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Review Article

The Line-Haul Feeder Vehicle Routing Problem: A Classification and Review

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Today, goods transportation is considered to be one of the most important activities of national economics. Logistics and supply chain play an important role in the industry and services, considering the needs of the people, while there is an increase in the population. In addition, the role of logistics in urban areas, especially in restaurants, grocery stores, etc., is clearly visible. Besides, the final price of the goods is the most important factor that is always considered in service and in production. Due to this important factor, transportation has been found to be one of the most significant and influential factors in determining the price of goods. For these reasons, the newest variant of the vehicle routing problem, called the line feeder vehicle routing problem (LFVRP), is considered in this paper, in which various types of vehicles (large and small vehicles) are used for providing services to customers. In this particular type of delivery issue, these vehicles must start from the warehouse, meet customers, and finally return to the depot. In fact, the issue of LFVRP is related to the fast customer service in urban areas because in this case, all that are considered to be of priority are to minimize transportation costs and overall distribution time for fast customer service, especially in urban areas. Due to the many applications of this problem in the real world, a general review of this problem is conducted, and the versions of this problem are described along with the algorithms for its solution in the paper.

1. Introduction

Transportation, as a critical part of human activities, helps and permits maximise other public and financial activities. Every time we use a telephone, keep at a grocery store, study our mail, or fly for any form of business or delight or study, we are the beneficiaries of a system that transfers messages, items, or humans from one position to another. Shipment transportation by means of vehicles is one of the most critical activities today. It is far conjecture that distribution costs account for approximately 1/2 of all logistics costs, and in several professions, which include the food and beverage change, repartition fees can tally for as much as 70 percent of the cost introduced value of products [1–3].

For the first time, Dantzig presented the vehicle routing problem (VRP) in 1959 [4], the vehicle routing problem is one of the maximum challenging combinatorial optimization tasks, that is of interest because of its practical relevance in addition to its vast problem. Given a set of customers in different geographic locations, each with demand for a given good, VRP has to discover a set of routes of minimum duration for a fleet of automobiles to start with located at a relevant depot, to the extent that every customer is visited identically once by vehicles. The goal is to meet the customer's needs in a way that does not exceed the intended capacity of the vehicle. According to the needs of customers and the limited capacity of each vehicle, the vehicle may periodically return to the central warehouse for reloading and revisit each customer [5, 6]. The increase in population around the world and the expansion of urbanization have created problems in daily life that are very important to solve. Logistics and supply chain are the kinds of topics that play a fundamental role in the industry, and state-of-the-art offerings consist of topics that begin from the start of the manufacturing manner of a product and retain till the shipping of the relevant product to the patron [7]. Considering the reduction of transportation problems in urban areas, urban logistics has been an important research topic. In order to stay and survive in the competitive market, logistics companies use different strategies to optimize vehicle routes. Accordingly, researchers are trying to reach a perfect path to deal with several strategies [8] to have the lowest cost because one of the significant parameters that is always considered in production and service delivery is to reduce the final cost of the product. In addition, increasing returns at the company's competitive gain in phases of manufacturing and offering calls for the enterprise's earnings' growth. In addition, another way to reduce costs is to minimize transportation costs, in cases goods are transported from one spot to another at a minimum cost [9].

Despite the abovementioned records, logistics is challenged to keep pace with customer demand as well as improve service to them. The significance of logistics in urban areas is to meet customer demand in an effective way in various departments such as electronic commerce companies [10] because the target customers demand fast and trustworthy transport services during a day or a few hours at the lowest cost. When the demand of all target customers goes beyond the allowed capacity of the vehicle, the number of return trips to the central warehouse plus the travel costs will be very high. To solve this problem, two strategies have been considered to minimize the total travel costs for returning to the warehouse and reloading, which are as follows: (1) creating more warehouses and (2) using a larger vehicle that has more capacity according to the needs of customers. [5]. There is a fundamental limitation in the first strategy. This limitation is that the cost of land and construction costs should be low, which is practically not possible because the price of land and the creation of a warehouse in it include expensive costs [5]. Development, expansion, and the preference of people to settle in the city is one of the main problems of increasing the price of land in the city. In fact, these tendencies have not only increased the price of land but the available space is also very limited [5, 8, 10, 11], especially when distribution warehouses are close to developed urban areas.

The second strategy seems more reasonable but may be associated with problems, specifically when neighborhood streets are slender and tough for large vehicles to pass [5]. To solve this problem, the type of customers can be labeled according to their demand and requests (type-I and type-II), which according to the customer's demand, the right vehicle and also the right route according to the available parameters such as the longitude of the route, the quality of the route, and the time to reach the destination are considered. According to this need, they classified into two types of vehicles to serve the two types of customers we mentioned earlier [5, 11]. In order to survive and stay in the market, logistics companies must look for new, effective, and intelligent ways to provide services to customers. To do this, various companies are adopting new strategies and technologies or modifying and improving their logistics strategy to reduce time and cost.

A completely new type of routing problem, the linear feeder vehicle routing problem (LFVRP), aims to provide fast service to customers in different urban areas. This type of VRP consists of a heterogeneous vehicle fleet with large and small vehicles. In this specific sort of issue, vehicles that have a lower capacity are actually sent as complementary vehicles to help prepare transportation, especially for small packages, with vehicles that have a larger capacity. Due to the fact that motorcycles are both highly affordable in terms of gasoline and side costs, and given their speed, they are capable of moving between motors, mainly at some point of visitors and bustle [10]. Within the LFVRP, small vehicles meet with large vehicles at certain client areas for reloading since these small vehicles have restricted capacity [10]. After loading from a large vehicle, the small vehicle can move immediately. In addition, before the end of the daily work shift, both vehicles return to the warehouse. The difference among LFVRP problem and other vehicle routing problems is that vehicles do not need to be returned to the depot because here large vehicles represent the same mobile warehouses that are present to reload the small vehicle at the junctions. As a result, due to the abovementioned structure, less distance is required, which leads to reduced costs.

