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Book Review

Armin Bunde and Shlomo Havlin, (Fractals and Disordered Systems). Second Revised and Enlarged Edition, 408 pages with index and with 165 Figures and 10 Color Plates (Springer-Verlag, Berlin, 1996).

Shortly after Mandelbrot's *Fractal Geometry of Nature*, there was explosive growth in fractal applications to a variety of fields. Much like the original development of "the calculus", fractal geometry is a branch of mathematics inextricably tied to natural problems. It provides one of the first tools for understanding complexity in both natural and man-made settings. In a sense, it gives us a "calculus for complexity", allowing infinitely jagged structures to be analyzed and interpreted.

The early stage in the growth of this new discipline was largely phenomenological. A diversity of spatial and temporal behavior was examined to see if scaling laws are obeyed. Fractal dimensions were ascribed to a range of structures. The fractal dimension in a very loose sense replaces the derivative of calculus. Fractal curves, such as the jagged path of a lightning bolt, show scale invariance. That is, they show a similar jaggedness at any level of magnification. It is impossible to take the derivative of such a curve because the jaggedness persists at all scales. Regardless of magnification, a smooth region for drawing a tangent does not exist. A consequence of this is that fractal functions are non-analytic and are governed by power law behavior. The fractal dimension appears in the exponent of these power law expressions and provides a means of characterizing the degree of jaggedness of the function.

A very heady result in those early days of fractals was the computer simulation by Witten and Sanders of a diffusion limited aggregate (DLA). Here was a computer experiment that followed a simple set of rules and produced a fractal structure. Not only that, it was the sort of image one sees in a variety of physical phenomena ranging from electrochemical deposition to the growth of bacterial colonies. This early result suggested that such computer experiments could lead from phenomenology to a deeper understanding of fractals. As the present volume will attest to, this is an unfulfilled promise.

Fractals and Disordered Systems represents not only the maturing of a field but also the inevitable confronting of deep and difficult problems. The era of phenomenology is over. It has been demonstrated that fractal structures pervade our lives. They exist virtually everywhere there is growth, be it in a plant such as cauliflower, in cracks and grains of rock formations, in river basins or even in urban sprawl. As H.E. Stanley points out in Chapter 1, the Metro of Paris has fractal-like tentacles. Interestingly, similar fractal dimensions exist for a broad range of growth processes. The focus now centers on why fractals are so pervasive in nature. Is there a biological advantage to fractal growth patterns? If so, why do similar patterns appear in inorganic chemical systems? Why do higher organizational structures have similar patterns? The goal now is to understand why such diverse structures are scale invariant. This question runs throughout the edited volume.

Scale invariance has been familiar to physicists and physical chemists for years. When a system is at a critical point between two phases, it will often show scale invariance. For instance, at the critical point between a gas and a liquid, the system will fluctuate wildly between the two states. Liquid-like clusters of all sizes will be constantly forming and breaking up. These clusters will exist on all length scales and are, therefore, considered to be scale invariant. They are said to have no "characteristic length". This form of scale invariance has been studied extensively and there are mature and deep theories that describe such phenomena.

The problem facing the fractal field is that a similar "criticality" occurs in growth patterns such as DLA. The growth will hit a critical point where scale invariant structures are formed. These are often dendritic structures whose branches appear similar to magnified versions of branches of higher generations. These patterns arise from frozen, non-equilibrium structures and our level of understanding of them is in its infancy. This is a central challenge revealed by *Fractals and Disordered Systems*. Much of the book describes various mathematical tools and approaches that brings us to grips with this problem.

The book begins with the chapter, *Fractals and Multifractals: The Interplay of Physics and Geometry*, by H. Eugene Stanley. This excellent chapter introduces the basic concepts and definitions of fractals. To approach the problem of criticality in growth phenomena, multifractals are used as a tool. Multifractals are a generalization of the fractal concept to probability distributions. The moments of a distribution of growth probabilities will often show power law scaling. One can then assign a fractal dimension for each moment and, hence, the term multifractals. The multifractal spectrum provides a means of characterizing the distribution. Stanley points out that the multifractal formalism provides parameters that are mathematical analogues of thermodynamic quantities. This allows the criticality of growth phenomena to be viewed in a similar context as phase transitions.

