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Research Article

Robust Decentralized Formation Tracking Control of Complex Multiagent Systems

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This article investigates the decentralized formation tracking problem for complex multiagent systems in finite settling time, subjected to output constraints and external disturbances. The barrier Lyapunov function is used to constrain the output of each agent. Therefore, starting inside a closed boundary, the agents are guaranteed to remain inside it for all future time. Under directed communication topology, decentralized time-varying formation tracking is achieved in finite time. Furthermore, in the proposed work, the linear sliding manifold is employed to mitigate the singularity problem that occurs in the conventional robust finite-time methods, i.e., terminal sliding mode-based control schemes. The stability properties of the proposed framework are established through the Lyapunov method which not only ensures the finite-time formation tracking of nonlinear multiagent systems under directed communication but also guarantees that for all time the agents remain inside a closed boundary if they are initially inside it. Consequently, the uniqueness of this article is that it presents a novel formation tracking control framework for multiagent systems that simultaneously considers three performance metrics of robustness, finite-time convergence, and output constraints while mitigating the singularity problem. The proposed topology is validated by implementing the numerical examples in MATLAB/SIMULINK.

1. Introduction

Multiagent systems have gained popularity as a result of their ability to perform distributed sensing and actuating operations. In comparison with a single agent, they are more flexible, cost-effective, reliable, and robust while operating in an uncertain environment and are capable of handling complex tasks. Owing to these benefits, they are being adopted in a variety of applications, such as rescue and retrieval operations, intelligence, surveillance, and reconnaissance (ISR) missions, precision agriculture, space and planetary exploration, cooperative transportation, communication networking. [1, 2]. Typically, in these applications, the agent can be a robotic manipulator, an unmanned aerial vehicle, an underwater vehicle, a ground vehicle, etc.

In order to accomplish the aforementioned tasks, a cooperative controller is required such that the multiagent system attains the anticipated group behavior based on individual dynamic models and the information sharing among these agents [3]. In literature, several linear, nonlinear, fuzzy, and observer-based techniques are employed to design cooperative controllers such as [4–10]. Cooperative control can either be centralized or decentralized [11]. The centralized approach entails the control of all agents by a central station; consequently, each agent shall perpetually have knowledge of the central station's states and inputs. The most fatal weakness of the centralized approach is its reliance on a central station, resulting in poor robustness, high computational requirements, and susceptibility to disruptions. In order to address these problems, the decentralized approach was introduced in which the central station is not present to manage the multiagent system [12]. A controller is assigned to each agent in accordance with its local interaction in order to execute a global group mission [13].

Recently, decentralized cooperative control has attracted much interest due to its stronger robustness, easy maintenance, and high flexibility [14]. The recent significant decentralized works include consensus control [15–20], formation control [21–23], consensus tracking control [24], formation tracking control [1, 25–27], containment control [28–30], and flocking [31]. In consensus control, agents share local information to establish an agreement on a common state (i.e., consensus point) in a decentralized manner. In formation control, each agent keeps a predetermined spatial gap from the other [3]. The particular spatial pattern is named the desired formation that can be time-varying or time-invariant, as discussed in [27]. Furthermore, formation tracking control deals with tracking a reference while maintaining multiagent formation [26]. The formation tracking control problem in multiagent systems is examined in this article.

In the formation tracking problem, convergence time is of significant importance in complex scenarios where there are constraints on the time available for accomplishing a particular task. For example, in some industrial applications, the operations are constrained by the production schedule; in ISR missions that often take place in hostile or challenging environments, such as battlefields or disaster zones, the task execution time becomes crucial for the success of the mission. One of the viable solutions in such circumstances is to design a controller that ensures finite-time stability, as discussed in [25, 32–36]. In finite-time control schemes, systems' states converge to a fixed value or a limit set within a finite time interval. Consequently, finite-time stability offers high control precision and improved disturbance rejection attributes [37]. The existing finite time control approaches employ the terminal sliding mode that suffers from singularity. This paper employs a linear sliding manifold to establish finite time formation tracking control, and as a result, the singularity problem is mitigated.

In practice, the complexity of control design arises when the multiagent systems are subjected to particular constraints. Typical examples include the current through a branch of the synchronous circuit, keeping a self-driving car platoon inside the road boundary, or the flights of an unmanned aerial system may be restricted by speed, angle of attack, or position limits. Moreover, it is common practice to deploy multiagent systems in a confined space, where it is necessary to impose constraints on the agents' states [38]. To handle this complexity, these constraints are fulfilled by introducing the barrier Lyapunov functions (BLFs) in nonlinear controllers [39]. In another article, the distributed consensus of the multiagent system under position constraints is studied using BLF in conjunction with adaptive backstepping [40]. In another research, the consensus problem was addressed under input saturation constraints [41]. BLF-based constrained nonlinear control along with a disturbance observer is presented for surface ships [9]. In another study, an output synchronization problem with prescribed constraints is incorporated for nonlinear strict-feedback multiagent systems by utilizing a high-gain observer and adaptive controller [8]. In another study, BLF is used in conjunction with sliding mode control (SMC) for finite-time tracking of a single quadrotor under output constraints [42]. The linear sliding manifold is used in this work to achieve a finite settling time of states. A similar approach is used in References [43–47] for finite and fixed settling time control of single agents under constraints. It is essential to note that recent work also includes fixed-time or predefined-time controllers derived from finite-time controllers [48–50]. However, these approaches do not constitute the consideration of constraints. In general, existing research focuses on either finite-time cooperative control or the performance of cooperative control under constraints. However, singularity-free finite-time cooperative control of multiagent systems with output constraints and the effect of external disturbances is still an open problem. In this paper, we present the singularity-free formation tracking of multiagents in finite settling time under output constraints.

