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Research Article

On a Discrete Epidemic Model

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We investigate the global asymptotic behavior of solutions of the difference equation $x_{n+1} = (1 - \sum_{j=0}^{k-1} x_{n-j})(1 - e^{-Ax_n})$, $n \in \mathbb{N}_0$, where $A \in (0,\infty)$, $k \in \{2,3,\dots\}$, and the initial values $x_{-k+1}, x_{-k+2}, \dots, x_0$ are arbitrary negative numbers. Asymptotics of some positive solutions of the equation are also found.

1. Introduction

The necessity for some techniques which can be used in investigating equations arising in mathematical models describing real-life situations in population biology and economics has increased the interest in studying nonlinear and rational difference equations for the last four decades, see, for example, [1–23] and the references therein.

In [8] the authors proposed the following research project.

Research project 6.71. Investigate the oscillatory behavior, the global stability, and periodic character of the solutions of the following equation:

$$x_{n+1} = \left(1 - \sum_{j=0}^{k-1} x_{n-j}\right)\left(1 - e^{-Ax_n}\right), \quad n \in \mathbb{N}_0, \tag{1.1}$$

where $A \in (0,\infty)$ and $k \in \{2,3,\dots\}$.

Equation (1.1) describes a discrete epidemic model. For some other biological models, see, for example, [6–9, 11, 13, 21] and the references therein.

In [23] Zhang and Shi address the research project for the case when all solutions are positive. It is easy to see that if $x_{-k+1}, x_{-k+2}, \dots, x_0$ are positive numbers such that

$\sum_{i=-k+1}^{0} x_i < 1$, then the corresponding solution of (1.1) is positive, that is, $x_n > 0$ for every $n = -k+1, \ldots, -1, 0, 1 \ldots$.

When $A \in (0,1]$, then it is easy to see that the equation $(1-x)(1-e^{-Ax}) = x$ has a unique solution $x = 0$ in the interval $[0,1]$, which implies that for each positive solution $(x_n)_{n\in\mathbb{N}}$ of (1.1),

$$x_{n+1} = \left(1 - \sum_{j=0}^{k-1} x_{n-j}\right)\left(1 - e^{-Ax_n}\right) < \left(1 - x_n\right)\left(1 - e^{-Ax_n}\right) < Ax_n \le x_n \tag{1.2}$$

for $n \ge 0$ (we have employed here that $\sum_{i=-k+1}^{0} x_i < 1$). Hence, every positive solution of (1.1) decreasingly converges to zero in this case.

They also show that the equation

$$\begin{aligned} 1 + \frac{A}{k} &= \exp\left(\frac{(2k+1)A + k - \sqrt{A^2 + 2k(2k+1)A + k^2}}{2k(2k+1)}\right) \\ &\quad + \frac{(2k+1)A + k - \sqrt{A^2 + 2k(2k+1)A + k^2}}{2k(2k+1)} \end{aligned} \tag{1.3}$$

has a unique root A^* for $A > 1$.

The main results in [23] can be formulated as follows.

Theorem 1.1. *Consider (1.1) with $A > 1$. The following statements are true.*

(a) *If $1 < A < A^*$, then every positive nonoscillatory solution of (1.1) eventually monotonically converges to a unique positive root $\overline{x}$, where $\overline{x} < 1/(k+1)$.*

(b) *If $A = A^*$, then every positive nonoscillatory solution of (1.1) eventually increasingly converges to the equilibrium $\overline{x}$.*

(c) *If $A > A^*$, then every nontrivial solution of (1.1) is strictly oscillatory about the equilibrium $\overline{x}$.*

Based on Theorem 1.1, it is interesting to answer the following question.

Question 1.2. Is there a monotone solution of (1.1) for the case $1 < A \le A^*$?

