Research Article

Application of Interval Arithmetic in the Evaluation of Transfer Capabilities by Considering the Sources of Uncertainty

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Total transfer capability (TTC) is an important index in a power system with large volume of inter-area power exchanges. This paper proposes a novel technique to determine the TTC and its confidence intervals in the system by considering the uncertainties in the load and line parameters. The optimal power flow (OPF) method is used to obtain the TTC. Variations in the load and line parameters are incorporated using the interval arithmetic (IA) method. The IEEE 30 bus test system is used to illustrate the proposed methodology. Various uncertainties in the line, load and both line and load are incorporated in the evaluation of total transfer capability. From the results, it is observed that the solutions obtained through the proposed method provide much wider information in terms of closed interval form which is more useful in ensuring secured operation of the interconnected system in the presence of uncertainties in load and line parameters.

1. Introduction

Due to the deregulation of the power industry, there has been an increasing interest in quantifying the transmission transfer capabilities of power systems. Interconnected systems are replacing isolated systems for economic and reliability reasons, but the resulting looped networks give rise to new technical concerns. Assessment of total transfer capability and available transfer capability (ATC) are such issues in a power system. ATC can be mathematically defined as the total transfer capability (TTC) minus the transmission reliability margin (TRM), minus the sum of existing commitments and the capacity benefit margin (CBM) [1].

In the past few years, a number of methods have been proposed for ATC determination. The literature of ATC calculation can be divided into two broad categories: deterministic methods and probabilistic methods. The deterministic methods can be further

divided into the continuation power flow (CPF) method [2], the repeated power flow (RPF) method [3], and the optimal power flow (OPF) methods [4, 5]. Starting from a solved base case, the CPF method obtains a series of power flow solutions by increasing the transfer parameter by using a prediction-correction scheme. RPF method starts from a base case and repeatedly solves the power flow equations, each time increasing the power transfer by a small increment until it reaches an operation limit. OPF method formulates the TTC calculation as an optimization problem, with equality constraints of power flow and inequality constraints from operation limits. The objective function, obviously, is to maximize power flow on the specified transmission route. These methods seem to be appropriate and efficient in management of the transmission system. However, because it fails to consider the probabilistic nature of the power system, the obtained ATC may be too conservative and therefore lead to costly and inefficient use of system resources.

The quantitative uncertainty is quantifiable in nature and is represented in numerical terms by a mathematical function with deterministic parameters. This could be achieved either by probabilistic variables or by interval variables. In the former, the uncertainty is defined by probability density functions [6, 7]. The application of the interval Arithmetic method for power flow analysis of transmission networks was first proposed in [8]. This concept has been applied for radial distribution system analysis [9, 10]. The basic idea in this paper is to extend the application of interval arithmetic in order to consider the probabilistic aspects of the system input parameters in the calculation of total transfer capability. The problem is formulated as an optimization problem, where the objective function is to maximize the power transfer between a specific set of generator(s) and load(s) without violating any of the system constraints.

The remainder of this paper is organized as follows: in Section 2, the problem formulation of TTC is presented; the functional procedure of the interval arithmetic is discussed in Section 3 and calculation of base case power flow is briefly discussed in Section 4; the results for two important case studies on transfer capability are discussed in Section 5 and finally the conclusions are summarized in Section 6.

2. Problem Formulation for Total Transfer Capabilty

The mathematical formulation of TTC can be expressed as follows.

The objective function for the OPF reflects the maximum power transfer from one bus/area to another bus/area.

Objective function is

$$f(x) = \text{Max} \left(\sum_{i \in S_D} \Delta P_{Di} \right). \tag{2.1}$$

The equality constraints of the OPF reflect the power system. The power system is enforced through the power flow equations which require that the net injection of real and reactive power at each bus is equal to zero.

Equality constraints are

$$P_{Gi} - P_{Di} - \sum_{j=1}^{n} |V_i||V_j|(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) = 0,$$

$$Q_{Gi} - Q_{Di} - \sum_{j=1}^{n} |V_i||V_j|(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) = 0. \tag{2.2}$$

The inequality constraints of the OPF reflect the limits created to ensure system security. The limiting constraints considered in this paper consist of three types: generation capacity, transmission line capacity, and voltage level.

