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Research Article

The Core Might Change Anyhow We Define It: The Instability of Key Actors in Longitudinal Social Network Data

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Central actors or opinion leaders are in the right structural position to spread relevant information or convince others about adopting an innovation or behaviour change. Who is a central actor or opinion leader might be conceptualised in various ways. Widely accepted centrality measures do not take into account that those in central positions in the social network may change over time. A longitudinal comparison of the set and importance of opinion leaders is problematic with these measures and therefore needs a novel approach. In this study, we investigate ways to compare the stability of the set of central actors over time. Using longitudinal survey data from primary schools (where the members of the social networks do not change much over time) on advice-seeking and friendship networks, we find a relatively poor stability of who is in the central positions anyhow we define centrality. We propose the application of combined indices in order to achieve more efficient targeting results. Our results suggest that because opinion leaders may change over time, researchers should be careful about relying on simple centrality indices from cross-sectional data to gain and interpret information (for example, in the design of prevention programs, network-based interventions or infection control) and must rely on more diverse structural information instead.

1. Introduction

Social networks turn out to be crucial for adopting practices that affect personal lives. This phenomenon was first described by [1–3], and the influential persons were referred to in the scientific literature as opinion leaders [4–7]. The role of opinion leaders was analysed in very different life situations, like political orientation, marketing, fashion, moviegoing, consumer behaviour, family planning, science, and agriculture [8]. Social network analysis investigating opinion leaders teaches us that they are the most embedded persons of a social group, the sociometric stars, with the most connections [6, 7]. The diffusion of innovation [4] also highlights the social network aspect of opinion leaders who have an important role to play in spreading the innovations among their multiple contacts. As Valente and Pumpuang [9] describe, a wide range of research suggests that community-based or health-related programs using opinion leaders can be more effective, as seen in tobacco prevention [10–12], medical practices [13], and HIV/STD risk reduction [14, 15] or in the cases where opinion leaders or key actors need to be removed from the network to avoid terrorist actions [16]. There are many potential uses beyond these examples, such as increasing cyber security awareness [17] or spreading the ideas of sustainable development [18]. Opinion leaders and their influence have been investigated not only at the actor level but also between social groups and institutions [19]. Scientific research also investigated the role of opinion leaders among classmates in schools. Some papers discuss differences between lead users (innovators) and opinion leaders [20–23], but there seems to be a consensus about the major role of opinion leaders in the diffusion of innovations and prevention programs [4, 24, 25].

The structure of the network where innovations are spread is also important: bridges or opinion brokers between different social groups encourage innovation [26]. Kratzer and Lettl [23] found that between schoolchildren, those whose betweenness centrality is higher are more likely to be identified as lead users (or in Burt's terms, opinion brokers), while those whose degree centrality and closeness centrality are higher are more likely to be identified as opinion leaders. As we can see, there seems to be a consensus referring to the central network position of opinion leaders, although the identification of opinion leaders is not always so obvious. Scientific literature specifies some methods to identify opinion leaders, such as celebrities, self-selection, self-identification, staff-selection, positional approach, judge's rating, expert identification, snowball method, sample sociometric, and sociometric [9].

A greater challenge to the literature on identifying central actors and opinion leaders is posed by the differentiation between simple and complex contagion [27–29]. Unlike for simple contagion, such as the spread of a viral disease, multiple sources of exposure are required before an individual adopts an innovation or behaviour in complex contagion. Most social dissemination processes are believed to be complex contagions that they require to reconsider which nodes in the network should be considered as central or influential [30].

A general question of network intervention studies is whom to target. Targeting is typically based on cross-sectional network surveys. However, the stability of occupying key positions in the social network and the time perspective of the intervention have not been taken into account so far. There have been only few discussions in the scientific literature of instability regarding the central network positions [31]. If it turns out that opinion leaders', key actors', or top influencers' position is very unstable, then the validity and effectiveness of social network interventions might be questioned. Moreover, depending on the intervention, the selection of opinion leaders or key actors should always depend on their individual characteristics as well as on structural characteristics (see, e.g., [32]).

To examine how a social network facilitates the flow of information, commonly used and simple measures are degree centrality [33], measuring the number of connections of an actor; betweenness centrality [34], measuring the number of shortest paths between all pairs of actors that pass through a given actor in the network; and closeness centrality [33], or the total graph-theoretic distance of an actor from all other actors. The difficulty is caused by the fact that those actors who are individually the most central are not necessarily central at the group level due to the redundancy of their connections. That is why Borgatti [35] and An and Liu [36] differentiate the centrality measures on the individual and group levels. Individual (micro)-level measures (or bottom-up measures) are as follows: degree centrality [37, 38] and two-hop neighbourhood and the core [38, 39]. Top-down measures [36] are as follows: fragmentation centrality [35] and diffusion centrality [40]. Network-level measures take into account the entire network structure [41]: betweenness centrality, closeness, and eigenvector centrality [39].

The question of how network centrality can be better measured arises not only in static networks but also when networks change. Some networks, such as online social networks, mobile phone networks, and multiplayer online game networks, and centrality therein change rapidly. Changes in these networks can be observed either with precise time-stamps or at different time points. Timing of how changes occur in the network and which individuals can be reached from which node is crucial for problems such as virus spread and the diffusion of innovations [42]. It is important not only who are the central actors but also when they are in an influential position. Temporal network analysis recognizes that in these cases, "there is usually no unique candidate for the temporal version of a static centrality measure" [43, 44]. Based on the findings of [45–48], a new measure of temporal betweenness centrality is developed. Moreover, the changing role of a person in such networks was also recognized [49]. If the interconnectivity of social members is taken into consideration, a member can play different roles in one or more communities [50–52]. Role detection is usually made with the blockmodelling, which identifies nodes with structural equivalence [53, 54] and could also be used to detect structural changes in a network [50]. These methods are very convenient when dynamic online communities [50] or social rating platforms [55] are investigated. While in the highly volatile online space the connections and community roles can be very unstable, one can suppose that in networks with relatively stable membership like workplaces or school classes, the community role structure would be more stable over time.

Recent studies draw attention to the fact that even in static networks, no single centrality measure has a consistent performance across different networks [56]. Bucur empirically examined in 60 networks how well a selected subset of classical centrality measures identifies super-spreaders. The super-spreaders of the networks were identified by simulated SIR (Susceptible-Infected-Recovered) spreading processes. She found that none of the selected centrality measures performed well on all the test networks. Applying supervised learning, however she could identify 6 centrality pairs, and using these pairs, she could find the super-spreaders with high accuracy almost in all test networks. In the pairs, there is a pattern. One of the participants is a local measure, and the other is a global one. A local centrality measure, like the degree, measures the density of the node's extended neighbourhood, while a global centrality such as PageRank indicates the location importance of the node (core or periphery). A local centrality may give a high ranking to nodes that are in a dense cluster but at a peripheral region of the network. The additional global centrality indicator gives higher scores to the nodes of more central regions. The super-spreaders usually have high values of both centrality types. Bucur used the supervised support vector machine algorithm to learn the decision boundary in her study.

However, there are unsupervised methods to aggregate different centrality measures. Ibnoulouafi et al. [57] defined m-centrality as the convex combination of two centrality measures, the core number and $\Delta\mathfrak{D}$, where $\Delta\mathfrak{D}$ is a local measure calculating the degree variation in the neighbourhood of the node. The coefficients of each measure are computed using the Shannon entropy. In their study [58], Madotto and Liu applied the Borda count method to aggregate the centrality measures and used a correlation and entropy-based heuristic to prune the set of the possible centralities.

In conclusion, although it is a common approach to identify opinion leaders as the most central nodes of a given network, centrality is defined in many different ways, from different aspects, and no single centrality can properly identify the opinion leaders in all possible network types, particularly if networks change over time. In this study, we develop a methodology to quantify the stability of key actors over time. Instead of using a single centrality as an "opinion leader measure," we apply a selection of classical centrality measures and observe the stability of the central actors. The centrality measures analyzed are in-degree, two-hop neighbourhood, core, closeness, betweenness, eigenvector centrality, and PageRank. In Section 2.1, we give an overview of these centrality measures. Comparison of ranked lists has a key role in our methodology; therefore, in Section 2.2, we review the problem of comparing rankings. We apply three methods to empirically quantify the differences between the centrality measures. Monotonicity, Kendall tau correlation, and a comparison method focusing on the top influencers are discussed in Section 2.2. We improve the identification performance of top influencers by aggregating over the centrality measures, using by the Borda count method [58–60], discussed in Section 2.3. For measuring the stability of key actors, we develop a method in Section 2.5. We illustrate our methodology using five waves of a longitudinal social network survey from Hungarian primary schools described in Section 3. The empirical results are presented in Section 4.

2. Methodology

Advice-seeking and friendship networks are examples of different relationships; both are networks with relatively stable membership, constituting directed graphs without self-loops and multiple edges. Such a G directed graph is given by a pair: $G = (V, E)$, where V is the set of nodes and E is the set of directed edges: $E \subset V \times V$. Sometimes, we need the undirected version of a directed graph. In this case, we just omit the direction and define an edge between the nodes a and b as a $\{a, b\}$ set rather than an ordered pair. We consider the opinion leader as someone occupying a central network position. In general, centrality measures describe the importance of each node. Centrality can be measured in many ways. The C centrality measure can be viewed as a $\phi_C\colon V \longrightarrow \mathbb{R}$ map, and $\phi_C(a) > \phi_C(b)$ indicates that node a is more important than node b (with respect to the C measure). We sort the nodes according to their centrality value in descending order. As a result, we get an ordered list of the nodes. We call this ordered list the induced ranking of the nodes by the centrality measure. Of course, ties occur regularly. In this case, we handle the induced ranking as an ordered list of sets rather than a list of nodes, where each set contains nodes with the same score value. We discuss the applied centrality measures in the next section. In our calculations, we used the igraph package [61] to compute the centrality measures.

2.1. Centrality Measures. The simplest centrality measure is degree centrality [33], which keeps track of the degree of the given node. In a directed environment, we can define the in-degree and the out-degree of a node as the number of incoming and outgoing edges. We use in-degree centrality because of the directed nature of the networks studied. A related centrality measure is two-hop neighbourhood. Two-hop neighbourhood is defined as the number of nodes that are at distance 1 or 2 from the focal node. In this paper, we use the "in" version of this centrality measure, understood as the number of nodes from which the focal node can be reached in maximum of 2 steps.

The coreness or core number [62] is related to the core-periphery structure of the graph. In a $G = (V, E)$ graph, a subgraph $H = (\widehat{V}, E \mid \widehat{V})$ induced by the $\widehat{V} \subset V$ set is a k-core or a core of order k if and only if for all $v \in \widehat{V}$ vertices, the degree of v in H is at least k, and H is the maximum subgraph with this property. From the definition, it is clear that the cores are nested. The core number of vertex v is the highest order of a core that contains this vertex. If the G graph is directed, we can define the in-degree and out-degree versions of the core number in a straightforward way. In our study, we use the in-degree version of core centrality.

The closeness centrality [33, 63] measures how close a given node is to the other nodes. The closeness of a node a is the inverse of the average distance between a and any other node:

$$\text{closeness}(a) = \frac{n-1}{\sum_{b \in V, a \neq b} l(a, b)}, \tag{1}$$

where n is the number of nodes and $l(a, b)$ is the number of links in the shortest path between a and b. There are various conventions for handling networks that are not connected. In our work, if there is no path between a and b, we set $l(a, b) = n$ [61]. The closeness centrality is defined both for directed and undirected graphs. In this paper, we always apply the undirected version; therefore, if we compute the closeness centrality for a directed graph, we first omit the direction of the edges.

Betweenness centrality [34, 63] measures how well situated a node is in terms of the paths that it lies on. Denote the number of geodesics (shortest paths) between the nodes b and c by $P(b, c)$ and the number of geodesics between b and c that a lies on by $P_a(b, c)$. If the ratio $P_a(b, c)/P(b, c)$ is close to 1, then a lies on most of the shortest paths between b and c; therefore, a is important in terms of connecting these nodes. Averaging across all pairs of nodes, we can get a sense of how important a is in connecting the nodes:

$$\text{betweenness}(a) = \frac{2}{(n-1)(n-2)} \sum_{b \neq c:\ a \notin \{b,c\}} \frac{P_a(b,c)}{P(b,c)}. \quad (2)$$

Similar to the closeness centrality, we always compute the betweenness centrality on the undirected version of the directed graphs.

