Research Article

A New Approach to Selecting Optimal Parameters for the Sliding Mode Algorithm on an Automotive Suspension System

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Stimuli from the road surface cause car vibrations. The suspension system ensures the vehicle’s stability and smoothness when moving. In this research, the author introduces the use of active suspension with a hydraulic actuator to improve the comfort of cars. Car oscillations are described based on a quarter-dynamics model with five state variables. Besides, the nonlinear system’s SM (sliding mode) control algorithm is used to control the system. The SM algorithm’s parameters are optimally selected using the multiple loop algorithm. The multiple loop algorithm allows the values to vary within a predefined limit and provides an optimal parameter based on the vehicle’s vibration conditions. This is a new point in the paper compared to previous studies. The output of the simulation problem includes displacements and accelerations of both the sprung and unsprung masses. In addition, the change of dynamics force at the wheel is also considered when evaluating the interaction between the wheel and the road surface. According to the paper’s results, the vehicle body’s acceleration and displacement values are strongly reduced when using the SM algorithm, compared with cars with only traditional suspension. In addition, the change in dynamics force at the wheel is not significant when using this algorithm. As a result, the car’s smoothness has been improved while the vehicle’s stability has not been lost. The phenomenon of “chattering” still occurs in some cases; however, its effect is negligible.

1. Introduction

The suspension system is an extremely important component of the car, playing a role in ensuring the smoothness of the vehicle when oscillating [1]. Vibrations are generated by stimuli from the road surface, transmitted through the wheels, suspension, and up to the vehicle’s body. The vibration of the car causes fatigue and discomfort for the passengers. Besides, continuous high-frequency oscillations can adversely affect the car’s chassis structure and reduce the quality of the goods that are transported by the car.

Today, the suspension system is much improved and more developed than before. The standard suspension system used on most cars is mechanical (traditional) suspension. The mechanical suspension consists of metal springs (coil springs, leaf springs, torsion bars, etc.), shock absorbers, and a lever arm (or multi-link bar, torsion beam, etc.). According to Satyanarayana et al., the stiffness of the shock absorbers and springs of the mechanical suspension cannot be changed. Therefore, it is called passive suspension [2]. The passive suspension system has a relatively simple structure and high durability. However, the effect on vibration suppression is insignificant, according to Jayachandran and Krishnapillai [3]. As a result, many other modern suspension systems are used to replace passive suspension to improve the vehicle’s smoothness. To change the stiffness of the elastic component, the metal spring must be replaced by a pneumatic spring [4]. According to Sun et al., air suspension helps change the vehicle’s height. Besides, the smoothness of the car is also better [5]. The stiffness of the pneumatic spring is changed based on the pneumatic pressure adjustment inside the chambers. The controller plays the role of providing voltage signals to perform the closing and opening of the pneumatic valves in the system [6]. Another solution for changing the stiffness of the suspension system is to use electromagnetic shock absorbers [7]. According to Tang et al., this is a “semi-active suspension” [8]. Shock absorbers work on changing
magnetic fields when current is applied to the inside of electrodes. Semi-active suspension is structurally simpler than air suspension because it only needs to replace mechanical dampers with electromagnetic ones. However, the smoothness effect that the semi-active suspension system brings is not good. In [9], Rath et al. introduced active suspension to improve ride comfort and maintain wheel traction. According to Liu et al., the active suspension has an additional hydraulic actuator located in place of the suspension system [10]. The fluid pressure inside the actuator is regulated by servo valves [11]. The impact force generated by the active suspension is greater than that of the semi-active and air suspensions. Besides, the system’s response to road surface changes is also better. However, the structure of the active suspension is more complex.

