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Research Article

Accurate Computation of Periodic Regions' Centers in the General M-Set with Integer Index Number

Wang Xingyuan, He Yijie, and Sun Yuanyuan

College of Computer Science and Technology, Dalian University of Technology, Dalian 116024, China

Correspondence should be addressed to Wang Xingyuan, ptxylr163@126.com

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This paper presents two methods for accurately computing the periodic regions' centers. One method fits for the general M-sets with integer index number, the other fits for the general M-sets with negative integer index number. Both methods improve the precision of computation by transforming the polynomial equations which determine the periodic regions' centers. We primarily discuss the general M-sets with negative integer index, and analyze the relationship between the number of periodic regions' centers on the principal symmetric axis and in the principal symmetric interior. We can get the centers' coordinates with at least 48 significant digits after the decimal point in both real and imaginary parts by applying the Newton's method to the transformed polynomial equation which determine the periodic regions' centers. In this paper, we list some centers' coordinates of general M-sets' k-periodic regions ($k = 3, 4, 5, 6$) for the index numbers $\alpha = -25, -24, \ldots, -1$, all of which have highly numerical accuracy.

1. Introduction

According to the idea of complex dynamic system theory presented by Julia and Fatou, the famous mathematician Mandelbrot constructed and studied the M-sets of complex mapping $z \leftarrow z^{\alpha} + c$ ($\alpha = 2$) utilizing computer graphics technologies [1]. During the last 20 years, people have researched the embedded-layer relationship and distribution of the bifurcation sequence and topological rule of periodic trajectories in the general M-sets with $\alpha \in \mathbb{R}$ and found there existed orderly structure within the M-sets [2–16]. For example, Álvarez et al. studied the location and number of each periodic region in M-sets [9]; Buchanan et al. studied the location of periodic region of the general M-sets with $\alpha = -2$ [11]; Geum and Kim analyzed the quantitative relationship of each periodic region in the general Mandelbrot sets with positive integer index number, and calculated the coordinates of periodic regions' centers [15]; The author studied the structure and distribution of the general M-sets with integer index number [16].

M-sets consist of different-period regions which constitute the fractal structures of the M-sets. The analysis on the stability of the maps uncovered new and unexpected algebraic properties of the periodic regions. The centers of the periodic regions are determined by the transformed polynomials we worked on. The motivation for computing the periodic regions' centers is provided by the need to consider the locations of the periodic regions and fractal structures of the M-sets, which are useful to understand the inner infinite structures of the M-sets.

On the basis of above research, we study the periodic region's centers in the general M-sets with integer index number, determine the relationship between each periodic region's number in the general M-sets with positive index number and negative index number, and then present a new method of calculating the coordinates of periodic regions' centers in the general M-sets with negative integer index number. Our research has good prospects in physics, information science, and other fields.

2. Periodic Region Theory of General M-Sets

Definition 2.1 (see [15]). Let $f_c(z) = z^\alpha + c$ for $\alpha \in \mathbb{Z}$ with $z, c \in \mathbb{C}$, then the general M-sets is defined to be the set

$$\mathrm{M} = \left\{ c \in \widehat{\mathbb{C}}, \lim_{k\to\infty} f_c^k(0) \neq \infty \right\}. \tag{2.1}$$

Definition 2.2 (see [15]). The sets defined by $P_m = \{c \in \widehat{\mathbb{C}} : c = re^{i\phi_m}, 0 \le r, \phi_m = m\pi/|\alpha - 1|\}$ for $m \in \{1, 2, \ldots, 2|\alpha - 1|\}$ are called the rays of symmetry. The set P_1 is called the principal ray of symmetry. As is shown in Figure 1(b).

Definition 2.3. The set $S_1 = \{c \in \widehat{\mathbb{C}} : c = re^{i\theta},\ 0 \le r,\ 0 < \theta \le \pi/|\alpha - 1|\}$ is called the principal symmetric sector, as is shown in Figure 1(b).

