








Research Article

Communities Detection in Multiplex Networks Using Optimization: Study Case—Employment in Mexico during the COVID-19 Pandemic

Edwin Montes-Orozco ¹, Roman Anselmo Mora-Gutiérrez ²,
Sergio Gerardo De-Los-Cobos-Silva ³, Roberto Bernal-Jaquez ¹,
Eric Alfredo Rincón-García ³, Miguel Angel Gutiérrez-Andrade ³,
and Pedro Lara-Velázquez ³

¹Departamento de Matemáticas Aplicadas y Sistemas, Universidad Autónoma Metropolitana Cuajimalpa, Av. Vasco de Quiroga 4871, Col. Santa Fe Cuajimalpa. C.P. 05348, Cuajimalpa, CDMX, Mexico

²Departamento de Sistemas, Universidad Autónoma Metropolitana Azcapotzalco, Av. San Pablo Xalpa 180, Azcapotzalco 02128, CDMX, Mexico

³Departamento de Ingeniería Eléctrica, Universidad Autónoma Metropolitana Iztapalapa, Av. San Rafael Atlixco 186, Iztapalapa 09340, CDMX, Mexico

Correspondence should be addressed to Edwin Montes-Orozco; emonteso@cua.uam.mx

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The detection of communities in complex networks offers important information about the structure of the network as well as its dynamics. However, it is not an easy problem to solve. This work presents a methodology based on the robust coloring problem (RCP) and the vertex cover problem (VCP) to find communities in multiplex networks. For this, we consider the RCP idea of having a partial detection based on the similarity of connected and unconnected nodes. On the other hand, with the idea of the VCP, we manage to minimize the number of groups, which allows us to identify the communities well. To apply this methodology, we present the dynamic characterization of job loss, change, and acquisition behavior for the Mexican population before and during the COVID-19 pandemic modeled as a 4-layer multiplex network. The results obtained when applied to test and study case networks show that this methodology can classify elements with similar characteristics and can find their communities. Therefore, our proposed methodology can be used as a new mechanism to identify communities, regardless of the topology or whether it is a monoplex or multiplex network.

1. Introduction

In recent years, the analysis of several characteristics of multilayer and multiplex networks has been of immense importance to scientists because most systems of daily life can be modeled as complex networks, such as social networks, transportation, electrical or communication networks, and epidemic propagation, among others [1–3].

Regarding the detection of communities, there are works that study the problem from various approaches; some treat

it as a static problem, while more recent ones treat it as a dynamic problem. However, both approaches are important to analyze diverse systems and real-life problems.

Since 2020, the world has been facing a pandemic caused by the disease known as COVID-19, and although most of the effects were related to the health of the population, the pandemic has caused various socioeconomic effects worldwide.

Therefore, in this work, we analyze the changes in the generation, acquisition, and loss of employment in Mexico

by proposing a new methodology to identify communities in multiplex networks as a dynamic problem. In the rest of this section, we present the main concepts of complex systems, complex networks, community detection, and the relationship between COVID-19 and employment in Mexico.

1.1. Complex Systems, Complex Networks, and Multiplex Networks. There is no precise and accurate definition of what a complex system is. However, the complex systems are composed of several elements connected in a certain way, where some properties arise, such as

- (1) **Emergence:** this is an essential characteristic of the complex systems, which arises from analyzing each of its components and the difference in analyzing the entire system. Emergent behavior arises as nascent properties from the interactions of all the elements that form the system and cannot be seen or predicted by analyzing each individual element.
- (2) **Self-organization:** this characteristic allows for the coordination and synchronization of all the elements that compose it, as well as all its processes, in an autonomous way
- (3) **Nonpredictability:** this property arises on the basis of the emergence and self-organization, as both cause the behaviors and dynamics of complex systems to be difficult to predict

The usual representation of complex systems is through complex networks. A complex network has nontrivial topological characteristics, such as degree distributions, high local cohesiveness (measured through clustering coefficients, etc.), community structures, and hierarchical structures, among others [4].

In recent years, there has been an emphasis on the study of multilayer networks, thanks to the fact that most real systems have structures with multiple types of links or interactions between nodes [5], such as the multimodal transport systems, biological systems, or the social networks [6–9].

Multiplex networks are a particular class of multilayer networks where all nodes (it is important to mention that in multiplex networks, each node belongs to all the M layers) are replicated across each layer and connected directly to their replicas to denote their relationships. Formally, let $GP = (\mathcal{G}\alpha, \mathcal{c}) \forall \alpha \in \{1, \dots, M\}$ be a multiplex network, where

- (i) $\mathcal{G}_\alpha = (X_\alpha, E_\alpha)$, is a single-layer network called layer α , where X_α and E_α are the set of nodes (it is important to mention that in multiplex networks, each node belongs to all the M layers) and the links in layer α , respectively
- (ii) $\mathcal{C} = \{E_{\alpha\beta} \subseteq X_\alpha \times X_\beta, \alpha, \beta \in \{1, \dots, M\}, \alpha \neq \beta\}$ is the set of interconnections between the nodes in different layers. The elements of \mathcal{C} are called cross layers, and the elements of each E_α are called intralayer connections of GP.

The importance of this type of network lies in the ability to work with distinct characteristics and relationships for each element, allowing for multicriteria or temporal analysis, as all nodes are present in all layers.

In this work, we leverage the structure of multiplex networks to analyze the dynamics and behavior of employment in Mexico over a four-year period (2018–2021). In the next section, we describe the significance of studying this case.

1.2. Employment and COVID-19. Since the end of March 2020, when the health emergency due to the COVID-19 epidemic began in Mexico, millions of workers have been forced to stay at home, telework, or face other consequences such as low wages or job loss. These consequences continue to affect many people in the country. Therefore, some researchers have focused on analyzing these problems from a social and economic point of view, using the information obtained from statistics and opinions of the population. Consequently, measuring the economy and employment during COVID-19 has led to a series of obstacles and prospects for the private sector and academia.