2. Literature Review

The virtual era provides many opportunities in urban logistics and has also changed our lifestyle. These days, customers can search for various types of items on their mobile and receive their order at any location they prefer [12]. In advancing the development of customer goals, other problems also appear. According to the report obtained by the U.N., the population of EU cities has reached its limit and has also increased by 20% or more in the last decade. The abovementioned statistics led to several problems, including more demand from people for their own needs, increased traffic, especially more congested ranges with constrained space, increased land prices, especially in more crowded areas, and other things [10, 12].

In a nutshell, a logistics company needs to stand numerous challenges right away due to noneconomical and high land prices in urban areas. In order to receive services, customers must receive services from warehouses that are located further away, which leads to a long tour time and distance. A main hassle is that, slim streets make it tough for large vehicles to move via the city. Due to this issue, using large vehicles is practically difficult and expensive, so it is reasonable to use small vehicles (such as cars, motorcycles, or bicycles). However, utilizing little vehicles comes about in another trouble because small vehicles have limited capacity [5, 8, 11].

Therefore, it is necessary for small vehicles to be loaded many times to provide services to customers in specific places in order to eliminate their limited capacity. As mentioned in the previous section, to improve customer service by vehicles (large and small), the type of each customer was labeled according to factors such as the amount requested and the place of delivery. Type-I customers can be serviced by using each huge and small automobiles, while type-II customers can simply be serviced with the aid of small automobiles [5, 8, 11–14]. Smaller vehicles that have limited loading capacity can reload from larger vehicles that stay in virtual VD depots to compensate for this problem.

Due to the high price of land, especially in urban areas, increasing the number of PDs is difficult and costly for logistics companies. The matter to hold in mind is that if the range of VDs and their locations is regular, the performance of the delivery operation will be much less. In addition, the instability of VDs is due to the truth that the region of VDs may additionally fluctuate every day, depending on the products obtained in addition to the delivered customers.

Consequently, it is reasonable to treat VDs as variables. This behavior means that if a type I customer is served by a large vehicle, it will additionally behave as a VD; in any other case, it can be serviced by either a small automobile or the big vehicle relying at the insertion value [8]. Moreover, PD belongs to a nongovernmental organization, while VD refers to a public parking space [5].

Decreasing the very last cost of products is one of the maximum essential things inside the current era, and one of the methods to reduce items is to limit the fee of transportation in order that the products wanted by customers may be transferred from one location to any other with the lowest cost. Consequently, logistics companies want to pick out new methods to provide smart logistics solutions and to compete in the market to satisfy those purchaser expectancies. To be able to accomplish that, a number of companies, in order to reduce time and costs, use new technologies or modify their logistics methods. For this purpose, to provide quick support and complete customer service in urban areas, a new type of vehicle dispatching method is created, especially for the LFVRP issue. LFVRP can be considered a version of the vehicle routing problem. VRP was first added by Dantzig et al. [4] and extended by Clarke [15] in the famous savings algorithm [11].

VRP and its editions have been widely studied over the last half of the century and are nevertheless a hot topic. The VRP has special extensions, inclusive of the multiple traveling salesman problem with time windows [16], the vehicle routing problem with time windows [3], the stochastic vehicle routing problem [17], the truck and trailer problem [18, 19], and the real-time vehicle routing problem [20]. Due to the limited capacity of vehicles in the VRP problem, vehicles must periodically return to the depot for reloading. If the PD is far from the demand market formed by customers, the journey to reload will be long. To obviate this issue, using multiple VDs in line feeder efficiency is a reasonable and economical option. Logistics companies dispatch smaller cars from physical depot (PD) to serve customers positioned on slim avenue. When the capacity of small vehicles is empty or almost empty, large vehicles that have more capacity transfer the goods to small vehicles (motorcycles) in a certain place. Thus, the use of small vehicles is of considerable help to reduce the traffic caused by large vehicles [13]. Chen et al. [5] proposed a two-stage heuristic to solve the linear feeder vehicle routing problem with virtual warehouses (LFVRP-VDs), in which, in the first stage, strategies are proposed to create initial solutions, and in the second stage, local search is adopted to improve the initial solution.

As the computational complexity of NP class belongs to VRP problem, this computational complexity is applied to different types of LFVRP problem and as a result, it provides research and development platform for heuristic approaches. One of the similarities between LFVRP and VRP is that VRP is one of the NP-hard problems (in fact, it expresses the complexity of the problem). As the computational complexity of NP class belongs to VRP problem, this computational complexity is applied to different types of LFVRP problem and as a result, it provides research and development platform for heuristic approaches and as a result, LFVRP is also one of the NP-hard problems [11, 12], [21]. Approximation algorithms for NP-hard problems (such as LFVRP) provide solutions that are close to optimal but do not guarantee optimality. They offer high-quality solutions that are computationally feasible. In addition, the approximation ratio of these algorithms varies depending on the problem's characteristics. For some problems, approximation algorithms with a ratio close to 1 may provide better solutions, while for others, the ratio may be larger.

Approximation algorithms use techniques such as greedy algorithms and linear programming to provide solutions that are close to optimal. The goal of these algorithms is to provide solutions that are computationally feasible and can be used as a substitute for optimal solutions for problems such as LFVRP. In general, approximation algorithms sacrifice optimality to provide solutions that are close to the optimal. The resulting solutions come with provable guarantees regarding their quality, which are measured by the approximation ratio [22, 23].

In the writings, approximate algorithms are divided into two separate parts as follows: the first part is heuristic algorithms and the second part is metaheuristic algorithms. The two main problems of the heuristic algorithms are the entanglement of the answers in the local optimization as well as the rapid convergence of these points. Heuristic algorithms are the proposed solutions to the problems of innovative algorithms. In fact, metaheuristic algorithms are one of the types of approximation algorithms that have the conditions to escape from local optimal points and can also be used in many large-scale problems. In a similar definition, metaheuristic algorithms are said to be a general framework of algorithms that can provide solutions to the same problem with little variation on various problems. The word was first coined by Glover to combine the Greek words "Meta" and "Heuristic." "Meta" means above or above the current level and "Heuristic" means to find. Some algorithms in this category are inspired by nature and some are not. In addition, some algorithms have memory, that is, they use the results obtained during the execution of the algorithm, and some are without memory.