Theorem 2.4. *In the $\widehat{\mathbb{C}}$ parameter plane, M is symmetric about P_m.*

Proof. Let $c = \rho e^{i\phi}$ with $\rho \ge 0,\ 0 \le \phi < 2\pi,\ i = \sqrt{-1}$. For all $k \in \mathbb{N}$, there exists the following recursive relations:

$$T_{k+1}(\phi) = T_1(\phi)\left(1 + T_k(\phi)\right)^\alpha, \tag{2.2}$$

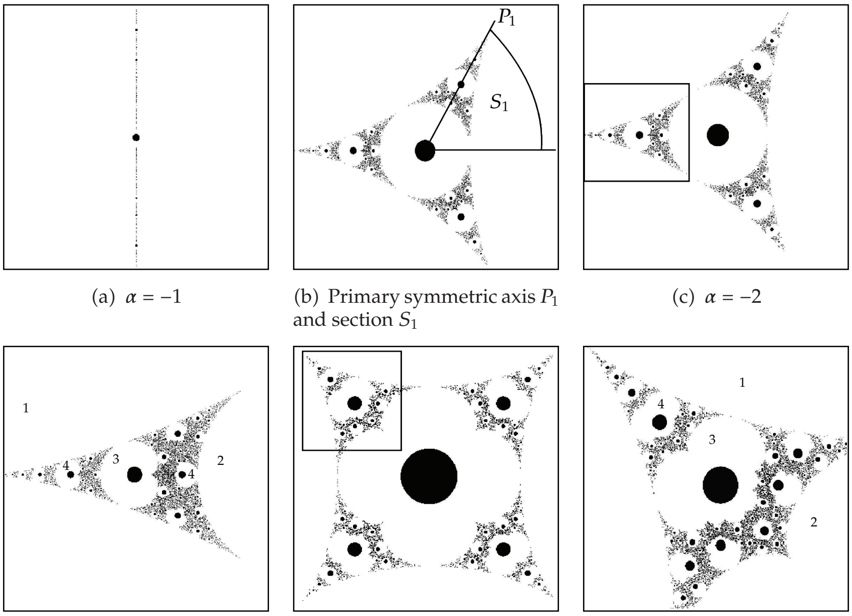
where $T_1(\phi) = c^{\alpha-1} = \rho^{\alpha-1}e^{i(\alpha-1)\phi}$. Using the mathematical induction, for all $k \in \mathbb{N}$ and $m \in \{1, 2, \ldots, 2|\alpha - 1|\}$,

$$\overline{T_k(\phi)} = T_k(-\phi) = T_k(2\phi_m - \phi) \tag{2.3}$$

exists. So we obtain $f_c^{k+1}(0) = c(1 + T_k)$ since

$$\begin{aligned} \left|f_c^{k+1}(0)\right| &= \rho\left|1 + T_k(\phi)\right| = \rho\left|\overline{1 + T_k(\phi)}\right| = \rho\left|1 + \overline{T_k(\phi)}\right| = \rho\left|1 + T_k(-\phi)\right| \\ &= \rho\left|1 + T_k(2\phi_m - \phi)\right| = \left|c^*\left(1 + T_k(2\phi_m - \phi)\right)\right| = \left|f_{c^*}^{k+1}(0)\right| \end{aligned} \tag{2.4}$$

with $c^* = \rho e^{i(2\phi_m - \phi)}$, we have $c^* \in \mathrm{M}$ whenever $c \in \mathrm{M}$. This completes the proof. □



(a) $\alpha = -1$

(b) Primary symmetric axis P_1 and section S_1

(c) $\alpha = -2$

(d) Partial enlargement of little block in Figure 1(c)

(e) $\alpha = -3$

(f) Partial enlargement of little block in Figure 1(e)

Figure 1: General M-sets and their periodic region.

Definition 2.5 (see [15]). An attracting k-periodic region is denoted by M'_k and is defined as a region of the set

$$\left\{c \in \mathbb{C} : \text{ there exist } z_0 \in \mathbb{C} \text{ such that } f_c^k(z_0) = z_0, \left|\frac{d}{dz} f_c^k(z)\right|_{z=z_0} < 1\right\}. \tag{2.5}$$

Definition 2.6. When α is a positive, if c_0 satisfying $f_{c_0}^k(0) = 0$ and $c_0 \in \mathrm{M}'_k$, then c_0 is called the center of an attracting k-periodic region.