On the basis of the abovementioned pandemic consequences, the following are the research questions: does the identification of communities in multiplex networks help verify the systems' dynamics? Specifically, if employment in Mexico is modeled as a complex system, can we analyze its behavior and the effects caused by the COVID-19 pandemic? The following are the proposed research questions: can the identification of communities in multiplex networks help analyze the behavior and effects caused by the COVID-19 pandemic on employment in Mexico, modeled as a complex system?

To respond to the research questions, we have divided the article into the following sections: in Section 2, we review the main works related to the main ideas of this paper, specifically those related to employment, community detection, and complex systems. In Section 3, we present the strategy that we used to model the employment networks and the development of the proposed methodology for detecting communities. In Section 4, we present the numerical classification of the nodes in the employment networks. Finally, in Section 5, we discuss the limitations of the study and provide a summary of the key findings.

2. Related Work

The components of a complex system are known to play distinct roles, and identifying the most influential nodes and the communities that form through their interactions is of great importance for real-world applications. In this section, we review the main works related to the community detection in complex systems and the analysis of employment during the COVID-19 pandemic, specifically in the health sector.

In Table 1, we show a brief summary on COVID-19 and the employment related to Mexico and the world.

TABLE 1: Related work about COVID-19 and employment.

Author	Approach	Main findings
Cohen [10]	Statistical, sociological, and economic for tracking employment	Existing and new methods for tracking employment statistics leave more questions than answers
Blustein et al. [11]	Psychological to analyze unemployment crisis caused by COVID-19	The results show that parallel loss of work and loss of life evokes a unique kind of pain
Llop-Gironés et al. [12]	Sociological analysis for working and employment conditions of nurses	The analysis shows that many nurses around the world face precarious working and employment conditions, which the COVID-19 pandemic has further exacerbated
Webb et al. [13]	Analysis of informal employment	During the pandemic, people in informal employment often receive less government support than those in formal employment
Gezici et al. [14]	An economy on the basis of the probability of unemployment among different groups in the US	There are significant racial disparities in the probability of being unemployed
Samaniego [15]	Economic analysis of job loss in Mexico	Immediate measures are necessary to protect the employment and income of workers who have become unemployed in Mexico
Ruiz-Ramirez [16]	Economic analysis of employment and underemployment in Mexico	The labor market in both the formal and informal sectors has been negatively impacted by COVID-19 and there is a reduction in the economic activity

On the other hand, in Table 2, we present the main works related to community detection in complex networks based on centrality measures, structural metrics, optimization, and artificial intelligence approaches.

On the other hand, we would like to highlight the following three articles that are closely related to the methodology proposed in this work, as they present novel and original approaches to solving the community detection problem with similar methods to the one proposed in this work.

In particular, the work by Berahmand et al. proposes a non-negative matrix factorization technique that uses node attribute information and adds regularization to the network structure to detect communities. On the other hand, the article by Nasiri et al. addresses the problem of link prediction in multiplex networks by using topologically biased random walks. Finally, the article by Berahmand et al. proposes an approach to detect communities in complex networks by detecting central nodes and expanding them in the network.

These three works present novel and original approaches to address this problem using techniques such as non-negative matrix factorization, deep learning, and central node detection in monoplex and multiplex networks, which make them comparable to our work.

3. Materials and Methods

This section presents the main characteristics used in the study, as well as the analysis of the structural metrics and the modeling of the networks.

3.1. Employment Networks. As we previously explained, this work aims to analyze the changes and characteristics over time. To achieve this, we use multiplex networks that treat each year as a layer and include all the states in each layer.

In these networks, intralayer links represent the relationships within each year, while interlayer links only occur between equivalent states in different years.

To accomplish our objective, we consider the number of jobs generated in each year, specifically for 2018 and 2019 (before the pandemic) and 2020 and 2021 (after the pandemic).

To model the employment networks, we utilized the information available on the INEGI web page and generated links for each year using the Mahalanobis distance. The Mahalanobis distance measures the similarity between two variables, taking into account the correlation between the random variables, in contrast to the Euclidean distance [30, 31].

On the basis of the calculation of this distance, we determined the relationships between states by evaluating the number of characteristics in which they are similar and quantified them using the following approach:

- (i) The Mahalanobis distance between each pair of states is calculated, and then the median of the Mahalanobis distances is computed

- (ii) For each pair of states with a distance lower than the median, a link is added

On the basis of this approach, we obtain links between the states with significant similarity per year. To analyze the dynamics and behavior for the established periods, we use the following multiplex network structure:

- (i) The nodes of each layer represent the states of the Mexican Republic, resulting in 32 nodes for each layer
- (ii) The intralayer relationships were obtained using the process described above. Interlayer relationships occur between replica nodes, as all the 32 states of the Mexican Republic are present in each period studied, and each state has a link with its replicas in each layer

3.2. Methodology. In the next section, we present our proposed methodology for identifying communities in multiplex networks, along with an example of a simple multiplex network and the corresponding resolution method.

3.2.1. Adaptation of Robust Coloring Problem and Vertex Cover to Identify Communities in Multiplex Networks. For this adaptation, we consider two well-known graph problems: the robust coloring problem (RCP) and the vertex cover problem (VCP).

To address the RCP, we assign penalties to edges on the basis of the distance between the connected states (i.e., the greater the distance, the greater the socioeconomic difference between the states). Then, using the complementary network, we aim to find the coloration with the lowest rigidity and the highest similarity.

For VCP, we apply it to the complementary graph to measure how similar the nodes (i.e., states of the Mexican Republic) are with respect to the analyzed characteristics. By avoiding the penalties of the nonedges (i.e., edges of the complementary graph) in RCP, we can generate communities of similar elements.