Many studies related to active suspension control have been published in recent years. The control algorithms are designed based on different perspectives from researchers. If the car vibration is assumed to be linear, a PID control algorithm can be applied. In [12], Li et al. proposed using a PID (proportional-integral-derivative) controller with three parameters for the active suspension system. The parameters $k_p, k_i$, and $k_d$ of the PID controller are optimally selected by the FNN (fuzzy neural network) algorithm [12]. These values can change in real time depending on the excitation conditions on the pavement. PID controller values can be tuned using a fuzzy algorithm [13]. In [14], Sweethamarai and Lakshmi proposed using the FOPID (fractional-order proportional-integral-derivative) algorithm for the suspension system. The results of Sweethamarai and Lakshmi [14] show that the response of the FOPID algorithm is better than that of the traditional PID. For systems with more inputs or more outputs, also known as MIMO systems (multiple-input and multiple-output), the LQ (linear quadratic) algorithm should be applied. The equations describing the oscillation states must be converted to the state matrix form when using the LQ algorithm. This algorithm aims to optimize the cost function, i.e., to make its value as small as possible. The parameters of the LQ algorithm can be tuned by the PSO algorithm [15] or the loop algorithm [16]. In fact, the oscillation of the suspension system is nonlinear. So, using algorithms like PID, LQ, and LQG is applicable only in a few specific cases. In [17], Nguyen introduced the SMC (sliding mode control) algorithm for an active suspension model with five variables. Four pavement excitation cases were used in [17], in which the author simulated three different situations. The SMC algorithm needs to use many parameters, so choosing the exact values is extremely difficult. According to Nguyen et al., the parameters of the SM controller can be optimally selected by a closed-loop algorithm based on the condition of finding the smallest average displacement [18, 19]. These values can also be optimized by the GA (genetic algorithm) [20], T2FNN (type-2 fuzzy neural network) [21], STA (super-twisting algorithm) [22], etc. Besides, many modern sliding mode algorithms, such as MFFOSMC (model-free fractional-order sliding mode control) [23], FDOSMC (fuzzy disturbance observer sliding mode control) [24], and ABSMC (adaptive backstepping sliding mode control) [25], have been applied to automotive suspension systems. In addition, many adaptive and robust control algorithms are also commonly used for automotive suspension systems [26–30].

The author proposes using the SM algorithm to control the quarter suspension model in this paper. This is a popular model that has been used in several previous studies. Besides, the SM algorithm is also applied in some studies related to the control of the suspension system. However, there are essential differences in selecting parameters for the SM controller between different publications. In this paper, the controller parameters are optimally selected by the multiple loop algorithm, which is proposed by the author. This algorithm is set up based on both conditions related to road holding and ride comfort requirements. Therefore, this is considered a new point in the paper compared to other publications. This research uses the numerical simulation method, which takes place in the MATLAB/Simulink environment. The structure of the paper consists of four main sections: Introduction, Dynamics and Control, Simulation and Result, and Conclusion.

2. Dynamics and Control

2.1. Sliding Mode Control Theory. Consider the system with an input signal $u = (u_1, u_2, \ldots, u_m)$ described by the following equation:

$$\dot{x} = f(x, u, d, t),$$  \hspace{1cm} (1)

where $x$: state vector in n-dimensional space, $x \in \mathbb{R}^n$, $f$: vector of continuous functions, and $d$: uncertain component, $d(x, u, t)$.

A smooth surface is described by a vector of $m$ smooth functions like (2). This sliding surface has a general form that depends on both the state vectors and time.

$$s(x, t) = \begin{bmatrix} s_1(x, t) \\ s_2(x, t) \\ \vdots \\ s_m(x, t) \end{bmatrix} = 0.$$  \hspace{1cm} (2)

In the ideal case, it is assumed that the smooth surface (2) is independent of time. Therefore, equation (2) can be rewritten in the following form:

$$s(x) = \begin{bmatrix} s_1(x) \\ s_2(x) \\ \vdots \\ s_m(x) \end{bmatrix} = 0.$$  \hspace{1cm} (3)

A task for the sliding mode controller is to determine the control signal $u(t)$ to bring system (1) towards the sliding surface (2) and fix its position on the sliding surface.

$$u(t) = \begin{cases} u_M(t); s(x, t) \neq 0, \\ u_{eq}(t); s(x, t) = 0, \end{cases}$$  \hspace{1cm} (4)
where \( u_M(t) \): signal that causes \( x(t) \) to move towards the sliding surface \((2)\) and \( u_{eq}(t) \): signal that helps to fix the position of \( x(t) \) on the sliding surface \((2)\).

According to \((4)\), at time \( t = t_0 \), the control signal \( u_M(t) \) must bring the object \( x(t) \) towards the sliding surface \((2)\). Then, the control signal \( u_{eq}(t) \) must generate \((6)\) to fix the position of the object.

\[
s(x(t_0), t_0) = 0,
\]
\[
\dot{s}(x(t), t) = 0 \quad \text{for} \quad t \geq t_0.
\]

If the sliding surface \((3)\) is used to replace the sliding surface \((2)\) and a positive definite function is described by \((7)\), the control signal \( u_M(t) \) needs to generate the sliding condition \((8)\); then, we can bring \( x(t) \) towards the sliding surface.

\[
V(s) = \frac{1}{2} s^T s,
\]
\[
\dot{V}(s) = \frac{1}{2} s^T \dot{s} = 0
\]

