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# Monitoring the Complexity of Ventricular Response in Atrial Fibrillation

H. KÄSMACHER\*, S. WIESE and M. LAHL

Clinic of Anaesthesiology, Medical Faculty of the University of Technology Aachen, Pauwelsstr 30, D-52074 Aachen, Germany

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Atrial fibrillation does not present a uniform extent of variability of the ventricular response exemplifying periodicities and more complex fluctuations, due to varying number and shape of atrial wavelets and aberrant conduction in the AV-junction. It was sought to categorise different degrees of complexity introducing an uncomplicated monitoring method for that objective.

The fluctuation of RR-intervals was investigated in 66 patients presenting with atrial fibrillation at different times of the day using conventional statistical methods as well as methods derived from non-linear dynamics. Specifically Poincaré-plots were employed to analyse the data. One hundred and seventy data sets consisting of 5000-8000 intervals each were considered.

Statistical methods were shown to describe the observed dynamics not adequately due to background noise in the acquired data as well as owing to intrinsic qualities of the data sets. Namely non-uniformly distributed data and trends within the data sets constituted limitations of statistical methods.

Poincaré-plots were proven to be an inexpensive and effective method to categorise the complexity of the ventricular response in atrial fibrillation. Periodical variation of interval lengths could be clearly differentiated from continuous variation in the considered data sets. The latter could be shown to exemplify either stochastic fluctuation or complex non-linear dynamics of RR-interval variation. Thus, different degrees of complexity could be clearly distinguished.

It could be shown exemplary that the observed dynamics remained nearly constant within the observation period.

Poincaré-plots, thus, provide a means to appreciate the fluctuation of RR-intervals semiquantitatively and to distinguish different patterns of fluctuation dynamics of the ventricular response in atrial fibrillation without being affected by contaminated or inconsistent data.

For a complete visualisation of the concerned dynamics relatively small data sets suffice. Thus, it is possible to observe the pattern of fluctuation essentially in real time which makes them ideally suited for future investigation.

Keywords: Atrial fibrillation, Ventricular response, Poincaré-plot, Complexity analysis

<sup>\*</sup> Corresponding author.

# **INTRODUCTION**

Since the beginning of the century extremely irregular ventricular activity known as *pulsus irregularis perpetuus* is recognised to possess a supraventricular origin namely atrial fibrillation or -flutter with irregular AV-conduction (Söderström, 1950). The disorder is quite frequent and its incidence is as high as 1% of the general population and even higher among the elderly (Cameron *et al.*, 1988; Heisel *et al.*, 1995).

The chief complaint is the unpleasant sensation of a highly irregular pulse, whereas the outcome is particularly determined by the hemodynamic consequences and thromboembolic complications of the disease (Ferguson and Cox, 1993).

Several investigations deal with the mechanisms of atrial fibrillation and -flutter itself whereas the ensuing ventricular response was rarely considered (Lammers and Allessie, 1993; Allesie *et al.*, 1990). It was recognised early that distinct intervals were more frequent than others, thus, constituting one or more peaks in the frequency distribution (Honzikova *et al.*, 1973; Söderström, 1950).

The interbeat interval in atrial fibrillation T can be legitimately considered to be the summation of the refractory period of the AV-junction  $\tau$ , the AV-conduction time c and the time t in which the AV-junction integrates enough temporal-spatial distinct influences of the atrial wavelets to reach a threshold and trigger the propagation of depolarisation through the AV-junction, thus,  $T = \tau + c + t$ (Cohen *et al.*, 1983; Zipes *et al.*, 1973).

Several electrophysiological investigations describe the dependence of a given refractory period  $\tau_i$  of the preceding interbeat interval  $T_{i-1}$  as  $\tau_i \approx \tau_{\infty} [1 - \exp(-T_{i-1}/\tau_{\infty})]$ , thus the longer the preceding interval is, the shorter the refractory period will be (Cohen *et al.*, 1983; Mendez *et al.*, 1956). However, the AV-conduction was shown to depend reciprocally on the preceding interval so that  $c_i \approx c_{\min} + \exp(-T_{i-1})$  (Shrier *et al.*, 1987).

