Hindawi
Complexity
Volume 2024, Article ID 3152363, 15 pages
https://doi.org/10.1155/2024/3152363

Research Article

The Complexities in the R&D Competition Model with Spillover Effects in the Supply Chain

Jianli Xiao [1] **and Hanli Xiao** [2]

[1] *School of Economics and Management, Yiwu Industrial & Commercial College, Yiwu 322000, China*
[2] *School of Tourism and Resources Environment, Qiannan Normal University for Nationalities, Duyun 55900, China*

Correspondence should be addressed to Jianli Xiao; fanping123@126.com

Received 4 October 2023; Revised 22 January 2024; Accepted 21 February 2024; Published 28 February 2024

Academic Editor: Siew Ann Cheong

This study aims to investigate the research and development (R&D) competition within the supply chain, focusing on two aspects: R&D competition at the manufacturing level and competition in pricing strategies. This paper establishes a dynamic game model of R&D competition, comprising two manufacturers and two retailers, with both manufacturers exhibiting bounded rationality. The key findings are as follows: (1) an increase in the adjustment speed positively affects the chaotic nature of the R&D competition system, leading to a state of disorder. This chaotic state has adverse implications for manufacturing profitability. (2) The spillover effect exhibits a positive relationship with the level of chaos in the R&D competition system. A greater spillover effect contributes to a more turbulent environment, which subsequently impacts the profitability of manufacturers. (3) R&D cost parameters exert a positive influence on the stability of the R&D competition system. When the system reaches a state of equilibrium, an escalation in the R&D cost parameters poses a threat to manufacturer profitability. (4) Retailer costs play a detrimental role in the stability of the R&D competition system. As retailer costs increase, there is a decline in R&D levels, thereby diminishing manufacturer profitability. (5) To mitigate the chaotic state, we propose the implementation of the time-delayed feedback control (TDFC) method, which reflects a more stable state in the R&D competition system.

1. Introduction

In the era of the knowledge economy and digitalization, businesses are increasingly emphasizing their technological innovation endeavors to adapt to external environmental changes and gain a competitive advantage. Numerous occurrences of cooperative innovation have manifested within various supply chains [1], as evidenced by partnerships between renowned automotive manufacturers GM and Benz with intermediaries from diverse nations. Likewise, in the context of China's community-based vegetable procurement, collaborative endeavors between vegetable suppliers and retailers have led to significant business model innovations. Technological innovation plays a vital role in driving economic growth at both national and regional levels, as well as boosting company profits [2]. Consequently, competition among firms in research and development (R&D) has become more intense. R&D activities are instrumental in enhancing firms' core competitiveness, reducing operational costs, and cultivating unique aspects of their business that provide an edge over competitors. However, engaging in R&D activities also presents decision-making challenges for firms [3], while the complexity associated with managing R&D activities leads to numerous management hurdles [4]. Moreover, the competitive nature of R&D strategies contributes to intricate behaviors within the entire system. In the real world, the spillover effect has emerged as a significant phenomenon in R&D activities, making it impossible for manufacturers to exclude competitors solely through their own R&D efforts.

As a consequence, the collaboration and R&D activities within the supply chain have led to an escalation of competitive dynamics [2]. In the event of competition failing, it precipitates detrimental implications for all involved entities within the supply chain. The losses incurred by enterprises such as Shi Hui Tuan and Ding Dong Maicai in 2021 substantially diminished the profits of vegetable suppliers and retailers, and in some cases, even resulted in their outright closure. Hence, given this scenario, it becomes imperative to address the ensuing questions concerning channel strategies: How do channels engage in competitive R&D? What are the ramifications of such competitive endeavors? How can the competitive behavior of channel partners be regulated?

The innovation of this article is comprised of the following facets: First, the present study presents an extension of the R&D competition model within the domain of the supply chain. By amalgamating R&D theory with the tenets of chaos theory, we systematically analyze the equilibrium points of the R&D competition system under the conditions of bounded rationality among participants, subsequently investigating the multifaceted dynamics that emerge in various scenarios. Second, we employ the time-delayed feedback control (TDFC) method as a strategic approach. By employing TDFC, our research endeavors to proficiently regulate chaotic phenomena. Our findings have revealed that TDFC not only diminishes the incidence of bifurcation and chaotic phenomena but also ensures the preservation of the original level of innovation, thereby safeguarding the efficacy of market mechanisms.

2. Literature Review

During the literature review, we have classified the papers into two distinct categories: the R&D stream that primarily focuses on investigating the spillover effects of research and development and the chaos theory stream that primarily focuses on the topics of bifurcation and chaos control.

The primary focus of the first stream lies in the realm of R&D within the supply chain. In this domain, corporations find themselves equipped with a competitive advantage through their ventures in R&D. The R&D activities of manufacturers exhibit a dual benefit, positively impacting both end-consumers and retailers [5]. Indeed, it is imperative to recognize that these R&D endeavors can give rise to the manifestation of diverse competitive strategies among enterprises [1]. In their insightful exploration of R&D collaboration within a supply chain, the authors of [6] assert that firms should diligently evaluate the spillover effect prior to embarking on cooperative R&D initiatives. Moreover, their research seeks to unravel the intricacies that underscore the occurrence of R&D-induced complexity, elucidating that firms tend to garner comparatively reduced profits within volatile environments. Significantly, the significance of R&D activities stems from their notable capacity to engender and propagate spillover effects throughout the supply chain. It is worthwhile to acknowledge that such activities can engender noticeable cost reductions for participating firms, a phenomenon often attributed to technological breaches or the sharing of erudite insights among researchers [7]. Market inquiry not only posits that R&D competition may yield greater financial gains vis-à-vis collaboration, as posited by [2], but also presents the inquiry into cooperative behaviors within a competitive R&D framework involving three prominent oligopolistic entities, as scrutinized by [8]. Nevertheless, it is incumbent upon us to underscore that R&D endeavors can yield intricate phenomena within the supply chain, as amply demonstrated by [9] investigation into the complexities that permeate a duopoly Stackelberg model of R&D competition. The contribution of R&D to the value creation process by precipitating conspicuous cost reductions across the entire supply chain remains an irrefutable fact. In this vein, an inquiry by [10] into the chaos that often pervades R&D initiatives within monopolistic firms and [11] exploration of the sophisticated dynamics at play within high-tech manufacturers substantiate this assertion.

The second stream of research focuses on the domain of chaos theory, which has garnered significant attention from scholars in recent years. Complexity is a pervasive phenomenon observed in economic and supply chain systems alike. Within chaotic systems, the sensitivity to initial conditions intensifies, leading to managerial challenges in decision-making processes [12]. Consequently, systems can display transitions between chaotic and stable states, underscoring the criticality of effective chaos control strategies. Notably, a growing number of researchers have adopted the framework of bounded rationality to investigate economic models, as exemplified by studies conducted by [13, 14]. The authors of [15] have analyzed the dynamic behavior of discretizing a continuous-time Leslie prey-predator model. The authors of [16] have examined complex behavior in investment patterns within spatial public goods games. These inquiries have shed light on the impact of bounded rationality on decision-making challenges within various market systems such as supply chains and platform ecosystems. Time delays and the influence of bounded rationality have been identified as key factors contributing to the intricate dynamics exhibited by these systems, including phenomena such as bifurcation and chaos. Consequently, research endeavors have proposed feedback control methods as viable means to address these complex behaviors. Notable studies by the authors of [17–19], the authors of [19] have thoroughly examined the application and effectiveness of feedback control methods. Additionally, time-delayed feedback control (TDFC) has emerged as a prominent technique used to stabilize unstable periodic orbits within non-linear dynamical systems, as demonstrated by the work of [20]. Furthermore, the authors of [21], have proposed the incorporation of upper or lower bounds to alleviate chaos in dynamical systems, offering an alternative approach to chaos control.

In summary, despite the existing research on supply chain innovation and chaos dynamics in academia, there are certain deficiencies in their integration. Given the widespread phenomena and highlighted significance of collaborative R&D within the supply chain, related studies hold crucial significance. The bounded rationality exhibited by supply chain participants necessitates exploring the occurrences of excessive or insufficient innovation, resulting in heightened market volatility.

The remaining sections of this paper are structured as follows: Section 3 presents the R&D competition model, while Section 4 explores the dynamic analysis of the game, specifically highlighting the stable regions within the R&D competition game. Section 5 utilizes numerical simulations to illustrate the bifurcation diagrams, maximum Lyapunov exponents, and strange attractors. We implement feedback control on systems that have already entered chaotic states. The conclusions and managerial recommendations are provided in Section 6.

3. Model

This investigation establishes a comprehensive model of supply chain R&D competition, as graphically displayed in Figure 1. In this context, the supply chain configuration encompasses two manufacturers and two retailers, where the manufacturers strategically formulate R&D policies to effectively compete for market share by engaging the retailers as intermediaries. The structural framework of this discourse aligns closely with the pioneering contribution by [1]. The systemic dynamics of the supply chain R&D competition model follow a sequential game sequence, commencing with the manufacturers' deliberation on their respective R&D levels. Subsequently, the manufacturers judiciously determine their wholesale prices, followed by the retailers' formulation of optimal retail prices. Lastly, consumers exercise discernment in selecting their consumption tendencies.

Following the study by the authors of [22], retailers are confronted with a counter-demand function that can be represented as follows:

$$p = a - b(q_1 + q_2), \tag{1}$$

where a denotes the magnitude of the market and q_i ($i = 1, 2$) represents the quantity supplied by retailer i.