If \((1)\) is well-defined and has an affine structure, it is rewritten as

\[
\dot{x} = f(x(t), t) + G(x(t))u.
\]

\( G(x, t) \) is the matrix defined by

\[
G(x, t) = (g_1(x(t), t), g_2(x(t), t), ..., g_m(x(t))).
\]

Taking the derivative of the sliding surface \( s(x, t) \):

\[
\dot{s}(x, t) = \frac{\partial s}{\partial x}(f(x(t), t) + G(x(t))u_{eq}(t)).
\]

According to condition \((6)\), the derivative signal of \((11)\) must be zero, i.e.,

\[
\dot{s}(x, t) = \frac{\partial s}{\partial x} f(x(t), t) + \frac{\partial s}{\partial x} G(x(t))u_{eq}(t) = 0.
\]

If the coefficient of the control signal \( u_{eq}(t) \) is not degenerated, this signal can be determined by

\[
u_{eq}(t) = \begin{bmatrix} \frac{\partial s}{\partial x} f(x(t), t) \\ \frac{\partial s}{\partial x} G(x(t)) \end{bmatrix}^{-1}.
\]

The control signal \( u_M(t) \) is determined by the signal \( u_{eq}(t) \) and the error signal \( \Omega \).

\[
u_M(t) = u_{eq}(t) \pm \Omega,
\]

\[
\Omega = \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \vdots \\ \Omega_m \end{bmatrix}.
\]

Taking the derivative of the sliding surface \( s(x, t) \):

\[
\dot{s}(x) = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} x + \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} f(x(t), t) + \frac{\partial s}{\partial x} G(x(t))u_{eq}(t) \pm \Omega.
\]

\[
\dot{s}(x) = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} f(x(t), t) + \frac{\partial s}{\partial x} G(x(t))u_{eq}(t) \pm \Omega.
\]

From \((6)\), \((12)\), and \((16)\), we have

\[
\dot{s}(x) = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} G(x(t))\Omega.
\]

Assume that the sliding surface \( s(x, t) = s(x) \) satisfies condition \((18)\).

\[
\frac{\partial s}{\partial x} G(x(t)) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\]

The error signal \( \Omega \) is rewritten as

\[
\Omega_i = -b_i \text{sign}(s_i(x)) \quad i = 1, m,
\]

\[
\text{sign}(s_i(x)) = \begin{cases} +1 & s_i > 0 \\ -1 & s_i < 0 \\ 0 & s_i = 0 \end{cases}
\]

The condition for the control signal \( u(t) \) to bring the vector \( x(t) \) back to the sliding surface \( s(t) \) is called the sliding condition. The sliding condition commonly used for nonlinear functions is described by

\[
\frac{\partial s}{\partial x} f(x, u, d, t) \leq -\lambda |s|^j.
\]

The differential \((21)\) is expressed in complex form. Therefore, it is necessary to come up with a simpler form of the original function that satisfies this condition.

\[
V(s) = \frac{1}{2} s^2.
\]

2.2. Dynamics Model. A quarter-dynamics model is used for simulating car vibrations. Compared with other models, such as the spatial model and the half model, the quarter model is simpler. Because the author designed a direct controller, which considers the influence of the hydraulic actuator, it is reasonable to use the quarter-dynamics model. This helps to reduce the complexity of the problem. Considering the suspension model with the sprung mass \( m_1 \) and the unsprung mass \( m_2 \) (Figure 1), these two masses perform two vertical displacements, \( x_1 \) and \( x_2 \).

The equations describing the vibrations of the suspension system have the following form:
\[ m_1 \ddot{z}_1 = F_{SK} + F_{SC} + F_{SA}, \]  
\[ m_2 \ddot{z}_2 = -F_{SK} - F_{SC} - F_{SA} + F_{TK}. \]  

The parts of (23) and (24) on the left are the inertial forces, and the parts on the right are the binding and actuating forces of the actuator.

Elastic force of the suspension spring, \( F_{SK} \):

\[ F_{SK} = S_{K}(z_2 - z_1). \]  

Damping force of the suspension damper, \( F_{SC} \):

\[ F_{SC} = S_{C}(\dot{z}_2 - \dot{z}_1). \]  

Elastic force of the tire spring, \( F_{TK} \):

\[ F_{TK} = T_{K}(z_3 - z_2). \]  

Substituting (25), (26), and (27) into (23) and (24), we get

\[ m_1 \ddot{z}_1 = S_{K}(z_2 - z_1) + S_{C}(\dot{z}_2 - \dot{z}_1) + F_{SA}, \]  
\[ m_2 \ddot{z}_2 = T_{K}(z_3 - z_2) - S_{K}(z_2 - z_1) - S_{C}(\dot{z}_2 - \dot{z}_1) - F_{SA}. \]  

