac{\alpha^{e^{\left(1-(g_n/x)^{\beta}\right)}} - 1}{\alpha - 1},\\
f(x) &= \frac{(g_n)^{\beta}\beta\log\alpha}{\alpha - 1}x^{-(\beta+1)}e^{\left(1-(g_n/x)^{\beta}\right)}\alpha^{e^{\left(1-(g_n/x)^{\beta}\right)}},\\
x_q &= \frac{g_n}{\left[1-\log\left[(1/\log\alpha)\left[\log\left[1+q[\alpha-1]\right]\right]\right]\right]^{1/\beta}},
\end{aligned}
\tag{1}
$$

where $0 < x \leq g_n$ and $\alpha > 0, \alpha > 1, \beta > 0$, are two shape parameters.

Note that, the ATOE-PF distribution is one of the particular members of the ATOE-G class of distributions. Therefore, this paper uses ATOE-PF distribution as a modeling framework, and our ongoing project's advanced complementary mathematical and reliability measures are under-processed.

2.7. Parameter Estimation. We use the maximum likelihood estimation technique for the parameter estimation of the ATOE-PF distribution. For this, we suppose $X_1, X_2, \ldots, X_n$ be a random sample of size n taken from X, then the log-likelihood function $(\operatorname{Log} L)$ of X is given by the following equation:

$$
\operatorname{Log} L = \begin{bmatrix} n\beta\log(g_n) + n\log(\beta) + n\log[\log(\alpha)] - n\log(\alpha-1) - \\ (\beta+1)\sum_{i=1}^{n}\log(x_i) + \sum_{i=1}^{n}\left[1-\left(\frac{g_n}{x_i}\right)^{\beta}\right] + \log(\alpha)\sum_{i=1}^{n}e^{\left[1-\left(\frac{g_n}{x_i}\right)^{\beta}\right]}\end{bmatrix}.
\tag{2}
$$

The partial derivatives of $\operatorname{Log} L$ for the parameters α and β are given by, respectively,

$$
\begin{aligned}
\frac{\partial l}{\partial\alpha} &= \frac{n\alpha}{\log(\alpha)} - \frac{n}{(\alpha-1)} + \frac{1}{\alpha}\sum_{i=1}^{n}e^{\left[1-(g_n/x_i)^{\beta}\right]}, \text{ and}\\
\frac{\partial l}{\partial\beta} &= n\log(x_n) + \frac{n}{\beta} - \sum_{i=1}^{n}\partial - \sum_{i=1}^{n}\left(\frac{g_n}{x_i}\right)^{\beta}log\left(\frac{g_n}{x_i}\right) + \left[e^{\left[1-(g_n/x_i)^{\beta}\right]}\right]\left(\frac{g_n}{x_i}\right)^{\beta}\log\left(\frac{g_n}{x_i}\right).
\end{aligned}
\tag{3}
$$

The ML estimates $(\widehat{\phi} = \widehat{\alpha}|^{\mathrm{MLE}}, \widehat{\beta}|^{\mathrm{MLE}})$ of the ATOE-PF distribution are derived by maximizing (2) or by solving the above nonlinear equations simultaneously. The following part has a detailed simulation with various parameter configurations to test the asymptotic capability of MLEs.

2.8. Simulation Study. The following algorithm discusses the performance of MLEs with the assistance of a simulation study:

Step-1: a random sample $x_1, x_2, x_3, \ldots, x_n$ of sizes $n = 100$, 150, 200, 250, 300, 350, 400, 450, and 500 are generated from $Q(q)$.

Step-2: the required results are obtained based on the different combinations of the model parameters for $g_n = 2$, placed in S-I $(\alpha = 1.9, \beta = 1.5)$, S-II $(\alpha = 1.1, \beta = 2.5)$, S-III $(\alpha = 1.5, \beta = 1.5)$, S-IV $(\alpha = 1.2, \beta = 1.9)$, S-V $(\alpha = 1.3, \beta = 1.7)$, S-VI $(\alpha = 1.7, \beta = 3.9)$, S-VII $(\alpha = 1.15, \beta = 4.75)$, S-VIII $(\alpha = 1.25, \beta = 7.75)$, and S-IX $(\alpha = 1.55, \beta = 5.95)$

Step-3: average estimate (AE), bias, mean square error (MSE), and variance (Var) are presented in Tables 2–4.

Step–4: each sample is replicated $N = 1000$ times.

Step-5: gradual decrease in AE(s), bias(es), MSE(s), and Var(s) with increases in the sample size is observed.

Step-6: finally, the estimates in Tables 2–4 help us specify that the method of maximum likelihood works consistently for the ATOE-PF distribution.