The idea behind the eigenvector centrality [39] is that the centrality of a given node is proportional to the sum of the centrality values of its neighbouring nodes. Mathematically, the vector of the centrality values of the nodes is defined as the eigenvector associated with the largest eigenvalue of the adjacency matrix of the graph. The eigenvector centrality can be computed both for directed and undirected graphs.

Another classical centrality measure is defined by the PageRank [64] algorithm. This method is a variant of eigenvector centrality that takes into account link direction and was originally used to rank web pages resulting from a web search. The algorithm takes into consideration that the relevance of a web page depends on the number of other web pages that link to it, as well as the relevance of those linked pages. We describe the algorithm in brief based on [65]. Let $G = (V = \{1, 2, \ldots, n\}, E)$ denote a directed graph. The PageRank value of the node a is defined as $\text{pg}(a) = \pi_a$, where $\pi = (\pi_i)_{i \in V}$ is the stationary distribution of a random walk on G. The random walk is defined in the following way: denote the out-degree of vertex i by $d_i^{(\text{out})}$ and $\alpha \in (0, 1)$ is a constant called dumping factor. If the random walker is at time t on the node i, then at time $t+1$, it jumps to any neighbour j of i with probability $(1-\alpha)/d_i^{(\text{out})}$ and with probability α to any uniform vertex in V. Thus, letting $(X_t)_{t \geq 0}$ denote the random walk, its transition probabilities are given as follows:

$$\mathbb{P}\left[X_{t+1} = j | X_t = i\right] = \frac{(1-\alpha)\mathbb{1}\{(i,j) \in E\}}{d_i^{(\text{out})}} + \frac{\alpha}{n}, \quad (3)$$

where $\mathbb{1}$ {condition} is the indicator function, in which value is 1 if the condition is true and zero otherwise. Since $(X_t)_{t \geq 0}$ is an irreducible Markov chain, the stationary distribution π exists. In our analysis, we fixed the α parameter to be 0.35, as suggested in [65].

We will often use the abbreviations of the centrality measures: betweenness: btw; closeness: cl; coreness: core; in-degree: iDg; eigenvector centrality: eign; PageRank: pg; and two-hop neighbourhood: 2Nbh.

2.2. Comparison of Ranked Lists. It is not easy to directly compare centrality measures, but we can apply a data-driven indirect way. If we compute a set of centrality measures on a given dataset, we can calculate the induced ranking lists to numerically describe the similarities and the differences between the centrality measures. In order to do this, we need a suitable similarity measure or similarity distance which compares ranked lists. We use rank distance measures not only to compare centrality measures but we use them also to quantify the temporal stability of top influencers.

In data science or information retrieval practice, the need to compare ranked lists is very common. The simplest and clearest situation is when we need to compare permutations of the same domain. In this case, each item has a fixed unique rank, there are no ties, and every permutation fully covers the domain. This ideal situation can be affected due to the numerous factors and perspectives that need to be taken into consideration [66]. One of them is the problem of ties, when more items may have the same rank. We say that two ranking lists are conjoint, if both lists consist of the same items. However, sometimes the lists what we need to compare are not conjoint. This is very common when we would like to compare top-k lists [67]; when longer, conjoint rankings are truncated to a fixed depth k.

The most common permutation metrics [66–68] are the Kendall tau metric, Spearman's footrule, and Spearman's rho. There are also correlation measures: Kendall's tau correlation [66–68] or Spearman's rho correlation [66–68]. These basic methods are often modified or extended to handle more complex cases. For example, Fagin et al. [67] extend the Kendall tau metric and Spearman's footrule to compare nonconjoint top-k rankings by inserting missing elements at rank $k+1$ and below. These extended top-k measures are denoted by $K^{(p)}$ and $F^{(l)}$, respectively.

These distance measures may or may not be a metric. A metric is defined as a bivariate function of a fixed domain, where for all x, y, and z in the domain, the symmetry $(d(x,y) = d(y,x))$, regularity $(d(x,y) = 0$ if and only if $x = y)$, and triangle inequality $(d(x,z) \leq d(x,y) + d(y,z))$ are satisfied. If only the symmetry and regularity conditions are satisfied, then d is a distance measure (for example, $F^{(l)}$ is a metric, but $K^{(p)}$ is not a metric [67]).

Given r ranked lists $W_1, \ldots, W_r$ (either full lists or top-k lists), the rank aggregation problem [69] with a distance measure d is to compute a list W such that $\sum_{j=1}^{r} d(W_i, W)$ is minimal. Dwork et al. [69] argued that Kendall tau and its variants are good measures for the aggregation, and Fagin et al. [67] have experimentally confirmed that. However, computing an optimal aggregation of several full or top-k lists is NP-hard for each of the measures from the Kendall family. Fagin et al. [67] also showed that the rank aggregation problem can be solved optimally in polynomial time for the $F^{(l)}$ metric (top-k extension of Spearman's footrule) and there is a polynomial time constant-factor approximation for the rank aggregation problem with respect to the Kendall measures.

2.2.1. Monotonicity of Rankings. A centrality measure induces a W ranking list of the nodes. However, a ranking may contain ties. To quantify the resolution of a ranking, we use the monotonicity [70] value given as follows:

$$M(W) = \left(1 - \frac{\sum_{r \in W} n_r (n_r - 1)}{n(n-1)}\right)^2, \quad (4)$$

where n is the number of the nodes and n_r is the number of ties for the same rank. This measure quantifies the fraction of ties in the ranking list. The M (W) monotonicity of the ranking list W is equal to one if the ranking list W is perfectly monotonic, and it is equal to zero if all nodes in W have the same rank. We use monotonicity because it allows quantifying the discrimination ability of a centrality measure.

2.2.2. Kendall's Tau Correlation. Kendall's tau correlation [68] coefficient measures the correspondence between two rankings. Consider two ranking lists that contain n elements: $X = (x_1, x_2, \ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$. Any pair of ranks (x_i, y_i) and (x_j, y_j) is said concordant if $x_i > x_j$ and $y_i > y_j$ or if $x_i < x_j$ and $y_i < y_j$. If $x_i > x_j$ and $y_i < y_j$ or if $x_i < x_j$ and $y_i > y_j$, then the pair is said to be discordant. In the case of tied pair, when $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant. Kendall's tau (τ) correlation coefficient is defined as follows:

$$\tau(X, Y) = \frac{n_c - n_d}{\sqrt{(n_0 - n_1)(n_0 - n_2)}}, \tau(X, Y) \in [-1, 1], \quad (5)$$

where $n_0 = n(n-1)/2$, $n_1 = \sum_i t_i (t_i - 1)/2$, $n_2 = \sum_j u_j (u_j - 1)/2$, n_c and n_d denote the number of concordant and discordant pairs, t_i is the number of tied values in the i-th group of ties for X, and u_j is the number of tied values in the j-th group of ties for Y. If $x_1 = x_2 = \ldots = x_n$ or $y_1 = y_2 = \ldots = y_n$, then τ (X, Y) is undefined. In this special case, we set τ (X, Y) to be 1 if $X = Y$; otherwise, we set it to zero (this rare situation occurs only in case of coreness centrality measure).

2.2.3. Jaccard Similarity of Top Influencers. The Jaccard similarity between two sets A and B is defined as follows:

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|}. \quad (6)$$

The Jaccard distance of sets A and B is derived from the Jaccard similarity: $d_J(A, B) = 1 - J(A, B)$. We define $J(\varnothing, \varnothing)$ as 1, so that $d_J(\varnothing, \varnothing) = 0$. It is well known that the Jaccard distance is a metric function [71, 72]. We use Jaccard similarity in several parts of the analyses: comparing centrality measures in terms of top influencers, tracking the wave-to-wave structural change in networks (Section 2.4), and describing the instability of top influencers over time when we care only the degree of overlap between the rankings (Section 2.5).

In most empirical cases, we are not interested in the full ranking of nodes but only in the ranking of top influencers. This is often the case in political competition, competition for assets, or competition for other scarce resources, such as social status or popularity in classrooms. Taking this into consideration, we focus on top influencers identified by the centrality measures.

In the beginning, we identify the set of top influencers using the following method: suppose we have a fixed N positive integer number which we define as the required minimum size of the set of the top influencers and initialize the T set of top influencers as empty, then we take the items with the highest score and add them to T. If $|T| \geq N$, then the identification was successful. Otherwise, take the items with the second-highest score and add them to T. We continue this procedure until $|T| \geq N$ or every item is added to T. If there are no ties, this method just returns the first N items in the sorted list. We use the Jaccard similarity to compare the set of top influencers induced by two centrality measures C_1 and C_2. Denote T_1 and T_2 the set of top influencers computed on a fixed network using the above described method induced by the centrality measures C_1 and C_2. We can quantify the similarity between the induced top influencers by $J(T_1, T_2)$. $J(T_1, T_2)$ is between 0 and 1, where 0 means that T_1 and T_2 do not have common elements, while 1 asserts that the two sets are the same. In this way, we can compare the set of opinion leaders of a network induced by different centrality measures. We set N to be 5. If there are ties, the size of T can be greater than N. For example, in case of in-degree centrality, if the highest in-degree value is 4, the second highest in-degree is 3, and there are 3 nodes with 4 in-degree and 4 nodes with 3 in-degree, and then the set of the top influencers contains 7 items.

2.2.4. Spearman's Footrule. A permutation σ is a bijection from a set $D_\sigma = \mathrm{D}(\sigma)$ (which we call the domain of the permutation) onto the set $[n] = \{1, \ldots, n\}$, where n is the size of D_σ. We interpret $\sigma(a)$ as the position or rank of element a. Spearman's footrule [68] metric is the L_1 distance between two permutations. If σ_1 and σ_2 are permutations of the same D domain, then

$$F(\sigma_1, \sigma_2) = \sum_{a \in D} |\sigma_1(a) - \sigma_2(a)|. \quad (7)$$

Fagin et al. [67] extended the notation of Spearman's footrule for nonconjoint top-k lists. A top-k list W is a bijection from a domain $D_W = \mathrm{D}(W)$ (the members of the top-k list) to $[k]$. Note that this definition does not allow ties. While calculating Spearman's footrule of two non-conjoint top-k lists W_1 and W_2, we do not assume that their domains are the same. Let l be a real number greater than k. The footrule distance with location parameter l, denoted by $F^{(l)}$, is obtained by placing all missing elements in each of the lists at position l and computing the usual footrule distance between them. More formally, given top-k lists W_1 and W_2, define functions W_1' and W_2' with domain $D_{W_1} \cup D_{W_2}$ by letting $W_1'(a) = W_1(a)$ for $a \in D_{W_1}$ and $W_1'(a) = l$ otherwise and similarly defining W_2'. The distance $F^{(l)}$ is then defined as follows:

$$F^{(l)}(W_1, W_2) = \sum_{a \in \left(D_{W_1} \cup D_{W_2}\right)} |W_1'(a) - W_2'(a)|. \quad (8)$$

Fagin et al. [67] showed that $F^{(l)}$ is a metric for every choice of location parameter l. We set l to be $k+1$. It is easy to see that $F^{(k+1)}$ reaches its maximum when D_{W_1} and D_{W_2} are disjoint, and in this case, $F^{(k+1)}(W_1, W_2) = k(k+1)$. Using this, we can define the normalized version of $F^{(k+1)}$:

$$f^*(W_1, W_2) = \frac{F^{(k+1)}(W_1, W_2)}{k(k+1)}. \quad (9)$$

The f^* normalized footrule distance with location parameter $k+1$ maps to the interval [0, 1], and it is also a metric.