The force, \( F_{SA} \), generated by the actuator depends on the pressure change inside the hydraulic cylinder (Figure 2).

\[ F_{SA} = (P_1 - P_2)S_p. \]  

The servo valves perform the role of changing the fluid flow that is supplied to the hydraulic cylinder. This process is described by (31) and (32).

\[ \left( \dot{P}_1 - \dot{P}_2 \right) = \frac{4\beta}{V} \left( Q - C_i (P_1 - P_2) - S_p (\dot{z}_1 - \dot{z}_2) \right). \]  
\[ Q = C_d w x_v \sqrt{\frac{P_0 - (P_1 - P_2) sgn(x_v)}{\rho}}. \]  

From (31) and (32), we can find the change in fluid pressure.

\[ \Delta P = P_1 - P_2 = -\frac{V}{4\beta C_t} \Delta \dot{P} + C_{di} w x_v \frac{C_t}{\rho} \left( \frac{P_0 - \Delta P sgn(x_v)}{\rho} - S_p \left( \dot{z}_1 - \dot{z}_2 \right) \right), \]  

where \( \beta \): fluid bulk module, \( C_i \): leakage parameter, \( Q \): hydraulic load flow, \( C_{di} \): discharge parameter, \( w \): valve area gradient, \( V \): cylinder effective volume, \( \rho \): liquid density, and \( P_0 \): initial pressure.

According to the (33), the liquid flow inside the cylinder depends on the displacement of the servo valve \( x_v \). This type of sliding valve performs displacement based on the voltage signal supplied by the controller.

\[ x_v = \frac{1}{\tau} \int (k_v u(t) - x_v) dt. \]  

Taking the derivative of (34), we get

\[ x_v = k_v u(t) - \tau \dot{x}_v. \]  

Substituting (34) and (35) into (30), the relationship of the acting force \( F_{SA} \) on other factors is shown by the following equation:

\[ F_{SA} = \frac{V S_p}{4\beta C_t} \Delta \dot{P} + \frac{C_{di} w (k_v u(t) - \tau x_v) S_p}{C_t} \frac{P_0 - \Delta P sgn(k_v u(t) - \tau x_v)}{\rho} - \frac{S_p^2}{C_t} (\dot{z}_1 - \dot{z}_2). \]
(36) is a very complex nonlinear differential equation. According to Nguyen et al., this equation is roughly linearized to become [16]

\[
\dot{F}_{SA} = \theta_1 u(t) - \theta_2 F_{SA} - \theta_3 (\dot{z}_2 - \dot{z}_1),
\]

(37)

where \( \theta \) are the coefficients of the equation (refer to Table 1).

Let state variables from \( x_1 \) to \( x_5 \) be

\[
x_1 = z_1, \\
x_2 = \dot{z}_1, \\
x_3 = z_2, \\
x_4 = \dot{z}_2, \\
x_5 = F_{SA}.
\]

Derivatives of state variables from \( x_1 \) to \( x_5 \) in turn:

\[
\dot{x}_1 = x_2, \\
\dot{x}_2 = \frac{1}{m_1} (-S_K x_1 - S_C x_2 + S_K x_3 + S_C x_4 + x_5), \\
\dot{x}_3 = x_4, \\
\dot{x}_4 = \frac{1}{m_2} (S_K x_1 + S_C x_2 - (S_K + T_K)x_3 - S_C x_4 - x_5), \\
\dot{x}_5 = -\theta_1 x_2 + \theta_3 x_4 - \theta_2 x_5 + \theta_1 u(t).
\]

Let \( y(t) \) be the output signal of the system:

\[
y(t) = z_1 = x_1.
\]

(38)

Table 1: Reference parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_K )</td>
<td>40500</td>
<td>Nm^{-1}</td>
<td>Suspension spring stiffness</td>
</tr>
<tr>
<td>( S_C )</td>
<td>3160</td>
<td>Nsm^{-1}</td>
<td>Suspension damper stiffness</td>
</tr>
<tr>
<td>( T_K )</td>
<td>175000</td>
<td>Nm^{-1}</td>
<td>Tire spring stiffness</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>400</td>
<td>kg</td>
<td>Sprung mass</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>37</td>
<td>kg</td>
<td>Unsprung mass</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>539561</td>
<td>Nm^{-3/2}kg^{-1/2}m^{-1/2}V^{-1}</td>
<td>Suspension actuator coefficient</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>1</td>
<td>s^{-1}</td>
<td>Suspension actuator coefficient</td>
</tr>
<tr>
<td>( \theta_3 )</td>
<td>5512500</td>
<td>Nm^{-1}</td>
<td>Suspension actuator coefficient</td>
</tr>
</tbody>
</table>

Take the first derivative of the output signal:

\[
\dot{y} = \dot{x}_1 = x_2.
\]

(41)

Take the second derivative of the output signal:

\[
\ddot{y} = -\frac{T_K}{\chi m_1} x_3,
\]

(42)

where \( \chi \) is the proportional constant. According to [18], the inertia forces of the unsprung mass and the sprung mass are assumed to be proportional.