Note that, Figure 5 is a useful visual representation of the density function curves for various choices of model parameters for simulated data. The figure provides researchers with valuable insights into the impact of different parameter values on the shape of the distribution, enabling them to make more informed modeling decisions.

$$AE(\widehat{\Xi}) = \frac{1}{N}\sum_{i=1}^{N}\widehat{\Xi}_i \ \mathrm{Bias}(\widehat{\Xi}) = \frac{1}{N}\sum_{i=1}^{N}\left(\widehat{\Xi}_i - \Xi\right),$$

$$\mathrm{MSE}(\widehat{\Xi}) = \frac{1}{N}\sum_{i=1}^{N}\left(\widehat{\Xi}_i - \Xi\right)^2, \tag{4}$$

$$\mathrm{Var}(\widehat{\Xi}) = \frac{1}{N}\sum_{i=1}^{N}\left(\Xi - \overline{\Xi}_i\right)^2.$$

3. Results and Discussions

Now, we report the application of the ATOE-PF distribution. For this, we focus on the agricultural sector and engage two suitable datasets. The ATOE-PF distribution is compared with well-known competitive models. The CDFs of competitive models are listed in Table 5. The parameter estimates and standard errors are presented in Tables 6 and 7 for both datasets, respectively. Some typical results from descriptive statistics for both datasets are shown in Tables 8 and 9. These descriptive statistics are minimum value, 1st quartile, mean, median, mode, standard deviation (SD), 3rd quartile, maximum value, 90%, 95%, and 99% confidence intervals.

The goodness-of-fit statistics for the ATOE-PF distribution and competing models are presented in Tables 10 and 11. A better fit model is one with the criteria of a minimum value of Anderson–Darling (AD), Cramer-von Mises (CVM), root mean square error (RMSE), and Kolmogorov–Smirnov (KS) with a higher P-value. Please note that a comprehensive list of standard measurement units and corresponding abbreviations can be found in Table 12 of this document.

The agriculture sector plays a crucial role in the economy of a country, and the ability to accurately predict crop yields is of utmost importance. In order to aid decision-makers in the farming industry, a new probability model was developed that is capable of accurately modeling agriculture data. This study utilized secondary data on pearl millet (Bajra) yields in Punjab Province, Pakistan and compared the alpha transformed odd exponential power function (ATOE-PF) distribution to its well-established rivals using various goodness of fit tests such as KS (P-value), AD, and CVM. The ATOE-PF distribution showed a better fit for the average yield of pearl millet (Bajra) in Punjab and Pakistan than any of its competitors. The P value (KS) was higher for the ATOE-PF distribution, indicating that it meets the minimal statistical value requirement for a better fit model.

Table 2: Bias, mean square error, variance, and average estimate.

n	Est	S-I		S-II		S-III	
		$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\alpha}$	$\widehat{\beta}$
100	Bias	0.1591	0.0349	0.2141	0.0136	0.1573	0.0212
	MSE	1.6324	0.0237	0.3585	0.0376	0.8554	0.0178
	Var	1.6071	0.0225	0.3126	0.0374	0.8306	0.0173
	AE	2.0591	1.5349	1.3141	2.5136	1.6573	1.5212
150	Bias	0.0598	0.0288	0.1394	−0.0038	0.0735	0.0192
	MSE	0.9004	0.0168	0.1808	0.0221	0.4627f	0.0127
	Var	0.8968	0.0159	0.1613	0.0221	0.4573	0.0123
	AE	1.9598	1.5288	1.2394	2.4961	1.5735	1.5192
200	Bias	0.0412	0.0227	0.1122	−0.0091	0.0496	0.0166
	MSE	0.5669	0.0134	0.1113	0.0175	0.2968	0.0105
	Var	0.5652	0.0129	0.0987	0.0174	0.2943	0.0103
	AE	1.9412	1.5227	1.2122	2.4908	1.5496	1.5165
250	Bias	0.0165	0.0196	0.0898	−0.0089	0.0269	0.0153
	MSE	0.4386	0.0108	0.0835	0.0143	0.2315	0.0089
	Var	0.4383	0.0105	0.0754	0.0143	0.2308	0.0087
	AE	1.9165	1.5196	1.1898	2.4911	1.5269	1.5152
300	Bias	0.0135	0.0183	0.0790	−0.0055	0.0213	0.0134
	MSE	0.3532	0.0088	0.0675	0.0123	0.1894	0.0068
	Var	0.3530	0.0085	0.0613	0.0122	0.1889	0.0067
	AE	1.9135	1.5183	1.1790	2.4944	1.5213	1.5134
350	Bias	0.0188	0.0148	0.0733	−0.0048	0.0226	0.0112
	MSE	0.3017	0.0075	0.0589	0.0108	0.1647	0.0059
	Var	0.3014	0.0073	0.0535	0.0108	0.1642	0.0058
	AE	1.9188	1.5148	1.1733	2.4951	1.5226	1.5112
400	Bias	0.0116	0.0142	0.0679	−0.0022	0.0159	0.0110
	MSE	0.2731	0.0066	0.0525	0.0098	0.1503	0.0053
	Var	0.2730	0.0064	0.0479	0.0098	0.1501	0.0052
	AE	1.9116	1.5142	1.1679	2.4977	1.5159	1.5110
450	Bias	−0.0004	0.0137	0.5781	0.0012	0.0051	0.0110
	MSE	0.2386	0.0059	0.0453	0.0091	0.1322	0.0047
	Var	0.2386	0.0057	0.0419	0.0091	0.1322	0.0046
	AE	1.8995	1.5137	1.1578	2.5012	1.5051	1.5110
500	Bias	0.0029	0.0109	0.0531	0.0024	0.0064	0.0087
	MSE	0.2247	0.0051	0.0434	0.0085	0.1248	0.0042
	Var	0.2247	0.0050	0.0405	0.0085	0.1248	0.0042
	AE	1.9029	1.5109	1.1531	2.5024	1.5064	1.5087

Table 3: Bias, mean square error, variance, and average estimate.

