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Research Article

Differential Game Model for a Dual-Channel Supply Chain's Optimal Strategy under the Reference Carbon Emission Effect

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Taking into account the impact of time factors on emission reductions and brand reputation, the reference carbon emission effect and dual-channel supply chain are incorporated into a unified analysis framework. We applied differential game theory to build models under centralized decision-making and decentralized decision-making. The aim of the research is to explore the strategies of a single manufacturer and a single retailer on product pricing, low-carbon production, and advertising. The research analyses the impact of reference carbon emission effects, cost coefficients, and interchannel substitutable coefficients on profits. In order to alleviate the double marginal effect brought about by decentralized decision-making, a cost-compensation coordination mechanism is proposed. The conclusions are as follows. First, centralized decision-making is the optimal decision-making mode, but further consultation is required to implement it voluntarily by both parties. Second, a cost-recovery contract occurs when the fixed fee that the retailer gives the manufacturer meets certain conditions. The contract can make the retailer's advertising investment reach the level of centralized decision-making and improve the member's profit under the decentralized decision-making. The coordination mechanism is effective. Third, the reference carbon emission effect can bring about an increase in the manufacturer's low-carbon production input and profits. The retailer's advertising investment is not affected by the reference carbon emission effect. Fourth, wholesale prices and online or offline retail prices are all positively correlated with the market share of the channel. The price-substitution coefficient between channels is positively correlated with both low-carbon inputs and profits.

1. Introduction

Environmental problems such as pollution and climate change are becoming increasingly severe and affecting people's lives. To meet the challenges brought by pollution, countries around the world are participating in various energy-saving and emission-reduction activities. China, for example, proposed a "dual carbon goal," to achieve the carbon peak by 2030 and carbon neutrality by 2060. This reflects the urgent need in China to find ways to save energy and reduce emissions to achieve a low-carbon supply chain. Meanwhile, with the rise of e-commerce, enterprises now participate in a dual-channel model comprising online sales and traditional offline sales [1]. This "online + offline" model has altered previous sales patterns, creating both opportunities and challenges for enterprises. The rising popularity of online sales among consumers has seriously affected the sales of physical stores. More recently, the spread of COVID-19 has brought about a wave of store closures. Data from China's National Bureau of Statistics show that online sales are climbing year by year, reaching 11.76 trillion yuan in 2020, that is, a 10.9% increase from the previous year, with physical goods accounting for 83.0%. In 2021, online sales increased to 13.1 trillion yuan, with a 14.1% increase from the previous year. Meanwhile, according to Zhiyan Consulting, the total volume of express deliveries in China reached 83.36 billion pieces in 2020, that is, a 31.2% from the previous year. Business volume in the first half of 2021 alone reached 49. 39 billion pieces, 80% of which came from e-commerce platforms. Against this background, a major practical problem that has emerged concerns is how to reduce emissions, coordinate upstream and downstream enterprises in dual-channel supply chains, and successfully transit from a non-low-carbon supply chain to a low-carbon one.

The literature review mainly covers the irrational behavior of dual-channel supply chain members, consumer preferences, emission-reduction decisions, and coordination mechanisms. It is divided into two aspects according to the specific research content. The first aspect is the study of low-carbon strategy of the dual-channel supply chain and the second aspect is the study of coordination mechanism of the dual-channel supply chain.

Research on low-carbon strategies in dual-channel supply chains can be divided into three main areas. First, from the perspective of dual-channel supply chain irrational members, studies have investigated the effects of equity concerns [2, 3], altruistic behavior [4, 5], and risk aversion [6] on emission-reduction strategies. Xue and Wang [7] simultaneously considered the impact of fairness concerns and risk avoidance on supply chain decisions. They calculated production and order quantities and further proposed revenue sharing and repurchase contracts. Second, from a consumer perspective, studies have investigated the effects of channel preferences [8] and low-carbon preferences [9] on the emission-reduction strategies of dual-channel supply chain members. Considering the stochastic demand for carbon emissions, Ghosh et al. [10] analyzed a two-level, dual-channel supply chain model under government regulation and consumers' low-carbon preferences. Usually, research on loss avoidance is conducted from the perspective of supply chain members. Zhang et al. [11] conducted research on consumer's loss avoidance behavior. In the research, it was found that the price differences between different channels can affect the purchasing behavior of loss-averse consumers. The third research perspective focuses on the effects of carbon tax policies [12] and subsidy policies [13, 14] on supply chain members' low-carbon strategies. Cao et al. [15] studied the optimal production in a dual-channel supply chain under remanufacturing subsidy policy and carbon tax policy.

In terms of consumers' low-carbon preferences, few studies have investigated the effects of reference carbon emission on supply chain emissions. Moreover, most studies have focused on traditional offline supply chains. By introducing low-carbon preferences into their model, Liang and Futou [16] considered the effect of low-carbon preference on emission reduction in supply chains. Low-carbon economy has brought about a change in consumption tendency. The psychological satisfaction brought by low-carbon consumption has become a new consumption orientation. Cheng et al. [17] also confirmed the existence of consumers' low-carbon preference and its impact on the decision-making of the supply chain. Liu et al. [18] found that both consumers' low-carbon preferences and governmental subsidies have an impact on firms' production decisions.

Meanwhile, research on supply chain coordination has mainly focused on cost-sharing and revenue-sharing contracts. By using cost-sharing and two-stage pricing contracts for supply chain coordination, Yang and Yao [19] considered the equilibrium decision problem for a two-level agricultural supply chain under carbon quota trading policies, consumer preferences for fresh agricultural products, and low-carbon preferences. Xu et al. [20] suggested that one-way cost-sharing contracts can achieve Pareto improvement and that two-way cost-sharing contracts can coordinate manufacturers and distributors. Parisa and Nasiri proposed discount and revenue-sharing contracts when studying pricing strategies in retail and chain stores [21].

Existing research on dual-channel, low-carbon supply chains has produced some useful findings. It should be noted that the existing literature mostly explores the problem of operational decision-making of dual-channel supply chains from a static perspective. The emission-reduction behavior of an enterprise is a long-term process, which makes it necessary to analyze the strategies of supply chain members from a dynamic perspective. A differential game is a game model that can determine the competition or cooperation decision-making of participants from a dynamic perspective. Studying the strategies of dual-channel supply chain members using differential games is therefore in line with practical needs. At the same time, environmental problems have become acute. Protecting the environment and producing low-carbon products have become the way for countries to solve environmental problems [22, 23]. Therefore, under the current situation in which China wants to achieve carbon peaking and carbon neutrality goals, how to realize the transformation of a non-low-carbon supply chain to a low-carbon supply chain by designing effective emission-reduction strategies has become a new topic in the field of supply chain research. In the context of the era of deepening the concept of sustainable development, the preference for low-carbon consumption will continue to strengthen. Consumers' final choice is the basis of enterprise decision-making. The production of low-carbon products by enterprises is a prerequisite for low-carbon consumption. In the face of the potential benefits brought by consumers' low-carbon preferences, enterprises should incorporate low-carbon preferences into their own business decisions. There are few studies in the literature on the reference carbon emissions on low-carbon supply chains. Based on consumers' low-carbon preference, this study considers the influence of reference carbon emissions on the decision-making of dual-channel supply chain and calculates their strategies for emission reduction, pricing, and promotion for both offline and online channels. It also designs contractual optimization schemes to enrich theories of dual-channel, low-carbon supply chain management and makes suggestions for corporate emission-reduction strategies.

2. Model Description

We consider a dual-channel, low-carbon supply chain consisting of a manufacturer (m), a retailer (r), and consumers. The manufacturer determines the product's low-carbon production effort level (E), online sales price (p_e), and wholesale price (w); the retailer determines the offline sales price (p_r) and advertising effort level (A). The manufacturer has both online and offline sales' channels. In traditional offline sales, the manufacturer first sells the product wholesale to the retailer, who then sells it to the consumer. In online sales, the manufacturer sells the product directly to the consumer. This study mainly uses differential games to analyze the problem. On the basis of theoretical analysis, parameters are selected to construct the objective function of the participants. The main parameters are the reference carbon emission level, the low-carbon production effort level, the promotion effort level, and the emission reduction. The specific emission process and mode of carbon emission are not studied. The model construction does not involve variables such as network constraints.