Definition 2.7. When α is a negative, c_0 is the center of a stable k-periodic region only if ∞ belongs to this k-periodic region and c_0 satisfies $f_{c_0}^k(0) = 0$.

3. Calculation of Periodic Regions' Centers in General M-sets with Integer Index Number

3.1. Calculation Method

The center of periodic region can be located by numerically solving the governing equation $f_c^k(0) = 0$ which is a polynomial of c. The equation can be written as

$$f_c^k(0) = c\left(1 + g_k(c)\right), \tag{3.1}$$

where g_k is a recursive function defined as

$$g_{k+1}(c) = c^{\alpha-1}\left(1 + g_k(c)\right)^{\alpha}, \quad g_1(c) = 0, \quad k \geq 1. \tag{3.2}$$

The c in (3.2) has a degree of $\alpha^{k-1}-1$ and thus will encounter a difficulty in obtaining accurate solutions as α and k increase. But the transformation $w = 1+c^{|\alpha-1|}$ reduces the degree by $|\alpha-1|$.

Let $H_k(w) = 1 + g_k(c)$.

(1) When α is a positive integer, (3.2) can be written as

$$H_{k+1}(w) = 1 + (w-1)H_k(w)^{\alpha}, \quad k \geq 1, \tag{3.3}$$

where $H_1(w) = 1$ and $w = 1 + c^{\alpha-1}$.

Definition 3.1 (see [15]). Let k_i $(1 \leq i \leq \nu)$ is a integer smaller than k and satisfying $1 < k_1 < k_2 < \cdots < k_\nu < k$, let

$$F_k(w) = \begin{cases} H_k(w), & \text{if } k = \text{prime}, \ \alpha \text{ is a positive integer}, \\ \dfrac{H_k(w)}{\prod_{i=1}^{\nu} F_{k_i}(w)}, & \text{if } k \neq \text{prime}, \ \alpha \text{ is a positive integer}. \end{cases} \tag{3.4}$$

(2) When α is a negative integer, (3.2) can be written as

$$H_{k+1}(w) = 1 + \frac{1}{(w-1)H_k(w)^{\alpha}}, \quad k \geq 2, \tag{3.5}$$

where $H_2(w) = 1$ and $w = 1 + c^{1-\alpha}$.

So we can solve the roots of (3.4) or (3.5) instead of (3.1). If α is a positive integer, we can use (3.4) to calculate the center of periodic region. If α is a negative integer, we can use (3.5) to calculate the center of periodic region.

3.2. Numerical Algorithm

Through solving the roots of $F_k(\omega) = 0$ or $H_k(w) = 0$, we can solve the roots of $1 + g_k(c) = 0$. Let ω_j $(j = 1, 2, \ldots, r)$ with $\mathrm{Im}(\omega_j) \geq 0$ be a root of $F_k(\omega) = 0$ or $H_k(w) = 0$. The transformation $\omega_j = 1 + c^{|\alpha-1|}$ with the symmetry of M-sets yields

$$c_j = \rho_j^{1/|\alpha-1|} e^{i(\theta_j/|\alpha-1|)}, \tag{3.6}$$

where $\rho_j = \sqrt{(\mathrm{Re}(\omega_j)-1)^2 + \mathrm{Im}(\omega_j)^2}$. If $\mathrm{Re}(\omega_j)-1 \geq 0$, then $\theta_j = \tan^{-1}(\mathrm{Im}(\omega_j)/(\mathrm{Re}(\omega_j)-1))$; If $\mathrm{Re}(\omega_j)-1 < 0$, then $\theta_j = \pi + \tan^{-1}(\mathrm{Im}(\omega_j)/(\mathrm{Re}(\omega_j)-1))$.

Among these c_j values, we select the ones in the interior of the primary symmetric S_1 and the ones on the primary symmetric axis P_1. Then by the rotation symmetry, we can get the center's coordinate of each periodic region.

Using the Newton method, the following algorithm can locate the center of periodic region [15].

(1) Set $n, k, \varepsilon_1, \varepsilon_2$, and $Digits$ (number of precision digits).

(2) Construct $F_k(w)$.