n	Est	S-IV $\widehat{\alpha}$	S-IV $\widehat{\beta}$	S-V $\widehat{\alpha}$	S-V $\widehat{\beta}$	S-VI $\widehat{\alpha}$	S-VI $\widehat{\beta}$
100	Bias	0.1932	0.0039	0.1763	0.0105	0.1536	0.0731
	MSE	0.4550	0.0203	0.5708	0.0188	1.2054	0.1398
	Var	0.4177	0.0203	0.5397	0.0187	1.1818	0.1345
	AE	1.3932	1.9039	1.4763	1.7105	1.8536	3.9731
150	Bias	0.1147	0.0035	0.0963	0.0109	0.0624	0.0627
	MSE	0.2349	0.0149	0.3008	0.0134	0.6633	0.1001
	Var	0.2217	0.0149	0.2915	0.0133	0.6594	0.0961
	AE	1.3147	1.9035	1.3963	1.7109	1.7624	3.9627
200	Bias	0.0538	0.0098	0.0690	0.0105	0.0424	0.0500
	MSE	0.0926	0.0087	0.1920	0.0112	0.4214	0.0804
	Var	0.0897	0.0086	0.1872	0.0110	0.4196	0.0779
	AE	1.2538	1.9098	1.3690	1.7105	1.7424	3.9500
250	Bias	0.0502	0.0080	0.0471	0.0112	0.0189	0.0463
	MSE	0.0811	0.0079	0.1480	0.0093	0.3280	0.0662
	Var	0.0785	0.0079	0.1458	0.0092	0.3276	0.0640
	AE	1.2502	1.9080	1.3471	1.7112	1.7189	3.9463
300	Bias	0.0444	0.0070	0.0371	0.0134	0.0154	0.0432
	MSE	0.0733	0.0069	0.1226	0.0083	0.2659	0.0545
	Var	0.0713	0.0069	0.1212	0.0082	0.2657	0.0527
	AE	1.2444	1.9070	1.3371	1.7134	1.7154	3.9432
350	Bias	0.0326	0.0055	0.0362	0.0082	0.0186	0.3610
	MSE	0.0645	0.0055	0.1069	0.0064	0.2295	0.0476
	Var	0.0634	0.0055	0.1056	0.0063	0.2291	0.0463
	AE	1.2326	1.9055	1.3362	1.7082	1.7186	3.9361
400	Bias	0.0035	0.0035	0.0287	0.0085	0.0120	0.0348
	MSE	0.0051	0.0051	0.0979	0.0057	0.2083	0.0423
	Var	0.0604	0.0051	0.0979	0.0057	0.2081	0.0411
	AE	1.2301	1.9035	1.3287	1.7085	1.7120	3.9348
450	Bias	0.0035	0.0035	0.0173	0.0086	0.0012	0.0331
	MSE	0.0051	0.0051	0.0866	0.0051	0.1819	0.0363
	Var	0.0604	0.0051	0.0863	0.0050	0.1819	0.0351
	AE	1.2301	1.9035	1.3173	1.7086	1.7012	3.9331
500	Bias	0.0035	0.0035	0.0174	0.0065	0.0032	0.0264
	MSE	0.0051	0.0051	0.0815	0.0047	0.1721	0.3221
	Var	0.0604	0.0051	0.0126	0.0046	0.1721	0.0315
	AE	1.2301	1.9035	1.3174	1.7065	1.7031	3.9264

Table 4: Bias, mean square error, variance, and average estimate.

n	Est	S-VII $\widehat{\alpha}$	S-VII $\widehat{\beta}$	S-VIII $\widehat{\alpha}$	S-VIII $\widehat{\beta}$	S-IX $\widehat{\alpha}$	S-IX $\widehat{\beta}$
100	Bias	0.2029	0.0131	0.1842	0.0434	0.1552	0.0932
	MSE	0.4038	0.1241	0.5102	0.3326	0.9364	2.8667
	Var	0.3626	0.1239	0.4763	0.3308	0.9123	0.2778
	AE	1.3529	4.7630	1.4342	7.7934	1.7052	6.0432
150	Bias	0.1262	−0.0010	0.1047	0.0409	0.0696	0.0834
	MSE	0.2064	0.0766	0.2666	0.2558	0.5094	0.2144
	Var	0.1904	0.0766	0.2556	0.2541	0.5045	0.2075
	AE	1.2762	4.7489	1.3547	7.7909	1.6196	6.0334
200	Bias	0.0987	−0.0015	0.0773	0.0329	0.0469	0.0657
	MSE	0.1283	0.0666	0.1689	0.2141	0.3262	0.1728
	Var	0.1185	0.0666	0.1630	0.2130	0.3240	0.1685
	AE	1.2487	4.7484	1.3273	7.7829	1.5969	6.0157

Table 4: Continued.