In this paper, we investigate the global stability of the negative solutions of (1.1). First, note that if $x_{-k+1}, x_{-k+2}, \ldots, x_0$ are negative numbers, then by the inequality $e^x > 1$ for $x > 0$, we have that $x_1 < 0$. If we assume that $x_j < 0$ for $-k+1 \le j \le n$, then from (1.1) and by the same inequality it follows that $x_{n+1} < 0$. Hence, by induction, we have that $x_n < 0$ for every $n \in \mathbb{N}$. Using the change $y_n = -x_n$, (1.1) becomes

$$y_{n+1} = \left(1 + \sum_{j=0}^{k-1} y_{n-j}\right)\left(e^{Ay_n} - 1\right), \tag{1.4}$$

where the initial values $y_{-k+1}, y_{-k+2}, \ldots, y_0$ are positive numbers.

Although (1.1) for the case of negative initial conditions (i.e., (1.4) for the case of positive initial conditions) perhaps does not represent a real life population model, it is important in its own right and addresses the above-mentioned research problem. It is also a prototype for the quite general equation appearing in Theorem 2.6, below.

Lemma 1.3. *The following statements are true.*

(a) *Assume that* $A \geq 1$. *Then the equation*

$$(1+kx)(e^{Ax}-1) = x \tag{1.5}$$

has a unique nonnegative root $\overline{x}_0 = 0$.

(b) *Assume that* $A \in (0,1)$. *Then (1.5) has a unique positive root* $\widehat{x}$.

Proof. (a) Let $g(x) = (1+kx)(e^{Ax}-1) - x$. Then $g(0) = 0$ and

$$g'(x) = (A+Akx+k)e^{Ax} - (k+1), \qquad g''(x) = A(A+Akx+2k)e^{Ax} > 0. \tag{1.6}$$

Hence $g'(x) > g'(0) = A-1 \geq 0$ for every $x > 0$, since $A \geq 1$, and consequently $g(x) > g(0) = 0$ when $x > 0$, from which the result follows in this case.

(b) If $A \in (0,1)$, then $g'(0) < 0$. Hence $g(x)$ is a convex function on the interval $[0,\infty)$, which decreases from 0 to a unique positive solution $\widetilde{x}$ of the equation $g'(x) = 0$, and increase from $\widetilde{x}$ to ∞, implying the result. □

2. Global stability of (1.4)

In this section, we prove some global convergence results concerning positive solutions of (1.4).

Theorem 2.1. *Assume that* $A \geq 1$. *Then every positive solution of (1.4) converges monotonically to* $+\infty$ *as* $n \to \infty$.

Proof. By the inequality $e^x - 1 > x$ for $x > 0$, it follows that

$$y_{n+1} = \left(1 + \sum_{j=0}^{k-1} y_{n-j}\right)(e^{Ay_n} - 1) > Ay_n \geq y_n, \tag{2.1}$$

that is, the sequence $(y_n)_{n\in\mathbb{N}}$ is monotonous. Hence there is a finite or infinite $\lim_{n\to\infty} y_n$. The former is impossible according to Lemma 1.3(a), from which the result follows. □

Remark 2.2. Note that from the proof of Theorem 2.1 we see that the following somewhat stronger result holds: assume that $A \geq 1$. Then every eventually positive solution of (1.4) converges eventually monotonically to $+\infty$ as $n \to \infty$.

Theorem 2.3. *Assume that* $A \in (0,1)$. *Then the following statements hold true.*

(a) *If* $0 < \max\{y_{-k+1}, y_{-k+2}, \ldots, y_0\} < \widehat{x}$, *then the solution of (1.4) converges to zero.*

(b) *If* $\min\{y_{-k+1}, y_{-k+2}, \ldots, y_0\} > \widehat{x}$, *then the solution of (1.4) converges to* $+\infty$.

(c) *If* $y_{-k+1} = y_{-k+2} = \cdots = y_0 = \widehat{x}$, *then* $y_n = \widehat{x}$ *for every* $n \in \mathbb{N}$.