Hypothesis 1. The manufacturer's abatement effort will bring about an increase in abatement. Meanwhile, there is a natural decay rate of abatement because of technology and other factors. The differential equation for product abatement is [24]

$$\dot{\chi}(t) = \alpha E(t) - \delta\chi(t); \chi(0) = \chi_0, \tag{1}$$

where χ_0 is the initial abatement; E is the manufacturer's level of effort to reduce emissions; α is the influence of the manufacturer's abatement effort level on the abatement amount and $\alpha > 0$; and δ is the natural decay rate of the abatement amount and $\delta > 0$.

Hypothesis 2. The unit carbon emissions of a product are clearly marked. Consumers are environmentally aware and will choose whether to purchase a product based on its emissions. Consumers will evaluate the product's carbon emissions based on its actual emissions. And consumers will compare the carbon emissions of the newly purchased product with the carbon emission level of the previously purchased product. For this, we refer to equation (2) [25], where $\theta > 0$, representing the consumer memory parameter. The larger θ is, the shorter the memory of the previous purchase experience is. χ is the actual emission reduction, and R is the carbon emission preference. When the actual emission reduction is greater than the carbon emission preference, the consumer senses a "gain"; otherwise, the consumer senses a "loss."

$$\dot{R}(t) = \theta(\chi(t) - R(t)); R(0) = R_0. \tag{2}$$

Hypothesis 3. Assume that the brand goodwill is positively affected by manufacturers' emission-reduction efforts and retailers' low-carbon advertising efforts, and that there is a natural decay of goodwill over time [24]. Goodwill is depicted as

$$\dot{G}(t) = aE(t) + bA(t) - \psi G(t); G(0) = G_0, \tag{3}$$

where a and b are the coefficients of the effects of the manufacturer's emission-reduction efforts and the retailer's advertising on goodwill, respectively, and ψ is the natural decay rate of goodwill, and $0 < \psi < 1$.

Hypothesis 4. Both price and nonprice factors affect quantity demand [26, 27]. Specifically, online and offline channels are competitive and are influenced by their own channel prices and each other's channel prices. Emission reduction and the level of advertising effort will both lead to an increase in demand. The product demand function is as follows:

$$\begin{aligned} D_e(t) &= [(1-s)D_0 - p_e(t) + \mu p_r(t)][\beta\chi(t) + \lambda(\chi(t) - R(t)) + \nu G(t)], \\ D_r(t) &= (sD_0 - p_r(t) + \mu p_e(t))[\beta\chi(t) + \lambda(\chi(t) - R(t)) + \nu G(t)], \end{aligned} \tag{4}$$

where s is the consumer loyalty to offline channels and $0 < s < 1$; D_0 is the potential product demand; μ is the price-substitution coefficient between channels and $0 < \mu < 1$; $\beta > 0$ is the coefficient of emission reduction on demand; λ is the coefficient of carbon emission preference effect on demand and $\lambda > 0$; and ν is the coefficient of goodwill on demand and $\nu > 0$.

Hypothesis 5. The manufacturer's emission reduction and the retailer's advertising and promotion costs have convex characteristics. The following is the setting for low-carbon input and advertising and promotion costs: f_m and f_r are coefficients of the manufacturer's and retailer's costs, and the cost of effort for both sides is

$$C(E(t)) = \frac{f_m}{2}E^2(t); C(A(t)) = \frac{f_r}{2}A^2(t). \tag{5}$$

Hypothesis 6. Both parties make decisions within an infinite time horizon, with a discount factor of ρ ($\rho > 0$). For simplicity, t is omitted here. The framework diagram is shown in Figure 1.

3. Model Construction

3.1. Centralized Decision-Making. Under centralized decision-making, the manufacturer and retailer cooperate with the objective of maximizing overall supply chain profits. Centralized decision-making is an ideal model for

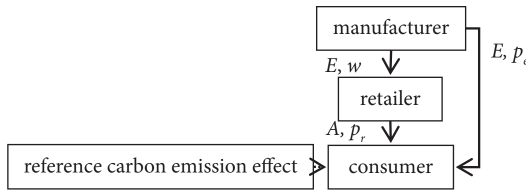

FIGURE 1: Structure diagram of a dual-channel supply chain under the reference carbon emission effect.

decision-making. The application of centralized decision-making in practice is further elaborated in Proposition 13. The objective function is as follows:

$$\max_{p_e,p_r,E,A} J_k^U = \int_0^\infty e^{-\rho t}\left\{p_e[(1-s)D_0 - p_e + \mu p_r][\beta\chi + \lambda(\chi - R) + \nu G] + p_r(sD_0 - p_r + \mu p_e)[\beta\chi + \lambda(\chi - R) + \nu G] - \frac{f_m}{2}E^2 - \frac{f_r}{2}A^2\right\}\text{dt}. \tag{6}$$

Theorem 7. *The equilibrium results of the centralized decision-making are as follows.*

(i) The optimal equilibrium strategies of manufacturers and retailers are represented as follows:

$$p_e^{U*} = \frac{(1-s+\mu s)D_0}{2(1-\mu^2)};\ p_r^{U*} = \frac{[s+\mu(1-s)]D_0}{2(1-\mu^2)};\ E^{U*} = \frac{D_0^2\triangle_1}{f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right);\ A^{U*} = \frac{D_0^2 vb\triangle_1}{f_r(\rho+\psi)}. \tag{7}$$

(ii) The optimal trajectory of emission reduction, carbon emissions reference, and goodwill is expressed as

$$\begin{aligned}
\chi^{U*} &= \chi_{RSS}^U + \left(\chi_0 - \chi_{RSS}^U\right)e^{-\delta t};\\
R^{U*} &= R_{RSS}^U + \frac{\theta}{\theta-\delta}\left(\chi_0 - \chi_{RSS}^U\right)e^{-\delta t} + \left[R_0 - R_{RSS}^U - \frac{\theta}{\theta-\delta}\left(\chi_0 - \chi_{RSS}^U\right)\right]e^{-\theta t};\\
G^{U*} &= G_{RSS}^U + \left(G_0 - G_{RSS}^U\right)e^{-\psi t},
\end{aligned} \tag{8}$$

where

$$\chi_{RSS}^U = \frac{\alpha D_0^2\triangle_1}{\delta f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right), R_{RSS}^U = \frac{\alpha D_0^2\triangle_1}{\delta f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right), G_{RSS}^U = \frac{aD_0^2\triangle_1}{\psi f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right) + \frac{b^2 D_0^2 v\triangle_1}{\psi f_r(\rho+\psi)}. \tag{9}$$

(iii) The optimal value of the profit of the dual-channel supply chain is

$$J_k^{U*} = e^{-\rho t}V_k^{U*}(\chi, R, G), \tag{10}$$

where

$$\begin{aligned}
V_k^{U*} &= D_0^2\triangle_1\triangle_2\chi^{U*} - \frac{\lambda D_0^2\triangle_1}{\rho+\theta}R^{U*} + \frac{\nu D_0^2\triangle_1}{\rho+\psi}G^{U*} + \frac{D_0^4\triangle_1^2}{2\rho f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right)^2 + \frac{D_0^4 v^2 b^2\triangle_1^2}{2\rho f_r(\rho+\psi)^2},\\
\triangle_1 &= \frac{(1-s)^2 + s^2 + 2\mu s(1-s)}{4(1-\mu^2)}, \triangle_2 = \frac{\beta\rho + \beta\theta + \lambda\rho}{(\rho+\theta)(\rho+\delta)}.
\end{aligned} \tag{11}$$

Proof. At any moment t, the HJB equation for the dual-channel supply chain equilibrium strategy is

$$\rho V_k^U = \max_{p_e,p_r,E,A}\Big\{p_e[(1-s)D_0-p_e+\mu p_r][\beta\chi+\lambda(\chi-R)+\nu G]+p_r(sD_0-p_r+\mu p_e)[\beta\chi + \lambda(\chi-R)+\nu G]-\frac{f_m}{2}E^2-\frac{f_r}{2}A^2+V_{k\chi}^{U'}(\alpha E-\delta\chi)+V_{kR}^{U'}(\theta\chi-\theta R)+V_{kG}^{U'}(aE+bA-\psi G)\Big\}. \tag{12}$$