(3) Construct *SolveCenter(k)* that does the following. (1) Using the *solve* function in *maple* to find the approximate roots of $F_k(w) = 0$; (2) Select the roots with nonnegative imaginary parts, and compute r and $\|F_k(w)\| = \max\{|F_k(w)|\}$.

(4) Construct *Newton* (f, w, N) that does the following. (1) Take the initial values w of about three decimal digits of accuracy from the results of *solve* to precede the Newton sequence $\{w_m\}$; (2) Set the maximum iteration number N; (3) Set ε satisfying $\|\operatorname{Re}(w_{m+1} - w_m)\| < \varepsilon$ and $\|\operatorname{Im}(w_{m+1} - w_m)\| < \varepsilon$.

(5) Construct *DefCenter(k)* finding coordinate of the periodic region's center: (1) Reset $Digits$ to a higher number *NtDigits*; (2) If $k = 2$, then $w^* = 0$; If $k \neq 2$, then call *Newton* (f, w, N) to get refined roots w^*; (3) Compute $c^* = (w^* - 1)^{1/|n-1|}$, m_1, m_2, and $\|F_k(w)\|$.

(6) Call *SolveCenter(k)*, Compute residual error $F_k(w_j)$ $(j = 1, 2, \ldots, r)$ and $\|F_k(w)\| = \max\{|F_k(w_j)|\}$.

(7) If $\|F_k(w)\| < \varepsilon_1$ is not met, increase *Digits* and call *SolveCenter* (k); If $\|F_k(w)\| < \varepsilon_1$ is met, then do the following. (1) Increase *NtDigits*; (2) Call *DefCenter(k)* to get refined roots w^*, and calculate $\|F_k(w^*)\|$; (3) If $\|F_k(w^*)\| < \varepsilon_2$ is not met, increase $NtDigits$ and call *DefCenter(k)*; If $\|F_k(w^*)\| < \varepsilon_2$ is met, the check the convergence of the sequence $\eta_m = \|e_{m+1}/e_m^2\|$, where $e_m = w_m - w^*$: If η_m is not convergent, increase *NtDigits* and call *DefCenter* (k); If η_m is convergent, then accept $c^* = (w^* - 1)^{1/|n-1|}$ as the desired solution and terminate the entire procedure.

The above algorithm is appropriate for α as a positive integer; if α is a negative integer, we can take $H_k(w)$ instead of $F_k(w)$. The algorithm is achieved by the *maple*. Tables 1 and 2 list typical coordinates of periodic region' centers for $\alpha = -1, -2, -5, -10, -25$ and $3 \leq k \leq 6$; the accuracy is of 48 precision digits, but only the first 40 precision digits were printed for the tabulation. Table 3 shows the residual error defined by $\|F_k(w)\|$ and the values of asymptotic error $\eta = \lim_{m\to\infty}\|e_{m+1}/e_m^2\|$. The parameters are defined as $\varepsilon = 0.5 \times 10^{-48}$, $\varepsilon_1 = 0.5 \times 10^{-3}$, and $\varepsilon_2 = 0.5 \times 10^{-90}$ in the experiment.

3.3. Numerical Results

Now, we study the relationship between the numbers of roots on P_1 and in the interior of S_1. According to the Rotation symmetry, we consider that c is and only is on P_1 and in the interior of S_1.

Let $W = \{w \in C : \operatorname{Im}(w) \geq 0\}$, we select roots having only nonnegative imaginary part and suppose r is the number of all such roots. Let d denotes the number of roots for $H_k(w) = 0$ in the complex plane $\mathbb{C}$, m_1 denotes the number of centers lying in the interior of S_1, m_2 denotes the number of centers lying on P_1, and $N_{k(\alpha)}$ denotes the total number of

Table 1: Typical coordinates (x, y) of periodic regions' centers in general M-sets for $\alpha = -1, -2$.