With these concepts in mind, we can define the following multiplex network $M = (V, L, P, M)$:

- (i) V is the set of nodes that represents the components of the system
- (ii) L is the set of layers representing the diverse types of system relationships or interactions
- (iii) P is the set of links that represent the relationships (we have $GPL \in P$ as the links of a certain layer)
- (iv) M is the set of networks of each monolayer system (networks of interactions of a particular type between the nodes)

Then, given the distance matrix $d_{i,j}^L$ and the penalty matrix $p_{i,j}^L \geq 0, \{i, j\} \in P^L$, the problem can be addressed. On the other hand, knowing that the rigidity of a k -coloration $(R(c))$ is $\sum p_{i,j}^L$ if the penalties are assigned to the $\{i, j\} \in P$.*i*.

TABLE 2: Related work about community detection.

Authors and year	Approach	Main findings
Li et al. (2019) [17]	Page rank centrality-based	This work shows that the proposed methodology improves the accuracy and efficiency of seed-expansion community detection methods
Gao et al. (2021) [18]	Page rank centrality-based	The authors propose a constrained personalized PageRank algorithm for detecting overlapping communities, which involves assigning each node a probability distribution over the communities and updating the distribution through iterations
Huang et al. (2022) [19]	Leader rank centrality-based	The authors also compare the results of their method with those of other centrality measures, such as degree centrality and eigenvector centrality, and find that their method outperforms these traditional measures in identifying influential nodes
Kiruthika et al. (2022) [20]	Eccentricity centrality-based	The authors describe that the performance of the algorithms varies depending on the characteristics of the network and that no single algorithm is universally best
Yang et al. (2018) [21]	Maximizing influence of nearby nodes	Development of a more efficient algorithm for community detection
Ma et al. (2019) [22]	Identification of cliques	This approach allows the identification of nodes that overlap in multiple communities
Zhang et al. (2021) [23]	Multiobjective optimization	Development of an efficient method for finding sets of nodes that form communities in large bipartite graphs, where each node has multiple attributes
Wang et al. (2021) [24]	Identification of dense subgraphs	Development of a new algorithm for community detection in large-scale networks
El Kouni (2020) [25]	Node importance	This work suggests that incorporating node importance measures into the labeling process can significantly improve the performance of label propagation-based algorithms for overlapping community detection in complex networks
Hesamipour et al. (2022) [26]	Multiobjective optimization on the basis of modularity, heterogeneity, and minimization of communities	The findings of this work have important implications for improving the accuracy and efficiency of community detection algorithms, particularly in cases where the network is large and complex
Yang et al. (2022) [27]	Multiobjective optimization on the basis of network structure	This work suggests that incorporating node classification information into community detection algorithms can significantly improve their performance and that MOEAs can be a useful approach for tackling the challenges of community detection in complex networks
Su et al. (2022) [28]	Convolutional neural networks	The main findings of the survey suggest that deep learning-based approaches have the potential to significantly improve community detection performance, but there are still much research studies to be conducted to fully realize this potential
Choong et al. (2020) [29]	Deep learning and multiobjective optimization	The authors suggest that the proposed dual optimization strategy for VGAE can significantly improve community detection performance and potentially be a useful approach to identifying communities

Thus, it is sought that the network has a minimum rigidity, and in addition to the idea of the coverage problem, it is sought that the difference of the states that belong to the color class is minimal. That is, they form the minimum subset S of $V(V) = \sum_{v \in V} y_v$, such that for each edge $\{i, j\}$ of the set P^L , either node i or node j belongs to S .

Therefore, we have the following mathematical programming model:

$$\text{Min } \sum p^L + \sum_{v \in V} y_v. \quad (1)$$

S.a.

$$\sum_{c=1}^k x_{i,c} = 1 \forall i \in \{1, \dots, n\}, \quad (2)$$

$$x_{i,k} + x_{j,k} \leq 1 \forall i, j \in P^L, \quad (3)$$

$$\sum_{c=1}^k \sum_{i=1}^n x_{i,c} \geq 1, \quad (4)$$

$$y_u + y_v \geq 1 \forall u, v \in \{1, \dots, n\}, \quad (5)$$

$$y_v \in \{0, 1\} \forall v \in V, \quad (6)$$

where each variable $x_{i,c}$, is equal to 1 if the color of the node i is c , that is, $(c(i) = c \forall i \in \{1, \dots, n\} \forall c \in \{1, \dots, k\})$ and y is zero if $c(i) \neq c$.

And each $y_u + y_v$ indicates that at least one of the nodes u or v is in the coverage of the nonedges $\{u, v\} \in P^L \forall i \in \{1, \dots, n\}$.

So, the constraint set (2) helps each node to be assigned to a color, the constraints (3) ensure that the adjacent links have different colors, and the constraint set (4) ensures that all k colors are used, the constraint set (5) ensures that each nonedge is covered and, finally, the constraint (6) indicates that each node can only be or not be in the coverage of the nonedges (\bar{P}).

To verify the operation of this model, we present an example of a multiplex network with 5 nodes and two layers, whose individual layers can be seen below.

Suppose we have the following multiplex network example:

(i) Layer 1 (Table 3)

(ii) Layer 2 (Table 4)

Then, the supra-adjacency matrix of the network is shown in Table 5

As can be seen in Table 5, the supra-adjacency matrix is composed of the adjacency matrices of the individual layers (Tables 3 and 4), which are related through the identity matrix for each node belonging to the multiplex network. Therefore, to graphically represent the multiplex network, we present Figure 1.

Therefore, the adjacency matrices and the complementary multiplex network can be viewed as follows:

TABLE 3: Adjacency matrix layer 1.

L1	1	2	3	4	5
1	0	1	0	1	0
2	1	0	0	1	0
3	0	0	0	0	1
4	1	1	0	0	1
5	1	0	1	1	0

TABLE 4: Adjacency matrix layer 2.

L2	1	2	3	4	5
1	0	0	0	0	1
2	0	0	0	1	0
3	0	0	0	1	0
4	0	1	1	0	0
5	1	0	0	0	0

TABLE 5: Multiplex network of layers 1 and 2.