The generators' maximum and minimum outputs for real and reactive powers are as follows:

$$
\begin{aligned}
|P_{Gi}|_{\min} &\leq |P_{Gi}| \leq |P_{Gi}|_{\max}, \\
|Q_{Gi}|_{\min} &\leq |Q_{Gi}| \leq |Q_{Gi}|_{\max}.
\end{aligned}
\tag{2.3}
$$

For the maintenance of system security, the transmission line MVA ratings are taken into account

$$
|S_{ij}| \leq |S_{ij}|_{\max}.
\tag{2.4}
$$

To maintain the quality of the electrical service and system security, bus voltage limits are considered

$$
|V_i|_{\min} \leq |V_i| \leq |V_i|_{\max},
\tag{2.5}
$$

where

ΔP_{Di}: is active power increment of load bus;

P_{Gi}, Q_{Gi}: are real and reactive power generation at bus i;

P_{Di}, Q_{Di}: are real and reactive load demand at bus i;

n: is bus number of the system;

V_i, V_j: are voltage magnitudes at bus i, j;

S_{ij}: is line MVA limit.

3. Interval Arithmetic Method

Interval arithmetic is a powerful tool to determine the effects of uncertain data. It can deal with numbers that vary within a range. The basic concepts of interval arithmetic are discussed as follows.

An interval number $X = [x_1, x_2]$ is the set of real numbers x such that $x_1 \leq x \leq x_2$. Here x_1 *and* x_2 are known as the lower limit and upper limit of the interval number, respectively. Let $X = [x_1, x_2]$ and $Y = [y_1, y_2]$ be the two interval numbers. The basic arithmetic operations of addition, subtraction, multiplication, and division of these two interval numbers are defined as follows [11]:

$$
\begin{aligned}
X + Y &= [x_1 + y_1, x_2 + y_2], \\
X - Y &= [x_1 - y_1, x_2 - y_2], \\
X * Y &= [\min(x_1 * y_1, x_1 * y_2, x_2 * y_1, x_2 * y_2), \\
&\qquad \max(x_1 * y_1, x_1 * y_2, x_2 * y_1, x_2 * y_2)], \\
X \div Y &= X * Y^{-1}.
\end{aligned}
\tag{3.1}
$$

However, for the purpose of power flow analysis, calculations are based on complex numbers rather than real numbers. The basic relations involving complex interval numbers are described as follows. In general, any complex number $Z = X + iY$—where i is the complex operator—is said to be a complex interval number if both its real and imaginary parts, X and Y, respectively, are interval numbers. Therefore X can be represented as $X = [x_1, x_2]$ and Y can be represented as $Y = [y_1, y_2]$.

Let $Z_1 = A_1 + iB_1$ and $Z_2 = A_2 + iB_2$ be the two complex interval numbers. The addition, subtraction, multiplication, and division of these two complex numbers are defined as follows:

$$
\begin{aligned}
Z_1 + Z_2 &= (A_1 + A_2) + i(B_1 + B_2), \\
Z_1 - Z_2 &= (A_1 - A_2) + i(B_1 - B_2), \\
Z_1 * Z_2 &= (A_1 * A_2 - B_1 * B_2) + i(A_1 * B_2 - A_2 * B_1), \\
Z_1 \div Z_2 &= C + iD,
\end{aligned}
\tag{3.2}
$$

where,

$$
\begin{aligned}
C &= (A_1 * A_2 + B_1 * B_2) \div (A_2{}^2 + B_2{}^2), \\
D &= (A_2 * B_1 + A_1 * B_2) \div (A_2{}^2 + B_2{}^2).
\end{aligned}
\tag{3.3}
$$

3.1. Interval Power Flow Analysis

The power flow analysis method used in this work is fast-decoupled load flow algorithm. However, to account for the uncertainty of the input load parameters, the real and reactive power loads are treated as interval numbers rather than fixed numbers. Similarly, to account for the uncertainty of the input line parameters, the resistance and reactance of the line are treated as interval numbers. Consequently, the complex arithmetic has been replaced by complex interval arithmetic

$$
\begin{aligned}
P(K) &= [P(k)_\ell, P(k)_u], \qquad Q(k) = [Q(k)_\ell, Q(k)_u], \\
R(jj) &= [R(jj)_\ell, R(jj)_u], \qquad X(jj) = [X(k)_\ell, X(jj)_u], \\
V(k) &= [V(k)_\ell, V(k)_u],
\end{aligned}
\tag{3.4}
$$

where,

$P(k)$ is real power load of kth bus;