Metaheuristic algorithms considerably grow the capability to find high first-rate solutions to difficult optimization problems. A commonplace feature among these algorithms is the departure from the local optimum. Approximate algorithms in operations research are algorithms for finding the approximate solutions to optimize problems. These algorithms are often used to approximate the solution of NP-HARD problems because many optimization problems are NP-HARD (in fact, checking the correct answer to such problems is tantamount to solving them in general). Approximation algorithms provide quasioptimal solutions with a coefficient for the approximation rate of the real answer and also guarantee the existence of their answer within the declared error range. For example, their answer is twice the optimal answer, with the difference that they produce their answer in polynomial time. Many optimization problems in mathematics, computer science, and engineering are NP-HARD, so it is improbable that optimal solutions to these problems will be obtained in polynomial time. Approximation algorithms make it possible to obtain solutions close to the optimal solution with a provable coefficient of approximation for this group of problems. Since the LFVRP is known as an NP-hard problem, finding optimal solutions computationally is infeasible, especially for large-sized instances due to the presence of time windows. Therefore, heuristic algorithms and metaheuristic approaches such as local search, population-based search, and learning mechanisms can help find near-optimal solutions for this problem.

Heuristic Algorithms. Heuristic algorithms are problem-solving techniques that aim to find good solutions in reasonable time (without guaranteeing optimality) considering the constraints and requirements of the problem. In LFVRP, heuristic algorithms can be used to generate initial solutions or improve existing solutions. For example, Bräysy and Rönnqvist developed two explorative heuristic algorithms for LFVRP that can provide near-optimal solutions. *Metaheuristic Algorithms.* Metaheuristic algorithms are higher-level strategies used especially for problems with large sizes. In a similar definition, metaheuristics are a general algorithmic framework that can provide problem-specific solutions with minor modifications (unlike heuristic algorithms that are specific to one problem). Generally, metaheuristic algorithms are a set of algorithms that are applied to heuristic algorithms and release them from local optimization, while also allowing the use of heuristic algorithms in a large number of problems. Some of the metaheuristic approaches commonly applied to LFVRP include the following: *Local Search.* Local search algorithms start with a proposed solution and then repeatedly move to neighboring solutions. This is only possible when neighborhood relations and adjacency are defined in the search space of the problem. The local search algorithms evaluate and modify only one or several "current states" instead of systematically and regularly examining paths from the "starting state." Local search algorithms are useful not only for finding objectives but also for solving optimization problems. In these problems, the goal is to find the best state based on the objective function.

2.1. Population-Based Search. Genetic algorithms first generate a set or population of initial solutions. Then, in successive generations, a set of modified solutions is generated. Usually, initial solutions are modified in a way that in each generation, the population of solutions converges towards the optimal solution. These algorithms simultaneously examine several solutions and can escape the local optima. *Learning Mechanisms.* Learning mechanisms refer to the combination of machine learning or reinforcement learning techniques in the solution process. In this algorithm, the optimal solution plays the role of a teacher, and other members play the role of students that the teacher teaches and helps to improve their status. These solutions also strive to improve themselves in the learning process. One of the significant advantages of learning mechanisms is the absence of various parameters. By combining learning mechanisms, algorithms can increase their search efficiency and effectiveness in finding near-optimal solutions [11, 12, 22, 24].

In the previous decades, the centralization has been primarily set on metaheuristic algorithms [3, 25]. In accordance to Laporte [26], metaheuristic algorithms' centralization is based on three basic principles, namely, local search, population search, and learning mechanisms [22]. Local search algorithms move from one solution to another solution in a space of possible solutions (search space) using limited variations until a solution seems favorable or time elapses. Outstanding instances are TS [27, 28], simulated annealing (SA) [29, 30], and adaptive large neighbourhood search (ALNS) [31, 32]. Another case of metaheuristic algorithms is the genetic algorithm (GA). Genetic algorithms represent the principle of population search [33, 34]. Genetic algorithms are one of the random search algorithms in which the genetic evolution of living organisms is simulated. Genetic algorithms for classical optimization methods have been very successful in solving linear, convex, and some similar problems, but it has to be stated that genetic algorithms are much more efficient in solving discrete and nonlinear problems. For example, we can mention the problem to the traveling salesman.

Another case for solving the problem LFVRP is the ant colony algorithm. A presentation of ACO can be found in the study by Reimann et al. [35], and an overview is given in the study of Dorigo and Blum [36].

In addition, Chen [14] exchanged samples with fifteen test samples taken from Solomon's [37] famous sample collection, and in addition, 4 topics including different solution algorithms, customer demands, VD candidates, and examined range of time windows in LFVRP. The wide variety of VDs and their places in a system might not be particular prematurely and need to be specified each time that a delivery and shipping processes are accomplished. Brandstätter and Reimann [11] defined the problem and solved it optimally for a small range of customers. Moreover, they provided efficient algorithms named Linkage approach and cut up approach. The motive of this practice became remedy to problems with large sizes. They examined these algorithms at the unique samples formerly presented by using the study by Chen et al. [5].

The research area of mathematical sciences is one of the new and modern study areas. The simple concept is to mix genuine solution techniques with heuristics. The end result is a hybrid heuristic that takes advantage of both solution techniques. A research by Doerner et al. [38] suggests that current hybrid heuristics, broadly speaking, fit into 3 divisions, namely, set masking, local branching, and decomposition techniques.

3. Time Windows

The time windows have observed a fantastic status in vehicle routing studies and hence consequently need to be addressed . A vital extension of the VRP is the car routing hassle with time windows (VRPTW), wherein all customers must be served inside a certain time window. Commonly, the time window is defined as the earliest start time (EST) and the latest start time (LST) [11]. Solomon [37] has proposed a few exploratory methods for VRPTW, at the side of benchmark instances. In addition, a terrific scrutiny of the solution way for the VRPTW can be observed in the study by Bräysy and Gendreau [39].