Continuing to take the third and fourth derivatives of the output signal \( y(t) \):

\[
y^{(3)} = \frac{T_K}{\chi m_1} x_4,
\]

(43)

\[
y^{(4)} = -\frac{T_K}{\chi m_1 m_2} (S_K x_1 + S_C x_2 - (S_K + T_K)x_3 - S_C x_4 - x_5).
\]

(44)

Finally, by taking the fifth derivative of the output signal, \( y^{(5)} \) is obtained.

\[
y^{(5)} = \frac{T_K}{\chi m_1 m_2} \left( \frac{S_K S_C (m_1^{-1} + m_2^{-1})}{m_1^{-1} + m_2^{-1}} + (S_C m_1^{-1} + S_C m_2^{-1} - S_K - \theta_3)x_2 + (S_C m_1^{-1} - (S_K + T_K)m_2^{-1})x_3 + (S_C m_1^{-1} - S_C m_2^{-1} + (S_K + T_K) + \theta_3)x_4 + (S_C m_1^{-1} - S_C m_2^{-1} - \theta_2)x_5 + T_K \theta_1 u(t). \right)
\]

(45)

(45) is in a very complicated form. It can be rewritten as (46) once the coefficients of the variables are compacted.
where

\[ a_1 = T_K \chi m_1 m_2, \]
\[ a_2 = T_K \theta x_1 \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix}, \]
\[ b_1 = S_K S_C \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix}, \]
\[ b_2 = S_C^2 \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} - S_K - \theta_3, \]
\[ b_3 = -S_C \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix}, \]
\[ b_4 = -S_C^2 \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} + \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} + \theta_3, \]
\[ b_5 = -S_C \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} - \theta_2. \]

Let \( e(t) \) be the error signal between the set point signal and the output signal of the system.

\[ e(t) = y(t) - y_{sp}(t), \] (48)

\( y_{sp}(t) \) is the set point signal.

The controller’s sliding surface, \( s(e) \), is the fourth derivative of the error signal.

\[ s(e) = \sum_{n=0}^{4} \lambda_n e^{(4-n)} = e^{(4)} + \lambda_1 e^{(3)} + \lambda_2 e + \lambda_3 \dot{e} + \lambda_4 \ddot{e}, \] (49)

where \( \lambda_i \) are the coefficients of the polynomial (50) such that it is a Hurwitz polynomial.

\[ p(\phi) = \sum_{j=0}^{n-2} \lambda_j \phi^j = \lambda_0 + \lambda_1 \phi + \lambda_2 \phi^2 + \lambda_3 \phi^3 + \lambda_4 \phi^4. \] (50)

Combined with (46), (49), and (50), the control signal \( u(t) \) is described by the following equation:

\[ u(t) = a_3 \left( y_{sp}^{(5)} + \sum_{n=1}^{4} \lambda_n e^{(4-n)} - \sum_{n=1}^{5} b_n x_n + P \text{sgn} \left( \sum_{n=0}^{4} \lambda_n e^{(4-n)} \right) \right), \] (51)

where \( P \): equation parameter and \( a_3 \): equation coefficient.

\[ a_3 = \chi m_1^{-1} m_2^{-1} T_K \theta_1. \] (52)

The controller parameters are selected by the multiple loop algorithm (Figure 3). According to this algorithm, the optimal computed values include \( P \) and \( \lambda_i \). Firstly, the limited values of \( P \) and \( \lambda_i \) need to be determined. This aims to limit the range of values to look for so that the calculation can happen faster. Secondly, the simulation run is done with the coefficients of \( P \) and \( \lambda_i \), respectively, from their smallest to maximum values. For each parameter, we will obtain the output values of the car, including displacement and acceleration. Besides, the criteria related to phase difference and chattering phenomena are also considered. This process is repeated many times until all values in the bounding range have been reached. Finally, optimal parameters are determined to ensure that the output values of the car oscillation are minimal.