The multifractal approach of Chapter 1 ties in closely with the more mathematical treatments in Chapter 4, *Fractal Growth* by Ammon Aharony and Chapter 10 *Exactly Self-similar Left-sided Multifractals* by Benoit B. Mandelbrot and Carl J.G. Evertsz (with appendices by Rudolf H. Riedi and Mandelbrot). These chapters focus on the nature of the multifractal spectrum of growth phenomena, especially DLA. They provide insightful views of the problems associated with the interpretation of multifractal spectrum. They grapple with the question: Is growth multifractal? These chapters provide a deep and technical overview of the use of multifractals.

Stanley's introductory chapter is followed by two fine chapters on percolation, simply entitled *Percolation I* by Armin Bunde and Shlomo Havlin and *Percolation II* by Shlomo Havlin and Armin Bunde. Percolation models have been used to describe such diverse phenomena as epidemics, forest fires and dielectric breakdown. Percolation has considerable theoretical appeal because it represents a "geometric phase transition". Additionally, it is an area where significant progress has been made. Again, scale invariance appears from computer models based on a simple set of rules. These two comprehensive and very readable chapters consecutively cover static and dynamic phenomena. These chapters also serve as introduction to material that appears throughout the book. The first four chapters comprise more than half of the book and lay a firm basis for later more specific chapters. They present a coherent overview of the theory of scale invariant structures and relate them to the underlying physics of disordered systems.

The following four chapters move from theory and computer simulations to a closer tie with the experimental world. *Fractures* by Hans J. Herrmann is a self-contained chapter of great technological importance. Herrmann details the development of models of fracture in solids, starting with a phenomenological description of elasticity and proceeding to lattice models. It is seen that certain fracture phenomena can be realistically modeled with lattice approaches. It is also seen how such models give rise to fractal structures. Fracture can be considered a growth phenomenon and as such has inherent fractal and multifractal characteristics.

Chapter 6 (*Transport Across Irregular Interfaces: Fractal Electrodes, Membranes and Catalysts* by Bernard Sapoval) and Chapter 7 (*Fractal Surfaces and Interfaces* Jean-François Gouyet, Michel Rosso and Bernard Sapoval) are another pair of complementary chapters. Unlike the percolation chapters, these chapters start with dynamics and then move to structure. Chapter 6 is concerned with transport processes across irregular surfaces. Biological examples of these are fluid exchange in the root system of a tree and the exchange of oxygen in the lung. A technologically important example is the optimization of the power of a car battery by using a porous electrode. These problems can be viewed as the "irrigation" of structures of large surface area and low volume. The chapter demonstrates that a wide range of these phenomena can be treated by a general formulation of Laplacian transfer across irregular surfaces. This problem is not a fractal one *per se*. Yet a fractal interface provides a mathematical convenience that actually makes the problem more tractable. One can then use these results to distinguish phenomena due to the fractal nature of the interface from general disorder phenomena.