In the context of manufacturers' R&D activities, spillover effects are present. To simplify the analysis, we assume equal spillover effects between the two firms. Consequently, the cost functions for the two retailers can be defined as follows:

$$\begin{cases} sc_1 = c_1 - x_1 - \alpha x_2, \\ sc_2 = c_2 - x_2 - \alpha x_1. \end{cases} \tag{2}$$

Here, α signifies the extent of spillover effects, while c_i ($i = 1, 2$) corresponds to the cost incurred by retailer i, and x_i ($i = 1, 2$) denotes the R&D level of manufacturer i. It is worth noting that the R&D endeavors of the manufacturer yield cost reduction effects on both retailers.

For computational ease, we make the assumption that the marginal cost of the manufacturer is negligible.

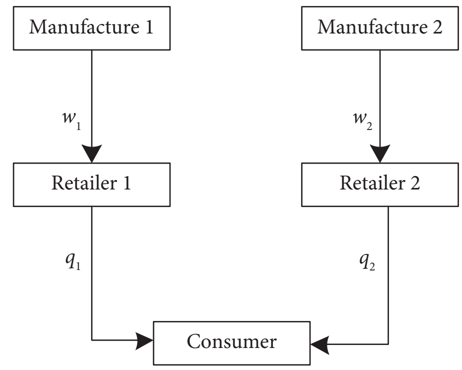

Figure 1: Supply chain R&D competition model.

The profit functions of the manufacturer can be defined as follows:

$$\begin{cases} \pi_{m1} = w_1 q_1 - \dfrac{1}{2}\beta_1 x_1^2, \\ \pi_{m2} = w_2 q_2 - \dfrac{1}{2}\beta_2 x_2^2, \end{cases} \tag{3}$$

where w_i ($i = 1, 2$) represents the price set by manufacturer i and β_i ($i = 1, 2$) denotes the parameter for R&D cost. We refer to the cost function as outlined in [23], setting it to be quadratic, indicating the phenomenon of diminishing returns in research and development benefits.

The profit function for retailers is given by

$$\begin{cases} \pi_{r1} = (p - w_1 - sc_1)q_1, \\ \pi_{r2} = (p - w_2 - sc_2)q_2. \end{cases} \tag{4}$$

By substituting equations (1) and (2) into equation (4), the retailer's function can be derived as follows:

$$\begin{cases} \pi_{r1} = (a - b(q_1 + q_2) - w_1 - (c_1 - x_1 - \alpha x_2))q_1, \\ \pi_{r2} = (a - b(q_1 + q_2) - w_2 - (c_2 - x_2 - \alpha x_1))q_2. \end{cases} \tag{5}$$

Taking the partial derivatives of π_{ri} ($i = 1, 2$) in equation (5) with respect to q_i ($i = 1, 2$), we obtain the necessary first-order conditions:

$$\begin{cases} \dfrac{\partial \pi_{r1}}{\partial q_1} = a - b(2q_1 + q_2) - w_1 - (c_1 - x_2 - \alpha x_1) = 0, \\ \dfrac{\partial \pi_{r2}}{\partial q_2} = a - b(q_1 + 2q_2) - w_2 - (c_2 - x_2 - \alpha x_1) = 0. \end{cases} \tag{6}$$

Solving these equations leads to the following solutions:

$$\begin{cases} q_1 = \dfrac{(2-\alpha)x_1 + (2\alpha-1)x_2 + a - 2c_1 + c_2 - 2w_1 + w_2}{3b}, \\ q_2 = \dfrac{(2-\alpha)x_2 + (2\alpha-1)x_1 + a - 2c_2 + c_1 - 2w_2 + w_1}{3b}. \end{cases} \tag{7}$$

Substituting equation (7) back into equation (5), the resulting expressions for the manufacturer's profit functions are as follows:

$$\begin{cases} \pi_{m1} = \dfrac{w_1\left((2-\alpha)x_1 + (2\alpha-1)x_2 + a - 2c_1 + c_2 - 2w_1 + w_2\right)}{3b} - \dfrac{1}{2}\beta_1 x_1^2, \\ \pi_{m2} = \dfrac{w_2\left((2-\alpha)x_2 + (2\alpha-1)x_1 + a - 2c_2 + c_1 - 2w_2 + w_1\right)}{3b} - \dfrac{1}{2}\beta_2 x_2^2. \end{cases} \tag{8}$$

Finally, differentiating π_{mi} $(i = 1, 2)$ in equation (8) with respect to w_i $(i = 1, 2)$, we obtain the first-order necessary conditions:

$$\begin{cases} \dfrac{\partial \pi_{m1}}{\partial w_1} = \dfrac{(2-\alpha)x_1 + (2\alpha-1)x_2 + a - 2c_1 + c_2 - 4w_1 + w_2}{3b} = 0, \\ \dfrac{\partial \pi_{m2}}{\partial w_2} = \dfrac{(2-\alpha)x_2 + (2\alpha-1)x_1 + a - 2c_2 + c_1 - 4w_2 + w_1}{3b} = 0. \end{cases} \tag{9}$$

The solutions for w_i $(i = 1, 2)$ are given by

$$\begin{cases} w_1 = \dfrac{(7\alpha-2)x_2 + (7-2\alpha)x_1 + 5a - 7c_1 + 2c_2}{15}, \\ w_2 = \dfrac{(7-2\alpha)x_2 + (7\alpha-2)x_1 + 5a - 7c_2 + 2c_1}{15}. \end{cases} \tag{10}$$

Substituting equation (10) back into equation (8), and differentiating π_{mi} with respect to x_i $(i = 1, 2)$, we obtain

$$\begin{cases} \dfrac{\partial \pi_{m1}}{\partial x_1} = \left(\dfrac{4(7-2\alpha)^2}{675b} - \beta_1\right)x_1 + \dfrac{4(7-2\alpha)^2}{675b}x_2 + \dfrac{4(7-2\alpha)(5a - 7c_1 + 2c_2)}{675b}, \\ \dfrac{\partial \pi_{m2}}{\partial x_2} = \dfrac{4(7-2\alpha)^2}{675b}x_1 + \left(\dfrac{4(7-2\alpha)^2}{675b} - \beta_2\right)x_2 + \dfrac{4(7-2\alpha)(5a - 7c_2 + 2c_1)}{675b}. \end{cases} \tag{11}$$

Taking into account the practical constraints faced by manufacturers, namely, information and resource limitations, it becomes evident that arriving at entirely rational decisions is unattainable. Therefore, in line with the research conducted by [24], we adopt the assumption that each manufacturer operates under bounded rationality. This implies that manufacturers rely on previous-stage information when making R&D decisions. As a result, a dynamic system can be derived as follows:

$$\begin{cases} x_1(t+1) = x_1(t) + \dfrac{\gamma_1 x_1(t)\partial \pi_{m1}(t)}{\partial x_1(t)}, \\ x_2(t+1) = x_2(t) + \dfrac{\gamma_2 x_2(t)\partial \pi_{m2}(t)}{\partial x_2(t)}. \end{cases} \quad (12)$$

In equation (12), γ_i $(i = 1, 2)$ represents the adjustment speed of manufacturer i.

4. Equilibrium

Upon reaching a state of equilibrium in the R&D competition system, it becomes necessary to satisfy the following conditions: $x_1(t+1) = x_1(t)$, $x_2(t+1) = x_2(t)$. Subsequent calculations yield four fixed points: $E1(0,0)$, $E2(0, (4(10a\alpha + 4\alpha c_1 - 14\alpha c_2 - 35a - 14c_1 + 49c_2)/16\alpha^2 - 675b\beta_2 - 112\alpha + 196))$, $E3((4(10a\alpha + 4\alpha c_2 - 14\alpha c_1 - 35a - 14c_2 + 49c_1)/16\alpha^2 - 675b\beta_1 - 112\alpha + 196), 0)$, $E4(x_1^*, x_2^*)$, where

$$x_1^* = \frac{4(7-2\alpha)\left(8a\alpha^2 - 75ab\beta_2 - 8\alpha^2 c_2 + 105b\beta_2 c_1 - 30b\beta_2 c_2 - 36a\alpha + 8\alpha c_1 + 28\alpha c_2 + 28a - 28c_1\right)}{64\alpha^4 + 240\alpha^2 b\beta_1 + 240\alpha^2 b\beta_2 - 10125b^2\beta_1\beta_2 - 448\alpha^3 - 1680\alpha b\beta_1 - 1680\alpha b\beta_2 + 720\alpha^2 + 2940b\beta_1 + 2940b\beta_2 + 448\alpha - 784}$$

$$x_2^* = \frac{4(7-2\alpha)\left(8a\alpha^2 - 75ab\beta_1 - 8\alpha^2 c_1 + 105b\beta_1 c_2 - 30b\beta_1 c_1 - 36a\alpha + 8\alpha c_2 + 28\alpha c_1 + 28a - 28c_2\right)}{64\alpha^4 + 240\alpha^2 b\beta_1 + 240\alpha^2 b\beta_2 - 10125b^2\beta_1\beta_2 - 448\alpha^3 - 1680\alpha b\beta_1 - 1680\alpha b\beta_2 + 720\alpha^2 + 2940b\beta_1 + 2940b\beta_2 + 448\alpha - 784}. \quad (13)$$

It is important to note that $E1$, $E2$, and $E3$ represent the corner solutions, bearing less significance in the context of R&D competition. Hence, our sole focus centers on the equilibrium solution $E4$, while analyzing the equilibrium conditions between the two manufacturers.

