EXPONENTIAL STABILITY OF A KIND OF STOCHASTIC DELAY DIFFERENCE EQUATIONS

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Received 22 February 2006; Accepted 1 May 2006

We present a Razumilchin-type theorem for stochastic delay difference equation, and use it to investigate the mean square exponential stability of a kind of nonautonomous stochastic difference equation which may also be viewed as an approximation of a nonautonomous stochastic delay integrodifferential equations (SDIDEs), and of a difference equation arises from some of the earliest mathematical models of the macroeconomic "trade cycle" with the environmental noise.

1. Introduction

The problem of stability of stochastic difference equation has been investigated in a number of papers. We refer the readers to [2, 3, 13–17]. Some results on the asymptotic behavior of the moments were obtained in [18]. But very few results on the Razumilchin-type theorem for stochastic delay difference equation have been published. In this paper, we present a Razumilchin-type theorem for stochastic delay difference equation, and use it to investigate the mean square exponential stability of a kind of nonautonomous stochastic difference equation.

We consider the equation

$$X_{n+1} = a(n, X_{n-m}, \dots, X_n) + b(n, X_{n-m}, \dots, X_n) \triangle\mu_n, \quad n \in \mathbb{Z}^+, \tag{1.1}$$

where $a \in \mathbb{R}$, $b \in \mathbb{R}$, $a(n,0,\dots,0) = b(n,0,\dots,0) = 0$, $\mathbb{Z}^+ = \{0,1,\dots\}$, and $X_n = \xi_n$ ($n \in I = \{-m, -m+1, \dots, -1, 0\}$) is the initial segment to be $\mathcal{F}_0$-measurable. $\triangle\mu_n$ are independent $N(0,1)$-distributed Gaussian random variables.

We denote by $(\Omega, \mathcal{F}, \{\mathcal{F}_n\}_{n\in\mathbb{N}}, \mathbb{P})$ a complete filtered probability space, where filtration $\{\mathcal{F}_n\}_{n\in\mathbb{N}}$ is naturally generated: $\mathcal{F}_{n+1} = \sigma\{\triangle\mu_{i+1} : i = 0,1,\dots,n\}$. Among all the sequences $\{X_n\}_{n\in\mathbb{N}}$ of the random variables, we distinguish those for which X_n are $\mathcal{F}_n$-measurable for all $n \in \mathbb{N}$.

Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2006, Article ID 94656, Pages 1–9
DOI 10.1155/DDNS/2006/94656

2. Main result

Definition 2.1. The stochastic difference equation (1.1) is said to be pth moment exponentially stable if there are positive constants γ and N such that with initial data ξ_n, $n \in I$,

$$E|X_n|^p \leq NE\|\xi\|^p e^{-\gamma n} \quad \text{on } n \in \mathbb{Z}^+, \tag{2.1}$$

where $\|\xi\| = \max_{n\in I}|\xi_n|$.

THEOREM 2.2. *Let all λ, p, c_1, c_2 be positive numbers and $q > 1$, suppose there exists a Lyapunov function $V : I \cup \mathbb{Z}^+ \times \mathbb{R} \to \mathbb{R}^+$ such that*

$$c_1|X|^p \leq V(t,X) \leq c_2|X|^p \quad \forall (n,X) \in I \cup \mathbb{Z}^+ \times \mathbb{R} \tag{2.2}$$

and for all $n \in \mathbb{Z}^+$,

$$E\triangle V(n,X_n) = E(V(n+1,X_{n+1}) - V(n,X_n)) \leq -\lambda EV(n+1,X_{n+1}) \tag{2.3}$$

if

$$EV(n+s,X_{n+s}) < qEV(n+1,X_{n+1}) \quad \text{on } s \in I. \tag{2.4}$$

Then for all ξ_n, $n \in I$, $\|\xi\| = \max_{n\in I}|\xi_n|$,

$$E|X_n|^p \leq \frac{c_2}{c_1}E\|\xi\|^p e^{-\gamma n} \quad \text{on } n \in \mathbb{Z}^+, \tag{2.5}$$

where $0 < \gamma = \min\{\log(1+\lambda), \log q/(1+m)\}$.

