Hindawi
Complexity
Volume 2024, Article ID 3016105, 20 pages
https://doi.org/10.1155/2024/3016105

Research Article

The Mesolevel Economy in Nineteenth-Century England and Wales: Applying Input-Output Accounting and Spatial Interaction Modelling to the Historical Study

Guy S. Solomon[1,2] **and Alan G. Wilson**[1]

[1]*The Alan Turing Institute, London, UK*
[2]*University of Glasgow, Glasgow, UK*

Correspondence should be addressed to Guy S. Solomon; guy.solomon@glasgow.ac.uk

Received 21 September 2023; Revised 1 February 2024; Accepted 24 February 2024; Published 28 March 2024

Academic Editor: Hiroki Sayama

This paper is offered as a contribution to understanding historical trade. The method of input-output modelling is utilised to evaluate the evolution of cities (and their associated regions) in England and Wales between 1851 and 1911. The construction of input-output accounts for each city region is based on "heroic" data assumptions, which enable the construction of a demonstration model illustrating a new iterative approach to historical analysis. In its current application, the model enables estimates to be made of mesolevel trade between cities, which enhances our analysis of urban evolution in this period.

1. Introduction

One of the major contributions of history is to examine change and evolution in systems of interest over time. In our case, we take this focus and apply it to cities in the nineteenth century—the process of their change and evolution, the key drivers of these processes, and the implications of these findings for wider understandings of the industrial revolution. In urban history, there have been extensive qualitative studies and a more limited number of quantitative approaches—in the latter case, often under the aegis of economic history (for example, [1–3]). There is also a significant literature which connects urban modelling developed in contemporary contexts to applications in history (for example, [4–7]). Our contribution is rooted in combining the output of the former with the analytical framework of the latter.

We take the—oft neglected—mesoscale as our focus. This means we are seeking to understand the economy of England and Wales in a way which allows us to concentrate on finer spatial scales (below the national level), while also simultaneously considering the relationships between these smaller units in the economy as a whole. We do this by constructing a set of input-output accounts for each town and city in England and Wales, by combining smaller "parishes" into urban units and their associated rural areas (which together we term a "city region"). This enables us to compute sector-specific trade flows between each pair of city regions in our dataset, which in turn facilitates analysis of their internal supply-demand relationships (including the balance-of-trade between a city region and the broader national economy), as well as geographic concentrations in the production and trade of different economic sectors. In doing so, we are able to examine the interdependence of the economies of towns and cities over the period 1851 to 1911—as well as how this evolved over a pivotal period of economic growth, which has important legacy effects for the British economy in the modern day.

In formal modelling terms, our ultimate goal is to build dynamic models—in which cities (and their associated regions) are nonlinear dynamical systems and as such are path dependent with change occurring through a series of phase transitions at critical values of model parameters (for a short overview, see Wilson [8]). Here, we present an important step in laying the foundations for this process: a demonstration model for a series of (mostly) decadal time points over the period 1851 to 1911. Representing this change through a time series of equilibrium models can fail to account for the phase changes present in a dynamic system, but it can nonetheless yield useful insights regarding the evolution of cities over the nineteenth century. By establishing equilibria at a series of points in time—which result from change in exogenous variables—we provide the basis for a future dynamic version (the algebraic basis of which we outline in Appendix 1, which draws on ideas in Wilson and Dearden [9] and applies them to this context).

The combination of historical analysis and computational modelling has significant potential for a mutually beneficial research framework. The former provides a starting point by identifying the drivers of change which need to be incorporated into the model. The model can then be used to assess the extent to which these drivers are interdependent, as well as highlight phase transitions. Further historical analysis at a microscale can be employed to examine how these changes came about. The outcomes of this research can then be used to further refine the model, and the process can be repeated.

A model-based analysis can therefore be employed to demonstrate points in time at which phase changes become possible, and historical study can then investigate their precise conditions. In the general area of our research, there have been several major contributions which might explain the growth trajectories of cities—entrepreneurs who recognised opportunities for innovation, pre-1850 industrial specialisation which facilitated later changes (either through creating necessary infrastructure or human capital formation), and the relationship between scientific developments and industrial applications—on both a macroscale and a microscale, but examining these factors systematically poses a significant research challenge [1, 2, 10, 11].

While others have focus on transport networks or endowments (for example, [12–14]), we focus more widely on input-output accounts for a system of cities—and then on trade as a feature of economic development. Both national accounting and international trade have been a major focus of historical research, and there are many examples of local- or firm-level studies of specific industries (for example, [15–19]), but estimating intercity trade (either by building a system of input-output accounts or some other means) remains a significant challenge for historians, primarily as a consequence of limited direct data. We therefore anchor our work in building a time series of input-output accounts and associated trade flows to fill this gap in the modelling base. Here, we adopt a broad definition of "cities" to mean "towns and cities" with more than 10,000 population and use the terms "urban" and "city" more or less interchangeably.

To understand this focus, it is necessary to note that the main subsystems representing cities in a comprehensive urban model are interdependent—as outlined by Figure 1. The left-hand (red) side of the diagram represents the demography of the city, generating individuals and households who live, work, and use a variety of services. The right-hand (green) side represents the economy of the city: generating housing, employment, products, and services. These subsystems are all linked through the (blue) transport and communications subsystems.

These models can be built at various spatial scales—from a system of cities in a regional or national setting to single city regions with a fine-grain spatial structure (for example, [22–24]). In our case, the economic model on the right-hand side of the diagram is based on input-output accounts that represent both the internal economy of a city and trade between cities in a broader-scale system of cities, but it is informed by the demographic information on the left-hand side. Adopting the spatial scale where cities (and their associated regions) are the focal units is the most effective way of adding a spatial dimension to a "whole country" analysis.

In the sections that follow, we outline the core concepts—with particular reference to input-output accounts and modelling at different scales—and explain the process of creating demonstration models that both offer insights in their own right and lay the foundations for future research. While we focus on a scale with towns and cities (including their "city regions") as the spatial units, the model specification allows for the potential of extending this research to finer spatial scales in the future.

2. Intercity Input-Output Accounts

2.1. Overview. The adaptation of the input-output accounting framework [25] developed here originates from a sketch in Oléron-Evans [26], which translates a model from Levy, Oléron-Evans, and Wilson [27] focused on the whole world (with countries as the spatial units) to a single country with cities (and their associated rural areas) as the spatial units. The specification—which we further develop here—roots the model in input-output accounts for each city region, including both a representation of trade flows between each pair of city regions in the network, and between these city regions and the rest of the world. The model itself is novel in terms of its methodology: we estimate city region input-output accounts, building on the information provided by national accounts in earlier work by linking these with other data sources [28, 29]. As might be expected, such an ambitious exercise involves making "heroic" assumptions, but we believe this to be worthwhile enabling us to build a distinctive demonstration model which provides a platform for further work. Our findings must therefore be interpreted with this in mind—and there are many refinements which might be made—but they nonetheless generate insights that add to conventional historical analysis due to their grounding in highly detailed demographic information.

2.2. The Accounting Framework. The key variables which will be the basis of our model (all for an unstated time period) are listed in Figure 2. Note that for any variable, if an index i is replaced with an asterisk, this implies summation over i.

Theoretically, E_i^m and M_i^m can be dealt with by adding a rest-of-the-world zone to the spatial interaction model discussed below. However, in this implementation, these flows are specified exogenously.

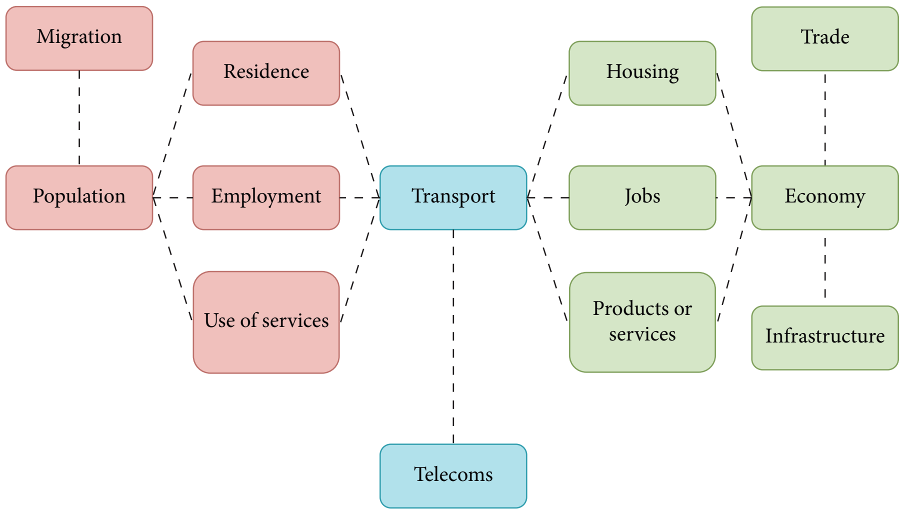

FIGURE 1: The main elements of a comprehensive model. This diagram draws on Figure 1.1 in Wilson [20]. It can also be found in Rees et al. [21].

Variable	Description
P_i	population of city i
Q_i^m	a demographically-weighted labour value of those employed in sector m in city i
$\boldsymbol{X_i^m}$	total production of sector m in city i: $m = 1, 2, 3, \ldots S$ and $i = 1, 2, 3, \ldots N$
$\boldsymbol{F_i^m}$	final demand for m in i
$\boldsymbol{e_i^m}$	total exports of m from i to other cities (excluding international exports)
$\boldsymbol{m_i^m}$	total imports of m into i from other cities (excluding international imports)
$\boldsymbol{E_i^m}$	international exports of m from city i
$\boldsymbol{M_i^m}$	international imports of m to city i
a_i^{mn}	the amount of m needed to produce a unit of n in i
$\boldsymbol{x_i^{mn}}$	the total demand for m to produce a unit of n in i
$\boldsymbol{y_{ij}^m}$	the trade flow (internal export) of m from city i to city j; y_{ij}^m is the internal import flow from j to i.
c_{ij}^k	the cost of transporting one unit from city i to city j, by mode k. Note that for simplicity mode k is kept constant across all sectors, but this could be relaxed in future implementations.

FIGURE 2: The main variables.