From the preceding discussion, it is evident that in the literature, robustness, settling time, and output constraints are the primary performance matrices that are considered when dealing with multiagent systems' control. Comparing the existing robust cooperative control methods, it is to be noted that the majority of the research is carried out without considering the finite convergence time or output constraints, as in references [15–19, 21–24, 27]. Only works of references [25, 26, 32–36, 51] investigated the coordinated control with convergence time consideration. References [8, 38, 40, 41] present the cooperative control under output constraints but only establish asymptotic convergence. A brief comparison of existing cooperative control methods is presented in Table 1. It demonstrates that none of the available techniques provide robust formation tracking in finite time while maintaining output constraints, i.e., the literature lacks to present a controller that considers three performance matrices of robustness, finite time, and output constraints, simultaneously. In this research, we investigate the formation tracking problem for nonlinear multiagent systems with output constraints under disturbances in finite settling time.

The subsequent points outline the main contributions of this paper:

(1) A decentralized formation tracking control topology is introduced for nonlinear multiagent systems that is robust against the matched disturbances.

(2) Using the proposed control, the output of all the agents is guaranteed to stay within the user-specified constraints under the directed communication graph.

(3) Singularity-free convergence of outputs within a finite time interval is guaranteed, and the upper bound on the settling time is predetermined.

In a nutshell, this article presents a new formation tracking control framework for multiagent systems that simultaneously considers the three performance metrics of robustness, finite-time convergence, and output constraints

Table 1: Comparison of existing cooperative control methods.

References	Method	Nonlinear dynamics	Robustness: Modeling uncertainties	Robustness: Matched disturbances	Finite time	Output constraint
[22]	SMC	Yes	Yes	Yes	No	No
[38]	Lyapunov redesign approach	No	Yes	No	No	Yes
[34]	Terminal SMC	Yes	Yes	Yes	Yes	No
[40]	Backstepping and BLF	Yes	Yes	No	No	Yes
[8]	Neuro-fuzzy control and BLF	Yes	Yes	No	No	Yes
[36]	Neural network and SMC	Yes	Yes	Yes	Yes	No
This paper	Linear SMC and BLF	Yes	Yes	Yes	Yes	Yes

while mitigating the singularity problem. To the best of the authors' knowledge, such a cooperative controller that considers these performance matrices simultaneously has not been reported in the literature.

This article is organized as follows: Section 2 presents the essential definitions and lemmas to be used in this work. The problem statements with considered assumptions are described in Section 3, whereas Section 4 presents the proposed finite settling time formation tracking control scheme. In Section 5, the MATLAB simulation-based results are discussed. The conclusion of this article is given in Section 6.

2. Preliminaries

In this section, the preliminaries are defined which will be used throughout this article. Moreover, a concise summary of previously published findings is also provided in this section of the paper.

2.1. Graph Theory. In multiagent systems consider that there are N agents. The communication graph $\mathbb{G}$ comprises a pair $(V(\mathbb{G}), E(\mathbb{G}))$, where $V(\mathbb{G}) = \{\nu_1, \nu_2, \ldots, \nu_N\}$ is the node set which denotes the vertex of the graph, and $E(\mathbb{G}) \subseteq \{(\nu_i, \nu_j); \nu_i, \nu_j \in V(\mathbb{G}), i \neq j\}$ is the edge set that defines the flow of information from the node ν_j to node ν_i. For an undirected graph, the set $E(\mathbb{G})$ must satisfy the condition $(\nu_i, \nu_j) = (\nu_j, \nu_i)$, $\forall E(\mathbb{G})$. On the other hand, if $(\nu_i, \nu_j) \neq (\nu_j, \nu_i)$ for any edge, then it is said to be a directed graph. In the case of an undirected graph, the degree of a node is defined as the number of edges that are incident to a node [30]. In graph theory, a communication path is defined to be the sequence of the edges among the nodes. Whenever there exists at least one communication path among all nodes of the graph, the graph $\mathbb{G}$ is connected. For communication graph analytics, some of the matrices are very significant. In a diagonal matrix $\mathbb{D}$, each diagonal element characterizes the degree of the respective node whereas in an adjacency matrix $\mathbb{A}$, the nondiagonal elements describe the information flows from the neighboring nodes. The communication graph is entirely defined by a semi-positive definite Laplacian matrix which is defined by $\mathbb{L} = \mathbb{D} - \mathbb{A}$ [52].

2.2. Notations. In the course of this article, the following notations will be used: A $n \times m$ dimensional set of real matrices is denoted by $\mathbb{R}^{n \times m}$; sets of negative, nonnegative, and positive real numbers are denoted by $\mathbb{R}^-$, $\mathbb{R}^{0+}$, and $\mathbb{R}^+$, respectively; the N-dimensional identity matrix is represented by I_N; the construction of a diagonal matrix is defined by diag{•}; and $\mathbb{I}_N$ denotes an N-dimensional column matrix with all entries equal to 1. The Kronecker product operator is denoted by $\otimes$ and $\operatorname{sgn}(x)$ defines the discontinuous scalar function, which is given by

$$\operatorname{sgn}(x) = \begin{cases} -1, & x < 0, \\ +1, & x \geq 0. \end{cases} \tag{1}$$

Also, if $\mathcal{X} = [x_1, x_2, x_3, \ldots, x_n]^T$ then $\operatorname{sgn}(\mathcal{X}) = [\operatorname{sgn}(x_1), \operatorname{sgn}(x_2), \operatorname{sgn}(x_3), \ldots, \operatorname{sgn}(x_n)]^T$ and $\operatorname{sgn}(\mathcal{X}^T) = [\operatorname{sgn}(\mathcal{X})]^T$.