The time t was explained to depend on the number of circulating atrial wavetets, on their

velocity and the direction in which they reach the AV-junction (Chorro *et al.*, 1990; Zipes *et al.*, 1973; Janse, 1969).

Whereas these electrophysiological properties are far from a complete characterisation of the actual AV-conduction process, it seems legitimate to state:

- There are influences of the AV-junction which are essentially stochastic as the bombardment by minute atrial excitations.
- There are influences of the AV-junction which are essentially deterministic, e.g. governed by conduction times and refractory periods of membranes.
- The described mechanisms exert divergent influences on a given interval.

These conditions seem ideally suited to cause extremely complex dynamics of interval fluctuation.

Several investigations were sought to analyse the ventricular response in atrial fibrillation by statistical means and considered the observed dynamics to be mainly stochastic, even though positive auto-correlation was demonstrated in some studies (Bootsma *et al.*, 1970).

Whereas phase-space-portraits have been introduced as method for the analysis of cardiac rhythms they have not been employed systematically for the analysis of the ventricular response in atrial fibrillation (Bigger *et al.*, 1993; Denton *et al.*, 1990).

In this investigation the Poincaré-plot as a specific example of phase-space-portrait was used to analyse systematically the interval-fluctuation during atrial fibrillation in order to distinguish different degrees of complexity, to investigate whether different reproducible patterns could be discovered and possibly to find a correlation with a given patient's diagnosis.

Furthermore the reliability of the method regarding background noise in the acquired data as well as non-uniformly distributed data and trends within the data sets was appreciated.

## METHODS

Patients of both sexes with atrial fibrillation presenting prior to cardiac catherisation or implantation of heart-valve prosthesis were included in the investigation after their informed consent. It was deliberately avoided to discriminate patients according to their underlying diseases. Patients presenting with acute cardiac decompensation and immediately after cardiac surgery were excluded. Due to the epidemiology of the disease the majority of patients with atrial fibrillation suffered from coronary heart disease or from a valvular heart defects. The underlying diseases were recorded as well as cardiac relevant drugs, the serum potassium concentration, and the time of the day of the particular registration. Even though it was sought to investigate at different times of the day no registration could be made during the night. Of the patients included for investigation 16 were considered frequently at different times of the day in order to analyse the stability of the pattern of the ventricular response. Thus, a total of 182 registrations of 66 patients were gathered.

Prior to each recording a conventional 12-lead-ECG was used to verify the diagnosis of atrial fibrillation which was additionally confirmed by two experienced cardiologists. The patients were required to rest in a supine position and refrain from all physical activities to reduce the chance of artefacts during the heart rate recordings. Then a conventional data monitor (Hewlett Packard<sup>©</sup>) as commonly employed for the monitoring of cardiovascular parameters was connected to the patient. An interface (Hewlett Packard Careplane<sup>®</sup>) linked the monitor to a conventional 80368 personal computer transmitting the ECG with a sampling rate of 500 Hz. A software specially designed for that purpose detected R-peaks with  $\pm 2 \,\mathrm{ms}$  precision and calculated interval lengths which were subsequently stored in the random access memory of the computer. Thus, they could be accessed by software analysis immediately allowing for an on-line representation of the particular heart rate dynamics. Typically 4000–7000 intervals were recorded for each registration. The activity of a particular patient (e.g. sleeping, reading) during the recording which typically lasted between 45 and 60 min was registered.