In the study by Laporte [37] and in another study by Laporte et al. [40], time windows role play a vital role in vehicle routing research, and as a result, many sorts of VRP have been analyzed with time windows. To display and analyze the time windows, we use TW for short. Time windows are related to the clients for most VRPTW versions. In other words, the vehicle must arrive within a predetermined time frame to begin service for the customer. In the meantime, other time windows such as the opening hours of supermarkets, delivery time windows may also be considered [12]. In general, time windows actually impose many constraints on the objective problem and also increase the complexity of VRP. It should be noted, especially for delivery, that if this time window is too narrow, the positive effect of routing will be significantly decreased [41]. The vehicle routing problem with time windows (VRPTW) is an important and practical problem that is used in many distribution systems.

The feeder line vehicle routing problem with time windows (LFVRPTW) is defined on $G = (N, A)$, where N represents the set of nodes starting from node 0 and continuing to node n, where node 0 represents the warehouse and other nodes represent the location of customers, also the link set $A = \{(i, j): i, j \in N, i \neq j\}$. Large and small vehicles are considered with capacity C_1 and C_2, respectively, and taking into account the parking space, accesses, and available routes, customers are classified into two types: type I customers and type II customers. The difference between type I and II customers is that, type-II customers are served only by small vehicles, while type I customers can be served by either large vehicles or small vehicles. The small vehicle can reload from PD or large vehicle parked at VD. Drivers can choose the reloading location purely based on costs, while the small vehicle load is almost empty or the next customer cannot be served en route, which reduces the range of subsequent trips between the small vehicle location and PD, which is reloadable and reduces operating costs [3, 13], [42].

The tours are related to the possible routes that begin and end in the depot. Some of VRPTW applications include postal package delivery, industrial waste collection, restaurant service, university bus routing, and JIT (simply in time) manufacturing [3]. Preliminary studies of solution techniques for VRPTW introduced by Golden and Assad [43], Desrochers et al. [44], Solomon and Desrosiers [45], Cordeau et al. [46], and Larsen [47] focus mainly on exact techniques. In the logistics market, it is not possible to serve customers 24 hours a day; they need to be serviced during a predistinct time window. In view of this need, a new research topic titled linear feeder vehicle routing problem with virtual warehouses and time windows, or LFVRPTW for short, is presented.

In LFVRPTW, the performance of the logistics system is improved by using large vehicles in VD and for reloading in PD. A type I client can act exactly like a VD when small vehicles need to reload from the VD. In contrast, a non-VD type I customer may be served by a small or large vehicle. With this, the number of vehicles used will decrease in proportion to VRPTW, and as a result, fixed costs will also decrease. In addition, each customer has a certain time (according to the predetermined time window) to receive the service. Vehicles cannot deliver items if the time window violation occurs [5, 13, 14].

LFVRPTW problem has similarities with problems of all its categories (types of VRP). One resemblance is to the heterogeneous fleet of vehicles. Although heterogeneous fleets are often present in real applications, most VRPTW issues consider a homogeneous fleet. A heterogeneous fleet consists of two types of vehicles that are classified based on specific characteristics and considered parameters (e.g., average speed, fixed, and variable costs, as well as vehicle capacity). In writing, this issue is often referred to as VRP (HFVRPTW). In the case of FVRPTW, two decisions must be made, these decisions concerning the routes and arrival times for transportation, as well as the decisions about the vehicles from which customers must receive services. In fact, the main goal is to minimize the additional waiting time for customers at the nodes.

Chen et al. [5] introduced a new two-stage heuristic algorithm with the addition of time windows (LFVRPVDTW), which included the tabu search method. For his or her very last paper (Chen [14]), several troubles for the LFVRPTW had been analyzed, which were distinct solution algorithms, modifications to patron needs, wide variety of type-I clients (VDs), and relaxed time windows. The number of VDs and their placement or service location in a system may not be specified in advance. In fact, the number is determined in each delivery operation [8]. When a VD uses vehicles to exhaust its capacity, it can be considered as a landfill in the GTRPTW time window constrained garbage truck routing problem. Given the time constraints, the goal in the GTRPTW problem is to find a path that minimizes the total cost of the tour while collecting garbage at predetermined stations. Each truck may also need to make more than one disposal trip in a day. When providing services, trucks must unload waste, even supposing the load is not always complete before returning to the depot. Also, researchers have proposed several studies of solution algorithms that have embedded metaheuristic algorithms to solve the GTRPTW problem [48–50].

One of the most important LFVRPTW similarities to the issues mentioned in this article is the use of all these issues with the heterogeneous vehicle fleet. A heterogeneous fleet consists of at least 2 types of vehicles based on certain parameters such as capacity, speed, fixed, and variable costs, which is called heterogeneous fleet VRP (HFVRPTW) problem in the literature (Baldacci et al. [51]; Subramanian et al. [52]; Penna et al. [53]; Kritikos and Ioannou [54]; Koç et al. [55]) as fleet size mix (FSM; Golden and Assad [43]; Liu and Shen [56]; Koç et al. [55]). In VRPTW, each vehicle can only be used for one tour, while in LFVRPTW, multiple tours are possible in each vehicle. However, according to the provision of service by vehicles, if the capacity of the existing vehicle is exhausted, it can return to the warehouse to reload and provide service to the target customers and enter the route again with a tour to provide service to customers. In the literature, this problem is known as multiroute VRP, VRP with multiple use of vehicles or VRP with multiple trips (MTVRP), and an overview is provided by Cattaruzza et al. [57], Cheikh et al. [58], François et al. [59], and Cattaruzza et al. [60].

4. Comparison of LFVRP with Other Variants of VRP

LFVRP can be enumerated as a special case of the heterogeneous HVRP routing problem [51, 52, 54, 61–64], where the combination of fleets with the mentioned characteristics is determined by minimizing costs. As seen in the study by Koç et al. [55] and Mancini [65], a key difference between HVRP and LFVRP is that in HVRP, a fleet of vehicles serves customers with predetermined demands and these vehicles do not return to the depot for reloading along their routes as in LFVRP (Figure 1.).

A connection of LFVRP with open HVRP (OHVRP) [66–70] is that, in both problems, heterogeneous vehicles start their route from the depot and terminate at the end customer without having to return to the central depot. LFVRP and OHVRP are similar due the fact that, they both use a heterogeneous set of vehicles to serve customers. Meanwhile, in LFVRP, large vehicles (trucks) and small vehicles (motorcycles) meet each other at the junction to reload motorcycles, and all vehicles (small and large) after completing their capacity, they return to the depot (Figure 2).