The optimization process is illustrated by (53) and (54).

\[ \begin{align*}
\lambda_1 & \rightarrow z_1, \Delta \phi_1 \\
\lambda_2 & \rightarrow z_2, \Delta \phi_2, \text{Min}(z, \Delta \phi) \lambda_{i, \text{optimal}} \\
\vdots \\
\lambda_k & \rightarrow z_k, \Delta \phi_k, \text{Min}(z, \Delta \phi) p_{i, \text{optimal}} \\
P_1 & \rightarrow z_1, \Delta \phi_1 \\
P_2 & \rightarrow z_2, \Delta \phi_2, \text{Min}(z, \Delta \phi) p_{i, \text{optimal}} \\
\vdots \\
P_k & \rightarrow z_k, \Delta \phi_k, \text{Min}(z, \Delta \phi) p_{i, \text{optimal}}
\end{align*} \] (53)

Previous algorithms often focused only on optimizing parameters according to one or two specific criteria, such as vehicle body displacement and acceleration values. These values are optimized according to the minimum or mean value conditions. Unlike the existing algorithms, the new algorithm described in this article helps ensure all four criteria simultaneously (as mentioned above). At the same time, the displacement and acceleration values are guaranteed under both conditions (including the maximum and mean values) rather than just one. This provides novelty to the article.

Using an in-loop algorithm can help select values optimally. Besides, many other solutions are also used to design control parameters for modern mechatronic systems. These solutions can be found in [31–33].

The optimal parameters found in this study can be applied to many different cases. However, if the input excitations differ too much, it can reduce the controller’s performance when using these parameters.

3. Simulation and Result

3.1. Simulation Cases. The input signal of the simulation is bumpy from the road surface. In this work, the author proposes three types of excitation signals from the pavement, corresponding to three specific cases. According to Figure 4, the first case uses a cyclic excitation signal. This signal is sinusoidal with a small amplitude and frequency. The cyclic excitation signal is commonly used in many problems simulating the oscillation of suspension systems. The mathematical function describing the type of cyclic excitation is shown by the following equation:

\[ z_3 = h_0 \sin 2\pi ft + \phi. \] (55)

The second case uses a step signal, also known as a pulse signal (Figure 4). This step signal has a larger amplitude than the periodic signal; the excitation pulse appears only once and is then maintained stably. This signal describes the state when the wheel goes up against an object and remains in this state for a certain amount of time without changing.
Equation (56) describes a mathematical function of the pulse signal, which is used in this paper. This is the Heaviside function with two steps.

\[
\begin{cases} 
  z_3 = 0; & t < t_0, \\
  z_3 = h; & t \geq t_0, 
\end{cases}
\]  

(56)

The pavement excitation in the second case can produce a significant acceleration, but it only lasts for a short time. Therefore, it is necessary to use another case to describe high-frequency oscillations in a continuous time interval. The third case uses a random function to describe the stimulus from the pavement. In this case, the frequency of the excitation changes continuously instead of being a fixed value like the sinusoidal excitation.

\[
z_3(t) = -2\pi \int_0^t \left[ f z_3(\tau) - \sqrt{Gv} \omega(\tau) \right] d\tau. 
\]  

(57)

Parameters for calculation and simulation are referenced in Table 1. CARSIM® software is used to reference automotive specifications, while [19] is used to cite actuator parameters.

There are three specific scenarios investigated in each case. In the first situation, the vehicle only employs a traditional passive suspension system with no controller (None). In the other two situations, the car is equipped with an active suspension with different controls, PID and SM. The simulation outputs include the values of displacement and acceleration of both the unsprung and sprung masses. Besides, the change in dynamics force at the wheel is also taken into account. The compared values include each object’s RMS value, maximum value, and minimum value.

3.2. Result and Discussion

3.2.1. The First Case. In the first case, the author proposes to use the excitation signal from the pavement, which has a sinusoidal shape. Neither the amplitude nor the frequency
of this excitation is large, so the car oscillation is small. Figure 5 depicts the change in vehicle body displacement during the simulation period. According to this result, the displacement in the first phase of the oscillation is the smallest (None). The displacement value in subsequent phases is stable, reaching a maximum of 66.20 mm. Meanwhile, the maximum value of the car body displacement when the vehicle uses active suspension is only 18.74 mm for the situation using the PID algorithm and 8.03 mm for the situation using the SM algorithm. Besides comparing the maximum value of the oscillation, the RMS value of the oscillation should also be considered (for continuous oscillations). According to the calculation results, the RMS value of the car body displacement reached 43.84 mm, 13.14 mm, and 5.59 mm, respectively, corresponding to the three investigated situations. The difference between SM and None results is up to 8.24 times (for the maximum value) and 7.84 times (for the RMS value).