L1-L2 multiplex	Layer 1					Layer 2				
	1	2	3	4	5	1	2	3	4	5
Layer 1	1	1	1	0	1	0	1	0	0	0
	2	1	1	0	1	0	0	1	0	0
	3	0	0	1	0	1	0	0	1	0
	4	1	1	0	1	1	0	0	0	1
	5	0	0	1	1	1	0	0	0	1
Layer 2	1	1	0	0	0	1	0	0	0	1
	2	0	1	0	0	0	1	0	1	0
	3	0	0	1	0	0	0	1	1	0
	4	0	0	0	1	0	0	1	1	0
	5	0	0	0	0	1	1	0	0	1

(i) Adjacency matrix of the complementary layer 1 (Table 6)

(ii) Adjacency matrix of the complementary layer 2 (Table 7)

Therefore, the supra-adjacency matrix of the multiplex network can be viewed as shown in Table 8.

In Table 8, we can see that the supra-adjacency matrix is composed of the adjacency matrices of the individual complementary layers related through the identity matrix for each node in the multiplex network. Therefore, to graphically represent the multiplex network, we present Figure 2.

On the basis of the previously presented model and the complementary multiplex network shown in Figure 2, we can consider the following penalty matrix of P^L (Table 9), which represents the inverse of the distance between each pair of nodes:

We proceed to conduct the following practical example:

(i) First, we find a valid coloring for the complementary multiplex network

As shown in Figure 3, the coloring is valid, considering the connections of the nodes in all the layers, 4 colors are necessary to carry it out. However, as mentioned above, the coloration should be performed by minimizing the

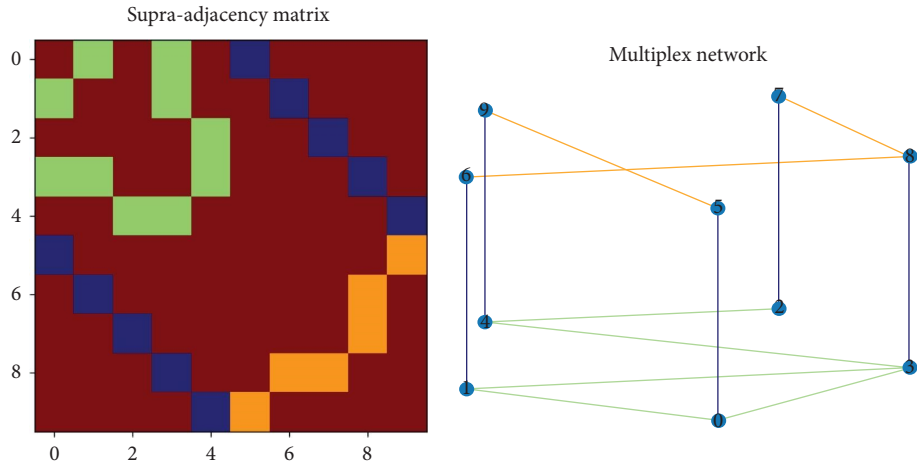


FIGURE 1: Adjacency matrix and multiplex network (graphic). Note: it is important to mention that node 5 is a replica of node 0, node 6 is a replica of node 1, node 7 is a replica of node 2, node 8 is a replica of node 3, and node 9 is a replica of node 4.

TABLE 6: Complementary layer 1 adjacency matrix.

Complementary $L1$	1	2	3	4	5
1	0	0	1	0	1
2	0	0	1	0	1
3	1	1	0	1	0
4	0	0	1	0	0
5	1	1	0	0	0

TABLE 7: Complementary layer 2 adjacency matrix.

Complementary $L2$	1	2	3	4	5
1	0	1	1	1	0
2	1	0	1	0	1
3	1	1	0	0	1
4	1	0	0	0	1
5	0	1	1	1	0

TABLE 8: Multiplex network of complementary layers 1 and 2.

Complementary $L1-L2$ multiplex	Layer 1					Layer 2					
	1	2	3	4	5	1	2	3	4	5	
Layer 1	1	0	0	1	0	1	1	0	0	0	0
	2	0	0	1	0	1	0	1	0	0	0
	3	1	1	0	1	0	0	0	1	0	0
	4	0	0	1	0	0	0	0	0	1	0
	5	1	1	0	0	0	0	0	0	0	1
Layer 2	1	1	0	0	0	0	0	1	0	1	
	2	0	1	0	0	0	0	0	1	0	
	3	0	0	1	0	0	1	1	0	1	
	4	0	0	0	1	0	0	0	1	0	
	5	0	0	0	0	1	1	1	0	0	

penalties with the original network. Then, on the basis of the information shown in Table 9 and the information on coloration penalties shown in Figure 3, we present Figure 4.

In Figure 4, we can see the penalties that are marked as X . Therefore, we sum the values for 1.4 (yellow link) and 0.3

(green link) for the penalties of color 1 (nodes 1, 3, 6, and 8 all with color C1).

Then, the idea of the coverage set problem for the complementary multiplex network is shown in Figure 5.