n	Est	S-VII $\widehat{\alpha}$	S-VII $\widehat{\beta}$	S-VIII $\widehat{\alpha}$	S-VIII $\widehat{\beta}$	S-IX $\widehat{\alpha}$	S-IX $\widehat{\beta}$
250	Bias	0.0761	0.0055	0.0551	0.0264	0.0241	0.0590
	MSE	0.0972	0.0561	0.1296	0.1675	0.2545	0.1394
	Var	0.0914	0.0561	0.1267	0.1669	0.2539	0.1359
	AE	1.2261	4.7555	1.3051	7.7764	1.5741	6.0090
300	Bias	0.0651	0.0148	0.0445	0.0361	0.0194	0.0567
	MSE	0.0793	0.0489	0.1071	0.1434	0.2074	0.1140
	Var	0.0751	0.0487	0.1051	0.1421	0.2070	0.1108
	AE	1.2151	4.7648	1.2945	7.7861	1.5669	6.0067
350	Bias	0.0605	0.0119	0.0421	0.0269	0.0211	0.0462
	MSE	0.0694	0.0449	0.0937	0.1249	0.1803	0.0972
	Var	0.0657	0.0448	0.0919	0.1242	0.1799	0.0950
	AE	1.2105	4.7619	1.2921	7.7769	1.5711	5.9962
400	Bias	0.0551	0.0088	0.0356	0.0294	0.0146	0.0465
	MSE	0.0622	0.0391	0.0852	0.1134	0.1640	0.0879
	Var	0.0591	0.0391	0.0839	0.1125	0.1638	0.8575
	AE	1.2051	4.7588	1.2856	7.7794	1.5646	5.9965
450	Bias	0.0439	0.0084	0.0236	0.0311	0.0038	0.0460
	MSE	0.0543	0.0333	0.0755	0.0967	0.1441	0.0772
	Var	0.0524	0.0332	0.0749	0.0958	0.1441	0.0750
	AE	1.1939	4.7584	1.2736	7.7810	1.5539	5.9960
500	Bias	0.0401	0.0026	0.0230	0.0235	0.0053	0.0371
	MSE	0.0519	0.0294	0.0713	0.0891	0.1361	0.0697
	Var	0.0503	0.0295	0.0707	0.0885	0.1361	0.0683
	AE	1.1900	4.7526	1.2730	7.7735	1.5553	5.9871

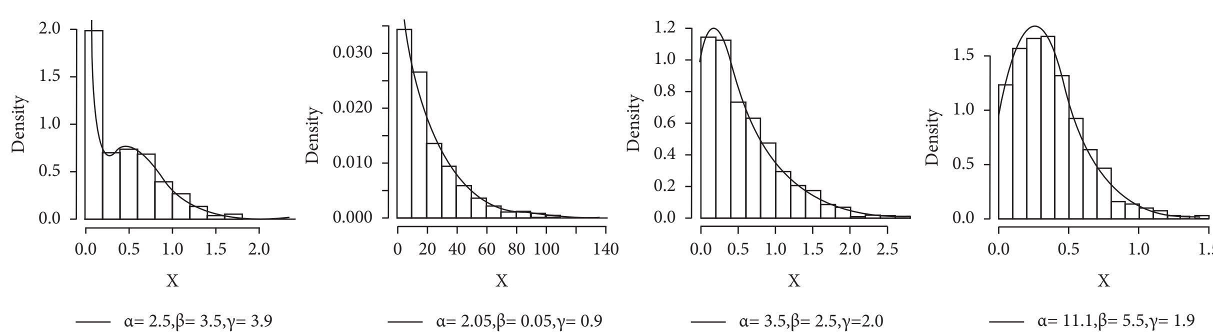


Figure 5: Density function curves for various choices of model parameters for simulated data.

Table 5: List of some competitive model's cumulative distribution functions.

Models	CDF's of model	Parameters	Support	Positions
HL-Exp	$P(x) = 1 - e^{-\alpha x}/1 + e^{-\alpha x}$	$\alpha > 0$	$0 < x < \infty$	Shape $= \alpha$
Exp	$P(x) = 1 - e^{-\alpha x}$	$\alpha > 0$	$0 < x < \infty$	Shape $= \alpha$
MO-Exp	$P(x) = 1 - \alpha e^{-x}/1 - (1-\alpha)e^{-x}$	$\alpha > 0$	$0 < x < \infty$	Scale $= \alpha$
NH-Exp	$P(x) = 1 - e^{1-(1+\alpha x)^{\beta}}$	$\alpha, \beta > 0$	$0 < x < \infty$	Scale $= \alpha$ Shape $= \beta$
Exp-Exp	$P(x) = e^{(1-e^{-\alpha x})} - 1/e - 1$	$\alpha > 0$	$0 < x < \infty$	Shape $= \alpha$
Alp-Exp	$P(x) = \alpha^{(1-e^{-\beta x})} - 1/\alpha - 1$	$\alpha, \beta > 0$	$0 < x < \infty$	Scale $= \alpha$ Shape $= \beta$
Pareto	$P(x) = 1 - (m_0/x)^{\alpha}$	$\alpha > 0$	$m_0 \leq x < \infty$	Shape $= \alpha$ Reflected $= m_0$
Gompertz	$P(x) = 1 - e^{-\alpha(e^{\beta x}-1)}$	$\alpha, \beta > 0$	$0 < x < \infty$	Shape $= \alpha$ Shape $= \beta$
Normal	$P(x) = \Phi(x - \alpha/\beta)$	$\alpha, \beta > 0$	$-\infty < x < \infty$	Location $= \alpha$ Scale $= \beta$

Table 5: Continued.

Models	CDF's of model	Parameters	Support	Positions
Burr-XII	$P(x) = 1 - (1 + x^{\alpha})^{-\beta}$	$\alpha, \beta > 0$	$0 < x < \infty$	Shape = α Shape = β
PF	$P(x) = (x/g_n)^{\alpha}$	$\alpha > 0$	$0 < x \leq g_n$	Shape = α

Table 6: Parameter estimates and standard errors for average yield (per acre) of Bajra in province Punjab.