We can derive the equilibrium $E4$ in the R&D competition system. The Jacobian matrix can be expressed as follows:

$$\begin{pmatrix} 1 + \gamma_1 x_1^*\left(\dfrac{4(7-2\alpha)^2}{675b} - \beta_1\right) & \dfrac{4\gamma_1 x_1^*(7-2\alpha)(7\alpha-2)}{675b} \\ \dfrac{4\gamma_2 x_2^*(7-2\alpha)(7\alpha-2)}{675b} & 1 + \gamma_2 x_2^*\left(\dfrac{4(7-2\alpha)^2}{675b} - \beta_2\right) \end{pmatrix}. \quad (14)$$

The trace of the Jacobian matrix is calculated as follows:

$$Tr = 2 + \gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b} - \beta_1\right) + \gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b} - \beta_2\right). \quad (15)$$

The determinant is determined as follows:

$$\mathrm{Det} = \left(1 + \gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b} - \beta_1\right)\right)\left(1 + \gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b} - \beta_2\right)\right) - \frac{16\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}{455625b^2}. \quad (16)$$

Consequently, the characteristic polynomial of the Jacobian matrix can be expressed as follows:

$$x^2 - \mathrm{Tr}x + \mathrm{Det}. \quad (17)$$

By evaluating the expression $Tr^2 - 4\mathrm{Det} = ((4(7-2\alpha)^2/675b) - (4(7-2\alpha)^2/675b) - \beta_1 + \beta_2)^2 + (64\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2/455625b^2) > 0$, we can conclude that equation (17) has two real roots [24].

According to the Jury rule [24, 25], the stability condition for the R&D competition system states that

$$\begin{cases} (1) 1 + Tr + \mathrm{Det} > 0, \\ (2) 1 - Tr + \mathrm{Det} > 0, \\ (3) 1 - |\mathrm{Det}| > 0. \end{cases} \quad (18)$$

The condition (1) can be expressed as follows:

$$
\begin{aligned}
1+Tr+\mathrm{Det} &= 1+2+\gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)+\gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)\\
&\quad+\left(1+\gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)\right)\left(1+\gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)\right)-\frac{16\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}{455625b^2}\\
&= 4+2\gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)+2\gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)+\gamma_1\gamma_2 x_1^* x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)\\
&\quad-\frac{16\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}{455625b^2}>0.
\end{aligned}
\tag{19}
$$

In order to satisfy this condition, it follows that

$$
\gamma_1<\frac{1822500b^2+1350b\gamma_2 x_2^*\left(4(7-2\alpha)^2-675b\beta_2\right)}{-1350bx_1^*\left(4(7-2\alpha)^2-675b\beta_1\right)-\gamma_2 x_1^* x_2^*\left(4(7-2\alpha)^2-675b\beta_1\right)\left(4(7-2\alpha)^2-675b\beta_2\right)+16\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}.
\tag{20}
$$

The condition (2) is given by

$$
\begin{aligned}
1-Tr+\mathrm{Det} &= 1-\left(2+\gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)+\gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)\right)\\
&\quad+\left(1+\gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)\right)\left(1+\gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)\right)-\frac{16\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}{455625b^2}\\
&= \gamma_1\gamma_2 x_1^* x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)-\frac{16\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}{455625b^2}>0.
\end{aligned}
\tag{21}
$$

We can conclude that when $(4(7-2\alpha)^2-675b\beta_1)(4(7-2\alpha)^2-675b\beta_2)>16(7-2\alpha)^2(7\alpha-2)^2$, condition (2) is satisfied.

When $\mathrm{Det}\geq 0$, the condition (3) can be represented as follows:

$$
\begin{aligned}
1-|\mathrm{Det}| &= 1-\left(\left(1+\gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)\right)\left(1+\gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)\right)-\frac{16\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}{455625b^2}\right)\\
&= -\gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)-\gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)-\gamma_1\gamma_2 x_1^* x_2^*\left(\frac{4(7-2\alpha)^2}{675b}-\beta_1\right)\left(\frac{4(7-2\alpha)^2}{675b}-\beta_2\right)\\
&\quad+\frac{16\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}{455625b^2}>0.
\end{aligned}
\tag{22}
$$

We can conclude that

$$\gamma_1 < \frac{-675b\gamma_2 x_2^*\left(4(7-2\alpha)^2 - 675b\beta_2\right)}{675bx_1^*\left(4(7-2\alpha)^2 - 675b\beta_1\right) + \gamma_2 x_1^* x_2^*\left(4(7-2\alpha)^2 - 675b\beta_1\right)\left(4(7-2\alpha)^2 - 675b\beta_2\right) - 16\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2} \tag{23}$$

When $\text{Det} < 0$,

$$\begin{aligned} 1 - |\text{Det}| &= 1 + \left(\left(1 + \gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b} - \beta_1\right)\right)\left(1 + \gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b} - \beta_2\right)\right) - \frac{16\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}{455625b^2}\right) \\ &= 2 + \gamma_1 x_1^*\left(\frac{4(7-2\alpha)^2}{675b} - \beta_1\right) + \gamma_2 x_2^*\left(\frac{4(7-2\alpha)^2}{675b} - \beta_2\right) + \gamma_1\gamma_2 x_1^* x_2^*\left(\frac{4(7-2\alpha)^2}{675b} - \beta_1\right)\left(\frac{4(7-2\alpha)^2}{675b} - \beta_2\right) \\ &\quad - \frac{16\gamma_1\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}{455625b^2} > 0. \end{aligned} \tag{24}$$

We can conclude that when $\text{Det} < 0$,

$$\gamma_1 < -\frac{901250b^2 + 675b\gamma_2 x_2^*\left(4(7-2\alpha)^2 - 675b\beta_2\right)}{675bx_1^*\left(4(7-2\alpha)^2 - 675b\beta_1\right) + \gamma_2 x_1^* x_2^*\left(4(7-2\alpha)^2 - 675b\beta_1\right)\left(4(7-2\alpha)^2 - 675b\beta_2\right) - 16\gamma_2 x_1^* x_2^*(7-2\alpha)^2(7\alpha-2)^2}. \tag{25}$$

Figure 2 illustrates the distinct areas of stability: Area I represents a stable region, while Area II is characterized as unstable. In Area II, both conditions (1) and (2) are satisfied. Area III, on the other hand, falls within the realm of instability, meeting condition (2) but failing to satisfy conditions (1) and (3). Similarly, Area IV is unstable but complies with conditions (2) and (1), albeit falling short in satisfying condition (1) completely. By examining Figure 2, we discern that a low adjustment speed of manufacturers leads to a stable state within the R&D competition system. However, when the adjustment speed surpasses a certain threshold, the R&D competition system undergoes bifurcations or even transitions into chaotic states. These circumstances, in turn, pose challenges in managing the R&D level of manufacturers, thus increasing the complexities of their management.

5. Simulation

From the previous discussion, it is apparent that the R&D competition system may exhibit nonlinear behavior and potentially undergo bifurcation or even enter a chaotic state. To elucidate the impact of various factors in the system, numerical simulation methods are employed. In this section, our primary focus is on examining the influence of decision-making behavior and R&D characteristics on dynamic behavior. Decision-making behavior comprises the adjustment speed of manufacturers and the marginal cost of retailers, which can also serve as a representation of the market conditions faced by manufacturers. R&D characteristics primarily encompass the influence of spillover effects and R&D costs. There are four factors that impact the R&D competition system: adjustment speed, spillover effect, R&D cost, and retailer marginal cost.

We shall utilize a parameter set based on actual industry conditions. These parameters shall be incorporated into the R&D competition system (12) and subsequently subjected to computational simulation via MATLAB software. In the event of nonlinear behavior observed in the simulation outcomes, bifurcation and chaos phenomena manifest in the bifurcation diagram. This occurrence is accompanied by fluctuations in the maximum Lyapunov exponent, signifying an unstable phenomenon. Consequently, the predictability of corporate behavior diminishes, imposing heightened challenges for enterprise managers and resulting in consequential losses due to nonlinear behavior.

5.1. Adjustment Speed Effects. In this subsection, we illustrate the effects of manufacturer's adjustment speed on stability. We utilize a fixed parameter set $\{a = 10, b = 1, \alpha = 0.8, c_1 = 2, c_2 = 1, \beta_1 = 0.8, \beta_2 = 0.4, \gamma_2 = 1\}$, with initial R&D levels set at $x_1(1) = 0.1$ and $x_2(1) = 0.2$. Figure 3 demonstrates that, holding other parameters constant, an increase in the adjustment speed γ_1 has a destabilizing effect.

Through numerical evidence of the complex dynamics in R&D competition system (12), we observe the bifurcation diagram of x_1 and x_2 concerning γ_1. Figure 3 reveals that the R&D competition system (12) remains stable when

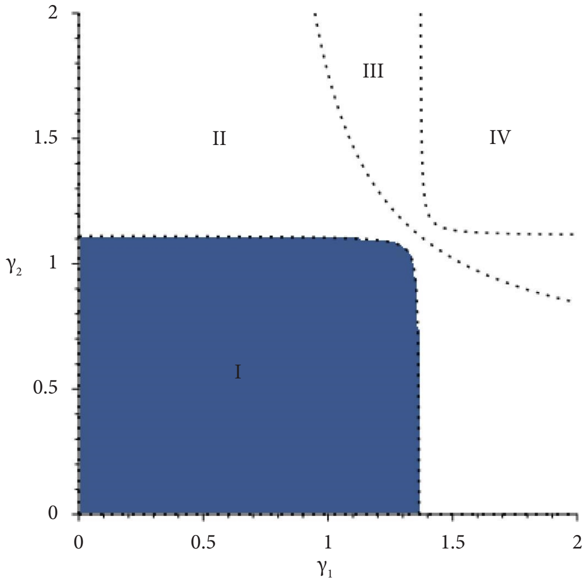

Figure 2: Stable area of the R&D competition system (12).