We then find the first-order partial derivatives of p_e, p_r, and A in equation (12) and make them equal to zero, as represented in the following equation:

$$p_e=\frac{(1-s)D_0+2\mu p_r}{2};\ p_r=\frac{sD_0+2\mu p_e}{2};\ E=\frac{\alpha V_{k\chi}^{U'}+aV_{kG}^{U'}}{f_m};\ A=\frac{bV_{kG}^{U'}}{f_r}, \tag{13}$$

where p_e and p_r are obtained as

$$p_e^{U*}=\frac{(1-s+\mu s)D_0}{2(1-\mu^2)};\ p_r^{U*}=\frac{[s+\mu(1-s)]D_0}{2(1-\mu^2)}. \tag{14}$$

Then, we substitute equations (13) and (14) into equation (12) and collate the results to obtain

$$\rho V_k^U=\left\{\frac{\left[(1-s)^2+s^2+2\mu s(1-s)\right](\beta+\lambda)D_0^2}{4(1-\mu^2)}-\delta V_{k\chi}^{U'}+\theta V_{kR}^{U'}\right\}\chi-\left\{\frac{\left[(1-s)^2+s^2+2\mu s(1-s)\right]\lambda D_0^2}{4(1-\mu^2)}+\theta V_{kR}^{U'}\right\}R+\left\{\frac{\left[(1-s)^2+s^2+2\mu s(1-s)\right]\nu D_0^2}{4(1-\mu^2)}-\psi V_{kG}^{U'}\right\}G+\frac{\left(\alpha V_{k\chi}^{U'}+aV_{kG}^{U'}\right)^2}{2f_m}+\frac{b^2\left(V_{kG}^{U'}\right)^2}{2f_r}. \tag{15}$$

Let $V_k^U(\chi, R, G)=a_1\chi+a_2R+a_3G+a_4$, which is the solution of the HJB equation, where a_1, a_2, a_3, and a_4 are constants.

$$a_1=D_0^2\triangle_1\triangle_2;\ a_2=-\frac{\lambda D_0^2\triangle_1}{\rho+\theta};\ a_3=\frac{\nu D_0^2\triangle_1}{\rho+\psi};\ a_4=\frac{D_0^4\triangle_1^2}{2\rho f_m}\left(\alpha\triangle_2+\frac{\nu a}{\rho+\psi}\right)^2+\frac{\nu^2b^2D_0^4\triangle_1^2}{2\rho f_r(\rho+\psi)^2}. \tag{16}$$

We then substitute equation (16) into equation (13) to find the equilibrium strategies of the manufacturer's emission reduction and the retailer's advertising under centralized decision-making, as in equation (7). Then, we substitute the optimal strategy (7) into equations (1)–(3) to obtain the optimal trajectory of emission reduction, carbon emission preference, and goodwill, as shown in equation (8). Finally, the total system profit is obtained by substituting equation (16) into $V_k^U(\chi, R, G)=a_1\chi+a_2R+a_3G+a_4$, and then substituting $V_k^{U*}(\chi, R, G)$ into $j_k^U(p_e, p_r, E, A)=e^{-\rho t}V_k^U(\chi, R, G)$. □

3.2. Decentralized Decision-Making. Under decentralized decision-making, the manufacturer decides the level of abatement effort E, the online channel retail price p_e, and the wholesale price w, first; subsequently, the retailer decides the offline channel retail price p_r and the advertising and promotion input A. For example, Haier and Gome, Walmart, and Procter and Gamble, as upstream and downstream companies in the supply chain, started out with decentralized decision-making. They aimed at maximizing their respective interests. At this point, the manufacturer and retailer's objective functions are

$$\max_{p_e,E} J_m^L = \int_0^\infty e^{-\rho t}\left(p_e(t)D_e(t) + w(t)D_r(t)\right) - C(E(t))\mathrm{d}t;$$
$$\max_{p_r,A} J_r^L = \int_0^\infty e^{-\rho t}\left(p_r(t)D_r(t) - w(t)D_r(t) - C(A(t))\right)\mathrm{d}t. \tag{17}$$

Theorem 8. *The equilibrium results of the decentralized decision-making are as follows.*

(i) The optimal strategies for manufacturers and retailers are expressed as

$$p_e^{L*} = \frac{(1-s+\mu s)D_0}{2\left(1-\mu^2\right)}; w^{L*} = \frac{[s+\mu(1-s)]D_0}{2\left(1-\mu^2\right)}; p_r^{L*} = \frac{\left[s\left(3-\mu^2\right)+2\mu(1-s)\right]D_0}{4\left(1-\mu^2\right)},$$
$$E^{L*} = \frac{D_0^2\triangle_3}{f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right); A^{L*} = \frac{D_0^2 s^2 vb}{16 f_r(\rho+\psi)}, \tag{18}$$

where $\triangle_2 = (\beta\rho + \beta\theta + \lambda\rho/(\rho+\theta)(\rho+\delta)), \triangle_3 = (2(1-s)(1-s+2\mu s) + s^2(1+\mu^2)/8(1-\mu^2))$.

(ii) The optimal trajectory of emission reduction, carbon emissions reference, and goodwill is as follows:

$$\chi^{L*} = \chi_{RSS}^L + \left(\chi_0 - \chi_{RSS}^L\right)e^{-\delta t};$$
$$R^{L*} = R_{RSS}^L + \frac{\theta}{\theta-\delta}\left(\chi_0 - \chi_{RSS}^L\right)e^{-\delta t} + \left[R_0 - R_{RSS}^L - \frac{\theta}{\theta-\delta}\left(\chi_0 - \chi_{RSS}^L\right)\right]e^{-\theta t};$$
$$G^{L*} = G_{RSS}^L + \left(G_0 - G_{RSS}^L\right)e^{-\psi t}, \tag{19}$$

where $\chi_{RSS}^L = (\alpha D_0^2\triangle_3/\delta f_m)\quad(\alpha\triangle_2 + (va/\rho+\psi))$, $R_{RSS}^L = (\alpha D_0^2\triangle_3/\delta f_m)(\alpha\triangle_2 + (va/\rho+\psi))$, and $G_{RSS}^L = (aD_0^2\triangle_3/\psi f_m)(\alpha\triangle_2 + (va/\rho+\psi)) + (b^2D_0^2s^2v/16\psi f_r(\rho+\psi))$.

(iii) The optimal value of the profit of both parties and the dual-channel supply chain as a whole is

$$J_m^{L*} = e^{-\rho t}V_m^{L*}(\chi,R,G); J_r^{L*} = e^{-\rho t}V_r^{L*}(\chi,R,G); J_k^{L*} = e^{-\rho t}\left(V_m^{L*}(\chi,R,G) + V_r^{L*}(\chi,R,G)\right), \tag{20}$$

where

$$V_m^{L*} = D_0^2\triangle_2\triangle_3\chi^{L*} - \frac{D_0^2\lambda\triangle_3}{\rho+\theta}R^{L*} + \frac{D_0^2 v\triangle_3}{\rho+\psi}G^{L*} + \frac{D_0^4\triangle_3^2}{2\rho f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right)^2 + \frac{D_0^4 s^2 v^2 b^2\triangle_3}{16\rho f_r(\rho+\psi)^2},$$
$$V_r^{L*} = \frac{D_0^2 s^2\triangle_2}{16}\chi^{L*} - \frac{D_0^2 s^2\lambda}{16(\rho+\theta)}R^{L*} + \frac{D_0^2 s^2 v}{16(\rho+\psi)}G^{L*} + \frac{D_0^4 s^2\triangle_3}{16\rho f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right)^2 + \frac{D_0^4 s^4 v^2 b^2}{512\rho f_r(\rho+\psi)^2}. \tag{21}$$

Proof. We now discuss the retailer strategy. We suppose that there is a differential function $V_m^L(\chi, R, G)$, then, $\chi \geq 0$, $R \geq 0$, and $G \geq 0$ satisfy the HJB equation represented as

$$\rho V_r^L = \max_{p_r, A}\{(p_r - w)(sD_0 - p_r + \mu p_e)[\beta\chi + \lambda(\chi - R) + \nu G] - \frac{f_r}{2}A^2 + V_{r\chi}^{L'}(\alpha E - \delta\chi) + V_{rR}^{L'}(\theta\chi - \theta R) + V_{rG}^{L'}(aE + bA - \psi G)]. \tag{22}$$