2.3. Aggregation of the Centrality Measures. The identification of top influencers can be improved by aggregation over the centrality measures. We suppose that an aggregation of centrality measures could be a straightforward way to reflect upon diverse advantages that single centrality indexes offer for network analysis and would give a more accurate estimation about who the opinion leaders are in a network. There are many possible ways to do the aggregation. Since we do not know the real opinion leaders or top influencers, we can do the aggregation only with an unsupervised method. Furthermore, we aggregate centrality measures of different network types (advice-seeking and friendship), so the structure of the networks cannot be used for aggregation. However, we may aggregate the implied rankings rather than the networks; therefore, we can consider the problem of centrality measure aggregation as a rank aggregation problem. One possible way to aggregate rankings is discussed in Section 2.2, where the aggregated ranked list is constructed as a W ranking such that the sum of distances of W from the constituent ranked lists is minimal. According to Fagin et al. [67], this problem can be solved, or a solution can be approximated in a reasonable time.

A second possible way for rank aggregation is the Borda count [58–60] method. The Borda count method consists of a set of ordered lists $\mathfrak{L} = \{l_1, \ldots, l_m\}$, where each list has M items in different orders. In the current context, each l_i in $\mathfrak{L}$ is an induced ranking list of the C_i centrality measure. Let us denote $l\,(z)$ the position of item z in the list l. We define a new $B\,(C)$ centrality measure, the Borda count aggregation of the centralities $C = \{C_1, \ldots, C_m\}$, where the score of item z is given as follows:

$$\phi_{B(C)}(z) = \sum_{l \in \mathfrak{L}} (N - l(z)). \tag{10}$$

If we consider the centrality measures as experts, then we get a simple intuitive interpretation, where each expert creates its own ranking based on its own view, and then the experts are voting to the items in each rank position. This method was further improved by Madotto and Liu [58]. They propose a heuristic pruning method based on correlation and entropy, where they select a $\mathfrak{L}^*$ subset of $\mathfrak{L}$ and apply the Borda count method to $\mathfrak{L}^*$. However, it is hard to intuitively interpret their method; therefore, we decided to use the original Borda count aggregation of centrality measures $C = \{C_1, \ldots, C_m\}$ given in equation (10). In Section 4, we show that according to the empirical Kendall's tau correlation values of $B\,(C)$ with the centrality measures in C, we can consider the Borda count aggregation as a suitable method.

2.4. Jaccard Similarity-Based Network Comparison. Our approach to identify opinion leaders is based on centrality measures, which only depend on the structure of the observed networks. Consequently, the source of the variability of all the derived quantities is the structural change in the networks. We use a Jaccard similarity-based method in order to quantify this structural network change over the waves, measuring the similarity between two directed graphs G_1 and G_2.

Let us denote $U = V\,(G_1) \cup V\,(G_2)$; therefore, U contains every vertex of G_1 and G_2. For a given a node of the G graph, we define the $N_{\text{in}}(a, G)$ set as the set of neighbours of a in G such that from every node in $N_{\text{in}}(a, G)$, there is a link to a in G. We use $N_{\text{in}}(a, G)$ because, in our analysis, we mostly use the in-degree version of the centrality measures. Now, we define the Jaccard network similarity measure simply as the average Jaccard similarity of the $N_{\text{in}}(a, G_1)$ and $N_{\text{in}}(a, G_2)$ sets over all the nodes:

$$J(G_1, G_2) = \frac{1}{|U|} \sum_{a \in U} J(N_{\text{in}}(a, G_1), N_{\text{in}}(a, G_2)). \tag{11}$$

2.5. Stability Measure of Key Actors: Sequence Instability. As we focus on the stability of key actors in time, in this section, we describe the method that we use to measure the change of observed and computed quantities over the waves. Since we are interested in the temporal stability of the opinion leaders of the empirical networks over time, we identified the top-k opinion leader ranks for each wave. This method reflects on the variability of the set of opinion leaders and also shows the rearrangement of top-k actors from wave to wave. The following example would clarify our methodological approach. If k is 5, then we consider the students in the first 5 ranking positions for each wave. Suppose that the order of items in the top-5 positions in the first wave is $W_1 = (\{A\}, \{B, C\}, \{D\}, \{E\}, \{F\})$.

We can see that W_1 is an ordered list of sets, because we allow ties in the list. This means, that in the first wave, student A is in the first place, students B and C are tied for second, D is third, E is forth, and F is fifth. We will use the notation $W_1(A)$ for the rank of A, so that $W_1(A) = 1$, $W_1(B) = W_1(C) = 2$, $W_1(D) = 3$, $W_1(E) = 4$, and $W_1(F) = 5$. Suppose now that in the second wave, there is rearrangement in the students of the top-5 list, and furthermore student D falls out from the top-5, but student G is included, and the order of the students in the top-5 list in the second wave becomes $W_2 = (\{C\}, \{A\}, \{B\}, \{F\}, \{E, G\})$. In the remaining 3 waves, the order of the students is as follows: $W_3 = (\{C, B\}, \{F\}, \{A\}, \{E\}, \{G\})$, $W_4 = (\{A, B\}, \{E\}, \{C\}, \{D\}, \{F\})$, and $W_5 = (\{E\}, \{A\}, \{C, F\}, \{G\}, \{B\})$. From this example, we can observe that between two waves, the set of the top-k students are not necessary the same, so that the rankings are nonconjoint. We can also observe that ties may occur in the rankings. In each wave, we rank students against a centrality measure and we restrict our attention only to the top-k ranks.

Our goal is to quantitatively describe the variability of the $W_1, W_2, \ldots, W_N$ sequence with a single number. We present the method in two steps. In the first step, we look at the case, when for each wave there is only a single top opinion leader (set $k = 1$), and we suppose that there are no ties. Then, in the second step, we extend the method for the top-k case.

2.5.1. Sequence Instability for Unique Opinion Leaders. When there is a single opinion leader ($k = 1$) for each wave and there are no ties, then the sequence could be ABABC, where A was opinion leader in the first wave, B in the second, A in the third again, B in the fourth, and C in the fifth wave. Suppose we have identified the opinion leaders of each wave. We also suppose that the opinion leader is unique in all waves. Depending on combinations, sequences could be like $AAAAA$ and $AABBB$, and so on. We assign a $v(S)$ number for each S sequence, which describes the instability of the sequence. One possible way to measure the instability of a sequence is Shannon entropy [73]. The drawback of using Shannon entropy is that it does not distinguish the sequences like AAABBB and ABABAB; hence, it does not satisfy requirement R3, formulated in (16).

Instead of using Shannon entropy, we postulate some requirements for the instability number $v(S)$ and we provide a formula for $v(s)$ which satisfies these requirements, using the following notations. Suppose we have a finite not empty set $\mathscr{A}$. An L-length sequence S over $\mathscr{A}$ is an item of the L-factor Descartes-product $\mathscr{A}^L = \prod_L \mathscr{A}$. If $S \in \mathscr{A}^L$ is an L-length sequence, then we denote the length of S by $|S| = L$, the i-th item of S by $S(i)$, the set of items in S by $\Lambda(S)$, and the number of different items in S by $\lambda(S) = |\Lambda(S)|$. We denote the number of occurrences when an item of the sequence is different from its immediately preceding item by $\mathrm{B}(S) = \sum_{i=2}^{|S|} \mathbb{1}\{S(i) \neq S(i-1)\}$, and its normalized version is $b(S)$ given by $B(S)/(|S|-1)$. Consider the following example: let the set $\mathscr{A} = \{A, B, C, D, E, F\}$, $L = 5$, and an 5-length sequence from $\mathscr{A}^5$ given as $Q = AABCB$. Using the introduced notations, $|Q| = 5$, $\Lambda(Q) = \{A, B, C\}$, $\lambda(Q) = 3$, and $b(Q) = 3/4$.

We formulate four requirements, supposing that S, S_1, and S_2 are sequences over a fixed $\mathscr{A}$.

(i) R1: If the sequence S consists of a single element (the same student is the opinion leader in every wave), then the instability should be 0.

$$v(S) = 0 \text{ if and only if } \lambda(S) = 1. \tag{12}$$

(ii) R2: If the length of the sequences S_1 and S_2 is the same and the S_1 sequence contains more items than the S_2 sequence, then the instability of S_1 should be more than the instability of S_2. For example,

$$\begin{aligned} v(\mathrm{AAAAAA}) < v(\mathrm{AAABAA}) < v(\mathrm{AABBAC}) \\ < \ldots < v(\mathrm{ABCDEF}). \end{aligned} \tag{13}$$

Formally,

$$\begin{aligned} &\text{If } |S_1| = |S_2| \text{ and} \\ &\lambda(S_1) > \lambda(S_2) \text{ then } v(S_1) > v(S_2). \end{aligned} \tag{14}$$

(iii) R3: If the length of the sequences S_1 and S_2 is the same and S_1 and S_2 sequences contain the same number of items, then the greater instability belongs to the more alterable one:

$$v(\mathrm{AAABBB}) < v(\mathrm{ABBBAA}) < v(\mathrm{ABABAB}). \tag{15}$$

Formally,

$$\begin{aligned} &\text{If } |S_1| = |S_2| \text{ and} \\ &\lambda(S_1) = \lambda(S_2) \text{ and} \\ &B(S_1) > B(S_2) \text{ then } v(S_1) > v(S_2). \end{aligned} \tag{16}$$

(iv) R4: $v(S)$ should be between zero and one ($0 \leq v(S) \leq 1$) and $v(S) = 1$ if and only if the elements in S are not repeated:

$$v(S) = 1 \text{ if and only if } \lambda(S) = |S|. \tag{17}$$

Based on these requirements, we propose the following formula to calculate the sequence instability of an L-length sequence S (where $L > 1$):

$$v(S) = \frac{\lambda(S) + \sigma b(S) - 1}{(|S| - 1)(1 + \sigma)}, \tag{18}$$

where σ is a non-negative real constant, satisfying the condition:

$$\sigma < \frac{1}{|S| - 1}. \tag{19}$$

The justification of formula (18) and condition (19) can be found in Appendix A. In our work, we set σ to $\sigma = (|S| - 1)^{-1} - \epsilon$ with $\epsilon = 0.0001$.

Ranges of $v(S)$ instability measure for different values of $\lambda(S)$ calculated on our dataset are presented in Table 1. In our empirical data set (see Section 3), there are 4 or 5 waves; therefore, we compute the ranges (minimum and maximum values) of $v(S)$ for $\lambda(S) \in \{1, 2, 3, 4, 5\}$.

2.5.2. Sequence Instability of Top-k Influencers. In the case of top-k influencers, we have a sequence $S^* = (W_1, W_2, \ldots, W_L)$, where each W_i item of the S^* sequence is a top-k list with a possibility of ties. We have several options to trace back this case to the method developed for unique opinion leaders. One possible way is applying a suitable clustering method to the rankings $W_1, W_2, \ldots, W_L$ and using formula (18) to the sequence $C(W_1), C(W_2), \ldots, C(W_L)$, where $C(W)$ denotes the cluster to which W belongs. However, difficulty arises when the length of the sequence S^* is small as in our case (4 or 5 waves). Consequently, we implement a heuristic method by reinterpreting formula (18).

We assume that Ω is a finite nonempty set, which represents the set of students appearing in the top-k lists, for example, $\Omega = \{A, B, C, D, \mathrm{E}, F, G\}$. We can define a W top-k list with possible ties of items from Ω as a $W: D_W \subseteq \Omega \longrightarrow [k]$ surjective function, where k is a non-negative integer. A $f: X \longrightarrow Y$ function is surjective if for every $y \in Y$, there exists at least one $x \in X$ with $f(x) = y$. We will use the notations $\mathrm{D}(W) = D_W$ for the domain of W. In the example above, $\mathrm{D}(W_1) = \{A, B, C, D, E, F\}$. Since we enable ties, we changed the bijective feature of the definition of top-k lists given in Section 2.2 to surjective, and we explicitly included the Ω

Table 1: Sequence instability ranges for unique opinion leaders depending on the number of different items in the sequence, when the sequence length is 4 or 5.

Number of students	Sequence length is 4	Sequence length is 5
1	[0, 0]	[0, 0]
2	[0.33, 0.5)	[0.25, 0.4)
3	[0.66, 0.75)	[0.5, 0.6)
4	[1, 1]	[0.75, 0.8)
5	—	[1, 1]

universe of the rankings to the definition. However, even a not truncated ranked list does not necessarily contain all the elements of Ω. We will denote the set of all top-k lists with possible ties over the finite nonempty Ω set with length $k > 0$ by $R_{\Omega,k}$, where $R_{\Omega,k} = \{W\colon D_W \subseteq \Omega \longrightarrow [k]\colon W$ is a surjective function$\}$.