While most of the variables listed in Figure 2 are self-explanatory, the X_i^m, F_i^m, and x_i^{mn} terms require some additional explanation. X_i^m can be understood as total production less imports for i (where i is a geographic unit). It is therefore a measure of what is actually produced in a location, but it is not the total value of the sector, which is made up of $X_i^m + m_i^m + M_i^m$.

F_i^m is more complex. "Final demand" in a national accounting framework has a defined meaning, and while the major elements are present in our input-output data, the list is not quite so extensive. F_i^m is taken to mean the sum of consumption and investment; as outlined below, exports are specified separately.

We can then define x_i^{mn} (the total demand for m to produce a unit of n in i) in relation to a_i^{mn} (the amount of m needed to produce a unit of n in i), such that

$$x_i^{\text{mn}} = a_i^{\text{mn}} X_i^n. \tag{1}$$

The core input-output accounting equation can then be given as

$$X_i^m + m_i^m + M_i^m = F_i^m + e_i^m + E_i^m + \sum_n a_i^{mn} X_i^n. \quad (2)$$

Cities (or any other geographic unit) balance their accounts through external imports and exports. We therefore introduce a balancing term V_i^m which can be used to identify the extent of the imbalance when (2) is in its initial state (prior to establishing domestic trading relationships, with $m_i^m = 0$, $e_i^m = 0$):

$$V_i^m = X_i^m + M_i^m - F_i^m - E_i^m - \sum_n a_i^{mn} X_i^n. \quad (3)$$

In other words, the "balance" is the total production plus imports, minus all demand. If the balance is positive, the geographic unit (i) can generate exports to balance its accounts (in the present application, the city region exports the surplus production to other city regions within England and Wales); if the balance is negative, instead imports (from other city regions) are necessary to meet the demand. For the sake of simplicity, we ignore the possibility of stockpiling.

We then predict the reconciliation of these imbalances through a doubly constrained spatial interaction model. This could be specified in a number of ways, but here, we assume that city regions either export when $V_i^m > 0$ or import when $V_i^m < 0$. In effect, this zeros out the attractiveness term where a city region does not require imports to balance the demands of a particular sector (and vice versa). We acknowledge that this is a drastic oversimplification, but it also creates a significantly less complex mathematical problem, as many potential "city pairs" are eliminated. The specific sectors included are given in Figure 3, and the number of towns and cities for each year is given in Figure 4—a full list of named towns and cities is provided in Smith and Bennett [30]—but in the most computationally intensive year (1911), there are approximately 2.1 million potential city pair-sector combinations; employing this approach reduces the actual city pair-sector combinations to be balanced through the spatial interaction model to much more manageable 425,000.

For the city level, we have both estimates of population (P_i) and a proxy (Q_i^m) for employment (weighted by demographic characteristics to account for the relative importance of women and children in the workforce of the mid-nineteenth century). As has been stated, "cities" are actually "city regions," as this allows for complete geographical coverage of England and Wales—and the process of estimating each "city region" is described below. We then use a national input-output table and scale each of the national variables down to a city level using the following equations—utilising population where the relationship is governed by demand and employment where this is determined by supply:

$$X_i^m = \frac{X_*^m Q_i^m}{Q_*^m}, \quad (4)$$

$$F_i^m = \frac{F_*^m P_i}{P_*}, \quad (5)$$

$$E_i^m = \frac{E_*^m Q_i^m}{Q_*^m}, \quad (6)$$

$$M_i^m = \frac{M_*^m P_i}{P_*}. \quad (7)$$

As noted, we assume that a city can only "export" (within England and Wales) if V_i^m is positive and can only "import" if V_i^m is negative. That is,

$$\text{if } V_i^m > 0, \text{then } e_i^m = V_i^m \text{ and } m_i^m = 0, \quad (8)$$

and

$$\text{if } V_i^m \leq 0, \text{then } m_i^m = |V_i^m| \text{ and } e_i^m = 0. \quad (9)$$

In other words, the imbalance is made up by exports or imports (as appropriate) from other cities. e_i^m and m_i^m are therefore determined by the initial conditions—although their destinations/origins (and therefore, the flows of trade) are determined by the spatial interaction element:

$$y_{ij}^m = A_i^m B_j^m e_i^m m_j^m \exp\left(-\beta^m c_{ij}^k\right), \quad (10)$$

where

$$A_i^m = \frac{1}{\sum_j B_j^m m_j^m \exp\left(-\beta^m c_{ij}^k\right)}, \quad (11)$$

$$B_j^m = \frac{1}{\sum_i A_i^m e_i^m \exp\left(-\beta^m c_{ij}^k\right)}, \quad (12)$$

which ensures that

$$\sum_i y_{ij}^m = m_j^m, \quad (13)$$

and

$$\sum_j y_{ij}^m = e_i^m. \quad (14)$$

The spatial interaction model is doubly constrained, to ensure that the total value of (domestic) import flows is equal to the total value of (domestic) export flows—in other words, because we control for international imports and exports in the national input-output table (and account for these in (6) and (7)), imbalances in the supply of, or demand for, particular sectors in an individual city region must be reconciled through the domestic economy.

1841 (HHW)	1907 (Thomas)
Agriculture	Agriculture, forestry, etc.
Mining and quarrying	Coal mining; Other mining; Coke ovens
Metal manufacture	Iron and steel; Non-ferrous metals
Metal goods	Engineering; Metal goods, n.e.s.; Shipbuilding; Motor and cycle; Railway rolling stock
Textiles, clothing, and leather goods	Cotton and silk; Woollen and worsted; Hosiery and lace; Other textiles; Jute, hemp, & linen; Textile finishing; Clothing; Boot and shoe; Leather and fur
Food, drink, and tobacco	Food processing; Drink; Tobacco
Soap, candles, and dyes	Soap and candle; Oils and paint
Other manufacturing; Bricks, pottery, and glass	Chemicals; Explosives; Paper; Printing and publishing; Rubber; Furniture; Other wood; Miscellaneous manufactures; Timber trades; Building materials
Construction	Building, etc.
Gas and water	Gas, electricity and water
Public administration and defence	Public utility
Distribution; Transport	Distributive services
Other service; Domestic service; Housing services	Other services; Laundry

Figure 3: Mapping Horrell, Humphries, and Weale's 1841 input-output table to Thomas' 1907 input-output table.

Year	Total number of towns and cities
1851	152
1861	172
1881	263
1891	306
1901	363
1911	404

Figure 4: The number of towns and cities include in the model, 1851–1911.

Equations (10)–(14) ensure that this is carried out in the most efficient manner—city regions will be predicted to import or export locally if they can do so but can also have a trading relationship with a distant city region if necessary. This is particularly important for sectors where production is geographically concentrated—such as mining or textiles—and examples of this are discussed below. Due to the limited information regarding transport infrastructure for the period of study, Euclidean distance between the centroid of each city (rather than the centroid of the city region, as the actual urban area is considered the organisational hub) is used as a proxy measure. As a further consequence of this limited information, we do not differentiate between different methods of transport in the current application of the model—although the model specification retains the flexibility to adopt this approach in future work.

This method also allows us to estimate "value added" at a city region level. Although we refer to this aspect of the generated input-output tables as "value added" as a shorthand, in this instance, it is in essence the difference between the total value of the sector and the sum of intermediate production and imports: all of which are ascertained from (2). While this is a fairly coarse measure, it nonetheless provides a good indicator of the general value of a sector to the economy of the city region.

3. Drivers and Interdependence

The model specification here stops short of a fully dynamic implementation, but it nonetheless contains many of the drivers associated with economic change. In Figure 5, we outline how these relate to the model variables. The first column notes the potential driver of change ("driver"), the second details how this driver can be observed ("observable aspects"), and the third maps this onto model variables ("representation"). Many of these aspects are currently proxied or implicitly modelled by variables (especially the a^{mn} coefficients), but it is possible that these could be specified directly (using submodels) in the future.

As is evident from Figure 5, the a^{mn} coefficients play an important role in defining the relationships between sectors [25]. The data sources employed for these are discussed in

Drivers	Observable aspects	Representation
Technology (both as exogenous change, and the *possibility* of exogenous change)	Machinery and mechanisation	Changing a^{mn} coefficients; changing Q_i^m size/composition.
	Industrial concentration and development	Changing X_i^m, in combination with c_{ij}^k.
	Infrastructure: transport (e.g. roads, canals, railways)	Changing unit costs (c_{ij}^k), although presently c_{ij}^k is proxied by distance and is therefore (mostly – barring boundary changes) constant
	Infrastructure: construction and utilities	Changing a^{mn} where m = construction
Demographic characteristics and change	Age structure and mortality	P_i and Q_i^m
	Regional concentration	
	Migration	
Economic factors	Employment	Changing Q_i^m
	Income	Mainly represented through a^{mn} but also through known quantities such as M_i^m and E_i^m.
	Prices	a^{mn} represents unit prices for supplies to n.
	Trade	
Provision and organisation of services	Number, size, and clustering of firms	Through a^{mn} for the service sectors.
	Cost, frequency, and form of passenger transport	Through Q_i^m where m includes services. Development of infrastructure associated with c_{ij}^k.
Technological impacts on agriculture	Yields-per-acre	Through a^{mn} where m = agriculture.
	Productivity per worker	
	Parliamentary enclosure	Q_i^m where m = agriculture.
	Changes in crop rotations	
	Arable: pasture ratio	
Organisation of industry	Factory system	Through a^{mn}
	Wheat storage	
Urban form	City boundaries	Through c_{ij}^k
	Land use	Changing geospatial boundaries, but proxied by Q_i^m
	Regional specialisation	
Government	Central (e.g. legislation, parliamentary commissions)	Through investment and taxes
	Local	

Figure 5: Variables and model representation.

Section 4, but—as outlined in equation (1)—the input-output method hinges upon a^{mn} terms accounting for the intermediate demands of each sector. By incorporating these intermediary sector flows, we are able to demonstrate the interdependence of different sectors—and this has important implications which are not immediately apparent when only final demand is considered.

First, some sectors used a notable proportion of their own output to meet their demands—this is particularly important for "textile, clothing, and leather goods" and "metal manufacturing." However, these proportions were not static and represent changes in manufacturing production processes. Notably, *increased* technological innovation did not inherently mean a *reduction* in these requirements—both previously mentioned sectors *increased* their internal consumption per unit production over the period as they became *more* technologically sophisticated, seemingly as an outcome of increasingly complex products and broader consumer choice.