Consider an autonomous single agent with a globally asymptotically stable origin defined as follows:

$$\dot{x} = f(x), \tag{2}$$

where $f: \mathbb{R}^{0+} \times \mathcal{Q} \longrightarrow \mathbb{R}^n$ is continuously defined inside the open set $\mathcal{Q} \in \mathbb{R}^n$ defined in the neighborhood of equilibrium $f(0) = 0$ and $f(0) \in \mathcal{Q}$. $x(t, x_0)$ defines the solution of (2) where $x_0 = x(0)$ denotes the initial condition.

The definitions below are taken from the literature and are provided here for the reader's convenience.

Definition 1 (finite-time stability) [53, 54]. For the agent (2), the origin is said to achieve global finite-time stability, if after time $T(x_0)$ its solution converges to the origin, i.e., $x(t, x_0) = 0, \forall t \geq T(x_0)$. $T(\bullet)$ denotes the settling time function.

Lemma 2 (see [54, 55]). *Let $V(t, x)$ be a radially unbounded and positive definite function that is continuously defined on the open neighborhood $\mathcal{Q}$ around the origin, i.e., $V(t, x)$: $\mathbb{R}^+ \times \mathcal{Q} \longrightarrow \mathbb{R}^n$, and satisfies*

(i) $V(t, x) = 0 \Rightarrow x = 0$

(ii) $\dot{V}(x) \leq -k\sqrt{V(x)}, k \in \mathbb{R}^+$

Then, the agent (2) is said to be finite-time stable.

Definition 3. (finite settling time stability) [45]: For the agent (2), the origin is said to achieve globally finite settling time stability if the solution $x(t, x_0)$ satisfies $x(t, x_0) \leq \epsilon \, \forall \, t \geq T(x_0)$ where $x_0 \in \mathbb{R}^- \cup \mathbb{R}^+$, ϵ is a bound such that $\epsilon \approx 0$, and finite settling time is denoted by $T(x_0)$.

Remark 4. According to Definition 3, an agent is considered to achieve finite settling time stability if the states of the agent approach the small bound $\epsilon \approx 0$ after some finite-time T, rather than being exactly 0. This assumption takes the practical aspect into account since in tracking control problems, it is typically acceptable to have a small user-defined error margin ϵ (e.g., order of 10^{-2}). In recent literature, this type of stability is also referred to as practical finite-time stability [56].

3. Problem Formulation

We consider N agents in a multiagent system, and each agent has single-input single-output second-order dynamics, as considered in [3]. The multiagent system is defined by

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= f(x_i, v_i) + g(x_i, v_i)u_i + w_i, \\ h_i &= x_i, \end{aligned} \tag{3}$$

where $X \coloneqq [x_i, v_i]^T \in \mathcal{Q} \subseteq \mathbb{R}^{2\times 1}$ defines the state vector; w: $\mathbb{R}^{0+} \times \mathcal{Q} \longrightarrow \mathbb{R}$ is the disturbance; f: $\mathcal{Q} \times \mathbb{R}^{0+} \longrightarrow \mathbb{R}$ and g: $\mathcal{Q} \times \mathbb{R}^{0+} \longrightarrow \mathbb{R}$ are continuously differentiable known functions with $g \neq 0$; and the origin of (3) lies inside the open set $\mathcal{Q}$. Moreover, x_i is position, v_i denotes the velocity, h_i and u_i represent the output and input, and w_i is the associated disturbance of the i^{th} agent; $i = 1, 2, \ldots, N$. The position vector χ is defined as $\chi = [x_1, x_2, \ldots, x_N]^T . x_0 \coloneqq [x_1(0), x_2(0), \ldots, x_N(0)]^T$ and $v_0 \coloneqq [v_1(0), v_2(0), \ldots, v_N(0)]^T$ denote the initial conditions of each agent in the multiagent system.

The main aim of this article is to propose a robust finite settling time decentralized formation tracking control for the multiagent system (3) under output constraints. For this purpose, we define the error variables as $p_i = x_i - r$ and $q_i = v_i - \dot{r}$ for reference trajectory r and $i = 1, 2, \ldots, N$. We further define $\eta = [p_1, p_2, \ldots, p_N]^T$, $\upsilon = \dot{\eta} = [q_1, q_2, \ldots, q_N]^T$, $u = [u_1, u_2, \ldots, u_N]^T$. Then,

$$\begin{aligned} \dot{\eta} &= \nu, \\ \dot{\nu} &= f^\star + \ddot{r}\mathbb{I}_N + g^\star u + w^\star, \end{aligned} \tag{4}$$

where the vectors $f^\star$ and $g^\star$ are defined by $f^\star = [f(x_1, v_1), f(x_2, v_2), \ldots, f(x_N, v_N)]^T$, $g^\star = \text{diag}\{g(x_1, v_1), g(x_2, v_2), \ldots, g(x_N, v_N)\}$, respectively. Consider the vector sliding surface s defined by

$$s = \upsilon + \dot{F} + \beta L[\eta + F], \tag{5}$$

where $L = \mathbb{L} + I_N$ is a positive definite matrix, $\beta \in \mathbb{R}^+$, and $F = [F_1, F_2, \ldots, F_N]^T$ denotes the desired formation. Furthermore, we define few variables that will be employed to construct the main results of this article, such as, $\mathscr{p} = \eta + F$, $\mathscr{q} = \dot{\mathscr{p}}$, $i = 1, 2, \ldots, N$, and $w^\star = [w_1, w_2, \ldots, w_N]^T$. It is pertinent to mention here that to avoid singularities in the nominal control and disturbance compensator, all diagonal entries of $g^\star$ must be different from zero. The block diagram to explain the notations and signal flow of the proposed cooperative controller is given in Figure 1.