Usual statistical methods were used to characterise each registration such as the calculation of mean, median, standard deviation, histograms and the derived parameters of distribution and kurtosis. Furthermore, each interval length  $T_i$  of a particular registration was plotted against its sequential number *i* in order to detect non-stationarities in the observed cardiac dynamics. Patients were grouped according to underlying disease, drugs, state (sleeping/non-sleeping) and an ANOVA was conducted to determine whether significant differences between the subgroups exist.

Furthermore the data was examined for trends and non-stationarity using Cox and Stuart's trend test.

The above mentioned physiological background suggests a certain dependence on a certain interval of its immediate predecessor even though a uniform influence seems unlikely. To characterise the correlation between the length of one RR-interval and subsequent RR-intervals the serial autocorrelation coefficients were computed. Furthermore the number of intervals elapsing until RR-intervals become independent of their predecessors was determined that is where the autocorrelation curve reaches zero.

A virtual two-dimensional plane was used to visualise the information contained in the RRinterval fluctuation. In a Cartesian co-ordinate system a particular point  $P_i$  is defined by the interval  $T_i$  and the *l* intervals subsequently following  $T_{i+1}$ , thus,  $P_i = (T_i, T_{i+1})$ . Likewise, the whole data set is depicted (see Fig. 1) forming more or less structured shapes. Ideally these shapes will represent underlying information about interval fluctuation and the interdependence of intervals. In this investigation l=1 was chosen as it has been done in similar investigations (Anan *et al.*, 1990, Garfinkel *et al.*, 1992). Described representation of the data is the



FIGURE 1 Construction of the Poincaré-plots: (a) Interval lengths are measured in an ECG recording  $(T_i, T_{i+1}, \text{ etc.})$ . Adjacent intervals form a couple which define a particular point  $(P_i, P_{i+1}, \text{ etc.})$  in a Cartesian co-ordinate system (b) the axis of which are  $T_i$  and  $T_{i+1}$  respectively. Likewise the whole registration is depicted. The dotted line is the line of identity, i.e.  $T_i = T_{i+1}$ .

Poincaré-plot which is applicable for discrete data as opposed to the conventional phase-space-plot where the concerned parameter is plotted against its first derivative which requires continuous data (Denton *et al.*, 1990). However, these maps are closely related because the x-axes are identical (parameter amplitude), while the y-axes are mathematically related (the y-axis of the Poincaré-plot is  $x + \Delta x/\Delta t$ , and the y-axis of the phase-plane- plot is dx/dt).

The inspection of the pattern of the resulting plots immediately informs about:

• the dispersion of  $T_{i+1}$  for a given  $T_i$  represented by the scattering of points along the abscissa,

- the change of dispersion for different  $T_i$ ,
- the RR-variability as represented by the scattering of points along ordinate and abscissa,
- the dynamics of a given RR-variability as the range of points vertical to the diagonal line representing  $T_i = T_{i+1}$  represents the accelerations and decelerations within the observed ventricular responses (Kamen and Tonkin, 1995).

Morphology and dimension of the resulting plots were examined and they were classified according to morphological criteria. It was sought to find reproducible common patterns among the Poincaré-plots in order to facilitate visual recognition of a given figure. Ten independent test-person were asked to classify 132 randomly selected plots according to previously determined morphological subgroups in order to verify the reproducibility of the classification. Furthermore, some indices were computed to quantify the morphology of each subgroup (e.g. number of points in each quadrant, relative distance from zero, dispersion at certain  $T_r$ -quantils).

# RESULTS

Of 50 single registrations of the ventricular response of patients suffering from atrial fibrillation four had to be disregarded due to technical difficulties or interruption of the registration. Thus, 46 registrations were considered.

It was contrived to obtain at least ten registrations of each of the remaining 16 patients. With four patients the sequence of registration was cut off because atrial fibrillation seized due to either spontaneous conversion to sinus rhythm or the necessity for the implantation of a pace maker. One patient died before ten registrations could be obtained and one patient did not cooperate until the end of the investigation. Thus, with ten patients a sequence of at least ten registrations could be obtained yielding a total of 136 registrations. Of these eight had to be disregarded due to technical difficulties or interruption of the registration.