$Q(k)$ is reactive power load of kth bus;

$V(k)$ is voltage magnitude at kth bus;

$R(jj)$ is resistance of the branch—jj;

$X(jj)$ is reactance of the branch—jj;

$P(k)_\ell$ and $P(k)_u$ are lower and upper limits of real power load of kth bus;

$Q(k)_\ell$ and $Q(k)_u$ are lower and upper limits of reactive power load of kth bus;

$R(jj)_\ell$ and $R(jj)_u$ are lower and upper limits of resistance of a particular branch—jj;

$X(jj)_\ell$ and $X(jj)_u$ are lower and upper limits of reactance of a particular branch—jj.
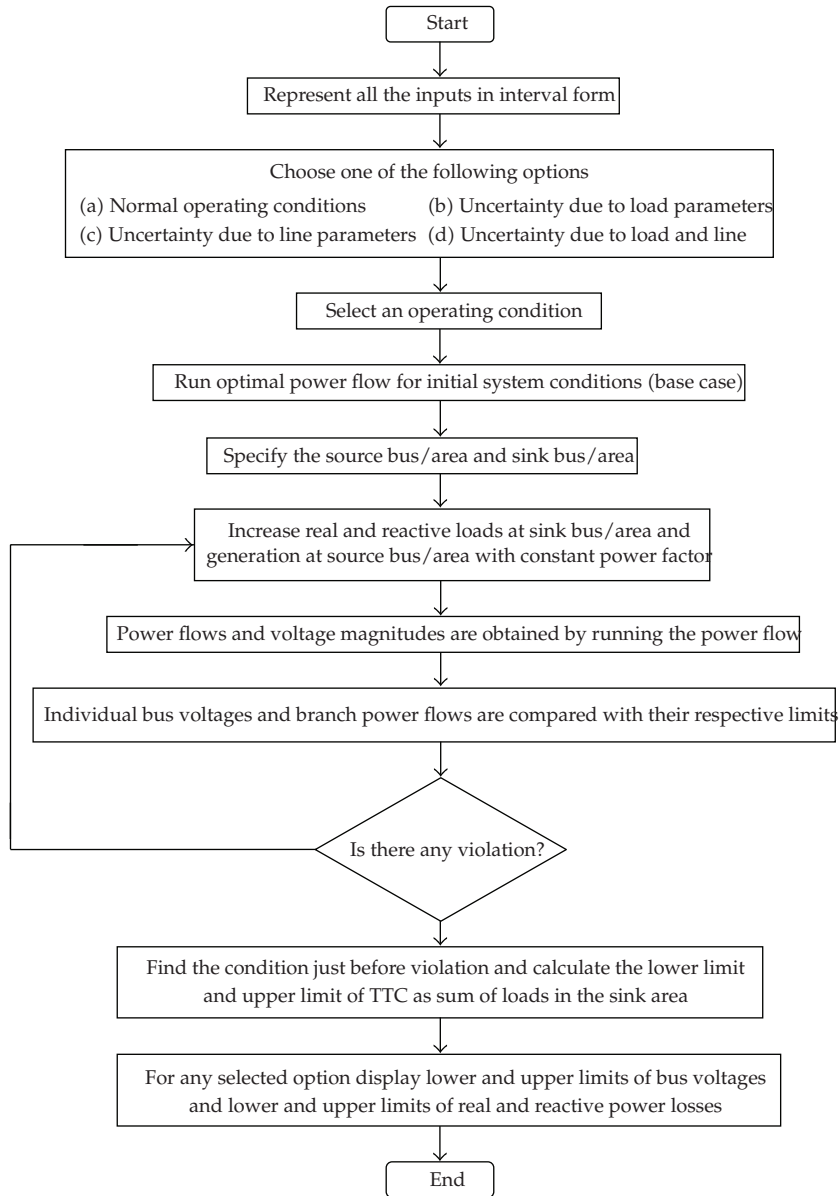
Figure 1: Flow diagram for the computation of ATC.