Second, some sectors were closely reliant on others. This is informative at a macrolevel, but at a mesolevel, it also incorporates a distinctly *spatial* element. For example, "metal manufacturing" understandably had a notable relationship with "mining and quarrying," which often led the two to be closely located to one another—indeed, in south Wales, coal pits and iron foundries often functioned under the same enterprise. However, if two related sectors were not closely geographically located, then this generates a trade flow—which would not be highlighted by measures of final demand/output.

4. Applications

We now turn to the task of applying the model to historical data. The model outlined in Section 2 requires us to specify time points, spatial units, and sectors. Much of the challenge is the assembly of these inputs from data sources which must be aligned with one another. Where data are not available with sufficient resolution or granularity, it is necessary to infer or estimate the required information from the closest available.

The simplest of these is perhaps time points: we take the decadal census years, starting with 1851 (the first surviving complete national census for England and Wales) and ending in 1911 (the most recent historical census available to academic researchers). We have to exclude 1871, as this was not digitised as part of the Integrated Census Micro-Data (I-CeM) project—which created digitised and codified records for each person in the census, including their basic demographic (name, age, sex, and family structure), geographic (the parish in which they were found at the time of the census), occupational, and other information [31]. The I-CeM database therefore provides us with population and occupation (used as the basis for our employment/labour measure) by parish for England and Wales. We presently exclude Scotland and Ireland—although there are complete data available for the former—due to the availability of other data sources.

I-CeM classifies individuals on the basis of their occupation and parish, but it does not inherently allow for analysis of city units. For this, coding from the BBCE (the British Business Census of Entrepreneurs)—which directly builds upon the I-CeM data—is employed. The BBCE identifies "urban" parishes and groups them based on a list of towns and cities identified by the Robson-Law database [32]. This is an imperfect method for our purposes as city boundaries did not usually conform to parish ones, but it does permit identification of predominantly urban areas.

By aggregating the individuals associated with each urban unit, we can therefore produce a labour estimate for each city identified in the data. As demographic factors (including child labour) played a more pronounced role in determining productivity in this period, each sector in each city is weighted into an "equivalent labour value"—which in essence accounts for the proportion of men, women, and children which constituted the labour force for each city. For the present model, these weightings are constant (see Appendix 2) although it is possible to vary these weightings by sector and city.

Notably, a significant proportion of national economic activity in this period took place outside of urban centres, particularly for certain industries (such as agriculture and mining and quarrying). One approach to solving this problem would be to create rural zones in addition to urban ones. However, this would drastically increase the size of the zonal-pair matrix; if we assume that each city had *at least* one rural zone, then the number of *ij* pairs would increase approximately fourfold—and often a single rural zone would fail to represent the complexity of the urban-hinterland exchange structure.

Instead, we opt to expand the geographical city unit to include its surrounding rural areas. There are several established methods for this type of operation. Within the geography literature, both market areas and central place theory might be applied [33, 34]. Percolation theory might also be applicable [35]. Here, given the paucity of sufficient data, we opt for a simpler approach. Therefore, each non-urban geographic unit (in this case, a historic census parish) is assigned to the city which maximises the function:

$$K\left(\frac{1}{c_{\mathrm{ij}}^{2}}\right)+L\left(\log\sum_{m}\mathrm{Q}_{\mathrm{i}}^{\mathrm{m}}\right), \tag{15}$$

where K and L are weighting terms. The weighting of $K = 8$ and $L = 0.003$ in this instance is determined experimentally to ensure that rural parishes are assigned to a city region primarily on the basis of their proximity to the urban boundaries of each city, but that larger cities (as determined by their productive capacity) have a longer reach. This is necessary because of the diversity in "urban" units in the dataset—the influence of a small town (the minimum for inclusion being 10,000 population) on its surrounding area is not necessarily equal to that of a large urban centre with population in millions. Nonetheless, weighting favours the former, and therefore, the calculated city regions are typically contiguous (although there are a very small number of exceptions). In areas with a high density of towns (which particularly occurs towards the end of the period), city regions may consist of a single (urban) parish. Conversely, in isolated areas, towns may provide the central point for multiple rural parishes in addition to the urban area of the town itself. This method has the notable shortcoming that each city region is distinct—in reality, these would have overlapped—but such a simplification is necessary with the available information. As there are limited accessible road data for the nineteenth century, major roads from the OpenStreetMap project are used as a proxy for the most likely transport routes between each parish and the centre of each urban unit, with the data obtained and routes determined using the OSMnx Python package [36].

An overview of the results of this exercise is visualised in Figure 6. By establishing which parishes can be classified as belonging to each city, we can then link these to the geospatial units provided by a set of census boundary shapefiles [37]. There are no parish boundary datasets for the entirety of our period, so in this instance, we approximate the position of parish units to their "registration subdistrict"—this is less precise than the parish but still provides a sufficiently accurate approximation for our purposes and allows us to avoid the caveats and complications of working with approximated "constant" parish units [38].

The distinction between a "city" and a "city region" is important to consider and is therefore discussed in relation to the results below. However, it is necessary to highlight that the cities likely played an important organisational role in this period—particularly with the limitations of the transport network—and were therefore very important to their local economies. As a consequence, although there are some city regions with a notable proportion of their population outside of the city boundaries—with Norwich being perhaps the most significant example—they still likely had an important economic organising effect.

This approach allows us to segment the entirety of the national economy (once Scotland and Ireland have been subtracted) —and because the parish units are tied to the Census data, this also allows us to infer population and occupational numbers for each city region. By classifying occupations into sectors which map onto our national input-output data (which we do by classifying the 797 "occupational codes" given to each employed individual in I-CeM to the sectors listed below), we can infer the economic contribution of each city region using the method outlined in Section 2.

Such an approach still requires national-level economic data. There is a significant body of literature concerning British industrial output during the nineteenth century. This addresses both national (typically, GDP) and sectoral levels. This debate has typically been framed around the rate of growth, in an attempt to understand the timing and speed of the industrial revolution [16, 39]. But comparatively little research has concentrated on intersectoral linkages within the British economy. Nonetheless, the literature provides two major contributions which (mostly) bookend our period of interest—Horrell, Humphries and Weale's work (HHW) for 1841 [29], and the Thomas thesis for 1907 [28]. Both are significant historical contributions, combining a vast array of published sources to create national input-output tables for their year of interest. We are therefore able to extract both the input-output tables and technical coefficients from their work and use them to inform our own estimates.

The first issue with this process is that—despite the HHW (1841) table being significantly influenced by Thomas' (1907) table—the sectors do not directly map onto each other. The latter originally contains 41 sectors and the former 17, likely as a reflection of the comparative complexity of the economy in both periods. It is therefore necessary to construct the sector map given in Figure 3). This is an imperfect process and comes with the following caveats:

(i) Thomas' table indicates small linkages without monetary values. These are replaced with 0.1 (essentially rounding up). This approach may overweight transactions including textiles due to small sector aggregation, but it does follow the HHW method.

(ii) "Domestic service" does not feature as a separate sector in Thomas' table, so needs to be combined into "other service" in 1841.

(iii) To match Thomas' work, "distribution" and "transport" in the HHW table are combined. This is nonoptimal, given their distinct roles in an economy.

(iv) With the lack of a suitable comparator in Thomas' table, "housing services" are added to general service sector in HHW. It is not entirely clear where the equivalent falls in Thomas' table.

(v) Public utility in Thomas' table has relatively little description and may not include defence spending (in contrast to HHW).

The second issue is that these input-output tables are constructed for the United Kingdom. Due to data limitations (outlined above), our work focuses on England and Wales. It is therefore necessary to deflate these values accordingly. We do this in accordance with the Scottish and Irish shares of GDP for this period, taken from Geary and Stark's regional GDP work [40]. Geary and Stark do not give a value for 1851; it is likely that the Irish figure was influenced by the famine for this year, but we adopt an estimate in-line with Geary and Stark's other years. In any case, the results are not heavily influenced by this figure, and it is only relevant when directly comparing the value of trade between 1851 and another year.

HHW and Thomas therefore provide much of the data necessary for dynamically estimating an input-output table for each year between 1841 and 1911. However, some further information is required:

(a) a^{mn} coefficients

(b) Value added, by sector

(c) Exports, by sector, for 1911

In the case of a), we assume a linear relationship between our two data points. It is likely that individual industries differed from this growth curve, and that this would also be reflected at the sector level (albeit to a lesser extent). However, this should provide a reasonable approximate over growth over the 70 year span. This can be expressed as follows:

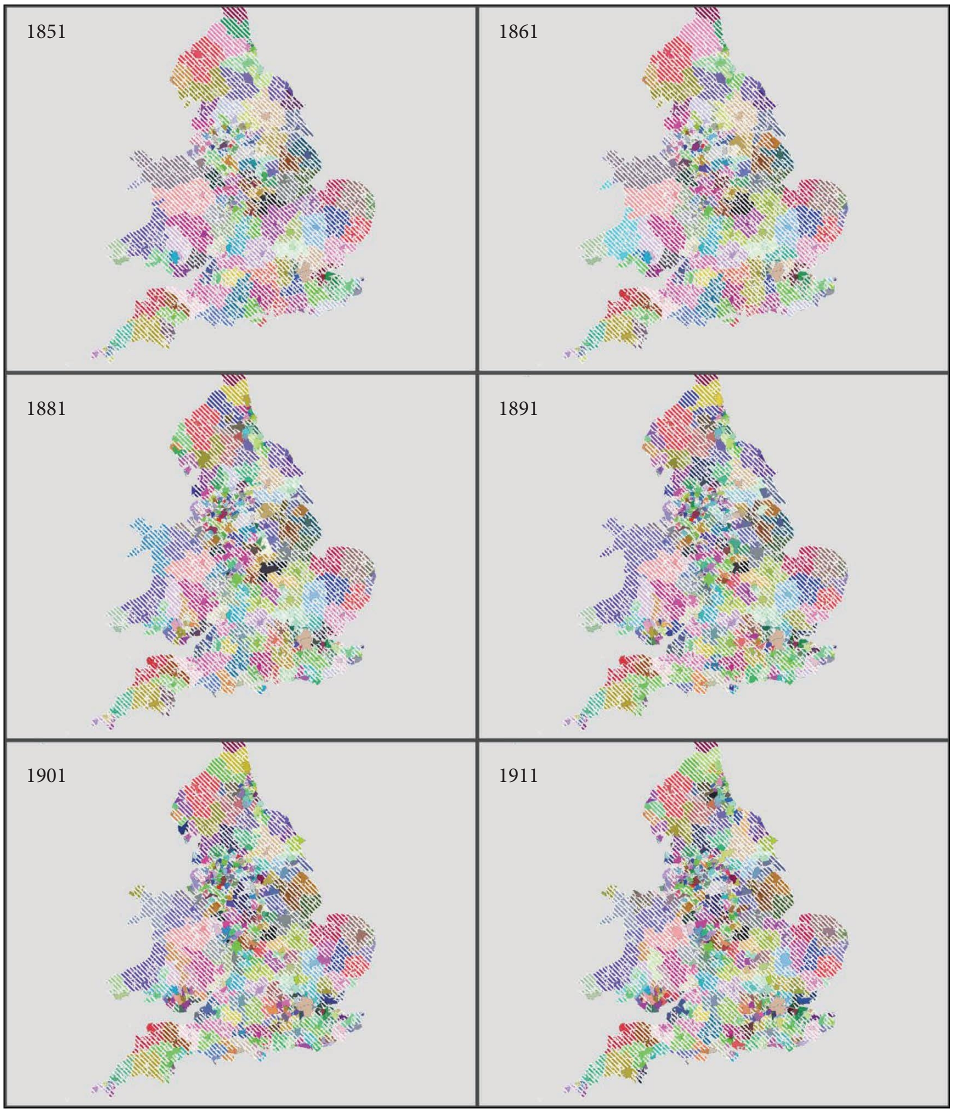

FIGURE 6: An illustration of calculated city regions, 1851–1911. The boundary data are provided by [37]. *Note.* The total number of city regions for each year is summarised in Figure 4.

$$a^{\mathrm{mn}}_{t(x)} = \max\left\{0,\ a^{\mathrm{mn}}_{t(1)} + \frac{\left(a^{\mathrm{mn}}_{t(2)} - a^{\mathrm{mn}}_{t(1)}\right)}{\left(t_{(2)} - t_{(1)}\right)}\left(t_{(x)} - t_{(1)}\right)\right\}, \quad (16)$$

where $a^{\mathrm{mn}}_{t(x)}$ is the amount of sector m required to produce one unit of sector n at time x, and $\mathrm{t}_{(x)}$ = the time point, where integers denote known datapoints (in years) and letters denote target years. $a^{mn}_{t(x)} < 0$ can potentially occur in cases where

$$t_{(x)} < t_{(1)} \text{ and } a^{\mathrm{mn}}_{t(1)} < a^{\mathrm{mn}}_{t(2)}, \quad (17)$$

or

$$t_{(x)} > t_{(2)} \text{ and } a^{\mathrm{mn}}_{t(1)} > a^{\mathrm{mn}}_{t(2)}. \quad (18)$$

In practice, this means that a few a^{mn} terms could be predicted as negative at the very end of the period; these are substituted to 0.

For b), we assume an exponential curve between each of the two known data points. With regards to c), HHW provides both imports and exports for 1841. Thomas provides imports for 1907 but not exports. The latter are therefore estimated using data from Mitchell's work on collating macroeconomic statistics, the MeasuringWorth datasets for price/cost comparisons, and other specialist historical sources; the specific sources are outlined in Figure 7. Decadal estimates are then calculating using the same method described for value added above.

These data can all be combined to iteratively generate an input-output table for any year in the period of interest (1851–1911). The technical coefficients are treated as fixed (once calculated, as above), but the ultimate production of a sector is not. The total for each sector is initially set to equal the sum of value added (as calculated above) and international imports, and this is multiplied by the technical coefficients to generate an initial value for each a^{mn} flow. The consumption and investment column can then be calculated as the sector total less the sum of exports and intermediate demand. We then enter an iterative cycle, where the

Sector	Export (million £)	Data
Agriculture	15.1	Mitchell [41], Agriculture 10: 2031 thousands of hundredweights exported of 'Wheat, Wheat meal and Flour'. Best estimate derived from Murphy [42] (weight per bushel) and Clark [43] Appendix 1 (shillings per bushel) for wheat.
Mining and quarrying	42.1	Mitchell [41] Overseas Trade 8: Coal, 42.1.
Food, drink, and tobacco	20.6	Baldwin [44]: UK exports in 1900: 71, 1913: 174 (in US dollars); therefore, 1907 is assumed to be approximately 100. MeasuringWorth [45] historical exchange calculator gives this value as 20.6.
Metal manufacture	58.3	Mitchell [41] Overseas Trade 8: Iron and Steel, 47.4; Non-ferrous Metals and Manufactures, 10.9.
Soap, candles, and dyes	3.5	This is a small industry, and literature is inconclusive on any growth/decline in exports. Therefore, assumed the same total:export ratio as HHW [29] for 1841.
Textiles, clothing, and leather goods	153.6	Mitchell [41] Overseas Trade 8: Cotton goods, 110.4; Wool goods, 28.2; Linen goods, 8.6; Silk goods, 2.2; Leather Manufactures, 4.0.
Metal goods	36.4	Mitchell [41] Overseas Trade 8: Hardwares and cutlery, 4.8; Machinery, 31.7.
Other manufacturing	40.3	Mitchel [41] Overseas Trade 8: Hats, Haberdashery, Apparel, 8.6; Chemicals, 17.1; Electrical goods, 2.5; Vehicles and Aircraft, 2.1; New Ships and Boats, 10.0.
Construction	0	Not a significant export industry.
Gas and water	0	Not a significant export industry.
Distribution & Transport	71.2	Mitchell [41] Overseas Trade 16: Proxied by invisible trade (142.4), and split equally with 'Domestic & Other Service'.
Domestic & Other Service	71.2	Mitchell [41] Overseas Trade 16: Proxied by invisible trade (142.4), and split equally with 'Distribution & Transport'.
Public administration and defence	0	Not a significant export industry.

Figure 7: Additional data sources for exports, 1907.

technical coefficients are multiplied by the previous sector total until this total reaches convergence (defined as all sector totals changing by less than one percent).

5. Findings

This all leads to a modelled set of cities (and their associated regions), sectors, and trade flows. The generality of the model means that this could be analysed in any number of ways. Here, we focus on the "major" centres identified by the model and their relationship to the national economy through trade. Section 5.1 considers the properties of the city regions contained within the system, Section 5.2 then employs these units to examine the spatial characteristics of the economy (with an emphasis of sectoral specificity), and Section 5.3 focuses on value added as a key indicator of economic specialisation and development. Together, these findings demonstrate how national accounting measures fail to adequately account for the distributed nature of the British economy in this period and highlight the long-term legacies of regional specialisation in explaining the uneven capabilities of the present-day economy.

5.1. The System of City Regions. As the only condition for including a town or city for any given year was a minimum population of 10,000 people, the number of town and city regions grows over the period, such that it more than doubled between 1851 and 1911 (Figure 4). This is a consequence of the combined effects of urbanisation and intrinsic population growth over the nineteenth century.

The population (P_i) of each city region is used to estimate an "equivalent labour value" (a proxy for employment)—i.e. (Q_i^m)—by weighting the population employed in each economic sector for the productive capacity of each demographic group, as outlined in Appendix 2. This accounts for the proportion of men, women, and children in

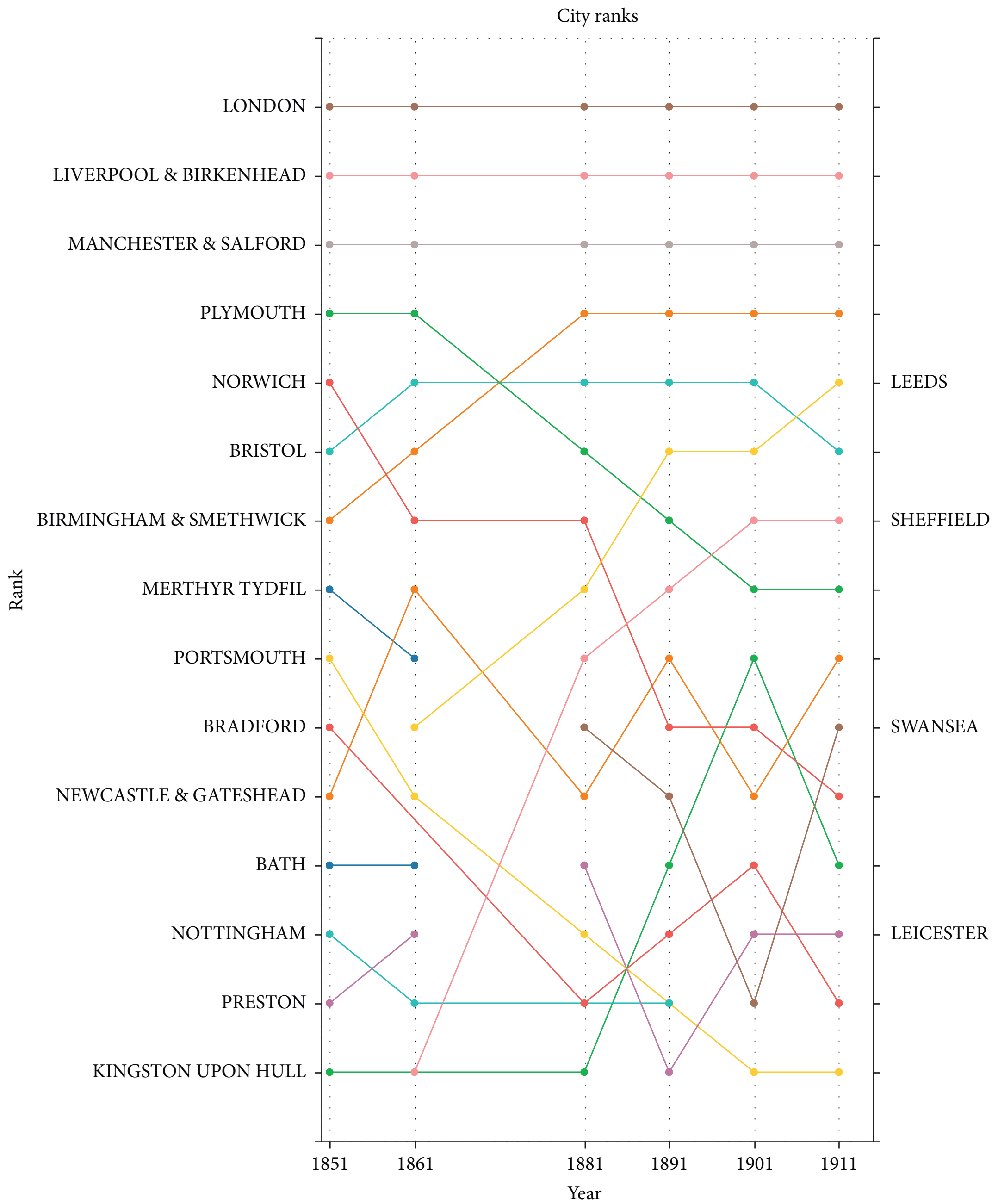

Figure 8: City region ranking by labour force size and year.