α	k	x	y
-1	3	0	1
	4	0	1.414213562373095048801688724209698078569
	5	0	0.6180339887498948482045868343656381177203
		0	1.618033988749894848204586834365638117720
	6	0	1.732050807568877293527446341505872366942
-2	3	0.5000000000000000000000000000000000000000	0.8660254037844386467637231707529361834714
		0.3627813151231631703818060700603897671678	0.6283556698299741263420784051580976696723
	4	0.6891203862446050962006675963540742790418	1.193591521507144896512098353766295629638
		0.4042236905404564572994748850027253568563	0.7001359696390695520682473983836636804246
		0.5865601716578127357562434844914253929171	0.5227469154574305782207221081920783006446
	5	0.6424754430209604509055886086291122942941	1.112800109927626758089719675886061908473
		0.7796925046167080079396216213156599757773	1.350467032276769692645172849662184857212

periodic regions' centers lying in the complex plane $\mathbb{C}$. Then, the following relation holds [15]:

$$m_1 + \frac{m_2}{2} = \frac{d}{2} = r - \frac{m_2}{2}. \tag{3.7}$$

Next we discuss the calculation of periodic regions' centers and the relation among m_1, m_2, d and $N_{k(\alpha)}$ when α is a positive integer and a negative integer, respectively.

(1) If α is a positive integer,

The general M-sets with positive integer index number are similar to flowers combined with $\alpha-1$ major petals, and stable region is embedded in unstable region. If $\alpha = 1$, the complex mapping f degenerates into linear mapping, the general M-sets is only a circular which is a trivial structure without complex boundary and self-similar structure. Once deviating from $\alpha = 1$, the complex structure appear immediately [3].

Theorem 3.2. *If α is a positive integer, then the relation $m_1 + m_2/2 = N_{k(\alpha)}/(2(\alpha - 1))$ (or $d = N_{k(\alpha)}/(\alpha - 1)$) yields the following [15].*

(1) *If $k = 2$, then $m_1 = 0$ and $m_2 = 1$ for any α;*

(2) *If $k \geq 3$, then $m_1 = N_{k(\alpha)}/(2(\alpha - 1))$ and $m_2 = 0$ for all odd α;*

(3) *If $k \geq 3$, then $m_1 \leq (1/2)\{N_{k(\alpha)}/(\alpha - 1) - 1\}$ and $m_2 \geq 1$ for all even α.*

(4) *If $k = 3$, then $m_1 = \alpha/2$ and $m_2 = 1$ for all even α;*

(5) *If $k = 4$, then $m_1 = \alpha(\alpha + 1)/2 - 1$ and $m_2 = 2$ for all even α;*

(6) *If $k = 5, 6, 7, 8, 9, 10$, then $m_2 = 3, 5, 9, 16, 28, 51$, respectively, for all even α.*

We can get the following from Theorem 3.2. Statement (1) implies that a two-periodic region's center lies on P_1; Statements (1) and (2) show that only a two-periodic region's center lies on P_1 when α is odd. Statements (2) and (3) show that every k-periodic region's center with $k \geq 2$ lies on P_1 when α is even. Statements (4) and (5) show that for any even integer $\alpha \geq 2$ one three-periodic and two four-periodic periodic region's centers lie on P_1, respectively.

Table 2: Typical coordinates (x, y) of periodic regions' centers in general M-sets for $\alpha = -5, -10, -25$.