We need a distance measure $d(W_1, W_2)$ to calculate the similarity between the items of $R_{\Omega,k}$. This distance measure is a map $d\colon R_{\Omega,k} \times \mathrm{R}_{\Omega,k} \longrightarrow [0,1]$. For any $W_1, W_2, W_3 \in R_{\Omega,k}$, the relation $d(W_1, W_2) < d(W_1, W_3)$ indicates that W_1 is closer to W_2 than W_3, or in other words, W_1 is more similar to W_2 than to W_3. At this point, we suppose that such a d distance measure is given.

We compute the instability of a S^* L-length sequence of items from $R_{\Omega,k}$ using a similar formula as equation (18), but we define the functions λ (S) and b (S) differently:

$$\nu^*(S^*) = \frac{\lambda^*(S^*) + \sigma b^*(S^*) - 1}{\left(|S^*| - 1\right)(1+\sigma)}. \tag{20}$$

Function b (S) quantifies the amount of step-by-step changes in the sequence $S \in \mathscr{A}^L$. We will do the same with $b^*(S^*)$. If we define the distance measure between items of $\mathscr{A}$ as $d_{\mathbb{1}}(x, y) = \mathbb{1}\{x \neq y\}$, then we can reformulate b (S) using $d_{\mathbb{1}}$ as $b(S) = \sum_{i=2}^{|S|} d_{\mathbb{1}}(S(i), S(i-1))$. We define $b^*(S^*)$ by replacing $d_{\mathbb{1}}$ to d in this formula:

$$b^*(S^*) = \sum_{i=2}^{|S^*|} d(S^*(i), S^*(i-1)). \tag{21}$$

Function $\lambda(S)$ means the number of different items in S. However, for the case when the sequence consists of top-k lists, the meaning of "number of different items in S" is not straightforward. Consider first again the unique opinion leader case. Suppose we have an L-length sequence $S \in \mathscr{A}^L$, and for this sequence, define the $\mathbf{M}$ similarity matrix with size $L \times L$ of the items of S as $M_{ij} = 1 - d_{\mathbb{1}}(S(i), S(j))$ for all $i, j \in [L]$. Then, $\mathbf{M}$ contains only zeros and ones, $M_{ij} = 1$ if and only if $S(i) = S(j)$, and the diagonal elements are all equal to one by definition. If $\lambda(S)$ is not known, we can calculate λ (S) from the $\mathbf{M}$ similarity matrix. We start with the first line of $\mathbf{M}$, which contains the similarity values of all $S(i)$ items (including $S(1)$) to the first item in the sequence, $S(1)$. If we sum all items in the first line, we get the number of occurrences of S (1) in S. Let us denote the sum of items in the i-th row of $\mathbf{M}$ by $s_i = \sum_j M_{\mathrm{ij}}$ so that s_i is the number of occurrences of item $S(i)$ in S. Because of the transitivity of equivalence, for all later j positions in the sequence such that $S(j) = S(i)$, $s_j = s_i$ holds. We use s_i to calculate $\lambda(S)$ by counting the cases when $s_i > 0$. In order to avoid double count, we need to adjust the rows of $\mathbf{M}$ corresponding to these items. Hence, for all $j > i$ such that $M_{ij} = 1$, we subtract 1 from all items of row j and take its positive part, so that for all $l \in [L]$, set M_{jl} to be $(M_{jl} - M_{ij})^+$, where $(x)^+$ is x if $x > 0$ and 0 otherwise. We start this method at the first row, then move to the second one, and repeat the process for all rows of the matrix. Since $s_i > 0$ indicates that $S(i)$ is a new item in the sequence, we get $\lambda(S)$ as $\lambda(S) = \sum_{i=1}^{|S|} \mathbb{1}(s_i > 0)$. We formally present this method in Algorithm 1.

We can use this method for the case when S^* is a L-length sequence of top-k lists. To calculate $\lambda^*(S^*)$, we use Algorithm 1, modifying the calculation of the similarity matrix. In this case, the similarity matrix is computed using the d rank distance measure rather than $d_{\mathbb{1}}$, so that the $\mathbf{M}$ similarity matrix is given as $M_{ij} = 1 - d(S^*(i), S^*(j))$. Because of the regularity of d, the diagonal elements of $\mathbf{M}$ are one; therefore, $s_i \geq 1$ for all i row indices. Before executing Algorithm 1, the value of s_i shows how similar $S^*(i)$ to the other items. If s_i is close to L, then $S^*(i)$ is very similar to the other items of S^*. On the other hand, if s_i is close to 1, then the degree of similarity to the other items is low. After executing Algorithm 1, s_1 quantifies the similarity of the initial element $S^*(1)$ to the other items of S^*, whereas for subsequent elements with indices $i > 1$, s_i characterizes how different the element $S^*(i)$ is compared to the preceding elements in S^*. If s_i $(i > 1)$ is close to zero, then $S^*(i)$ is close to one or more items of $P(i) = \{S^*(1), \ldots, S^*(i-1)\}$. If $\min(s_i, 1)$ is close to one, then the distance of $S^*(i)$ is high from all the items of $P(i)$. Hence, the sum $\min(s_1, 1) + \ldots + \min(s_L, 1)$ shows the degree to which the elements are different.

For the unique opinion leader case, the transitivity property guarantees the correctness of Algorithm 1. For any $a, b, c \in \mathscr{A}$, if $d_{\mathbb{1}}(a, b) = 0$ and $d_{\mathbb{1}}(b, c) = 0$, then $d_{\mathbb{1}}(a, c) = 0$. It is easy to see that $d_{\mathbb{1}}$ is a metric; in particular, it satisfies the triangle inequality. If d is a rank distance measure of the items of $R_{\Omega,k}$, then the regularity property guarantees its transitivity: for any $W_1, W_2, W_3 \in R_{\Omega,k}$, if $d(W_1, W_2) = 0$ and $d(W_2, W_3) = 0$, then $d(W_1, W_3) = 0$. If we require d to be a metric, the triangle inequality is also satisfied. However, in the context of heuristic methods, we refrain from asserting correctness, but these properties of d support the idea behind the heuristic.

Up to this point, we assumed that d is an arbitrary rank distance metric of $R_{\Omega,k}$. In our study, we implemented the sequence instability measure using two different distance metrics. The first is the d_J Jaccard distance (Section 2.2), when we do not care the order rearrangement in the top-k lists, just the extent of the overlap. The second is f^*, the normalized Spearman's footrule with location parameter $l = k + 1$, when we would like to take into consideration the change in order. However, f^* cannot handle ties; therefore, we combine f^* with Hausdorff distance. We denote this new rank distance by d_{SFH} and discuss the details of d_{SFH} in the next section.

2.5.3. The Combination of Spearman's Footrule f^* with Hausdorff Distance. The rank similarity measure f^* (defined in Section 2.2), the normalized footrule distance with

```
λ = 0
for i = 1 to L do
  s_i = 0
  for j = i to L do
    s_i = s_i + M_ij
    if (j > i) and (M_ij > 0) then
      for l = j to L do
        M_jl = (M_jl − M_ij)^+
      end for
    end if
  end for
  λ = λ + min(s_i, 1)
end for
return λ
```

Algorithm 1: λ^* (**M**).

location parameter $k+1$, is a suitable metric to compare nonconjoint top-k lists, but it cannot handle ties. However, we need a similarity measure what is capable to compare top-k lists with ties. In our approach, we extend the f^* metric using the Hausdorff distance [67, 74]. Let Γ be a nonempty set, d be a metric of distances between objects of Γ, and A, $B \subseteq \Gamma$ are finite sets. The Hausdorff distance between sets A and B is given as follows:

$$d_{\text{Haus}}(A, B) = \max\left\{\max_{f\in A}\min_{g\in B} d(f,g), \max_{f\in B}\min_{g\in A} d(f,g)\right\}. \quad (22)$$

The Hausdorff distance has an intuitive interpretation. The quantity $\min_{g\in B} d(f,g)$ is the distance between g and the set B. Therefore, the quantity $\max_{f\in A}\min_{g\in B} d(f,g)$ is the maximal distance of a member of A from the set B. Similarly, the quantity $\max_{f\in B}\min_{g\in A} d(f,g)$ is the maximal distance of a member of B from the set A. Therefore, $d_{Haus}(A,B)$ is the maximal distance of a member of A or B from the other set. Thus, A and B are within Hausdorff distance s of each other if every member of A and B is within distance s of some member of the other set. The Hausdorff distance is a metric.

When using the Hausdorff distance to compare ranked lists with ties, we need to express the W ranked list with ties as a set. We do this by collecting all ranked lists without ties determined by W. For example, if $W = (\{A, B\}, \{C\}, \{D\}, \{E, F\})$, then we represent W with the set $t(W) = \{ACDE, BCDE, ACDF, BCDF\}$. To define it formally, we introduce the notation $r(R, i)$, the set of items with rank i: $r(R, i) = \{a: R(a) = i\}$. For example, $r(W, 1) = \{A, B\}$, $r(W, 2) = \{C\}$, $r(W, 3) = \{D\}$, and $r(W, 4) = \{E, F\}$, so that $W = (r(W, 1), r(W, 2), r(W, 3), r(W, 4))$. Using this, we define the set representation of $W \in R_{\Omega,k}$ as follows:

$$t(W) = r(W,1) \times r(W,2) \times \ldots \times r(W,k), \quad (23)$$

where operator $\times$ denotes the Descartes-product. We define the distance of $W_1, W_2 \in R_{\Omega,k}$ ranked lists with potential ties as follows:

$$H(W_1, W_2; d) = d_{\text{Haus}}(t(W_1), t(W_2); d), \quad (24)$$

where d is a suitable metric on top-k lists, and finally, we define d_{SFH}, the Hausdorff distance with Spearman's footrule, as $d_{\text{SFH}}(W_1, W_2) = H(W_1, W_2;\ f^*)$. Since both f^* and d_{Haus} are metrics, therefore d_{SFH} is metric too.

We explain the instability calculation on a hypothetical example in Appendix B.

3. Sample and Data

We illustrate the method using data collected on the students' social networks between 2013 and 2017 in Hungarian primary school classes [75]. Primary school classes are very similar to networks with relatively stable membership, like workplace networks, where the members are working in the same place or office and the membership of working group does not change very much in time (for example, in public administration or educational sector, research projects, regular volunteering, and so on). Our dataset was constituted by a sample of 61 classes, which were selected for longitudinal investigation. The sampling procedure was stratified by the settlement location and type in the central part of Hungary. We investigated primary school grades 5$^{\text{th}}$, 6$^{\text{th}}$, and 7$^{\text{th}}$ in five data collection waves. Data has been collected in the autumn and spring in the 5th and 6th grades. The fifth data collection wave was at 7$^{\text{th}}$ grade in spring. We must note that in the Hungarian school system, some secondary schools start in grade 7, while most secondary schools start in grade 9. This means that some students, in case of successful admission exam, can change their institution from primary school to secondary school.

In each wave, we asked the following network questions with the same wording. Referring to the advice-seeking network, the question was “Whose opinion do you listen to?” According to the answers, we constructed a directed network for each class in every data collection time (wave). The nodes of the network are the students of the class, and the (A, B) directed link is part of the network if and only if student A listens to the opinion of student B. In consequence, we have advice-seeking networks and friendship networks at most for 5 waves. From the entire sample of 61 classes, 9 classes had inconsistent data (not enough mentions for opinion leaders and friends); for another 5 classes, we do not have data for at least four data collection waves, and there were some cases (4 classes) where the advice-seeking or friendship network was missing for a given wave, so these classes were excluded from analysis. Our analysis is limited to a total number of 44 classes, of which 21 classes where we have advice-seeking and friendship networks for 4 waves and 23 classes for 5 waves. The number of students included in advice-seeking networks was 1 166, the average network size was 20 persons, the smallest network size was 11, the largest was 31, and the standard deviation was 4.66. Referring to friendship networks, the question was “Who is your friend?” The total number of classes that have friendship data is 52, the number of classes which has data in 5 waves is 37, and 11 which have data for 4 waves. Like in the case of advice-seeking networks, we used only classes with data for at least 4 waves. The number of students included in friendship networks was 1138, the average network size was 19 persons, the smallest network size was 10, the largest was 30, and the standard deviation was 4.61. In this paper, we use the abbreviations “as” for advice-seeking networks and “fr” for friendship networks.