the labour force of each city region and their relative output. Q_i^m is primarily used in (4) and (6), but independent analysis of $\sum_m Q_i^m$ by city region facilitates a sector-agnostic overview of the labour force of England and Wales over this period. It shows that London is by far the biggest labour centre throughout the period, being (approximately) three times as large as the second biggest city region—Liverpool (and Birkenhead)—in 1851, and growing to almost six times as large by 1911. Liverpool is consistently predicted to have a large city region in the early years of the model. This is due to its geographical positioning and the lack of large towns in north Wales until 1881—this is consistent with the cultural importance of the former to the latter (being often known as "the Capital of North Wales"), even well into the mid-twentieth century [46]. Although relatively sparsely populated by comparison, the size of the Liverpool city region diminishes over the period of study, so that the labour value term is much closer to those of third placed Manchester (and Salford) and fourth placed Birmingham (and Smethwick) by 1911.

Figure 8 show the top 15 city regions for each year by estimated labour value (i.e., it ranks $\sum_m Q_i^m$ by city region). While the gap between fourth and seventh is small in 1851, Birmingham (seventh in that year) had established itself as far bigger than fifth placed Leeds by 1911 and was very comparable to Liverpool and Manchester. Plymouth and

Sector	Total production – excluding international imports (millions of 2015 £)	Total flows (millions of 2015 £)	Flows/production
Mining and quarrying	1669	785	0.47
Public administration and defence	1097	323	0.29
Metal goods	1693	457	0.27
Agriculture	12512	2841	0.23
Soap, candles, and dyes	935	184	0.20
Gas and water	432	78	0.18
Food, drink, and tobacco	14648	2231	0.15
Metal manufacture	1615	240	0.15
Other manufacturing	2182	310	0.14
Textiles, clothing, and leather goods	11077	1496	0.14
Distribution & Transport	10528	1146	0.11
Domestic & Other Service	8739	891	0.10
Construction	2071	134	0.06

FIGURE 9: Sectors by total production and intercity flows, 1851.

Norwich—ranking fourth and fifth in 1851—were quickly surpassed by Bristol, Leeds, and (later) Sheffield as the century progressed—the latter two on the back of their growing metal industries. Both Plymouth and Norwich were historically important regional centres for the relatively isolated agricultural production of Devon and Norfolk, and while the general city regions remained broadly consistent over the period, the significance of this trade appears to have diminished.

This emphasises the importance of understanding the different roles that cities played within the economy. While urban centres are often associated with service sectors, the idea of cities as a gateway to hinterland economies complicates this urban-rural dichotomy. The method employed here suggests that there were "agricultural" cities (even if the *urban* areas themselves did not create agricultural products), just as there were cities with little surrounding farmland that specialised in other areas. Similarly, while Newcastle contained urban collieries, its historic association with coal comes as much from its surrounding area as the city itself. Of course, it has long been the case that certain industries were associated with particular places—Nottingham was synonymous with lace, just as Sheffield was with steel—but the extent to which such relationships should be quantified and analysed at scale is more limited.

5.2. The Mesolevel Economy. The relative size of different city regions—and their trajectory over the period—allows us to make a number of inferences regarding the nature of the system (i.e., the economy of England and Wales) as a whole. While these findings are both useful and valuable, they do not rely on the main analytical power provided by the model. A particularly insightful application is assessing the necessary mobility of goods within each sector. A national-level input-output table can highlight the flow of goods from one sector to another within the economy, but by scaling this to the mesolevel, we are also able to consider the *geographic* flow of goods in addition.

The flows discussed here are the output of the doubly constrained spatial interaction model—in other words, y_{ij}^{m} from (10). For simplicity, these are deflated (using [47]), with a base year of 2015. As has been outlined above, the flows

Sector	Production – excluding international imports (millions of 2015 £)	Total flows (millions of 2015 £)	Flows/production
Mining and quarrying	14666	5305	0.36
Agriculture	19107	5777	0.30
Public administration and defence	2292	590	0.26
Metal goods	28184	6338	0.22
Soap, candles, and dyes	1812	387	0.21
Food, drink, and tobacco	19117	2389	0.12
Domestic & Other Service	74718	8195	0.11
Metal manufacture	13461	1472	0.11
Distribution & Transport	53513	5665	0.11
Gas and water	5045	443	0.09
Other manufacturing	15230	1191	0.08
Textiles, clothing, and leather goods	32824	2317	0.07
Construction	7965	404	0.05

Figure 10: Sectors by total production and intercity flows, 1911.

between different cities should be considered the minimum necessary to balance the demands of the economy, and trade may well have taken place beyond this. Nonetheless, understanding this "minimum" allows us to ascertain the extent to which production was evenly distributed (in relation to demand) throughout the country, or—conversely—the extent to which demand in certain areas outstripped supply in that region.

There are notable continuities between 1851 (Figure 9) and 1911, (Figure 10) albeit with some significant differences. In both cases, the "mining and quarrying" sector shows the highest trade flow per unit of production, although the actual value fell significantly over the period—likely due to the rise of the mining industry outside the major regions in the second half of the nineteenth century. "Agriculture"—one of the largest sectors by production—also exhibits a high tendency to move its produce in both years, which intuitively makes sense given the concentration of agricultural land in certain regions (although this only increased marginally over the period as a whole). "Public administration and defence" show a similar pattern, which is a consequence of its concentration in administrative centres.

At the other end of the scale, "construction" remains a low-movement sector in both years; this makes sense, given the nature of construction in this period (and note that some *quasi*-construction projects—such as shipbuilding—are allocated to other sectors). "Distribution and transport"—although increasing its flows/production rank in relation to other sectors between 1851 and 1911—remains relatively constant in terms of flows/production value. While this might initially seem odd given the focus of the sector on the movement of people and goods, this can be explained by two main factors: (a) transport workers were relatively few compared to those concerned with the distribution of goods, most of which took place within city regions, and (b) although the nature of the sector involved intercity movement, this does not mean that flow of value needed to move between cities; they were relatively good at exporting their own goods and services.

There are two major surprises in these findings: first, the relatively low movement of goods produced by the "textile, clothing, and leather goods" sector and second, the static

Sector	1851	1861	1881	1891	1901	1911
Agriculture	Norwich	Norwich	Norwich	Norwich	Norwich	Norwich
Construction	London	London	London	London	London	Bournemouth & Poole
Distribution & Transport	London	London	London	London	London	London
Domestic & Other Service	London	London	London	London	London	London
Food, drink, and tobacco	London	London	London	London	London	London
Gas and water	London	London	London	London	London	London
Metal goods	Birmingham & Smethwick	Birmingham & Smethwick	Birmingham & Smethwick	Birmingham & Smethwick	Birmingham & Smethwick	Birmingham & Smethwick
Metal manufacture	Birmingham & Smethwick	Birmingham & Smethwick	Birmingham & Smethwick	Birmingham & Smethwick	Birmingham & Smethwick	Birmingham & Smethwick
Mining and quarrying	Merthyr Tydfil	Newcastle & Gateshead	Newcastle & Gateshead	Rhondda	Rhondda	Rhondda
Other manufacturing	London	London	London	London	London	London
Public administration and defence	London	Aldershot	Portsmouth	Aldershot	Portsmouth	London
Soap, candles, and dyes	London	London	London	London	Liverpool & Birkenhead	Liverpool & Birkenhead
Textiles, clothing, and leather goods	Bradford	Manchester & Salford	Bradford	Bradford	Bradford	Bradford

Figure 11: Largest single exporter by sector.

flows/production value for the "domestic and other service" sector—despite its increase in ranking between 1851 and 1911. In the case of the former, this may be explained by the relatively high value of international exports; if textiles were in large part an export good, then lower internal movement might be expected.

For the latter, although there is significant growth in the *size* of the sector between 1851 and 1911, flows/production remains remarkably constant throughout the period, hovering between 0.10 and 0.12. This consistency may suggest that the service sector grew largely in proportion to geographic demand—in other words, it was largely created and consumed at the local level—or it may indicate that the significant diversity within the service sector meant that the movement of its value is masked by false equivalency. The latter might be corrected by an entropy-maximising "internal-likeness" coefficient, but such an exercise lies beyond the immediate scope of this work.

This analysis has significant implications for understanding market integration in this period. By estimating the extent to which goods might be expected to move internally within the economy, the model demonstrates the importance of internal trade networks to economic growth and regional specialisation. To further this analysis, we can consider the geographic concentrations in trade flows.