α	k	x	y
	3	0.8660254037844386467637231707529361834714	0.5000000000000000000000000000000000000000
		0.9396926207859083840541092773247314699362	0.3420201433256687330440996146822595807630
	4	1.094712686823523213316928247829558702952	0.6320326644228596128930478581312699907336
		0.7967257215431642519980222116419896073370	0.3953814333548772060210763599797199632167
		0.7360864941961324414193625731274047816387	0.4249797355709849680198964443168932113036
		0.7856338587327582965401249656709551480181	0.4120235779535403654938100729256015955341
		0.8004446001570064882132416909959852686047	0.4092017895850238127786028651284440809001
		0.8094119533872972312608524004319383353154	0.4018233097208282401257748308404147230355
		0.8145813321090840057351012061818996720571	0.3879152634894074458735794003658374945526
		0.8149958514928554223788166213145277073539	0.3399448426590958688275107331963268401499
		0.9044125863898232379201681820588098667113	0.3455698635303133428003524080560280276196
−5	5	0.9993362715743393720427440998655224791940	0.2835926532018401463511538519480040901230
		0.9255117037432101887468727385681042741597	0.3620033396828505808335124008216734556202
		0.9451946379157873907177917158114553649862	0.3656567095496140760775285440549985909933
		0.9694641741107180623575027320447562316133	0.3560080961927332150121341389871777368834
		1.116580526363573971834197527235703888800	0.5742412863507625718458386348765152916070
		1.069955621295139064555534263833642005509	0.5962126122905732351174863126710137613208
		1.193991828397960001427726051713155781130	0.6893515035357756591531775646851958338755
	3	0.9594929736144973898903680570663276990624	0.2817325568414296977114179153466168990357
		0.8978574653776339543388693405463208894818	0.2636347387173656727162596943859324534622
		0.9104989079407236885753517227873123437832	0.2367915193754428000032888830923803306147
−10		0.9334500946949781808039823343794415287503	0.2123497779324388912409043034218249820178
	4	0.9727622997546039520643435173301435568623	0.1918489833048745070912754338127184654426
		1.049709676698295730342527736899250962728	0.1891462127512417587632393395560253753949
		1.130312524668417811908520163810539185497	0.3318897025426986348883996121722492432445
	3	0.9927088740980539928007516494925201793436	0.1205366802553230533490676874525435822736
		0.9664323195132830361308380363378676828569	0.1148937164822188020505801496289032369093
		0.9676238283276849427272723258657042546928	0.1101302140885335128510533436733831564012
		0.9694433016226217291495915766991654290410	0.1054329018630946912670388018574858083463
		0.9719398783923833692895002322210280317735	0.1008010956452396592463960335789059436667
		0.9751959506314899516234021845256167661412	0.09623846922724149376992938643944612574927
		0.9793414975878867446486836531597425442696	0.09175563494485486517255996206353137944357
−25	4	0.9845812519718114260081505085282976395498	0.08737586254212876216122055789839027979179
		0.9912488732365256071521286888973731367570	0.08314879742066106441534388013424937098513
		0.9999255170838100020444386490494074709296	0.07918810138395086586130693931721388479083
		1.011737534937634869467650731889283987347	0.07579806851536097318670223194619562054147
		1.029258691458743926993593711963441023480	0.07406101804917325155983540234868545886878
		1.059396476738641643674942308389869077720	0.08045677368903782206077846838233504471072

Table 3: Residual errors and asymptotic error constant for $\alpha = -1, -2 - 5, -10, -25$.

α	k	$\|H_k(w)\|$	η
-1	3	0	not available
	4	0	not available
	5	$0.100000e-99$	2.4472136 at $c = 0.61803398874989484820i$
	6	0	not available
-2	3	0	not available
	4	$0.892511e-100$	2.7888544 at $c = 0.36278131512316317038 + 0.62835566982997412263i$
	5	$0.150000e-99$	3.4739187 at $c = 0.64247544302096045091 + 1.11280010992762 6758i$
-5	3	0	not available
	4	$0.101663e-97$	7.9352875 at $c = 0.79672572154316425199 + 0.39538143335487720602i$
	5	$0.8609622e-98$	78.011649 at $c = 0.81458133210908400573 + 0.38791526348940744587i$
-10	3	0	not available
	4	$0.188919e-98$	16.988191 at $c = 0.91049890794072368857 + 0.23679151937544280000i$
-25	3	0	not available
	4	$0.375119e-98$	39.779704 at $c = 0.97193987839238336928 + 0.10080109564523965924i$

(2) If α is a negative integer

The general M-sets with negative integer index number have planetary configuration consisting of a central planet with $|\alpha - 1|$ major satellite structures, and the unstable region is embedded in the stable region, as is shown in Figure 1. The numbers and locations of the periodic region's centers in the general M-sets with negative integer index number can be calculated using the same method as α being a positive integer.