We take the derivative of p_r and A in equation (22) to obtain

$$p_r = \frac{sD_0 + \mu p_e + w}{2}; A = \frac{bV_{rG}^{L'}}{f_r}. \tag{23}$$

Similarly, we solve the manufacturer's emission-reduction decision. It is assumed that there is a differential function $V_m^L(\chi, R, G)$. All χ, R, and G in $V_m^L(\chi, R, G)$ satisfy the HJB equation, and $\chi \geq 0$, $R \geq 0$, and $G \geq 0$.

$$\rho V_m^L = \max_{p_e, w, E}\{p_e[(1-s)D_0 - p_e + \mu p_r][\beta\chi + \lambda(\chi - R) + \nu G] + w(sD_0 - p_r + \mu p_e)[\beta\chi + \lambda(\chi - R) + \nu G] - \frac{f_m}{2}E^2 + V_{m\chi}^{L'}(\alpha E - \delta\chi) + V_{mR}^{L'}(\theta\chi - \theta R) + V_{mG}^{L'}(aE + bA - \psi G)]. \tag{24}$$

We substitute equation (23) into equation (24) and derive the derivatives for p_e and w to obtain

$$p_e^{L*} = \frac{(1 - s + \mu s)D_0}{2(1-\mu^2)}; w^{L*} = \frac{[s + \mu(1-s)]D_0}{2(1-\mu^2)}. \tag{25}$$

Similarly, we substitute equation (25) into equation (23) to obtain

$$p_r^{L*} = \frac{[s(3-\mu^2) + 2\mu(1-s)]D_0}{4(1-\mu^2)}. \tag{26}$$

We then find the first-order derivative of the manufacturer's abatement effort E for equation (24) as

$$E = \frac{\alpha V_{m\chi}^{L'} + aV_{mG}^{L'}}{f_m}. \tag{27}$$

We substitute p_e, p_r, w, E, and A into equations (24) and (22) and obtain the following:

$$\rho V_m^L = \left[D_0^2(\beta + \lambda)\triangle_3 - \delta V_{m\chi}^{L'} + \theta V_{mR}^{L'}\right]\chi - \left(D_0^2\lambda\triangle_3 + \theta V_{mR}^{L'}\right)R + \left(D_0^2\nu\triangle_3 - \psi V_{mG}^{L'}\right)G + \frac{\left(\alpha V_{m\chi}^{L'} + aV_{mG}^{L'}\right)^2}{2f_m} + \frac{b^2 V_{mG}^{L'}V_{rG}^{L'}}{f_r}; \rho V_r^L = \left[\frac{s^2D_0^2(\beta + \lambda)}{16} - \delta V_{r\chi}^{L'} + \theta V_{rR}^{L'}\right]\chi - \left(\frac{s^2D_0^2\lambda}{16} + \theta V_{rR}^{L'}\right)R + \left(\frac{s^2D_0^2\nu}{16} - \psi V_{rG}^{L'}\right)G + \frac{\left(\alpha V_{m\chi}^{L'} + aV_{mG}^{L'}\right)\left(\alpha V_{r\chi}^{L'} + aV_{rG}^{L'}\right)}{f_m} + \frac{b^2\left(V_{rG}^{L'}\right)^2}{2f_r}. \tag{28}$$

Let $V_m^L(\chi, R, G) = b_1\chi + b_2R + b_3G + b_4$ and $V_r^L(\chi, R, G) = c_1\chi + c_2R + c_3G + c_4$ be the solutions of HJB equation, where b_1, b_2, b_3, b_4, c_1, c_2, c_3, and c_4 are constants.

$$
\begin{aligned}
&b_1 = D_0^2\triangle_2\triangle_3; b_2 = -\frac{D_0^2\lambda\triangle_3}{\rho+\theta}; b_3 = \frac{D_0^2\nu\triangle_3}{\rho+\psi}; b_4 = \frac{D_0^4\triangle_3^2}{2\rho f_m}\left(\alpha\triangle_2 + \frac{\nu a}{\rho+\psi}\right)^2 + \frac{D_0^4 s^2\nu^2 b^2\triangle_3}{16\rho f_r(\rho+\psi)^2};\\
&c_1 = \frac{s^2D_0^2\triangle_2}{16}; c_2 = -\frac{s^2D_0^2\lambda}{16(\rho+\theta)}; c_3 = \frac{s^2D_0^2\nu}{16(\rho+\psi)}; c_4 = \frac{s^2D_0^4\triangle_3}{16\rho f_m}\left(\alpha\triangle_2 + \frac{\nu a}{\rho+\psi}\right)^2 + \frac{s^4D_0^4\nu^2b^2}{512\rho f_r(\rho+\psi)^2}.
\end{aligned} \tag{29}
$$

According to equation (29), the equilibrium strategies of manufacturers and retailers under decentralized decision-making can be further obtained, such as equations (18)–(20). □

Proposition 9

(i) The retail price of an online or offline channel positively correlates with the market share held by that channel

(ii) The manufacturer's offline channel wholesale price is positively influenced by consumer loyalty to the offline channel (i.e., the higher the consumer's offline loyalty, the higher the wholesale price)

(iii) An increase in both the interchannel price-substitution coefficient and the reference carbon emission effect on the demand coefficient can enhance the manufacturer's emission reduction

(iv) An increase in the interchannel price-substitution coefficient enhances the level of the retailer's advertising effort

Proof. Observing equations (7) and (18), $p_e^{U*} = p_e^{L*} = [(1-s+\mu s)\ D_0]/[2\ (1-\mu^2)]$, $p_r^{U*} = [s+\mu\ (1-s)]D_0/[2(1-\mu^2)]$, $p_r^{L*} = [s(3-\mu^2) + 2\mu(1-s)]\ D_0/[4(1-\mu^2)]$. We find the first-order derivative of $(1-s)$ for p_e^{U*} and p_e^{L*}. Then, we find the first-order derivative of s for p_r^{U*} and p_r^{L*} as

$$
\begin{aligned}
&\frac{d(p_e^{U*})}{d(1-s)} = \frac{d(p_e^{L*})}{d(1-s)} = \frac{D_0}{2(1+\mu)} > 0; \frac{d(p_r^{U*})}{ds} = \frac{D_0}{2(1+\mu)} > 0;\\
&\frac{d(p_r^{L*})}{ds} = \frac{D_0(3-\mu^2-2\mu)}{4(1-\mu^2)}, 0<\mu<1,\ 0<\mu^2<1,\\
&0<2\mu<2,\ 0<\mu^2+2\mu<3, 0<3-(\mu^2+2\mu)<3, \frac{d(p_r^{L*})}{ds} = \frac{D_0}{4(1-\mu^2)}\cdot(3-\mu^2-2\mu) > 0.
\end{aligned} \tag{30}
$$

According to equation (30), the first-order derivatives of both online and offline channel prices with respect to their market share are greater than 0. Online and offline retail prices are each positively correlated with their market share. Thus, the proof of (i) in Proposition 9 is complete. By continuing to observe equations (7) and (18), we find the first-order derivative of s for w^L; the first-order derivatives of μ and λ for E^{U*}; the first-order derivatives of μ and λ for E^{L*}; and the first-order derivatives of μ for A^{U*} and A^{L*}, respectively. Thus, (ii)–(iv) in Proposition 9 can be proved.

First, according to Proposition 9, the higher the consumers' acceptance of a channel, the higher the wholesale and retail prices on that channel. When a product has a large presence in a channel, the merchant will increase the wholesale and retail prices to increase revenue.

Second, the larger the price coefficient between channels, the smaller the difference between channels. In this case, manufacturers will attract consumers by increasing emission-reduction investment and retailers will attract consumers by increasing advertising investment (i.e., the larger the substitution coefficient between the channels, the more it will promote the effort levels of manufacturers and retailers). When the difference between online and offline channels is small, manufacturers and retailers can increase their revenues by raising their low-carbon inputs. Manufacturers increase their level of emission reductions and retailers increase their low-carbon promotion efforts.