Proof. (a) Let $M_0 = \max\{y_{-k+1}, y_{-k+2}, \ldots, y_0\}$ and

$$M_1 = (1+kM_0)(e^{AM_0} - 1). \tag{2.2}$$

From this and (1.4), we have

$$y_1 \le M_1 < \hat{x}. \tag{2.3}$$

On the other hand, by Lemma 1.3(b) $g(x) < 0$ when $x \in (0,\hat{x})$, which implies that $g(M_0) < 0$, that is, $M_1 < M_0$.

From this and (2.3), we have

$$y_2 \le (1+kM_0)(e^{AM_0}-1) = M_1 < \hat{x} \tag{2.4}$$

and similarly

$$y_i \le (1+kM_0)(e^{AM_0}-1) = M_1 < \hat{x}, \tag{2.5}$$

for every $i \in \{2,3,\dots,k\}$.

Now define a sequence $(M_n)_{n\in\mathbb{N}}$ inductively by

$$M_{n+1} = (1+kM_n)(e^{AM_n}-1). \tag{2.6}$$

Similar to above, by induction, we can obtain that

$$0 < y_{kn+i} \le M_{n+1} < M_n < \hat{x} \quad \text{for } n \in \mathbb{N} \tag{2.7}$$

and for every $i \in \{1,2,\dots,k\}$. As a monotonous and bounded sequence M_n converges, say to M. By Lemma 1.3, M is equal to zero, which implies that y_n converges to zero.

(b) Let $m_0 = \min\{y_{-k+1}, y_{-k+2},\dots,y_0\}$. Then similar to (a), we obtain

$$y_i \ge (1+km_0)(e^{Am_0}-1) = m_1 > \hat{x}, \quad i = 1,\dots,k. \tag{2.8}$$

Since $g(x) > 0$ for $x > \hat{x}$, we have that $m_1 > m_0$. Define a sequence $(m_n)_{n\in\mathbb{N}}$ by

$$m_{n+1} = (1+km_n)(e^{Am_n}-1). \tag{2.9}$$

It is easy to see, by induction, that

$$y_{kn+i} \ge m_{n+1} > m_n > \hat{x} \quad \text{for } n \in \mathbb{N} \tag{2.10}$$

and for each $i \in \{1,\dots,k\}$. Since m_n tends to $+\infty$ as $n \to \infty$ (note that by Lemma 1.3(b), $\hat{x}$ is a unique equilibrium of (2.9)), we have that $\lim_{n\to\infty} y_n = +\infty$, as desired.

(c) This statement is trivial. □

Remark 2.4. By some slight modification of the proofs of Theorem 2.3(a) and (b), it can be proved that if $\max\{y_{-k+1}, y_{-k+2},\dots,y_0\} = \hat{x}$ and there is an index $i_0 \in \{-k+1,-k+2,\dots,0\}$ such that $y_{i_0} < \hat{x}$, then Theorem 2.3(a) holds. Also if $\min\{y_{-k+1}, y_{-k+2},\dots,y_0\} = \hat{x}$ and there is an index $i_0 \in \{-k+1,-k+2,\dots,0\}$ such that $y_{i_0} > \hat{x}$, then Theorem 2.3(b) holds.

Remark 2.5. Note that Theorem 2.3(a) holds if we allow initial conditions to be nonnegative, that is, if $0 \le \max\{y_{-k+1}, y_{-k+2},\dots,y_0\} < \hat{x}$, then the solution of (1.4) converges to zero.

Similar to Theorem 2.3, it can be proved that the following extension of the theorem holds. The proof will be omitted.

Theorem 2.6. *Consider the difference equation*

$$x_{n+1} = f(x_n, \dots, x_{n-k+1}), \tag{2.11}$$

where f is a positive continuous function on $(0,\infty)^k$ increasing in each variable, the function $h(x) = f(x,\dots,x) - x$ has a unique positive root x^*, and

$$h(x)(x - x^*) > 0, \quad x \neq x^*. \tag{2.12}$$

Then the following statements hold true.