Assumption 5. $\mathbb{G}$ defines a strongly connected, directed graph.

Assumption 6. The control direction, i.e., $\text{sgn}(g(x_i, v_i))$ is always known.

Assumption 7. For $A_0, A_1, A_2, \mathscr{B}, \mathscr{F}_{\max} \in \mathbb{R}^+$, we assume the bound on a smooth reference $|r| \leq A_0 < \mathscr{B}, |\dot{r}| < A_1$, $|\ddot{r}| < A_2$ and formation $|F| < \mathscr{F}_{\max}$; $\dot{F}, \ddot{F}$ remain bounded. The error bound is defined by $\hbar = \mathscr{B} - (A_0 + \mathscr{F}_{\max})$ and initially, all agents satisfy the symmetric constraint $|p_i(0) - F_i| < \hbar$. The pictorial illustration is given in Figure 2.

Assumption 8. The uniform upper bound $\mathscr{W}(\text{t})$ on the matched perturbation is known, i.e.,

$$|w_i| \leq \mathscr{W}(\text{t}), i = 1, 2, \ldots, N. \tag{6}$$

Remark 9. This paper deals with consensus-based formation tracking. Therefore, virtually, the consensus point of multiagent is assumed to track the reference r.

3.1. Problem Statement. Under directed communication, propose a decentralized formation tracking control for a multiagent system (3) to

(i) Achieve the desired formation within finite settling time

(ii) Constraint the output of all agents, i.e., $-\mathscr{B} < x_i < \mathscr{B}, \forall t \in \mathbb{R}^{0+}$ and $i = 1, 2, \ldots, N$

Provide the predetermined upper bound on the settling time, depending upon the initial conditions of the agents.

4. Finite-Time Formation Tracking Control

In this section, the main contribution of this article is presented. The finite-time formation tracking control is presented in Theorem 10. Figure 3 presents the overview of the proposed formation tracking controller. The settling time function is derived in Corollary 11.

Theorem 10. *Under Assumptions 5–8 the multiagent system (3) is guaranteed to achieve the desired decentralized formation tracking within a finite settling time using the following control protocol:*

$$u = u_n + u_c, \tag{7}$$

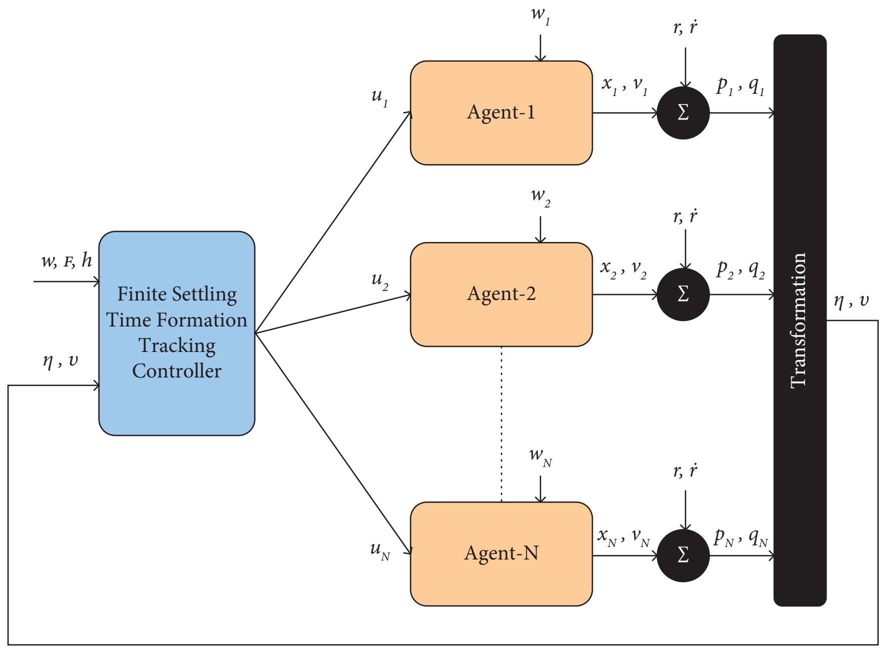

Figure 1: Block diagram of the proposed formation tracking controller.