Proof. Let

$$U(n) = \sup_{s\in I}\{e^{\gamma(n+s)}EV(n+s,X_{n+s})\}, \quad n \in \mathbb{Z}^+. \tag{2.6}$$

For any $n \in \mathbb{Z}^+$, we affirm that

$$\triangle U(n) = U(n+1) - U(n) \leq 0. \tag{2.7}$$

Otherwise, there exists an $n \in \mathbb{Z}^+$ such that

$$e^{\gamma(n+1)}EV(n+1,X_{n+1}) > U(n), \tag{2.8}$$

that is,

$$e^{\gamma(n+1)}EV(n+1,X_{n+1}) > e^{\gamma(n+s)}EV(n+s,X_{n+s}), \quad s \in I. \tag{2.9}$$

For any $s \in I$, the inequality (2.4) implies

$$EV(n+s,X_{n+s}) < e^{\gamma(m+1)}EV(n+1,X_{n+1}) < qEV(n+1,X_{n+1}) \tag{2.10}$$

since $\gamma < \log q/(1+m)$. Thus, by condition (2.3),

$$E\triangle V(n,X_n) = E\big(V(n+1,X_{n+1}) - V(n,X_n)\big) \le -\lambda EV(n+1,X_{n+1}). \tag{2.11}$$

We obtain that

$$EV(n+1,X_{n+1}) \le \frac{1}{1+\lambda}EV(n,X_n). \tag{2.12}$$

Multiplying both sides of above inequality by $e^{\gamma(n+1)}$ and noting that $\gamma \le \log(1+\lambda)$, we can get

$$e^{\gamma(n+1)}EV(n+1,X_{n+1}) \le e^{\gamma(n+1)}\frac{1}{1+\lambda}EV(n,X_n) \le e^{\gamma n}EV(n,X_n). \tag{2.13}$$

From the definition of $U(n)$, we have

$$e^{\gamma(n+1)}EV(n+1,X_{n+1}) \le U(n). \tag{2.14}$$

It is a contradiction with assumption (2.8), therefore, (2.7) holds. Thus, it follows from (2.7) immediately that

$$U(n) \le U(0), \quad \forall n \in \mathbb{Z}^+. \tag{2.15}$$

From the definition of $U(n)$ and the condition (2.2), we obtain that the inequality (2.5) holds. □

3. Application

3.1. A numerical approximation to SDIDEs. We consider the stochastic delay difference equation

$$\begin{gathered} X_{n+1} = X_n + h\left[a(n)X_n + h\sum_{i=1}^{m}K(n,i)X_{n-i}\right] + \sqrt{h}c(n)X_{n-m}\triangle\mu_n, \quad n \in \mathbb{Z}^+, \\ X_n = \xi_n, \quad n \in I, \end{gathered} \tag{3.1}$$

where $h > 0$ is a nonrandom parameter. For the functions $a(\cdot)$, $b(\cdot)$, and $K(\cdot,\cdot)$, suppose that

$$\begin{gathered} A(m,n,h) = 2a(n) + a^2(n)h + m\,|\,1+a(n)h\,|\,h^2, \\ B(m,n,h) = c^2(n) + h\big(mh^2 + |\,1+a(n)h\,|\big)\sum_{i=1}^{m}K^2(n,i) \end{gathered} \tag{3.2}$$

satisfy the conditions (H1) and (H2) as the following:

(H1) $A(m,n,h)+B(m,n,h) \le -\delta(m,h) < 0$ for $n \in \mathbb{Z}^+$;

(H2) $-1 < h\mu(m,h) \le hA(m,n,h)$.

Let

$$q(m,h) = \inf_{n\in\mathbb{Z}^+}\left\{\frac{B(m,nh)-A(m,n,h)}{2B(m,n,h)}\right\},$$
$$\lambda(m,h) = -\inf_{n\in\mathbb{Z}^+}\left\{\frac{A(m,n,h)+B(m,n,h)}{2(1+hA(m,n,h))}\right\}. \tag{3.3}$$

From the conditions (H1) and (H2), we know that

$$q(m,h) > 1, \qquad \lambda(m,h) > 0. \tag{3.4}$$

Equation (3.1) may also be viewed as an approximation of the stochastic delay integrodifferential equation

$$dX(t) = \left[a(t)X(t) + \int_{t-\tau}^{t} K(t,t-\theta)X(\theta)d\theta\right]dt + c(t)X(t-\tau)dW(t), \quad t > 0,$$
$$X(t) = \xi(t), \quad t \in [-\tau,0], \tag{3.5}$$