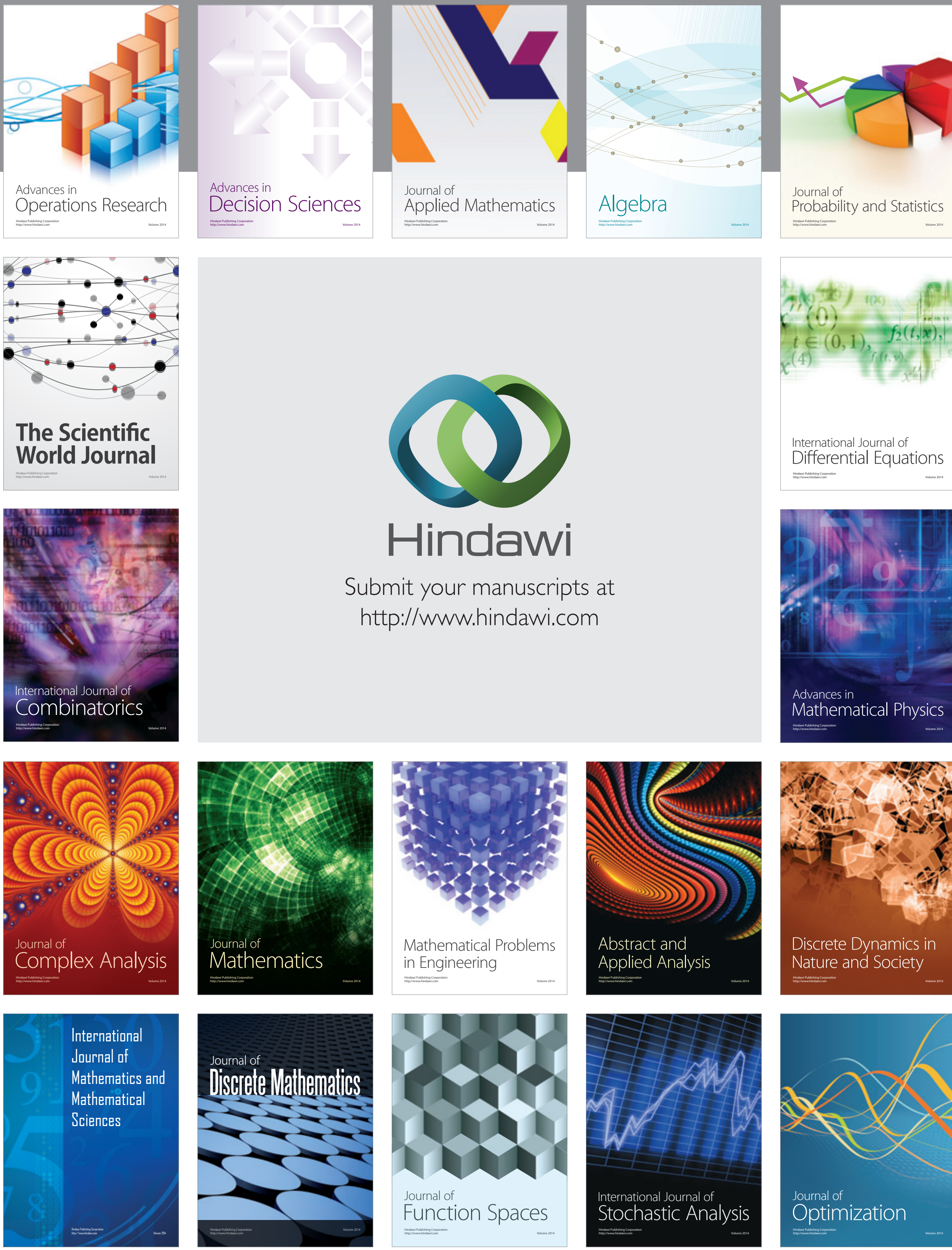
Fractals appear again as a useful tool in Chapter 7. This chapter deals with the description and properties of surfaces and interfaces, a topic of considerable recent attention. Additionally, the natural question of what creates specific interface geometries is asked. Interfaces reflect a competition between forces such as surface tension that tend to smooth them with forces that drive the system to disorder. The fractal formalism offers an obvious tool in handling objects of such complexity. Again, it is seen that for structures that are far from equilibrium, simple physical processes such as diffusion or aggregation give rise to fractal structures. A variety of physical examples of such behavior are discussed. It is speculated that the combination of random processes and non-equilibrium conditions is ubiquitous in fractal formation.

Chapter 8, *Fractals and Experiments* by Jorgen K. Kjems picks up themes of previous three chapters. This is an easy reading, qualitative chapter. It describes how you make fractals in the laboratory under controlled conditions, how you measure their fractal dimension and what physical properties they have. It covers a number of different experimental scenarios that are physical realizations of the theoretical models, such as DLA, discussed in earlier chapters. There is a nice exposition of the use of scattering techniques to determine the fractal dimension. Fractality has strong implications for the mechanical, vibrational and thermal properties of a structure. This chapter collects a variety of results from the literature, providing a useful survey.

In 1969 Stuart Kauffman proposed a model to describe cell differentiation in developing organisms. This model has grown into a whole class of models known as cellular automata. This is the topic of Chapter 9, *Cellular Automata* by Dietrich Stauffer. As in percolation, these models also show geometric phase transitions. They also show hidden fractal and multifractal behavior. This is a very brief chapter that may not be as tractable to the layman. Nevertheless, there are a range of interesting biological applications that enhance the appeal of the subject.

This book brings together leaders in the field in a comprehensive survey of recent developments. It is surprisingly cohesive for a multi-author work and the editors should be given credit for maintaining the uniformity of presentation. This is a sophisticated volume and not all of the chapters will appeal to the layman. My main criticism of the text is that it is not substantially different from the first edition. Regardless, this volume is an excellent entree to fractals and disordered systems and captures the excitement of the field.

T. GREGORY DEWEY
Department of Chemistry and Biochemistry, University of Denver, Denver, CO 80208, USA
Tel.: 303-871-3100. Fax: 303-871-2254.
E-mail: gdewey@du.edu.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