4. Results

The goal of the empirical illustration is to see how stable the opinion leaders’ position is in advice-seeking and friendship networks in a network with relatively stable membership over time, like in the case of primary school classes.

4.1. Jaccard Similarity of Networks. For quantifying the wave-to-wave variability of the networks for a given T network type (advice-seeking or friendship), we computed the Jaccard similarity of the networks for given types between the consecutive waves. More precisely, for all S school classes and all $(y, y+1)$ pairs, if the given network type T exists in the waves y and $y+1$ (denoted by $G^T_{S,y}$ and $G^T_{S,y+1}$), we computed all the $J(G^T_{S,y}, G^T_{S,y+1})$ Jaccard similarity values discussed in Section 2.4. In the case of advice-seeking networks, we got a mean value of 0.3005 with a variance of 0.0139, and for the friendship networks, the mean was 0.3247 with a variance of 0.0197. The wave-to-wave statistics and the histogram of the Jaccard similarity values can be found in the Supplementary material (S1–S3). The analysis shows us that there is a huge wave-to-wave rearrangement in both network types, but in the case of friendship networks, the rearrangement is more moderate, except between the 4 and 5 waves, where there is a significant drop in the mean Jaccard similarity value. This drop can be explained by the peculiarities of the Hungarian education system. In Hungary, some secondary schools start in grade 7, while the majority of secondary schools start in grade 9. This means that some students, in case of successful admission exam, can change their institution from primary school to secondary school. Therefore, in our target schools, the highest composition change is between grade 6 and 7, that is, between waves 4 and 5. This is reflected not only in composition change but also in the considerable drop in the Jaccard similarity of the friendship network.

Based on the volume of the wave-to-wave network reorganization in both network types, we can predict that centrality might at least equally change; hence, the opinion leader position will not be stable and derived stability metrics will show low stability as well.

4.2. Centrality Measures Computed on the Advice-Seeking Networks. If we interpret the opinion leader role as the student who got the maximum number of nominations to the question “Whose opinion do you listen to?”, then we get the in-degree centrality measure. If we would like to take into account not only the number of incoming nominations but the wider environment of the nodes or the prestige of the nominators, then we can try two-hop neighbourhood, core number, eigenvector centrality, or PageRank. In the case of betweenness and closeness centralities, it is hard to find a meaningful interpretation for the advice-seeking networks; therefore, we do not compute them. Nominations (links) in advice-seeking networks are rarely symmetric; therefore, we use the directed version of these measures. In addition to these classical centrality measures, we also calculated the Borda count aggregation (Bca; see Section 2.3) of these measures (iDg, pg, 2Nbh, core, and eign) on the advice-seeking networks.

We computed the average monotonicity of each centrality measures for all school classes in every wave. The eigenvector and PageRank centralities have the best discrimination ability. Their average monotonicity values are 0.9841 and 0.9385, which means that ties are very rare for these measures. On the other extreme, the average monotonicity of the coreness is very low; it is 0.2616. The average monotonicity of the aggregated measure is also high: its value is 0.9515. S4 in the Supplementary materials contains the average monotonicity values and the variances for all the centrality measures computed on the advice-seeking networks.

We calculated the mean and the variance of the set of opinion leader (top-1 ranking) sizes for each centrality measure and collected the results in Table 2. If the size of a top-1 set is one, then the opinion leader can be uniquely identified. We also added the number of cases when the opinion leader is unique to Table 2. We can observe that the opinion leaders induced by eigenvector centrality are always unique, and opinion leaders induced by PageRank are unique in the 95% of the cases, while coreness has the highest

Table 2: Size statistics of opinion leaders (top-1 influencers) for the centrality measures computed on the advice-seeking networks (221 networks).

Centrality	Mean	Variance	No. of unique cases
In-degree	1.5782	0.7557	128
PageRank	1.0047	0.0047	210
Two-hop neighbourhood	1.9383	2.1621	114
Coreness	11.6825	29.193	0
Eigenvector	1.0	0	221
Borda count aggregation	1.085	0.0875	194

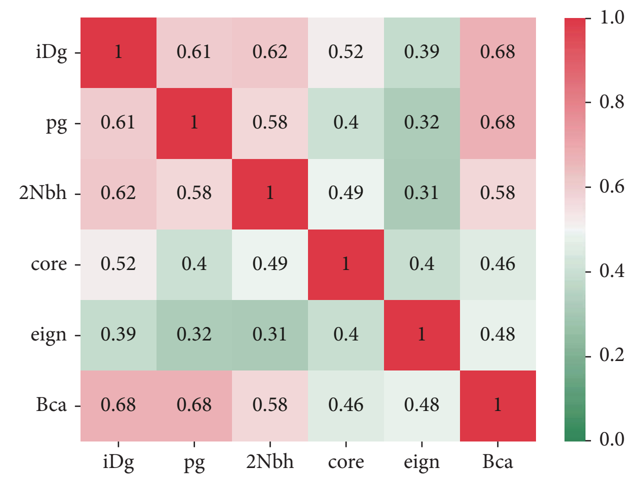

Figure 1: The average Jaccard similarity values of the top influencers computed on the advice-seeking networks, setting the N parameter values to be 5. The variance is between 0.0244 and 0.0601. iDg: in-degree, pg: PageRank, 2Nbh: two-hop neighbourhood, core: coreness, eign: eigenvector centrality, and Bca: Borda count aggregation.

average opinion leader group size with value 11.6825. This is not a surprise if we consider this group as the core of the network and the remaining students as the periphery.

We compared the rankings generated over advice-seeking networks by centrality measures with Kendall tau correlation. The mean Kendall tau correlation values (Supplementary material S5) describe to what extend the individual centralities agree on the order in average. The results show that the mean correlation of the Borda count aggregation measure with the other centralities is between 0.64 and 0.8; therefore, we really can consider this measure as the aggregation of the others. In order to test the consistency of our data, we also made a wave-to-wave correlation analysis. The results (Supplementary material S21) show that the mean of correlations is between 0.32 and 0.57.

Similarly to Kendall tau correlation, we compared the centrality measures calculated over advice-seeking networks in terms of Jaccard similarity of the top influencers (Section 2.2). The results are summarised in Figure 1. These values show the average degree to which the individual centralities agree in the set of top influencers. We can observe that the values are between 0.31 and 0.68. The minimum value belongs to the eign—2Nbh pair, while the maximum belongs to iDg-Bca. We can also observe that in every column of Figure 1, except of the column of core, the Borda count aggregation value is the highest. This indicates that in terms of top influencers, the aggregated centrality measure can predict the outcome of the other centralities with the highest accuracy (in average), which also leads to the conclusion that Borda count aggregation methodology is appropriate for aggregating the other centrality measures.

We counted the returning frequency of opinion leaders (how many times a student becomes opinion leader) in Tables 3(a) and 3(b), separately for the school classes with available data for 4 and 5 waves. We can see in the table that the majority of the opinion leaders only hold their position once or twice. For example, with 5 available waves, for the PageRank algorithm, there are 49 students who become opinion leaders only once, 18 who become opinion leaders twice, and only 9 students who become opinion leaders more than twice. Only the core shows some stability: with 5 available waves, there are 270 students who were part of the core at least 3 times and 165 at least 4 times.

The variability of the opinion leaders was also described by using the sequence instability metric (Section 2.5). Making the interpretation of the results easier, similarly to the returning frequency of the opinion leaders, we separated the classes into two groups: in the first group, we have observation data for 4 waves; in the second group, we have observation data for 5 waves. First, we calculated the sequence instability only for the opinion leaders with equation (20), setting $k = 1$ and the d rank distance measure to the d_J Jaccad-distance. Using the d_J metric, we could properly handle the ties appearing in the first rank position. Averaging the sequence instability over the classes, we could grasp the variability (of a fixed centrality measure) with a single number, presented in the Supplementary material (S6 and S7). Second, we calculated the sequence instability for the top-k influencers, where $k > 1$. For each centrality measure, we determined the value of k from the mean observed monotonicity value. If the mean monotonicity is high (more than 0.85), then we set k to 5. If the mean monotonicity is moderate (between 0.5 and 0.85), then we set k to 3, and for low mean monotonicity (below 0.5), we set k to 1. According to this, we set k to 5 for PageRank, eigenvector centrality, and Borda count aggregation measure, 3 for in-degree and two-hop neighbourhood, and 1 for core, so that we do not calculate sequence instability for core when $k > 1$. We calculated the top-k sequence instability for both d_J and d_{SFH}. The average top-k sequence instability values are given in Tables S8–S11 of Supplementary material. We have presented the distribution of the sequence instability values on box plots. Figure 2 shows the sequence instability values of the Borda count aggregation for the cases when the parameters are $\{k = 1,\ d = d_J\}$, $\{k = 5,\ d = d_J\}$, and $\{k = 5,\ d = d_{\mathrm{SFH}}\}$. The box plots for the other centrality measures (iDg, pg, 2Nbh, core, and eign) are in the Supplementary material (S12 and S13). In case of the sequence instability of the opinion leaders (when $k = 1$ and $d = d_J$), the median values are between 0.488 and 0.75. For example, the median sequence instability of the Borda count aggregation measure with available data in 5 waves is 0.674, the 25th percentile is 0.5, and the 75th percentile is 0.8. This covers the [0.5, 0.6)

Table 3: (a) Returning frequency of the opinion leaders by different centrality measures for advice-seeking (as) and friendship (fr). Observation data for 4 waves. (b) Returning frequency of the opinion leaders by different centrality measures for advice-seeking (as) and friendship (fr). Observation data for 5 waves.

Type	Centrality	1	2	3	4	5
(a)						
as	iDg	62	16	11	3	
as	pg	52	12	4	2	
as	2Nbh	88	25	11	2	
as	Core	135	105	104	118	
as	eign	46	20	2	1	
as	Bca	52	12	2	5	
fr	iDg	44	13	4	0	
fr	pg	26	9	0	0	
fr	2Nbh	54	16	2	0	
fr	Core	42	46	56	54	
fr	eign	17	6	5	0	
fr	cl	36	7	1	1	
fr	btw	27	5	1	1	
fr	Bca	20	6	2	2	
as & fr	Bca	46	11	7	0	
(b)						
as	iDg	73	25	12	5	3
as	pg	49	18	6	2	1
as	2Nbh	96	23	18	6	2
as	Core	96	93	105	86	79
as	eign	63	18	4	1	0
as	Bca	59	19	4	2	2
fr	iDg	163	37	18	5	0
fr	pg	123	22	7	0	0
fr	2Nbh	230	83	27	4	1
fr	Core	190	170	127	174	138
fr	eign	128	16	7	1	0
fr	cl	148	36	6	1	0
fr	btw	117	21	8	1	0
fr	Bca	122	26	7	1	0
as & fr	Bca	70	20	5	0	0

iDg: in-degree, pg: PageRank, 2Nbh: two-hop neighbourhood, core: coreness, eign: eigenvector centrality, cl: closeness, btw: betweenness, and Bca: Borda count aggregation.

interval which means that 3 students take turns as opinion leaders and the [0.75, 0.8) interval that belongs to 4 students (for the meaning of the intervals, see Table 1). The minimum and maximum values are 0.0 and 1.0; hence, the whole range of possible values is covered. When $k > 1$, we can observe that the median values do not change so much, but the range of the values has shrunk. For the Borda count aggregation measure with available data in 5 waves with $k = 5$ and $d = d_J$, the 25th percentile is 0.55, the 75th percentile is 0.66, and the minimal and the maximal values are 0.1667 and 0.8188. We can observe similar effect for the case when we use the d_{SFH} distance metric.

4.3. Centrality Measures Computed on Friendship Networks. Friendship networks differ from advice-seeking networks in the sense that friendships are supposed to be more stable relations over time [76]. Friendship is also depending on the length and content of the connection (how much time they spend together and the reason why), homophilia and sympathy between them, socioeconomic background, neighbourhood, common interests, activities, and so on. We applied the analysis of centrality measures for friendship networks in order to see the differences compared with advice-seeking networks.