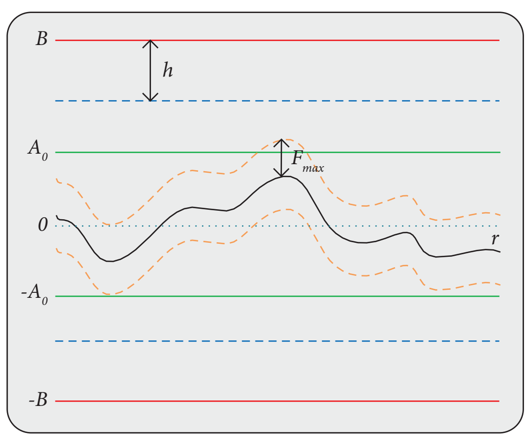

Figure 2: Pictorial illustration of Assumption 7.

where nominal control u_n and disturbance compensator u_c are, respectively, given by

$$u_n = -[g^\star]^{-1}\left[f^\star + \ddot{r}\mathbb{1}_N + \ddot{F} + \beta L[\nu + \dot{F}]\right],$$

$$u_c = -[g^\star]^{-1}\left[\frac{\wp^{\mathrm{T}} L^T L q}{\hbar^2 - \wp^{\mathrm{T}} L^T L \wp}\cdot\frac{\operatorname{sgn}(s)}{N} + \frac{\kappa}{N}\left(\frac{\sqrt{\wp^{\mathrm{T}} L^T L \wp}}{\sqrt{\hbar^2 - \wp^{\mathrm{T}} L^T L \wp}} + \sqrt{2\operatorname{sgn}\left(s^T\right)s}\right)\operatorname{sgn}(s) + s\mathscr{W}(t)\right], \tag{8}$$

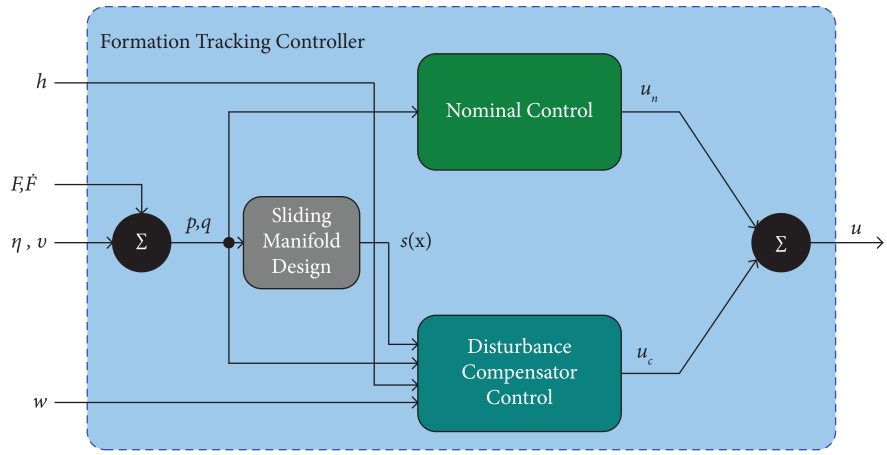

Figure 3: Finite-time formation tracking control.

where s denotes the sliding surfaces defined in (5) and $\kappa \in \mathbb{R}^{+}$*. Moreover, the outputs of all agents will satisfy the constraint* $|\chi| < \mathscr{B}\ \forall\, t \geq 0$.

Proof. Consider a Lyapunov function

$$V = \operatorname{sgn}\left(s^{T}\right)s + \frac{1}{2}\ln\left(\frac{\hbar^{2}}{\hbar^{2} - p^{T}L^{T}Lp}\right). \quad (9)$$

The derivative of the Lyapunov function results in

$$\dot{V} = \operatorname{sgn}\left(s^{T}\right)\dot{s} + \frac{p^{T}L^{T}Lq}{\hbar^{2} - p^{T}L^{T}Lp}. \quad (10)$$

Solving for $\dot{s}$

$$\dot{s} = \dot{\nu} + \ddot{F} + \beta L[\dot{\eta} + \dot{F}]. \quad (11)$$

Plugging $\dot{\nu}$ and $\dot{\eta}$ from (4)

$$\dot{s} = f^{\star} + \ddot{r}\,\mathbb{1}_{N} + g^{\star}u + w^{\star} + \ddot{F} + \beta L[\nu + \dot{F}], \quad (12)$$

and hence,

$$\dot{V} = \operatorname{sgn}\left(s^{T}\right)\left[f^{\star} + \ddot{r}\,\mathbb{1}_{N} + g^{\star}u + w^{\star} + \ddot{F} + \beta L[\nu + \dot{F}]\right] + \frac{p^{T}L^{T}Lq}{\hbar^{2} - p^{T}L^{T}Lp}. \quad (13)$$

Plugging the control law [7],

$$\dot{V} = \operatorname{sgn}\left(s^{T}\right)w^{\star} - \kappa\left(\frac{\sqrt{p^{T}L^{T}Lp}}{\sqrt{\hbar^{2} - p^{T}L^{T}Lp}} + \sqrt{2\operatorname{sgn}\left(s^{T}\right)s}\right) - \operatorname{sgn}\left(s^{T}\right)s\mathscr{W}(t). \quad (14)$$

Note that the second term in (14) is scalar and therefore $\operatorname{sgn}(s^{T})\operatorname{sgn}(s) = N$. Moreover, the Assumption 8 results in $\operatorname{sgn}(s^{T})w^{\star} - \operatorname{sgn}(s^{T})s\mathscr{W}(t) \leq 0$ and hence,

$$\dot{V} \leq -\kappa\left(\frac{\sqrt{p^{T}L^{T}Lp}}{\sqrt{\hbar^{2} - p^{T}L^{T}Lp}} + \sqrt{2\operatorname{sgn}\left(s^{T}\right)s}\right). \quad (15)$$