3.2. Uncertainty Parameters in Load and Line Variation Modeling

The system is assumed to operate under normal conditions but line and load parameters vary within a certain range. In this paper, ±10% variations in load parameters and ±3% for line parameters are considered from their rated nominal value

$$P(k)_\ell = 0.90P(k), \qquad P(k)_u = 1.10P(k),$$
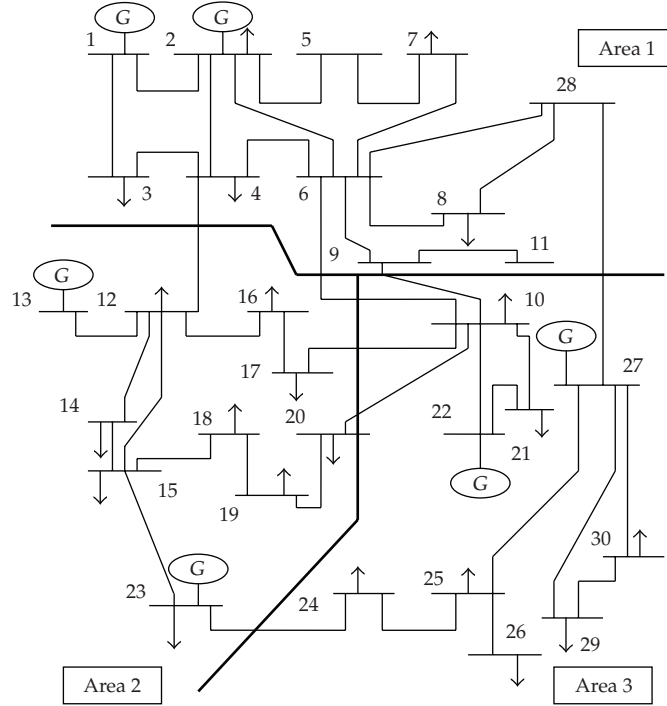$$Q(k)_\ell = 0.90P(k), \qquad Q(k)_u = 1.10Q(k),$$

Figure 2: IEEE 30 bus test system.

$$R(jj)_\ell = 0.97R(jj), \qquad R(k)_u = 1.03R(jj),$$

$$X(jj)_\ell = 0.97P(jj), \qquad X(k)_u = 1.03X(jj).$$

$$(3.5)$$

Various case studies have been conducted and the results of the following cases are presented:

   (i) the system under normal operating conditions,

  (ii) uncertainties due to load parameters only,

 (iii) uncertainties due to line parameters only,

 (iv) uncertainties due to both load and line parameters.

The basic steps used for computing the TTC using interval arithmetic for each transaction are given in Figure 1.

4. Basecase Power Flow Calculation

The IEEE 30 bus system shown in Figure 2 is adopted to illustrate the proposed method. The system is divided into 3 areas. The system has six generators, with two generators in each area. We assume each area as a utility. The utility in a certain area wants to import power from another area. Hence TTC is evaluated between areas. The base case load flow for fixed values of input is given in Table 1.
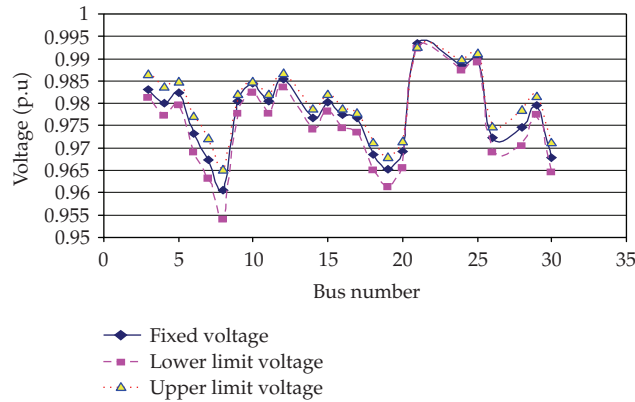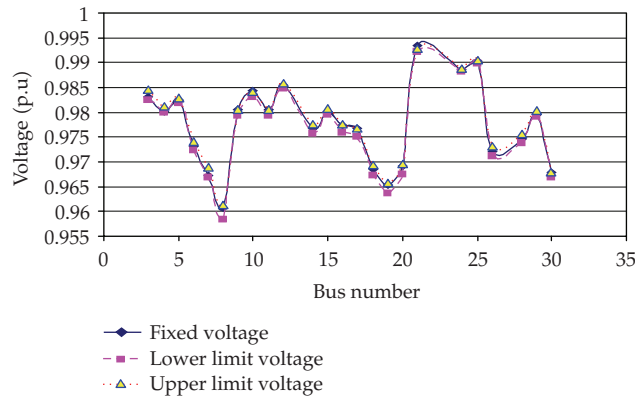
Figure 3: Voltage profile for load uncertainties.