Therefore, as we can see in Figure 5, we obtain a classification of the elements (nodes) that considers the several

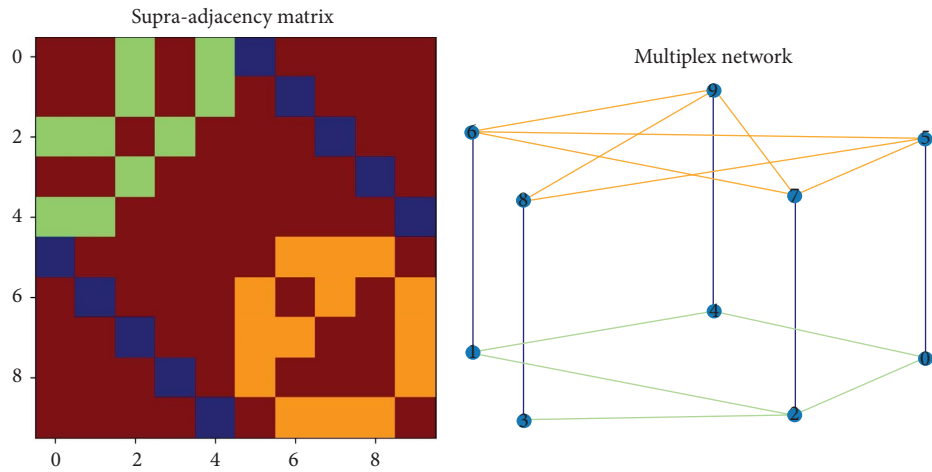


FIGURE 2: Adjacency matrix and complementary (graphical) multiplex network. Note: it is important to mention that, as in the previous case, node 5 is a replica of node 0, node 6 is a replica of node 1, node 7 is a replica of node 2, node 8 is a replica of node 3, and node 9 is a replica of node 4.

TABLE 9: Matrix of penalties $p_{i,j}$.

Penalty matrix $p_{i,j}$					
Complementary $L1$	1	2	3	4	5
1	0	0.3	0.2	0.5	0.2
2	0.3	0	0.4	0.4	0.3
3	0.2	0.4	0	0.5	0.3
4	0.5	0.4	0.5	0	0.4
5	0.2	0.3	0.3	0.4	0
Complementary $L2$	1	2	3	4	5
1	0	0.2	0.3	0.5	0.3
2	0.2	0	0.1	0.4	0.4
3	0.3	0.1	0	0.2	0.3
4	0.5	0.4	0.2	0	0.5
5	0.3	0.4	0.3	0.5	0

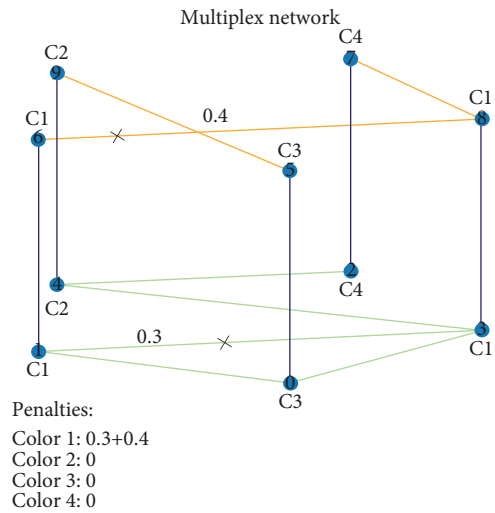


FIGURE 4: Penalties of the original multiplex network with the coloring of the complementary multiplex network.

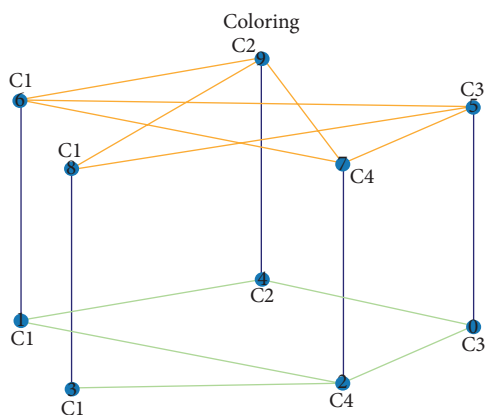


FIGURE 3: Valid coloring for the complementary multiplex network.

types of relationships or changes in time between the connections of the elements (given by the layers of the network). However, we can observe that the number of

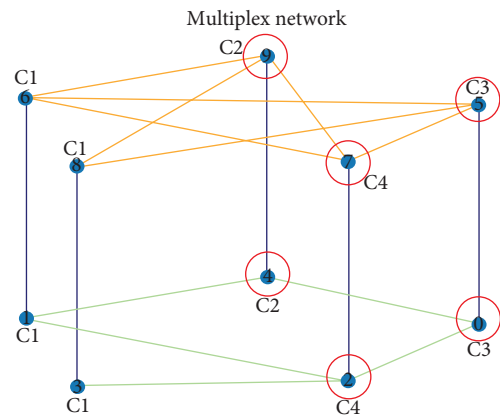


FIGURE 5: Nodes that belong to the coverage set of the complementary multiplex network.

groups (colors) needed to classify the communities is fewer than that obtained by the PCR. This is achieved through a classification on the basis of the multicriteria analysis (using PCR), and the dynamic over time is obtained thanks to the multiplicity of layers. On the other hand, on the basis of the VCP, the number of groups is minimized, which allows us to improve the classification on the basis of similar characteristics between the groups formed by PCR.

3.2.2. *Resolution Method.* In this work, the adaptation of RCP and VCP was solved by using an adaptation of the genetic algorithm (GA) [32] developed in Python language with the following set of control parameters:

- (i) The number of generations = 100
- (ii) Size of population = 10
- (iii) Crossover rate = 0.65
- (iv) Mutation rate = 0.3

On the basis of the previous parameters, the number of offspring generated per generation is equal to 10 (population size) multiplied by the sum of the crossover rate (0.65) and the mutation rate (0.33), resulting in 9.25.

Therefore, the total number of evaluations in 100 generations equals 100 times the number of evaluations per generation, which is 10 multiplied by 9.25, resulting in a total of 9,250. Since the objective function can be computed in $O(n^2)$ time as it is a matrix operation, the complexity of this algorithm is $O(n^2)$. This implies that if we assume that the size of the problem is 1,000 and the number of layers is 4, and considering that the operations are simple and take one clock cycle each, we have $4 \times (1,000)^2 = 4,000,000$ operations, which a 1 GHz computer could perform in 4 seconds.