One similarity between the LFVRP problem and the CVRP problem [69, 71–76] is that, they both use the same depot. In addition, each customer needs a certain quantity of goods that have to be brought by a fixed fleet of vehicles [77]. They should be delivered with the purpose of a series of trips that start from the depot and end at the same warehouse. In addition, the most simple version of the CVRP problem is the traveling salesmen problem (TSP) which was solved by an efficient metaheuristic algorithm (Figure 3) [67, 78–82].

The LFVRP is related to the truck and trailer routing problem (TTRP), which has been studied over the recent years by Chao [19], Lin et al. [83], Scheuerer [18], and Villegas et al. [84]. According to Lin et al. [83], VRPT and TTRP have many similarities, yet differ in the constraints related to customer demand, fixed and variable costs, and trailer parking location. The LFVRP is not like each of those problems in that heterogeneous vehicles leave and go back to the depot after serving all clients, and meet at joints for reloading the small vehicle (Figure 4).

LFVRP is very similar to VRP with more than one synchronizing constraint (VRPMS), where there is a multivehicle interdependency between their routes to provide service as mentioned in the study by Drexl [85]. Drexl [86] first introduced VRP with trailers and transport (VRPTT). In VRPTT, trucks are autonomous vehicles that pull nonautonomous vehicles, i.e., trailers. Moreover, a trailer can be used as a mobile warehouse to transfer goods to one or more trucks. In addition, motorcycles meet each other in special places called connection nodes (joints) to provide services to customers from a truck (which is already loaded from trailers), while in LFVRP, all vehicles are heterogeneous (Figure 5).

The LFVRP can be enumerated from many other VRP variants based on the transportation between the large and small vehicle class. To wit, small and large vehicles can meet each other at contact points and move goods to provide services to customers. As an end result, the small vehicle does no longer need to go back to the depot for a reload [10–12, 87–90]. The important point is that, when two vehicles meet each other for loading, they must be in the same place at the same time to carry out the transportation (Figure 6).

5. Solution Algorithm

Chen et al. [5] presented a two-stage heuristic problem in their first study in 2011. Within the first stage, preliminary routes are constructed for the use of a 3-module heuristic and within the 2nd stage, the initial routes are improved for the use of the local search technique provided. In module (1) preconstruction duction of giant route and in the module (2), choosing type-I customers to serve as VDs is a challenging project. The choice needs to bring about minimal added fees, such as travel fees for the large vehicle and also facet journeys for the small vehicle. Regrettably, such costs incurred at every VD cannot be exactly calculated until all reloading operations were completed. To deal with this difficulty, two strategies have been proposed to select a VD, namely, the first case is the threshold method and the module (3), and the second case is the cost-sharing method.

Chen et al. [8] in the same year and in the second study, presented a time window constraint to the problem of LFVRP. In reality, small and large vehicles provide service to customers in a specific time window. Small vehicles on their route may load goods from a physical warehouse or from a virtual warehouse. A 2-stage solution heuristic related to tabu-seek is proposed to resolve this problem. Within the first stage, preliminary routes are constructed and within the 2nd stage, the preliminary answer is improved.

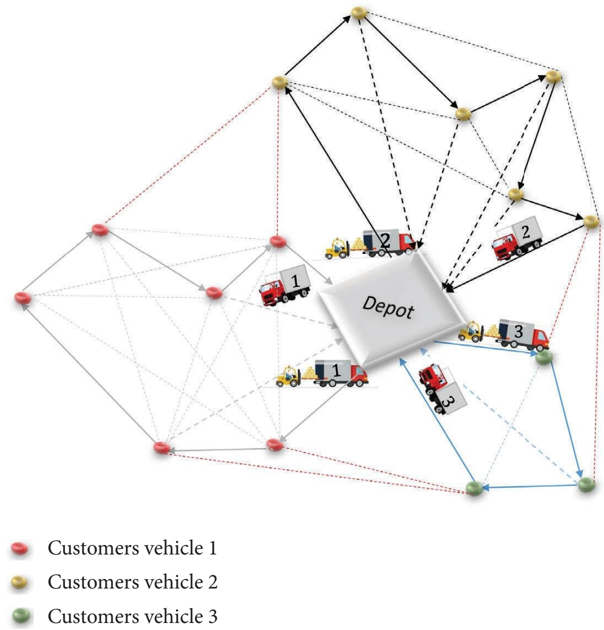

Figure 1: Heterogeneous vehicle routing problem with three types of vehicles.

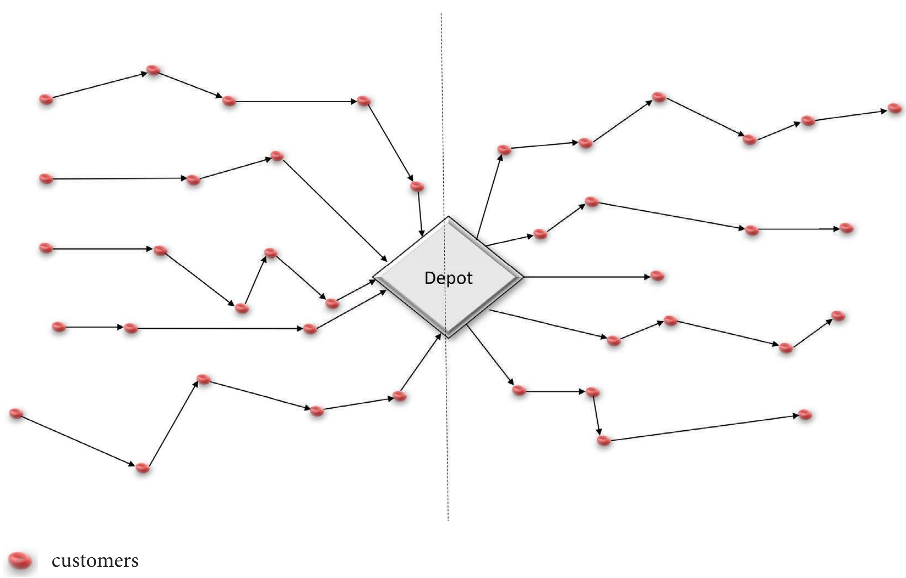

Figure 2: Open vehicle routing problem.