Third, when the current emission reduction of a product is higher than the carbon emission preference, consumers will favor the product more, which in turn leads to higher demand and increased revenue, which further motivates manufacturers to reduce emissions. Therefore, manufacturers and retailers can further use advertising and other means to promote consumers' low-carbon consumption behavior and increase revenues. When consumers have low-carbon preferences, low-carbon products can stimulate consumers' desire to buy and thus increase product sales. Therefore, merchants should strive to improve the low-carbon level of their products to meet the low-carbon demand of consumers, so as to realize their own profits (according to equations (7) and (18)). □

Proposition 10

(i) The relationship between online and offline retail prices: $p_e^{U*} = p_e^{L*}$, $p_r^{L*} > p_r^{U*}$

(ii) Manufacturers' level of effort to reduce emissions and retailer's level of effort to promote low-carbon: $E^{U*} > E^{L*}$, $A^{U*} > A^{L*}$

(iii) Emission reductions, carbon emissions reference, and goodwill: $\chi^{U*} > \chi^{L*}$, $R^{U*} > R^{L*}$, and $G^{U*} > G^{L*}$

(iv) Optimal revenue of the dual-channel supply chain system: $J_k^{U*} > (J_m^{L*} + J_r^{L*})$

Proof. By observing equations (7) and (18), it is clear that $p_e^{U*} = p_e^{L*}$.

$$p_r^{L*} - p_r^{U*} = \frac{\left[s\left(3-\mu^2\right) + 2\mu(1-s)\right]D_0}{4\left(1-\mu^2\right)} - \frac{[s+\mu(1-s)]D_0}{2\left(1-\mu^2\right)} = \frac{SD_0}{4} > 0, \tag{31}$$

where $p_r^{L*} > p_r^{U*}$. Thus, (i) in Proposition 10 is proved, and (ii)–(iv) in Proposition 10 are obtained in this way.

Proposition 10 suggests that the online retail price is the same under both decisions, and the offline retail price is higher under decentralized decision-making. Emission reduction, carbon emission preference, goodwill, emission-reduction effort, and overall supply chain benefits are higher under centralized than decentralized decision-making. For example, Haier and Gome did not cooperate in the early days. In the continuing competition, Haier's market share was affected and gradually launched cooperation with Gome. Eventually, both parties realized a win-win situation. Decentralized decision-making leads to double marginal effects. To mitigate this situation, further coordination mechanisms need to be designed to promote supply chain coordination and enhance overall benefits. □

4. Coordinating Mechanism

The contract has two phases. First, to incentivize the retailer's low-carbon promotion, the manufacturer shares a portion of the advertising costs as η $(0 \le \eta \le 1)$. Second, the retailer pays the manufacturer a fixed compensation fee h. This cost-compensation contract redistributes the profits so that both the manufacturer and the retailer are incentivized. The profits of the manufacturer and the retailer under the cost-compensation contract H (η) are as follows:

$$\begin{aligned}
\max_{p_e,w,E} J_m^B &= \int_0^\infty e^{-\rho t}\left(p_e(t)D_e(t) + w(t)D_r(t)\right) - C(E(t)) - \eta C(A(t)) + h)\mathrm{d}t;\\
\max_{p_r,A} J_r^B &= \int_0^\infty e^{-\rho t}\left(p_r(t)D_r(t) - w(t)D_r(t) - (1-\eta)C(A(t)) - h\right)\mathrm{d}t.
\end{aligned} \tag{32}$$

Proposition 11. *Supply chain coordination is achieved when the manufacturer's cost-sharing rate to retailers satisfies equation (33). The manufacturer's share of low-carbon promotional costs for retailers is negatively correlated with the offline channel's market share.*

$$\eta^{B*} = 1 - \frac{s^2}{16\Delta_1}, \tag{33}$$

where $\triangle_1 = \left((1-s)^2 + s^2 + 2\mu s(1-s)/4(1-\mu^2)\right)$.

Proof. First, for the retailer's decision problem, we assume that there is a differential function V_r^B (χ, R, G) that satisfies the HJB equation for all $\chi \ge 0$, $R \ge 0$, and $G \ge 0$.

$$\begin{aligned}
\rho V_r^B = \max_{p_r,A} \{&(p_r - w)(sD_0 - p_r + \mu p_e)[\beta\chi + \lambda(\chi - R) + \nu G] - (1-\eta)\frac{f_r}{2}A^2 - h\\
&+ V_{r\chi}^{B'}(\alpha E - \delta\chi) + V_{rR}^{B'}(\theta\chi - \theta R) + V_{rG}^{B'}(aE + bA - \psi G)].
\end{aligned} \tag{34}$$

We then find the first-order derivatives of p_r and A for equation (34) and make them zero to obtain the following equation:

$$p_r = \frac{sD_0 + \mu p_e + w}{2}; A = \frac{bV_{rG}^{B'}}{(1-\eta) f_r}. \tag{35}$$

Then, we solve it for the manufacturer's decision problem. The function construction method is similar to the equation for retailers.

$$\begin{aligned} \rho V_m^B = \max_{p_e, w, E} \{ & p_e [(1-s)D_0 - p_e + \mu p_r] [\beta\chi + \lambda(\chi - R) + \nu G] + w(sD_0 - p_r + \mu p_e)[\beta\chi + \lambda(\chi - R) + \nu G] \\ & - \frac{f_m}{2} E^2 - \frac{\eta f_r}{2} A^2 + h + V_{m\chi}^{B'}(\alpha E - \delta\chi) + V_{mR}^{B'}(\theta\chi - \theta R) + V_{mG}^{B'}(aE + bA - \psi G)]. \end{aligned} \tag{36}$$

We substitute equation (35) into equation (36) and derive the derivative for p_e and w to get

$$p_e^{B*} = \frac{(1-s+\mu s)D_0}{2(1-\mu^2)}; w^{B*} = \frac{[s+\mu(1-s)]D_0}{2(1-\mu^2)}. \tag{37}$$

We also substitute equation (37) into equation (35) to obtain p_r as

$$p_r^{B*} = \frac{[s(3-\mu^2) + 2\mu(1-s)]D_0}{4(1-\mu^2)}. \tag{38}$$

Then, we find the first-order derivative of the manufacturer's abatement effort E for equation (36) as

$$E = \frac{\alpha V_{m\chi}^{B'} + a V_{mG}^{B'}}{f_m}. \tag{39}$$

By substituting p_e, p_r, and A into equations (36) and (34), we get

$$\begin{aligned} \rho V_m^B = & \left[D_0^2 (\beta + \lambda)\triangle_3 - \delta V_{m\chi}^{L'} + \theta V_{mR}^{L'} \right]\chi - \left(D_0^2 \lambda \triangle_3 + \theta V_{mR}^{L'} \right) R + \left(D_0^2 \nu \triangle_3 - \psi V_{mG}^{L'} \right) G \\ & + \frac{\left(\alpha V_{m\chi}^{B'} + a V_{mG}^{B'}\right)^2}{2 f_m} - \frac{b^2 \left(V_{rG}^{B'}\right)^2 \eta}{2 f_r (1-\eta)} + \frac{b^2 V_{mG}^{B'} V_{rG}^{B'}}{f_r (1-\eta)} + h; \rho V_r^B = \left[\frac{s^2 D_0^2 (\beta + \lambda)}{16} - \delta V_{r\chi}^{B'} + \theta V_{rR}^{B'} \right] \chi \\ & - \left(\frac{s^2 D_0^2 \lambda}{16} + \theta V_{rR}^{B'} \right) R + \left(\frac{s^2 D_0^2 \nu}{16} - \psi V_{rG}^{B'} \right) G + \frac{\left(\alpha V_{m\chi}^{B'} + a V_{mG}^{B'}\right)\left(\alpha V_{r\chi}^{B'} + a V_{rG}^{B'}\right)}{f_m} + \frac{b^2 \left(V_{rG}^{B'}\right)^2}{2 f_r (1-\eta)} - h. \end{aligned} \tag{40}$$

From equation (40), let $V_m^B(\chi, R, G) = d_1\chi + d_2 R + d_3 G + d_4$ and $V_r^B(\chi, R, G) = g_1\chi + g_2 R + g_3 G + g_4$ be the solutions of the HJB equation, where d_1, d_2, and d_3 are constants, to find

$$\begin{aligned} d_1 = & D_0^2 \triangle_2 \triangle_3; d_2 = -\frac{D_0^2 \lambda \triangle_3}{\rho + \theta}; d_3 = \frac{D_0^2 \nu \triangle_3}{\rho + \psi}; d_4 = \frac{D_0^4 \triangle_3^2}{2\rho f_m} \left(\alpha \triangle_2 + \frac{\nu a}{\rho + \psi} \right)^2 - \frac{D_0^4 s^4 \nu^2 b^2 \eta}{512 \rho f_r (1-\eta)(\rho + \psi)^2} \\ & + \frac{D_0^4 s^2 \nu^2 b^2 \triangle_3}{16 \rho f_r (1-\eta)(\rho + \psi)^2} + \frac{h}{\rho}; g_1 = \frac{D_0^2 s^2 \triangle_2}{16}; g_2 = -\frac{D_0^2 s^2 \lambda}{16(\rho + \theta)}; g_3 = \frac{D_0^2 s^2 \nu}{16(\rho + \psi)}; \\ g_4 = & \frac{s^2 D_0^4 \triangle_3}{16 \rho f_m} \left(\alpha \triangle_2 + \frac{\nu a}{\rho + \psi} \right)^2 + \frac{s^4 D_0^4 \nu^2 b^2}{512 \rho f_r (1-\eta)(\rho + \psi)^2} - \frac{h}{\rho}. \end{aligned} \tag{41}$$