On the other hand, because GA is a technique on the basis of the genetic operators, it is necessary to establish the structure of mutation. In this work, we use mutations that consider the changes in the color of the selected gene (chosen randomly) with another color. For example, if we have a solution with 3 colors for a network of 9 nodes, then we have

3	2	1	3	2	2	1	1	2
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We consider one gene at a time, and on the basis of the mutation rate, we generate a random number, r_1 , between 0 and 1 (continuous). If r_1 is less than the mutation rate, we change the value of the gene by selecting a number between 0 and the number of nodes at random. For example, using the vector mentioned above, a mutation would be as follows:

7	2	2	3	3	3	1	1	2
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where the mutated vector has 4 colors.

On the other hand, the crossover is an operator that uses two parents and works at a specific point. A random position between 1 and the number of nodes is obtained, and a cut is made in the vector of each parent. Then, the first part of the first parent and the second part of the second parent produce child 1, while the second part of the first parent and the first part of the second parent produce child 2.

For example, if we take the abovementioned vectors and position “4,” then we can produce the following two childrens:

- (i) Child 1:

3	2	1	3	3	3	1	1	2
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- (ii) Child 2:

7	2	2	3	2	2	1	1	2
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On this basis, we can see that the genetic operators satisfy the sexual reproduction and adaptation of individuals.

4. Analysis of Results

In this section, we present the values of the main structural metrics and provide an analysis of the communities in the Mexican employment networks. To better understand the results presented in the following subsections, each multiplex network consists of four layers, with the first layer representing the year 2018, the second representing 2019, the third representing 2020, and the fourth representing 2021.

Figure 6 displays the adjacency matrix and graph of the multiplex network composed of these four networks.

It is worth noting that in Figure 6, every node has its duplicates. For instance, node 1 has its duplicates shown as 0, 32, 64, and 96, node 2 has its duplicates shown as 1, 33, 65, and 97, and so on until node 31 (as the identifiers start at 0 and end at 31 due to the 32 states in Mexico) with their duplicates shown as 31, 63, 95, and 127.

- (i) Note: in order to identify the Mexican states in Figures 6 and 7, Table 10 shows the corresponding numerical ID for each one.

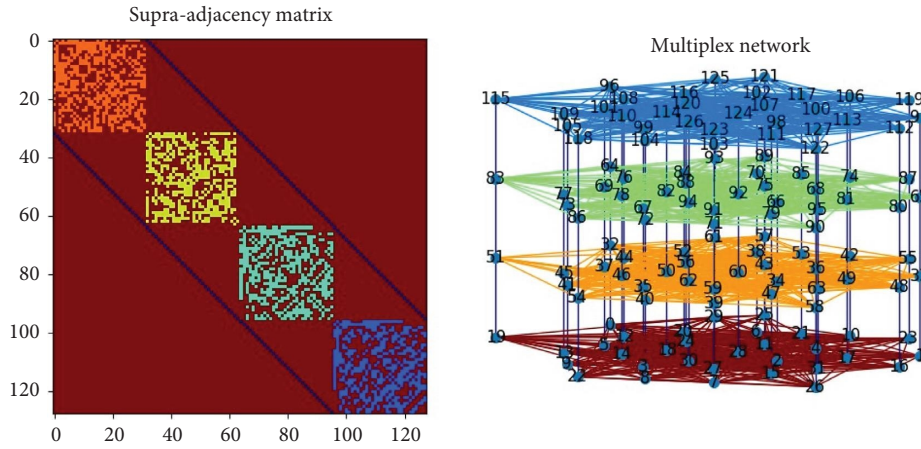


FIGURE 6: Original multiplex network.

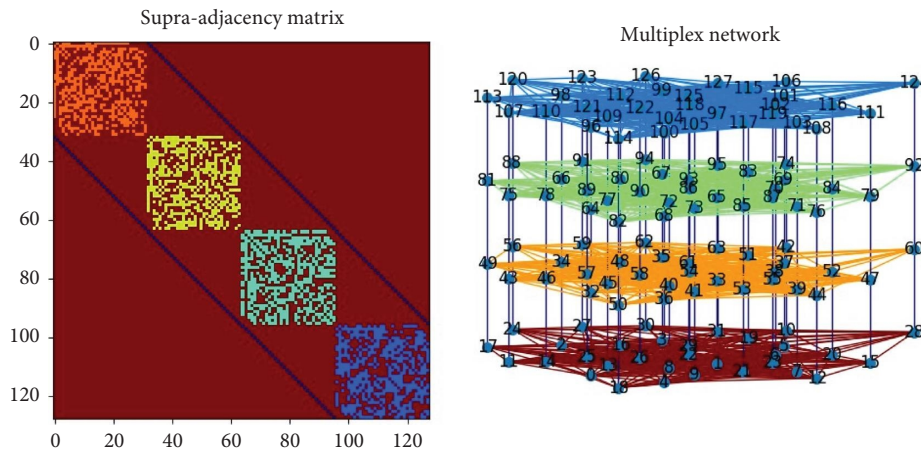


FIGURE 7: Complementary multiplex network.

TABLE 10: Mexican states and numerical IDs.

Name	ID
Michoacán de Ocampo	0
Aguascalientes	1
Zacatecas	2
Veracruz de Ignacio de la Llave	3
Tlaxcala	4
Quintana Roo	5
Hidalgo	6
Puebla	7
Chiapas	8
Mexico	9
Ciudad de Mexico	10
Jalisco	11
Colima	12
Chihuahua	13
Oaxaca	14
Baja California	15
Coahuila de Zaragoza	16
Guerrero	17
Yucatán	18
Nuevo León	19

TABLE 10: Continued.

Name	ID
Tabasco	20
Durango	21
Morelos	22
Sinaloa	23
Campeche	24
Querétaro	25
Tamaulipas	26
Nayarit	27
Sonora	28
Baja California Sur	29
San Luis Potosí	30
Guanajuato	31

4.1. *Communities Using RCP-VC.* In our study, the objective is to obtain a valid coloring for both the original network and its complementary network. Therefore, Figure 7 shows the multiplex complementary network of the original network presented in Figure 6.