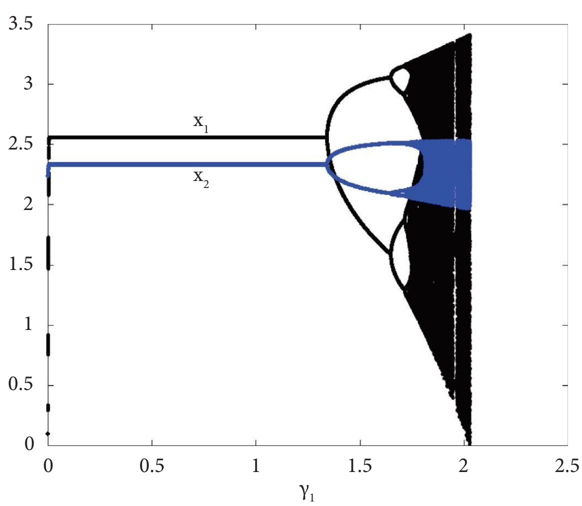

Figure 3: Bifurcation diagram with respect to γ_1 when $\{a = 10, b = 1, c_1 = 2, c_2 = 1, \beta_1 = 0.8, \beta_2 = 0.4, \gamma_2 = 1\}$, $x_1(1) = 0.1, x_2(1) = 0.2$.

$\gamma_1 < 1.342$. In this stable state, the R&D level of manufacturer 1 exceeds that of manufacturer 2, possibly due to cost pressures leading to higher R&D level for manufacturer 1. When γ_1 ranges from 1.342 to 1.649, the R&D competition system (12) undergoes a period-2 bifurcation. Subsequently, as $\gamma_1 > 1.649$, the R&D competition system (12) experiences a period-4 bifurcation. Ultimately, when $\gamma_1 > 1.734$, a state of chaos emerges. It is notable that within the stable regime, variations in the adjustment speed utilized by manufacturers do not significantly impact the changes in R&D level. However, when the adjustment speed surpasses an optimal threshold, the R&D competition system exhibits nonlinear behaviors, thereby rendering predictability of R&D actions arduous and exacerbating managerial complexities within manufacturers.

The largest Lyapunov exponent (LE) exhibits bifurcations and chaotic dynamics, whereby positive LE values indicate the occurrence of chaotic behavior. The LE can be understood as the exponential rate at which the infinitesimally small separation between two nearby initial states in the evolving phase space expands over time [26]. In Figure 4, the LE remains negative when $\gamma_1 < 1.342$, thus indicating the stability of the R&D competition system (12) due to the low adjustment speed of x_1. When γ_1 approximates 1.342 and 1.649, the LE attains a zero value, suggesting the onset of bifurcation within the R&D competition system (12). Beyond $\gamma_1 \approx 1.734$, the LE primarily assumes positive values, signifying the entrance into a chaotic regime for the R&D competition system (12). Figure 5 visually presents the manifestation of strange attractors in the R&D competition system (12) at $\gamma_1 = 1.830$. Additionally, Figure 5 demonstrates the partial synchronization of x_1 and x_2 during the state of chaos within the R&D competition system (12).

Figure 6 depicts the aggregate profits of Manufacturer 1 over 100 iterations based on the adjustment speed. As the adjustment speed increases, Manufacturer 1 eventually enters a chaotic state. It is evident that the adjustment speed has no impact on profit variations when the R&D competition system is in a stable state. However, following the occurrence of bifurcation, Manufacturer 1 experiences a significant decline in profits. This observation implies that while the adjustment speed does not affect the R&D level to a certain extent, the nonlinear behavior results in profit losses throughout the entire supply chain. Moreover, it introduces greater management challenges for manufacturers within the supply chain.

5.2. Retailer Marginal Cost Effect. In this section, we elucidate the influence of retailer marginal cost c_1 on the stability of R&D competition system (12). The simulation in this section aligns closely with the previously presented data. We constrain the parameters to the following values: $\{a = 10, b = 1, \alpha = 0.8, c_2 = 1, \beta_1 = 0.8, \beta_2 = 1, \gamma_1 = 1, \gamma_2 = 1.4\}$, and initialize the R&D level with $x_1(1) = 0.1$ and $x_2(1) = 0.2$. With the purpose of providing a fully representation of the impact of market capability on dynamic behavior, we judiciously adjust the prior adjustment speed, accentuating the increase in retailer 2's adjustment speed. Consequently, the R&D competition system's precariousness rises, which enhances the manifestation of chaotic phenomena, while preserving its evolutionary tendencies intact. Figure 7 substantiates the realization of stability in R&D competition system (12) in response to inflated retailer marginal costs.

Within Figure 7, we peruse the comprehensive bifurcation diagram depicting x_1 and x_2 as nuanced by retailer 1's marginal cost c_1. Ascertained therein is the R&D competition system's manifestation of chaotic behavior for c_1 values less than 0.605, transmuting into a bifurcation regime within the range of $0.605 < c_1 < 2.251$. Profoundly, when c_1 exceeds the threshold of 2.251, the R&D competition system (12) stabilizes. It is pertinent to note that, during this phase of stability, retailer 1's marginal cost escalation precipitates

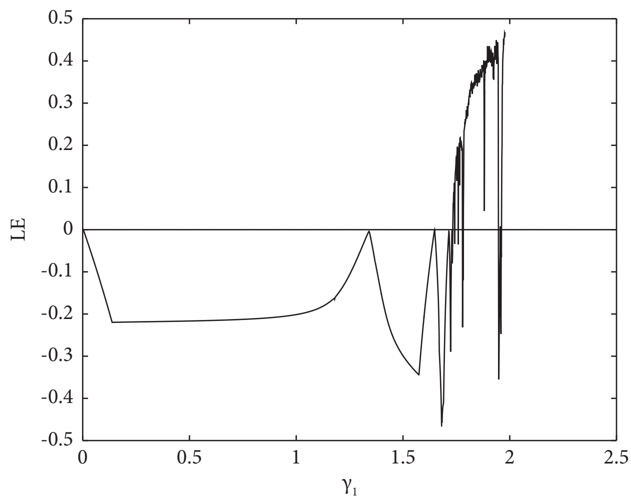

Figure 4: The maximum Lyapunov exponent with respect to γ_1 corresponding to Figure 3.

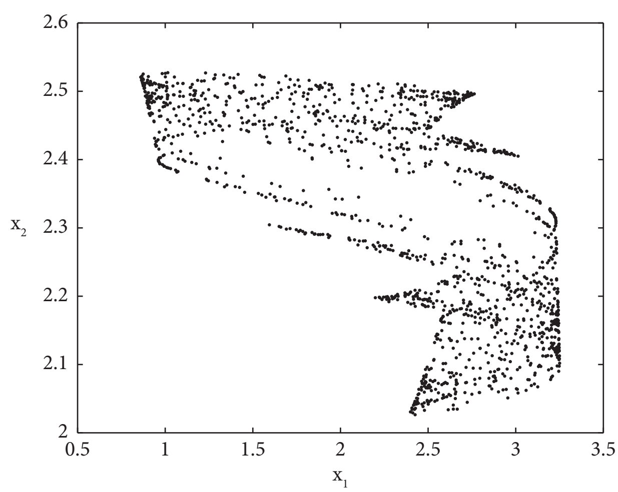

Figure 5: The strange attractors of R&D competition system (12) when $\gamma_1 = 1.830$ corresponding to Figure 3.

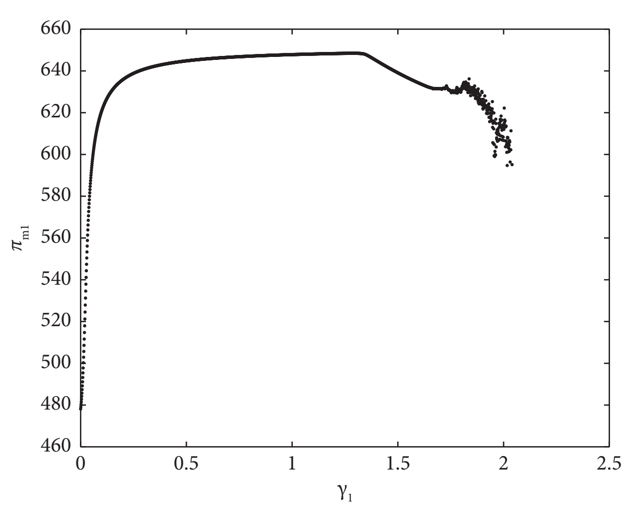

Figure 6: The manufacturer 1's aggregate profits according to γ_1 for 100 iterations.