By substituting equation (41) into equation (35), we obtain the retailer's equilibrium strategy for advertising under the cost-compensation mechanism as

$$A^{B*} = \frac{s^2 D_0^2 vb}{16 f_r (\rho+\psi)(1-\eta)}. \quad (42)$$

Under this contract, the dual-channel supply chain can reach coordination. Thus, the retailer's advertising effort can reach the effort level of the centralized decision, that is,

$$A^{B*} = A^{U*} = \frac{D_0^2 vb\triangle_1}{f_r(\rho+\psi)}. \quad (43)$$

The manufacturer's share of the cost for the retailer is obtained by using equations (42) and (43) and is denoted as η:

$$\eta^{B*} = 1 - \frac{s^2}{16\Delta_1}. \quad (44)$$

We then substitute equation (41) into equation (39) to obtain the manufacturer's optimal level of effort to reduce emissions as

$$E^{B*} = \frac{D_0^2\triangle_3}{f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right). \quad (45)$$

By substituting equations (43) and (45) into equations (1)–(3), we obtain the optimal trajectory of emission reduction, carbon emission preference, and goodwill, as follows:

$$\begin{aligned}
\chi^{B*} &= \chi_{RSS}^{B} + \left(\chi_0 - \chi_{RSS}^{B}\right)e^{-\delta t};\\
R^{B*} &= R_{RSS}^{B} + \frac{\theta}{\theta-\delta}\left(\chi_0 - \chi_{RSS}^{B}\right)e^{-\delta t} + \left[R_0 - R_{RSS}^{B} - \frac{\theta}{\theta-\delta}\left(\chi_0 - \chi_{RSS}^{B}\right)\right]e^{-\theta t};\\
G^{B*} &= G_{RSS}^{B} + \left(G_0 - G_{RSS}^{B}\right)e^{-\psi t},
\end{aligned} \quad (46)$$

where $\chi_{RSS}^{B} = (\alpha D_0^2\triangle_3/\delta f_m)\quad(\alpha\triangle_2 + (va/\rho+\psi)), R_{RSS}^{B} = (\alpha D_0^2\triangle_3/\delta f_m)(\alpha\triangle_2 + (va/\rho+\psi)), G_{RSS}^{B} = (aD_0^2\triangle_3/\psi f_m)(\alpha\triangle_2 + (va/\rho+\psi)) + (b^2 D_0^2 v\triangle_1/\psi f_r(\rho+\psi))$.

Finally, the respective profits and the total system profits are obtained as

$$J_m^{B*} = e^{-\rho t}V_m^{B*}(\chi,R,G); J_r^{B*} = e^{-\rho t}V_r^{B*}(\chi,R,G); J_k^{B*} = e^{-\rho t}\left(V_m^{B*}(\chi,R,G) + V_r^{B*}(\chi,R,G)\right), \quad (47)$$

where

$$\begin{aligned}
V_m^{B*} &= D_0^2\triangle_2\triangle_3\chi^{B*} - \frac{D_0^2\lambda\triangle_3}{\rho+\theta}R^{B*} + \frac{D_0^2 v\triangle_3}{\rho+\psi}G^{B*} + \frac{D_0^4\triangle_3^2}{2\rho f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right)^2 + \frac{D_0^4 v^2 b^2\left(512\triangle_1\triangle_3 - 16s^2\triangle_1 + s^4\right)}{512\rho f_r(\rho+\psi)^2} + \frac{h}{\rho},\\
V_r^{B*} &= \frac{D_0^2 s^2\triangle_2}{16}\chi^{B*} - \frac{D_0^2 s^2\lambda}{16(\rho+\theta)}R^{B*} + \frac{D_0^2 s^2 v}{16(\rho+\psi)}G^{B*} + \frac{D_0^4 s^2\triangle_3}{16\rho f_m}\left(\alpha\triangle_2 + \frac{va}{\rho+\psi}\right)^2 + \frac{D_0^4 s^2 v^2 b^2\triangle_1}{32\rho f_r(\rho+\psi)^2} - \frac{h}{\rho}.
\end{aligned} \quad (48)$$

Equation (47) shows that introducing the contract yields higher maximum manufacturer returns than in the case of decentralized decision-making (i.e., $V_m^{B*} > V_m^{L*}$). However, the retailer's optimal returns might not always be enhanced because $V_r^{B*} - V_r^{L*} \geq 0$, which needs to satisfy certain conditions to hold. It is necessary, therefore, to further explore the range of fixed fees offered by the retailer that will maximize members' returns under this contract. □

Proposition 12. *The coordination mechanism optimizes the benefits of the overall supply chain and of each member when* $h \in [h_1, h_2]$. h_1 *and* h_2 *are expressed as*

$$h_1 = 0, h_2 = \frac{D_0^2 s^2 vb\rho}{16\psi f_r(\rho+\psi)}\left(A^{B*} - A^{L*}\right)\left(1 - e^{-\psi t}\right) + \frac{D_0^4 s^2 v^2 b^2\left(16\triangle_1 - s^2\right)}{512 f_r(\rho+\psi)^2}. \quad (49)$$

Introducing this cost-compensation contract H (η) enables supply chain coordination and brings the members' optimal strategies to the level of centralized decision-making, according to Propositions 11 and 12. Although manufacturers have two sales channels (online and offline) and compete with retailers, such competition can be coordinated. Supply chain members should cooperate to enhance profits. However, this cost-compensation model can only be realized when the cost-sharing ratio and the fixed return fee meet certain conditions, thus realizing the coordination of the supply chain. Manufacturers and retailers cannot cooperate blindly. Only when the conditions are met, can both parties cooperate effectively, thus eliminating the double marginal effect brought about by decentralized decision-making.

Proof. Since the manufacturer has achieved revenue improvement ($V_m^{B*} - V_m^{L*} \geq 0$), it is necessary to ensure that the optimal revenue of each member under the contract is not less than the result under decentralized decision-making (i.e., $V_r^{B*} - V_r^{L*} \geq 0$). According to equation (47), we get

$$V_r^{B*} - V_r^{L*} = \frac{D_0^2 s^2 v b}{16\psi f_r(\rho+\psi)}\left(A^{B*} - A^{L*}\right)\left(1 - e^{-\psi t}\right) + \frac{D_0^4 s^2 v^2 b^2\left(16\triangle_1 - s^2\right)}{512\rho f_r(\rho+\psi)^2} - \frac{h}{\rho} \geq 0. \tag{50}$$

According to equation (50), the range of h is solved as follows:

$$h \in \left[0, \frac{D_0^2 s^2 v b\rho}{16\psi f_r(\rho+\psi)}\left(A^{B*} - A^{L*}\right)\left(1 - e^{-\psi t}\right) + \frac{D_0^4 s^2 v^2 b^2\left(16\triangle_1 - s^2\right)}{512 f_r(\rho+\psi)^2}\right]. \tag{51}$$