Since, $\ln(\hbar^{2}/(\hbar^{2} - (Lp)^{2})) \leq p^{T}L^{T}Lp/(\hbar^{2} - p^{T}L^{T}Lp)$ (Rens' inequality [57]), therefore,

$$\dot{V} \leq -\kappa\left(\sqrt{\ln\left(\frac{\hbar^{2}}{\hbar^{2} - p^{T}L^{T}Lp}\right)} + \sqrt{2\operatorname{sgn}\left(s^{T}\right)s}\right). \quad (16)$$

Using inequality, $\sqrt{a+b} \leq \sqrt{a} + \sqrt{b},\ \forall\, a, b \geq 0$

$$\dot{V} \leq -\kappa\sqrt{2}\sqrt{\operatorname{sgn}\left(s^{T}\right)s + \frac{1}{2}\ln\left(\frac{\hbar^{2}}{\hbar^{2} - p^{T}L^{T}Lp}\right)}. \quad (17)$$

Defining $\kappa = \kappa^{\star}/\sqrt{2}$, $\kappa > 0$, then

$$\dot{V} \leq -\kappa^{\star}\sqrt{V}. \quad (18)$$

Therefore, by virtue of Lemma 2, the sliding mode is attained in finite time. Since the error is bounded by an upper bound $\hbar$ and exponential convergence of states is guaranteed after attaining sliding mode, therefore, it is straightforward to determine the maximum settling time that states require after attaining sliding mode. This results in finite settling time stability. Moreover, by the construction of the Lyapunov function, it is evident that $|\wp| < \hbar$, which eventually leads to $|\chi| < \mathscr{B}$ based on Assumption 7. The mathematical expressions for finite settling time are provided in Corollary 11. □

Corollary 11. *The settling time function T of the closed-loop multiagent system using control law (6) is upper bounded by*

$$T(x_0) < \max\left[\left[\ln\frac{(\hbar\otimes\mathbb{I}_N/\epsilon)}{\beta}\right]L^{-1}\right] + \max\left(\frac{1}{\kappa}\left(\sqrt{2V_0}\right)\right), \tag{19}$$

where V_0 denotes the value of Lyapunov function V at time $t = 0$ and is dependent on the initial conditions of agents, Laplacian matrix, and initial value of the reference. ϵ denotes the error margin as explained in Definition 3.

Proof. Simplify the inequality (19) to solve the reaching time T_r of states. By straightforward evaluation of ordinary differential inequality $dV/\sqrt{V} \le -\kappa^\star dt$, we get $2(\sqrt{V} - \sqrt{V_0}) \le -\kappa^\star t$, where V_0 denotes the initial values of the Lyapunov function. By virtue of Definition 1 and Lemma 2, $V = 0; \forall t \ge T_r(x_0)$, which implies that $T_r \le 2\sqrt{V_0}/\kappa^\star$. Simplifying these expressions result in $T_r \le \max(1/\kappa(\sqrt{2V_0}))$. Once the states of each agent reach the respective sliding manifold, i.e., $s = 0$, it is guaranteed by the construction of the sliding surface that the output converges exponentially to the origin, i.e., $\wp = \wp_{10}^\star \exp(-\beta \mathrm{Lt})$, where $\wp_{10}^\star$ denotes the value of $\wp$ when $s = 0$ is achieved. Subsequently, $\wp_{10}^\star < \hbar \otimes \mathbb{I}_N$ implies that the maximum settling time T_s to establish $\wp_1 < \epsilon$ becomes $T_s < \max[[\ln(\hbar\otimes\mathbb{I}_N/\epsilon)/\beta]L^{-1}]$. Therefore, $T = T_r + T_s$ satisfies Corollary 11. □

Remark 12. It is noticeable that the settling time function T, of Corollary 11, can be predetermined, depending upon the initial conditions, communication graph, disturbance bound, and the control parameters. The settling time function of Corollary 11 exhibits relatively more overestimation of the settling time, and it can be improved by knowing the exact knowledge of states at the time of attaining the sliding mode.

Remark 13. Table 2 elaborates on the selection criteria as well as the essence of all the formation tracking controller parameters:

Remark 14. Excitation in the unmodeled closed loop dynamics, known as chattering, is caused by the high-frequency switching that occurs during the sliding phase (using the signum function). Actuators in the system can experience wear and tear due to chattering. While it is impossible to completely eliminate the chattering, the techniques outlined in [58] can help reduce it to a manageable level. The interesting fact about these methods is that the controller is initially designed using the signum function, and then its approximations are used to diminish chattering. In this article, however, the chattering suppression is not taken into account, and the focus is kept on the design of a decentralized finite settling time formation tracking controller with output constrained in the presence of disturbances.

Remark 15. By systematic extension of Theorem 10 and Corollary 11, the validation of the results of this paper can be employed to the square MIMO multiagent system; with each agent having n^{th} order dynamics which are characterized by $n/2$ block diagonals, each block having a relative degree 2.

5. Simulations and Results

In order to validate the theoretical developments, two numerical examples are presented here. Four frictionless carts (agents) under external force and disturbances are considered, as illustrated in Figure 4. Assuming $M = 1$, the dynamics can be written as follows:

$$\ddot{x}_i = u_i + w_i. \tag{20}$$

The directed communication graph is given in Figure 5. The resulting Laplacian matrix $\mathbb{L}$ is as follows:

$$\mathbb{L} = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & 0 & 2 \end{pmatrix}. \tag{21}$$

In Example 1, a formation controller is devised for multiagent cart dynamics while the solution is extended to time-varying formation tracking control in Example 2.

5.1. *Example 1* (Formation of Cart-Multiagent Dynamics).