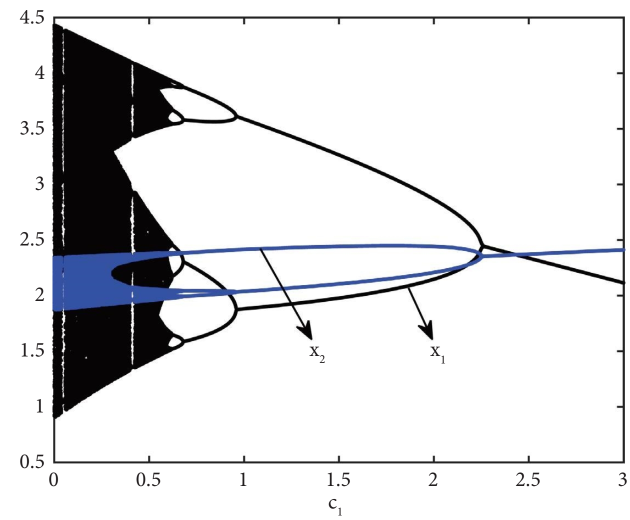

Figure 7: Bifurcation diagram with respect to c_1 when $\{a = 10, b = 1, c_2 = 1, \beta_1 = 0.8, \beta_2 = 1, \gamma_1 = 1, \gamma_2 = 1.4\}$, $x_1(1) = 0.1$, $x_2(1) = 0.2$.

the decrement of x_1 and the concomitant increment of x_2. Evidently, greater retailer marginal costs instigate R&D competition system stability, implying that a stronger market advantage paradoxically engenders a heightened propensity towards chaotic dynamics.

Figure 8 presents the Lyapunov exponent (LE) in relation to retailer marginal cost. For c_1 values less than 0.605, the LE exhibits negative values, indicating the R&D competition system (12) displays chaotic behavior. At approximately $c_1 \approx 0.680$ and $c_1 \approx 0.960$, the LE assumes a value of zero, signaling a transition to bifurcation in R&D competition system (12). Beyond $c_1 \approx 0.960$, the LE reverts to negative values, suggesting the R&D competition system (12) attains stability. Figure 9 showcases the strange attractors of R&D competition system (12) at $c_1 = 0.345$.

Figure 10 portrays the aggregate profit of Manufacturer 1 over 100 iterations in relation to the impact of Retailer 1's marginal cost. It is evident that as Retailer 1's marginal cost escalates, Manufacturer 1's aggregate profit diminishes. While reducing marginal cost may seem economically advantageous, it proves deleterious to system stability. As previously expounded, the market influence of R&D manufacturers directly affects their R&D levels. Greater market influence corresponds to higher levels of R&D, resulting in maximal profits. However, it is crucial to recognize that such a scenario also engenders a heightened inclination towards system instability. Under these circumstances, manufacturers must strive to strike a balance among their R&D decisions, profitability, and market stability.

5.3. Spillover Effects. This analysis examines the influence of spillover effects on the profitability and stability of R&D competition systems. Utilizing the fixed parameter set $\{a = 10, b = 0.7, c_1 = 2, c_2 = 1, \beta_1 = 0.8, \beta_2 = 1, \gamma_1 = 0.8, \gamma_2 = 0.7\}$, we consider the initial R&D levels as $x_1(1) = 0.1$ and $x_2(1) = 0.2$. We adjust the speed of adaptation to yield an uninterrupted visualization of the occurrence of chaotic phenomena.

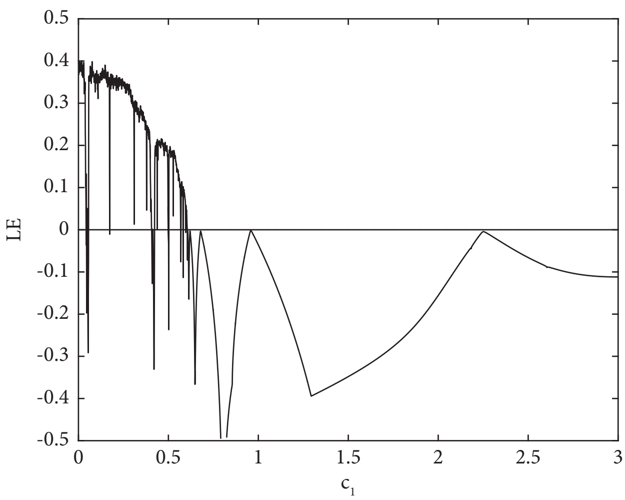

FIGURE 8: The maximum Lyapunov exponent with respect to c_1 corresponding to Figure 7.

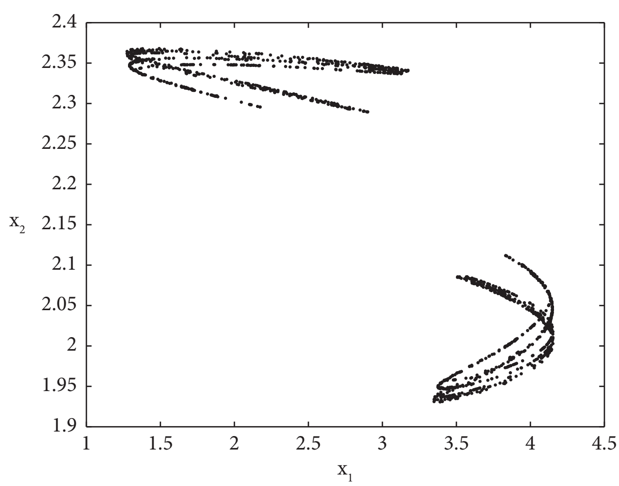

FIGURE 9: The strange attractors of R&D competition system (12) when c_1 = 0.345 corresponding to Figure 7.

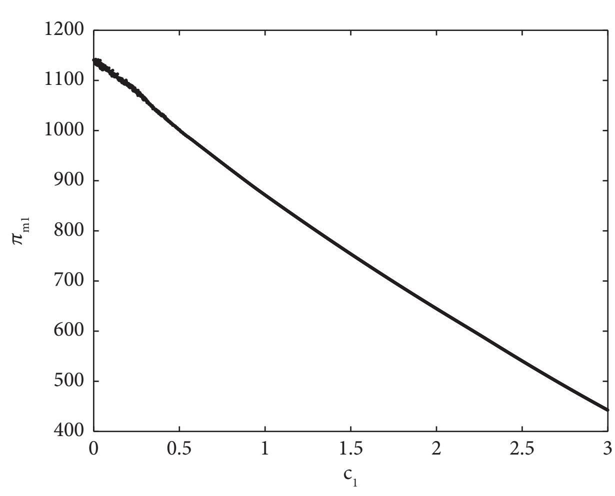

FIGURE 10: The manufacturer 1's aggregate profits according to c_1 for 100 iterations.

Figure 11 presents the bifurcation diagram of x_1 and x_2 with respect to the spillover effects, providing insight into the stability of the R&D competition system (12). Stability is achieved when $\alpha < 0.437$, during which the Nash equilibrium point E4 remains locally stable. Specifically, for $\alpha 0.437 < \alpha < 0.802$, the R&D competition system (12) undergoes a transition into a 2-period bifurcation, while for $\alpha > 0.802$, it exhibits chaotic behavior. Notably, due to their lower R&D costs, Manufacturer 1 gains a competitive advantage in innovation, surpassing Manufacturer 2 in terms of R&D levels. However, as spillover effects increase, Manufacturer 1 experiences a decline in R&D level, while Manufacturer 2 sees an improvement. Figure 11 illustrates that, while holding other parameters constant, an increase in spillover effects leads to instability in the R&D competition system (12). Noteworthy is the supply chain system's bifurcation and resulting instability when spillover effects are high, culminating in intermittent chaos within the R&D competition system [27].

Figure 12 showcases the Lyapunov exponent (LE) as it pertains to α. Notably, the LE denoted values below zero when $\alpha < 0.437$, shedding light on the inherent stability of the R&D competition system (12) amidst low spillover effects. A threshold is observed at $\alpha \approx 0.437$, where the LE converges to zero, implying the emergence of a two-period bifurcation within the R&D competition system (12). Upon exceeding $\alpha > 0.802$, the LE predominantly assumes positive values, signifying the R&D competition system's entry into a chaotic domain. Figure 8 effectively captures the intermittent chaos traits distinctly manifested in the R&D competition system. In parallel, Figure 13 visually delineates the strange attractors of the R&D competition system (12) at $\alpha = 0.910$. Moreover, Figure 13 underscores the coalescence between x_1 and x_2 under chaotic circumstances within R&D competition system (12).

Figure 14 visually represents the aggregate profit of Manufacturer 1 over 100 iterations, elucidating its dependency on the spillover effect. Notably, a positive correlation is observed between Manufacturer 1's aggregate profit and the magnitude of spillover. Contrary to alternative scholarly findings, an increase in spillover effects among distinct manufacturers yields a collective rise in their profits. However, as the spillover effect intensifies, the R&D competition system experiences bifurcations, impeding the rate of profit escalation. Simultaneously, the occurrence of these bifurcations leads to an augmented burden of managerial costs. Consequently, prudent supply chain managers are compelled to impose limitations on the intensity of spillover effects.

5.4. R&D Cost Effects. This section presents a comprehensive analysis of the influence of R&D costs on the stability of R&D competition system (12). The specified parameter set $\{a = 10, b = 1, c_1 = 2, c_2 = 1, \beta_2 = 1, \gamma_1 = 1.2, \gamma_2 = 1\}$ is held constant, while the initial R&D levels are set as $x_1\ (1) = 0.1$ and $x_2\ (1) = 0.2$. Consistent with the preceding discourse, adaptations are made to the adjustment speed, while maintaining the constancy of the remaining coefficients.