Twenty-nine patients were male and 33 female. Forty-two patients suffered from valvular heart defects, 17 had coronary heart disease, while the rest presented with various other underlying diseases. All patients could be classified in more or less advanced states of congestive heart failure (NYHA criteria II and higher). Twenty-nine patients had cardiac surgery and all patients were antiarrhythmically treated with digitalis being used in 51 patients.

The mean interval lengths  $\overline{T}$  of patients who underwent cardiac surgery were significantly longer than those of patients without previous surgical intervention. Systematic statistical analysis with ANOVA and parametric tests did not reveal any more significant differences in mean interval length, standard deviation and the other above mentioned statistic parameters between the different subgroups of underlying diseases, used medication, and activity of the patients. Furthermore, it could not be shown that serum potassium concentration correlated with the observed parameters.

It was shown that the majority of the recorded data sets was not stationary, i.e. a significant trend was detected in 76.5% of the sequential registrations. There was no favourite direction of a given trend, in 53.5% the heart rate decelerated and in 46.5% it accelerated in the course of registration.

The inspection of the obtained interval histograms showed relatively broad distributions and the standard deviation  $s_{\rm T}$  was quite great in comparison with  $\overline{T}$ , thus, the coefficient of variation  $s_{\rm T}/\overline{T}$  was generally greater than 0.2 which corresponds with literature data for this parameter (Cohen et al., 1983). Moreover, the shape of the histograms was quite variable (Fig. 2). The distribution could be unimodal with a single smooth peak, or may have a single narrow peak superimposed on a smooth background, or it may be characterised by multiple peaks tributed. It could be shown, however, that the shape of the interval histogram varied greatly within non-stationary data sets which constituted the majority of registrations (see Fig. 3).

First autocorrelation coefficients  $r_1$  were usually relatively small albeit significant in the data sets of the sequential registrations. 91.7% of registrations had significant  $r_1$  typically  $-0.2 < r_1 < 0.2$ . Each coefficient  $r_i$  denotes the degree of correlation between one RR-interval and the RR-interval occurring *j* beats later. The autocorrelation curve that is  $r_i$  considered as a function of *j* typically shows that the RR-intervals during atrial fibrillation are essentially statistically independent of each other, except for a slight correlation between the duration of immediate subsequent beats (see Fig. 4). However, only 50% of the autocorrelation functions derived from sequential registrations displayed the above criteria. The remainder showed strangely distorted autocorrelation functions where



FIGURE 2 Different interval length histograms: (a) single smooth peak, (b) single narrow peak superimposed on a smooth background, (c) multiple peaks.



FIGURE 3 Shape of histogram of interval changes within a non-stationary registration: (a) depicts the histogram of interval lengths 1-1000 as bimodal, whereas in (b) the histogram of interval 3001–4000 is unimodal with a steeper left slope.

the whole autocorrelation curve seemed set off from the abcissa. The occurrence of "strange" autocorrelation functions was highly significantly linked to non-stationarity of a given data set with  $r_1$ being highly significantly higher in non-stationary data.

Poincaré-plots were inspected and classified according to morphological criteria. These classes were distinguished with capital Latin letters as shown in Table I. Note that roughly 400 intervals sufficed to yield the same pattern as the total data

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FIGURE 4 Typical autocorrelation curve: The dotted lines represent the treshold beyond which  $r_j$  is significant (p < 0.001).  $r_j$  is highly significant only for j = 1.

set (see Fig. 5). The overall reproducibility of the classification as verified with ten test persons was as high as 76.5% which is comparable to other investigations involving visual pattern recognition (Reidborg and Redington, 1992).

The patterns were distributed as shown in Table II among the investigated collective. They could not be significantly correlated to underlying disease, state, or employed medication.