where $W(t)$ is a standard Brownian motion. Here, setting $h = \tau/m$ and approximating the differential part of (3.5) with the Euler-Maruyama method and the integral part with composite left-side rectangle rule [12], $t_n = nh$, write X_n for an approximate value to $X(nh)$, and use X_{n-m} to approximate the delayed argument $X(t_n - \tau)$. When $n \in I = \{-m,-m+1,\dots,-1,0\}$, we have $X_n = \xi(t_n)$. Moreover, the increments $\sqrt{h}\triangle\mu_n := W(t_{n+1}) - W(t_n)$ are independent $N(0,h)$-distributed Gaussian random variables, so $\triangle\mu_n$ are independent $N(0,1)$-distributed Gaussian random variables. We assume X_n to be $\mathcal{F}_n$-measurable at the mesh-points t_n. It is therefore to be hoped for h sufficiently small that solutions of (3.1) have similar asymptotic properties to those of (3.5). A statement of these asymptotic results for stochastic delay differential equations can be found in, for example, [1, 4–11].

Here, we use the above Razumilchin-type Theorem 2.2 to study the moment exponential stability of (3.1).

Theorem 3.1. *Assume conditions (H1) and (H2) are satisfied, the solution sequence $\{X_n\}$ produced by the difference equation (3.1) satisfies*

$$E|X_n|^2 \le E\|\xi\|^2 e^{-\gamma(m,h)nh} \quad \text{on } n \in \mathbb{Z}^+, \tag{3.6}$$

where $0 < \gamma(m,h) = \min\{\log(1+h\lambda(m,h))/h, \log q(m,h)/h(1+m)\}$.

Proof. Define a Lyapunov function $V(n,X) = |X|^2$. Clearly, the condition (2.2) in Theorem 2.2 is satisfied naturally with $c_1 = c_2 = 1$, $p = 2$. We see from (3.1) that

$$\begin{aligned}
X_{n+1}^2 &= (1+a(n)h)^2X_n^2 + h^4\left(\sum_{i=1}^{m}K(n,i)X_{n-i}\right)^2 + (\sqrt{h}c(n)\triangle\mu_n)^2X_{n-m}^2 \\
&\quad + 2(1+a(n)h)h^2\sum_{i=1}^{m}K(n,i)X_{n-i}X_n + 2(1+a(n)h)\sqrt{h}c(n)\triangle\mu_nX_nX_{n-m} \\
&\quad + 2h^2\sqrt{h}c(n)\triangle\mu_n\sum_{i=1}^{m}K(n,i)X_{n-i}X_{n-m} \\
&\le (1+a(n)h)^2X_n^2 + mh^4\sum_{i=1}^{m}K^2(n,i)X_{n-i}^2 + (\sqrt{h}c(n)\triangle\mu_n)^2X_{n-m}^2 \\
&\quad + |1+a(n)h|h^2\sum_{i=1}^{m}(K^2(n,i)X_{n-i}^2 + X_n^2) + 2(1+a(n)h)\sqrt{h}c(n)\triangle\mu_nX_nX_{n-m} \\
&\quad + 2h^2\sqrt{h}c(n)\triangle\mu_n\sum_{i=1}^{m}K(n,i)X_{n-i}X_{n-m} \\
&= \left[(1+a(n)h)^2 + m|1+a(n)h|h^2\right]X_n^2 \\
&\quad + (\sqrt{h}c(n)\triangle\mu_n)^2X_{n-m}^2 + \sum_{i=1}^{m}\left[mh^4 + |1+a(n)h|h^2\right]K^2(n,i)X_{n-i}^2 \\
&\quad + 2(1+a(n)h)\sqrt{h}c(n)\triangle\mu_nX_nX_{n-m} + 2h^2\sqrt{h}c(n)\triangle\mu_n\sum_{i=1}^{m}K(n,i)X_{n-i}X_{n-m}.
\end{aligned} \tag{3.7}$$

Note that $E(\triangle\mu_n) = 0$, $E[(\triangle\mu_n)^2] = 1$ and X_n, X_{n-k} are $\mathcal{F}_n$-measurable, Hence

$$\begin{aligned}
E(\triangle\mu_nX_{n-k}X_{n-m}) &= E(X_{n-k}X_{n-m}E(\triangle\mu_n \mid \mathcal{F}_n)) = 0, \\
E(\triangle\mu_nX_{n-m}^2) &= E(X_{n-m}^2E(\triangle\mu_n \mid \mathcal{F}_n)) = 0, \\
E(\triangle\mu_n^2X_{n-m}^2) &= E(X_{n-m}^2E(\triangle\mu_n^2 \mid \mathcal{F}_n)) = E(X_{n-m}^2).
\end{aligned} \tag{3.8}$$