Models	$\widehat{\alpha}$		$\widehat{\beta}$	
	Estimate	Std. error	Estimate	Std. error
ATOE-PF	0.0324	0.0216	2.8176	0.1534
Normal	5.2154	0.0939	0.7914	0.0664
Gompertz	0.0027	0.0005	1.0561	0.0336
MO-Exp	178.85	32.008	—	—
PF	2.9609	0.3514	—	—
Pareto	2.5048	0.2973	—	—
Alp-Exp	143.58	56.144	0.4184	0.0284
NH-Exp	0.0045	0.0020	33.936	14.896
HL-Exp	0.2947	0.0270	—	—
Exp-Exp	0.2583	0.0253	—	—
Exp	5.2164	0.6192	—	—
B-XII	6.4565	12.036	0.0944	0.1764

Table 7: Parameter estimates and standard errors for average yield (per acre) of Bajra in Pakistan.

Models	$\widehat{\alpha}$		$\widehat{\beta}$	
	Estimate	Std. error	Estimate	Std. error
ATOE-PF	0.0371	0.0244	2.4341	0.1348
Normal	4.8428	0.1018	0.8580	0.0720
Gompertz	0.0048	0.0015	1.0144	0.0550
MO-Exp	122.02	22.020	—	—
PF	2.5878	0.3071	—	—
Pareto	2.1107	0.2505	—	—
Alp-Exp	263.67	110.42	0.4724	0.0309
NH-Exp	0.0047	0.0023	34.591	16.662
HL-Exp	0.3168	0.0291	—	—
Exp-Exp	0.2779	0.0273	—	—
Exp	4.8434	0.5749	—	—
B-XII	7.5194	19.042	0.0851	0.2158

Table 8: Descriptive statistics for average yield (per acre) of Bajra in province Punjab.

Data	Minimum	1st quartile	Mean	Median	Mode	SD	3rd quartile	Maximum
	3.510	4.635	5.216	5.100	5.700	0.797	5.585	7.230
		Confidence interval			Skewness		Kurtosis	
Punjab	90%		(5.057, 5.373)					
	95%		(5.027, 5.404)		0.611		3.033	
	99%		(4.965, 5.466)					

The empirical fitted PDF, CDF, Probability-Probability, and box plots of the ATOE-PF distribution are presented in Figures 6 and 7, which visually demonstrate the model's adequacy. All numerical results and model estimates were obtained using the free statistical software R Studio version 1.2.5033 (cited therein) and its exclusive package AdequacyModel. This new probability model provides decision-makers in the farming industry with a reliable tool to aid in predicting crop yields. By utilizing the ATOE-PF distribution, farmers and related departments can begin implementing more effective predictive measures. The model's superiority over its competitors in accurately

Table 9: Descriptive statistics for average yield (per acre) of Bajra in Pakistan.

Data	Minimum	1st quartile	Mean	Median	Mode	SD	3rd quartile	Maximum
	3.020	4.305	4.843	4.780	6.380	0.863	5.075	7.020
		Confidence interval			Skewness		Kurtosis	
Pakistan	90%		(4.672, 5.014)					
	95%		(4.638, 5.047)		0.646		3.159	
	99%		(4.571, 5.114)					

Table 10: The goodness of fit statistics for average yield (per acre) of Bajra in province Punjab.

Models	CVM	AD	K-S	K-S (P-val)	RMSE
ATOE-PF	0.0763	0.5023	0.0766	0.7981	4.9176
Normal	0.1541	1.0181	0.0888	0.6305	4.9282
Gompertz	0.4668	2.8004	0.1615	0.0493	4.9946
MO-Exp	0.1379	0.9312	0.2264	0.0014	5.0594
PF	1.9546	10.6645	0.3287	0.0012	5.0473
Pareto	0.1194	0.8793	0.3497	0.0009	5.0903
Alp-Exp	0.0887	0.6193	0.3906	0.0111	5.1469
NH-Exp	0.1352	0.9010	0.5483	0.0015	5.1801
HL-Exp	0.0989	0.6814	0.5107	0.0080	5.1921
Exp-Exp	0.0945	0.6542	0.5072	0.0050	5.1957
Exp	0.0886	0.6172	0.5153	0.0009	5.2064
B-XII	0.0454	0.3479	0.5498	0.0010	5.1197

Table 11: The goodness of fit statistics for average yield (per acre) of Bajra in Pakistan data.

Models	CVM	AD	K-S	K-S (P-val)	RMSE
ATOE-PF	0.1940	1.0670	0.1209	0.2497	4.5849
Normal	0.2885	1.7279	0.1515	0.0768	4.5966
Gompertz	0.6920	3.8658	0.2004	0.0067	4.6576
MO-Exp	0.2556	1.5727	0.2068	0.0046	4.7069
PF	1.9931	10.7507	0.3356	0.0051	4.7037
Pareto	0.2897	1.6829	0.3545	0.0161	4.7515
Alp-Exp	0.1966	1.1906	0.3530	0.0111	4.7824
NH-Exp	0.2629	1.5708	0.5265	0.0150	4.8169
HL-Exp	0.2106	1.2712	0.4931	0.0115	4.8295
Exp-Exp	0.2048	1.2361	0.4906	0.0111	4.8333
Exp	0.1973	1.1891	0.5012	0.0109	4.8446
B-XII	0.1500	0.8773	0.5352	0.0011	4.9182