The patterns of any individual of whom sequential registrations were available remained nearly constant during the observation period with the exception of the occurrence of a separate "angle" in the type A or B, thus, yielding an example of type C. As will be discussed below, this does, however, probably not constitute a fundamental change in the observed dynamics. Two patients experienced circumstances which were apt to modify the physical cardiac entity within the observation period: cardiopulmonal resuscitation in one case and cardiac surgery in the other. These occurrences altered the observed pattern as shown in Fig. 6.

#### DISCUSSION

The majority of the considered data sets was non-Gaussian distributed and non-stationary. These properties must be inherent to biological processes as the system will never be stable over a longer period of time. Thus, to reach stationary biological data registrations ought to be infinitesimally small which on the other hand is not feasible because the time-scale of the process in question is primarily unknown. The direction of a trend within the data is not known, therefore, they cannot be excluded by filtering from the data sets. These features constitute limitations for any kind of statistical analysis as previously recognised (Vibe-Reymer *et al.*, 1996; Webber and Zbilut, 1994).

With these limitations in mind we might regard the significantly longer  $\overline{T}$  in patients who had been submitted to cardiac surgery prior to the investigation as the result of the denervation of the cardiac entity due to surgical preparation. This in turn leads to a decreased influence of vagal tone which otherwise shortens the refractory period in the atria (Schuessler *et al.*, 1992; Allesie *et al.*, 1984). Thus, lesser mean heart frequencies and longer interval lengths ensue.

The same restrictions as for statistical analysis apply for the calculation of the autocorrelation coefficient as shown in Fig. 7. The larger the number of intervals included in the computation of the autocorrelation function the more the influence of non-stationarity increases in increasingly damping the curve and shifting it to larger autocorrelation coefficients. The shortcoming of autocorrelation analysis for biological data is described elsewhere (Webber and Zbilut, 1994; Zbilut *et al.*, 1991).

#### **Poincaré-plots**

Poincaré-plots as employed in the registrations of RR-intervals in this investigation are used widely to investigate the complex dynamics of non-linear processes (Denton *et al.*, 1990). They were originally designed to detect complex periodic rhythms in

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TABLE I

| Туре | Description   | Example   |
|------|---|---|
| A    | "Cluster"<br>Round shape centered<br>around the $T_i = T_{i+1}$<br>(mainly stochastic)  | $\begin{bmatrix} 1000 \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $   |
| В    | "Angle"<br>Cluster with apparent<br>shortest interval length<br>(stochastic with shortest<br>interval)                                      | $\mathbf{T}_{I} \begin{bmatrix} \mathbf{s}_{1000} \\ \mathbf{s}_{1000} \\ \mathbf{s}_{1000} \\ \mathbf{s}_{1000} \end{bmatrix}$   |
| С    | "Cluster and detached<br>angle"<br>Similar shapes as above<br>with detached angular<br>shape (mainly stochastic<br>with ventricular ectopy) |   |
| D    | "Points"<br>Discontinuous point-shaped<br>cluster on and on both<br>sides of $T_i = T_{i+1}$<br>(complex periodic)                          |   |
| Ε    | "Shapes"<br>Linear shapes which cross<br>$T_i = T_{i+1}$<br>(non-linear deterministic)  | $\begin{bmatrix} \mathbf{J} \\ \mathbf{J} $ |

The dotted line represents the line of identity where  $T_i = T_{i+1}$ .



(all units are [ms])

FIGURE 5 Excerpts of a full registration yield similar pattern as the full registration: (a) depict 4000 intervals of the registration, whereas (b) represents interval 500-1000 and (c) characterises interval 2000-2500. The smaller portions exemplify the same behaviour as the full registration.