Centrality in the case of friendship networks gives us information about the influence of the students in the class. In this case, all the centrality measures discussed in Section 2.1 make sense and give us a different description of the node's position. However, in the case of the closeness and betweenness centralities, we applied the measure to the undirected version of the friendship networks, because in a school class, the friendship network is not a "who-knows-who" question (since in such a small community everybody knows everybody) but is more about cliques within the class. That is why closeness and betweenness are related to the information flow between the cliques, which were supposed to be bidirectional. In contrast, for example, PageRank is much more about prestige in this environment; therefore, we still compute it on directed networks. We also applied the Borda count aggregation (Section 2.3) method to aggregate these centrality measures (iDg, pg, 2Nbh, core, eign, cl, and btw) in a similar way as applied to the advice-seeking networks.

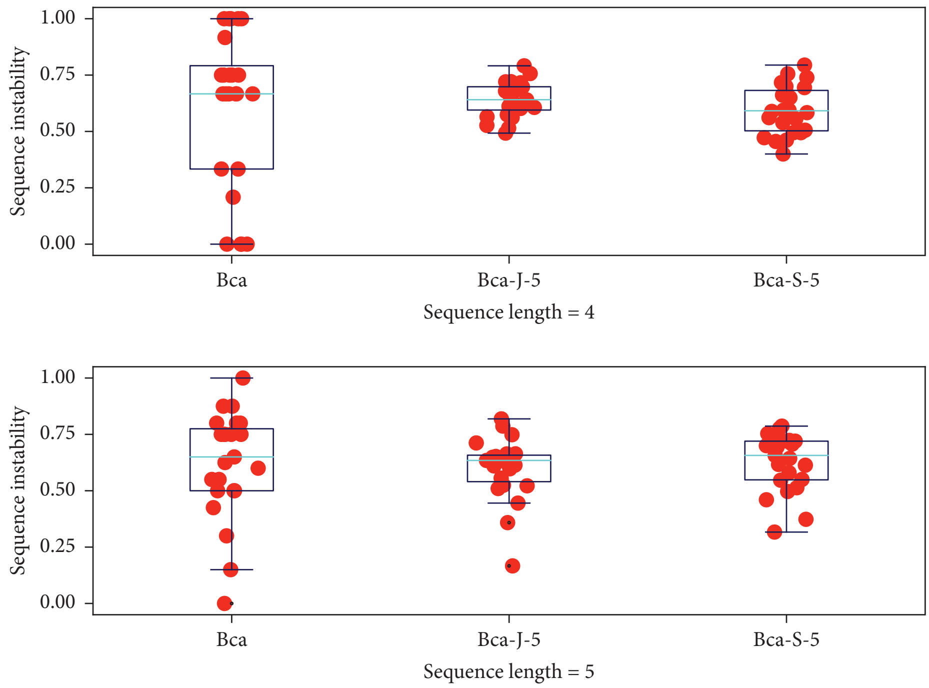

Figure 2: Box plot of sequence instability values computed on the advice-seeking networks for the Borda count aggregated measure (Bca). We separated the school classes into two groups: on the top plot, we have observation data for 4 waves, on the bottom for 5 waves. Bca: sequence instability of Borda count aggregated measure with parameters $\{k=1,\ d=d_J\}$, Bca-J-5: sequence instability of Borda count aggregated measure with parameters $\{k=5,\ d=d_J\}$, and Bca-S-5: sequence instability of Borda count aggregated measure with parameters $\{k=5,\ d=d_{\mathrm{SFH}}\}$.

The monotonicity of the centrality measures computed on the friendship networks behaves similar to the monotonicity computed on the advice-seeking networks. Eigenvector centrality has the highest monotonicity value of 0.9957, while coreness has the lowest value of 0.2433. The numerical details can be found in S14 of the Supplementary material.

We computed the mean and variance of the set of opinion leaders (top-1 rankings) sizes for each centrality measures on the friendship networks. The details can be found in Table 4. Similar to the advice-seeking networks, the opinion leaders are unique in almost all the cases for the eigenvector, betweenness, and PageRank centralities.

The mean group size of the students with the highest core number is 12.2926.

We compared the centrality measures using the Kendall tau correlation in a similar way as we did for the advice-seeking networks. The mean correlation values can be found in S15 of the Supplementary material. The mean correlation values are between 0.2 and 0.75, while the mean correlation of the Borda count aggregation measure with the other individual measures is between 0.49 and 0.71; therefore, we can consider that the Borda count aggregation measure covers well the other individual measures. We also made a wave-to-wave correlation analysis for the friendship networks. The results (S21) show that the means of correlations are between 0.21 and 0.52 in each wave, except between the 4 and 5 waves, where the values are much smaller. The results are similar with those discussed in Section 2.4 (Jaccard similarity).

According to the average Jaccard similarity values of the top influencers, presented in Figure 3, we can observe that the mean Jaccard similarity of the Borda count aggregated measure with the other centrality measures is relatively high.

The returning frequency of opinion leaders in Tables 3(a) and 3(b) shows that majority of the opinion leaders hold their position once or twice; only the core of the networks shows some stability.

Sequence instability measures for friendship networks were computed in the same way as in the case of advice-seeking networks. The average sequence instability values for the opinion leaders can be found in the Supplementary material (S6 and S7), and the average top-k sequence instability numbers are given in Tables S8–S11 of Supplementary material. We provide the box plot of the sequence instability values for the Borda count aggregation of the other individual centrality measures on Figure 4 where the parameters are $\{k=1, d=d_J\}$, $\{k=5, d=d_J\}$, and $\{k=5, d=d_{\mathrm{SFH}}\}$. The box plots of the other measures are in the Supplementary material (S16–S18). We can observe a similar behaviour as the case of advice-seeking networks. The median sequence instability of the Borda count aggregation measure with available data in 5 waves is 0.75, the 25th percentile is 0.6 and the 75th percentile is 0.9. This covers the [0.75, 0.8) interval that belongs to 4 students (for the meaning of the intervals, see Table 1). The minimum and maximum values are 0.25 and 1.0. When $k>1$, we can observe that the median values do not change so much, but the range of the values has shrunk. For the Borda count aggregation measure with available data in 5 waves with

Table 4: Size statistics of the opinion leaders (top-1 influencers) for the centrality measures computed on the friendship networks (229 networks).

Centrality	Mean	Variance	No. of unique cases
In-degree	1.7162	1.2862	142
PageRank	1.0131	0.0129	226
Two-hop neighborhood	2.5764	4.7944	104
Coreness	12.2926	19.0541	0
Eigenvector	1.0087	0.0087	227
Closeness	1.3057	0.5092	192
Betweenness	1.0087	0.0087	227
Borda count aggregation	1.066	0.0699	215

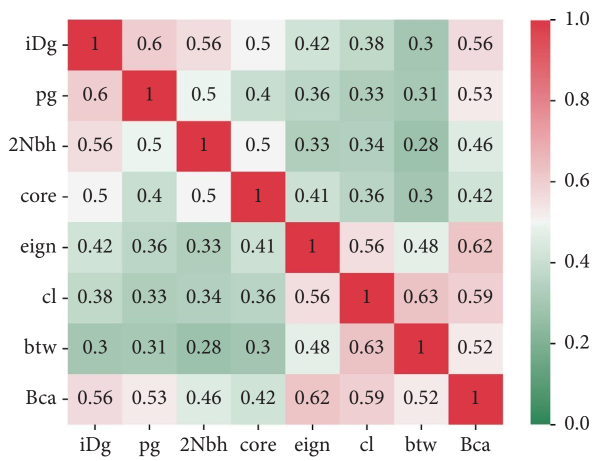

Figure 3: The empirical mean of the Jaccard similarity values of the top influencers computed on the friendship networks, setting the N parameter value to be 5. The variance is between 0.012 and 0.076.

$k = 5$ and $d = d_J$, the 25th percentile is 0.58, the 75th percentile is 0.71, and the minimal and the maximal values are 0.3964 and 0.8119. We can observe similar effect for the case when we use the d_{SFH} distance metric.

4.4. Aggregation of Centrality Measures Computed on Advice-Seeking and Friendship Networks. The Borda count aggregation method can be used not only for aggregation the centrality measures of the same network type but also for aggregation the centrality measures of multiplex networks. We illustrate this by examining the set of central actors with aggregating friendship and advice-seeking networks. Although the correlation between the two types of networks (see Kendall tau correlation in S19 where the range varies between [0.017, 0.097] is low, the centralities computed on these different network types agree in the top influencers surprisingly frequently (see Jaccard similarity mean values in Figure 5). This also shows that the cores of the two kinds of networks highly overlap each other. After the Borda count aggregation of all the centrality measures computed on the advice-seeking networks and the friendship networks, we got an aggregated centrality measure that can be considered as the aggregation of the centrality measures over the network types.

Let us examine how the aggregated measure behaves. First, we discuss the monotonicity of the aggregated measure. Its mean value appears to be 0.9755 with a variance of 0.003. This shows that the discrimination power of the aggregated centrality measure is high. Using the aggregated measure, the mean size of the top performers is 1.075, with a variance of 0.08. The opinion leader role is unique in 185 out of 199 cases.

Comparing the aggregated centrality measure to the other measures computed on both network types by the Kendall tau correlation, the mean correlation values are in S20 of the Supplementary material. The mean correlations are in the range [0.39, 0.65]. We can, therefore, interpret the aggregated measure as a new centrality. In order to compare the aggregated measure in terms of the top influencers, we computed the Jaccard similarity of the top influencers with parameter $N = 5$. Table 5 contains the results. In the case of the advice-seeking networks, the mean similarity is between 0.35 and 0.43, while in the case of the friendship networks, the values are in the range [0.39, 0.55].

Regarding the returning frequencies of the opinion leaders (in Tables 3(a) and 3(b)), for the classes with 4 available waves, only 7 students were opinion leader at least 3 times, and for the classes with 5 available waves, only 5 students were opinion leader at least 3 times.

The sequence instability values are in Figure 6. We can observe a similar behaviour to the advice-seeking and friendship networks.

4.5. Analysis of the Core. The core centrality measure alone is not suitable for finding the top influencers, but it provides very important information about a node's position and embeddedness. Staying with our example of primary school classes, one might question how exactly opinion leaders are getting into central network position. If someone is not necessarily opinion leader measured by centrality measures, but is "almost" there, being member of a popularity group, then maybe this person has bigger chances to become opinion leader. The analysis of the core is justified from a practical point of view, in order to be considered in the research regarded stability of the opinion leaders.

Tables 2 and 4 show that the size of the core is very large comparing with the average class size which is 20 for advice-seeking networks and 19 for friendship networks. The average core size is 11.68 and 12.29 for the advice-seeking and friendship networks, respectively. However, it seems that (relative to the other centrality measures) the core shows some level of stability (see Tables 3(a) and 3(b) and Figures S12 and S16). In this section, we investigate the core in more detail to have a better picture of the extent of the core, its stability, and the connection with the other centrality measures.

We can get a more detailed picture if we observe the core size for each wave. S22 of the Supplementary material contains the mean core and the relative mean core size values. We can see that, on average, the core is roughly 60% of the class size for the advice-seeking networks and 67% of the class size for the friendship networks in each wave.