(a) *If $\max\{x_{-k+1}, x_{-k+2}, \dots, x_0\} \le x^*$, and there is an index $i_0 \in \{-k+1, -k+2, \dots, 0\}$ such that $x_{i_0} < x^*$, then the solution of (2.11) converges to zero.*

(b) *If $\min\{x_{-k+1}, x_{-k+2}, \dots, x_0\} \ge x^*$, and there is an index $i_0 \in \{-k+1, -k+2, \dots, 0\}$ such that $x_{i_0} > x^*$, then the solution of (2.11) converges to $+\infty$.*

(c) *If $x_{-k+1} = x_{-k+2} = \cdots = x_0 = x^*$, then $x_n = x^*$ for every $n \in \mathbb{N}$.*

Question 2.7. What can we say about global stability of those solutions of (1.4) whose initial values do not satisfy any of the three conditions mentioned in Theorem 2.3, that is, if some of them are strictly below $\hat{x}$ and some of them are strictly above $\hat{x}$?

A partial answer to this question is given by the following result. Before formulating the result, we define x_1^* as a unique positive root of the equation

$$(1+x)(e^{Ax} - 1) - x = 0, \tag{2.13}$$

where $A \in (0,1)$. The existence and uniqueness of the root follows from the proof of Lemma 1.3(b).

Theorem 2.8. *Assume that $A \in (0,\infty)$ and that (y_n) is a solution of (1.4) such that there is an $n_0 \ge 0$ such that $y_{n_0} \ge x_1^*$. Then $y_n \to \infty$ as $n \to \infty$.*

Proof. Let a sequence (z_n), $n \ge n_0$, be defined as follows:

$$z_{n+1} = (1+z_n)(e^{Az_n} - 1), \tag{2.14}$$

$z_{n_0} = y_{n_0}$.

Assume first that $y_{n_0} > x_1^*$. From this, (1.4), and (2.14), it follows that $y_{n_0+1} > z_{n_0+1}$. By induction, it can be easily proved that

$$y_n > z_n \quad \text{for } n \ge n_0 + 1. \tag{2.15}$$

Since $z_{n_0} > x_1^*$, by Theorems 2.1 and 2.3, case $k = 1$, it follows that $\lim_{n\to\infty} z_n = +\infty$. From this and (2.15), the result follows in the case.

If $y_{n_0} = x_1^*$, then from (1.4) we have that

$$y_{n_0+1} = \left(1 + \sum_{j=0}^{k-1} y_{n_0-j}\right)(e^{Ay_{n_0}} - 1) > (1+y_{n_0})(e^{Ay_{n_0}} - 1) = x_1^*. \tag{2.16}$$

So we can choose a sequence (z_n), $n \geq n_0 + 1$ defined by (2.14) with $z_{n_0+1} = y_{n_0+1}$ and repeat the above procedure.

We now return to consideration of positive solutions to (1.1).

3. Asymptotics of some positive solutions of (1.1) for the case $A \in (0,1]$

As we noted in Section 1, when $A \in (0,1]$ all positive solutions of (1.1) are decreasing and converge to zero. Moreover, we know that $x_{n+1} < Ax_n$ for $n \geq 0$, hence $x_n < A^n x_0$. An interesting question is whether there is a solution which satisfies the following asymptotic relationship:

$$x_n \sim A^n. \tag{3.1}$$

This will be considered by making use of a recent inclusion theorem due to Berg [3]. For closely related results, see, for example, [1–5, 10–17, 19].