TABLE II

| Туре                           | Number of registrations |
|--------------------------------|-------------------------|
| A "Cluster"                    | 39                      |
| B "Angle"                      | 52                      |
| C "Cluster and detached angle" | 30                      |
| D "Points"                     | 26                      |
| E "Shapes"                     | 23                      |
| Total                          | 170                     |

discrete data (Anan *et al.*, 1990; Crutchfield *et al.*, 1986; Packard *et al.*, 1980). Unlike phase-spaceportraits they are quite apt to withstand noisy data. They were appreciated as suitable means to detect



(all units are [ms])

FIGURE 6 Occurrences which change the cardiac entity alter the appearance of the Poincaré-plots: (a) is a registration prior to cardiac surgery which the patient underwent on the 12 Nov 1993; (b) shows the different aspect of the resulting plot. Both times the ECG diagnosis was atrial fibrillation.

structural properties which would escape statistical analysis (Denton *et al.*, 1990). As method integrating time they provide a means to characterise features of numerous sequential data (Kamen and Tonkin, 1995; Woo *et al.*, 1992). It allows to consider a given data set qualitatively and semiquantitatively in compact visual patterns.



FIGURE 7 Dependence of the autocorrelation function of the size of the data set: The larger the extract of the considered data used for the computation of the autocorrelation function the more evident becomes the influence of the nonstationarity present in the registration. The autocorrelation function with larger excerpt seems damped and shifted to larger autocorrelation coefficients. Only for j=1 the autocorrelation seems quite resistent against the trend contaminated data.

Poincaré-Plots were previously employed for the appreciation of atrial fibrillation (Anan *et al.*, 1982). These investigators mentioned the possibility to describe those properties of the disease which concern the coupling interval of the preceding interval which they consider in the functional refractory period of the AV-node in atrial fibrillation (Suyama *et al.*, 1993; Chishaki *et al.*, 1991). Systematical analyses of the dynamics of the ventricular response in particular patients over the time, however, were not undertaken previously.

If different types of patterns of interval fluctuation can clearly be distinguished within the same diagnosis of atrial fibrillation it can be assumed that the observed irregularities of ventricular response possess different degrees. Intraoperative mapping studies indicate that a wide range of distinct cardiac behaviour is subsumed to that diagnosis (Cox *et al.*, 1991). In this investigation it was sought to point out different degrees of complexity of ventricular behaviour which is mainly determined by the function of AV-node and the conduction below it as well as their particular refractory periods.

#### Type A (Mainly Stochastic)

This type represents most likely the predominance of stochastic processes which lead to a clustering of intervals to all sides of a mean interval length. The interval fluctuation does not seem to have a particular direction. This type was thought to be the archetype of ventricular behaviour in atrial fibrillation in numerous investigations. The resulting interval lengths seem to depend solely of the random arrival of minute excitations by atrial wavelets. It was sought to find out whether these registrations were comparable to white noise data. Thus, the data of type A registrations were exemplary submitted to the calculation of the largest Lyapunov exponent (LLE) (Wolf et al., 1985) using the SANTISalgorithm (Vandenhouten et al., 1996). It could be shown that this type possesses a LLE which is positively different from those of a computer generated series of random numbers.

#### Type B (Stochastic with Shortest Interval Length)

This type is characterised by a histogram which possesses a steep left slope and which smoothes towards longer interval lengths. A limiting shorter interval length seems to exist from which the succeeding interval is more likely to become successively longer. This could be the result of a cut-off interval length determined by the refractory period of the AV-node. This assumption was also made by other authors which could also simulate the observed ventricular dynamics by random pacing of dog atria (Chishaki et al., 1991). These and other authors (Suyama et al., 1993; Anan et al., 1982; 1990) regard this type as typical pattern of ventricular response in atrial fibrillation but fail to mention other types. In this investigation this type is the most frequently occurring type and possesses the largest  $\overline{T}$  among the unimodal distributed registrations. Possibly these registrations are the result of a particular state dependent aspect (Lydic, 1987) in which a high vagal tone prevails thereby lengthening the functional refractory period of the AV-node. However it could not be demonstrated that this type is predominant among sleeping patients who ought to have a quite high vagal tone.