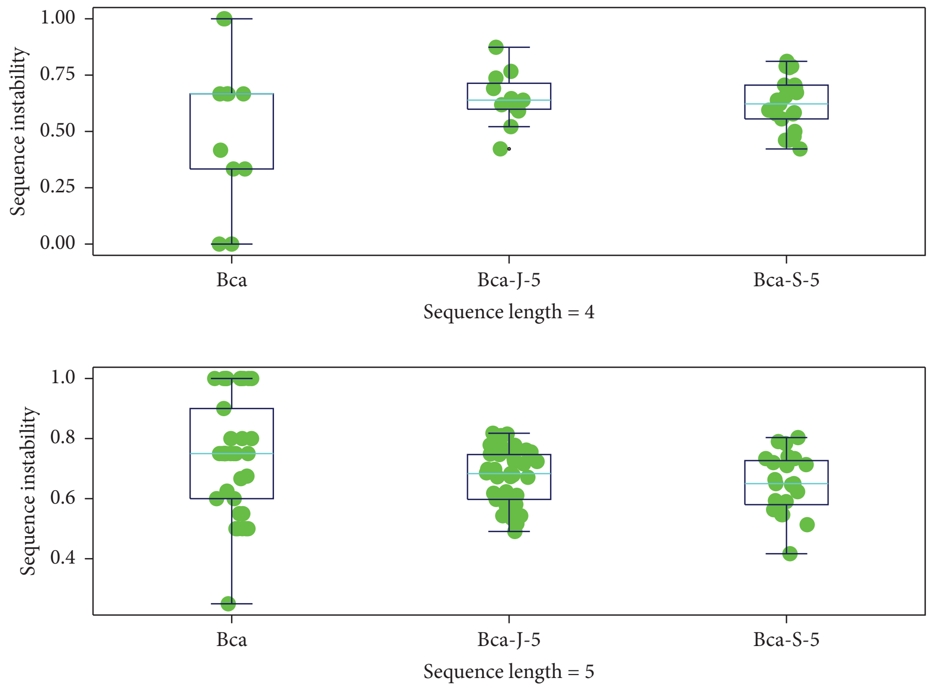

FIGURE 4: Box plot of sequence instability values computed on the friendship networks for the Borda count aggregated measure (Bca). We separated the school classes into two groups: on the top plot, we have observation data for 4 waves, on the bottom for 5 waves. Bca: sequence instability of Borda count aggregated measure with parameters $\{k = 1, d = d_J\}$, Bca-J-5: sequence instability of Borda count aggregated measure with parameters $\{k = 5, d = d_J\}$, and Bca-S-5: sequence instability of Borda count aggregated measure with parameters $\{k = 5, d = d_{\mathrm{SFH}}\}$.

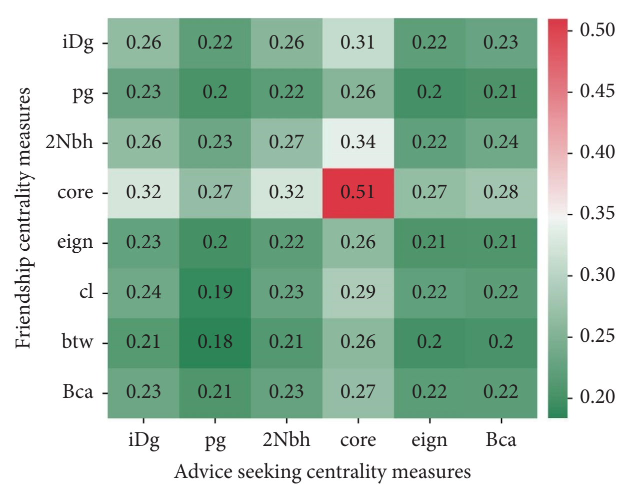

FIGURE 5: The empirical mean of the Jaccard similarity values of the top influencers computed between the advice-seeking and the friendship networks. The variance is between 0.0142 and 0.0462. iDg: in-degree, pg: PageRank, 2Nbh: two-hop neighbourhood, core: coreness, eign: eigenvector centrality, Bca: Borda count aggregation, cl: closeness, and btw: betweenness.

TABLE 5: The empirical mean of the Jaccard similarity values computed between the aggregated measure over both advice-seeking and friendship networks and the classical centrality measures.

Network type	iDg	pg	2Nbh	Core	eign	cl	btw	Bca
Advice-seeking	0.43	0.38	0.39	0.38	0.35	—	—	0.43
Friendship	0.47	0.43	0.4	0.39	0.48	0.46	0.42	0.55

The variance is between 0.0151 and 0.1081. iDg: in-degree, pg: PageRank, 2Nbh: two-hop neighbourhood, Core: coreness, eign: eigenvector centrality, cl: closeness, btw: betweenness, and Bca: Borda count aggregation.

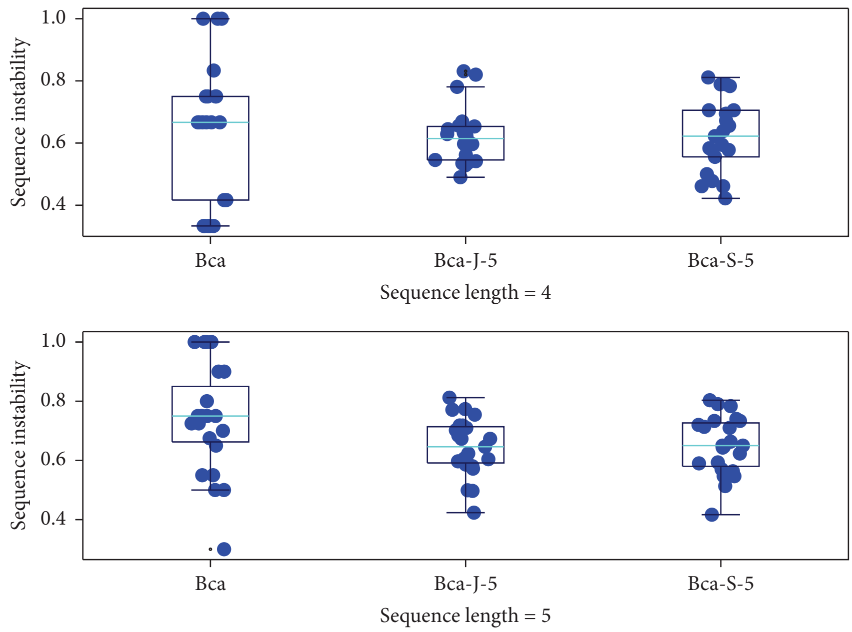

Figure 6: Box plot of sequence instability values computed on both the advice-seeking and friendship networks for the Borda count aggregated measure (Bca). We separated the school classes into two groups: on the top plot, we have observation data for 4 waves, on the bottom for 5 waves. Bca: sequence instability of Borda count aggregated measure with parameters $\{k = 1,\ d = d_J\}$, Bca-J-5: sequence instability of Borda count aggregated measure with parameters $\{k = 5,\ d = d_J\}$, and Bca-S-5: sequence instability of Borda count aggregated measure with parameters $\{k = 5,\ d = d_{\mathrm{SFH}}\}$.

Table 6: Example sequence with full ranks.

—	S (1)	S (2)	S (3)	S (4)	S (5)
1	A	J	F	E	G
2	B	I	G	D	A
3	C	H	H	C	E
4	D	G	I	B	B
5	E	F	J	A	C
6	F	E	A	J	I
7	G	D	B	I	D
8	H	C	C	H	F
9	I	B	D	G	H
10	J	A	E	F	J

Table 7: Example sequence instability values evaluated using the metrics d_J (first column) and d_{SFH} (second column) with $k = 1, \ldots, 10$.

k	d_J	d_{SFH}
1	1.0	1.0
2	0.8667	0.8667
3	0.86	0.85
4	0.6181	0.705
5	0.3611	0.5733
6	0.5069	0.5214
7	0.4736	0.4929
8	0.3644	0.4667
9	0.16	0.4089
10	0.0	0.3345

The finding that the core stands out is not that surprising as these networks might be quite segregated/clustered, e.g., by gender or ethnicity, and the core would indicate centrality within the cluster. We computed the mean of the number of common items in the core for each school class and the relative number of the common items of the core for each school class summarised in S23 of the Supplementary material. For the advice-seeking networks, the average rate of the common items is 24% and 18% for 4 and 5 waves, respectively, while for friendship networks, these are 30% and 20%.

Finally, we looked that the frequency of opinion leaders induced by the other centrality measures is in the core. For a given centrality measure and network type, we refer to the fraction of opinion leader cases in the core divided by the total number of opinion leaders as the in-core rate. These numbers are collected separately for the advice-seeking and the friendship networks in S24 and S25 of the Supplementary material. In the case of the advice-seeking networks, the in-core rate computed on the same network type is above 90%. When the centrality measures are computed on the friendship networks, the in-core rate is between 59% and 67%, and the in-core rate of the Borda count aggregation computed on both network types is 90%. For the friendship networks, we got similar results. The in-core rate of the centrality measures computed on the friendship networks is above 90%, except for the closeness (71%) and betweenness (72%). The in-core rate of the Borda count aggregation computed on both network types is 92%.

5. Conclusions and Discussion

Network interventions target central individuals who are able to cause an intended diffusion or spillover. Targeting in these interventions is typically based on a single centrality measure. This is problematic for at least two reasons. First, who are the central actors in the network might significantly change over time anyhow we define centrality. Second, a single centrality measure in addition to its instability is unable to incorporate multiple structural characteristics that might be important for initiating strong influence and a diffusion process.

Therefore, in our research, we have developed a methodology to track and quantify the stability of the set of individuals who are occupying central positions based on single centrality measures. Furthermore, we proposed that an aggregation of centrality measures could be a straightforward way to reflect upon diverse advantages that single centrality indexes offer for network interventions and, potentially, to decrease temporal instability in comparison to single centrality measures. We have illustrated that aggregation can be carried out not only over different centrality measures but also over different network types measured in the same context.

The study introduced the methodology and illustrated its use on a data set collected from Hungarian primary schools (Section 3). The identification of opinion leaders is based on network structure: we rank individuals by their "importance," where "importance" is quantified by a centrality measure. Importance can be defined from different points of view, and each definition reflects upon a structural concept that could be important for the identification of actors who are important for strong influence and dissemination. Therefore, we used a selection of classical centrality measures (Section 2.1) rather than picking one as the best.

We applied the Borda count method (Section 2.3) to aggregate different centrality measures. The Borda count method [59, 60] consists of a set of ordered lists $\mathcal{L} = \{l_1, \ldots, l_m\}$, where each list has M items in different orders. In this paper, each l_i in $\mathcal{L}$ is an induced ranking list of the C_i centrality measure. We can interpret the Borda count as a voting procedure, where the centrality measures can be considered as experts who are voting to the items of the list in each rank position.

An actor can be part of different networks, where each network is built using a different relation, just like the phenomenon described by [77] and later discussed by [78] distinguishing "local leaders" (students popular as friends) from "system leaders" (students popular as role models), the latter having a larger impact on the school community as a whole. The study [78] shows that network items organised into clusters overlap but did not fully coincide with the network measures most often used in adolescent research, such as friendship and dislike. In our data set, we used two network types: advice-seeking and friendship networks. The question is self-evident: can we aggregate over the different network types? There are multiple methods to do this, and which is desirable depends on researcher needs and questions. When data from several different ties are available in the same context, then researchers might want to condense information with cluster analysis and find the latent dimensions that are measured by similar networks, such as friendship and affection [78, 79]. Another way is to simply add values up in different sparse networks or to use thresholds for cell sums to create a composite network [75]. One possible way to do this is the Borda count aggregation since this method worked on the induced ranking list rather than the networks.

In order to capture the similarities and differences between the centrality measures, we used three methods: monotonicity of rankings (Section 2.2), Kendall tau correlation (Section 2.2), and Jaccard similarity of the top influencers (Section 2.2). All these methods operate on the ranking of the individuals induced by the centrality measures. The monotonicity of the rankings quantifies the flatness of the hierarchy defined by a given centrality measure. Kendall's tau correlation coefficient measured the correspondence between two rankings of the same set of items. In this paper, we focused on the top influencers or key actors; therefore, we separately examined the similarity of the top influencers induced by the different centrality measures. This was what the "Jaccard similarity of the top influencers" tried to capture.

Since the influence of an agent was determined by the agent's position in the network(s) defined by the agent's relations, the source of the variability of the top influencers was the temporal change of the underlying network structure. We used a Jaccard similarity-based network comparison method (Section 2.4) to quantify the wave-to-wave structural change in the networks.