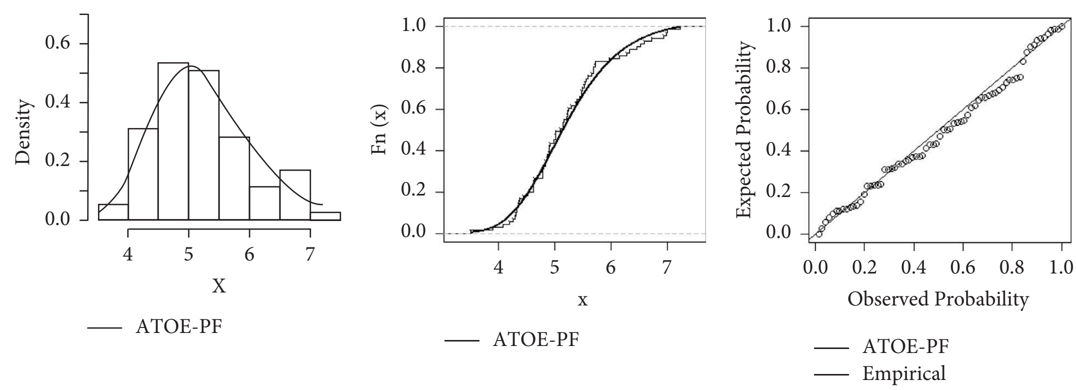
Table 12: List of standard measurement units and other abbreviations.

Full names	Symbol/description
Half logistic exponential	HL-Exp
Nadarajah-Haghighi exponential	NH-Exp
Marshall-Olkin exponential	MO-Exp
Alpha power exponential	Alp-Exp
Exponential-exponential	Exp-Exp
Power function	PF
Exponential	Exp
Per acre, thousand tonnes	Per acre/000 tonnes
Anderson-darling test	AD = a test to detect the departure of sample distribution from normality
Kolmogorov-Smirnov test	K-S = a test to detect the departure of CDF from empirical CDF
Cramer-von mises test	CVM = a test to compare two empirical CDFs
Root mean square error	RMSE = a measure to describe data around the best fit line
Mean square error	MSE = a risk function, which measures the discrepancy between the estimated and real values

Table 12: Continued.

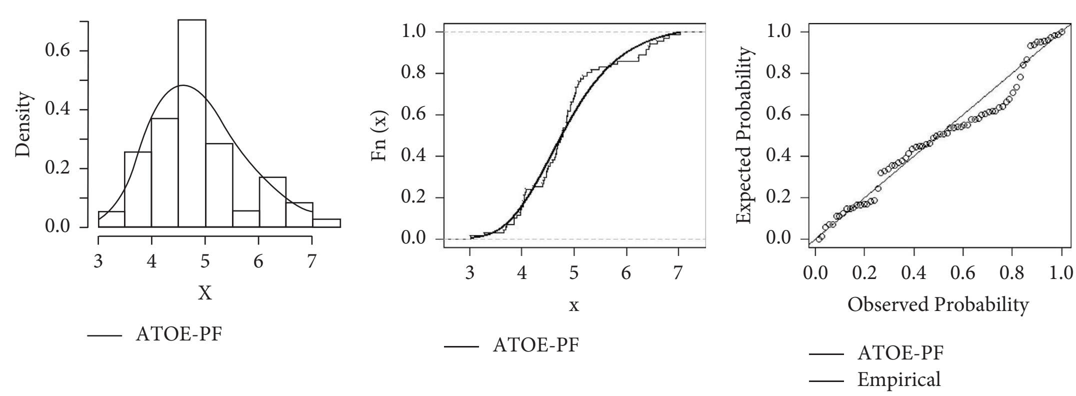
Full names	Symbol/description
Bias	Bias = the term “bias” refers to a consistent departure from the true value. It is the discrepancy between the parameter’s actual value and its intended value
Variance	Var = the term “variance” refers to measuring how widely apart a group of numbers is from another
Average estimate	AE = a point estimate of a mean of an unknown distribution

Electronic address of data set https://www.amis.pk/Agristatistics/Data/HTML%20Final/Bajra/Production.html.



(a) (b) (c)

Figure 6: Empirically fitted plots for an average yield of Bajra in Punjab, Pakistan.



(a) (b) (c)

Figure 7: Empirically fitted plots for an average yield of Bajra in Pakistan.

modeling agriculture data provides valuable information for agriculture bodies. In addition, the use of various goodness of fit tests ensures that the model provides an adequate fit. Overall, the ATOE-PF distribution presents a promising solution for researchers and practitioners in the agriculture sector.

4. Conclusions

In this work, a novel model called the alpha transformed odd exponential power function (ATOE-PF) distribution was established, and we introduced its PDF and CDF. A simulation study was carried out using the maximum likelihood estimation technique. To prove the superiority of the proposed model, we fitted two pearl millet datasets. The ATOE-PF distribution was considered the best fit model among the well-known rivals after passing the various goodness of fit tests. Referring to Tables 10 and 11, we found that the (ATOE-PF) distribution has the lowest K-S value and the highest *P* value, proving the ATOE-PF distribution’s superiority. The efficiency and applicability of the ATOE-PF distribution are discussed over the provinces of Punjab (with RMSE = 4.9176) and Pakistan (with RMSE = 4.5849). Furthermore, outperforming estimates made it more relevant and encouraging for pearl millet farm decision-makers and other agriculture agencies.