Theorem 3.3. *If α is a negative integer, then the relation $m_1 + m_2/2 = N_{k(\alpha)}/(2|\alpha - 1|)$ (or $d = N_{k(\alpha)}/|\alpha - 1|$) yields the following:*

(1) *If $\alpha = -2$ and $k \geq 3$, then $N_{k(\alpha)} = N_{k(-\alpha)}$;*

(2) *If $\alpha \leq -3$ and $k \geq 3$, then $N_{k(-\alpha)} = |\alpha + 1| N_{k(\alpha)}$, namely $N_{k(\alpha)}$ with negative integer α is equal to d with $-\alpha$;*

(3) *If $k = 2$ or 3, then $m_1 = 0, m_2 = 1, d = 1, N_{k(\alpha)} = |\alpha - 1|$ for all α;*

(4) *If $k = 4$, then $m_2 = 2$ and $m_1 = |\alpha|/2 - 1$ for $|\alpha|$ being even; $m_2 = 1$, $m_1 = (|\alpha| - 1)/2$ for $|\alpha|$ being odd;*

(5) *If $k = 4$, then $d = |\alpha|$ for all α.*

We can get the following from Theorem 3.2. Statement (3) implies that one two-periodic and one three-periodic region's center lies on P_1; Statement (4) state that two four-periodic region's center lies on P_1 when $|\alpha|$ is even, as is shown in Figure 1(d); When $|\alpha|$ is odd, only one four-periodic region's center lies on P_1, as is shown in Figure 1(f). Table 4 shows the relationship among m_1, m_2, d and $N_{k(\alpha)}$ when α is a negative integer, which testifies Theorem 3.3 and (3.7). The numbers in Table 4 are m_1/m_2. The corresponding d and $N_{k(\alpha)}$ satisfy Theorem 3.2.

Table 4: (m_1/m_2) versus (α, k).

α	k								
	2	3	4	5	6	7	8	9	10
−1	2/1	2/1	2/1	4/2	4/2	6/3	6/3	8/4	8/4
	0/1	0/1	0/1	0/2	0/2	0/3	0/3	0/4	0/4
−2	3/1	3/1	6/2	15/5	27/9	63/21	120/40	252/84	
	0/1	0/1	0/2	1/3	2/5	6/9	12/16	28/28	
−3	4/1	4/1	12/3	40/10	116/28				
	0/1	0/1	1/1	4/2	14/1				
−4	5/1	5/1	20/4	85/17					
	0/1	0/1	1/2	7/3					
−5	6/1	6/1	30/5	156/26					
	0/1	0/1	2/1	12/2					
−6	7/1	7/1	42/6	259/37					
	0/1	0/1	2/2	17/3					
−7	8/1	8/1	56/7	400/50					
	0/1	0/1	3/1	24/2					
−8	9/1	9/1	72/8	585/65					
	0/1	0/1	3/2	31/3					
−9	10/1	0/1	90/9	820/82					
	0/1	0/1	4/1	40/2					
−10	11/1	11/1	110/10						
	0/1	0/1	4/2						
−11	12/1	12/1	121/12						
	0/1	0/1	5/1						
−12	13/1	13/1	156/12						
	0/1	0/1	5/2						
−13	14/1	14/1	182/13						
	0/1	0/1	6/1						
−14	15/1	15/1	210/14						
	0/1	0/1	6/2						
−15	16/1	16/1	240/15						
	0/1	0/1	7/1						
−16	17/1	17/1	272/16						
	0/1	0/1	7/2						
−17	18/1	18/1	306/17						
	0/1	0/1	8/1						
−18	19/1	19/1	342/18						
	0/1	0/1	8/2						
−19	20/1	20/1	380/19						
	0/1	0/1	9/1						
−20	21/1	21/1	420/20						
	0/1	0/1	9/2						
−21	22/1	22/1	462/21						
	0/1	0/1	10/1						
−22	23/1	23/1	506/22						
	0/1	0/1	10/2						
−23	24/1	24/1	552/23						
	0/1	0/1	11/1						

Table 4: Continued.

α	k 2	3	4	5	6	7	8	9	10
−24	25/1	25/1	600/24						
	0/1	0/1	11/2						
−25	26/1	26/1	650/25						
	0/1	0/1	12/1						

Table 5: Expression of $h_k(c)$ when $\alpha = -1$.

k	$h_k(c)$
2	c
3	c^2+1
4	c^3+2c
5	c^4+3c^2+1

4. Calculation of Periodic Regions' Centers in General M-sets with Negative Integer Index Number

Let $f_c^k(0) = h_k(c)/h_{k-1}(c)^{|\alpha|}$, then

$$h_k(c) = h_{k-2}(c)^{|\alpha|^2} + ch_{k-1}(c)^{|\alpha|} \quad (k \geq 2), \tag{4.1}$$

where $h_0(c) = 0$ and $h_1(c) = 1$. The coordinates of periodic regions' centers can be obtained by solving the equation $h_k(c) = 0$ with c satisfying $f_c^k(0) = 0$.