In Section 2.5, we developed a measure to describe the temporal (wave-to-wave) instability of the top influencers with a single number. We have made this in two steps: first, we supposed that the opinion leader is unique in all waves. We postulated four intuitive requirements that our instability measure should meet and proposed a simple formula (equation (18)) that meets the requirements. We called this measure the "sequence instability." Second, we have extended this method to handle sequences, where the elements of the sequence are top-k lists. This extended algorithm requires a distance measure of top-k lists that can handle ties. When we were only interested in the degree of overlap between the rankings, we used the d_J Jaccard distance (Section 2.2), but when we wanted to take into account the change in order between the rankings, we applied the d_{SFH} distance (Section 2.5). The distance measure d_{SFH} is the combination of the normalized Spearman's footrule distance (Section 2.2) with the Hausdorff distance.

The main lesson that we learnt from our data was that the opinion leaders are quite unstable in time. The returning frequency of the opinion leaders was very rarely higher than 2 (Tables 3(a) and 3(b)), and for the vast majority of the classes, for all centrality measures (except the core value), the sequence instability (see Figures 2, 4, and 6 and Supplementary material) was very high. Furthermore, the Borda count aggregation of classical centrality measures did not improve the stability.

We investigated these in both advice-seeking and friendship networks which allowed us to compare the rankings and especially the top influencers or key actors of these networks. We used Kendall's tau correlation to compare the induced rankings on the different network types. The mean values of the Kendall tau correlations were in the range [0.017, 0.097] (S19 of the Supplementary material). Hence, our conclusion was that the centrality measures computed on the different network types were inducing different rankings. The mean values of the Jaccard similarity of the top influencers (with the parameter value $N = 5$) shown in Figure 5 were mostly between 0.18 and 0.34, except between the core on friendship networks and the core on advice-seeking networks, where we can observe a higher value of 0.51. Although the correlation was low, the centralities computed on the different network types agreed on the top influencers surprisingly frequently but not so frequently to consider them identical.

In Section 4.5, we separately discussed the core. In general, the relative size of the core (compared to the class size) was very high: on average, this was about 60% for the advice-seeking networks and 67% for the friendship networks. Thanks partly to being large, the core shows some level of stability: lots of students were part of the core in all the available waves (Tables 3(a) and 3(b)). For advice-seeking networks, the average rates of the common items were 24% and 18% for 4 and 5 waves, respectively, while for friendship networks, these were 30% and 20%. It remains an open question for the future if the common items of the core can be considered as students with high influence and is it possible to characterise these students somehow.

The spread of opinions or the coolness of some friends in a school class is particularly interesting for targeted prevention programs [80] (see 80 for these data). As our results suggest, long-time prevention programs should not be based only on those students who have central positions in a school class (often mentioned in many ways by scientific literature, like superhubs, key actors, opinion leaders, and influencers) because their central position is temporarily unstable. However, for prevention and pedagogical programs, it is useful to know that school classes have a relatively stable core of students, in which the most central key actors can vary in time. Our results suggest that targeted interventions must be designed keeping this evolution into account.

5.1. Limitations of the Research

5.1.1. Limitations of Data. Our analysis is based on a sample of Hungarian primary school classes that overrepresented students from the Roma minority. Some classes were from the rural area of Hungary, where the number of students was lower than in major cities or the capital. Social relations in a class can be very different depending on the number of classmates or locality type (villages vs. bigger cities). Also, the number of students in a class can vary from wave to wave, especially in the 6th grade. In Hungary, students can enroll in a 6-year high school if they achieve high admission scores. This is a more popular phenomenon in cities. These parameters can influence the interpretation of our research. Another limitation regarding to data is that the algorithm was tested on survey dataset (name generation questions of personal survey). Although this is the main strength of our research (since similar research about stability of network positions are focusing on online data, for example [81, 82] or [31]), in our further work, we plan to test our methodology and algorithms also on online datasets, such as social media networks.

5.1.2. Limitations Due to the Focus on Central Actors. Our research meant to discover the patterns of key actors between opinion leaders and friends in time, so we do not investigate the socio and economic status of the students. Of course, opinion leaders can maintain or lose their social network position for socio, economic, and demographic reasons, too. Although, partly based on other relevant research (for example), we suppose that peripherical positions are much more influenced by a student's socio and economic status or their family than opinion leadership, this aspect will need further investigation. Another aspect that can be investigated is whether achievement influences the stability of opinion leaders. Our research did not take into consideration the school results or even behaviour notes of the students like. This also has to be explored in the next research phase.

5.1.3. Limitation of Approach. Many scientific studies highlight that opinion leaders may differ depending on the topic. For example, someone who is an opinion leader in political questions may not be an opinion leader in professional questions or technical innovation themes, well summed up by. We did not take into consideration the topic of opinion leadership; we only wanted to find a good mechanism to identify the key actors in a network and to describe their position' stability in time. In our approach, which is quite similar to the diffusion of innovation sociological approach [9–16], the social networks were pathways along which "social contagion" can spread, as defined by [28]. Guilbeault et al. also distinguished between simple contagions (like infectious diseases, where a simple contact can be sufficient for transmission) and complex contagions (like transmission of behaviours, beliefs, or attitudes, where multiple contacts are necessary for transmission). However, we did not include the specific behaviours, beliefs, and attitudes transmitted in our analysis. We only measured the central position of networks and their stability in time.

Appendix

A. Justification of Sequence Instability Formula for Unique Opinion Leader

In our approach, we assign a non-negative penalty for each element in $\Lambda(S)$ and we define the $v(S)$ instability of a $S \in \mathscr{A}^L$ sequence as the normalized sum of the penalty values:

$$v(S) = \frac{V(S)}{V_{\max}} = \frac{1}{V_{\max}} \sum_{a\in\Lambda(S)} \text{penalty}(S, a), \tag{A.1}$$

where penalty (S, a) is the penalty assigned to item a and $V_{\max}$ is the maximum value of $V(S)$ over all possible L-length sequences:

$$V_{\max} = \max_{R\in\mathscr{A}^L} V(R). \tag{A.2}$$

We introduce some further notations. We denote the number of occurrences of element $a \in \mathscr{A}$ in a sequence $S \in \mathscr{A}^L$ by $c(S, a)$, and the relative frequency of a is S by $f(S, a) = c(S, a)/|S|$. Furthermore, for any $a \in \mathscr{A}$, we define the frequency of case in which element a is different from the element immediately preceding it in the sequence by $B(S,a)$, so that $B(S,a) = \sum_{i=2}^{|S|} \mathbb{1}\{S(i) = a\}\mathbb{1}\{S(i) \neq S(i-1)\}$. The normalized version of $B(S,a)$ is $b(S,a)$, given by $b(S,a) = B(S,a)/(|S| - 1)$. It is clear that $B(S) = \sum_{a\in\Lambda(S)} B(S,a)$, and there is a similar relationship between $b(S)$ and $b(S,a)$. For example if the sequence Q is given by $Q = AABCB$, then $c(Q,A) = c(Q,B) = 2$, $c(Q,C) = 1$, $b(Q,A) = 0$, $b(Q,B) = 0.5$, and $b(Q,C) = 0.25$.

We suppose that penalty (S,a) depends on the relative frequency $f(S,a)$ of a in S and the $b(S,a)$ relative frequency of cases when an a item is different from its immediate preceding item, and we also suppose that this dependence is linear; therefore, we are looking for penalty (S, a) in the following form:

$$\text{penalty}(S, a) = \alpha + \beta f(S, a) + \gamma b(S, a), \tag{A.3}$$

where α, β, and γ are nonzero real constants.

Consider now requirement R1. Suppose that $\lambda(S) = 1$. In this case, $\Lambda(S) = \{a\}$; hence, the relative frequency of this single item a is 1, and $b(S,a) = 0$. In order to satisfy (12), consider first the case $\lambda(S) = 1$, and we need $V(S) =$ penalty $(S,a) = \alpha + \beta = 0$. From this, we immediately get that $\beta = -\alpha$ and penalty (S,a) has the form as follows:

$$\text{penalty}(S, a) = \alpha - \alpha f(S, a) + \gamma b(S, a). \tag{A.4}$$

Since $\alpha \neq 0$, denoting $\sigma = \gamma/\alpha$, we can write

$$\text{penalty}(S, a) = \alpha(1 - f(S, a) + \sigma b(S, a)). \tag{A.5}$$

Since the not normalized instability number is just the sum of the penalties assigned to each item in S, we get $V(S)$ as follows:

$$V(S) = \sum_{a\in\Lambda(S)} \text{penalty}(S, a) = \alpha(\lambda(S) + \sigma b(S) - 1). \tag{A.6}$$

To get equation (A6), we have used that the sum of the $f(S,a)$ relative frequencies is 1, i.e., $\sum_{a\in\Lambda(S)} f(S, a) = 1$ and $\sum_{a\in\Lambda(S)} b(S, a) = b(S)$. From equation (A6), we can also see that if $v(S) = 0$, then $\lambda(S) = 1$. Indeed, $v(S) = V(S) = 0$ can be true only if $b(S) = 0$ and $\lambda(S) = 1$.

Suppose now that the conditions of requirement R2 hold, so that $|S_1| = |S_2|$ and $\lambda(S_1) > \lambda(S_2)$. We denote the maximal instability number if the number of items in an L-length sequence is N by $\overline{V}_N$:

$$\overline{V}_N = \max_{R\in\mathscr{A}^L}\{V(R)\colon \lambda(R) = N\}, \tag{A.7}$$

and similarly, we denote the minimal instability number if the number of items in an L-length sequence is N by $\underline{V}_N$:

$$\underline{V}_N = \min_{R\in\mathscr{A}^L}\{V(R)\colon \lambda(R) = N\}. \tag{A.8}$$

In order to ensure that (14) is met, we need $\underline{V}_{N+1} > \overline{V}_N$ for all $N \in \{1, \ldots, L - 1\}$. It is easy to see that $\overline{V}_N = \alpha(N + \sigma(|S| - 1) - 1)$ and $\underline{V}_N = \alpha(N + \sigma(N - 1) - 1)$. From the inequality $\underline{V}_{N+1} > \overline{V}_N$, we get the following condition:

$$\sigma < \frac{1}{|S| - N + 2}. \tag{A.9}$$

Since $N \geq 1$, this condition surly holds if

$$\sigma < \frac{1}{|S| - 1}. \tag{A.10}$$

Let us continue with requirement R3. Suppose the conditions of R3 are hold, so that $|S_1| = |S_2|$, $\lambda(S_1) = \lambda(S_2)$, and $b(S_1) > b(S_2)$. From equation (A6), we can directly see condition (16); therefore, requirement R3 is satisfied.

Finally, let us consider requirement R4. In order to scale the instability measure to the [0, 1] interval, we need to find $V_{\max}$. It is easy to see from equation (A6) that $V_{\max} = \max_{R\in\mathscr{A}^L} V(R) = \alpha(L + \sigma(L - 1) - 1) = \alpha(L - 1)(1 + \sigma)$. As a result, we get the formula for $v(S)$ as follows:

$$v(S) = \frac{\lambda(S) + \sigma b(S) - 1}{(|S| - 1)(1 + \sigma)}, \tag{A.11}$$

and σ must satisfy condition (A10). From equation (A11), we can directly get that $v(S) = 1$ can hold only if $\lambda(S) = |S|$, so that we verified that condition (17) is satisfied.

B. An Example Instability Calculation

We illustrate the sequence instability method with an example calculation. Suppose that $\Omega = \{A, B, C, D, E, F, G, H, I, J\}$, and the S example sequence with full ranks is given in Table 6. We calculate the sequence instability using the distance measures: Jaccard distance (d_J) and Spearman's footrule combined with Hausdorff distance (d_{SFH}) for each $k \in \{1, \ldots, 10\}$. The results are collected in Table 7. We expect that for $k = 1$, the sequence instability takes the maximal value for both distance measure because the first items of each lists of S are different. We also expect a decreasing trend (but not monotonic) as k is increasing for both distance measures. At $k = 10$, we see that the instability becomes zero for the case of Jaccard distance, since every item is identical with respect to the Jaccard distance; however, the order of the items is different; therefore, $v(S) > 0$ for $k = 10$, when we use d_{SFH}.

Data Availability

All data used in this study have been collected by the Research Center for Educational and Network Studies (RECENS), supported by Lendület Program of Hungarian Academy of Sciences. The datasets used and the algorithms are available on request or at https://github.com/rpethes/ol/.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Supplementary Materials

Supplementary material contains additional numerical results. (*Supplementary Materials*)

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