The first thing to note—which has already been suggested above—is the dominant position of London in the economy of England and Wales during the whole of this period. As outlined in the model specification, cities (and their associated hinterlands) were *either* exporters or importers in each sector, determined by their shortfalls and surpluses. London can only be described as a service powerhouse. The sectors in which it imported were generally those producing raw materials: "agriculture," "mining and quarrying," and "metal manufacture"—the exceptions being

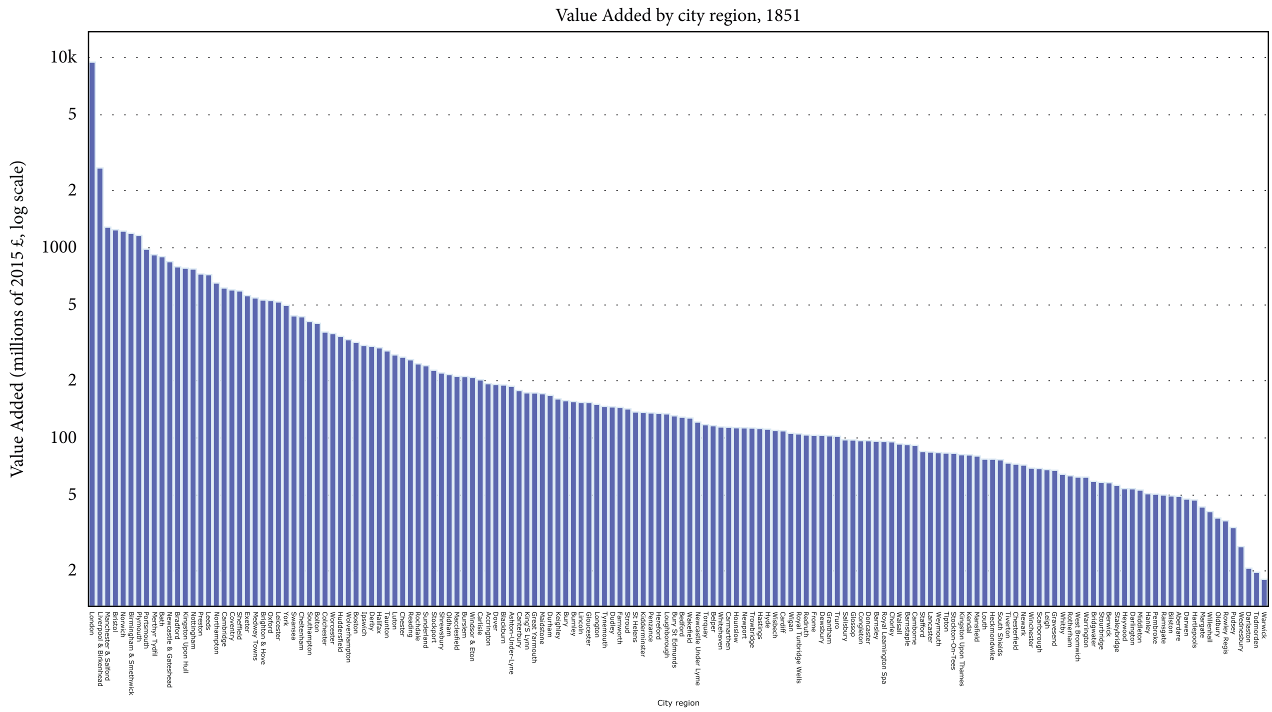

Figure 12: Value added by city region, 1851, log scale.

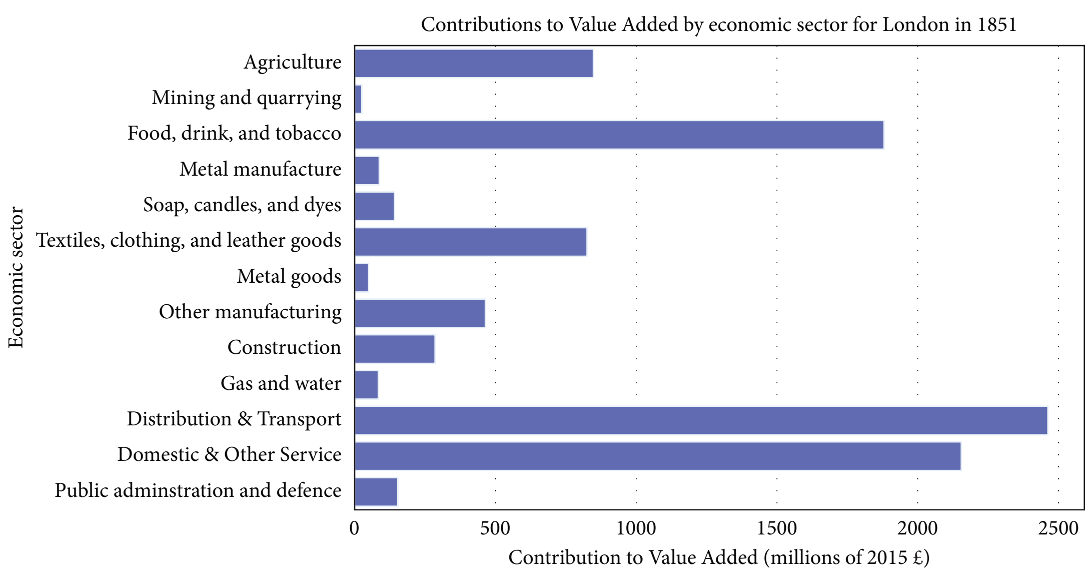

Figure 13: London value added by economic sector in 1851.

the manufacturing sectors of "metal goods" (which was a fairly small sector throughout the period) and "textiles, clothing, and leather goods" (which was heavily concentrated in the north-west of England, and "internal" London production—although significant—was likely outstripped by the sheer size of the population).

On the other hand, London's largest export sectors were generally service-based. The "domestic and other service" sector, although large in the earlier years of the study (third largest export in 1851 and second in 1861), really took off towards the end of the century; in 1881, it (narrowly) became London's largest export but totalled roughly the same value as the exports of *all* other sectors combined by 1911. "Distribution and transport"—a sector with a significant service component—was the second largest export sector for every year other than 1861 (when it was third). "Food, drink, and tobacco"—again with a notable, albeit comparatively smaller, service element (e.g., through hospitality)—were the other notable export, the largest in 1851 and 1861, but largely stagnant (with a slight decline) from 1881 onwards. The other sectors ("other manufacturing," "gas and water," "public administration and defence," "construction," and

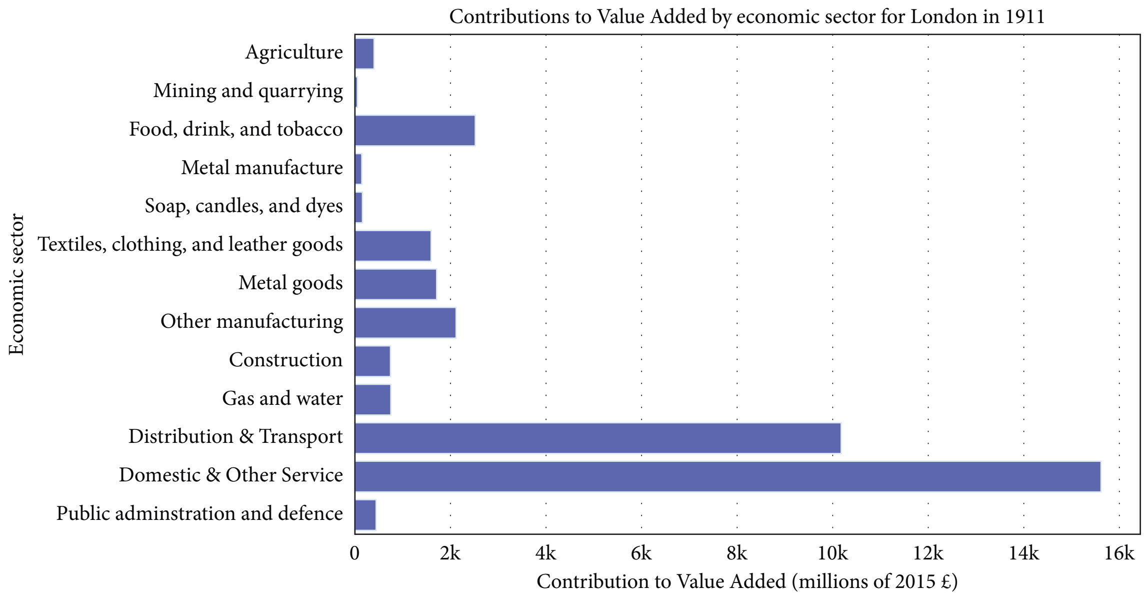

Figure 14: London value added by economic sector in 1911.

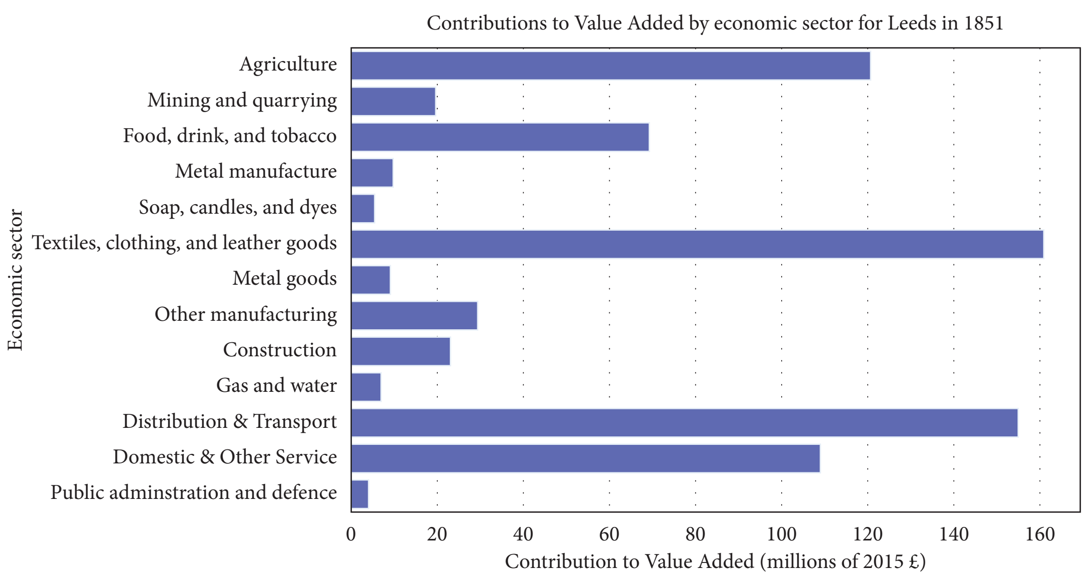

Figure 15: Leeds value added by economic sector in 1851.

"soaps, candles, and dyes") were much smaller exporters, but this was more a consequence of the size of the sector than London's role in them.