□

Proposition 13. *Centralized decision-making is the optimal model. However, in practice, it is difficult to get manufacturers and retailers to voluntarily participate in centralized decision-making. Centralized decision-making requires manufacturers and retailers to maximize the interests of the supply chain as a whole. For example, in the face of fierce competition in the market, Walmart and Procter and Gamble want to cooperate, but Procter and Gamble could not accept the harsh demands made by Walmart. Procter and Gamble argue that under the current scheme, the supply chain's overall profitability can be enhanced, but its own interests are damaged. Thus, the process of cooperation between the two parties was stopped. Afterwards, the two parties negotiate so that their interests are safeguarded and cooperation is finally realized. Under centralized decision-making, manufacturers and retailers do not make decisions based on maximizing their own interests. At this point, the system is the most profitable, but the manufacturer's and retailer's respective profits are not necessarily better than the decentralized decision-making and cost-compensation models. This makes manufacturers and retailers not necessarily willing to implement centralized decision-making even if the overall profitability of the system is the highest. The fundamental reason is that maximizing overall profit is not the same as maximizing self-interest. If centralized decision-making is implemented, both the manufacturer and the retailer will voluntarily engage in centralized decision-making only if their respective interests outweigh those under the decentralized decision-making and cost-compensation models. That is, they choose this model only when their profits satisfy equation (52), where* $\triangle J_m$ *and* $\triangle J_r$ *are the incremental profits of the manufacturer and the competitive retailers, respectively, which are affected by the members' negotiation power.*

$$\triangle J_m = J_m^{U*} - J_m^{L*} \geq 0; \triangle J_m = J_m^{U*} - J_m^{B*} \geq 0; \triangle J_r = J_r^{U*} - J_r^{L*} \geq 0; \triangle J_r = J_r^{U*} - J_r^{B*} \geq 0. \tag{52}$$

5. Simulation Analysis

The model's validity and parameter sensitivity are further analyzed by using MATLAB software. Referring to reference [24], the parameters are assigned as follows: $\alpha = 2$, $\theta = 0.9$, $a = 2$, $b = 1$, $f_m = 1$, $f_r = 1$, $s = 0.4$, $\mu = 0.05$, $\beta = 0.2$, $\lambda = 0.5$, $v = 0.2$, $D_0 = 10$, $\chi_0 = 10$, $R_0 = 10$, $G_0 = 10$, $\delta = 0.1$, $\psi = 0.1$, $\rho = 0.8$, $t = 1$, and $h = 0.7$. The centralized decision-making, decentralized decision-making, and cost-compensation models are denoted by U, L, and B, respectively.

Figures 2 and 3 show the optimal trajectories of emission reduction χ and carbon emission preferences R for the three scenarios. We can see in Figures 2 and 3 that emission reductions and carbon emission preferences tend to be stable over time in all three decision-making models. Emission reductions and carbon emission preferences are equal under decentralized decision-making and cost-compensation mechanisms, and both are lower than those under centralized decision-making. Initially, consumers' carbon emission preference values are relatively low. Over time,

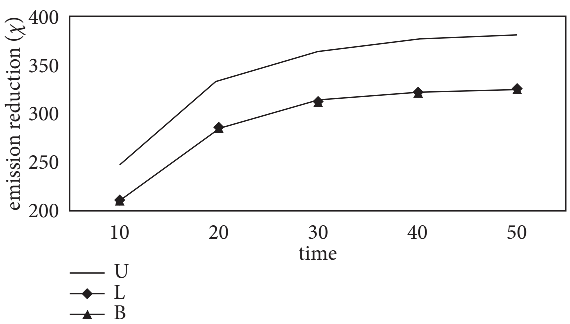

Figure 2: Optimal trajectory of emission reduction.

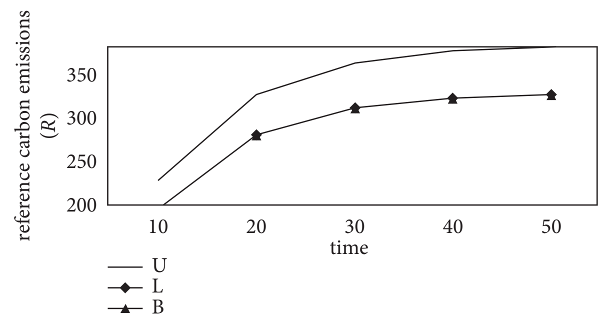

Figure 3: Optimal trajectory of reference carbon emissions.

however, technology improves, actual product emissions are reduced, and the carbon emission preference values gradually rise, tending toward stable values. L and B represent decentralized decision-making and cost-compensation models, respectively. It is worth noting that the lines represented by L and B overlap since the emission reductions are equal in the decentralized decision-making and cost-compensation models, and the reference carbon emissions are also equal.

Figure 4 shows the variations in total system profits over time under the three decision-making models. We can see in Figure 4 that the total profits are the highest under centralized decision-making and the lowest under decentralized decision-making. Introducing a cost-compensation mechanism can coordinate the dual-channel supply chain, improve profits under decentralized decision-making, and eliminate the double marginal effect. Figure 5 shows the influence of λ on total system profit under the three decision modes. λ is the coefficient of influence of reference carbon emissions on demand. The greater the influence of the reference carbon emission effect on demand quantity, the higher the total system profits. In other words, the higher the consumers' emission preferences are, the more beneficial it is for profits. At this time, consumers are highly aware of environmental protection, and manufacturers can increase product demand by increasing emission-reduction investment, which in turn will increase profits.

Figures 6 and 7 illustrate the variations in supplier and retailer profits over time under the decentralized and cost-compensation mechanisms, respectively. We can see in the figures that the coordination mechanism is effective as the profits of both the manufacturer and the retailer increase after introducing the cost-compensation mechanism. At this time, the manufacturer bears part of the retailer's promotion costs, the retailer gives fixed compensation to the manufacturer, and both parties are incentivized.

Figure 8 shows the effect of parameters a and b on the total profit of the system, where a and b are the coefficients of the impact of low-carbon production effort level and low-carbon publicity effort level on goodwill, respectively. We can see in Figure 8 that the greater the effect of both parties' effort levels on goodwill, the higher the total system profit. Since goodwill can lead to increased profits, supply chain members will engage in low-carbon efforts to enhance the brand goodwill and contribute to increased profits. Figure 9 shows the effect of both sides of cost coefficients on total system profits under centralized decision-making. As shown in Figure 9, the total system profit decreases as the cost of low-carbon effort increases for both parties. Manufacturers and retailers should therefore reduce costs as much as possible to enhance profits.

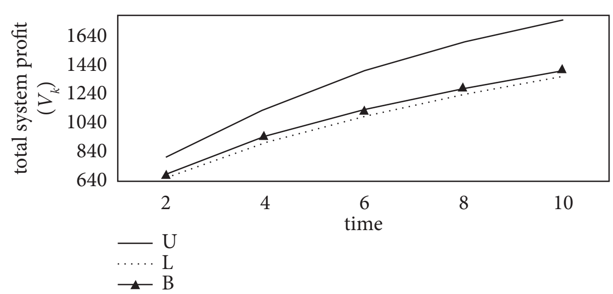

Figure 4: Change of the total system profit over time.

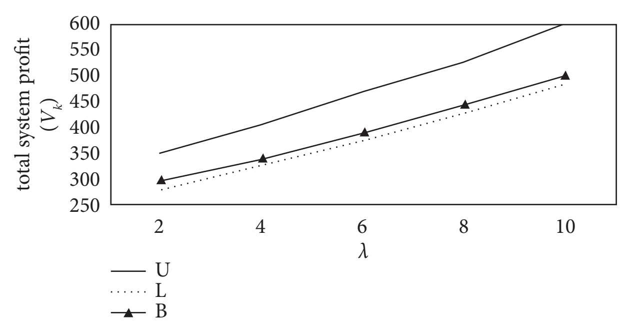

Figure 5: The influence of λ on total system profit.

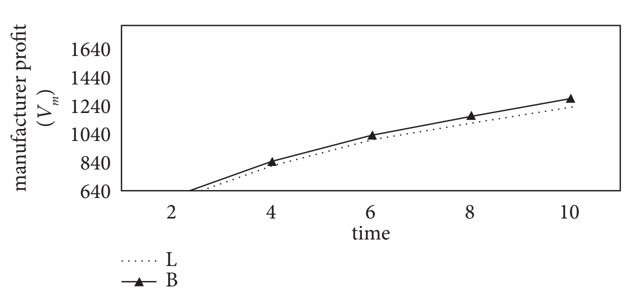

Figure 6: Change of manufacturer's profit over time.

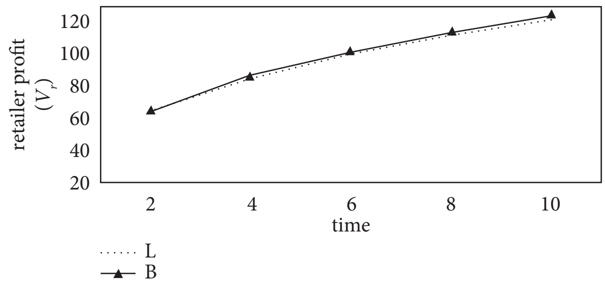

Figure 7: Change of retailer's profit over time.