Vehicle body acceleration is often mentioned when considering the smoothness of a car. If the value of acceleration is too large, the vehicle may lose its smoothness. In the first phase, the vehicle suddenly oscillates (Figure 6), so the acceleration increases rapidly. In subsequent phases of the oscillation, the value of the acceleration is more stable, reaching a maximum of 2.25 (m/s²), 1.74 (m/s²), and 1.03 (m/s²), respectively, corresponding to None, PID, and SM. The values of the acceleration, when calculated according to the RMS criterion, are 1.62 (m/s²), 0.49 (m/s²), and 0.21 (m/s²), respectively, in the above order. Compared with the vehicle with only passive suspension (None), the average value of acceleration is only 12.96% if the vehicle uses active suspension (SM) and 30.25% (PID). The car’s smoothness can be further improved using the SM algorithm to control the suspension.

Although the SM algorithm is a robust control algorithm, it can still cause “chattering.” This phenomenon causes the acceleration signal to be noisy before the phase transition (Figure 6). However, because this fluctuation is not too large, it does not have a negative impact on the smoothness of the car. Investigating the “chattering” phenomenon in other complex oscillation cases is necessary to comprehensively assess the system’s stability when using the SM algorithm.

Two criteria are considered when evaluating the suspension system: ride comfort and road holding. In many oscillation cases, improving the quality of only one criterion is expected, while the other is not guaranteed. If the smoothness is increased, the actuator must work harder, producing more force. This can cause the unsprung mass to oscillate more, resulting in reduced holding between the wheels and the road surface, causing instability when traveling. Conversely, smoothness can be lost if the suspension strictly guarantees the interaction between the wheels and the road. It is challenging to meet both of the above criteria simultaneously. Therefore, it is necessary to investigate the vibration of the unsprung mass further to assess the interaction between the wheel and the road surface when the vehicle body vibrates strongly.

The variation of the vertical displacement of the unsprung mass is shown in Figure 7. According to this finding, the displacement values for all three situations are nearly the same; the difference between them is negligible. Even the displacement of the unsprung mass when a car uses a controlled suspension is smaller than that of an uncontrollable suspension.

The acceleration of the unsprung mass is obtained by taking the second derivative of the displacement. As shown in Figure 8, the value of the acceleration reaches its maximum at the first phase of the oscillation (because of the sudden oscillation of the vehicle). In the subsequent phases, the acceleration varies periodically with time and obeys the sinusoidal law. As mentioned above, “chattering” can occur when an oscillating signal is about to transition to the next phase. However, the effect of this phenomenon is relatively small. The smoothness of the car is still well guaranteed in this case.

The oscillation of the car causes a change in the dynamics force at the wheel. If the car oscillates more strongly (both the unsprung mass and the sprung mass), the change in the
dynamics load will be more significant. Once the force at the wheel is reduced to zero, the wheel can easily be lifted off the road, causing instability. According to the results of Figure 9, the car situation without a controlled suspension has the most significant variation in dynamics forces. If the active suspension is used, dynamics force changes are smaller. The minimum value of dynamics force in all three situations is 3163.57 (N), 3423.89 (N), and 3625.58 (N), in the order (None), (PID), and (SM). The RMS values of the dynamics force in this order are 4357.79 (N), 4298.90 (N), and 4291.47 (N). It can be seen that the change in dynamics force, in this case, is not large. This is because the excitation signal that is used for simulation has a periodic form with a small amplitude and frequency. It is necessary to use excitation signals of higher frequency and amplitude to simulate vehicle oscillations.

3.2.2. The Second Case. In the second case, the excitation signal from the pavement takes the form of a single pulse (step form). According to the results shown in Figure 10, the vehicle body is suddenly shifted to a new step (corresponding to the stimulus). In the case of a car with only passive suspension, the new body height is equal to the height of the step, reaching 50.0 mm. Meanwhile, the body’s height is reduced even more to only 23.9 mm and 10.0 mm when using the active suspension system with PID and SM algorithms, respectively.

In this case, the stimulus from the road comes on suddenly. Therefore, the value of the vehicle body acceleration increases rapidly (Figure 11). The maximum values of acceleration are 13.69 (m/s²), 13.17 (m/s²), and 9.57 (m/s²), respectively, for the three situations (None), (PID), and (SM). After the vehicle body has reached a steady state in the new position, the acceleration will decrease to zero. Because this is not a continuous oscillation, the values associated with the RMS will not be considered.