It is significant that of the four major cities (London, Manchester and Salford, Liverpool and Birkenhead, and Birmingham and Smethwick), none share the same import-export profile. Liverpool's exports were heavily weighted towards "distribution and transport," exceeding the sum total of all other sectors for each year. Manchester, despite its geographic closeness to Liverpool, also primarily exported "distribution and transport" (at least, after 1861) but also had a notable "textile, clothing, and leather goods" component, although the latter declined over the period of study. Birmingham was perhaps the most distinct of the four, with a clear emphasis on "metal goods" and "metal manufacturing," particularly after 1861. This indicates that although these cities (in the case of Manchester and Liverpool, somewhat jointly) played a significant role in the economies of their local regions, they did not fulfil precisely the same function, and thus, the importance of intercity trade to balance accounts becomes of particular importance.

This is not to say that the largest exports did not come from these cities (Figure 11). London accounted for the largest export flow for six to eight (out of the thirteen) sectors (depending on the year) and Birmingham a further two; Liverpool and Manchester rank as the largest exporters for any of the given sectors more rarely, but this is primarily due to their largest export industry ("distribution and transport") being overshadowed by London, and they are therefore relegated to smaller industries (in the case of

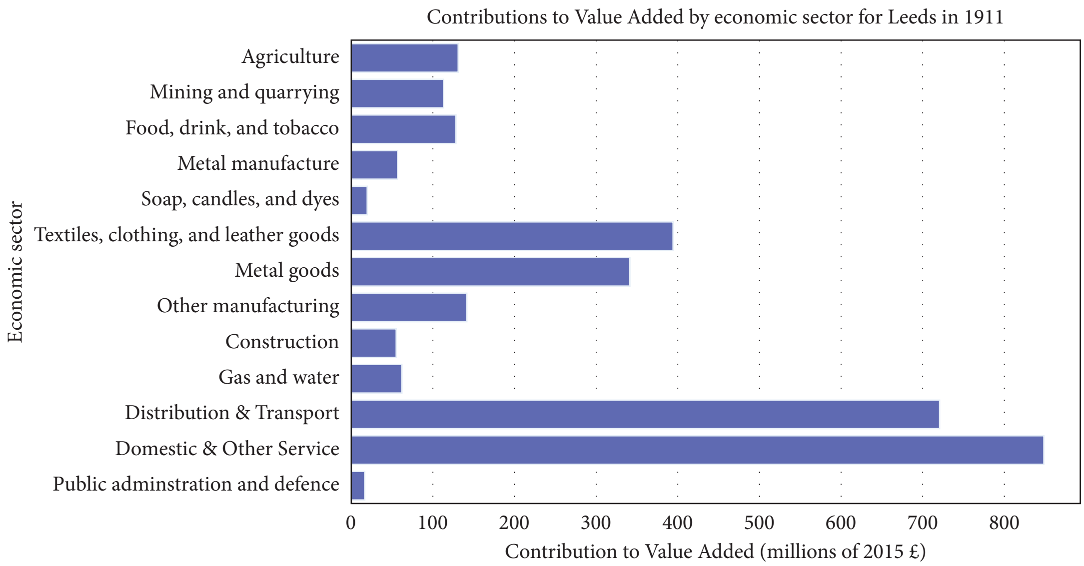

Figure 16: Leeds value added by economic sector in 1911.

Liverpool) or fighting with Bradford over textile exports (in the case of Manchester). The remainder were more specialist producers; Newcastle representing the Great Northern Coalfield in the "mining and quarrying" sector—which was then subsequently displaced by Rhondda in south Wales—should be no surprise (although the relatively early importance of Merthyr Tydfil might be considered unexpected) nor should the role of Aldershot and Portsmouth in "public administration and defence." Norwich as a large agricultural exporter is also perhaps unsurprising.

The relative importance of Bradford in the textile sector is more unexpected; while its production was smaller than that of Manchester in each year (sometimes by as much as 25 percent), the relative populations meant that Bradford had a greater surplus to export. As for services, it is important to note that there are some imperfect substitution effects to which the model (in its present specification) is not very sensitive—Bradford was known for its woolens, whereas Manchester was primarily known for producing cotton products. The model presents these to be directly comparable and interchangeable, but these products were to some extent consumed by different markets. With any level of aggregation, this type of issue is likely to occur; a more specific sectoral classification would help to avoid this issue, but the data required to do this are not presently available until 1907.

5.3. City-Level Economic Indicators. All of this brings us to an estimate of value added by city. This has enormous potential for understanding the nature of the economy in England and Wales during this period. We estimate value added by drawing data from the input-output table for each city region and calculating the difference between the total value of the sector and the sum of intermediate flows used in its production and imports. This is a fairly coarse measure of value added, and it is also influenced by sectoral composition—in part, as a consequence of the model assumptions—but identifies several significant trends.

First, the scale of London's economy continues to show through. As Figure 12 demonstrates, it was three and a half times as large as the next biggest city region—Liverpool and Birkenhead. This is broadly in line with the size of the workforce. Indeed, the highest value added city units for 1851 largely correspond to the biggest producers, with London, Liverpool and Birkenhead, Manchester and Salford, Bristol, Norwich, and Birmingham and Smethwick shown as the leaders.

City-level value added having the potential to become particularly insightful is to understand the evolution of city-level economies over the period. To continue with London as an example, in 1851 (Figure 13), its most valuable sector (in terms of value added) was "distribution and transport," followed by "domestic and other service" and "food, drink, and tobacco." In 1911 (Figure 14), these remain the biggest contributors to value added, but the order is shuffled, with the relative importance of other smaller sectors also altered.

If we compare this to a major established northern city—such as Leeds (Figures 15 and 16)—we are still able to observe growth in the importance of "distribution and transport" and "domestic and other service" as well as a decline in the contributions of "agriculture" (common to many of the larger economic centres), but the relative importance of manufacturing sectors (in this case, "textiles, clothing, and leather goods" and "metal goods") is much greater.

6. Conclusions

The paucity of data for intercity trade in the nineteenth century hinders understandings of regional economic development. This is important because the legacies of this pivotal period survive into the present day—the major cities in 1851 by and large remain the major cities over 150 years later. But more than this, in many cases, the growth trajectory of a city or city region was established by its specialisation and development in the second half of the nineteenth century. Understanding the origins of this growth is therefore key to explaining geographic socioeconomic divisions in modern Britain.

Unsurprisingly, the mesolevel economy demonstrates a complex picture over the period of study. The demonstration model presented here is a first step in unravelling this complexity, but it is only the beginning of the work necessary to adequately understand this phenomenon. Calibration is particularly challenging, and it is reliant on very general data and heroic assumptions—both of which can be superseded and refined by specialists in the future. However, despite the potential future refinements and modifications, several major themes emerge.

First, the spatial intricacy of the nineteenth century mesolevel economy is a severely underexplored theme, but it is central to explaining regional economic development. While some related sectors were located to take advantage of proximity, in each sector, there was still significant transportation of goods around the city system in order to fulfil the local supply and demand requirements in each instance. Historians have often focused on transport networks or firm-level movement of goods, but sector-level flows are difficult to directly observe. This is where the method outlined here can make a valuable contribution: we demonstrate how certain sectors (for example, textiles) were largely centralised, and others (for example, mining and quarrying) were heavily concentrated in multiple regions—both of which necessitated notable movement of goods around the domestic economy. This is not in itself a new idea, but the *interactions* between different sectors in different places have hitherto received very little attention and rarely at scale. These interchanges of goods are not something which can be ignored—there were associated frictional costs, mesolevel competitive advantages leading to particular paths of economic development, and sociopolitical ramifications.

Second, the general significance of London is often acknowledged in the literature, but the findings here demonstrate just how important it was to the economy of England and Wales in this period. Its relative economic (and associated political and cultural) power dwarfed competing regional centres, and it played a significant role (either as an importer or an exporter) in every sector of the economy in England and Wales. There are other major centres which played key roles in particularly sectors, but none with the economic breadth displayed by London. The general dominance of London over the south-east (and south England in general) can also be contrasted with the relationship of other major centres (such as Birmingham and Manchester) with their respective regions: although the latter were undoubtedly important, the economic diversity and relative population size meant that their ability to pull in trade was more limited by comparison.

Finally, we move towards establishing value added statistics at a city region level. While these may benefit from future refinement, this is nonetheless an important step in establishing the regional wealth disparity characteristic of the modern day "north-south divide." We demonstrate that some sectors were greater contributors to city-level value added than others and that these differences go some way to explaining the relative wealth of cities.

There are many areas in which this model might be developed—this is the nature of attempting to specify something so general. However, we would conclude by highlighting the areas in which we perceive the most significant gains might be made. First, relaxing the relationship between employment and output to allow for regional variations in productivity would facilitate a better representation of geographic disparities in production, as well as demonstrate greater sensitivity in city-level metrics—these are important for understanding the roots of geographic inequality in modern Britain. Second, highlighting the role of investment (both government and private) would improve the capacity of the model to demonstrate the influence of capital formation. Third, an integrated transport network—and costs varied by transport method—would better account for the economic penalties to the movement of goods and integrate the rapid infrastructural development taking place in this period. Finally, removing the calculation of international imports and exports and allowing these flows to be modelled dynamically (including the specification of port cities as points of ingress and egress) would allow for an improvement in the realism of both international and domestic flows. In the longer run, a dynamic model (such as that specified in Appendix 1) should be the ultimate goal—but incremental refinements to the work presented here can yield new insights into the development of the economy in England and Wales during this pivotal period.

Data Availability

The census data used in this study were supplied by UKDS [SN:7481] under license and so cannot be made freely available. Requests for access to these data should be made to UKDS. The RSD boundary data were supplied directly by Joe Day at the University of Bristol and Alice Reid at the University of Cambridge. They are in the process of being deposited with UKDS, and the citation may change to reflect this in due course.

Disclosure

A preprint version of this paper was made available as Solomon and Wilson [48].