TABLE 11: Classification by communities of the 32 Mexican states.

Name	Color
Michoacán de Ocampo	1
Aguascalientes	2
Zacatecas	2
Veracruz de Ignacio de la Llave	3
Tlaxcala	4
Quintana Roo	5
Hidalgo	6
Puebla	7
Chiapas	8
Mexico	8
Colima	9
Chihuahua	9
Jalisco	10
Ciudad de Mexico	11
Oaxaca	12
Baja California	13
Coahuila de Zaragoza	13
Guerrero	14
Yucatán	14
Nuevo León	15
Tabasco	16
Durango	17
Morelos	18
Sinaloa	18
Campeche	19
Querétaro	19
Tamaulipas	20
Nayarit	21
Sonora	21
Baja California Sur	22
San Luis Potosí	22
Guanajuato	23

It is important to mention that in the multiplex complementary network (Figure 7), the intralinks for each layer are complementary to those in the original network; however, the replicas are maintained. If we consider the penalty matrix (which shows the distance between the states on the basis of their characteristics), we can look for a valid coloration for the complementary multiplex network.

For our study case, we need 23 colors to satisfy the coloration requirement. It is important to note that the number of colors is high compared to the number of nodes because the coloration must be valid for both the original network and its complementary network, across all layers. Therefore, the classification for the 32 Mexican states is as follows.

On the basis of Table 11, we can see the distribution of similarities (using RCP). However, if we apply the idea of VCP, we only need 9 colors. It is important to mention that if we consider these 9 colors, the obtained coloring is invalidated. However, it is the basis for a correct classification. In other words, the 23-coloration is the first approximation of the formation of subsets by links, and once VCP is applied,

TABLE 12: Classification by communities obtained from the adaptation of RCP and VCP.

Name	Community
Michoacán de Ocampo	1
Aguascalientes	1
Zacatecas	1
Veracruz de Ignacio de la Llave	2
Tlaxcala	2
Quintana Roo	2
Hidalgo	3
Puebla	3
Chiapas	3
Mexico	3
Ciudad de Mexico	3
Jalisco	3
Colima	4
Chihuahua	4
Oaxaca	4
Baja California	4
Coahuila de Zaragoza	4
Guerrero	4
Yucatán	5
Nuevo León	5
Tabasco	5
Durango	5
Morelos	5
Sinaloa	6
Campeche	6
Querétaro	6
Tamaulipas	7
Nayarit	7
Sonora	8
Baja California Sur	8
San Luis Potosí	9
Guanajuato	9

we obtain a classification of the elements on the basis of the similarity, which causes a decrease in the number of groups.

Therefore, we can verify that community detection is achieved without considering the calculation of the structural or topological metrics of the network. Now, Table 12 shows the classification by communities obtained using the proposed methodology in the study case.

On the basis of the information in Table 12, we can observe that the classification is now performed in 9 communities. Therefore, we can verify which characteristics the elements of each of them share and thus analyze the dynamics (changes) of employment in Mexico during the COVID-19 pandemic.

- (i) States such as Mexico City, the State of Mexico, Jalisco, and Hidalgo were likely affected by the COVID-19 pandemic in terms of job creation and preservation due to several factors. These states are among the most populous and urbanized in Mexico, with high levels of economic activity and employment in sectors such as manufacturing, services, and

TABLE 13: Mexican population and employment (numerical data).

Year	Population (millions) [33–35]	Population change (%)	Formal employment (net) [36–38]	Unemployment rate
2018	129.2	3.6	660,910	3.3
2019	130.2	0.8	−382,210	3.5
2020	131.1	0.7	−647,710	4.4
2021	132.2	0.8	774,133	4.8

TABLE 14: Population and employment in Mexico (State by State).

State	Population 2018	Population 2019	Population 2020	Population 2021	Formal employment 2018	Formal employment 2019	Formal employment 2020	Formal employment 2021
Aguascalientes	1,425,592	1,435,866	1,455,933	1,461,294	295,124	294,943	295,029	295,215
Baja California	3,376,557	3,404,909	3,423,516	3,440,028	880,462	870,944	868,903	866,446
Baja California Sur	798,447	811,486	817,238	824,627	183,620	184,930	186,146	186,860
Campeche	899,931	907,229	907,227	907,097	164,849	165,132	166,086	165,329
Chiapas	5,290,918	5,347,818	5,408,695	5,465,382	1,182,908	1,201,747	1,212,748	1,227,390
Chihuahua	3,573,751	3,602,607	3,630,747	3,658,711	770,715	773,334	776,253	779,981
Ciudad de Mexico	8,918,653	8,918,846	8,841,916	8,891,865	3,972,013	3,987,318	3,965,517	3,938,963
Coahuila	2,954,915	2,970,953	2,983,044	2,995,611	672,612	670,744	671,463	670,571
Colima	731,391	735,853	737,681	740,522	154,270	157,029	155,759	156,763
Durango	1,754,754	1,767,795	1,780,647	1,793,202	373,222	374,345	375,624	376,141
Estado de Mexico	17,430,925	17,568,972	17,614,888	17,662,617	4,787,321	4,840,029	4,822,468	4,867,475
Guanajuato	5,853,677	5,926,546	6,000,940	6,075,463	1,276,914	1,292,898	1,302,431	1,305,434
Guerrero	3,533,251	3,566,251	3,574,750	3,583,251	893,123	902,365	896,103	902,509
Hidalgo	2,858,359	2,893,957	2,907,514	2,921,453	725,566	735,191	729,102	735,979
Jalisco	8,238,908	8,297,383	8,322,776	8,348,113	2,079,294	2,105,998	2,088,524	2,105,044
Michoacán	4,625,986	4,662,418	4,675,745	4,689,338	1,127,499	1,140,098	1,132,698	1,140,721
Morelos	1,954,046	1,972,370	1,977,329	1,982,554	457,321	464,534	458,008	462,383
Nuevo León	5,484,709	5,529,147	5,542,810	5,556,480	1,371,468	1,383,951	1,374,789	1,384,667
Oaxaca	3,967,889	4,007,526	4,019,446	4,031,479	981,237	993,678	986,198	994,190
Puebla	6,190,327	6,247,995	6,265,158	6,282,617	1,607,947	1,626,590	1,615,277	1,627,956
Querétaro	2,038,372	2,099,834	2,117,908	2,136,147	586,909	602,824	597,831	604,020
Quintana Roo	1,501,562	1,524,511	1,532,359	1,540,828	445,997	453,246	449,738	454,013
San Luis Potosí	2,822,255	2,862,730	2,878,693	2,894,938	711,438	722,661	716,759	723,889
Sinaloa	2,966,321	3,000,040	3,014,902	3,029,943	740,231	749,569	743,399	750,180
Sonora	2,908,092	2,938,434	2,951,471	2,964,693	724,571	734,270	728,154	734,870
Tabasco	2,387,479	2,414,184	2,421,238	2,428,858	518,913	526,200	522,889	526,977
Tabasco	2,387,479	2,414,184	2,421,238	2,428,858	518,913	526,200	522,889	526,977
Tamaulipas	3,526,594	3,572,634	3,586,577	3,600,785	901,004	914,174	907,032	914,860
Tamaulipas	3,526,594	3,572,634	3,586,577	3,600,785	901,004	914,174	907,032	914,860
Tlaxcala	1,279,857	1,290,719	1,294,229	1,297,921	321,639	326,310	323,410	326,210
Veracruz	8,048,743	8,117,363	8,147,817	8,178,016	2,183,693	2,213,685	2,196,082	2,214,009
Zacatecas	1,622,138	1,636,579	1,641,340	1,646,289	418,032	424,056	419,709	424,970

