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Random Graphs, by V. F. Kolchin, 1999, Cambridge, New York, Melbourne: Cambridge University Press, 252 pp., subject index, 5 figures, US $80.00

The author, Valentin Fedorovich Kolchin, is a leading researcher in the area of probabilistic combinatorics, an associate at the Steklov Institute and a professor at the Moscow Institute of Electronics and Mathematics (MIEM). This is the fifth book he has produced, printed by Cambridge University Press as Volume 53 of the series edited by G.-C. Rota *Encyclopedia of Mathematics and Its Application*.

In the probabilistic approach, numerical characteristics of a random combinatorial object are random variables but when probability spaces with uniform distribution are used in spite of the probabilistic terminology the problems considered are in essence enumeration problems of combinatorial analysis. The book explores the role of probabilistic methods for solving enumerative combinatorial problems – finding expressions for number of objects (with special emphasis on random graphs) possessing a property under investigation. There exists a large body of literature on random graphs, including fundamental books by Bollobas and Palmer [1, 2]. In the series *Encyclopedia of Mathematics and Its Application* itself there are already several excellent books on subjects related to random graphs – the editor lists more than 30 such books. This book differs from other treatments of random graphs in the systematic use of a method called the generalized scheme of allocation that has been widely used in probabilistic combinatorics for the last 15 years. The scope of the book concerns not only random graphs but also related problems – systems of random linear equations in GF(2), random permutations and simple equations involving permutations – so the title of the book is to some extent misleading.

The book consists of 5 chapters. Each chapter ends with a very useful section Notes and References, summarizing the chapter and giving information about past development of the discussed subjects.

Chapter 1, "The generalized scheme of allocation and the components of random graphs", describes the method and its applications to a random forest of nonrooted trees, a random graph consisting of unicyclic components, and a random graph with a mixture of trees and unicyclic components. The method is so named because of its connection with the problem of assigning n objects randomly to N cells. Let $\eta_1, \dots, \eta_N$ be random variables that are, for example, the sizes of components of a graph. If there are independent random variables $\xi_1, \dots, \xi_N$ so that the joint distribution of $\eta_1, \dots, \eta_N$ for any integer $k_1, \dots, k_N$ can be written as

$$\mathbf{P}\{\eta_1 = k_1, \dots, \eta_N = k_N\} \\ = \mathbf{P}\{\xi_1 = k_1, \dots, \xi_N = k_N\, \xi_1 + \cdots + \xi_N = n\}$$

where n is a positive integer, then one says that $\eta_1, \dots, \eta_N$ satisfy the generalized scheme of allocation with parameters n and N and independent random variables $\xi_1, \dots, \xi_N$. So, the approach based on the generalized scheme of allocation reduces the investigation of equiprobable graphs to some problems con-cerning sums of independent random variables. For nonequiprobable graphs few results have been obtained because of the lack of effective methods. Nonequiprobable graphs are shortly discussed in Chapter 2.

In Chapter 2, "Evolution of random graphs", these results are applied to the study of the evolution of random graphs. Graph evolution is the random process of sequentially adding new edges to a graph. Studies of the evolution of random graphs were started by P. Erdös and A. Rényi in Hungary and by V. E. Stepanov in Russia in 1960's. In the book it is demonstrated that applying generalized schemes of allocation makes it possible to analyze random graphs at different stages of their evolution and to obtain limit distributions. The parameter $\theta = 2T/n$, where n and T denote the number of vertices and the number of edges in the labeled random graph under consideration, plays a decisive role in the behavior of random graphs, and it may be interpreted as time in the evolution of the graphs. Many of the graph characteristics change abruptly at the "critical point" $\theta = 1$; For example, the giant component appears – in chemical system appearance of the giant component corresponds to appearance of gel fraction in the solgel transition point (*cf.* [3]).

Chapter 3, "Systems of random linear equations in GF(2)", is devoted to an important branch of probabilistic combinatorics, developed mainly by Russian researchers like V. L. Goncharov, S. N. Bernstein, N. V. Smirnov, and V. E. Stepanov, who continue the famous Russian school of probability, founded by A. A. Markov, P. L. Lyapunov, A. Ya. Khinchin, and A. N. Kolmogorov. GF(2) denotes the field with elements 0 and 1. The matrix of a system of linear equations in GF(2) with random coefficients corresponds to a random graph or hypergraph, so results on random graphs help to study such systems in particular and systems of random equations in finite fields in general. The theory of random graphs provides a basis for obtaining the results on the system of random equations with coefficients taking their values with equal probabilities; only a few results have been obtained for systems with nonequiprobable coefficients.

Chapter 4, "Random permutations", and Chapter 5, "Equations containing an unknown permutation", contain some interesting results on random permutations, also obtained mainly by Russian researchers. A random permutation may be represented by a random graph whose vertex set is the set of the elements that undergo permutation and the edge set consists of arcs connecting each element with the one it is mapped onto; such graph consists of connected components that are cycles of the permutation. The generalized scheme allocation applied to the connected components of these graphs is used to study of random permutations, *e.g.*, to study the number of permutation cycles, while solving a simple permutation equation helps *e.g.*, in studying the order of a permutation. As other books of the *Encyclopedia of Mathematics and Its Applications* the book contains a thorough Bibliography on its subjects, listing 156 entries.

This book has been written for specialists working in the area of combinatorics and probability theory and partly those applying probabilistic combinatorics in fields like mathematical genetics, cryptology, communication theory. The book is far from being self-contained – reader's familiarity with graph theory and probabilistic combinatorics is silently assumed.

Probabilistic hypergraphs are only shortly mentioned in Chapter 3.1. despite the fact that probabilistic theory of hypergraphs has found applications in several problems touched in the book *e.g.*, concerning critical points in system evolution like sol-gel transition (*cf.* [3, 4]).

The math is not presented graphically, but rather in heavy mathematics of theorems and equations. Graph theory has found many applications in Physics, Chemistry, Economy exactly because the graphs used have had a graphical representation and so operations on the system modeled by the graphs might be represented as graphical transformations of the graphs and combinatorial problems as enumerations of certain classes of graphical objects instead of much more abstract operations on algebraic representations.

Nothing of this figurativeness is left in the book. There are only 5 figures in the whole volume, between them such like Figure 5.1.1. – graphs of elementary functions $\cos\varphi$ and $\cos 2\varphi$, which seems humorous in such an advanced book. On the other hand, no graphs presenting the considered systems are given.

This monograph should not be considered as a systematic lecture on probabilistic combinatorics but rather as a unifying collection of material on that subject. The book is based on results obtained primarily by Russian mathematicians. This is often the first English-language presentation of many of the results. Moreover, the presented materials is spread in various journals and therefore not well known on the whole. So the reviewer is convinced that books such as this should be written.

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