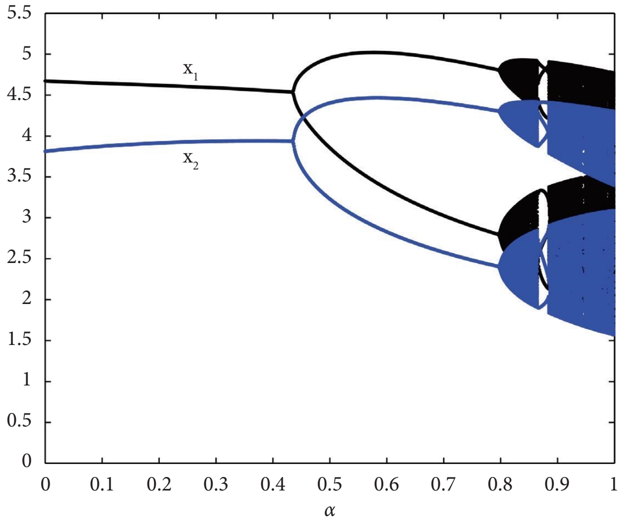

Figure 11: Bifurcation diagram with respect to α when $\{a = 10, b = 0.7, c_1 = 2, c_2 = 1, \beta_1 = 0.8, \beta_2 = 1, \gamma_1 = 0.8, \gamma_2 = 0.7\}$, $x_1(1) = 0.1, x_2(1) = 0.2$.

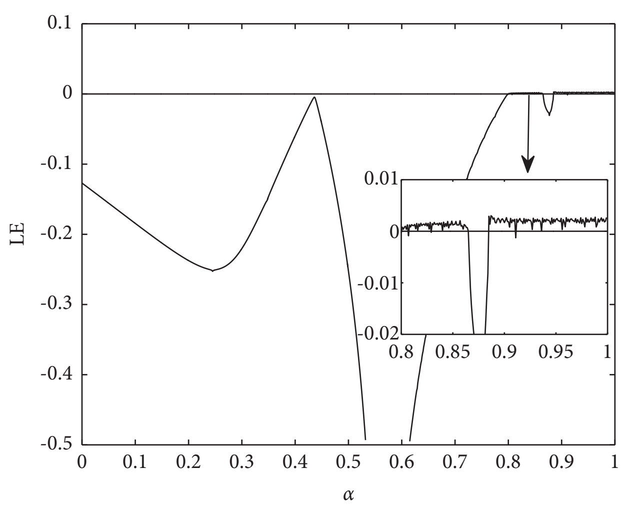

Figure 12: The maximum Lyapunov exponent with respect to α corresponding to Figure 11.

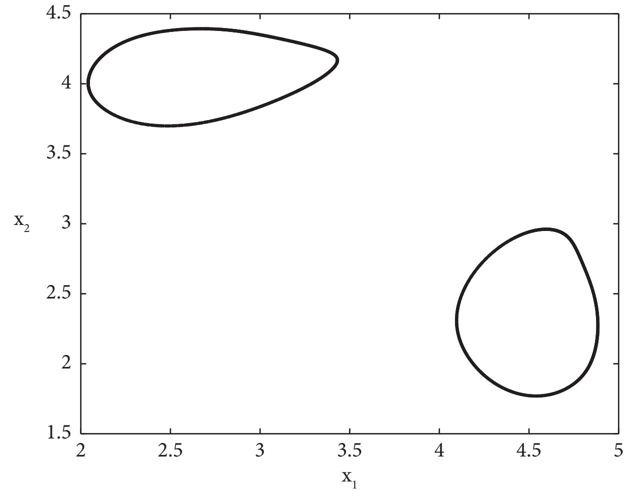

Figure 13: The strange attractors of the R&D competition system (12) when $\alpha = 0.910$ corresponding to Figure 11.

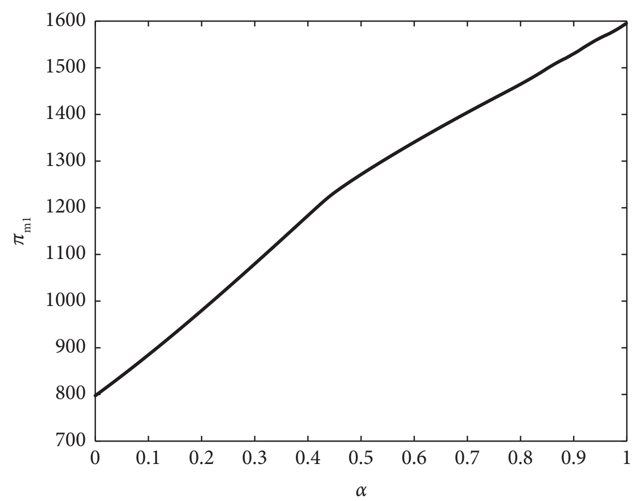

Figure 14: The manufacturer 1's aggregate profits according to α for 100 iterations.

Figure 15 visually portrays the R&D competition system's response to varying R&D costs, revealing the attainment of stability as these costs escalate.

In Figure 15, the bifurcation diagram of x_1 and x_2 is presented in relation to Manufacturer 1's R&D cost. It is evident that the R&D competition system (12) manifests a state of chaos when $\beta_1 < 0.295$, transitions into a state of bifurcation for $0.446 > \beta_1 > 0.295$, and ultimately achieves stability when $\gamma_1 > 0.295$. Under stable area, an increase in Manufacturer 1's R&D cost leads to a decrease in x_1 and an increase in x_2. Notably, this observation reflects the occurrence of intense market competition and potential chaos when R&D costs are low, ultimately giving way to a more stable system when R&D costs are relatively high. Moreover, it underscores the significance of enhancing R&D capabilities for manufacturers striving to maintain a competitive edge.

Figure 16 depicts the Lyapunov exponent (LE) in relation to β_1: for $\beta_1 < 0.295$, the LE exceeds zero, indicating chaotic behavior in R&D competition system (12). At approximately $\beta_1 = 0.446$, the LE reaches zero, signifying the onset of bifurcation in the system. As β_1 surpasses the threshold of 0.446, the LE becomes negative, indicating R&D competition system (12) attains stability. Figure 17 illustrates the emergence of strange attractors in R&D competition system (12) when $\beta_1 = 0.293$.

Figure 18 depicts the progressive variation of manufacturer 1's aggregate profit over 100 iterations in response to changes in its R&D cost. A discernible pattern emerges,

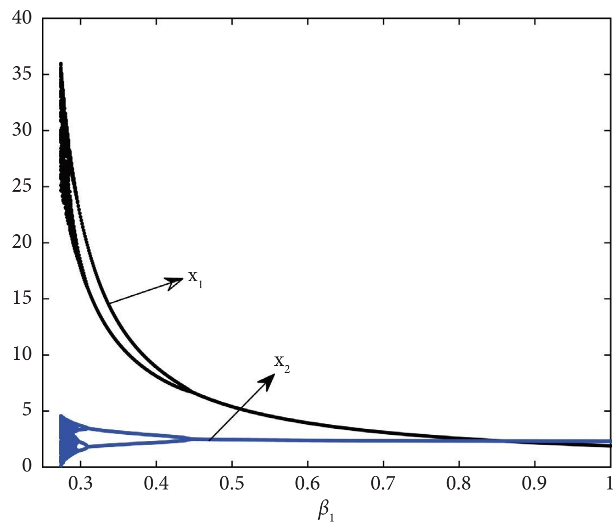

FIGURE 15: Bifurcation diagram with respect to β_1 when $\{a = 10, b = 1, c_1 = 2, c_2 = 1, \beta_2 = 1, \gamma_1 = 1.2, \gamma_2 = 1\}$, $x_1(1) = 0.1, x_2(1) = 0.2$.

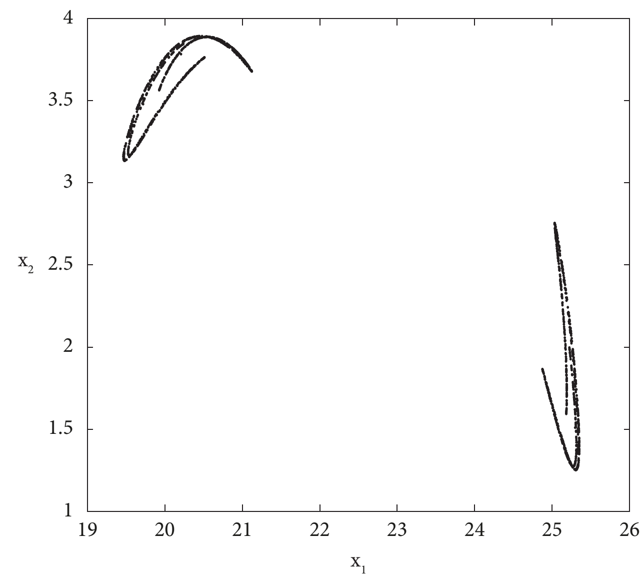

FIGURE 17: The strange attractors of R&D competition system (12) when $\beta_1 = 0.293$ corresponding to Figure 15.

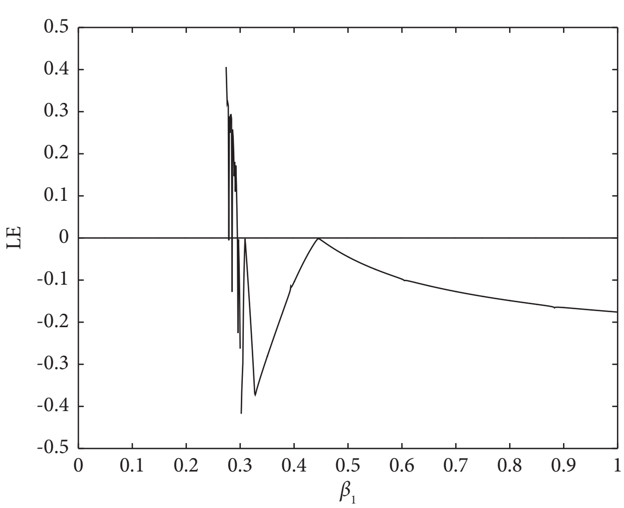

FIGURE 16: The maximum Lyapunov exponent with respect to β_1 corresponding to Figure 15.