5. Future Directions

The proposed technique would hopefully be adopted by agriculture experts and concerned agencies and implemented on maize, soybeans, rice, sugarcane, cotton, moong, mash, and jowar for a more appropriate prediction and a respectable predicted yield. Also, we have another critical future work: the study of COVID-19 infections and the mortality rate of the infected. Another expansion will be the competing risk resulting from death, whether it is from the disease or another cause.

Appendix

The first data presents the average yield of Bajra in Punjab (1947-48 to 2017-18) (Per Acre/000 Tonnes). 4.79, 4.64, 4.84, 4.92, 3.90, 3.51, 4.80, 4.30, 4.26, 4.31, 4.09, 4.44, 4.39, 4.62, 5.02, 5.48, 4.90, 5.16, 4.88, 4.91, 5.22, 4.63, 4.84, 5.02, 5.16, 5.24, 5.10, 4.99, 5.25, 5.23, 5.54, 5.30, 5.40, 5.38, 5.45, 5.48, 5.60, 5.66, 5.53, 5.70, 4.62, 4.20, 4.34, 4.37, 4.36, 4.34, 4.53, 4.63, 4.79, 4.81, 4.87, 4.91, 5.03, 5.57, 5.17, 5.50, 5.72, 5.70, 5.73, 5.98, 6.15, 6.47, 6.28, 6.94, 6.99, 7.00, 6.54, 6.59, 6.34, 6.73, 7.23.

The second data relates to the average yield (Per Acre/000 Tonnes) of Bajra in Pakistan (1947-48 to 2017-18). 3.7, 3.65, 3.91, 4.02, 3.28, 3.02, 4.47, 3.98, 3.86, 3.97, 3.71, 3.86, 4.07, 4.08, 4.43, 4.94, 4.86, 4.88, 4.39, 4.41, 4.51, 4.47, 4.76, 4.78, 4.79, 5.03, 4.85, 4.93, 4.99, 4.85, 5.02, 4.88, 5.00, 5.33, 4.93, 5.08, 4.69, 4.74, 4.66, 4.63, 4.68, 3.99, 4.04, 4.04, 4.49, 4.22, 4.59, 4.54, 4.02, 4.86, 4.65, 4.66, 5.03, 5.17, 5.25, 5.48, 5.13, 5.70, 5.07, 4.78, 5.82, 6.38, 6.23, 6.38, 6.71, 6.81, 6.42, 6.45, 6.24, 6.58, 7.02.

Data Availability

The data used to support the study are included in the paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study is supported via funding from Prince sattam bin Abdulaziz University project number (PSAU/2023/R/1444).

References

[1] A. G. Awan and A. Alam, "Impact of agriculture productivity on economic growth: a case study of Pakistan," *Industrial Engineering Letters*, vol. 5, no. 7, pp. 27–33, 2015.

[2] A. Azam and M. Shafique, "Agriculture in Pakistan and its impact on economy," *Rev. Int. J. Adv. Sci. Tech*, vol. 103, pp. 47–60, 2017.

[3] F. Bhatti, "A model based study of rice production: a case study of district Lodhran," M. Phil thesis, Dep. Stat. Islamia Univ. Bahawalpur, Pakistan, 2015.

[4] M. A. Raza, L. Y. Fenga, A. Manaf, A. Wasaya, and M. Ansar, "Sulphur application increases seed yield and oil content in sesame seeds under rainfed conditions," *Field Crops Research*, vol. 51, p. 58, 2018.

[5] Anonymous, *Agricultural Statistics of Pakistan*, Food and Agriculture Division. Planning Unit, Islamabad, Pakistan, 2011.

[6] Icrisat, "International crops research institute for the semiarid tropics," 2016, https://www.cgiar.org/research/center/icrisat/.

[7] Gop (Government of Pakistan), *Economic Survey of Pakistan*, Ministry of Finance, Pakistan, 2015.

[8] M. Ayub, M. A. Nadeem, A. Tanveer, M. Tahir, and R. M. A. Khan, "Interactive effect of different nitrogen levels and seeding rates on fodder yield and quality of pearl millet," *Pakistan Journal of Agricultural Sciences*, vol. 44, pp. 592–596, 2007.

[9] S. R. Chughtai, J. Fateh, M. H. Munawwar, M. Aslam, and H. N. Malik, "Alternative uses of cereals-methods and feasibility: Pakistan perspective," *CFC and ICRISAT, 2004. Alternative uses of Sorghum and Pearl Millet in Asia: Proc. Expert Meeting, ICRISAT, Patancheru, Andhra Pradesh, India*, vol. 34, pp. 210–220, 2004.

[10] S. Nedumaran, P. Abinaya, and M. C. S. Bantila, "Sorghum and millets futures in asia under changing socio-economic and climatic scenarios," *ICRISAT Socio-Economic Discussion Paper Series*, International Crops Research Institute for the Semi-arid Tropics, Patancheru, India, 2013.

[11] R. H. Day, "Probability distributions of field crop yields," *Journal of Farm Economics*, vol. 47, no. 3, pp. 713–741, 1965.

[12] J. R. M. Hosking, J. R. Wallis, and E. F. Wood, "Estimation of the generalized extreme value distribution by the method of probability-weighted moments," *Technometrics*, vol. 27, no. 3, pp. 251–261, 1985.