While moving on a single axis under the effect of external force, the cart dynamics are in the form of double integrator dynamics (i.e., in (3), $f(x_i, v_i) = 0$; $g(x_i, v_i) = 1$). First, the simulations are carried out for formation control only (Example-2 is presented for evaluation of performance for the formation tracking control), and the initial positions of four carts are considered as $x_0 = [-2, 2, -0.9, -0.1]^T$ and speeds as $v_0 = [1, -1, 0.5, -0.5]^T$, respectively.

The desired formation is $F = [1, -1, 3, -3]^T$ while reference is set to $r = 0 \Rightarrow A_0 = 0$. Moreover, for simulations, the disturbance is assumed to be $w_i = [0.25, 0.1, 0.5, 0.1]^T$. The controller parameters are set to $\beta = 2; \kappa = 1; \mathscr{W} = 5; \hbar = 3$. From (18), the settling time function yields a maximum time of 14.11 s. The resulting plots are given in Figures 6 and 7; it is evident from these

Table 2: Controller parameters.

Control parameter	Selection criteria	Description
κ	Positive real value, i.e., $\kappa > 0$	Tuning parameter that varies the reaching time of the controller
β	Positive real value, i.e., $\beta > 0$	Tuning parameter that varies the sliding time of the controller
$\mathscr{W}$	Positive real value specified by the user	The upper bound on the disturbance
$\mathscr{B}$	Positive real value specified by the user	The bound on the output of each agent that eventually transforms into $\hbar$ used in the controller
ϵ	Positive real value specified by the user	The acceptable error margin

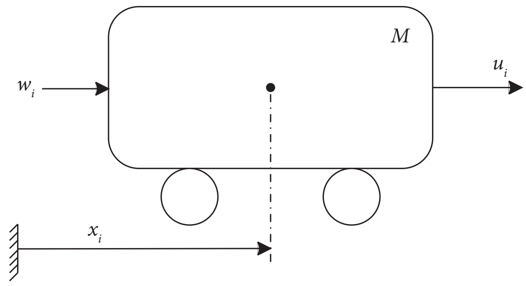

FIGURE 4: Frictionless carts under the effect of external forces and disturbances $i = 1, 2, 3, 4$.

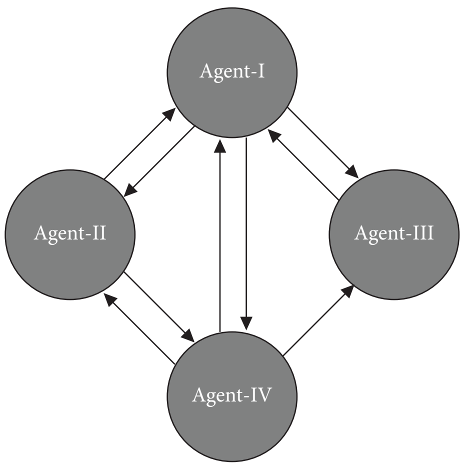

FIGURE 5: Communication graph for multiagent cart.

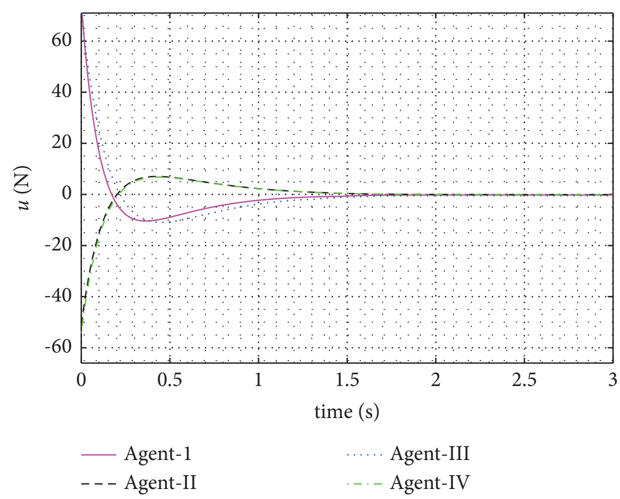

FIGURE 7: Control inputs for formation of the cart-multiagent system.

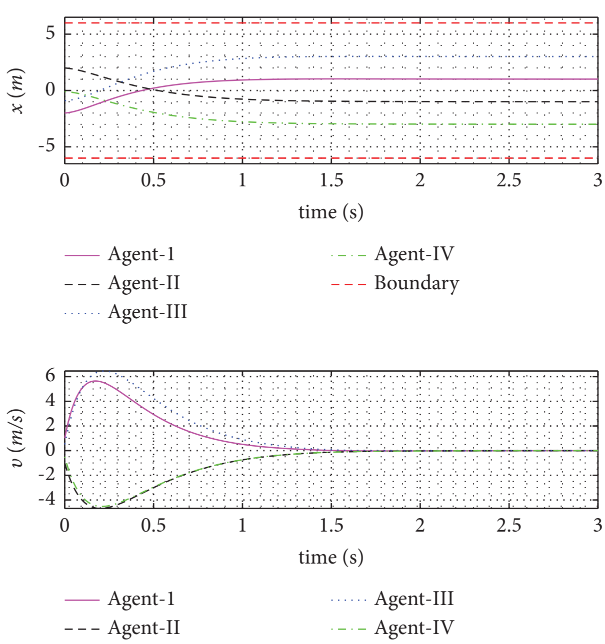

FIGURE 6: States of the cart-multiagent system for constant formation.