Figure 4: Voltage profile for line uncertainties.

4.1. Base Case Power Flow Incorporating Various Uncertainties Using Interval Arithmetic

The transfer capability is computed from a base case constructed from the system information available at a given time. Usually, there is some uncertainty or inaccuracy in this computation because the transfer capability computed at the base case does not reflect evolving system conditions. These uncertainties include inaccurate or incorrect network parameters, effects neglected in the data and approximations in transfer capability computations. As already discussed in Section 3, the uncertainties in load and line parameters have been taken into account by assuming that they vary over a range. When the load demands in the system vary within some intervals, the bus voltages and other quantities such as line flows and line losses also vary within certain intervals. The intervals of variation of bus voltages for the system under consideration have been calculated for all the three uncertainties and are presented in Table 2. In this table, the symbols $V_{\ell\ell}$ and $V_{u\ell}$ denote the lower limit and upper limit of the voltage, respectively.

Figures 3 and 4 show the voltage profile for load uncertainty and line uncertainty for fixed, lower, and upper limit voltages. One can see that the load uncertainty has a considerable effect on the voltage profile compared with the line uncertainty. However, when
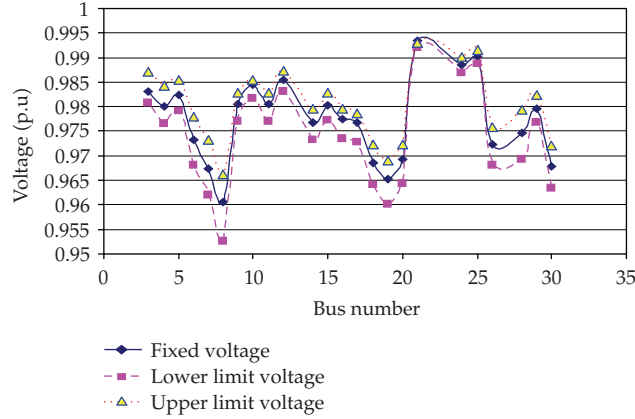
Figure 5: Voltage profile for load and line uncertainties.

both load and line uncertainties are taken into account, the effect is much more and is reflected in the voltage profile shown in Figure 5.

5. Results and Discussions

Several cases on the transfer capability are studied and from them the results of two important case studies are presented and discussed below.

5.1. Case 1: The Transfer Capability from Area 2 to 1

Using the proposed interval arithmetic method, TTC is evaluated for both fixed input parameters and for various uncertainties. The loads are modeled as constant power factor loads. The active loading (in MW) of area 1 before and after the transaction is shown in Table 3.

The generation of area 2 increases from 56.2 to 70.0 MW and the load at area 1 from 84.5 to 98 MW and the limiting condition was the overloading of lines 6–8. The sum of the loads at the sink area is taken as the TTC. Table 4 gives the total load of area 1 and total real and reactive losses in interval parameters. One may observe that the range of the total loss is higher for load uncertainty compared to line uncertainty. Obviously, the losses increase when both load and line uncertainties are taken into account.

Table 5 shows the values of different operating parameters after the transaction has been carried out. The TTC values are presented in interval form for all the categories of uncertainties. It is observed that the interval arithmetic method indicates a wider range of intervals.

5.2. Case 2: The Transfer Capability from Area 3 to 1

The generation of area 3 is increased proportionally when the loads in area 1 are increased. The generation at area 3 is increased from 84.5 to 94 MW. The load active power vector of area 1 in bus number sequence is [0.0, 27.7, 3.06, 9.70, 0.0, 0.0, 29.1, 38.29, 0.0, 0.0]. The limiting

Table 1: Base case load flow.