From (3.7), (3.8) we get that

$$\begin{aligned}
EV(n+1,X_{n+1}) \le{}& \left[(1+a(n)h)^2 + m|1+a(n)h|h^2\right]EV(n,X_n) \\
&+ (\sqrt{h}c(n))^2EV(n-m,X_{n-m}) \\
&+ h^2(mh^2 + |1+a(n)h|)\sum_{i=1}^{m}K^2(n,i)EV(n-i,X_{n-i}).
\end{aligned} \tag{3.9}$$

Let $q = q(m,h)$, from assumption (2.4), we also have

$$\begin{aligned} EV(n+1,X_{n+1}) &\le \left[(1+a(n)h)^2 + m\,|1+a(n)h|\,h^2\right]EV(n,X_n) \\ &\quad + hq(m,h)\left[c^2(n) + h\left(mh^2 + |1+a(n)h|\right)\sum_{i=1}^{m} K^2(n,i)\right]EV(n+1,X_{n+1}) \\ &= (1+hA(m,n,h))EV(n,X_n) + hq(m,h)B(m,n,h)EV(n+1,X_{n+1}) \\ &\le (1+hA(m,n,h))EV(n,X_n) + h\frac{B(m,n,h)-A(m,n,h)}{2}EV(n+1,X_{n+1}), \end{aligned} \tag{3.10}$$

that is,

$$E\triangle V(n,X_n) \le h\frac{B(m,n,h)-A(m,n,h)}{2(1+hA(m,n,h))}EV(n+1,X_{n+1}). \tag{3.11}$$

By the definition of $\lambda(m,h)$ and (3.4), we get

$$E\triangle V(n,X_n) \le -h\lambda(m,h)EV(n+1,X_{n+1}). \tag{3.12}$$

Therefore, the inequality (3.6) holds by Theorem 2.2. □

3.2. Models of macroeconomics. Consider the following nonlinear delay difference equation:

$$x(n+1) = cx(n) + f(x(n) - x(n-m)) + \varepsilon x(n)\triangle\mu_n, \tag{3.13}$$

where $c \in [0,1)$ and ε are constants, m is a positive integer. $\triangle\mu_n$ are independent $N(0,1)$-distributed Gaussian random variables. We assume that $x(n)$ are $\mathcal{F}_n$-measurable for all $n \in \mathbb{N}$, and we have $x(n) = \xi_n$ when $n \in I$. $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(0) = 0$, $f(u) \neq 0$ for $u \neq 0$, and there exists a constant α such that

$$|f(u)| \le \alpha\,|u|. \tag{3.14}$$

Such equation arises from some of the earliest mathematical models of the macroeconomic "trade cycle" with the environmental noise.

Theorem 3.2. *Assume that the conditions (3.14) and*

$$0 \le c < -2\alpha + \sqrt{2\alpha^2 - \varepsilon^2 - 2\alpha + 1} \tag{3.15}$$

are satisfied. Then there exists positive constants γ such that with initial data ξ_n, $n \in I$,

$$E|X(n)|^2 \le E\|\xi\|^2 e^{-\gamma n} \quad on\ n \in \mathbb{Z}^+. \tag{3.16}$$

Proof. Also define a Lyapunov function $V(n,x) = |x|^2$. Similar to the proof of Theorem 3.1, let

$$q = \frac{1 - c^2 - 2\alpha c - 2\alpha^2 - \varepsilon^2 + 2\alpha}{2\alpha(2+c)},$$
$$\lambda = \frac{[2(2+c)\alpha - 1](c^2 + 2\alpha^2 + \varepsilon^2 + 3c\alpha) + 1}{2(2+c)\alpha(c^2 + 2\alpha^2 + \varepsilon^2 + 3c\alpha)}, \tag{3.17}$$

the inequality (3.16) with $\gamma = \min\{\log(1+\lambda), \log q/(1+k)\}$ can be completed by Theorem 2.2 easily. □

4. Examples

4.1. A numerical approximation to autonomous SDIDEs. Consider the autonomous stochastic delay integrodifferential equation as (3.5),

$$dX(t) = \left[aX(t) + b\int_{t-\tau}^{t} X(\theta)d\theta\right]dt + \sqrt{h}cX(t-\tau)dW(t), \quad t > 0,$$
$$X(t) = \xi(t), \quad t \in [-\tau, 0]. \tag{4.1}$$