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The authors would like to thank Griffith Rees and Bowen Zhang for their contributions to the broader project of which this work is a part. The Python implementation of the core mesolevel input-output framework was developed in collaboration with Griffith Rees and Bowen Zhang. For further details (and application to other contexts), see the *estios* package (https://github.com/griff-rees/estios) and Rees et al. (forthcoming), but note that the version employed here is tailored for historical usage (including significant additional data processing steps and an adapted model specification). This article makes extensive use of data and coding schemes provided by the I-CeM and BBCE projects: Schurer and Higgs [31]; Smith and Bennett [30]. The authors would like to thank Joe Day and Alice Reid for supplying the RSD Boundary data and lookups prior to their deposit with the UKDS [37] and Sara Horrell for making available supporting documentation for [29]. This work was primarily undertaken as part of the "Mathematics of Cities" [TU/ASG/R-SPEU-112] and "Living with Machines" [AH/S01179X/1] projects. Living with Machines, funded by the UK Research and Innovation (UKRI) Strategic Priority Fund, was a multidisciplinary collaboration delivered by the Arts and Humanities Research Council (AHRC), with the Alan Turing Institute, the British Library and King's College London, East Anglia, Exeter, and Queen Mary University of London. One author also received support from the UKRI Future Leaders Fellowship "Indicative Data: Extracting 3D Models of Cities from Unavailability and Degradation of Global Navigation Satellite Systems (GNSS)" [MR/S01795X/2].

Supplementary Materials

Appendixes 1 and 2 (referenced in the text) are supplied as supplementary materials. (*Supplementary Materials*)

References

[1] H. Smith, R. J. Bennett, and D. Radicic, "Towns in victorian England and Wales: a new classification," *Urban History*, vol. 45, no. 4, pp. 568–594, 2018.

[2] J. G. Williamson, *Coping with City Growth during the British Industrial Revolution*, Cambridge University Press, Cambridge, UK, 1990.

[3] C. J. Simon and C. Nardinelli, "The talk of the town: human capital, information, and the growth of English cities, 1861 to 1961," *Explorations in Economic History*, vol. 33, no. 3, pp. 384–413, 1996.

[4] T. E. Rihll and A. G. Wilson, "Spatial interaction and structural models in historical analysis: some possibilities and an example," *Histoire & Mesure*, vol. 2, no. 1, pp. 5–32, 1987.

[5] T. E. Rihll and A. G. Wilson, "Model-based approaches to the analysis of regional settlement structures: the case of ancient Greece," in *History and Computing*, P. Denley and D. Hopkin, Eds., Manchester University Press, Manchester, UK, 1987, https://books.google.co.uk/books?id=uXi7AAAAIAAJ.

[6] T. E. Rihill and A. G. Wilson, "Modelling settlement structures in Ancient Greece: new approaches to the polis," in *City and Country in the Ancient World*, Routledge, New York, NY, USA, 1991.

[7] A. Bevan and A. Wilson, "Models of settlement hierarchy based on partial evidence," *Journal of Archaeological Science*, vol. 40, no. 5, pp. 2415–2427, 2013.

[8] A. Wilson, "Boltzmann, lotka and volterra and spatial structural evolution: an integrated methodology for some dynamical systems," *Journal of The Royal Society Interface*, vol. 5, no. 25, pp. 865–871, 2007.

[9] A. G. Wilson and J. Dearden, "Tracking the evolution of regional DNA: the case of Chicago," in *Understanding Population Trends and Processes*, J. C. H. Stillwell, Ed., Springer, Berlin, Germany, 2011.

[10] R. C. Allen, "The British industrial revolution in global perspective," in *New Approaches to Economic and Social History*, Cambridge University Press, Cambridge, UK, 2009.

[11] J. Mokyr, *The Enlightened Economy: An Economic History of Britain*, Yale University Press, New Haven, CT, USA, 2009.

[12] N. Crafts and A. Mulatu, "What explains the location of industry in Britain, 1871–1931?" *Journal of Economic Geography*, vol. 5, no. 4, pp. 499–518, 2005.

[13] N. Crafts and A. Mulatu, "How did the location of industry respond to falling transport costs in Britain before world war I?" *The Journal of Economic History*, vol. 66, no. 03, pp. 575–607, 2006.

[14] D. Bogart, X. You, E. J. Alvarez-Palau, M. Satchell, and L. Shaw-Taylor, "Railways, divergence, and structural change in 19th century England and Wales," *Journal of Urban Economics*, vol. 128, Article ID 103390, 2022.

[15] S. Broadberry, B. M. S. Campbell, A. Klein, M. Overton, and B. van Leeuwen, *British Economic Growth, 1270–1870*, Cambridge University Press, Cambridge, UK, 2015.

[16] N. F. R. Crafts and C. K. Harley, "Output growth and the British industrial revolution: a restatement of the crafts-harley view," *The Economic History Review*, vol. 45, no. 4, pp. 703–730, 1992.

[17] R. Church, A. Hall, and J. Kanefsky, "History of the British coal industry," *Victorian Pre-Eminence*, OUP Oxford, Oxford, UK, 1st edition, 1986.

[18] J. A. Hassan, "The growth and impact of the British water industry in the nineteenth century," *The Economic History Review*, vol. 38, no. 4, pp. 531–547, 1985.

[19] E. Hopkins, "The trading and service sectors of the Birmingham economy 1750–1800," in *Business in the Age of Reason*, R. P. T. Davenport-Hines, Ed., Routledge, London, UK, 1987.

[20] A. Wilson, "The science of cities and regions: lectures on mathematical model design," in *SpringerBriefs in Geography*, Springer, Berlin, Germany, 2012.

[21] G. Rees, G. Solomon, B. Zhang, and A. G. Wilson, "Input-output analytics for urban systems: explorations in policy and planning," in *31st Annual Geographical Information Science Research UK Conference (GISRUK)*, University of Glasgow, Glasgow, Scotland, 2023.

[22] M. Batty and R. Milton, "A new framework for very large-scale urban modelling," *Urban Studies*, vol. 58, no. 15, pp. 3071–3094, 2021.

[23] M. Birkin, W. James, N. Lomax, and A. Smith, "Chapter 2: data linkage and its applications for planning support systems," in *Handbook of Planning Support Science*, S. Geertman and J. Stillwell, Eds., Edward Elgar Publishing, pp. 22–36, Cheltenham, UK, 2020.

[24] N. Lomax and A. Smith, "'DAFNI hosts population forecast model', data & analytics facility for national infrastructure-DAFNI," 2023, https://www.dafni.ac.uk/project/dafni-hosts-population-forecast-model/.

[25] W. Leontief, *Input-Output Economics*, Oxford University Press, Oxford, UK, 1986.
[26] A. G. Wilson and T. P. Oléron-Evans, *A Two-Tier National Economic Model*, 2015.
[27] R. G. Levy, T. P. Oléron-Evans, and A. Wilson, "A global inter-country economic model based on linked input–output models," 2016, https://onlinelibrary.wiley.com/doi/abs/10.1002/9781118937464.ch4.
[28] M. Thomas, *An Input-Output Approach to the British Economy, 1890-1914*, University of Oxford, Oxford, UK, 1984, https://solo.bodleian.ox.ac.uk/permalink/44OXF_INST/35n82s/alma990115960680107026 Ph.D.
[29] S. Horrell, J. Humphries, and M. Weale, "An input-output table for 1841," *The Economic History Review*, vol. 47, no. 3, pp. 545–566, 1994.
[30] H. Smith and R. J. Bennett, "Research data supporting wp 6: urban-rural classification using census data," 2019, https://doi.org/10.17863/CAM.44400.
[31] K. Schurer and E. Higgs, *Integrated Census Microdata (I-CeM), 1851-1911*, UK Data Service, England, UK, 2020.
[32] Bbce, "Cambridge » towns and urban areas," 2023, https://www.bbce.uk/entrepreneurship/urban/.
[33] J. P. Rodrigue, "Market area analysis the geography of transport systems," 2023, https://transportgeography.org/contents/methods/market-area-analysis/.
[34] J. Malczewski, "Central place theory," in *International Encyclopedia of Human Geography*, R. Kitchin and N. Thrift, Eds., Elsevier, Oxford, UK, 2009.
[35] E. Arcaute, C. Molinero, E. Hatna et al., "Cities and regions in Britain through hierarchical percolation," *Royal Society Open Science*, vol. 3, no. 4, Article ID 150691, 2016.
[36] G. Boeing, *OSMnx: new methods for acquiring, constructing, analyzing, and visualizing complex street networks, Computers, Environment and Urban Systems*, vol. 65, pp. 126–139, 2017.
[37] J. D. Day, *Registration Sub-District Boundaries for England and Wales 1851-1911*, 2021.
[38] essex, "Integrated census microdata | university of essex," 2023, https://www.essex.ac.uk/research-projects/integrated-census-microdata.
[39] M. Berg and P. Hudson, "Rehabilitating the industrial revolution," *The Economic History Review*, vol. 45, no. 1, pp. 24–50, 1992.
[40] F. Geary and T. Stark, "Regional GDP in the UK, 1861-1911: new estimates," *The Economic History Review*, vol. 68, no. 1, pp. 123–144, 2015.
[41] B. R. Mitchell and P. Deane, *Abstract of British Historical Statistics*, Cambridge University Press, Cambridge, UK, 1962.
[42] W. J. Murphy, *'Tables for Weights and Measurements—Crops.', Science And Technology Guide*, University of Missouri-Columbia, Columbia, MO, USA, 1981.
[43] G. Clark, "The price history of English agriculture, 1209–1914," *Research in Economic History*, vol. 22, pp. 41–123, 2004.
[44] R. E. Baldwin, "The commodity composition of trade: selected industrial countries," *The Review of Economics and Statistics*, vol. 40, no. 1, pp. 50–68, 1958.
[45] L. H. Officer and S. H. Williamson, "Computing "real value" over time with a conversion between U.K. Pounds and U.S. Dollars, 1774," https://www.measuringworth.com/calculators/exchange/.
[46] O. Sykes, J. Brown, M. Cocks, D. Shaw, and C. Couch, "A city profile of Liverpool," *Cities*, vol. 35, pp. 299–318, 2013.
[47] R. Thomas and S. H. Williamson, "'Notes on three centuries of UK GDP and seven centuries of English GDP', MeasuringWorth," 2020, https://www.measuringworth.com/datasets/ukgdp/Source4.php.
[48] G. Solomon and A. Wilson, "History through computer models: the evolution of cities in the nineteenth century," *SocArXiv*, vol. 15, 2023.