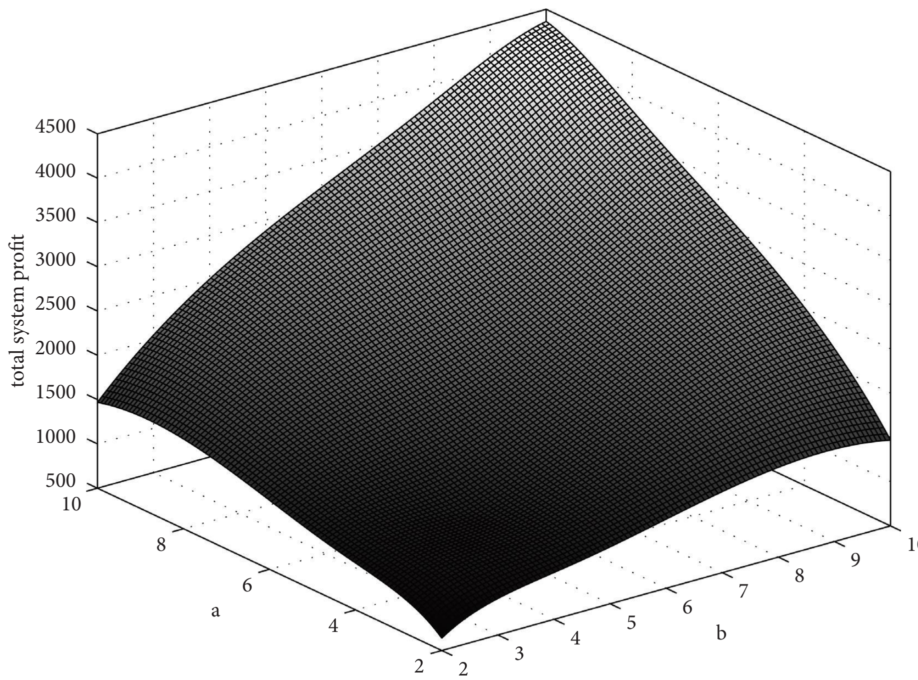

Figure 8: Effect of a and b on total system profit.

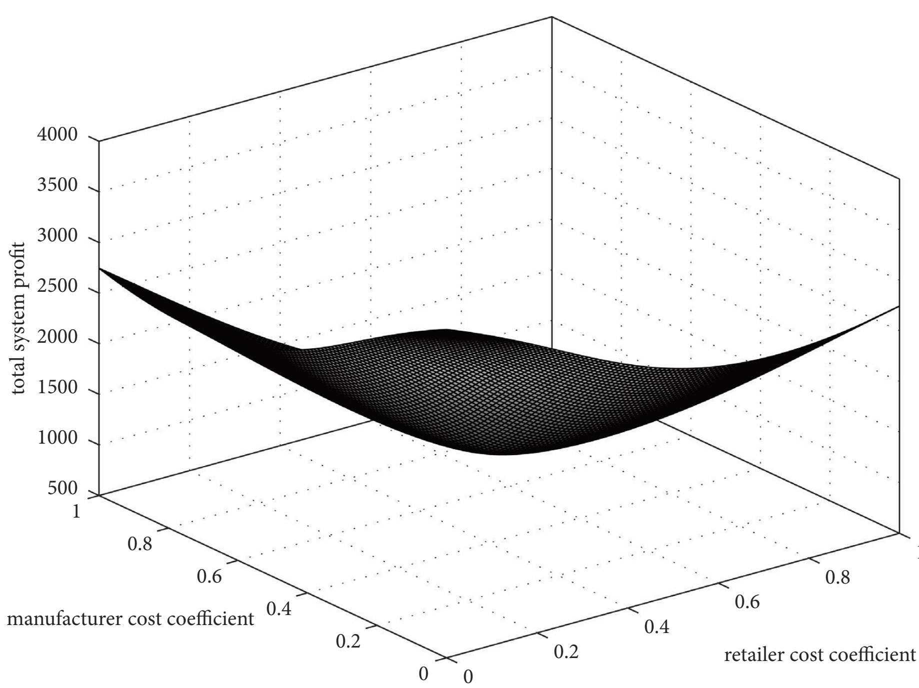

Figure 9: Effect of cost coefficients on total system profit.

6. Conclusions

This study investigates a single manufacturer and a single retailer, in which the manufacturer has both online and offline sales' channels and the retailer conducts offline sales. Adopting a dynamic perspective, a differential game model is constructed under centralized and decentralized decision-making. Dual-channel supply chain member pricing, low-carbon production effort level, and low-carbon advertising effort level are studied. Then, a cost-compensation coordination mechanism is designed to achieve supply chain coordination. The main findings are as follows:

(i) Centralized decision-making is the optimal decision model. It achieves the optimal low-carbon input, emission reduction, goodwill, individual profits, and total system profits. However, further negotiation is required for voluntary implementation by both parties. Coordination is inseparable from negotiation between manufacturers and retailers. Members of the dual-channel supply chain need to focus on their negotiation and information-gathering skills to improve their own utility. Manufacturers and retailers may choose centralized decisions. At this point, the total profit is the highest from the point of view of the system as a whole. However, both parties also have to give a proposal on the distribution of their respective profits. If their respective profits are higher than those in the decentralized decision-making and cost-compensation models, then centralized decision-making is the preferred mode.

(ii) A cost-compensation mechanism is introduced to coordinate the supply chain. At this point, consumers are affected by the reference carbon emission effect, and this cost-compensation mechanism provides a new cooperative solution for supply chain members. The contract can help the retailer's advertising and promotion investment reach the level of centralized decision-making and improve the member's profit under decentralized decision-making. Thus, this coordination mechanism is effective. However, the cost-compensation contract occurs when the cost-sharing ratio and the fixed compensation fee meet certain conditions. Under this contract, the manufacturer bears part of the advertising and promotion costs for the retailer, giving the manufacturer a certain fixed fee. The cost-sharing ratio is jointly influenced by the market share and interchannel substitution coefficient, that is, the higher the online product share, and the larger the interchannel substitution coefficient, the higher the cost-sharing ratio. Manufacturers and retailers can calculate the cost-sharing ratio and fixed costs based on the findings of this study when choosing the decision-making model. If the conditions are satisfied, they can choose this cost-compensation pact. Cooperation and competition coexist between channels, and coordination mechanisms should be explored to improve profits and coordinate the supply chain.

(iii) The reference carbon emission effect can lead to low-carbon production investment and improved profits for manufacturers. When the reference carbon emission effect on product demand is greater, the incentive for manufacturers to invest in low-carbon production is greater, which in turn leads to increased profits. The retailer's advertising and promotion investment is not affected by the reference carbon emission effect. In a setting where consumers' environmental awareness gradually increases, manufacturers should engage in low-carbon production, continuously upgrade low-carbon technology, and further reduce emissions. Reducing carbon emissions is the common theme faced globally. For manufacturers, while participating in low-carbon production, they have to consider factors such as costs and benefits. Based on consumers' low-carbon preferences, this study confirms the impact of reference carbon emission effects on consumers' purchasing decisions and supply chain members' profits. Although, manufacturers have to pay costs for low-carbon production, overall, low-carbon production can enhance their profits under the premise that consumers have reference carbon emission characteristics. Manufacturers actively engaging in low-carbon production can achieve the dual goal of enhancing economic and environmental benefits.

(iv) Pricing strategy is influenced by the channel's market share. Wholesale, online, and offline retail prices are positively correlated with the market share of the channel. The price-substitution coefficient between the channels is positively correlated with the low-carbon input and profits of both parties. In practice, the market share of the online sale channels gradually increases, increasing the manufacturer's say in pricing. The online retail price of the product is the same under centralized and decentralized decision-making while the offline retail price is higher under decentralized decision-making. Product pricing decisions are unaffected by the introduction of the cost-compensation mechanism and are consistent with the results under decentralized decision-making. Manufacturers and retailers primarily consider the market share of a product when making pricing decisions. If a product has a high market share in a particular channel, both the retail and wholesale prices in that channel can be increased appropriately.

Data Availability

The data used to support the findings of this study are included in the simulation analysis section.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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