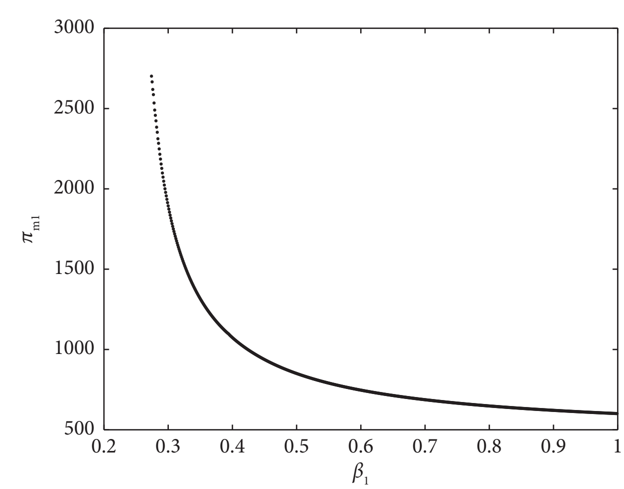

FIGURE 18: The manufacturer 1's aggregate profits according to β_1 for 100 iterations.

revealing a negative relationship between Manufacturer 1's R&D cost and its aggregate profit. Specifically, as the R&D cost increases, the aggregate profit experiences a consistent decline. This finding underscores the intricate trade-off between profitability and stability within the context of R&D investment. Notably, when the R&D cost is low, the R&D competition system achieves a relatively higher profit level. However, this regime also coincides with a greater propensity for bifurcation and chaotic phenomena. Conversely, when the R&D cost is set at a higher value, the R&D competition system gravitates towards stability but at the expense of diminished profitability. Thus, it becomes imperative for manufacturers to diligently strike a delicate equilibrium between sustaining profitability and ensuring system stability.

5.5. Time-Delayed Feedback Control. Based on the preceding discourse, it is discernible that under specific circumstances, the R&D competition system manifests phenomena characterized by bifurcation and chaos. These phenomena yield a deleterious impact on the profitability of the R&D competition system, thereby posing formidable challenges to the management of the supply chain. To effectively govern the chaos and curtail financial losses with the manufacturers, the adoption of control mechanisms emerges as a customary choice. Within the context of Section 5.1, the R&D competition system (12) transitions into a state of chaos. To ameliorate this condition, we employ the Time-Delay Feedback Control (TDFC) method, as expounded by [27], as a means of mitigating the chaotic state prevalent in R&D competition system. Mathematically, the TDFC mechanism can be characterized by the equation

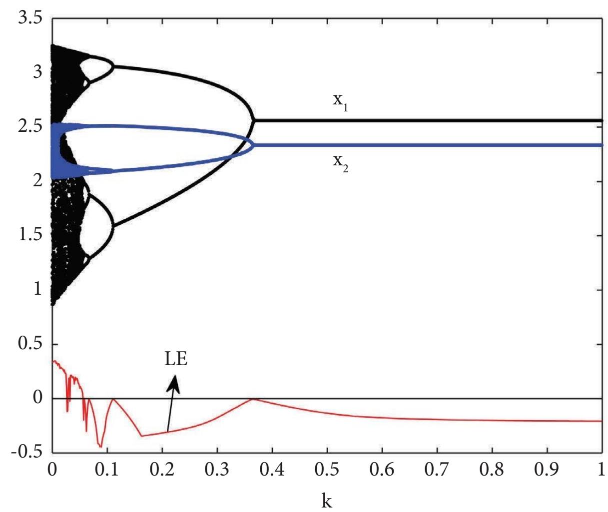

FIGURE 19: Bifurcation diagram and LE with respect to k when $\{a = 10, b = 1, c_1 = 2, c_2 = 1, \beta_1 = 0.8, \beta_2 = 0.4, \gamma_1 = 1.830, \gamma_2 = 1\}$, $x_1(1) = 0.1, x_2(1) = 0.2$.

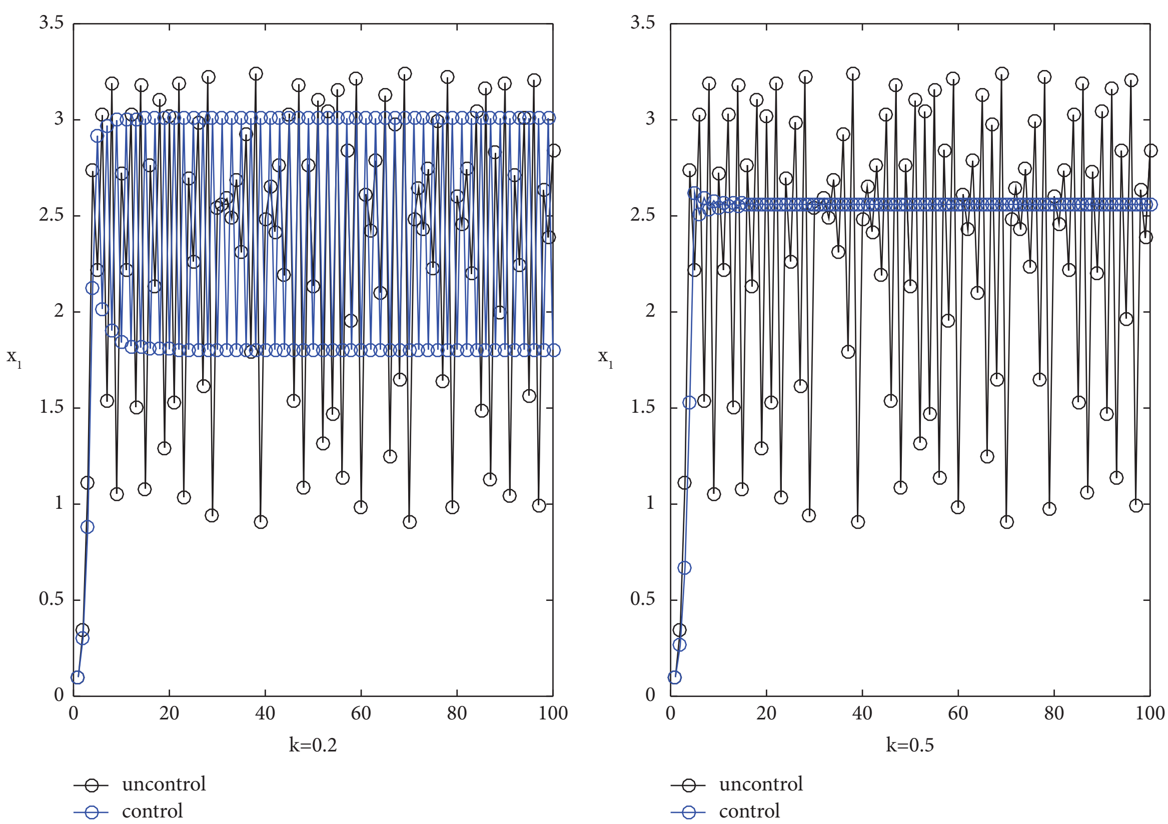

FIGURE 20: The time series difference between the R&D competition system (12) and under TDFC method.

$G(t+1) = k(x_1(t) - x_1(t+1))$, where the parameter k assumes the role of the chaos control parameter. As a consequence, the R&D competition system (12) undergoes a transformation. Notably, the ensuing R&D TDFC system can be succinctly presented as follows:

$$\begin{cases} x_1(t+1) = x_1(t) + \dfrac{\gamma_1 x_1(t)\partial\pi_{m1}}{\partial x_1(t)} + G(t+1), \\ x_2(t+1) = x_2(t) + \dfrac{\gamma_2 x_2(t)\partial\pi_{m2}}{\partial x_2(t)}. \end{cases} \tag{26}$$

So we get the R&D TDFC system as follows:

$$\begin{cases} x_1(t+1) = x_1(t) + \dfrac{\gamma_1 x_1(t)\partial\pi_{m1}}{(1+k)\partial x_1(t)}, \\ \\ x_2(t+1) = x_2(t) + \dfrac{\gamma_2 x_2(t)\partial\pi_2}{\partial x_2}. \end{cases} \tag{27}$$

In accordance with Section 4.1, the parameter set $\{a = 10, b = 1, c_1 = 2, c_2 = 1, \beta_1 = 0.8, \beta_2 = 0.4, \gamma_1 = 1.830, \gamma_2 = 1\}$ was established as fixed, while the initial R&D levels assumed values of $x_1(1) = 0.1$ and $x_2(1) = 0.2$. As an outcome of this configuration, the R&D competition system demonstrated a state of chaos. Figure 19 effectively portrays the bifurcation diagram and the corresponding Lyapunov exponent (LE) in relation to the parameter k. A detailed scrutiny unveils that the R&D competition TDFC system devolves into a state of chaos when k assumes values below the threshold of 0.056, exhibits bifurcation within the parameter range of $0.056 < k < 0.366$, and ultimately achieves stability when k surpasses the value of 0.366. Employing the TDFC method proves instrumental in attenuating intricate occurrences, namely bifurcation and chaos, without imposing any perceivable detriment upon the equilibrium state and its correlated innovation quotient.

In Figure 20, the absence of the TDFC mechanism elicits a chaotic state with x_1. Continuing along this line of inquiry, subsequent explorations are undertaken for two distinct scenarios, where k is set at 0.2 and 0.5, respectively. The acquisition of empirical evidence conclusively affirms that upon setting k equal to 0.2, the R&D competition TDFC system undergoes a two-period bifurcation, whereas an unequivocally stable state is attained as k acquires the value of 0.5.