Bus number	Area number	V (p.u)	Angle (deg)	Generation		Load	
				P	Q	P	Q
1	1	1	0	25.97	0.99	0	0
2	1	1	−0.415491	60.97	31.99	21.7	12.7
3	1	0.983138	−1.522074	—	—	2.4	1.2
4	1	0.980093	−1.794728	—	—	7.6	1.6
5	1	0.982406	−1.863823	—	—	0	0
6	1	0.973184	−2.266957	—	—	0	0
7	1	0.967355	−2.651837	—	—	22.8	10.9
8	1	0.960624	−2.725769	—	—	30	30
9	1	0.980506	−2.996933	—	—	0	0
10	3	0.984404	−3.374936	—	—	5.8	2
11	1	0.980506	−2.996933	—	—	0	0
12	2	0.985468	−1.536912	—	—	11.2	7.5
13	2	1	1.476163	37	11.36	0	0
14	2	0.976677	−2.308035	—	—	6.2	1.6
15	2	0.980229	−2.311835	—	—	8.2	2.5
16	2	0.977396	−2.644486	—	—	3.5	1.8
17	2	0.976865	−3.392339	—	—	9	5.8
18	2	0.96844	−3.478388	—	—	3.2	0.9
19	2	0.965287	−3.958205	—	—	9.5	3.4
20	2	0.969166	−3.871024	—	—	2.2	0.7
21	3	0.993383	−3.488393	—	—	17.5	11.2
22	3	1	−3.392729	21.59	39.56	0	0
23	2	1	−1.589228	19.2	7.95	3.2	1.6
24	3	0.988566	−2.631461	—	—	8.7	6.7
25	3	0.990215	−1.689989	—	—	0	0
26	3	0.972194	−2.139346	—	—	3.5	2.3
27	3	1	−0.828439	26.91	10.54	0	0
28	1	0.974715	−2.265929	—	—	0	0
29	3	0.979597	−2.128498	—	—	2.4	0.9
30	3	0.967883	−3.041524	—	—	10.6	1.9

conditions are overloading in lines 6–8 and 21-22. It is observed that the TTC lower limit and upper limit vary over a wide range when both load and line uncertainties are taken into account. Table 6 shows the values of TTC and other system parameters in interval form after the transaction has been carried out.

6. Conclusion

The need for efficiency in electrical power deregulation has increased the need for improved calculations of TTC by incorporating the uncertainties in transmission. This paper proposes a probabilistic approach using interval arithmetic to estimate the TTC. The solution obtained from the interval arithmetic method is found to be more informative in qualitative terms about the system analysis when compared to the conventional deterministic approach. It is conjectured that power planners and operators may benefit not only from the expected

Table 2: Bus voltage magnitudes with different uncertainties.

Bus number	Fixed values	Load uncertainty		Line uncertainty		Load & line uncertainties	
		Lower limit $V_{\ell\ell}$ (p.u)	Upper limit $V_{u\ell}$ (p.u)	Lower limit $V_{\ell\ell}$ (p.u)	Upper limit $V_{u\ell}$ (p.u)	Lower limit $V_{\ell\ell}$ (p.u)	Upper limit $V_{u\ell}$ (p.u)
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
3	0.983138	0.981306	0.986537	0.982488	0.984535	0.980683	0.986972
4	0.980093	0.9773	0.983761	0.979993	0.981257	0.976548	0.984285
5	0.982406	0.979675	0.984989	0.981824	0.982977	0.978998	0.985475
6	0.973184	0.96903	0.977216	0.972369	0.974102	0.968013	0.977945
7	0.967355	0.963214	0.972314	0.966832	0.9689	0.962015	0.973196
8	0.960624	0.95401	0.965198	0.958474	0.961055	0.952516	0.966304
9	0.980506	0.977627	0.982225	0.979353	0.980634	0.9769	0.982787
10	0.984404	0.982294	0.984897	0.983113	0.984147	0.981728	0.98537
11	0.980506	0.977627	0.982225	0.979353	0.980634	0.9769	0.982787
12	0.985468	0.983673	0.986949	0.984867	0.985837	0.983129	0.98738
13	1	1	1	1	1	1	1
14	0.976677	0.974063	0.978931	0.975781	0.977289	0.973224	0.979605
15	0.980229	0.978056	0.982241	0.979542	0.980819	0.977344	0.98281
16	0.977396	0.974298	0.97883	0.975853	0.977355	0.973467	0.979506
17	0.976865	0.973539	0.977869	0.974976	0.976508	0.972696	0.978563
18	0.96844	0.965117	0.971416	0.967297	0.969315	0.963998	0.972322
19	0.965287	0.961235	0.968089	0.963586	0.965822	0.959998	0.969093
20	0.969166	0.965531	0.971485	0.967554	0.969544	0.964431	0.972382
21	0.993383	0.992167	0.992787	0.992253	0.992718	0.991922	0.993009
22	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1
24	0.988566	0.987247	0.989803	0.988173	0.988881	0.986853	0.990118
25	0.990215	0.989156	0.991279	0.989914	0.990527	0.988815	0.991552
26	0.972194	0.969092	0.974968	0.97117	0.972908	0.968128	0.975744
27	1	1	1	1	1	1	1
28	0.974715	0.970329	0.978686	0.973798	0.975448	0.969353	0.979372
29	0.979597	0.977483	0.981787	0.979001	0.980292	0.976763	0.98236
30	0.967883	0.964467	0.97125	0.966859	0.967859	0.963333	0.972154

Table 3: Active loading of area 1.