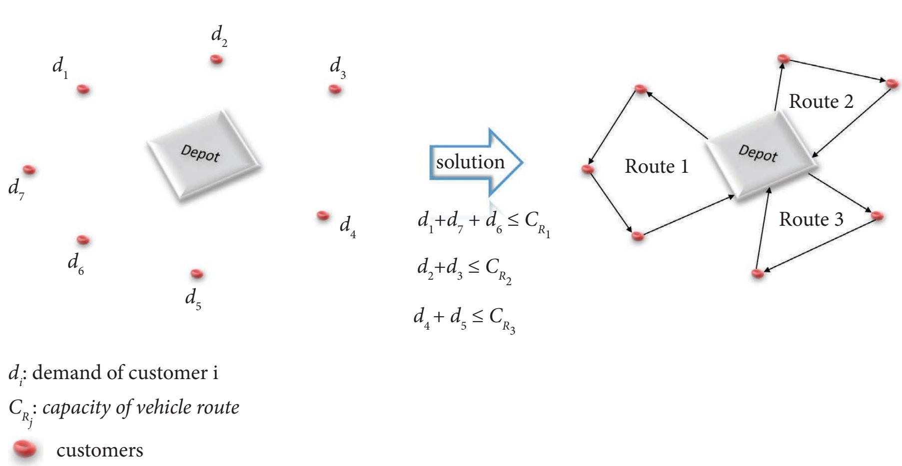

Figure 3: Capacitated vehicle routing problem.

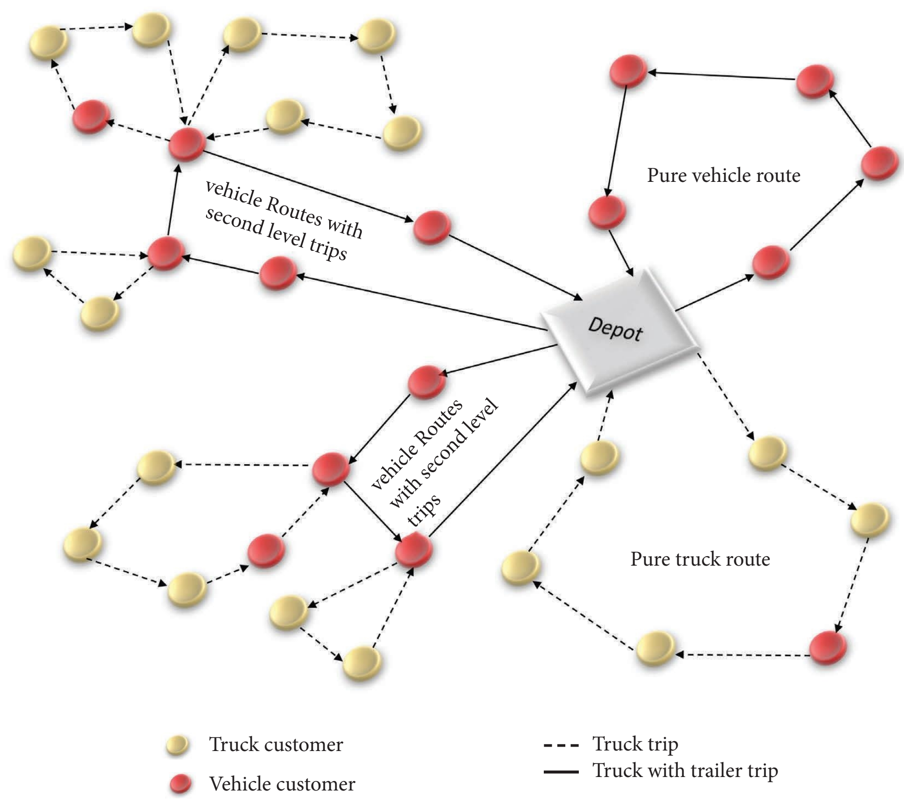

Figure 4: A feasible solution of LFVRP.

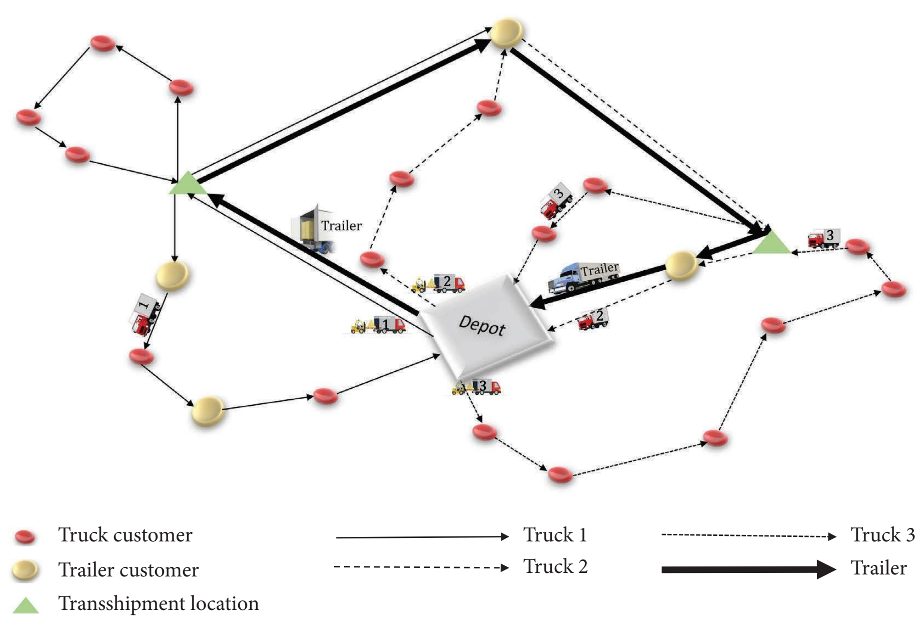

Figure 5: VRP with trailers and trans-shipments.