Tables 5 and 6 list the expressions of centers' coordinates from two-periodic to five-periodic region.

For the M-set constructed from the complex mapping $f : z \leftarrow z^{-1} + c$ (shown in Figure 1(a)), one-periodic region's center is ∞ for $f^1(0) = \infty$. Two-periodic region's center reaches the one-periodic region's center by one iteration, so the origin is the center of two-periodic region, that is, $h_0(c) = 0$ in Table 5 deduces $c = 0$. Similarly, three-periodic region's centers can be located by the roots of $c^2 + 1 = 0$.

For the M-set constructed from the complex mapping $f : z \leftarrow z^{-2} + c$ (shown in Figure 1(c)), one-periodic region is a huge area whose boundary is defined by $c = 2^{1/3}e^{-i\theta} - 2^{-2/3}e^{2i\theta}$. Two-periodic stable region divides the area outside of one-periodic region into three parts. One-periodic region's center is ∞. Two-periodic region's center is the origin. Similarly, three-periodic region's centers can be located by the roots of $c^3 + 1 = 0$.

The discussion above indicates that k-periodic region's centers can be located by the roots of $h_k(c) = 0$. The results of centers' coordinates are almost the same as the results obtained by the first method described in Section 2 when α is a negative integer.

5. Conclusions

(1) In this paper, we proposed two methods for calculating the periodic regions' centers of the general M-sets. The first method fits for calculating the periodic regions' centers in the general M-sets with integer index number, which is to transform the polynomial

Table 6: Expression of $h_k(c)$ when $\alpha = -2$.

k	$h_k(c)$
2	c
3	$c^3 + 1$
4	$c^7 + 3c^4 + c$
5	$c^{15} + 7c^{12} + 5c^9 + 12c^6 + 5c^3 + 1$

equation that governs the periodic regions' centers, obtain high precision of the coordinates by the simple method, and analyze the relationship between the number of each periodic regions' centers on the principal symmetric axis and in the principal symmetric interior, then comparatively analyze the relation of periodic regions' number in general M-sets with opposite integer index number. The second methods as discussed in Section 3 suits for calculating the periodic regions' centers in the general M-sets with negative integer index number, which also transforms the polynomial equation. The results of centers' coordinates obtained by the second method are almost the same as that of the first method when α is a negative integer which is described in Section 2.

(2) The investigation of the periodic regions' centers of the M-sets can help us explore the distribution of periodic regions of the M-sets, which can further help us to study the fractal structures of the M-sets. The centers of the periodic regions are located as the roots of certain polynomials, which are shown to coincide with solutions of the Douady and Hubbard formula [17]. In addition, the methods we proposed are helpful for solving polynomial equations, especially of high degree.

(3) This research is some inspiration for the people studying on the difficult problems in their professional and interdisciplinary fields.

As a classical example of physics, Brownian movement is the most simple and typical random movement. The Langevin equation can depict the rule of a charged particle under the circularly successive influence of the impulse functions. However, it is difficult to visually depict the trajectory and dynamics of these systems with many random variables. If we construct the complex general M-sets using the rules of Langevin equation, the fractal structure characteristics of the general M-sets can reveal the changing rule of the particle velocity visually [18]. This study makes it possible to depict complex Brownian movement more accurately.

In addition, the theories of M-sets have potential applications on image processing. We have known that general M-sets are illustrated dictionary of the corresponding general Julia sets [16], which means a single point on the M-sets can represent the huge amount image data of the general Julia sets with manifold shapes and complicated structures. This research provides the technology support for determining rapidly the coordinates of the points on the M-sets by the Julia images. On the basis of the above research results, future work includes establishing dictionary of fractal compression and studying the corresponding coding algorithm to improve the transmission and memory of the information, which could provide the new thoery for fractal compression technology.

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