Approximating the differential part of (3.5) with the Euler-Maruyama method and the integral part with composite left-side rectangle rule, we get the difference equation as follows:

$$X_{n+1} = X_n + \left(aX_n + bh\sum_{k=1}^{m} X_{n-k}\right)h + \sqrt{h}cX_{n-m}\triangle\mu_n, \quad n \in \mathbb{Z}^+. \tag{4.2}$$

Here, $h = \tau/m$ and

$$A(m,n,h) = 2a + a^2h + \tau h|1 + ah|,$$
$$B(m,n,h) = c^2 + b^2\tau(\tau h + |1 + ah|). \tag{4.3}$$

Since

$$1 + hA(m,n,h) = (1 + ah)^2 + \tau h|1 + ah| > 0, \tag{4.4}$$

so that the (H2) is satisfied, and if

$$2a + c^2 + b^2\tau + (a^2 + \tau + b^2\tau^2 - ab^2\tau)h - a\tau h^2 < 0, \tag{4.5}$$

then (H1) also is satisfied.

By letting

$$q(m,h) = \frac{B(m,nh) - A(m,n,h)}{2B(m,n,h)},$$
$$\lambda(m,h) = -\frac{A(m,n,h) + B(m,n,h)}{2(1 + hA(m,n,h))}. \tag{4.6}$$

From Theorem 3.1 we know that the inequality (3.6) holds.

From the above analysis and (4.5), we can get the theorem as follows.

Theorem 4.1. *Assume condition $2a+c^2+b^2\tau<0$ is satisfied, then there exists an $h^*>0$ such that for all $0<h<h^*$, the solution sequence $\{X_n\}$ produced by (4.2) satisfies*

$$E\left|X_n\right|^2 \leq E\|\xi\|^2 e^{-\gamma(m,h)nh} \quad \text{on } n\in\mathbb{Z}^+, \tag{4.7}$$

where $0<\gamma(h)=\min\{\log(1+h\lambda(h))/h,\log q(h)/(h+\tau)\}$, *and*

$$h^* = \frac{a^2+\tau+b^2\tau^2-ab^2\tau-\sqrt{\left(a^2+\tau+b^2\tau^2-ab^2\tau\right)^2+a\tau\left(2a+c^2+b^2\tau\right)}}{2a\tau}. \tag{4.8}$$

Corollary 4.2. *Assume condition $2a+c^2+b^2\tau<0$ is satisfied, then for any given $\varepsilon>0$, there exists an $h(\epsilon)>0$ such that for all $0<h<h(\epsilon)$, the numerical solution sequence $\{X_n\}$ produced by the numerical scheme (4.2) satisfies*

$$E\left|X_n\right|^2 \leq E\|\xi\|^2 e^{-(\gamma-\epsilon)nh} \quad \text{on } n\in\mathbb{Z}^+, \tag{4.9}$$

where $\gamma=\min\{\lambda,\log q/\tau\}$ *with* $\lambda=-(a+(1/2)(c^2+b^2\tau))$ *and* $q=1/2-a/(c^2+b^2\tau)$.

Proof. From Theorem 4.1, the corollary is a consequence of the fact that

$$\begin{aligned}\frac{\log(1+h\lambda(h))}{h} &= \lambda+O(h),\\ \frac{\log q(h)}{(h+\tau)} &= \frac{\log q}{\tau}+O(h).\end{aligned} \tag{4.10}$$

□

4.2. An example of macroeconomics models. Consider the difference equation

$$x(n+1)=cx(n)+0.25\sin\left(x(n)-x(n-3)\right)+0.25x(n)\triangle\mu_n, \tag{4.11}$$

we can get that if the condition

$$0\leq c\leq 0.25 \tag{4.12}$$

is satisfied, then the inequality (3.6) holds, with $\gamma=\min\{\log(1+\lambda),\log q/4\}$. Here

$$\begin{aligned}\lambda &= \frac{21/8-2c^2-c}{2+c},\\ q &= \frac{2+c\left(c^2+(3/4)c+3/16\right)}{(2+c)\left(c^2+(3/4)c+3/16\right)}.\end{aligned} \tag{4.13}$$

Acknowledgment

This work is supported by the NSF of China (no.10271036) and of HIT(200518).

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