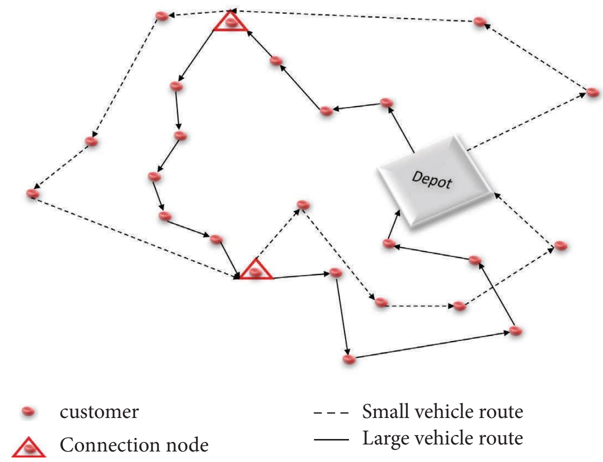

Figure 6: Line-haul feeder vehicle routing problem.

Chen and Wang in 2012 [13] offered an extended linear feeder vehicle routing problem with time windows (ELFVRPTW). To solve this problem, a 2-stage solution heuristic, in which the first stage of the initial routes are constructed using the general insertion method, and in the second stage, the initial solution is constructed using the 1-1 node exchange method and Glover's displacement movement in the improved TS combination [27]. According to Chen in 2015 [14], the LFVRPTW problem can generally be solved by a two-stage solution algorithm [3, 39]. Brandstätter and Reimann in 2018 [11] for the primary time presented a preferred mathematical model for the problem LFVRP. As nicely, they used heuristics inspired by the way of some approximately structural insights to the problem.

In addition to this, they used two techniques, namely, split approach and linkage approach. Brandstätter and Reimann [22], within the identical year and within the 2nd study, proposed and justified numerous upgrades to original algorithms, inclusive of the application of ant colony optimization, different local search operators, in addition to the precise solution of a subproblem. They evaluated every algorithmic element personally as well as their mixture and statistically confirmed the advantages. Brandstätter in 2019 [12] conducted a comprehensive and complete study around time windows. In addition, Brandstätter considered the previously introduced algorithm for the LFVRP problem and also presented a series of separate results for each of the VRPTW and HFVRPTW problems with LFVRPTW, resulting in improved solutions for large time windows. He also acknowledged that for medium time windows, the HFVRPTW problem is slightly superior to the LFVRPTW problem because the LFVRPTW problem requires quantitatively more large vehicles. Finally, all items are summarized in Table 1 [5, 8, 11–14, 22].

5.1. Sensitivity Analysis. As cited inside the structural analysis, LFVRP procedures as compared to HFVRP procedures ought to use from a greater faraway warehouse because it reduces the possibility of transporting cargo by small vehicles for reloading to a physical warehouse. Brandstatter and Reimann [11] also changed service and shipping times. It is clear that shorter shipping times should be in favor of LFVRP approaches. They also studied the effect of small vehicle capacity. The smaller the capacity, the more retransformation should be required, thus requiring more synchronization in LFVRP. Furthermore, the lower capacity of type-II vehicles needs to typically imply that HFVRP calls for more of these vehicles.

Therefore, it is not clear whether the lower requirement of LFVRP compared to HFVRP benefits or harms LFVRP approach types. Finally, the number of type-I and type-II customers ought to play a crucial function. The guess should be that increasing the number of type-I customers should increase the benefits of LFVRP over HFVRP. When type-I customers are very few, it may be difficult to find effective transportation facilities for small vehicles with large vehicles. In particular, they allow all three approaches, namely, HFVRP, linkage, and split to be run once for each instance. For example, when shipping time increases, the superiority of LFVRP disappears, while a warehouse in a more remote location determines the superiority of LFVRP over HFVRP. Increasing the capacity of small vehicles significantly improves the relative overall performance of LFVRP over HFVRP for the linkage approach, while for the split approach, this advantage is the greatest when the SV capacity is not too high. The split approach, beginning with the giant tour, affords greater options for syncing, considering any pair of a success customer at the giant excursion yields an affordable breakpoint for a detour to a virtual depot. This indicates that synchronization has greater efficiency inside the split approach. In the linkage approach, routes are predefined and can only be interconnected at the corresponding end, leaving less space for efficient transportation. However, when the capacity of small vehicles is larger, it is more effective for starting initial journeys and increases its transportation benefits. On the grounds that any pair of successive customers at the giant tour yields an affordable breakpoint for a detour to a virtual depot. Summarizing, the linkage approach outperforms the

Table 1: The improved algorithms.

Author	LFVRP	LFVRPTW	HLFRPTW	ELFRPTW	Tabu search	Objective value	Min cost	Sequential insertion	Cost-sharing method	Threshold method	Linkage method	Split method	Results	Imp. rate (%)	CPU (GHz)	Ref
Chen et al.									Initial solve: 16769 The best solve: 15521	Initial solve: 16849 The best solve: 15337			(1) Number of small vehicles: LFVRP-VD performs best (2) Objective values: the VRP approach, followed by the LFVRP-VD performs best (3) Local search improvement: the LFVRP-VD performs best, followed by the VRP			[5]
Chen et al.		✓				LFVRPTW: 22970 VRPTW: 25101							(1) Number of small vehicles: LFVRPTW are better than those for the VRPTW| (2) Objective values: LFVRPTW are better than those for the VRPTW	12.55 2.60	2.53	[8]
Chen and Wang				✓		LFVRPTW: 18719 ELFRPTW: 18300							(1) Number of small vehicles: ELFVRPTW is advantageous over the LFVRPTW (2) Objective values: ELFVRPTW is advantageous over the LFVRPTW	7.8 6.0	2.53	[13]
Chen		✓					4type I: 23771.7 8type I: 23370.9	4type I: 23080.5 8type I: 22561.32					(1) LFVRPTW usually yields better results than the VRPTW (2) LFVRPTW is beneficial compared with VRPTW (3) Less restrained time window constraints can yield vast gain to the LFVRPTW	−1.72 −2.3	2.53	[14]

Table 1: Continued.