There is not much difference in displacement for the unsprung mass between the three situations examined (Figure 12). In this case, the hydraulic actuator generates a large impact force to maintain the body’s height. Therefore, the displacement of the unsprung mass when using the SM algorithm is more significant than that in the standard suspension. This only happened for a short time.

The value of the acceleration of the unsprung mass in the second case is very large. This is caused by the sudden oscillation of the vehicle (Figure 13). This sudden oscillation causes the wheel’s dynamics force to drop drastically (Figure 14). According to the results shown in Figure 14, the wheel has been separated from the road surface. However, the separation of the wheel also only happens for a short time. Under the action of the spring’s elastic force and the damping’s resistance, the wheel can quickly return to contact with the road surface.

3.2.3. The Third Case. The last case uses a random high-frequency pavement excitation. Furthermore, the excitation amplitude is more extensive than that in the previous two cases. Because this is a random oscillation, the displacement and acceleration of the body do not follow any particular laws.
Figure 15 depicts the change in vehicle body displacement over simulation time. Looking at the graph more closely, one can see that car body displacement is most significant when the vehicle does not have a controlled suspension. Meanwhile, displacement value decreases sharply when cars use modern suspension systems with the SM algorithm, which was proposed earlier. The maximum value of automobile body displacement can be up to 75.11 mm, 32.14 mm, and 13.72 mm, while their RMS values are 35.71 mm, 15.79 mm, and 6.64 mm. Compared to cars using the traditional suspension, the body displacement when using the active suspension, controlled by the SM algorithm, is only 18.59%. This result contributes to increased confidence in the SM algorithm’s superiority.

The acceleration of the car body in the last case is relatively large (Figure 16). Besides, the change in acceleration is continuous. According to the results obtained from the simulation, the maximum acceleration and RMS acceleration of the (SM) situation are 3.06 (m/s²) and 1.03 (m/s²), respectively.
respectively, while those of the situation (None) are 4.38 (m/s²) and 1.48 (m/s²). The result obtained for the situation (PID) is also close to the situation (None).

The displacement of the unsprung mass (Figure 17) between all three situations is roughly the same, while the acceleration of the unsprung mass using the SM algorithm is larger than the other two scenarios (Figure 18). This is because the hydraulic actuator generates a more considerable impact force to help stabilize the oscillation (in the case of random oscillation). Therefore, increasing the force causes the unsprung mass to oscillate more. The result is an increase in the acceleration of the unsprung mass while the acceleration and displacement of the car body are still well maintained. This issue can be solved by flexibly changing the controller parameters rather than having them have fixed values. However, this is quite complicated because it is necessary to combine fuzzy algorithms to accomplish this. Besides, this does not affect the interaction between the
### Table 2: Simulation results (the first case).

<table>
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<tr>
<th></th>
<th>Sprung mass</th>
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<td>Acceleration (m/s²)</td>
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<td><strong>Max</strong></td>
<td><strong>RMS</strong></td>
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<td>Passive</td>
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### Table 3: Simulation results (the second case).

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<td>Static</td>
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### Table 4: Simulation results (the third case).

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wheel and the road surface. According to Figure 19, the change of dynamics force in the car situation using the SM algorithm is the smallest (according to the RMS criterion) compared to the other two situations. Therefore, if the SM algorithm is used, the vehicle’s smoothness can be improved while the car’s stability is not affected.

The results obtained from the simulation are listed in Tables 2–4.

4. Conclusions

The smoothness of a car depends on the performance of the suspension system. Active suspension helps provide better performance than traditional passive suspension systems. In this paper, the author proposes using an active suspension system with the SM algorithm to improve the vehicle’s oscillating smoothness.

Automotive vibrations are described by a quarter model that is equipped with an additional actuator (for active suspension). The primary oscillation source is the excitation from the road, which is the input to the simulation problem. The output of the simulation problem includes the values of displacement and acceleration of both the unsprung and sprung masses, as well as the change in the dynamics load.

The simulation is performed with three specific cases (in each case, three different situations are created). According to the research results, both displacement and acceleration of the car body were significantly reduced when using the SM algorithm to control the suspension. Besides, the acceleration and displacement of the unsprung mass do not change much compared to the other two situations. This helps to keep the wheels in good contact with the road surface.

The phenomenon of “chattering” still occurs in some survey cases; however, its influence is not considerable. The car’s smoothness can be improved using the SM algorithm to control the suspension while the wheel’s road holding is still guaranteed. Future studies may aim to minimize the influence of the “chattering” phenomenon when using the SM algorithm.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

References