tourism. The pandemic caused widespread business closures, reduced consumer demand, and restrictions on mobility and gatherings, which likely led to job losses and reduced hiring in many industries.

- (ii) States such as Michoacán, Aguascalientes, and Zacatecas were not necessarily immune to the COVID-19 pandemic's effects on job creation and preservation, but they may have had some advantages. These states are known for producing and exporting agricultural and perishable products such as fruits, vegetables, and meat, which were likely in

demand during the pandemic as people focused on buying essential goods. However, the pandemic's effects on global supply chains, transportation, and logistics could still have had some impact on these industries.

- (iii) States such as Veracruz, Tlaxcala, Guerrero, Baja California, and Oaxaca were likely affected by the drop in tourism, which is a major source of employment and economic activity in these regions. The pandemic's travel restrictions, border closures, and the fear of infection likely led to a significant decline in tourist arrivals and spending, which could

have affected hotels, restaurants, entertainment, and other related sectors. However, these states may have had some resilience due to their production of food supplies, such as coffee, sugar, seafood, and other agricultural products, which were still in demand during the pandemic.

- (iv) States such as Nuevo León, Tabasco, Sinaloa, Sonora, and Baja California Sur were likely affected by the reduced manufacturing production due to pandemic's disruptions of supply chains, labor shortages, and reduced demand. However, these states may have had some diversification in their economies, with significant employment and production in other sectors such as livestock, fishing, and agriculture. These industries may have benefited from the pandemic's effects on consumer demand and the shift towards locally produced goods. However, the pandemic's effects on global trade and markets could still have had some impact on these sectors.

On the other hand, in order to verify the results obtained in our methodology for employment and population, we present Table 13 with information on the population and the employment in Mexico (general and State by State).

From Table 13, we can see that during this period (2018 to 2021), Mexico faced significant challenges in terms of job creation. In 2019, the country experienced a net loss of formal jobs [36–38], and in 2020, the COVID-19 pandemic exacerbated the situation, resulting in a net loss of around 647,710 formal jobs. However, in 2020 and 2021, Mexico managed to recover most of the jobs lost due to the pandemic, with a net increase of around 774,133 formal jobs in 2021 and 500,000 (estimated) in 2021.

- (i) Note: Table 14 shows the information State by State.

Therefore, we can see that using the complex network approach to identify elements with similar characteristics (on the basis of identifying communities) can help classify and observe the phenomena from different fields.

5. Conclusions and Future Work

In this work, we proposed a novel methodology to identify communities in multiplex networks using the robust coloring problem (RCP) and the vertex cover (VCP) algorithms. We demonstrated the efficacy of this approach ($O(n^2)$) in identifying similarity relationships between nodes in a multiplex network, even in the presence of multiple replicas of each node.

To evaluate the real-world applicability of our methodology, we used employment data from Mexico before and during the COVID-19 pandemic. The results showed that our methodology can effectively identify communities in multiplex networks and can classify them on the basis of the multicriteria analysis (with RCP) and the dynamic overtime

(due to the multiplicity of layers). In addition, we used the VCP algorithm to minimize the number of groups and improve the classification on the basis of the similar characteristics between the groups formed by RCP.

Moving forward, we plan to conduct a more detailed analysis of the productive sectors (primary, secondary, and tertiary) across all states in Mexico to gain insight into which sectors were most affected or benefited by the COVID-19 pandemic. Moreover, we aim to apply this methodology to networks with diverse characteristics and structural properties to test its generalizability.

Overall, our methodology represents a promising approach for identifying communities in multiplex networks and has potential applications in various fields, including social networks, transportation systems, and ecological networks.

Data Availability

The databases used to support the findings of this study are available from the corresponding author upon request.

Disclosure

Thanks to Research Square because a preprint has previously been published

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

The authors Edwin Montes Orozco, Roman Anselmo Mora-Gutiérrez, Sergio Gerardo De-Los-Cobos-Silva, Roberto Bernal-Jaquez, Eric Alfredo Rincón-García, Miguel Ángel Gutiérrez-Andrade, and Pedro Lara-Velázquez equally contributed to this work.

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