#### Type C (Stochastic with Ventricular Ectopy)

This type is not clearly distinct from the above mentioned characteristics with the exception of the "angle" which additionally appears towards smaller interval lengths. Most likely this "angle" is the substrate of ventricular extrasystoly with a consecutive recovery pause.

### **Type D (Complex Periodic)**

This type generally displays bi- or multimodal histograms. Different from the previously mentioned types here discontinuous patterns are found in the Poincaré-plots which are separated by "forbidden zones" (Denton et al., 1990). The intervals cluster to certain distinct interval lengths and successive intervals follow discrete interval differences instead of continuous interval variation. Furthermore, from a certain interval length the next interval can only have a finite number of different durations which are multiples of the previous interval length. It appears, thus, that the succeeding interval length is strongly determined by the length of the previous interval. Not surprisingly this type possessed the largest  $r_1$ . This behaviour suggests a dependence of interval lengths of certain AV-blocks which permit only conduction of atrial impulses in fixed conduction ratios (e.g. 2:1-, 3:1-, 3:2-, [...] conduction ratio) (Shrier et al., 1987). These blockades of the AV-junction usually occur at high atrial fibrillation frequencies so that high atrial stimulation frequencies are inversely related to the resulting ventricular heart rate (Chorro et al., 1990). Elsewhere it was shown that an infinite number of conduction ratios exist for each given ratio allowing for extremely complex interval fluctuation dynamics (Keener, 1981). However, note that certain stable interval lengths exist along the "line of identity" where  $T_i = T_{i+1}$ .

#### **Type E (Non-linear Deterministic)**

This type produces Poincaré-plots which have less defined point-shaped cluster as the previously mentioned dynamic. Instead we find linear elements which cross the "line of identity" temporarily equal successive interval lengths occur. The development of interval lengths along certain trajectories is usually associated with directions which approach each other infinitesimally in certain fixed points in order to diverge along other directions. Elsewhere these directions were called stable and unstable manifolds to describe the local geometry around a local recurrence point (Garfinkel et al., 1992; Liebert, 1991). Consider Fig. 8 where certain exemplary interval lengths display a typical development: Until P908 interval lengths stay sufficiently close to a local recurrence point (RP) to remain stable. Interval 909 is somewhat shorter and the resulting P909 is already outside the RP. The successive interval lengths now diverge from the RP along an unstable manifold until P912 comes close to the stable manifold and the system approaches the RP along P913. This behaviour has been regarded to be typical of non-linear determinism (Denton et al., 1990,1991; Glass and Mackey, 1988). It was mentioned previously that the directions of mentioned manifold vary over the time (Garfinkel et al., 1992). However, in the investigation the employed Poincaré-plots integrated ventricular dynamics over a period of approximately one hour or in the case of several successive registrations even over several days. Thus, it can be postulated that a given dynamics remains stable for a considerable period of time.

Poincaré-plots are not subjective to non-Gaussian distributions and quite resistant against random noise in the acquired data. Furthermore, the registration of interval lengths is fairly effortless and little time-consuming using a common monitoring device in a single session. Moreover, they could be used in certain experimental settings. Thus, they represent an inexpensive method of data visualization.

It can be assumed that a single registration of interval lengths suffices to characterise the





# (all units are [ms])

FIGURE 8 Example of a non-linear deterministic registration with evidence of a stable (SM) and unstable manifold (UM): (a) shows the concerned part of the registration. Note that relatively stable regions are interspersed by interval lengths which undergo rapid changes. (b) is the specific Poincaré-plot of the data.

complexity of ventricular response using Poincaréplots. Apparently the observed dynamics are quite heterogeneous ranging from mainly stochastic processes to determined non-linear dynamics.

The implications of the differences in complexity could be relevant for a differential therapy as well as for the prognosis as future investigations have to show.

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