[13] O. A. Ramirez, "Estimation and use of a multivariate parametric model for simulating heteroskedastic, correlated, non Normal random variables: the case of corn belt corn, soybean and wheat yields," *American Journal of Agricultural Economics*, vol. 79, no. 1, pp. 191–205, 1997.

[14] B. K. Goodwin and A. P. Ker, "Nonparametric estimation of crop yield distributions: implications for rating group-risk crop insurance contracts," *American Journal of Agricultural Economics*, vol. 80, no. 1, pp. 139–153, 1998.

[15] J. Atwood, S. Shaik, and M. Watts, "Are crop yields normally distributed? A reexamination," *American Journal of Agricultural Economics*, vol. 85, no. 4, pp. 888–901, 2003.

[16] M. H. Munawwar, J. Fateh, H. I. Javed, H. N. Malik, and M. Hussain, "Stability analysis of Millet varieties across diverse environments in Pakistan," *Sarhad Journal of Agriculture*, vol. 3, pp. 645–648, 2007.

[17] T. Górski, "A probability distribution for crop yields in Poland," *Geographia Polonica*, vol. 82, pp. 61–67, 2009.

[18] D. A. Hennessy, "Crop yield skewness and the normal distribution," *Journal of Agricultural and Resource Economics*, vol. 1, pp. 34–52, 2009.

[19] K. M. Lee, P. R. Armstrong, J. A. Thomasson, R. Sui, M. Casada, and T. J. Herrman, "Application of Binomial and Multinomial probability statistics to the sampling design process of a global grain tracing and recall system," *Food Control*, vol. 22, no. 7, pp. 1085–1094, 2011.

[20] W. Xiang, L. Yunxian, Q. Zhenwei, and S. Zeliang, "Estimation of crop yield distribution: implication for crop engineering risk," *Systems Engineering Procedia*, vol. 3, pp. 132–138, 2012.

[21] A. Holzkämper, P. Calanca, and J. Fuhrer, "Statistical crop models: predicting the effects of temperature and precipitation changes," *Climate Research*, vol. 51, no. 1, pp. 11–21, 2012.

[22] M. J. Yusuf, G. Nabi, A. Basit, S. K. Husnain, and L. H. Akhtar, "Development of high yielding Millet variety "Sargodha Bajra-2011" released for general cultivation in Punjab Province of Pakistan," *Pakistan Journal of Agricultural Sciences*, vol. 3, pp. 275–282, 2012.

[23] M. Hassan, A. Ahmad, S. Zamir et al., "Growth, Yield and Quality Performance of Pearl Millet (<i>Pennisetum americanum</i> L.)," *American Journal of Plant Sciences*, vol. 5, no. 15, pp. 2215–2223, 2014.

[24] K. Cristian and G. Camelia, "Use of maximum entropy in estimating production risks in crop farms," in *Proceedings of the 6th Edition of the International Symposium*, The Research Institute for Agricultural Economy and Rural Development (ICEADR), Bucharest, Romania, November 2015.

[25] M. M. Rahman and A. J Robson, "A novel approach for sugarcane yield prediction using landsat time series imagery: a case study on bundaberg region," *Advances in Remote Sensing*, vol. 5, no. 2, pp. 93–102, 2016.

[26] U. Asmat, A. Ahmad, T. Khaliq, and J. Akhtar, "Recognizing production options for Pearl Millet in Pakistan under changing climate scenarios," *Journal of Integrative Agriculture*, vol. 4, pp. 762–773, 2017.

[27] Z. I. Muhammad, M. Riaz, and W. Nasir, "Multivariate outlier detection: a comparison among two clustering techniques," *Pakistan Journal of Agricultural Sciences*, vol. 1, pp. 227–231, 2017.

[28] A. Ullah, N. Salehnia, S. Kolsoumi, A. Ahmad, and T. Khaliq, "Prediction of effective climate change indicators using statistical downscaling approach and impact assessment on pearl millet (Pennisetum glaucum L.) yield through Genetic Algorithm in Punjab, Pakistan," *Ecological Indicators*, vol. 90, pp. 569–576, 2018.

[29] O. P. Yadav, S. K. Gupta, M. Govindaraj et al., "Genetic gains in pearl millet in India: insights into historic breeding strategies and future perspective," *Frontiers of Plant Science*, vol. 12, Article ID 645038, 2021.

[30] T. I. Masenya, V. Mlambo, and C. M. Mnisi, "Complete replacement of maize grain with sorghum and pearl millet grains in Jumbo quail diets: feed intake, physiological parameters, and meat quality traits," *PLoS One*, vol. 16, Article ID e0249371, 2021.

[31] S. Gul, J. Ren, N. Xiong, and M. A. Khan, "Design and analysis of statistical probability distribution and nonparametric trend analysis for reference evapotranspiration," *PeerJ*, vol. 9, Article ID e11597, 2021.

[32] A. M. C. Heureux, J. Alvar-Beltrán, R. Manzanas et al., "Climate trends and extremes in the Indus river basin, Pakistan: implications for agricultural production," *Atmosphere*, vol. 13, no. 3, p. 378, 2022.

[33] V. Vadez, "Root hydraulics: the forgotten side of roots in drought adaptation," *Field Crops Research*, vol. 165, pp. 15–24, 2014.

[34] R. K. Uppal, S. P. Wani, K. K. Garg, and G. Alagarswamy, "Balanced nutrition increases yield of pearl millet under drought," *Field Crops Research*, vol. 177, pp. 86–97, 2015.