Consider a general real nonlinear difference equation of order $m \geq 1$, of the form

$$F(x_n, \ldots, x_{n+m}) = 0, \tag{3.2}$$

where $F : \mathbb{R}^{m+1} \to \mathbb{R}$, $n \in \mathbb{N}_0$. Also, let φ_n and ψ_n be two sequences such that $\psi_n > 0$ and $\psi_n = o(\varphi_n)$ as $n \to \infty$. Then, under some conditions posed on the function F, for arbitrary $\varepsilon > 0$, there exist a solution x_n of (3.2) and an $n_0(\varepsilon) \in \mathbb{N}$, such that

$$\varphi_n - \varepsilon\psi_n \leq x_n \leq \varphi_n + \varepsilon\psi_n, \tag{3.3}$$

for $n \geq n_0(\varepsilon)$. The set of all sequences x_n satisfying (3.3) is called an asymptotic stripe $X(\varepsilon)$, that is, $y_n \in X(\varepsilon)$ implies the existence of a real sequence C_n with $y_n = \varphi_n + C_n\psi_n$ and $|C_n| \leq \varepsilon$ for $n \geq n_0(\varepsilon)$. Hints for the construction of the pairs φ_n, ψ_n can be found in [1–3].

The next theorem is the main result in [3]. (See also [4], for a correction of the proof.) □

Theorem 3.1 (see [3, Theorem 2.1]). *Let $F(w_0, w_1, \ldots, w_m)$ be continuously differentiable when $w_i = y_{n+i}$, for $i = 0, 1, \ldots, m$, and $y_n \in X(1)$. Let the partial derivatives of F satisfy*

$$F_{w_i}(y_n, \ldots, y_{n+m}) \sim F_{w_i}(\varphi_n, \ldots, \varphi_{n+m}) \tag{3.4}$$

as $n \to \infty$ uniformly in C_j for $|C_j| \leq 1$, $n \leq j \leq n+m$, so far as $F_{w_i} \not\equiv 0$. Assume that there exist a sequence $f_n > 0$ and constants $A_0, A_1, \ldots, A_m$ such that both

$$\begin{gathered} F(\varphi_n, \ldots, \varphi_{n+m}) = o(f_n), \\ \psi_{n+i} F_{w_i}(\varphi_n, \ldots, \varphi_{n+m}) \sim A_i f_n \end{gathered} \tag{3.5}$$

for $i = 0, 1, \ldots, m$ as $n \to \infty$, and suppose there exists an integer l, with $0 \leq l \leq m$, such that

$$|A_0| + \cdots + |A_{l-1}| + |A_{l+1}| + \cdots + |A_m| < |A_l|. \tag{3.6}$$

Then, for sufficiently large n, there exists a solution $(x_n)_{n \in \mathbb{N}_0}$ of (3.2) satisfying (3.3).

Using Theorem 3.1, now we find the asymptotics of some solutions of (1.1) for the case $A \in (0,1)$.

Theorem 3.2. *Let $A \in (0,1)$. Then (1.1) has a solution with the following asymptotics:*

$$x_n = t^n + bt^{2n} + o(t^{2n}) \tag{3.7}$$

for some $t \in (0,1)$ and some $b \neq 0$.

Proof. In order to apply Theorem 3.1, we write (1.1) in the following form:

$$F(x_n,\dots,x_{n+k}) = x_{n+k} - \left(1 - \sum_{j=1}^{k} x_{n+k-j}\right)\left(1 - e^{-Ax_{n+k-1}}\right) = 0. \tag{3.8}$$

Assume that a solution of (1.1) has the first two members in its asymptotic as follows:

$$\varphi_n = t^n + bt^{2n}. \tag{3.9}$$

Thus, we calculate $F(\varphi_n,\varphi_{n+1}\dots,\varphi_{n+k}) = F_n$. We have

$$\begin{aligned} F_n &= t^{n+k} + bt^{2n+2k} - \left(1 - \sum_{j=0}^{k-1}\left(t^{n+j} + bt^{2(n+j)}\right)\right) \\ &\quad \times \left(A\left(t^{n+k-1} + bt^{2(n+k-1)}\right) - \frac{A^2}{2}t^{2(n+k-1)} + o\left(t^{2(n+k-1)}\right)\right) \\ &= t^{n+k-1}(t-A) + t^{2n}\left(bt^{2k} - \left(Ab - \frac{A^2}{2}\right)t^{2k-2} + A\sum_{j=0}^{k-1} t^{k+j-1}\right) + o(t^{2n}). \end{aligned} \tag{3.10}$$