Author	LFVRP	LFVRPTW	HLFRPTW	ELFRPTW	Tabu search	Objective value	Min cost	Sequential insertion	Cost-sharing method	Threshold method	Linkage method	Split method	Results	Imp. rate (%)	CPU (GHz)	Ref
Brandstatter and Reimann											Cost average: 10778	Cost average: 11375	(1) Number of small vehicles: LFVRP is advantageous over the HFVRP (2) They investigated the impact of random aspects of the algorithms by repeatedly running the approaches 10, 100, and 1000 times (3) The linkage approach outperforms the break split approach when the depot is much less far off, or SV capacity is big, we discover that the split approach benefits greater strongly from its larger synchronization potential while the depot is further afield or small	Linkage: 40.3 Split: 32.8	3.1	[11]
Brandstatter and Reimann											Average solomon: 1822 Average chen: 2244 Average total: 2054	Average solomon: 1915 Average chen: 2362 Average total: 2161	They propose and justify several improvements to original algorithms including metaheuristic strategy (MS), metaheuristic algorithm (MA), multiple solutions strategy (MS), and local search strategy (LS) Best found results from both approaches: Average solomon: 1637 Average chen: 2015 Average total: 1845	9	3.1	[24]

Table 1: Continued.

Author	LFVRP	LFVRPTW	HLFRPTW	ELFRPTW	Tabu search	Objective value	Min cost	Sequential insertion	Cost-sharing method	Threshold method	Linkage method	Split method	Results	Imp. rate (%)	CPU (GHz)	Ref
Brandstatter		✓	✓										They able to handle time windows by four improvement strategies including metaheuristic strategy (MS), metaheuristic algorithm (MA), enerating multiple solutions (MS), and local search (LS).		3.1	[12]

split approach, while the depot is much less faraway, or the SV capacity is large.

6. Conclusion

Cities today face new challenges. Some of these challenges are dramatic increases in land prices and customer demand, especially in urban areas, cities with limited space, and increased traffic or additional environmental regulations. Existing types of VRP issues address some of these challenges but not all of them can be claimed. New VRP issues arise in the field of urban procurement, and heuristic approaches must be developed to master these issues. Therefore, new types arise with many challenges. One of these issues is the issue of LFVRP. The LFVRP issue was introduced by Chen et al. about 10 years ago and has since become a rich research area. Several studies have been conducted on this issue, and it is used in many fields in applications. Over the years, the LFVRP problem has been approached optimally by heuristic or metaheuristic methods. Moreover, in recent years, most research efforts have been made to study LFVRP-rich developments such as time windows. Time windows play a special and important role in the city's logistics because the delivery of goods often depends on time (for example, working hours, delivery only in the morning, noon or night, and finally the ban on driving trucks due to air pollution and insufficient visibility). Therefore, due to the new features and also the increase in customer expectations, time windows are highly sensitive and should be considered in vehicle routing research accordingly.

Recent research in recent years has shown that LFVRP uses a heterogeneous fleet of vehicles (small and large) to serve two types of customers. Type-I customers have ample space and can access both classes of vehicles, but type-II customers can only access the SV class due to space constraints. The main and important feature that distinguishes the LFVRP issue from other types of VRP issues is the synchronization between vehicles. In fact, if a small and large vehicle meets after customer service, they have performed the reload operation and it can be said that synchronization has been completed. In general, constructive exploration combined with two-stage exploration involving the search for tabu is the main method of these studies. In the first stage, the initial paths are created. In the second stage, the initial solution is improved by using the tabu search or local search method [23]. As mentioned in the previous sections, two exploratory approaches, linkage approach and split approach have been developed based on the insights of structural analysis. Linkage approach was further developed using four improvement strategies. Metaheuristic strategy (ME) creates small tours for all type 2 clients, while metaheuristic strategy (MA) tries to find the optimal solution for each tour. Furthermore, multiple solution strategy (MS) creates multiple solutions using different construction and reloading techniques. Finally, the local search strategy (LS) uses the destroy-and-repair mechanism to further improve the solution. There are several key challenges and complexities associated with LFVR [8, 11, 22].

(1) Vehicle routing and planning: optimizing feeder vehicle routing is a complex task that involves selecting the best route among the available routes based on the locations of the distribution centers and customers. Factors such as travel time, distance, vehicle capacity, and delivery time windows are taken into account. The challenge in LFVRP is that, several vehicles with varying capacities need to be considered for travel.

(2) Capacity optimization: in LFVRP, vehicles have limited capacities, meaning that the consolidation of loads must be such that each type of cargo is transported by the appropriate vehicle type, and all vehicle capacity is utilized optimally.

(3) Time windows: time windows are of utmost importance in LFVRP. Predefined time windows within a specific time frame require vehicles to deliver services to customers within that time. In addition, the time window of the vehicle itself considers the maximum time that a vehicle can spend on a route.

(4) Balancing cost and service: in LFVRP, there needs to be a balance between the costs associated with meeting customer needs. This means that routes must be designed to provide the best service in the shortest time to minimize costs.

Transportation has become a major issue due to the development of businesses and the competitiveness of the market [42]. Vehicle routing problems (VRPs) have been researched for several years, but it remains a hot topic that researchers continue to explore. The complexity of solving this problem and its various types still exist in the literature [91]. FVRP is highly complex due to the combinatorial nature and the maximum number of possible routes, even for small cities. Logistics companies can improve their service levels, capacity utilization, and customer satisfaction within an acceptable time frame by utilizing all the capacities [8]. In general, solving the feeder vehicle routing problem (FVRP) in urban logistics and supply chain management requires advanced algorithms and mathematical modeling due to its various complexities. Moreover, due to the dynamic nature of routes [91] and the complexities of the supply chain [42], this problem will always remain a challenge that requires continuous updates and improvements. In this study, the dynamics of routes and supply chain characteristics, such as time windows, vehicle types, inventory management, product quality, costs, and flexibility in delivery patterns in urban transportation systems are examined [92]. Another challenge in FVRP is energy consumption, which can be addressed through artificial and alternative delivery methods with battery power [93]. Reducing carbon emissions and improving air quality in areas with high energy consumption is a crucial challenge in urban logistics. Challenges related to transportation costs, fuel, and labor can be higher in areas with faster growth rates than in other areas. For example, traffic in urban areas can cause time wastage and increased energy consumption, which can make FVRP more challenging [92].

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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