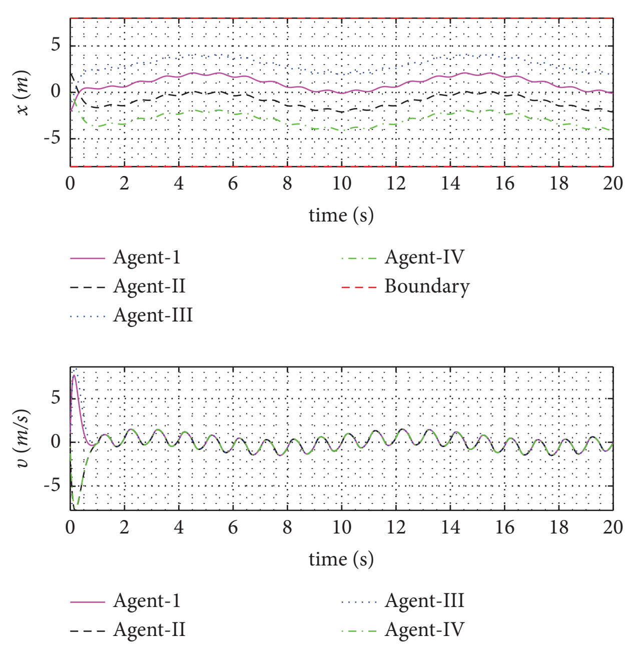

FIGURE 8: States of the cart-multiagent system for time-varying formation tracking.

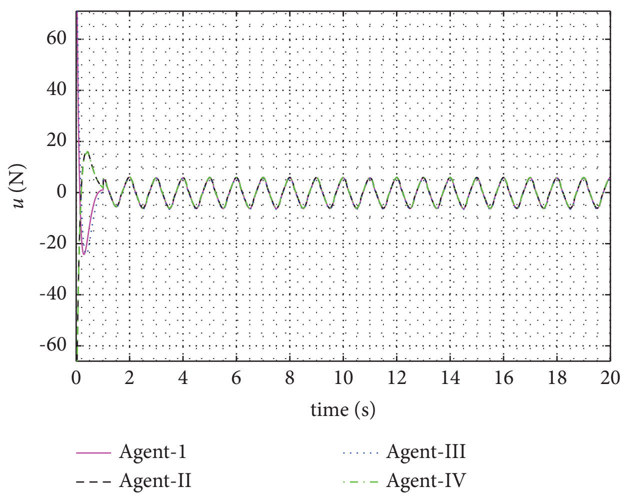

FIGURE 9: Control inputs for time-varying formation tracking of cart-multiagent system.

results that the settling time of the multiagent is 1.6 s, therefore, the upper bound evaluated using the settling time function is over-estimated. The plot in Figure 6 shows the position and speed state of each cart starting from different initial conditions. The position of each cart attains the desired formation and then sustains it for all future time. Furthermore, the position state satisfies the output constraints for all time. Figure 7 displays the control input for all agents. The chattering is observed with the presented controller but is of a very small amplitude, and it is inevitable when working with sliding mode control as discussed in Remark 14.

5.2. Example 2 (Time-Varying Formation Tracking Control). In this section, the time-varying formation tracking control is presented for cart-multiagent dynamics of Example-1. Here, the desired formation is time-dependent and given as $F = 0.1\sin(2\pi t) \times [1, -1, 3, -3]^T$ while reference is set to $r = \cos(0.2\pi t) \Rightarrow A_0 = 1$. The perturbation is assumed to be $w_i = \sin(t) \times [0.25, 0.1, 0.5, 0.1]^T$. The controller parameters are set to $\beta = 2; \kappa = 1; \mathscr{W} = 5; \hbar = 3$. From (18), the upper bound on settling time shall be 14.30 s. The results (see Figures 8 and 9) show that there is no degradation in the performance of the proposed control for varying formations. The constraint on the output is also satisfied for all time.

The time-varying reference formation F is kept similar with just bias involved in the provided simulations for clear visualization; nonetheless, the controller performs equivalently well for alternative formations as well. Moreover, the reference r is exactly in the center of the solid-magenta and the dashed-black agent and the multiagent system attains the formation around that reference according to desired F. Figure 9 represents the control effort for the proposed time varying formation controller. From the position plot of Figure 8, it is to be noted that the cart multiagent system is not only tracking the low-frequency reference but also has attained the high-frequency time varying formation.

6. Conclusion

A novel control topology with robustness and finite-time formation tracking for complex multiagent systems, subjected to output constraints, is designed in this paper. It is established using BLF that the proposed controller ensures that the output state never leaves that preassigned bound, provided it starts from that bound. As a result, the output bound during reaching phase of SMC is known and consequently, the finite convergence time is achieved using linear sliding surface. Therefore, no singularity occurs in the proposed formation tracking controller unlike in the existing literature. Contrarily, nonlinear sliding surfaces are utilized in existing literature to achieve finite time stability of the agents that causes the singularity in the control input. The simulations are conducted for cart-multiagent dynamics, and the results show that the proposed decentralized controller performs well for the formation tracking of constrained agents in the presence of unwanted disturbances. Thus the strength of this work lies in providing a unique solution to the formation tracking control of multiagent systems that offers output constraints, robustness, and finite time convergence simultaneously without singularity issues. However, the settling time function exhibits relatively more overestimation and is the weakness of this cooperative control framework. The future directions for this study include exploring the nonsymmetric constraints, predefined settling time, and higher-order multiagent systems. Moreover, a parallel study will be carried out to design a finite-time distributed cooperative control under output constraints.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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