Taking $t = A$ and putting it into (3.10), we have that

$$F_n = A^{2n}\left(bA^{2k-1}(A-1) + \frac{1}{2}A^{2k} + \sum_{j=0}^{k-1} A^{k+j}\right) + o(A^{2n}). \tag{3.11}$$

If we choose

$$b = \frac{(1/2)A^{2k} + \sum_{j=0}^{k-1} A^{k+j}}{A^{2k-1}(1-A)}, \tag{3.12}$$

we obtain that

$$F_n = o(A^{2n}). \tag{3.13}$$

The partial derivatives of the function

$$F = F(w_0, w_1, \ldots, w_k) = w_k - \left(1 - \sum_{j=1}^{k} w_{k-j}\right)(1 - e^{-Aw_{k-1}}) \tag{3.14}$$

are

$$\begin{gathered} F_{w_i} = (1 - e^{-Aw_{k-1}}), \quad i \in \{0, 1, \ldots, k-2\}, \\ F_{w_{k-1}} = (1 - e^{-Aw_{k-1}}) - A\left(1 - \sum_{j=1}^{k} w_{k-j}\right)e^{-Aw_{k-1}}, \qquad F_{w_k} = 1. \end{gathered} \tag{3.15}$$

From this, it is easy to see that

$$F_{w_i}(y_n, \ldots, y_{n+k}) \sim F_{w_i}(\varphi_n, \ldots, \varphi_{n+k}) \tag{3.16}$$

as $n \to \infty$, uniformly in C_j for $|C_j| \le 1$, $n \le j \le n+k$, for every $y_n = \varphi_n + C_n\psi_n$, where $\psi_n = A^{2n}$.

Further we have

$$\psi_{n+i}F_{w_i}(\varphi_n, \ldots, \varphi_{n+k}) \sim A^{3n+2i+k}, \tag{3.17}$$

when $i \in \{0, 1, \ldots, k-2\}$,

$$\psi_{n+k-1}F_{w_{k-1}}(\varphi_n, \ldots, \varphi_{n+k}) \sim -A^{2n+2k-1}, \tag{3.18}$$

$$\psi_{n+k}F_{w_k}(\varphi_n, \ldots, \varphi_{n+k}) = A^{2n+2k}. \tag{3.19}$$

Hence, it is natural to choose $f_n = A^{2n}$, from which it follows that $A_i = 0$, $i \in \{0, 1, \ldots, k-2\}$, $A_{k-1} = -A^{2k-1}$, and $A_k = A^{2k}$.

From (3.16)–(3.19), we see that all conditions of Theorem 3.1 are satisfied with $\varphi_n = A^n + bA^{2n}$, $\psi_n = A^{2n}$, and $l = k-1$, which implies that (1.1) has a solution x_n satisfying the inequalities in (3.3) with such chosen φ_n and ψ_n. From this, it follows that the following asymptotics hold:

$$x_n = A^n + bA^{2n} + o(A^{2n}), \tag{3.20}$$

where b is given by (3.12).

In view of our results in [17], the following question is interesting for the case $A = 1$. □

Question 3.3. Assume that $A = 1$. Is there a solution of (1.1) with the following asymptotic:

$$x_n = \frac{a}{n} + \frac{b\ln n + c}{n^2} + \frac{d\ln^2 n + e\ln n}{n^3} + o\left(\frac{1}{n^3}\right) \tag{3.21}$$

for some real numbers a, b, c, d, and e?

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