6. Conclusions and Discussion

6.1. Conclusions. This study explores the stability of the R&D competition model within the context of the supply chain. It has been observed that four influential factors, namely adjustment speed, spillover effect, R&D cost effect, and retailer marginal cost, significantly impact the stability of the R&D system. Through simulated simulation of these factors, it has been ascertained that an escalation in adjustment speed and spillover effects precipitates a state of bifurcation, leading ultimately to the onset of a chaotic state in the R&D competition system. The adjustment speed does not exert any influence on the equilibrium R&D level; however, the occurrence of complex phenomena, such as bifurcation, yields a decline in manufacturer profits. Additionally, the spillover effect heightens the R&D level, resulting in augmented profits. Nevertheless, when this effect surpasses a certain threshold, R&D competition system instability ensues, thereby impeding profit growth rates. Conversely, elevations in R&D cost and retailer marginal cost promote the R&D competition system stability; nonetheless, they concomitantly give rise to diminished manufacturer profits. To combat the issue of chaotic dynamics, the TDFC mechanism has been introduced, which effectively mitigates chaos.

6.2. Theoretical Contributions. Our study makes theoretical contributions in the following aspects: (1) We investigate the stability of R&D competition within the context of the supply chain, extending upon prior works such as [23, 28], by integrating R&D competition, chaos theory, and the supply chain. (2) We enhance our understanding of the stability of the R&D competition system in the supply chain, shedding light on the origins of R&D competition system instability and providing valuable insights for further advancement of R&D practices. (3) We provide a foundation for managerial decision-making by examining critical factors influencing R&D behavior, including adjustment speed, spillover effects, R&D costs, and retailer marginal costs. (4) Recognizing the trade-off between profit and stability, we propose the TDFC mechanism as an effective tool to alleviate chaos, aiming to enrich R&D research with the integration of chaos theory.

6.3. Managerial Implications. Our research yields managerial implications for supply chain decision-making: First, we find that the R&D competition system becomes chaotic and loses control when the retailer's marginal cost reaches a sufficiently low level. This indicates that supply chain innovation must necessarily take into account the retailer costs. When retailer costs are excessively reduced, such as in the case of team buying, it can lead to complex behaviors. At this time, the business model represented by team buying, which excessively relies on the cost-driven operating model, should be moderately controlled. Retailers with cost advantages may introduce complexities to the R&D competition system. Therefore, managers should carefully balance manufacturer profitability and stability. Second, although chaos may emerge in simulations, we conclude that the chaotic state can be effectively managed through the implementation of the TDFC method. Managers should consider the adoption of similar control mechanisms to mitigate chaotic behaviors and enhance system stability.

6.4. Limitations and Further Research. This paper acknowledges several limitations. First, our R&D competition model assumes symmetry in the spillover effects among firms. However, in the real world, different firms may exhibit diverse spillover effects with each other, leading to potentially more interesting conclusions. Second, our assumption of collaboration between manufacturers and retailers overlooks power dynamics and possible competition between them. It is essential to pay closer attention to this aspect, as it is likely to yield more intriguing findings.

Data Availability

The data that supports the findings of this study are available upon request from the author Jianli Xiao at fanping123@126.com.

Disclosure

During the preparation of this work, the authors used ChatGPT in order to improve language. After using this service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Authors' Contributions

Conceptualization was done by J.X. and H.X.; methodology was proposed by J.X.; software was handled by J.X.; J.X. wrote the original draft; J.X. reviewed and edited the paper; visualization was done by H.X.; and funding acquisition was done by J.X. All authors have read and agreed to the published version of the manuscript.

Acknowledgments

This research was supported by the Research Initiation Foundation Project of Yiwu Industrial & Commercial College (RCKY202309).

References

[1] Z. Ge, Q. Hu, and Y. Xia, "Firms' R&D cooperation behavior in a supply chain," *Production and Operations Management*, vol. 23, no. 4, pp. 599–609, 2014.

[2] R. Amir and J. Wooders, "Cooperation vs. Competition in R&D: the role of stability of equilibrium," *Journal of Economics*, vol. 67, no. 1, pp. 63–73, 1998.

[3] L. M. B. Cabral, "R&D competition when firms choose variance," *Journal of Economics and Management Strategy*, vol. 12, no. 1, pp. 139–150, 2003.

[4] J. E. R. Mote, "Ecology: using 2-mode network analysis to explore complexity in R&D environments," *Journal of Engineering and Technology Management*, vol. 22, no. 1–2, pp. 93–111, 2005.

[5] F. Wu and J. Ma, "The equilibrium, complexity analysis and control in epiphytic supply chain with product horizontal diversification," *Nonlinear Dynamics*, vol. 93, no. 4, pp. 2145–2158, 2018.

[6] T. Li and J. Ma, "Complexity analysis of the dual-channel supply chain model with delay decision," *Nonlinear Dynamics*, vol. 78, no. 4, pp. 2617–2626, 2014.

[7] J. Hinloopen, "More on subsidizing cooperative and noncooperative R&D in duopoly with spillovers," *Journal of Economics*, vol. 72, no. 3, pp. 295–308, 2000.

[8] T. Li and J. Ma, "The complex dynamics of R&D competition models of three oligarchs with heterogeneous players," *Nonlinear Dynamics*, vol. 74, no. 1–2, pp. 45–54, 2013.

[9] Y. Peng, Q. Lu, and Y. Xiao, "A dynamic Stackelberg duopoly model with different strategies," *Chaos, Solitons and Fractals*, vol. 85, pp. 128–134, 2016.

[10] X. Yuan and R. Nishant, "Understanding the complex relationship between R&D investment and firm growth: a chaos perspective," *Journal of Business Research*, vol. 129, pp. 666–678, 2021.

[11] S. Jayanthi and K. K. Sinha, "Innovation implementation in high technology manufacturing: a chaos-theoretic empirical analysis," *Journal of Operations Management*, vol. 16, no. 4, pp. 471–494, 1998.

[12] D. Levy, "Chaos theory and strategy: theory, application, and managerial implications," *Strategic Management Journal*, vol. 15, no. S2, pp. 167–178, 2007.

[13] Y. Peng, Q. Lu, X. Wu, Y. Zhao, and Y. Xiao, "Dynamics of hotelling triopoly model with bounded rationality," *Applied Mathematics and Computation*, vol. 373, Article ID 125027, 2020.

[14] Z. Ge, Q. Hu, and Y. Xia, "Firms' R&D cooperation behavior in a supply chain," *Production and Operations Management*, vol. 23, no. 4, pp. 599–609, 2014.

[15] S. Akhtar, R. Ahmed, M. Batool, N. A. Shah, and J. D. Chung, "Stability, bifurcation and chaos control of a discretized Leslie prey-predator model," *Chaos, Solitons and Fractals*, vol. 152, Article ID 111345, 2021.

[16] X. Ma, J. Quan, and X. Wang, "Effect of reputation-based heterogeneous investment on cooperation in spatial public goods game," *Chaos, Solitons and Fractals*, vol. 152, Article ID 111353, 2021.

[17] A. L. Fradkov, R. J. Evans, and B. R. Andrievsky, "Control of chaos: methods and applications in mechanics," *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 364, no. 1846, pp. 2279–2307, 2006.

[18] Y. Guo, W. Jiang, and B. Niu, "Bifurcation analysis in the control of chaos by extended delay feedback," *Journal of the Franklin Institute*, vol. 350, no. 1, pp. 155–170, 2013.

[19] S. M. A. Pahnehkolaei, A. Alfi, and J. A. Tenreiro Machado, "Chaos suppression in fractional systems using adaptive fractional state feedback control," *Chaos, Solitons and Fractals*, vol. 103, pp. 488–503, 2017.

[20] V. Pyragas and K. Pyragas, "Act-and-Wait time-delayed feedback control of autonomous systems," *Physics Letters A*, vol. 382, no. 8, pp. 574–580, 2018.

[21] J. Du, T. Huang, Z. Sheng, and H. Zhang, "A new method to control chaos in an economic system," *Applied Mathematics and Computation*, vol. 217, no. 6, pp. 2370–2380, 2010.

[22] R. Amir, C. Halmenschlager, and J. Jin, "R&D-Induced industry polarization and shake-outs," *International Journal of Industrial Organization*, vol. 29, no. 4, pp. 386–398, 2011.

[23] C. D'Aspremont and A. Jacquemin, "Cooperative and noncooperative R&D in duopoly with spillovers," *The American Economic Review*, vol. 78, no. 5, p. 1133, 1988.

[24] A. A. Elsadany, "Dynamics of a cournot duopoly game with bounded rationality based on relative profit maximization," *Applied Mathematics and Computation*, vol. 294, pp. 253–263, 2017.

[25] J. Ma and H. Ren, "Influence of government regulation on the stability of dual-channel recycling model based on customer expectation," *Nonlinear Dynamics*, vol. 94, no. 3, pp. 1775–1790, 2018.

[26] D. Dai, X. Wu, F. Si, Z. Feng, and W. Chu, "The impact of tariff policies on vaccine supply chains: short-term and evolutionary game behaviors based on uncertain utility," *Applied Mathematical Modelling*, vol. 115, pp. 754–777, 2023.

[27] R. Amir and J. Wooders, "Cooperation vs. Competition in R&D: the role of stability of equilibrium," *Journal of Economics*, vol. 67, no. 1, pp. 63–73, 1998.

[28] J. Hinloopen, "Subsidizing cooperative and noncooperative R&D in duopoly with spillovers," *Journal of Economics*, vol. 66, no. 2, pp. 151–175, 1997.