Bus number	2	3	4	7	8
Before	21.7	2.4	7.6	22.8	30.0
After	23.5	5.1	11.2	25.8	32.4

TTC evaluation, but also from an evaluation of confidence intervals of the calculation. The transfer capability from area 2 to 1 and 3 to 1 of the IEEE 30 bus test system is computed with fixed line and load, with line uncertainty, with load uncertainty, and with both line and load uncertainties. The total loss for the lower limit and upper limit is computed for all four types and compared. The results clearly elucidate that the total loss occurring is minimum when the bus is operated with line uncertainty. The obtained results are useful to ensure a secured

Table 4: Base case for Cases 1 and 2.

Varying parameters	Real load $P_{\ell\ell}$ (MW)	Real load $P_{u\ell}$ (MW)	Reactive load $Q_{\ell\ell}$ (MVAr)	Reactive load $Q_{u\ell}$ (MVAr)	Total real loss $P_{\ell\ell}$ (MW)	Total real loss $P_{u\ell}$ (MW)	Total reactive loss $Q_{\ell\ell}$ (MVAr)	Total reactive loss $Q_{u\ell}$ (MVAr)
Fixed line and load	84.5	84.5	56.4	56.4	2.44	2.44	8.99	8.99
Load uncertainty	76.05	92.95	50.86	62.14	1.96	3.215	7.34	11.47
Line uncertainty	84.5	84.5	56.4	56.4	2.382	2.542	8.72	9.30
Load and line uncertainty	76.05	92.95	50.86	62.14	1.845	3.322	7.10	11.85

Table 5: Interval values for the operating parameters in case 1.

Varying parameters	Total transfer capability Lower limit (MW)	Upper limit (MW)	Reactive load $Q_{\ell\ell}$ (MVAr)	$Q_{u\ell}$ (MVAr)	Total real loss $P_{\ell\ell}$ (MW)	$P_{u\ell}$ (MW)	Total reactive loss $Q_{\ell\ell}$ (MVAr)	$Q_{u\ell}$ (MVAr)	Total loss $S_{\ell\ell}$ (MVA)	$S_{u\ell}$ (MVA)
Fixed line and load	98	98	63.37	63.37	2.68	2.68	11.52	11.52	11.83	11.83
Load Uncertainty	88.89	108.65	59.45	72.64	2.22	3.874	8.78	14	9.06	14.53
Line Uncertainty	90.49	104.23	59.9	73.4	2.808	2.99	10.27	13.44	10.65	13.77
Load and Line Uncertainty	86.45	106.43	56.02	65.99	2.65	3.3	10.75	14.86	11.07	15.22

Table 6: Interval values for the operating parameters in case 2.

Varying parameters	Total Transfer capability Lower limit (MW)	Upper limit (MW)	Reactive load $Q_{\ell\ell}$ (MVAr)	$Q_{u\ell}$ (MVAr)	Total real loss $P_{\ell\ell}$ (MW)	$P_{u\ell}$ (MW)	Total reactive loss $Q_{\ell\ell}$ (MVAr)	$Q_{u\ell}$ (MVAr)	Total loss $S_{\ell\ell}$ (MVA)	$S_{u\ell}$ (MVA)
Fixed line and load	107.85	107.85	71.98	71.98	2.6	2.6	10.04	10.04	10.37	10.37
Load uncertainty	97.058	118.63	64.9	79.31	2.05	3.39	8.8	12.2	9.04	12.66
Line uncertainty	102.35	114.52	66.33	80.41	2.51	2.69	9.71	10.36	10.03	10.70
Load and line uncertainty	95.03	116.28	65.83	73.73	1.98	3.5	8.52	12.63	8.75	13.11

operation of the interconnected network when dealing with extensively large uncertainties in load and line parameters.

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